1. Methodology

In this study, debris-covered ice ablation data for the Satopanth glacier, estimated by measuring surface height change using ablation stakes, was used for validation (Shah et al., 2019). For Dunagiri glacier, data from the Geological Survey of India records were utilized (Srivastava and Swaroop 1987). This data is used as a reference to develop and calibrate various extensions of Temperature-index models (degree-day models) given as follows:

Model-1:

$$M = TF \times PDD$$

TF $(mm\ °C^{-1}\ day^{-1})$ is the temperature factor calculated as

$$TF = TF_1 \times d^{TF_2}$$ *d is in meters here*

Model-2:

$$M = (DDF \times PDD)/(1 + d/d_0)$$

Model-3:

$$M = (DDF \times PDD)/(1 + (d/d_0)^2)^a$$

Where M is melt (mm) for *n*th time intervals, PDD is Positive degree day for each time interval, d (cm) is the thickness of debris cover. $DDF(mm\ °C^{-1}\ day^{-1}$, degree-day factor), TF_1, TF_2, d_0 (cm) and a are model parameters. $PDD(\ °C\ \ day\ )$ is given by

$$PDD = \sum_{i=1}^{n} T^{+} \Delta t$$

Where T^{+}(℃) is the sum of positive air temperature for *n* time intervals Δt(day) (Hock, 2003). Model-1 is taken from Steiner et al., 2021. We neglected the $lag_T * d$ from Steiner's, where lag_T is the time per unit distance it takes for the energy flux to travel through the debris pack.

1.1 Temperature index models using observed temperature for Satopanth glacier

Temperature data from two Automatic Weather stations (AWS-1 and AWS-2) is utilized. AWS-1 , on the glacier with an elevation of 4376m, is typically located in the middle of the glacier. AWS-2, elevation 3870m, is close to the glacier's snout. The 12-hours, 1-day, 2-days, and 5-day average lapse rate was calculated using these two stations for Julian days 155 to 296 of the year 2017 (Figure 1). The result does not change using different lapse rates. We used 1-day average lapse rates for further calculations.

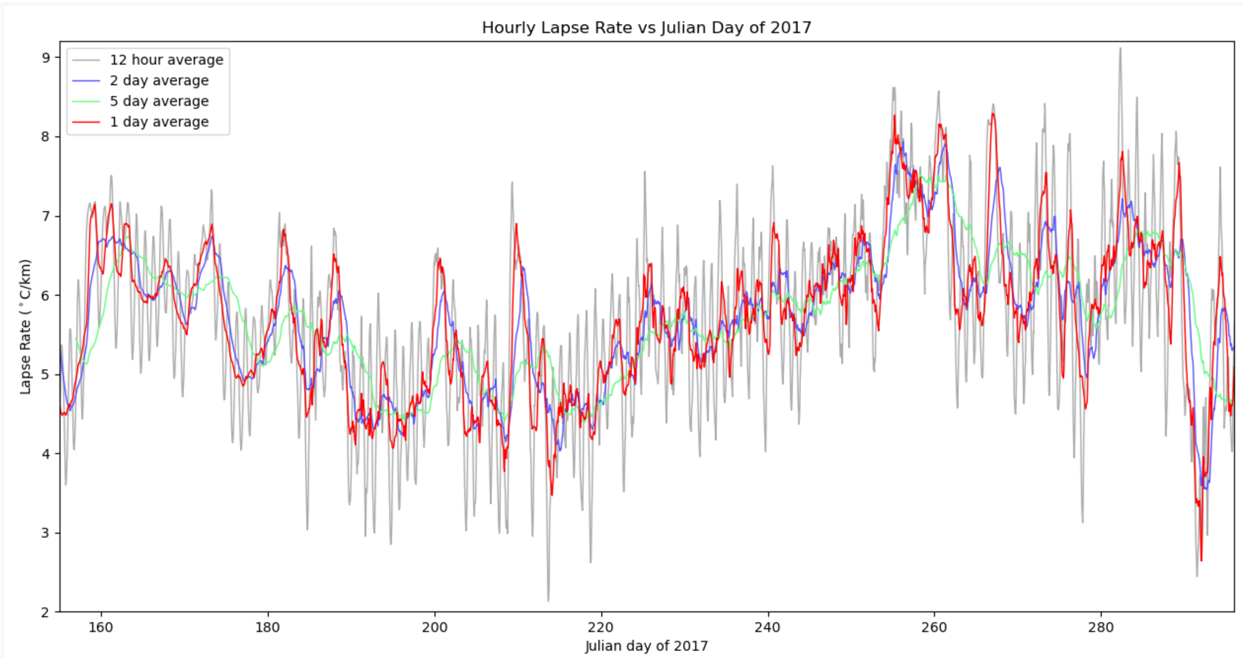
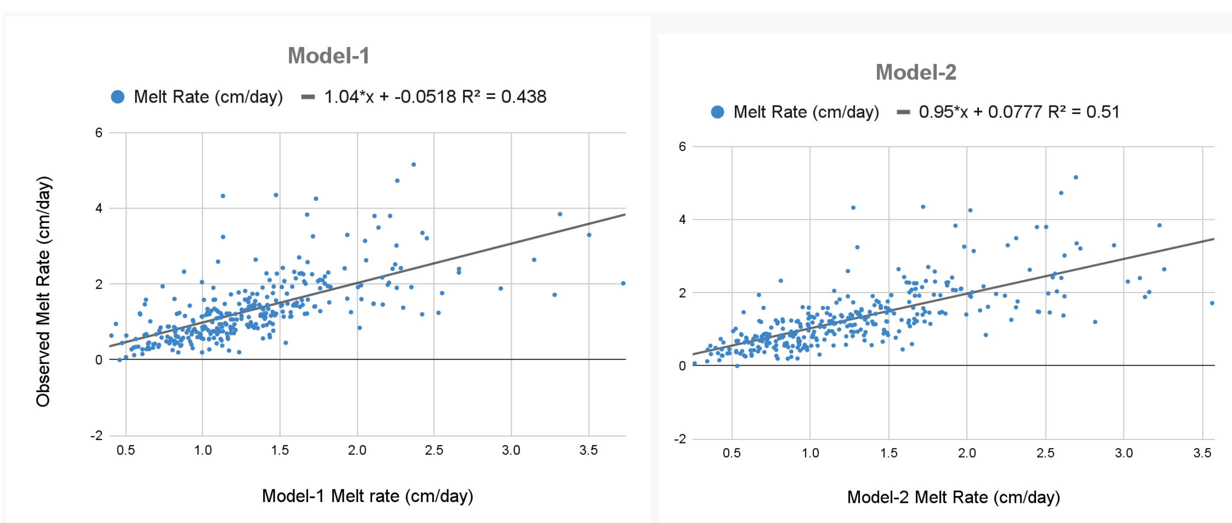


Figure: 1

This lapse rate was then used to find the PDD, and above, three degree-day models were fitted to the point scale stake ablation data of 2017 from Shah et al. (2019). Figure 2 shows that all the models are equally good. RMSE for models 1, 2, and 3 are 0.62, 0.60, and 0.59 (in cm/day), respectively.



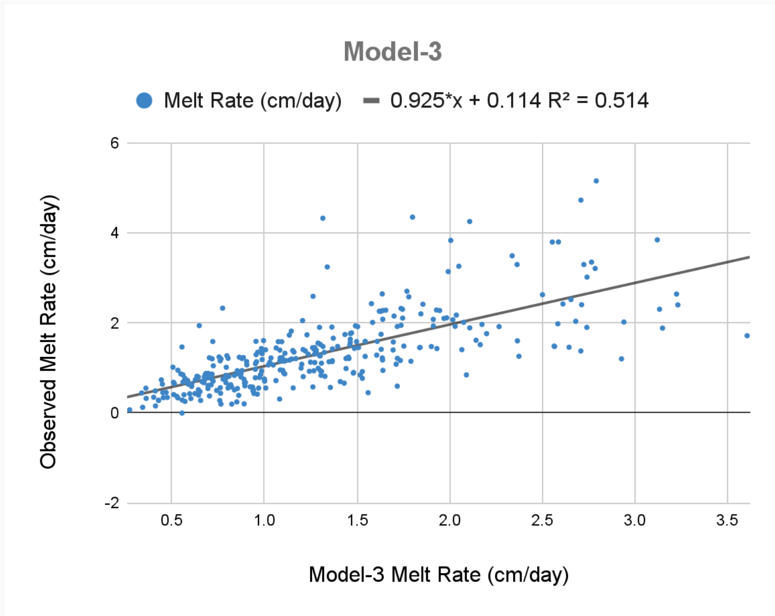


Figure: 2

Table- 1

Monte Carlo (1000)			
Model-1			
	TF1	**TF2**	
Fitted Value	0.82	-0.5	
Uncertainty	0.015	0.005	
Model-2			
	DDF	**d0**	
Value	5.04	12.76	
Uncertainty	0.05	0.16	
Model-3			
	DDF	**d0**	***a***
Fitted Value	5.27	4.9	0.36
Uncertainty	0.047	0.114	0.005

To obtain the total volumetric melt, we divided the glacier into *j* debris thickness bands (d_1, d_2, d_3…., d). To calculate the total melt for the *nth* time period, we computed melt for each debris thickness band separately for the *nth* period. We then summed it up, considering the area frequency distribution of each band.

Total melt for the *nth* time period is given by

$$Melt_{n,}(m^3) = \sum_j M_{n,j} \times Area_j(in\, m^2) \times f_{d_j}$$

Where f_{d_j} is the fraction of area covered by d_j band, $Area_j$ is the total debris-covered area (12.096 sq km) and $M_{n,j}(m)$ is the melt for nth time interval and jth debris thickness band, obtained from above three degree-day models using area-weighted PDD. RMSE for periodic total melt is 0.45, 0.38, and 0.38 (in m^3 10^6), respectively, for model 1, 2, and 3.

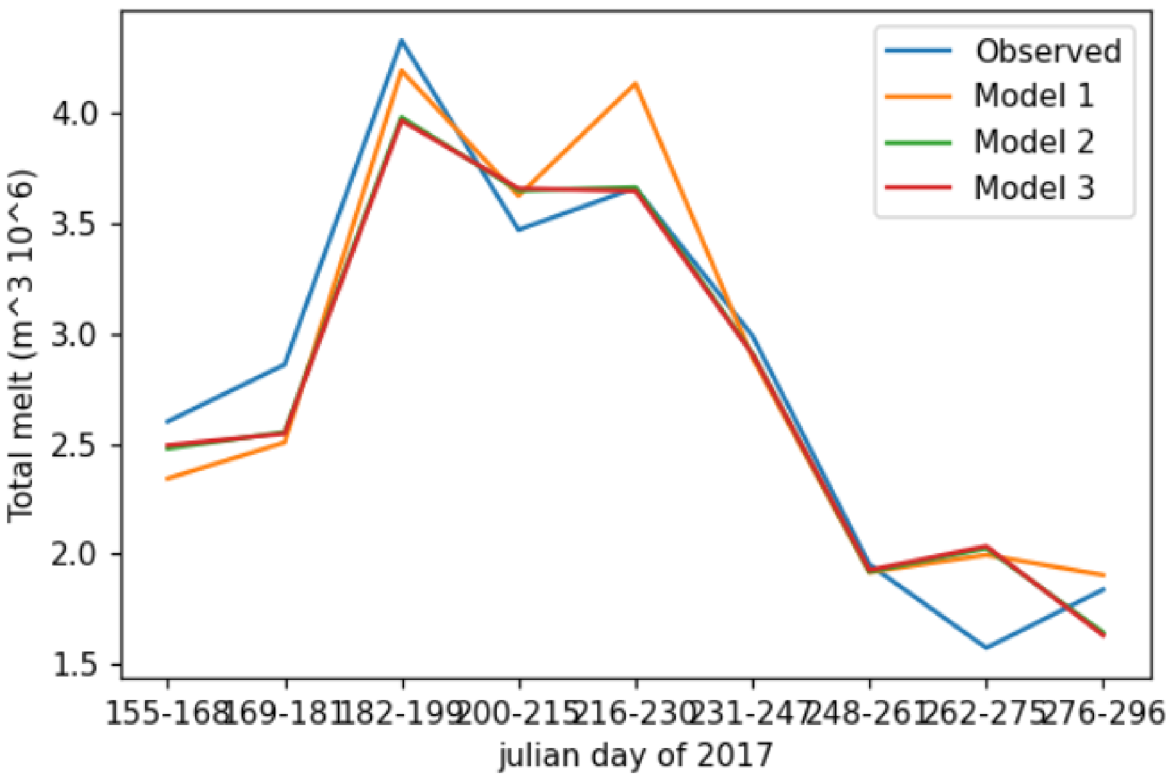


Figure: 3

1.2 TI models using reanalysis data for Satopanth Glacier

We used 9 tiles of ERA5-Land hourly temperature data and ERA5-Land geopotential height data and calculated the lapse rate using a linear fit model. Using this lapse rate and AWS-1 temperature data, a 15-day average bias in the reanalysis temperature data was calculated. We have also checked 1, 2,5,10 days average bias correction, and the result doesn't change.

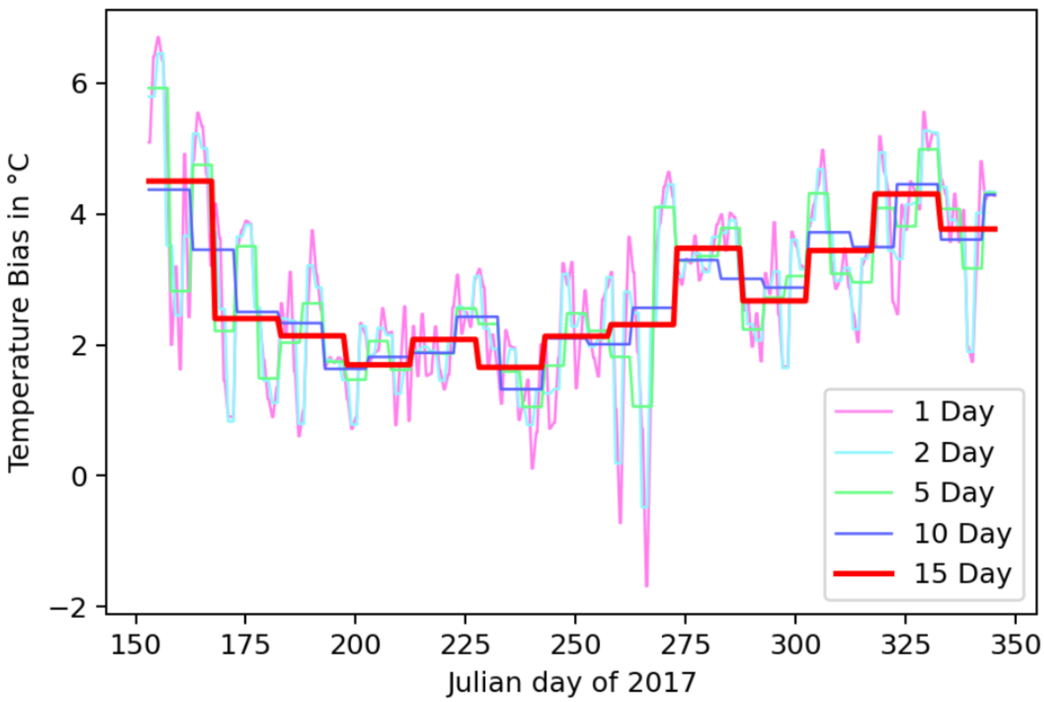
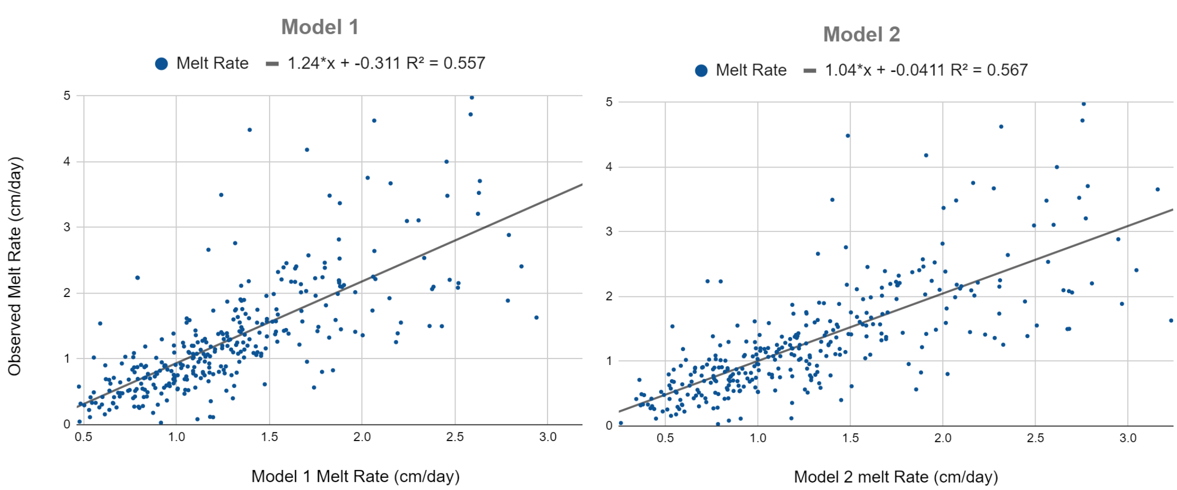


Figure: 4

Using a 1-day average lapse rate and after correcting the bias, we calculated PDD and fitted above three degree-day models using the same approach as in section 1.1. All 3 models are equally good.



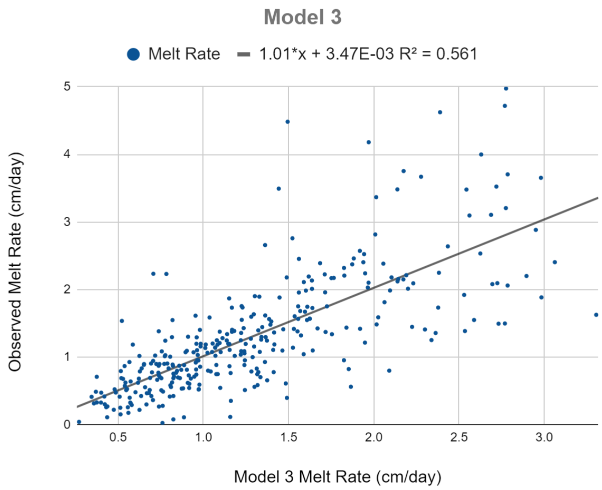


Figure: 5

Table-2

Monte Carlo (1000)			
Model-1			
	TF1	**TF2**	
Fitted Value	0.96	-0.41	
Uncertainty	0.015	0.004	
Model-2			
	DDF	**d0**	
Value	5.66	9.78	
Uncertainty	0.05	0.16	
Model-3			
	DDF	**d0**	***a***
Fitted Value	5.24	3.82	0.32
Uncertainty	0.044	0.113	0.004

Total Volumetric ablation was calculated using the method discussed in section 1.1. RMSE for periodic total melt is 0.48, 0.43, and 0.44 (in m^3 10^6), respectively, for model 1, 2, and 3.

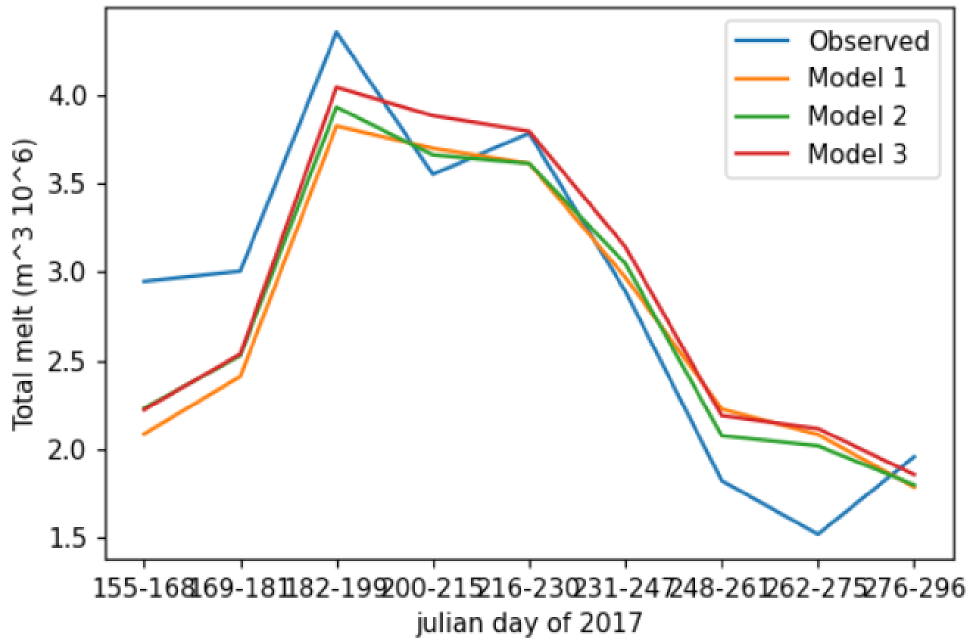


Figure: 6

1.3 Validating TI models for Dunagiri Glacier

We used 9 tiles of ERA5-Land hourly temperature data and ERA5-Land geopotential height data and fitted these 9 tiles in a linear model to calculate lapse rate. As a result, does not change using a 1-day, 2-day, 5-day, and 15-day average lapse rate, we used a 1-day average lapse rate for further calculations. The 1-day average lapse rate for the year 1986-92 (except 1991) is shown in figure 7.

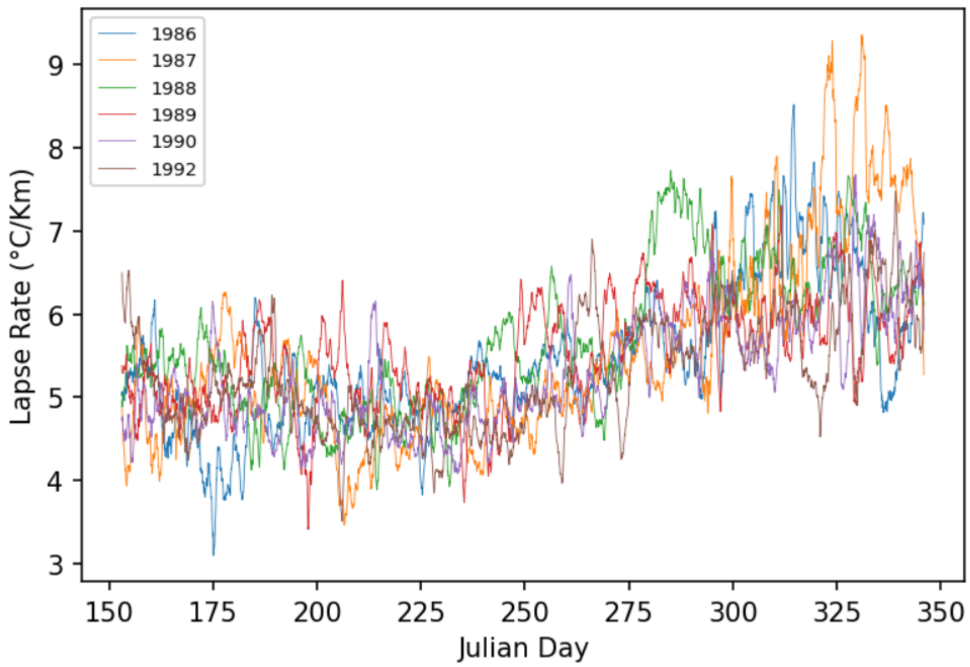


Figure: 7

Using this lapse rate, we calculated the temperature at the snout of the glacier (4200 m), where meteorological data (15-16 days mean Temperature data) was collected by Srivastava et al. We calculated the mean temperature bias for *n* time periods and calculated total melt for the same time period following the approach discussed in section 1.1 using Model parameters from Tabel 2. We used an area elevation histogram of the Dunagiri glacier to find the area-weighted *PDD*. The area elevation histogram was prepared by using SRTM 30-meter DEM data. We got the debris frequency distribution data for the Dunagiri glacier from Rounce et al. (2021). The total area of the Dunagiri glacier is 2.56 sq Km (GIS report by Deepak Srivastava and Siddarth Swaroop).

Table 3 shows the RMSEs for each year using three models.

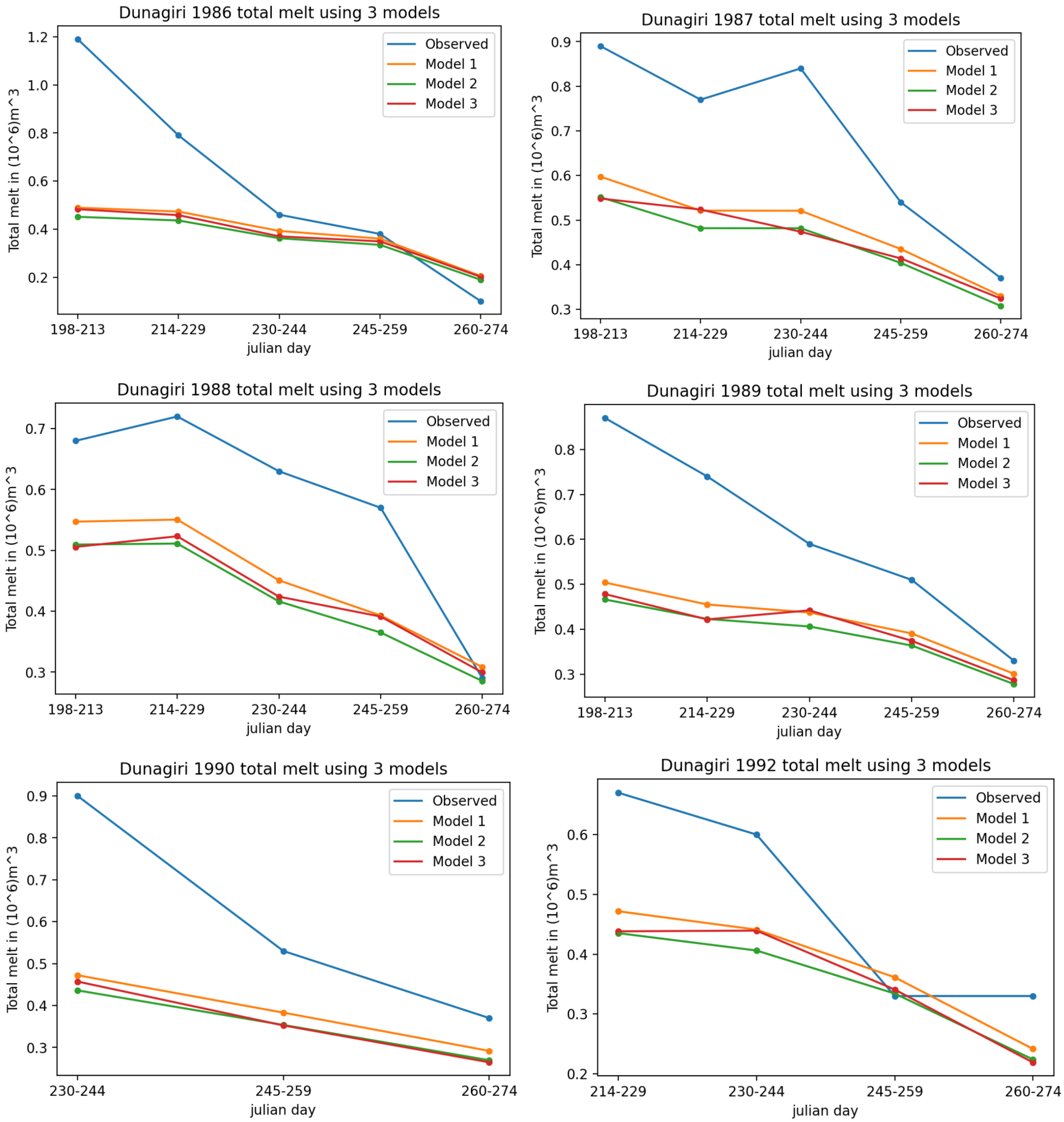


Figure: 8

Table 3

	1986	1987	1988	1989	1990	1992
Model 1	0.34	0.22	0.14	0.22	0.26	0.13
Model 2	0.37	0.26	0.18	0.26	0.29	0.16
Model 3	0.35	0.25	0.17	0.24	0.29	0.15

1.4 Validating TI models for North Changri Nup Glacier

We used 13 Stake ablation data of North Changri Nup Glacier from Patrick Wagnon. We used air temperature data from an AWS situated at 5470m height.

We used 9 tiles of ERA5-Land hourly temperature data and ERA5-Land geopotential height data and fitted these 9 tiles in a linear model to calculate the lapse rate. As a result, it does not change using a 1-day, 2-day, 5-day, or 15-day average lapse rate. We used a 1-day average lapse rate for further calculations. The 1-day average lapse rate from November 2014 (Julian Day 1) to Nov 2015 (Julian day 365) is shown in figure 9

Using this lapse rate and AWS temperature data 15-day average bias in the reanalysis temperature data was calculated. We have also checked the 1, 2,5,10 days average bias, and the result doesn't change.

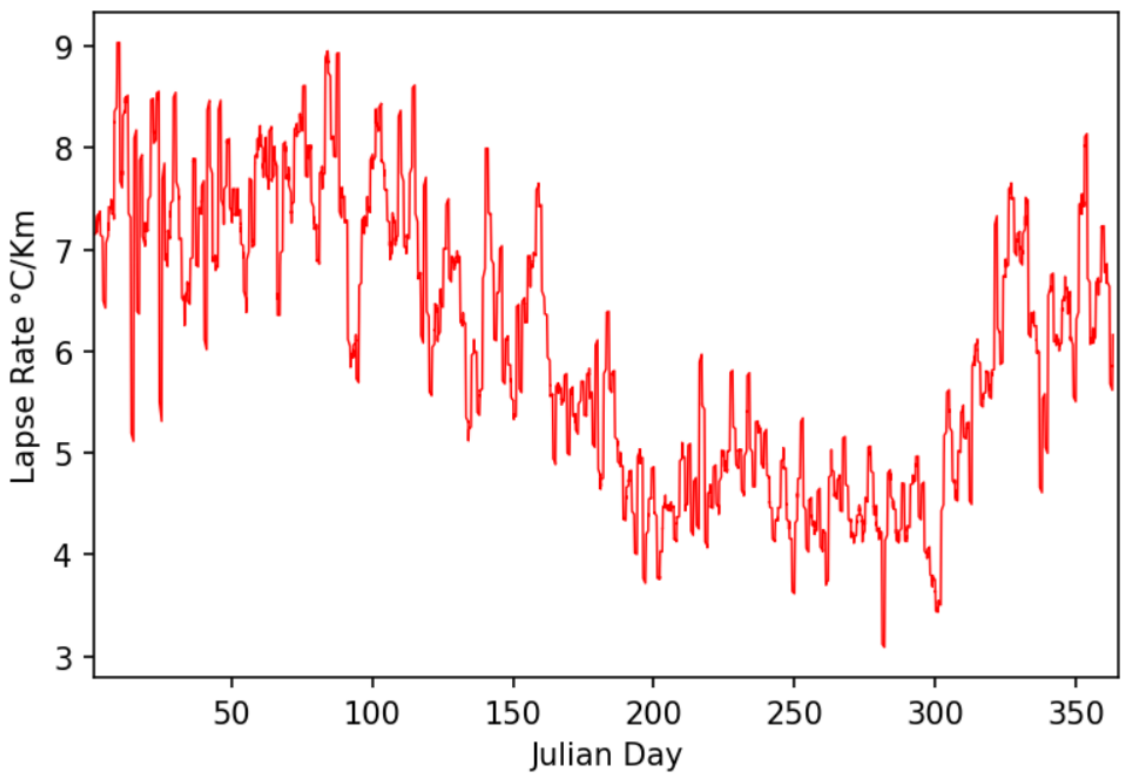


Figure: 9

We used an area elevation histogram of the Changri Nup glacier to find the area-weighted *PDD*. The area elevation histogram was prepared by using SRTM 30-meter DEM data. We got the debris frequency distribution data for the Changri Nup glacier from Rounce et al. (2021). The total area of the North Changri Nup glacier is 1.78 sq Km (Vincent et al., 2016).

We calculated the modeled melt for each stake (for the given period) using Model parameres from Table 2. RMSE for models 1, 2, and 3 are 0.91, 0.89, and 0.89 (in cm/day), respectively.

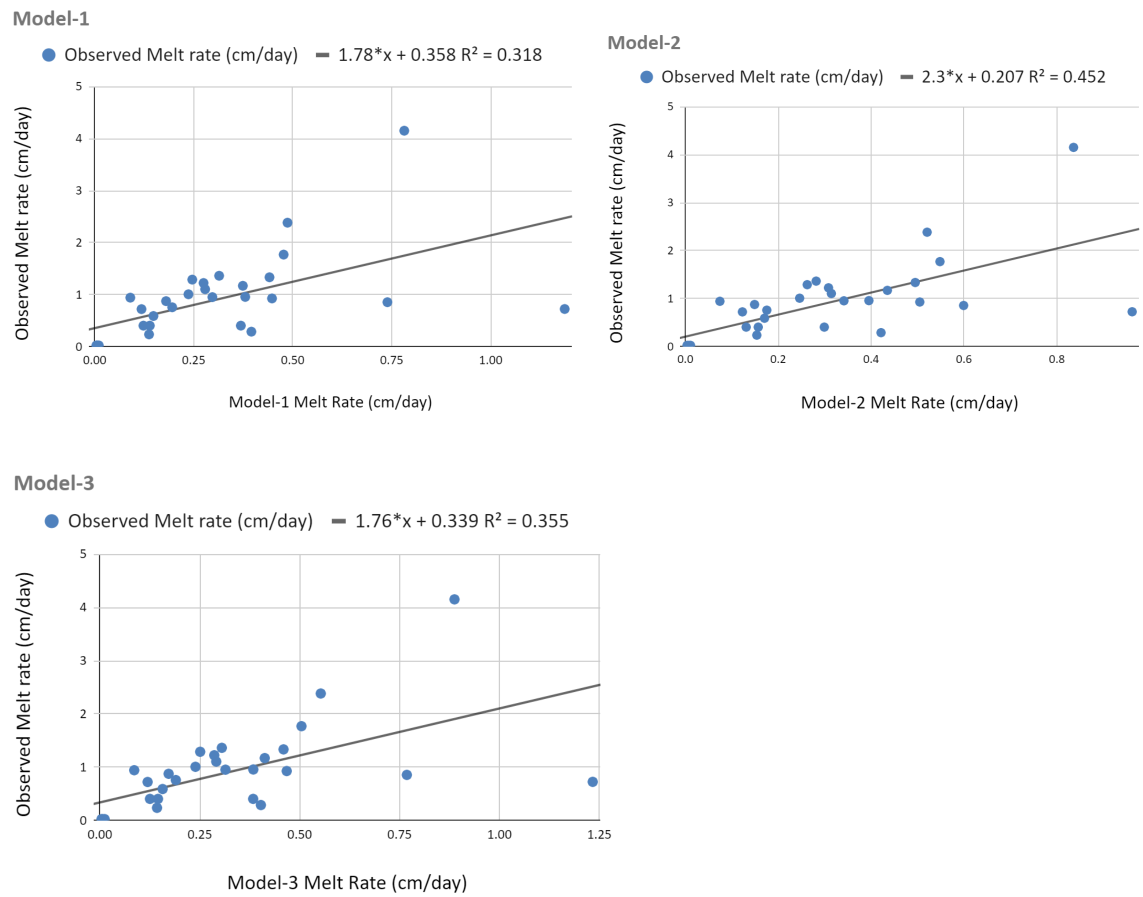


Figure: 10

Total Volumetric ablation was calculated using the method discussed in section 1.1. RMSE for periodic total melt is 0.208, 0.213, and 0.206 (in m^3 10^6), respectively, for models 1, 2, and 3.

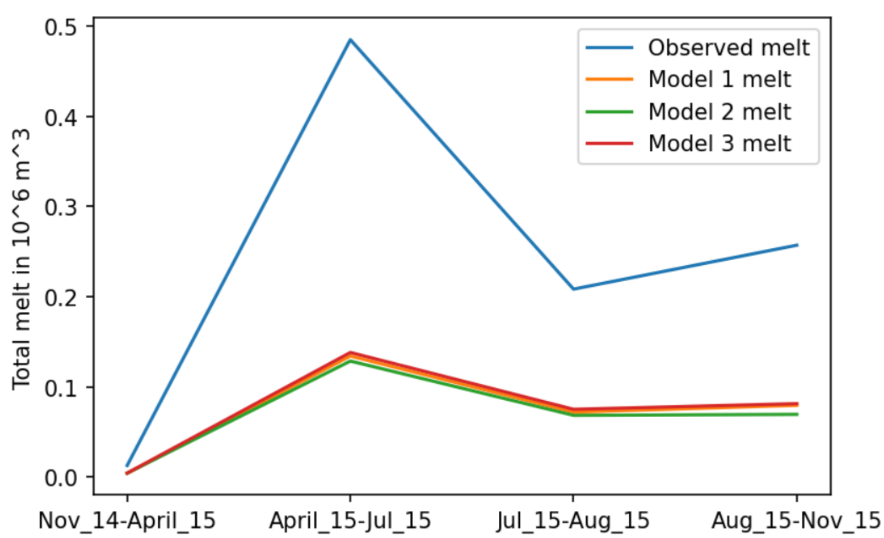


Figure: 11

1.5 Uncertainty

There are many uncertainties using these models: uncertainty in observed stake ablation data (4 cm, from Shah et al., 2019), observed debris thickness data (4 cm, from Shah et al., 2019), and associated with meteorological data (0.2 °C, Mention source). We ran 1000 Monte Carlo simulations adding Gaussian noise to each of these variables and fitted it. Using fitted parameters, we calculated the weighted standard deviation (sd_w) using the following formula for each parameter.

$$sd_w = \sqrt{\frac{\sum_{i=1}^{N} w_i (x_i - \bar{x}_w)^2}{\frac{(N' - 1)\sum_{i=1}^{N} w_i}{N'}}}$$

Where x_i is the model parameter, w_i is the weight of *i*th step ($w_i = e^{-MSE}$, MSE is mean square deviation), $\bar{x}_w$ is the weighted mean of x_i(mean of $x_i * w_i$), and N' is the number of non-zero weights.