

DATA557 Homework 2

[YOUR NAME HERE]

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Problems

Question 1. (This is taken from Q3 of Exercise 2.)

A researcher is interested in measurements of a pollutant in water samples. In particular, there is a question about whether the value changes if the sample is tested when it is older compared with being tested right after it is collected. The researcher does not know whether aging could increase or decrease the pollutant concentration. To test the hypothesis 15 samples of water were taken from a lake. Each sample was divided into 2 aliquots, one to be analysed right away and the other to be analysed 1 month later. The difference between pollutant concentrations was recorded for each of the samples. The values obtained for the differences (fresh sample - aged sample), arranged from smallest to largest, were as follows: -5, -2, -1, -1, 0, 0, 2, 3, 4, 4, 5, 5, 6, 6, 11.

1.1 Calculate a 95% confidence interval for the mean difference in concentration between aged and fresh samples. Also perform a test of the null hypothesis that the mean difference is equal to 0. Do not assume a known variance. What is the name of the test? Compare results with the results from Exercise 2, Question 2. Explain how and why they differ.

1.2. Conduct a simulation study to assess the validity of the test used in 1.1 for this experiment. Choose a non-normal distribution for the population that is a plausible distribution for the differences in pollutant measurements (it is not necessary to require that the variance is equal to 15). Estimate the type I error of the test. Is the test valid? Compare to the results for Exercise 2, Question 2.4. Explain the differences that you observe.

1.3. Propose a second plausible distribution for the population that you think might give different results for the type I error. Estimate the type I error probability of the test under this distribution and compare with results of the previous simulation. Explain any differences.

1.4. Suppose that it was determined that the last data value (11) was an error due to failure of the measuring equipment. Re-run the test and confidence interval for the mean with this value excluded. How did the results change?

Question 2

A researcher wants to evaluate the sensitivity of their assay for measuring urinary mercury. The standard is to have a 99% probability of detecting a sample with mercury concentration 1 ppm. Therefore, they wish to test the null hypothesis $H_0: p=0.99$, where p is the probability of a positive test when the concentration is

1 ppm. The alternative hypothesis is that p is not equal to 0.99. The researcher created 500 samples with a 1ppm mercury concentration and tested them. The number of samples that tested positive was recorded.

2.1. Suppose that they decide to reject H_0 if the number of positive samples is 494 or less. What is the type I error probability of this rejection rule?

2.2. Find a different rejection rule for the test that gives a type I error probability that is less than or equal to 0.05 and as close to 0.05 as possible.

2.3. Suppose that the number of positive samples was 490, i.e., 98% of 500. Using your rejection rule from Q2, would you reject the null hypothesis?

2.4. Now suppose that the lab had performed only 100 samples and had found the same proportion of positive tests, i.e., 98 out of 100 positive tests? What is the rejection rule for the test in this situation and what would be the outcome of the test? Explain any differences in the outcomes between this test and the test based on 500 samples.

2.5. Calculate p-values for the tests based on 500 samples and 100 samples. Explain the difference between the p-values.

Question 3 (Data set: 'iq.csv'. See HW1 for background).

3.1. Test the null hypothesis that the mean IQ score in the community is equal to 100 with a 2-sided significance level of 0.05. Use the 2-sided 1-sample t-test. State whether or not you reject the null hypothesis at significance level 0.05 and also give the p-value for your test.

3.2. Compute a 95% confidence interval for the mean IQ. Do the confidence interval and hypothesis test give results that agree or conflict with each other? Explain. (2 points)

3.3. Repeat Q3.1 and Q3.2 using a significance level of 0.01 and a 99% confidence interval.

3.4. Perform a simulation study to assess the type I error probability of the test. For the simulation, generate samples of IQ scores using the normal distribution with mean 100 and SD 15. The sample size should be the same as for the data set. Report the observed type I error based on your simulation and comment on how well it agrees with theory.

3.5. Perform a simulation study to estimate the power of the test to detect an alternative mean value for the mean IQ equal to 95. Generate samples (of same size as the data set) from a normal distribution with SD equal to 15.

3.6. Find the largest value of the alternative hypothesis mean that would be rejected with power of approximately 0.9. (Consider only values less than 100 for the alternative hypothesis mean.)