DATA 557 - Homework 5 Question 2

Will Wright

2/13/2019

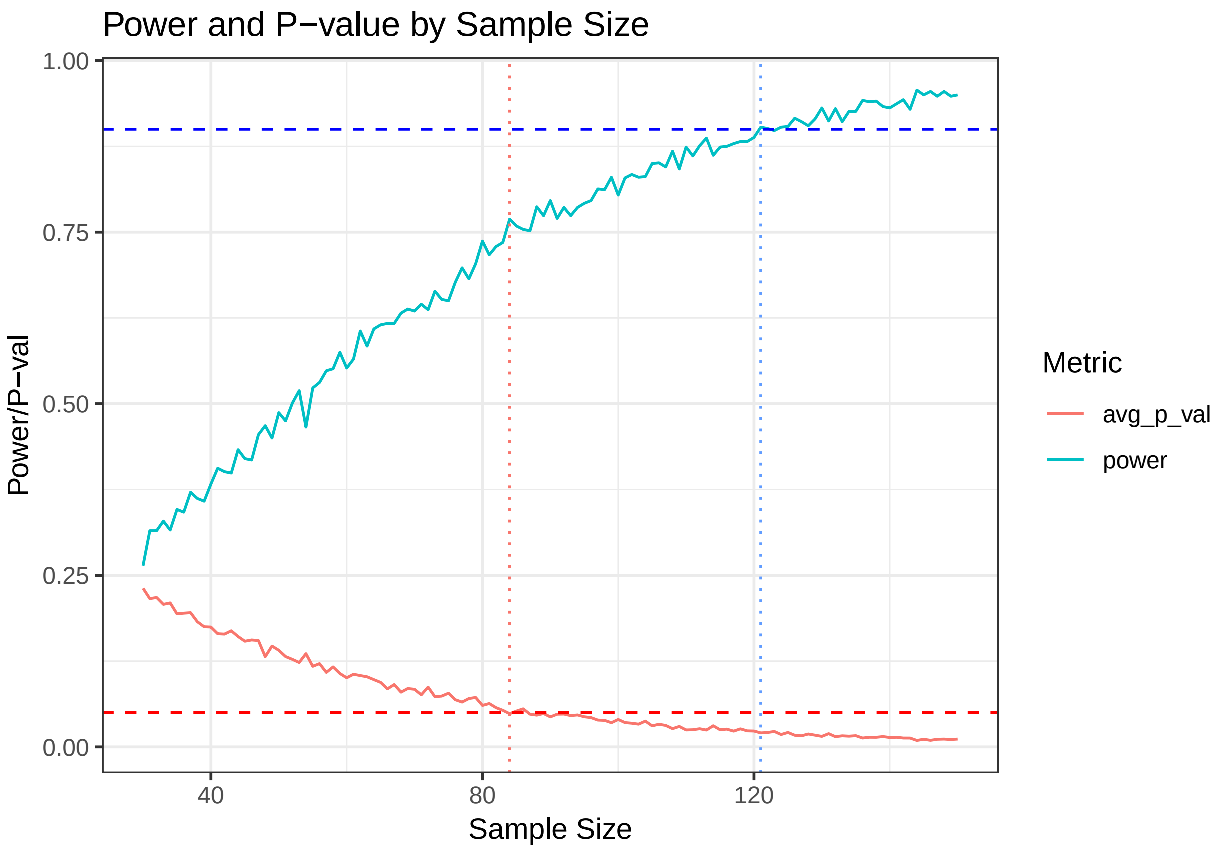
Question 2

A researcher is designing an experiment to compare the mean life length of three different types of cell-phone batteries (A, B, and C). The null hypothesis to be tested is that the mean life length is the same for all three battery types. Based on previous experiments of the life length of similar batteries it is expected that life length has an exponential distribution with mean approximately 3 days. There are two alternative hypotheses of particular interest. For the first alternative hypothesis, the mean life-length is 3 days for A and B, but only 2 days for C. For the second alternative hypothesis of interest, the mean life-lengths are 2, 3, and 4 days for A, B, and C, respectively. Your assignment is to design the experiment, i.e., determine appropriate sample sizes for each battery type, in such a way that the following two requirements are met: 1. The ANOVA F-test is a valid test of the null hypothesis with type I error rate of approximately 0.05 2. Power of 90% to detect the first alternative hypothesis of interest 3. Power of 90% to detect the second alternative hypothesis of interest

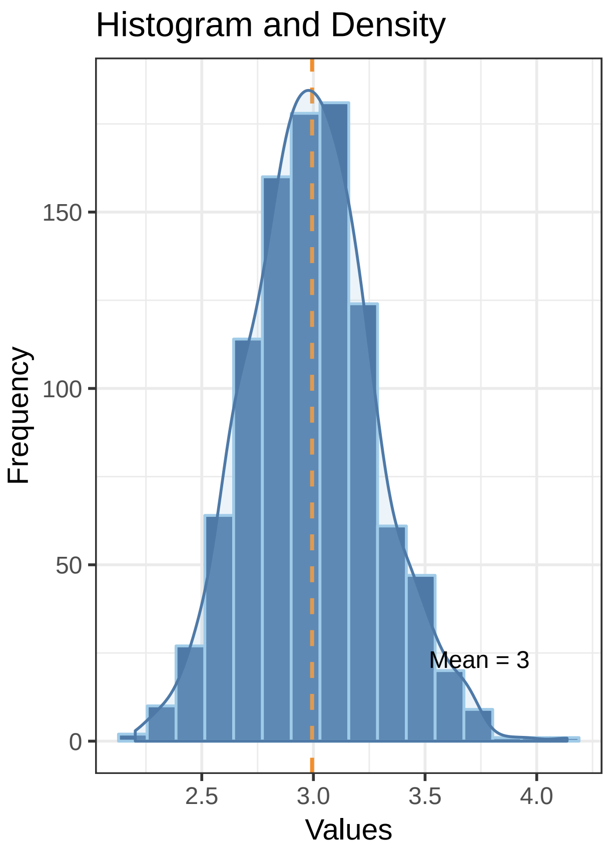
Answer

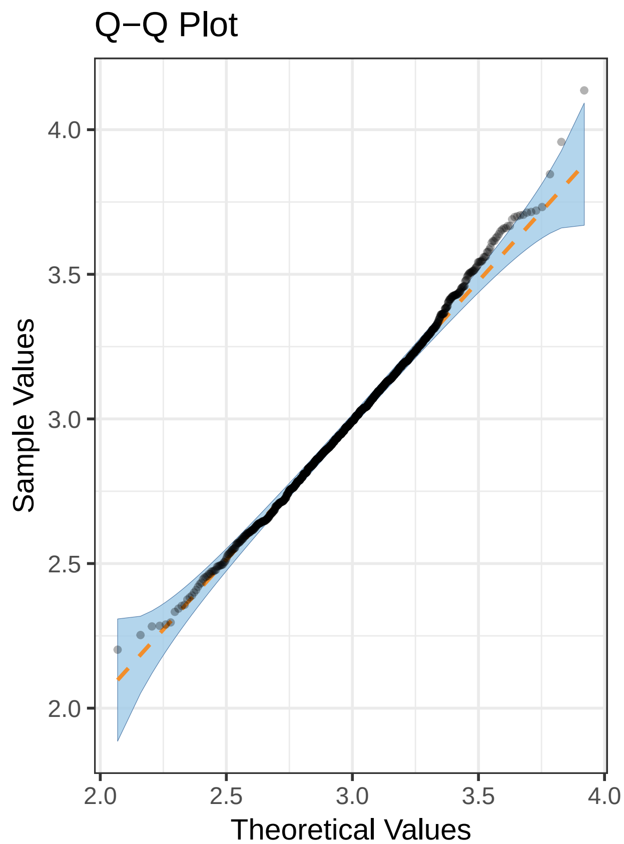
In order for the ANOVA F-test to be valid, we must ensure that there is approximately equal variance between samples, that the sample mean distributions are approximately normal, and that the samples are independent. Because we'll be simulating sample means from the exponential distribution rather than from real data, we can assume that there will be equal variance. We'll check normality after we've selected a sample size. Independence is also guaranteed by virtue of the rexp() function, where each sampled data is independent.

To satisfy the first alternative hypothesis, sample sizes of 30 to 150 were simulated from the exponential distribution with means of 3, 3, and 2. This was repeated 1000 times for each sample size to generate the type I error rate and power, as can be seen below:

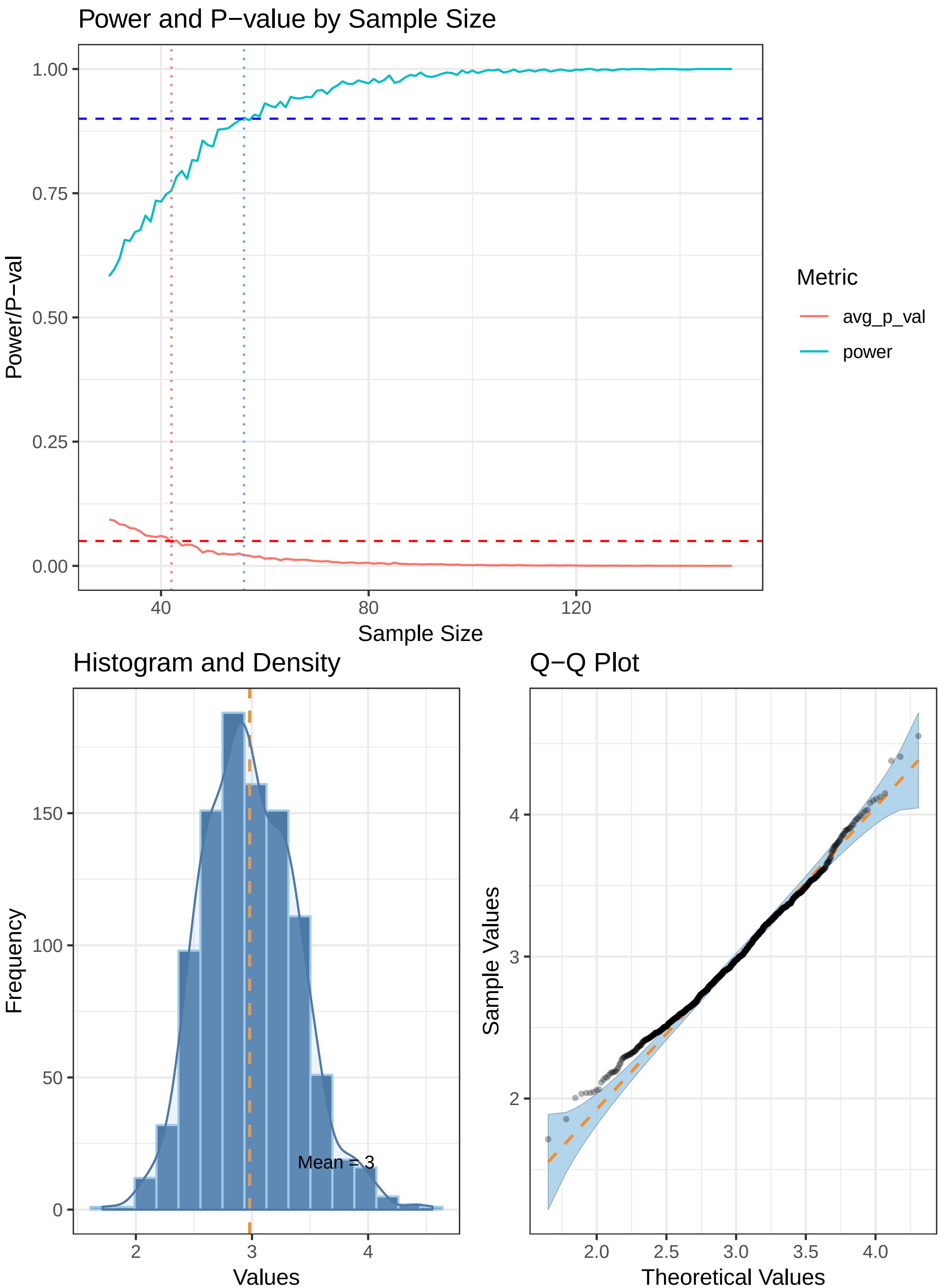


According to this, we can achieve a p_val less than 0.05 at n=88 for each battery, but we don't achieve a power of 0.9 until n=122. At n=122, we get the following distribution of sample means (satisfying the normality assumption):





For the second alternative hypothesis a simular procedure is done with means of 2, 3, and 4 for the different batteries.


Power and P−value by Sample Size
Power/P−val
1.00
0.75
0.50
0.25
0.00
40
80
120
Sample Size
Metric
avg_p_val
power
Histogram and Density
Frequency
150
100
50
0
Mean = 3
2
3
4
Values
Q−Q Plot
Sample Values
4
3
2
2.0
2.5
3.0
3.5
4.0
Theoretical Values

In this case, we'd need a minimum sample size of 42 to get significance and a minimum sample size of 56 to ensure a power of 0.90. The cause for the smaller sample requirements is that the means differences are more extreme and therefore, easier to detect.