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DATA 556 **Discrete Random Variables**

Problem 1
Let X be a random variable with CDF F, and $Y = \mu + \sigma X$, where μ and σ are real numbers with $\sigma > 0$. Find the CDF of Y, in terms of F.

$Y = \mu + \sigma X$

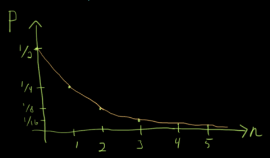
$$F(Y) = P(Y \le y)$$
SUBSTITUTE
$$= P(\mu + \sigma X \le y)$$
$$= P(\sigma X \le y - \mu)$$
$$= P\left(X \le \frac{y-\mu}{\sigma}\right)$$

Problem 2
(a) Show that $p(n) = \left(\frac{1}{2}\right)^{n+1}$, for $n = 0, 1, 2, \ldots$ is a valid PMF for a discrete random variable.

(b) Find the CDF of a random variable with PMF from (a).

a) A VALID PMF SATISFIES TWO CRITERIA:

CRITERIA 1: NON-NEGATIVE FOR ANY n



n	P
0	1/2
1	1/4
2	1/8
3	1/16
⋮	⋮
∞	0

THIS FUNCTION APPROACHES 0, BUT IS NEVER NEGATIVE

CRITERIA 2: MUST SUM TO 1

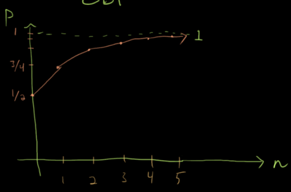
$$\int_0^\infty \left(\frac{1}{2}\right)^{n+1} dn \quad \text{FOR } n = 0, 1, 2, \ldots \infty$$

I CAN'T RECALL HOW TO DO THIS WITH POSITIVE INTEGERS SO I'LL SHOW IT WITH A SIMPLE SUM OF A TABLE:

A	B	C	D
n	(1/2)^(n+1)		
0	0.5000	Sum(B:B) =	1.0000
1	0.2500		
2	0.1250		
3	0.0625		
4	0.0313		
5	0.0156		
6	0.0078		
7	0.0039		
8	0.0020		
9	0.0010		
10	0.0005		
11	0.0002		
12	0.0001		
13	0.0001		
14	0.0000		
15	0.0000		

b) PMF: $p(n) = \left(\frac{1}{2}\right)^{n+1}$

CDF:



n	$P(N \le n)$
0	1/2
1	3/4
2	7/8
3	
4	15/16
5	31/32

$$P(N \le n) = \sum_{n=0}^{n} \left(\frac{1}{2}\right)^{n+1}$$

Problem 3
Let X, Y and Z be discrete random variables such that X and Y have the same conditional distribution given Z, i.e., for all a and z we have

$$P(X = a \mid Z = z) = P(Y = a \mid Z = z).$$

Show that X and Y have the same distribution (unconditionally, not just when given Z).

$$P(X = a)$$
$$= \sum_z P(X = a \mid Z = z) P(Z = z)$$
$$= \sum_z P(Y = a \mid Z = z) P(Z = z)$$
$$= P(Y = a)$$

FROM THE LAW OF TOTAL PROBABILITY (WITH SOME EXTRA CONDITIONING):

$$P(B) = \sum_{i=1}^{n} P(B \mid A_i) P(A_i)$$

Problem 4
(a) Let $X \sim \text{DUnif}(C)$, and B be a nonempty subset of C. Find the conditional distribution of X, given that X is in B.

(b) If $X \sim \text{HGeom}(w, b, n)$, what is the distribution of $n - X$?

a) $$P(X \mid X \in B) = \frac{P(X \in C \cap X \in B)}{P(X \in B)} = \frac{P(X \in B)}{P(X \in B)} = 1$$

b) $X \sim H\,Geom(w, b, n)$

THE PMF OF X IS $$P(X = k) = \frac{\binom{w}{k}\binom{b}{n-k}}{\binom{w+b}{n}}$$

NO IDEA. NOTE TO SELF: GO TO OFFICE HOURS...