



AAE1001 Introduction to Artificial Intelligence and Data Analytics in Aerospace and Aviation Engineering

Week 10 (Introduction to Path Planning)

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Introduction to A* Path Planning Algorithm





Daily Path Planning

MTR stations – available node to go

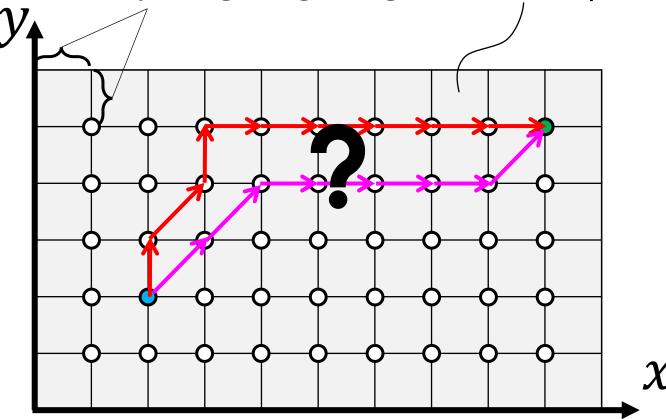






Definition of Path Planning

1 m spacing for griding 2D free space



(0,0) • Start node • Goal node

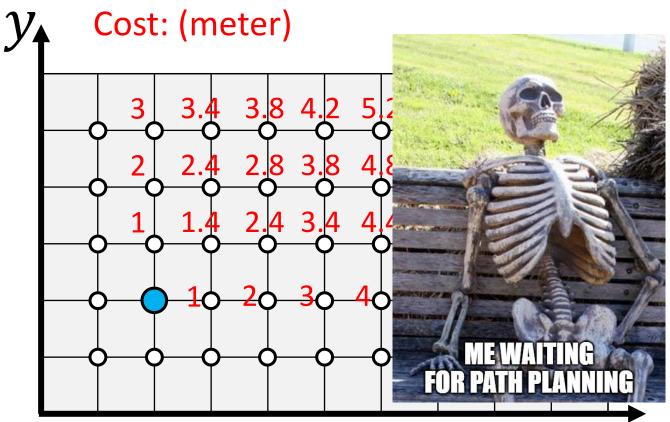
- •Node All potential position you can go across with a unique position (x, y)
- •Search space A collection of nodes, like all board positions of a board game.
- •Objective of path planning— Find the <u>shortest</u> routes with <u>smallest cost</u> from start node to goal node.

Which one is better?





Path Planning by Checking All Available Nodes



(0,0) • Start node • Goal node 1 m spacing for griding Test each possible nodes one-by-one from Start node



Record its shortest path between Start node

Reach Goal

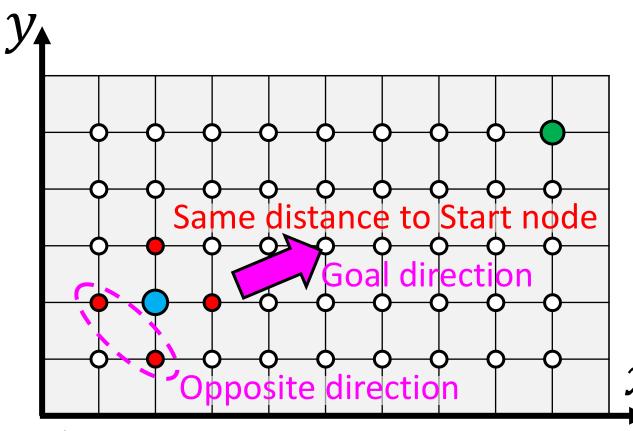
Retrieve the shortest path (Dijkstra's algorithm)

Need higher efficiency!





Path Planning by Checking NOT All Available Nodes



0,0) • Start node • Goal node 1 m spacing for griding

Test each possible nodes one-by-one from Start node



Record its shortest path between Start node

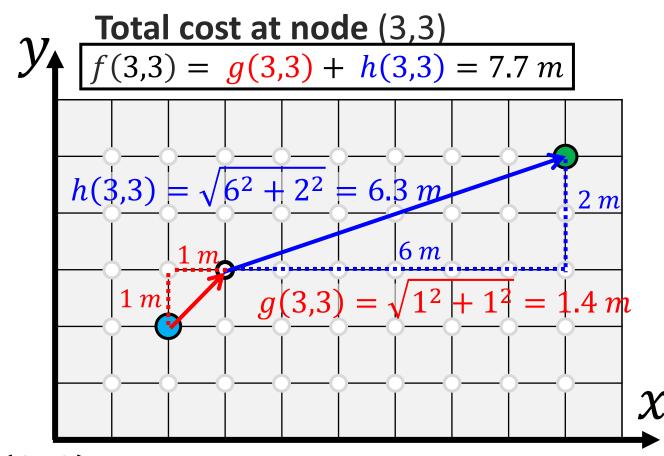








Path Planning by A-star (A*) Search Algorithm



- (0,0) \bullet Start node
 - Goal node 1 m spacing for griding

Definition of cost:

g(x,y) — this represents the **exact cost** of the path **from** the **Start** node to node (x,y)

h(x, y) — this represents the heuristic <u>estimated cost</u> from node (x, y) to the Goal node

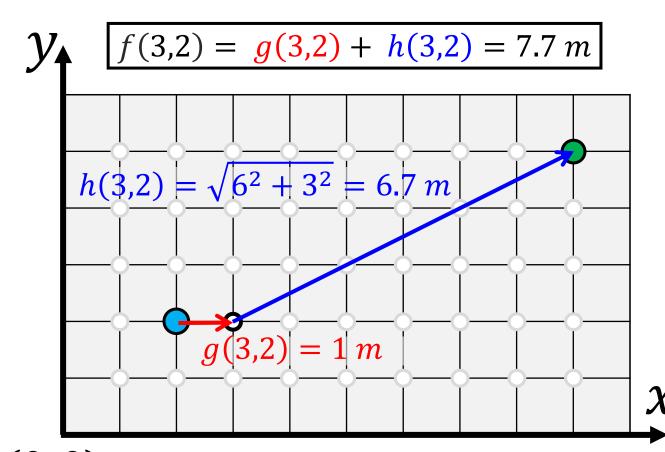
$$f(x,y) = g(x,y) + h(x,y)$$

 $- \underline{\text{total}} \text{ cost of a neighboring}$
 $\text{node } (x,y)$





A-star (A*) Path Planning – Cost at Node (3,2)



(0,0) • Start node • Goal node 1 m spacing for griding

Definition of cost:

g(x,y) — this represents the **exact cost** of the path **from** the **Start** node to node (x,y)

h(x, y) — this represents the heuristic <u>estimated cost</u> from node (x, y) to the Goal node

f(x,y) = g(x,y) + h(x,y)— <u>total</u> cost of a neighboring node (x,y)





A-star (A*) Path Planning – Cost at Node (2,1)

$$y = f(2,1) = g(2,1) + h(2,1) = 8.2 m$$

$$h(2,1) = \sqrt{6^2 + 4^2} = 7.2 m$$

$$g(2,1) = 1 m$$

(0,0) • Start node • Goal node 1 m spacing for griding

Definition of cost:

g(x,y) — this represents the **exact cost** of the path **from** the **Start** node to node (x,y)

h(x, y) — this represents the heuristic <u>estimated cost</u> from node (x, y) to the Goal node

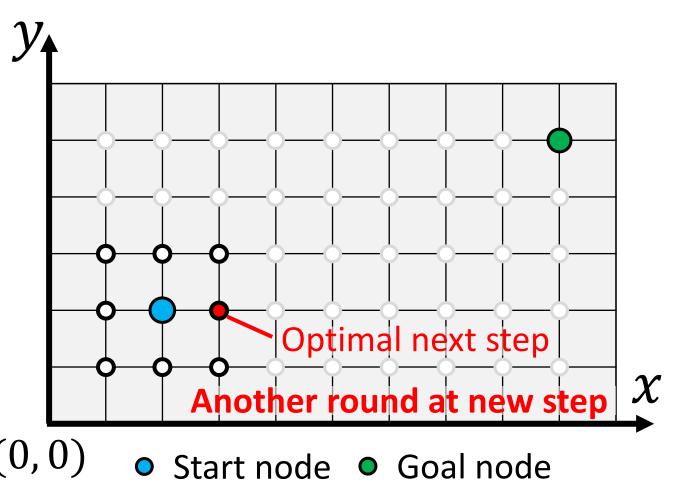
$$f(x,y) = g(x,y) + h(x,y)$$

— total cost of a neighboring
node (x,y)





Total Costs at Neighbouring Nodes



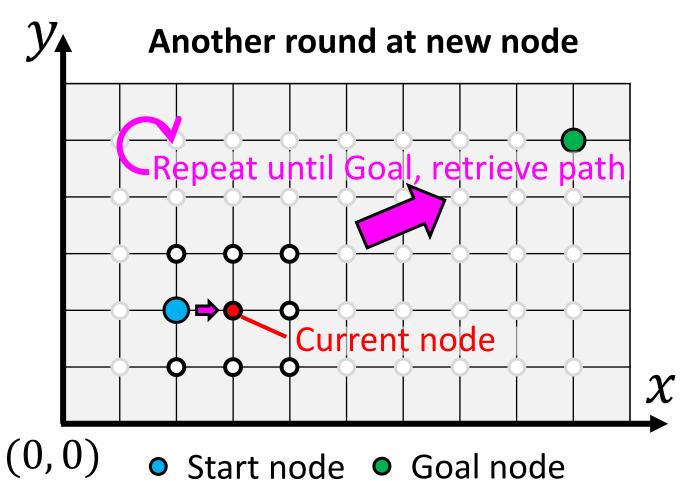
Exact cost Estimated from Start cost to Goal Total cost

Node ID	g(x,y)	h(x,y)	f(x,y)
(1,1)	1.4	8.9	10.3
(1,2)	1	8.5	9.5
(1,3)	1.4	8.2	9.6
(2,1)	1	8.1	9.1
(2,3)	1	7.3	8.3
(3,1)	1.4	7.2	8.6
(3,2)	1	6.7	7.7
(3,3)	1.4	6.3	7.7





Total Costs at Neighbouring Nodes – Round 2



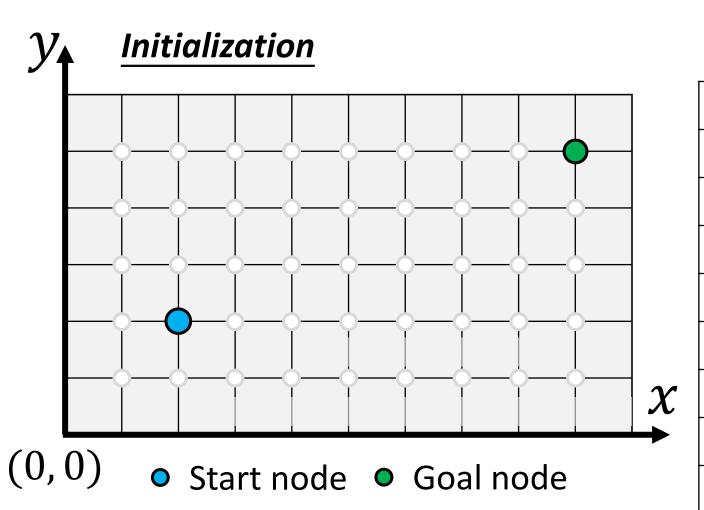
Exact cost Estimated from Start cost to Goal Total cost

Node ID	g(x,y)	h(x,y)	f(x,y)
(2,1)			
(2,2)			
(2,3)			
(3,1)			
(3,3)			
(4,1)			
(4,2)			
(4,3)			

How does computer search step by step and got path?







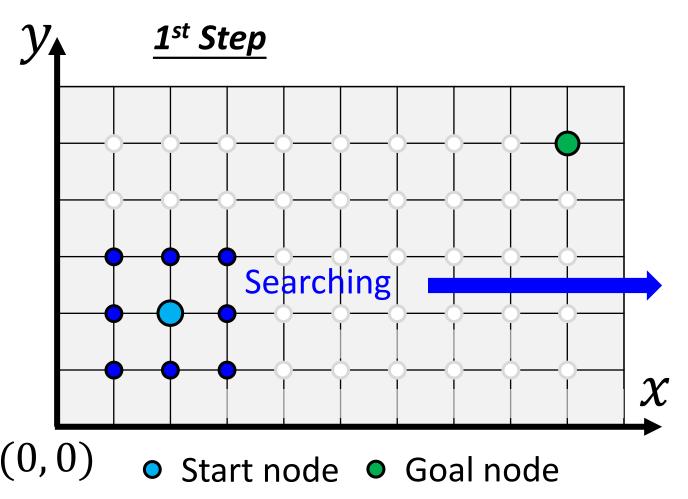
Open List (searched nodes) (arrived nodes)

Close List

Node	f	Source	Node	f	Source
			Start	-	-







Open List (searched nodes) (arrived nodes)

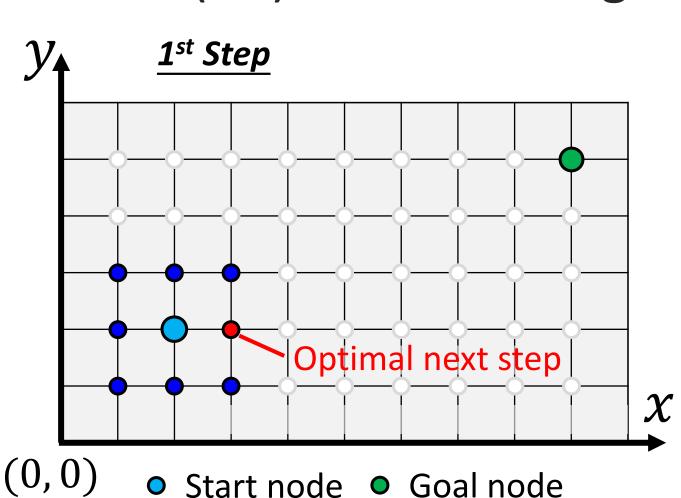
Close List

Node	f	Source
(1,1)	8.9	(2,2)
(1,2)	8.5	(2,2)
(1,3)	8.2	(2,2)
(2,1)	8.1	(2,2)
(2,3)	7.3	(2,2)
(3,1)	7.2	(2,2)
(3,2)	6.7	(2,2)
(3,3)	6.3	(2,2)

Node	f	Source
Start	-	-







Open List (searched nodes)

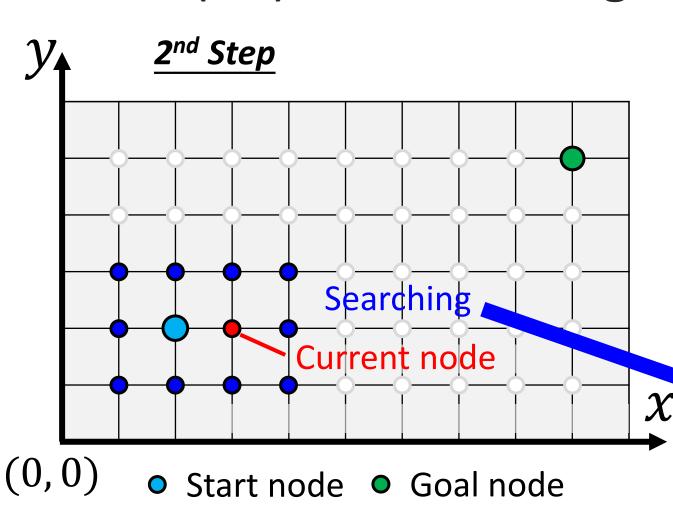
C	ose l	List .		
(arrived nodes)				
	C	C		

Node	f	Source
(1,1)	8.9	(2,2)
(1,2)	8.5	(2,2)
(1,3)	8.2	(2,2)
(2,1)	8.1	(2,2)
(2,3)	7.3	(2,2)
(3,1)	7.2	(2,2)
(3.2)	6.7	(2.2)
(3,2)	0.7	(2,2)
(3,3)	6.3	(2,2)

Node	f	Source
Start	ı	-
(3,2)	6.7	(2,2)







Open List (searched nodes)

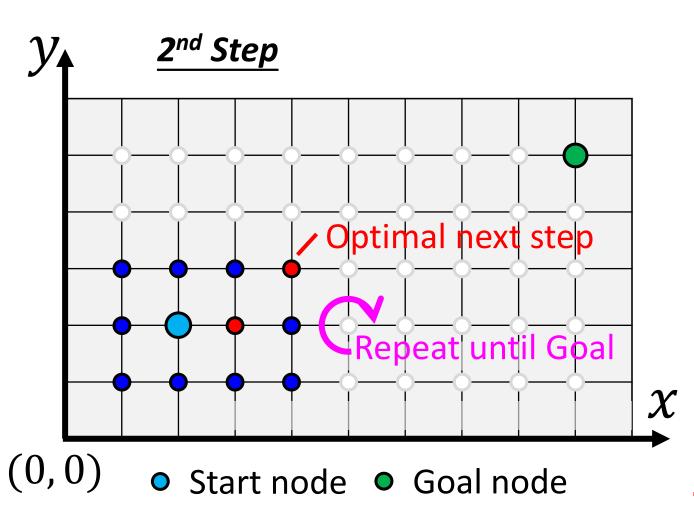
Close	? Li	ist	
(arrived	no	ode	25)

Node	f	Source
(1,1)	8.9	(2,2)
•	•	:
(3,1)	7.2	(2,2)
(3,3)	6.3	(2,2)
•••	•	:
(4,1)	7.8	(2,2)
(4,2)	7.8	(2,2)
(4,3)	8.8	(2,2)

f	Source
1	-
6.7	(2,2)
	<i>f</i> - 6.7







Open List (searched nodes) (arrived nodes)

Close List

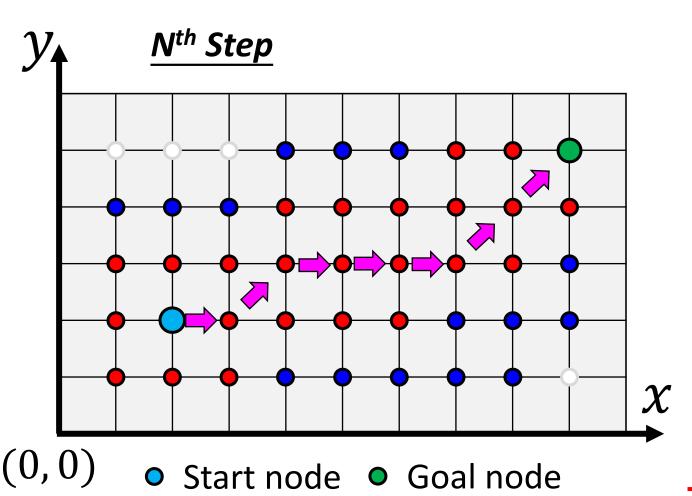
Node	f	Source
(1,1)	8.9	(2,2)
•	• • •	•
(3,1)	7.2	(2,2)
(3,3)	6.3	(2,2)
•	• • •	•
(4,1)	8.8	(3,2)
(4,2)	7.8	(3,2)
(4,3)	7.8	(3,2)
(- / - /		(-,-,

Node	f	Source
Start	ı	1
(3,2)	6.7	(2,2)
(4,3)	7.8	(3,2)





A-star (A*) Path Planning – Retrieve Best Path



Open List Close List (searched nodes) (arrived nodes)

Node	f	Source
(1,1)	8.9	(2,2)
••••	• • •	:
(3,1)	7.2	(2,2)
(3,3)	6.3	(2,2)
•••	•	:
•••	•	:
:	•	:
Goal	X	(x,y)

<u> </u>		
Node	f	Source
Start	-	-
(3,2)	6.7	(2,2)
(4,3)	7.8	(3,2)
	i	:
	i	:
Goal	Х	(x,y)
-	Trace	back
ar	rived	nodes





A-star Example

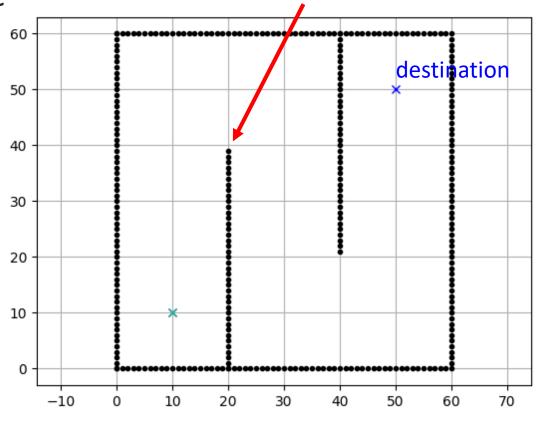
Each time A* enters a node, it calculates the cost: f(n) - n being the neighboring node

It travel to all of the neighboring nodes, and then enters the node with the lowest value of f(n)

These values we calculate using the following formula:

$$f(x,y) = g(x,y) + h(x,y)$$

Wall (obstacles) cannot go through!





Trip Cost of Flight





Trip Cost of Flight

The fundamental rationale of the cost index concept is to achieve minimum **trip cost** by means of a trade-off between operating costs per hour and incremental fuel burn.

$$C = C_F \cdot \Delta F \cdot T + C_T \cdot T + C_C$$

* Related to travelling time

With

- C_F =cost of fuel per kg
- C_T =time related cost per minute of flight
- C_c =fixed cost independent of time
- ΔF =trip fuel (e.g., 3000kg/h)
- T=trip Time (e.g., 8 hours from Hong Kong to Paris)

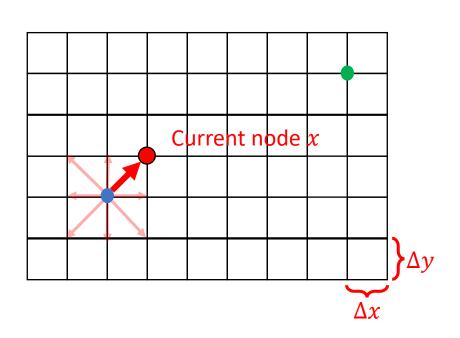
Consider this relationship to imitate the path planning for flights?







Conventional Path Planning Task



Total cost:
$$f(x,y) = g(x,y) + h(x,y)$$

Distance information (conventional)

Example for one step:

$$g(x,y) = \sqrt{\Delta x^2 + \Delta y^2}$$

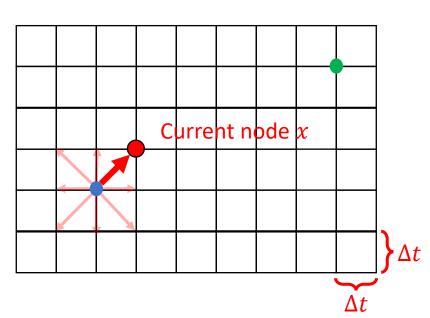
Objective: Find the path with the lowest traveling <u>distance</u>. (in the unit of meter)

- Goal node
- Start node





Path Planning Task for Out Flight



Total cost:
$$f(n) = g(n) + h(n)$$

Traveling time (our case for flight)

Time cost on each step:

$$s(n) = \begin{cases} \Delta t, & vertical/horizontal\ motion \\ \sqrt{2}\Delta t, & diagonal\ motion \end{cases}$$

Example for first step:

$$g(n) = s(n) = \sqrt{2}\Delta t$$

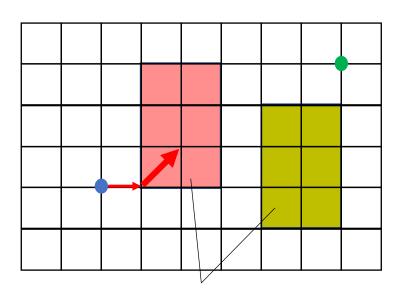
Objective: Find the path with the lowest traveling <u>time</u>. (in the unit of min)

• Goal node • Start node n – index of nodes





Flight Planning Considering Cost Intensive Areas



Cost intensive area (addition time)

Cost Intensive Areas: The cost for flying through such area is increased due to airflow, legal restrictions and other reasons.

Total cost for one node:

$$f(n) = g(n) + \alpha \cdot s(n) + h(n)$$

Previous cost from start

Additional time at current step

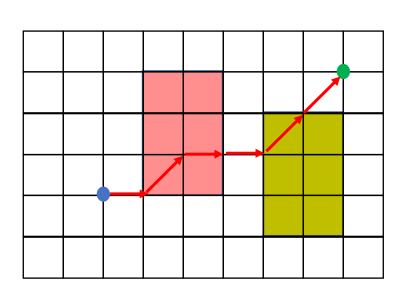
 α – additional time factor (equal zero for normal area)

- Goal node
- Start node





Flight Path Planning Trip Cost





Cost Intensive Areas: The cost for flying through such area is increased due to

airflow, legal restrictions and other reasons.

Total cost for one node:

$$f(n) = g(n) + \alpha \cdot s(n) + h(n)$$

$$T_{best} = \min[f(goal)]$$

$$T_{best} = \min[f(goal)]$$
 Additional cost on each node
$$= \min[g(goal) + \alpha_1 s(n_1) + \alpha_2 s(n_2) + \dots + \alpha_{goal} s(n_{goal})]$$

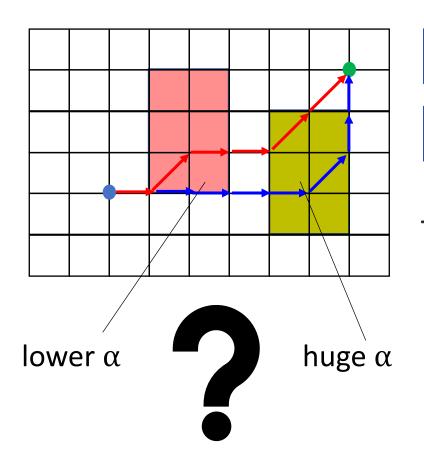
Trip cost for planned path:
$$C = C_F \cdot \Delta F \cdot T_{best} + C_T \cdot T_{best} + C_c$$

- Goal node
- Start node α additional time factor





How it choose different routes?



Cost Intensive Areas: The cost for flying through such area is increased due to airflow, legal restrictions and other reasons.

Time for the planned path:

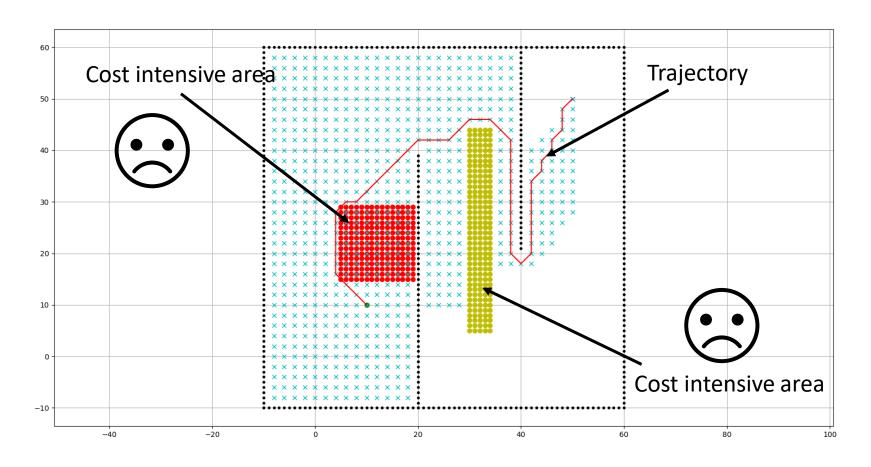
$$T_{best} = \min \left[g(goal) + \underline{\alpha_1 s(n_1)} + \underline{\alpha_2 s(n_2)} + \dots + \underline{\alpha_{goal} s(n_{goal})} \right]$$

Depending on the extra time accumulated more specifically, the additional time factor





Example of Flight Path Planning

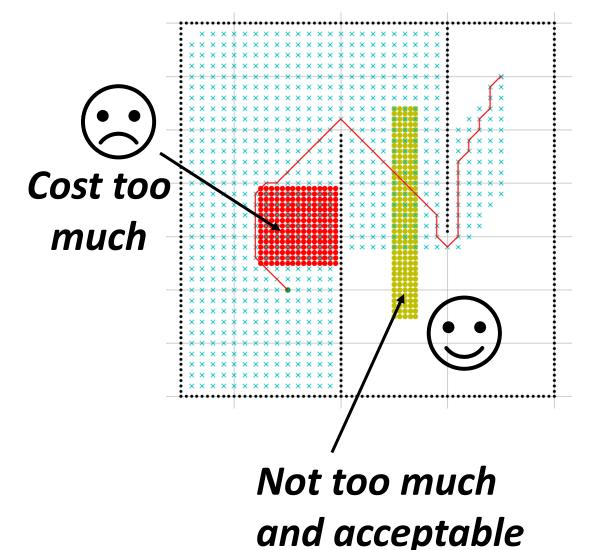


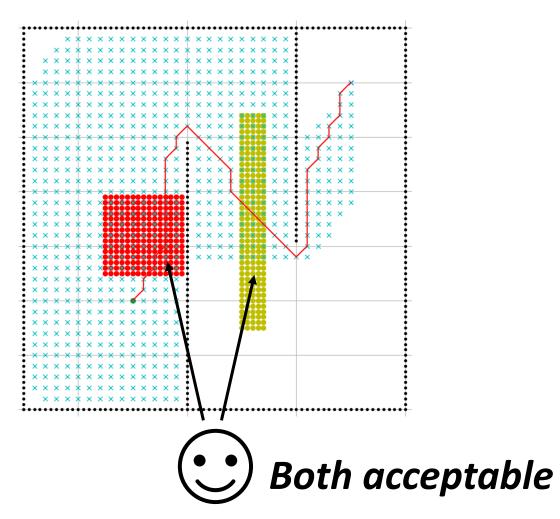
Cost is way too high for going through, better to avoid!





Example of Flight Path Planning









Flight Path Planning Project



You will be creating and completing your own path planning program based on groups



You can find the project tasks / requirements in the GitHub homepage



Additional resources could be found inside the course GitHub repository

- Video tutorial
- Tutorial slides