

NORTH-WEST UNIVERSITY

HONOURS THESIS

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# A cellular automata approach to model informal settlement growth

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*Author:*

Affaan MUHAMMAD

*Supervisor:*

Prof. Hennie KRUGER

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*“The main problems in life can only be solved when you know what works, what doesn’t and why”*

Charlie Munger, American investor and billionaire

NORTH-WEST UNIVERSITY

# *Abstract*

Faculty of Natural and Agricultural Sciences  
School of Computer Science and Information Systems

Bachelor of Science Honours

**A cellular automata approach to model informal settlement growth**

by Affaan MUHAMMAD

Abstract will come here in the future.



# *Acknowledgements*

Acknowledgements will come here in the future.



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# List of Abbreviations

<b>CA</b>	<b>Cellular Automata</b>
<b>UN</b>	<b>United Nations</b>
<b>StatsSA</b>	<b>Statistics South Africa</b>



# Chapter 1

## Introduction

### 1.1 Project description

The process of urbanisation has been a double edged sword whereby many a people have benefited from it while a lot more have not. The issue of informal settlements is one that covers areas such socio-economic, governance, climate, politics, healthcare, and resource management to name a few.

One of the few approaches to tackle these issues is through good modelling, and understanding the growth of such informal settlements.

This project will employ a cellular automata technique, specifically John Conway's 'Game of Life' to model informal settlement growth in South Africa.

### 1.2 Problem description and background

From the 1950s to the early 2000s the rate of urbanisation has increased 20%, however the amount of people living in inadequate housing, informal settlements, and slums globally is still about 1 billion.[25]

According to the United Nations (UN) the definition of an informal settlement is a dwelling with a lack of security, sanitation, water, living area, and housing durability.[9]

The UN also has a list of 17 Sustainable Development Goals of which number 11 is to create 'Sustainable Cities and Communities'. These goals are such that if even a few can be achieved the other will become easier to achieve as well. Discuss Informal settlement framework(what, where, when, how of informal settlements)

Cellular Automata (abbreviated as CA) is a discrete computational model which is studied in automata theory. The basic components of such a model is a grid which contains cells. Each cell can have a finite number of states it

can take on. An initial state at time ( $t = 0$ ) is assigned to the grid as a whole. For each time interval thereafter the cells change their states according to a predefined set of rules.[20]

Conway's *Game of Life* also known as *Life* was created by the British Mathematician John H Conway in the 1970s and first appeared in the *Scientific American* magazine.[8]

The states the cells in Life can take on are either alive or dead. The rules that govern the states of Life are as follows:

1. Due to underpopulation a cell will die if it has less than 2 neighbours<sup>1</sup>.
2. If a cell has 3 or 2 neighbours it will remain alive in the next cycle.
3. If a cell has more than 3 neighbours it will die.
4. If a dead cell is surrounded by 3 alive cells it will become alive in the next cycle.

Using such a set of rules this project will embark on creating a model that will accurately predict informal settlement growth in South Africa.

Some limitations have been noted in using CA to model urban growth. These include creating tradeoffs between flexibility and simplicity for the transitional rules. At the same time other opportunities in CA models also present themselves for study such as calibration, stochastic components, and cell types.[18]

## 1.3 Aims and objectives

### 1.3.1 Aims

- Create a sound mathematical and statistical model
- Apply model to Life to predict growth of informal settlements

### 1.3.2 Objectives

- Conduct the relevant literature reviews
- Get access to relevant maps needed regarding informal settlements

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<sup>1</sup>These are cell which are alive and are located around the current cell



- If above step fails, create own maps
- Create an application that allows a map to be added and a grid to be placed for simulating Life.
- Create a mathematical and statistical model to provide an initial state
- Apply initial state to the map
- Iterate and monitor growth output
- Calculate accuracy of model by comparing maps from different time periods

## 1.4 Procedures and methods

### 1.4.1 Paradigms

When a research project is conducted it usually fall under specific paradigms. A research paradigm is defined as "a set of commonly held beliefs and assumptions within a research community about ontological, epistemological, and methodological concerns." [12]

The research paradigms are as follows:

- Positivism
- Interpretivism
- Critical realism
- Critical theory

*Positivism* stipulates that reality exists independent of human experiences and actions. It further states that knowledge can be gained from experimentation and observation. Lastly, its strategy focuses on surveys and experiments.

*Interpretivism* stipulates that a 'social world' is created by humans who conduct social actions and give meaning to the actions. It further states that knowledge can be gained by actively partaking in the phenomenon and the people who create it. Lastly, its strategy focuses on ethnography, action research, and case studies.

*Critical realism* stipulates that science is not only about observation and there exist mechanisms that are not observable but generate observable behaviours and events. It prefers reduction which is a methodological approach. It begins from unexplained phenomenon and moves to proposing different mechanisms that explain the phenomenon.

*Critical theory* stipulates that understanding should not be the only goal and human emancipation should be another driving force of research. It strives to expose oppressing effects of political ideologies, cultural practices, and academic theories.

This project will fall under a combination of Positivism and Interpretivism. The study will be on social factors combined with infrastructure and other related factors that will assist in creating a suitable or viable model.

### 1.4.2 Artefact Life Cycle Philosophies

When an artefact is developed it can take on different methodologies in its approach. Below is a table that has a few such approaches. The table is a modified example from another source[11].

Method	Description
Crystal Clear	Emphasis on 'osmotic communication'
Disciplined agile delivery (DAD)	Incremental and iterative
Extreme Programming (XP)	Concrete commitments
Kanban	Incremental improvements
Lean Software Development	Deliver value
Scrum	Feedback and self management, iterative
Rational Unified Process (RUP)	Iterative cycles. Each cycle has 4 phases Inception, Elaboration, Construction, and Transition.
Waterfall	Linear sequential phases
Top-down or Bottom-up	Composition and Decomposition

TABLE 1.1: Overview of development methodologies

The approach for this project will be Scrum. As it is perfect for incremental changes made to the model that will be created.

### 1.4.3 Collaboration

A comparative approach will also be taken by integrating work from a fellow student's project. The student will be focusing on CA modelling from a swarm intelligence approach.

## 1.5 Project management

### 1.5.1 Project Plan

Self management will be key in this project and all its endeavours.

Important dates include:

- 18 April: Submission of project planning and research proposal
- 13 June: Submission of literature study
- 21 October: Demonstrations of artefact and poster
- 1 November: Submission of complete documentation

A basic Gantt chart with overview of the project is shown below

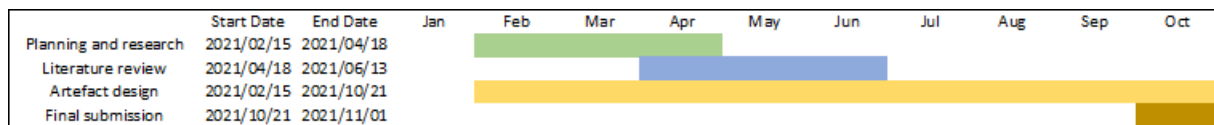


FIGURE 1.1: Project plan as represented by a Gantt chart

### 1.5.2 Scope and Limitations

The scope for this project is the creating, testing, and computing accuracy of the model. Further research will have to be done in other projects at a later stage. The current limitations for this project is time as the deadline for the final presentations and artefact design is in November 2021. No other limitations with regards to financing or resources exists.

Risk analysis was conducted and all the risks involved are listed below:

- Model is not viable
- Model is not accurate enough
- Time is not enough to conclude research

## 1.6 Development platform, resources, and environments

Datasets: Still under procurement from third parties (AfriGIS, StatsSA, GeoTerra Image).

Operating system: Windows and Linux

Programming language: Python will be used

IDE: No specific. However current tools include Jupyter Notebooks, Google Colaboratory, and basic text editors.

The reason why these tools were chosen is for their versatility, ease of access, plethora of libraries and additional packages that can be imported as needed.

## 1.7 Ethical and legal implications

The only legal and ethical implications for this project is in the form of third party data access. The ethics form is attached in Appendix [A](#)

## 1.8 Provisional chapter division

Chapter 1 - Introduction

This will cover the outline of the project.

Chapter 2 - Literature Review

The focus of this section is to delve deeper into the relevant literature for this project and give a thorough analysis.

Chapter 3 - Artefact Design

The focus of this section will be on the complete processes involved in creating the model for this project.

Chapter 4 - Discussion

The focus in this section will be to analyse the results gained from the model and compare them to the relevant literature discussed before. Further assumptions and claims are also discussed here. Lastly, comparisons with other students' work will be discussed.

Chapter 5 - Conclusion

The research project is closed off and further insights for the future can also be given. Aim and objectives are re-examined to ensure if they were achieved.

## Chapter 2

# Literature Review

## 2.1 Cellular Automata and Game of Life

Cellular Automata can be modelled in a number of dimensions including anything from one, two, three, four, or more dimensions.[1] In this project the focus will be on the two dimensional approach using a  $n$ -dimensional lattice (will be referred to as a grid). The grid can be of infinite size, however will be limited to a finite space i.e. a predetermined set of cells superimposed over a map of an informal settlement.

An individual square in the grid (which will be referred to as a cell) can have upto eight neighbours. This is shown in the diagram below.

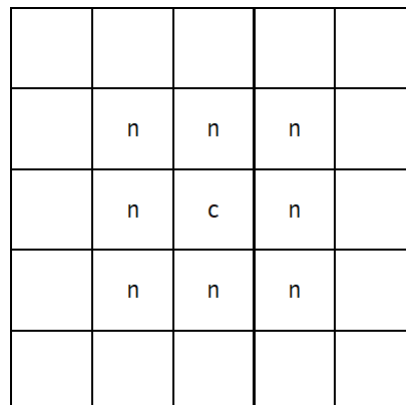


FIGURE 2.1: A cell 'c' and its neighbours 'n'

Source: Own Creation (2021)

A cell can have two discrete states for any given discrete time unit (referred to as generation). These states can be 'alive' or 'dead'. A cell is shown to be alive by having it *shaded* and if it is dead it is left blank.[1] This is shown in the figure below.

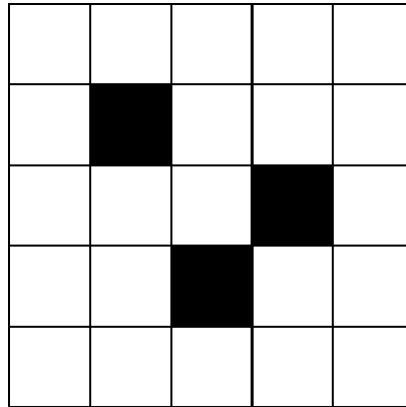
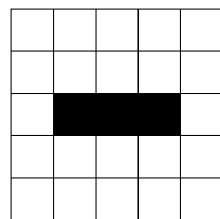
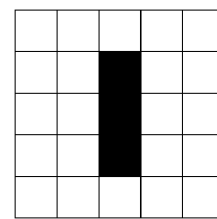


FIGURE 2.2: A grid showing alive and dead cells

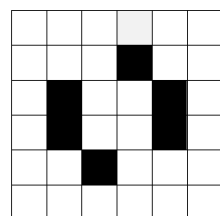
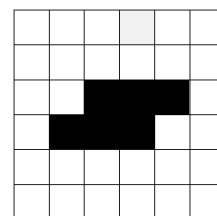
Source: Own Creation (2021)

As shown in Figure 2.1 a cell can have either lateral or diagonal neighbours. The rules of *Life* were discussed briefly above in Section ?? . A graphical representation of these is shown below.

(A) Generation  $t = 0$ (B) Generation  $t = 1$ FIGURE 2.3: A simple 2 generation iteration of *Life*

Source: Own Creation (2021)

Another slightly more advanced setup of *Life* is shown below.

(A) Generation  $t = 0$ (B) Generation  $t = 1$ FIGURE 2.4: An advanced 2 generation iteration of *Life*

Source: Own Creation (2021)

In a seminal paper by Stephen Wolfram, four classes of CA were proposed according to their behaviour given an initial random condition .[24] These four classes are as follows:

- Class 1 - After a finite number of generations a unique homogeneous state is reached i.e. all cells become the same eventually.
- Class 2 - Simple structures are generated which are either periodic, or stable (also called persistent).
- Class 3 - An aperiodic (or chaotic) patterns emerge which carry on indefinitely.
- Class 4 - Capable of universal computations i.e. can exhibit complex behaviour.

Due to the nature of the rules of *Life* certain patterns emerge. This is thanks to the cycles of repeated states which evolve over certain number of generations. These patterns include still-lives, period two (or *blinkers*), gliders, oscillators, glider guns, and puffer trains.[1]

The applications of CA and, or *Life* have sparked a number of research papers in fields such as physics, music, complexity, and computation.

Examples in physics include, interaction between a complex system and electromagnetic radiation [4], an implementation of *Life* with quantum features.[7]

Implementations in music include the development of CAMUS (Cellular Automata Music).[15]

In the fields of complexity, and computation a vast array of work has been done therefore a few examples include; Universal Computer-Constructor in CA [10], and creation of a Turing Machine in *Life*. [16]

## 2.2 Informal settlements

Informal settlements are housing dwellings that are part of urban districts or neighbourhoods that arise and develop without oversight or control from the state. They are synonymous with 'slums' or 'squatters', though are not the same. They form an integral element of urban sustainability whereby developing cities can not develop without them. The connotations with 'informal', 'slums', and 'squatter' have always been seen in a negative light. This is not seen as beneficial as the growth of urbanisation is highly intertwined with

informal settlements.[5]

Across the globe informal settlements are known colloquially by their own variety of terms. [25] In this project the umbrella term informal settlement will be utilised.

The process or principles of informal settlements growth can be grouped into three categories namely;[5]

- *settling* - simply settling down on what is usually unclaimed land
- *inserting* - usually into urban areas that are abandoned, or uninhabited
- *attaching* - informal settlements that grow out of existing urban settlements

In morphological terms, informal settlements can be classified into eight different types. This refers to the urban conditions rather than the process mentioned above, however it is not to say that the two are mutually exclusive. The types are not mutually exclusive from each other either. The types are as follows:[5]

- Districts - most popular where over long periods of time the settlement grows to encompass mixed-use districts with both retail, and industrial functionality.
- Waterfronts - settlements between land and water, whether it is a lake, harbour, river, canal to name a few. Prone to water issues such as flooding if the climate is such.
- Escarpments - settlements on urban topography that is usually too steep to build formal structures. an example location would be an area between a mountain and a city. Can be prone to earthquakes, landslides, and mudslides if the climate is such. Transportation is another key concern.
- Easements - Major infrastructure in urban cities such as roads, railways, pipelines to name a few offer 'buffer' zones which can become informal settlements.
- Sidewalks - These settlements emerge when public area such as sidewalks have an area where people can set up dwellings, even if for temporary usage.
- Adherences - Related to the principle mentioned above.



- Backstages - Settlements hidden from the public's gaze. Becomes more informal the 'deeper' one goes from a formal street frontage.
- Enclosures - Settlements where the 'shell' is contained within another urban building.

Informal settlement residents just like the residents of formal settlements have a desire for a good quality of life. Factors that influence this quality of life include food storage and preparation, water, sanitation, air quality and pollution, electricity, health risks, access to public facilities and amenities, among other things.[17]

Richards et al. (2007), further states that everyday problems such as unemployment, and crime also influence the quality of life. The paper further stipulates that the research can be expanded further to look into spatial and temporal factors that influence informal settlements.

The characteristics of informal settlements transcend the physical and are closely knitted with socio-economic as well as political conditions.[23]

## 2.3 Modelling Urban Development

The phenomenon of urbanisation in the last century has drastically increased from a mere 13% in 1900 to a 49% in 2005. The current projected estimates for the year 2030 in 60%. The majority of this urbanisation is still going to occur in the developing nations.[21]

One of the three factors that is going to influence humans directly in the near future is the land-use/land cover change. This will affect policy makers, geographers, and planners all alike. This is thanks to the socio-environmental consequences of the spatio-temporal process of urban development.[14, 22]

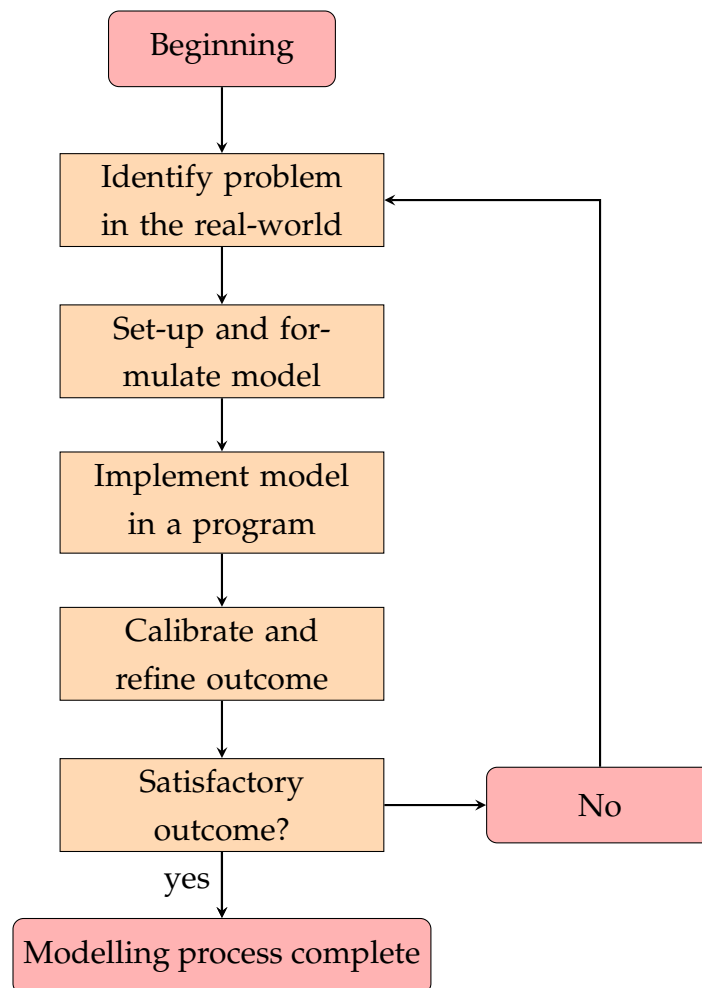
When it comes to geographic models they can be categorised into three categories of increasing abstraction.[19] The models are as follows:

- Scale
  - Iconic
  - Analogue
- Conceptual or Diagrammatical

- Mathematical
  - Probability
  - Deterministic

The mathematical models are therefore the highest level of abstraction. Scale models mimic reality but are miniaturised copies of the real world. The main difference between Iconic and Analogue is the later besides being replicas they tend to transform certain properties of the real world system, e.g. using wool for clouds. Conceptual models are concerned with the relationships that exist between components in the system e.g. a sewage system can be denoted with arrows and boxes on a diagram. Mathematical models if in their purest form translate a conceptual model in to pure, formal, and symbolic logic of mathematics.[19]

The flow chart of the modelling process is shown below.



Source: Own Creation (2021) adapted from [14]

Before continuing it should be noted that one of the major pitfalls of models are they are a simplified view of reality, hence they can tend to leave certain facts behind of reality.[14]

One of the earliest models created in geography and urban modelling dates back to the 1800s where Johann Heinrich von Thünen in his book titled *Der isolierte Staat* translated to mean *The Isolated State* discussed maximising agricultural production.[3]

Other seminal research included the like of Alfred Weber in the early 1900s published the title *Über den Standort der Industrien* which translates to *Theory of the Location of Industries*. This work created models based on real-world conditions.[6]

The seminal work on the three classical models of urban structure, growth, and urban land usage patterns was done by a number of authors. The models include Concentric Zone Theory, Sector model, and the Multiple nuclei model.[14] These models laid the foundations for later computer based models to emerge in the field of urban modelling. The early works include techniques on simulation, linear analysis, and mathematical programming.[13] Among the first studies carried out on simulations of real-world cases using urban CA was done by [2].

The meta-analysis carried out by Sante et al. (2010) showed that a number of characteristics are utilised in each of the studies that resulted in a varied range of accuracy for the models implemented in CA for urban growth.[18] A few characteristics are:

- Cell space
- States
- CA relaxations (Neighbourhood, Transition rules, Constraints)
- Calibration
- Validation

Additional factors were also implemented which also helped in creating a varied range of accuracies. Some of these factors include:

- Accessibility of transport
- Distance to railways, airports, and urban centres
- Social services

- Slope & Elevation
- Environmental factors
- Hazard lands
- Agricultural value
- Urban suitability
- Zoning
- Population density
- Land value
- Construction year
- Water supply
- Social housing

All of the above with a combination of additional research will now help the project move to the next step which is the development of the artefact, which in this project's case is a model.

# Appendix A

## Ethics Form

ITRI 671 Research ethics form: Honours project

**Name:** Affaan Muhammad

**Title of project:** A cellular automata approach to model informal settlement growth

**Supervisor:** Prof Hennie Kruger

**Starting and end dates of project:** 15 Feb 2021 – 1 Nov 2021

1. Have you read the information available related to research ethics (Chapter 5 of Researching Information Systems and Computing; BJ Oates and Chapter 13 of Writing for computer science, J Zobel; Manual for post graduate studies, available on efundi)?	Yes ✓	No
--	-------	----

2. Do you make use of people as source of data in your project (for example the completion of questionnaires or evaluation of products)?	Yes	No ✓
--	-----	------

3. Are there any aspects of your research that you need permission from another party to use (for example use of property or tools)? If yes, provide more detail.	Yes ✓	No
---	-------	----

4. Describe your research question and give a short description of your plans for the collection of data.  <b>Access for maps from third parties such as AfriGIS, StatsSA, and GeoTerra Image.</b>
--

5. Describe how you plan to provide information about yourself and the goals of your research to participants.

**N/A**

6. Describe what methods you will use to get permission from participants in your study.

**N/A**

7. Will you be able to ensure that participants' information will be used in an anonymous, private and confidential way? How?

**N/A**

Yes

No ☒

8. Are there any foreseeable risks of damage (physical, social or psychological) to participants or the environment? If you answer yes, give detail of the preventative measures you will follow.

**N/A**

Yes

No ☒

9. Are there any foreseeable risks to the NWU, for example lawful actions that may follow the research, or damage to the image of the university? If yes, give detail.  <b>N/A</b>	Yes	No <input checked="" type="checkbox"/>
10. Are there any other ethical issues that may occur during the execution of the research (for example conflicting interests)? If yes, provide detail and explain how you plan to handle them.  <b>N/A</b>	Yes	No <input checked="" type="checkbox"/>

I declare that the information contained in this form is accurate. I have attempted to identify the risks that may arise in conducting this research and acknowledge my obligations and the rights of the participants. I confirm that the research will be conducted in line with all University, legal and ethical standards.

**Name of student:** Affaan Muhammad  
**Signature:** *Signed Affaan Muhammad*  
**Date:** 18 April 2021

**Name of study leader:** Prof Hennie Kruger  
**Signature:**  
**Date:**

**Name of additional moderator:**  
**Signature:**  
**Date:**





# Research Proposal

## Research Proposal for Honours project

The Subject Group requires that the research proposal will be submitted through the use of this form and in the format below. Please complete using a computer.

Initials	A	Surname	Muhammad	Student number	33016763
----------	---	---------	----------	----------------	----------

Bachelor of Science Honours in  
Computer Science and Information  
Technology

Initials and surname	H Kruger
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Title (preferably not more than 12 words)	A cellular automata approach to model informal settlement growth
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<p>Aims are to create a sound mathematical and statistical model thereafter apply the model to Life to predict growth of informal settlements.</p> <p>Objectives are firstly to get access to the relevant maps and datasets. If this is not achievable other methods such as creating it ourselves will be employed. Thereafter creating and applying the model to the maps to predict growth. Lastly compare accuracy.</p>
--

N/A
-----

## 8 Method of investigation

### 8.1 Literature study

Provide an indication only of which literature will be used in the study with a few key references. A summary of the literature is not required here.

### 8.2 Methods of investigation

The proposed design, data acquisition, procedures, data processing, funding sources (but not a budget), mathematical methods, computer methods, etc.

## 9 Provisional chapter division

Here it should be clear that there was proper reflection on the appearance of the final product (mini dissertation). Provide provisional titles of the various chapters, with a brief outline of the planned content of each.

1. Introduction
2. Literature Review
3. Artefact Design
4. Discussion
5. Conclusion

## 10 Literature references

Provide complete references to the literature referenced to in this proposal only.



Student

Supervisor

18 April 2021

Date

# Bibliography

- [1] A. Adamatzky. *Game of Life Cellular Automata*. Springer London, 2010. ISBN: 9781849962179. URL: <https://books.google.co.za/books?id=5iz6C0zzWKcC>.
- [2] Michael Batty and Yichun Xie. "From cells to cities". In: *Environment and planning B: Planning and design* 21.7 (1994), S31–S48.
- [3] Colin Clark. "Von Thunen's isolated state". In: *Oxford Economic Papers* 19.3 (1967), pp. 370–377.
- [4] Claudio Conti. "The Enlightened Game of Life". In: *Game of Life Cellular Automata* (Oct. 2008). DOI: [10.1007/978-1-84996-217-9\\_22](https://doi.org/10.1007/978-1-84996-217-9_22).
- [5] Kim Dovey and Ross King. "Forms of informality: Morphology and visibility of informal settlements". In: *Built Environment* 37.1 (2011), pp. 11–29.
- [6] David Fearon. "Alfred Weber: theory of the location of industries, 1909". In: *R Center for Spatially Integrated Social Science* (2006).
- [7] Adrian Flitney and Derek Abbott. "Towards a Quantum Game of Life". In: Jan. 2010, pp. 465–486. ISBN: 978-1-84996-216-2. DOI: [10.1007/978-1-84996-217-9\\_23](https://doi.org/10.1007/978-1-84996-217-9_23).
- [8] Martin Gardner. "MATHEMATICAL GAMES". In: *Scientific American* 223.4 (1970), pp. 120–123. ISSN: 00368733, 19467087. URL: <http://www.jstor.org/stable/24927642>.
- [9] *Goal 11: Make cities and human settlements inclusive, safe, resilient and sustainable - SDG Indicators*. URL: <https://unstats.un.org/sdgs/report/2016/goal-11/>.
- [10] Adam Goucher. "Universal Computation and Construction in GoL Cellular Automata". In: *Game of Life Cellular Automata*, ISBN 978-1-84996-216-2. Springer-Verlag London Limited, 2010, p. 505 (Jan. 2010). DOI: [10.1007/978-1-84996-217-9\\_25](https://doi.org/10.1007/978-1-84996-217-9_25).

- [11] “ISO/IEC/IEEE International Standard - Systems and software engineering — Developing information for users in an agile environment.” In: *ISO/IEC/IEEE 26515:2018(E)* (2018), pp. 1 –32. ISSN: 978-1-5044-5297-7.
- [12] Paul Johannesson and Erik Perjons. *An introduction to design science*. Springer, 2014. ISBN: 9783319106328.
- [13] M Kilbridge, R O’Block, and P Teplitz. “Population Density: Concepts and Measures”. In: *Urban Analysis, Harvard U. Press, Cambridge, Mass* (1970).
- [14] Y. Liu. *Modelling Urban Development with Geographical Information Systems and Cellular Automata*. Environmental engineering. CRC Press, 2008. ISBN: 9781420059908. URL: <https://books.google.co.za/books?id=6jELer0fbJsC>.
- [15] Eduardo Miranda and Alexis Kirke. “Game of Life Music”. In: *Game of Life Cellular Automata*, ISBN 978-1-84996-216-2. Springer-Verlag London Limited, 2010, p. 489 (Jan. 2010). DOI: [10.1007/978-1-84996-217-9\\_24](https://doi.org/10.1007/978-1-84996-217-9_24).
- [16] Paul Rendell. “A Simple Universal Turing Machine for the Game of Life Turing Machine”. In: *J. Cellular Automata* 6 (Jan. 2011), pp. 323–340. DOI: [10.1007/978-1-84996-217-9\\_26](https://doi.org/10.1007/978-1-84996-217-9_26).
- [17] Robin Richards, Brian O’Leary, and Kingstone Mutsonziwa. “Measuring quality of life in informal settlements in South Africa”. In: *Social indicators research* 81.2 (2007), pp. 375–388.
- [18] Inés Santé et al. “Cellular automata models for the simulation of real-world urban processes: A review and analysis.” In: *Landscape and Urban Planning* 96.2 (2010), pp. 108 –122. ISSN: 0169-2046.
- [19] Reginald William Thomas and Richard J Huggett. *Modelling in geography: a mathematical approach*. Rowman & Littlefield, 1980.
- [20] Tommaso Toffoli and Norman Margolus. *Cellular automata machines : a new environment for modeling*. Cambridge, Mass: MIT Press, 1987. ISBN: 9780262200608.
- [21] UNPD. *World Urbanization Prospects: The 2005 Revision*. 2005.
- [22] Peter M Vitousek. “Beyond global warming: ecology and global change”. In: *Ecology* 75.7 (1994), pp. 1861–1876.

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- [23] B.W. Wekesa, G.S. Steyn, and F.A.O. (Fred) Otieno. "A review of physical and socio-economic characteristics and intervention approaches of informal settlements". In: *Habitat International* 35.2 (2011), pp. 238–245. ISSN: 0197-3975. DOI: <https://doi.org/10.1016/j.habitatint.2010.09.006>. URL: <https://www.sciencedirect.com/science/article/pii/S0197397510000652>.
- [24] Stephen Wolfram. "Universality and complexity in cellular automata". In: *Physica D: Nonlinear Phenomena* 10.1-2 (1984), pp. 1–35. DOI: [10.1016/0167-2789\(84\)90245-8](https://doi.org/10.1016/0167-2789(84)90245-8).
- [25] *World Cities Report 2016 Urbanization and Development: Emerging Futures*. United Nations Pubn, July 2016. 260 pp. ISBN: 9211327083.