AM205: Final Project - Instrument Identification

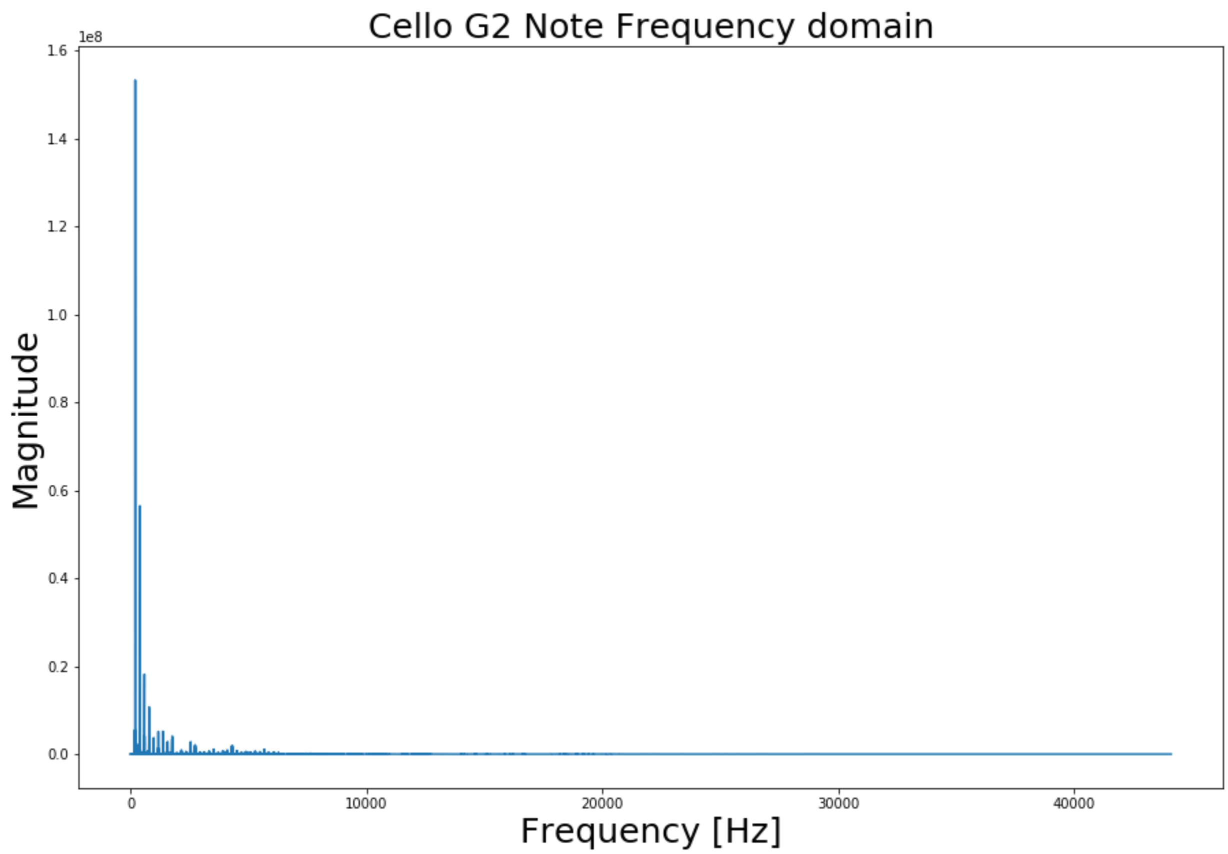
Simon Warchol and Matthieu Meeus - Fall 2019

In [1]:
```
import matplotlib.pyplot as plt
from scipy.fftpack import rfft, ifft
from scipy.io import wavfile
import numpy as np
from scipy.interpolate import lagrange
```

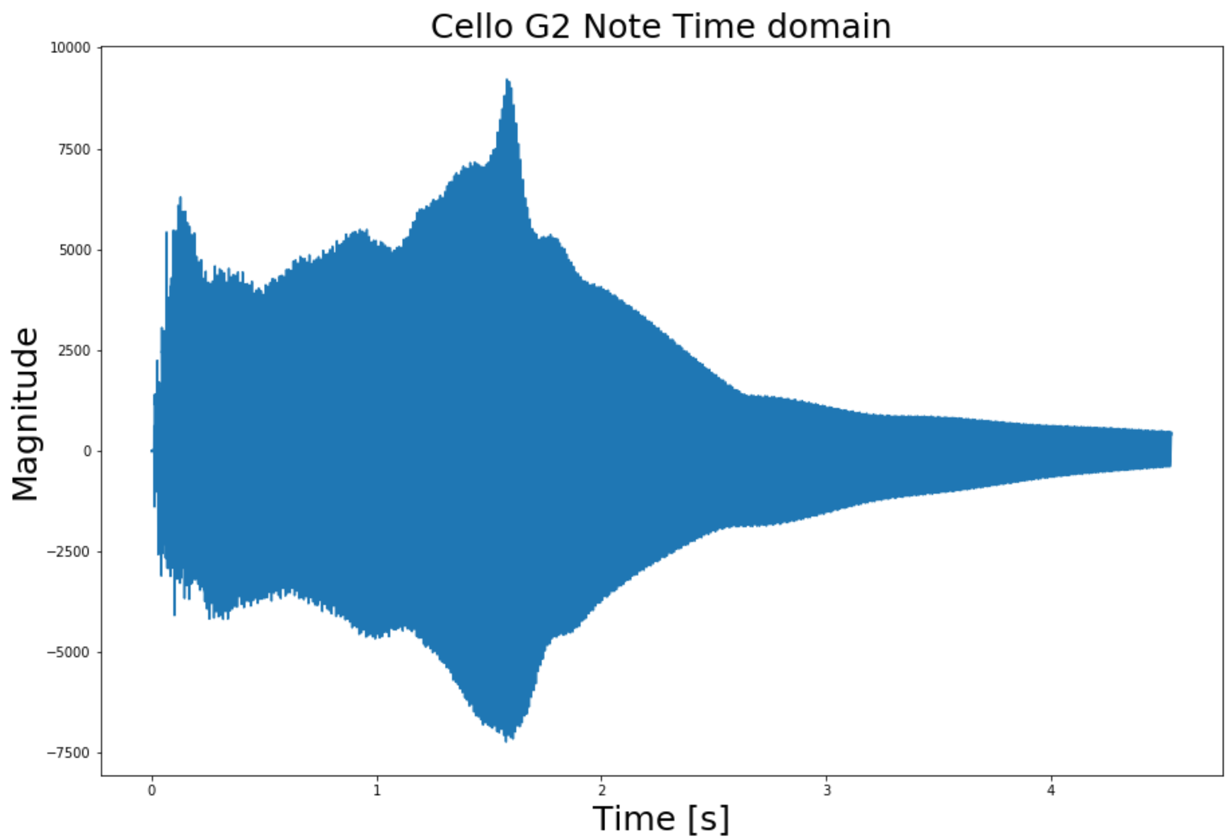
1. FFT on simple, single-note sound files of different instruments

1.1 Test

In [2]:
```
fs, data = wavfile.read('48028__smoseson__g2.wav') # load the data
data = data[:200000,:]
a = data.T[0] # this is a two channel soundtrack, I get the first track
#b=[(ele/2**8.)*2-1 for ele in a] # this is 8-bit track, b is now normalize
c = rfft(a) # calculate fourier transform (complex numbers list)
plt.figure(figsize = (15,10))
plt.plot(np.linspace(0,fs,len(c)), abs(c))
plt.title('Cello G2 Note Frequency domain', fontsize = 25)
plt.xlabel('Frequency [Hz]', fontsize = 25)
plt.ylabel('Magnitude', fontsize = 25)
plt.show()
```



In [3]:
```
plt.figure(figsize = (15,10))
plt.plot(np.linspace(0,200000/fs, 200000), data.T[0])
plt.title('Cello G2 Note Time domain', fontsize = 25)
plt.xlabel('Time [s]', fontsize = 25)
plt.ylabel('Magnitude', fontsize = 25)
plt.show()
```



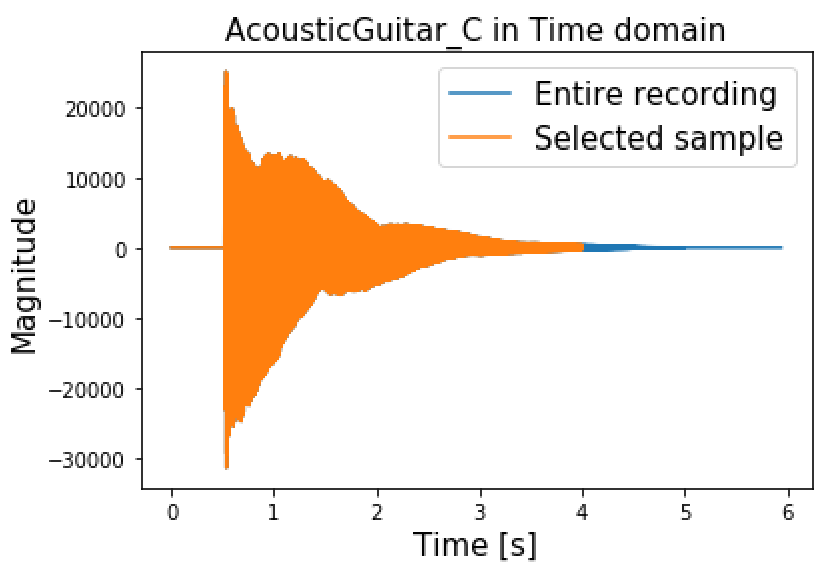
1.2 Multiple instruments

In [4]:
```
instruments = ['AcousticGuitar_C', 'CathedralOrgan_C', 'Clarinet_C', 'Frenc
               'RockGuitar_C', 'Saxophone_C']
```

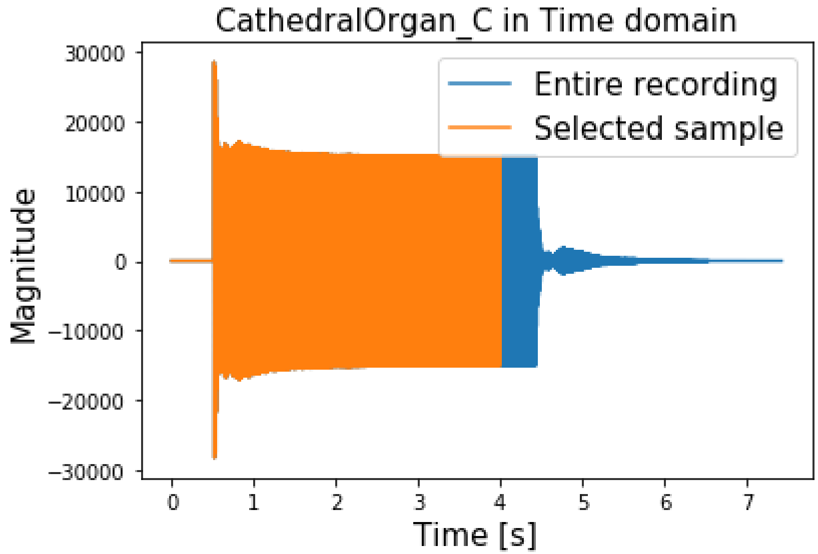
In [5]:

```
# sampling a representative time frame for each instrument
samples = dict()

# saving the frequencies
freqs = dict()

for instrument in instruments:

    plt.figure()
    print(instrument)
    fs, data = wavfile.read('sounds/'+instrument[:-2]+'.wav') # load the da
    freqs[instrument] = fs

    # sample selected = 1 to 2 sec
    start = 0*fs
    end = 4*fs
    samples[instrument] = data.T[0][start:end]
    t_length = data.shape[0]
    plt.plot(np.linspace(0,t_length/fs, t_length), data.T[0], label = 'Enti
    plt.plot(np.linspace(start/fs, end/fs, (end-start)), samples[instrument
    plt.title(instrument + ' in Time domain', fontsize = 15)
    plt.xlabel('Time [s]', fontsize = 15)
    plt.ylabel('Magnitude', fontsize = 15)
    plt.legend(fontsize = 15)
    plt.show()
```

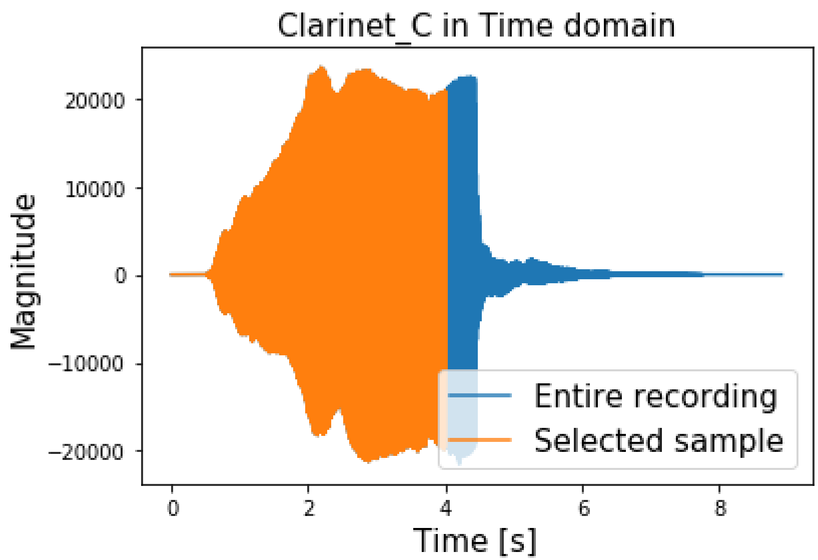
```
AcousticGuitar_C

C:\Users\Matthieu\Pictures\Anaconda3_2\lib\site-packages\scipy\io\wavfil
e.py:273: WavFileWarning: Chunk (non-data) not understood, skipping it.
  WavFileWarning)
```



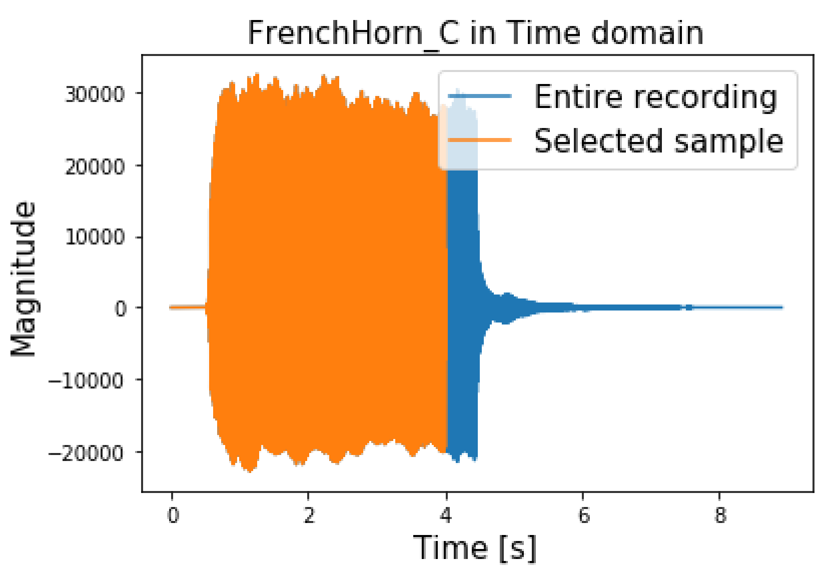

```
CathedralOrgan_C
```



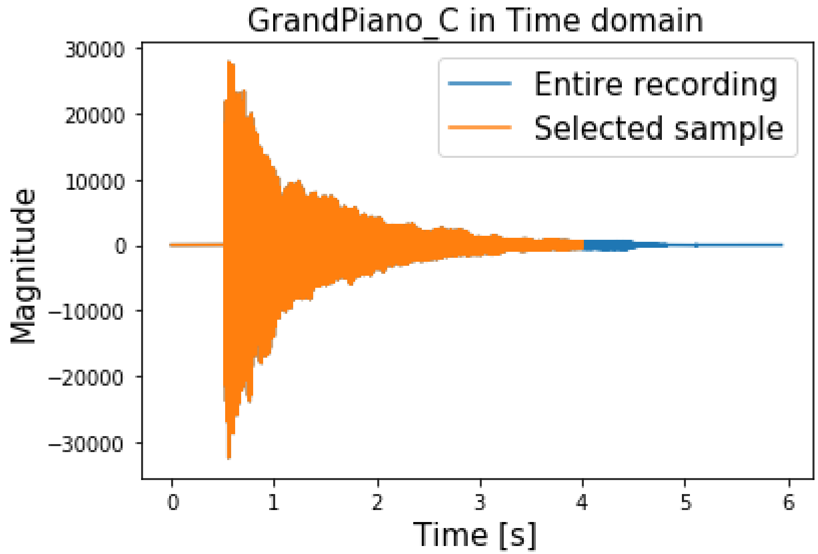
Clarinet_C



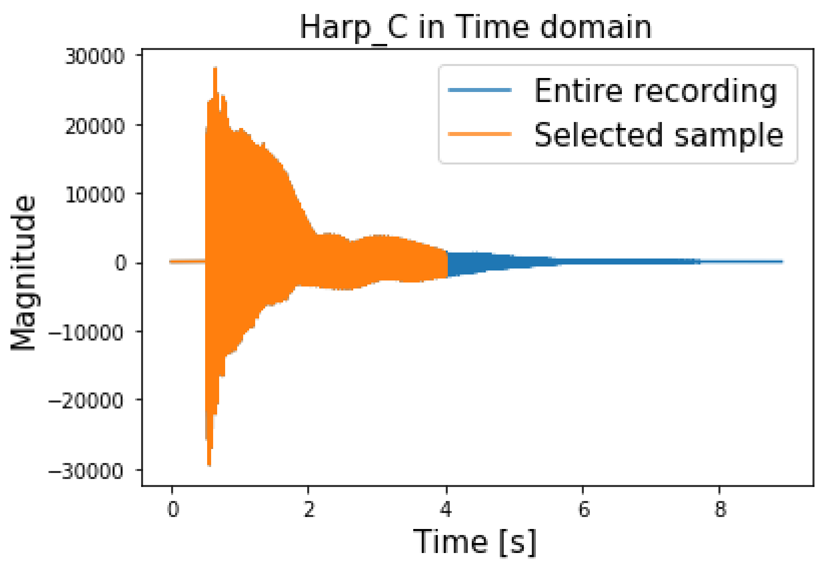
FrenchHorn_C



GrandPiano_C



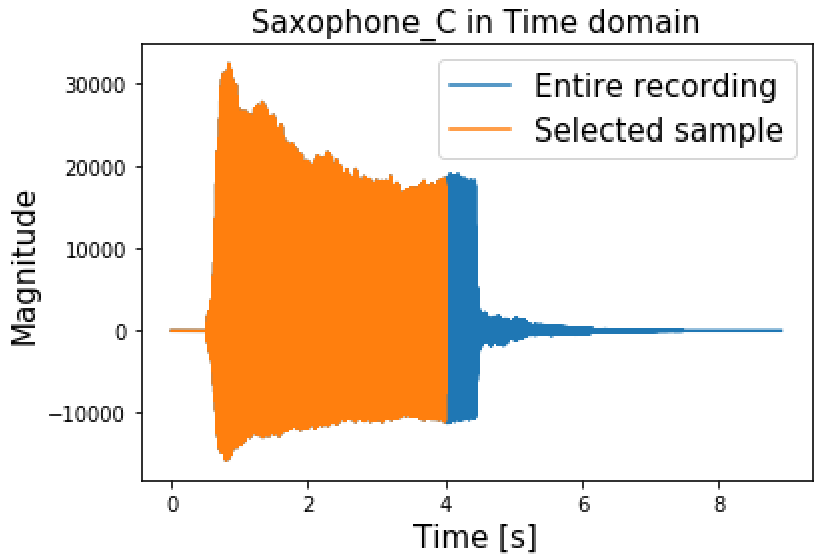
Harp_C



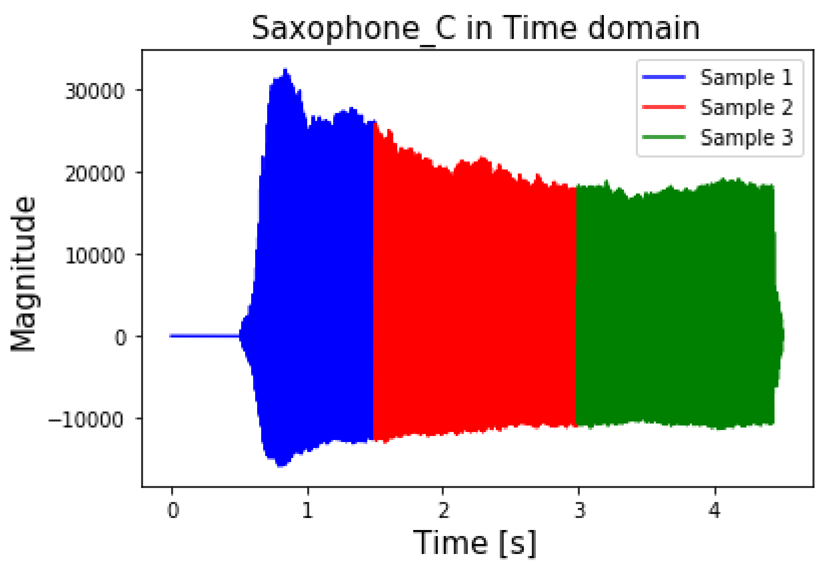
RockGuitar_C


RockGuitar_C in Time domain
20000
Entire recording
Selected sample

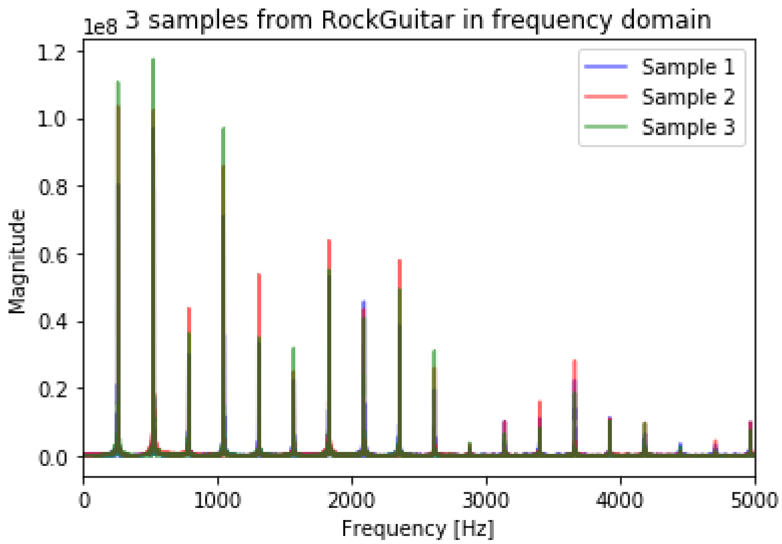
Saxophone_C


Saxophone_C in Time domain
Entire recording
Selected sample
30000
20000
10000
0
−10000
Magnitude
0
2
4
6
8
Time [s]

In [6]:
```
# check influence of different samples

#1
start = 0*fs
end = int(1.5*fs)
sample1 = data.T[0][start:end]
plt.plot(np.linspace(start/fs, end/fs, (end-start)), sample1, label = 'Samp
start = int(1.5*fs)
end = 3*fs
sample2 = data.T[0][start:end]
plt.plot(np.linspace(start/fs, end/fs, (end-start)), sample2, label = 'Samp
start = 3*fs
end = int(4.5*fs)
sample3 = data.T[0][start:end]
plt.plot(np.linspace(start/fs, end/fs, (end-start)), sample3, label = 'Samp
t_length = data.shape[0]

plt.title(instrument + ' in Time domain', fontsize = 15)
plt.xlabel('Time [s]', fontsize = 15)
plt.ylabel('Magnitude', fontsize = 15)
plt.legend()
plt.show()
```

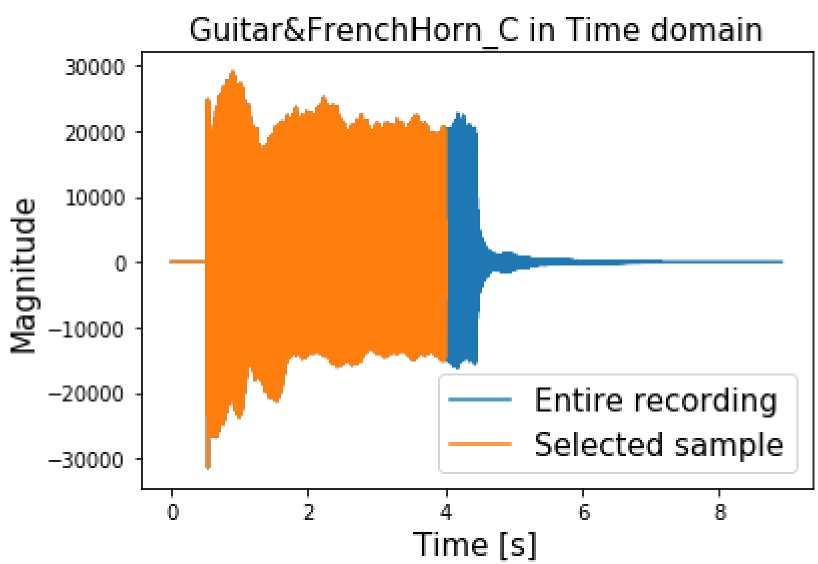


In [7]:
```
c = rfft(sample1) # calculate fourier transform (complex numbers list)
plt.plot(np.linspace(0,fs, len(c)), abs(c), label = 'Sample 1', c = 'blue',
c = rfft(sample2) # calculate fourier transform (complex numbers list)
plt.plot(np.linspace(0,fs, len(c)), abs(c), label = 'Sample 2', c = 'red',
c = rfft(sample3) # calculate fourier transform (complex numbers list)
plt.plot(np.linspace(0,fs, len(c)), abs(c), label = 'Sample 3', c = 'green'
plt.title('3 samples from RockGuitar in frequency domain')
plt.xlabel('Frequency [Hz]')
plt.ylabel('Magnitude')
plt.legend()
plt.xlim(0,5000)
plt.show()
```



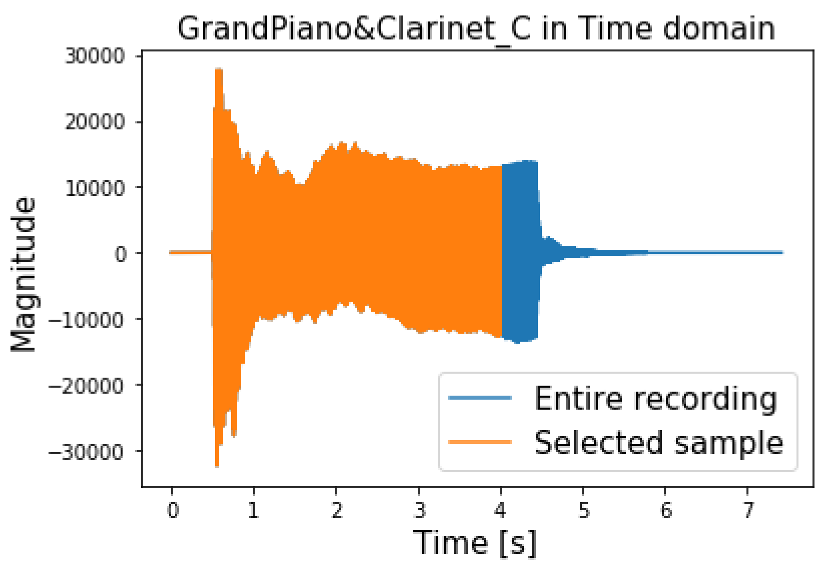
In [8]:
```
# let's look at multiple instruments

mix_samples = dict()
plt.figure()
instrument = 'Guitar&FrenchHorn_C'
print(instrument)
fs, data = wavfile.read('sounds/'+instrument[:-2]+'.wav') # load the data
freqs[instrument] = fs

# sample selected = 1 to 2 sec
start = 0*fs
end = 4*fs
mix_samples[instrument] = data.T[0][start:end]
t_length = data.shape[0]
plt.plot(np.linspace(0,t_length/fs, t_length), data.T[0], label = 'Entire r
plt.plot(np.linspace(start/fs, end/fs, (end-start)), mix_samples[instrument
plt.title(instrument + ' in Time domain', fontsize = 15)
plt.xlabel('Time [s]', fontsize = 15)
plt.ylabel('Magnitude', fontsize = 15)
plt.legend(fontsize = 15)
plt.show()
```

```
Guitar&FrenchHorn_C
```

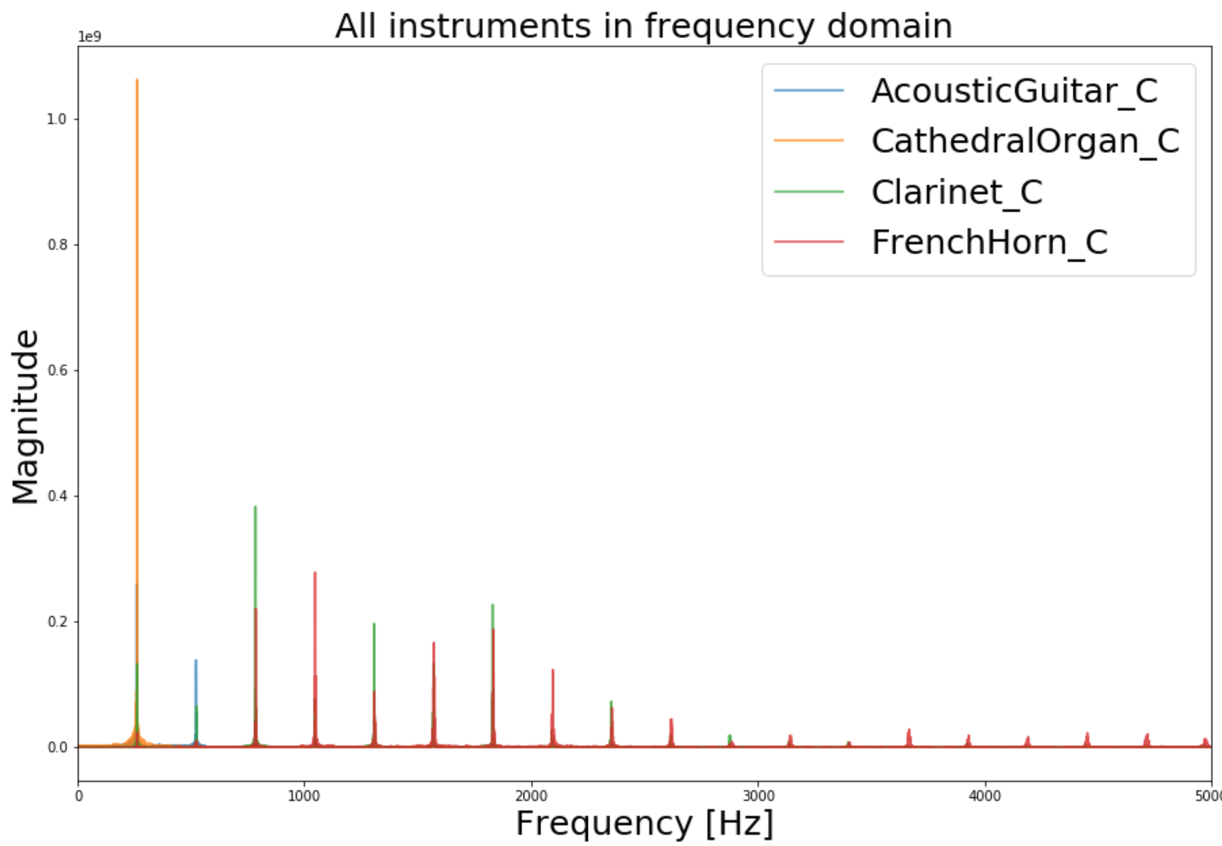


In [9]:
```
plt.figure()
instrument = 'GrandPiano&Clarinet_C'
print(instrument)
fs, data = wavfile.read('sounds/'+instrument[:-2]+'.wav') # load the data
freqs[instrument] = fs

# sample selected = 1 to 2 sec
start = 0*fs
end = 4*fs
mix_samples[instrument] = data.T[0][start:end]
t_length = data.shape[0]
plt.plot(np.linspace(0,t_length/fs, t_length), data.T[0], label = 'Entire r
plt.plot(np.linspace(start/fs, end/fs, (end-start)), mix_samples[instrument
plt.title(instrument + ' in Time domain', fontsize = 15)
plt.xlabel('Time [s]', fontsize = 15)
plt.ylabel('Magnitude', fontsize = 15)
plt.legend(fontsize = 15)
plt.show()
```

```
GrandPiano&Clarinet_C
```

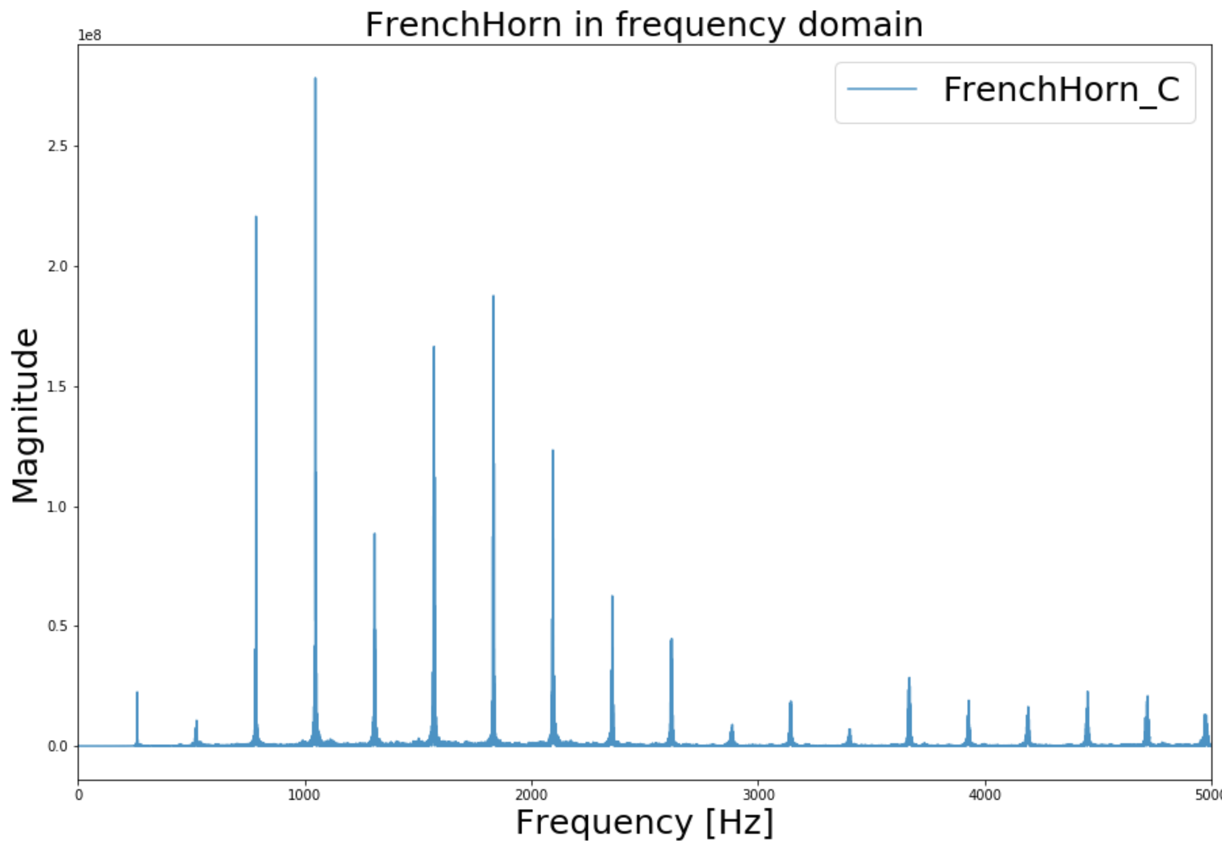


In [10]:
```
plt.figure(figsize = (15,10))

for instrument in instruments[:4]:
    sample = samples[instrument]
    #b=[(ele/2**8.)*2-1 for ele in sample] # this is 8-bit track, b is now
    c = rfft(sample) # calculate fourier transform (complex numbers list)
    fs = freqs[instrument]
    plt.plot(np.linspace(0,fs, len(c)), abs(c), label = instrument, alpha =

plt.title('All instruments in frequency domain', fontsize = 25)
plt.xlabel('Frequency [Hz]', fontsize = 25)
plt.ylabel('Magnitude', fontsize = 25)
plt.legend(fontsize= 25)
plt.xlim(0,5000)
plt.show()
```



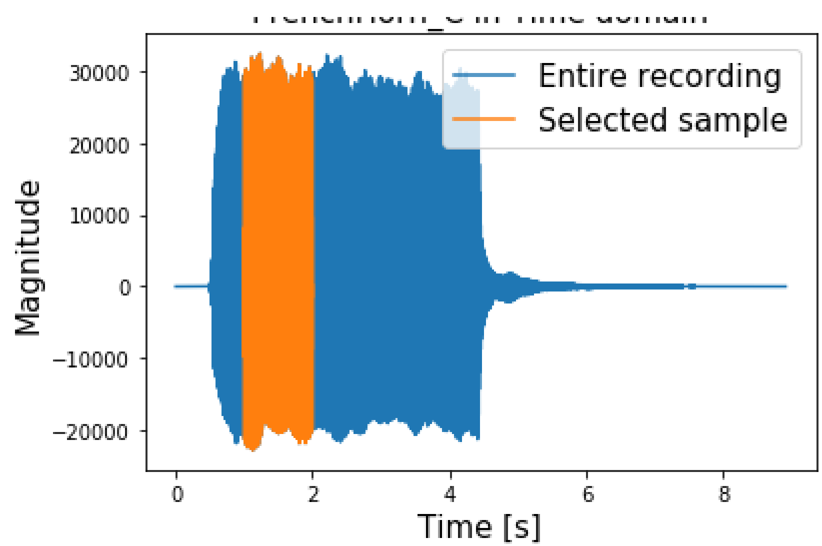
Dive into FrenchHorn

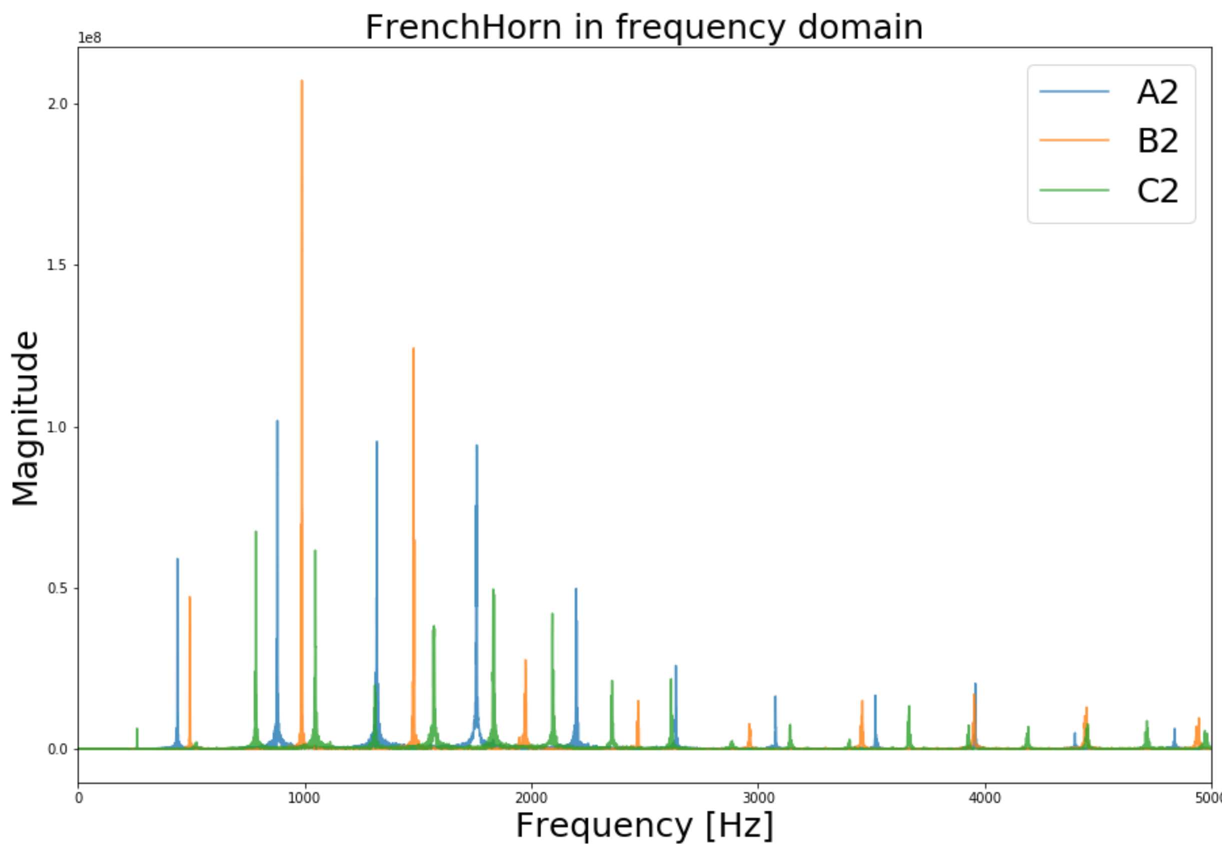
In [11]:
```
plt.figure(figsize = (15,10))
instrument = 'FrenchHorn_C'
sample = samples[instrument]
c = rfft(sample) # calculate fourier transform (complex numbers list)
fs = freqs[instrument]
plt.plot(np.linspace(0,fs, len(c)), abs(c), label = instrument, alpha = 0.8
plt.title('FrenchHorn in frequency domain', fontsize = 25)
plt.xlabel('Frequency [Hz]', fontsize = 25)
plt.ylabel('Magnitude', fontsize = 25)
plt.legend(fontsize= 25)
plt.xlim(0,5000)
plt.show()
```



In [12]:
```python
fs, data = wavfile.read('sounds/FrenchHornOctave/A.wav') # load the data
start = 1*fs
end = 2*fs
t_length = data.shape[0]
A2_sample = data.T[0][start:end]
# plt.plot(np.linspace(0,t_length/fs, t_length), data.T[0], label = 'Entire
# plt.plot(np.linspace(start/fs, end/fs, (end-start)), A2_sample, label = '
# plt.title(instrument + ' in Time domain', fontsize = 15)
# plt.xlabel('Time [s]', fontsize = 15)
# plt.ylabel('Magnitude', fontsize = 15)
# plt.legend(fontsize = 15)
# plt.show()

fs, data = wavfile.read('sounds/FrenchHornOctave/B.wav') # load the data
start = 1*fs
end = 2*fs
t_length = data.shape[0]
B2_sample = data.T[0][start:end]
# plt.figure()
# plt.plot(np.linspace(0,t_length/fs, t_length), data.T[0], label = 'Entire
# plt.plot(np.linspace(start/fs, end/fs, (end-start)), B2_sample, label = '
# plt.title(instrument + ' in Time domain', fontsize = 15)
# plt.xlabel('Time [s]', fontsize = 15)
# plt.ylabel('Magnitude', fontsize = 15)
# plt.legend(fontsize = 15)
# plt.show()

fs, data = wavfile.read('sounds/FrenchHornOctave/C.wav') # load the data
start = 1*fs
end = 2*fs
t_length = data.shape[0]
C2_sample = data.T[0][start:end]
plt.figure()
plt.plot(np.linspace(0,t_length/fs, t_length), data.T[0], label = 'Entire r
plt.plot(np.linspace(start/fs, end/fs, (end-start)), C2_sample, label = 'Se
plt.title(instrument + ' in Time domain', fontsize = 15)
plt.xlabel('Time [s]', fontsize = 15)
plt.ylabel('Magnitude', fontsize = 15)
plt.legend(fontsize = 15)
plt.show()


plt.figure(figsize = (15,10))
c_A2 = rfft(A2_sample) # calculate fourier transform (complex numbers list)
plt.plot(np.linspace(0,fs, len(c_A2)), abs(c_A2), label = 'A2', alpha = 0.8
c_B2 = rfft(B2_sample) # calculate fourier transform (complex numbers list)
plt.plot(np.linspace(0,fs, len(c_B2)), abs(c_B2), label = 'B2', alpha = 0.8
c_C2 = rfft(C2_sample) # calculate fourier transform (complex numbers list)
plt.plot(np.linspace(0,fs, len(c_C2)), abs(c_C2), label = 'C2', alpha = 0.8
plt.title('FrenchHorn in frequency domain', fontsize = 25)
plt.xlabel('Frequency [Hz]', fontsize = 25)
plt.ylabel('Magnitude', fontsize = 25)
plt.legend(fontsize= 25)
plt.xlim(0,5000)
plt.show()
```

FrenchHorn C in Time domain

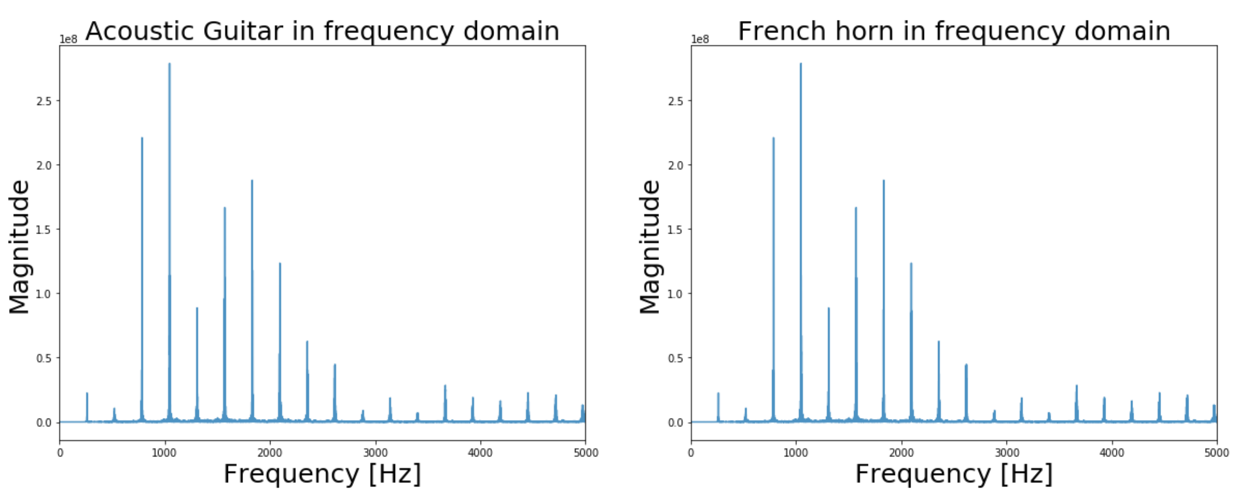

FrenchHorn_C in Time domain
Entire recording
Selected sample
Magnitude
Time [s]

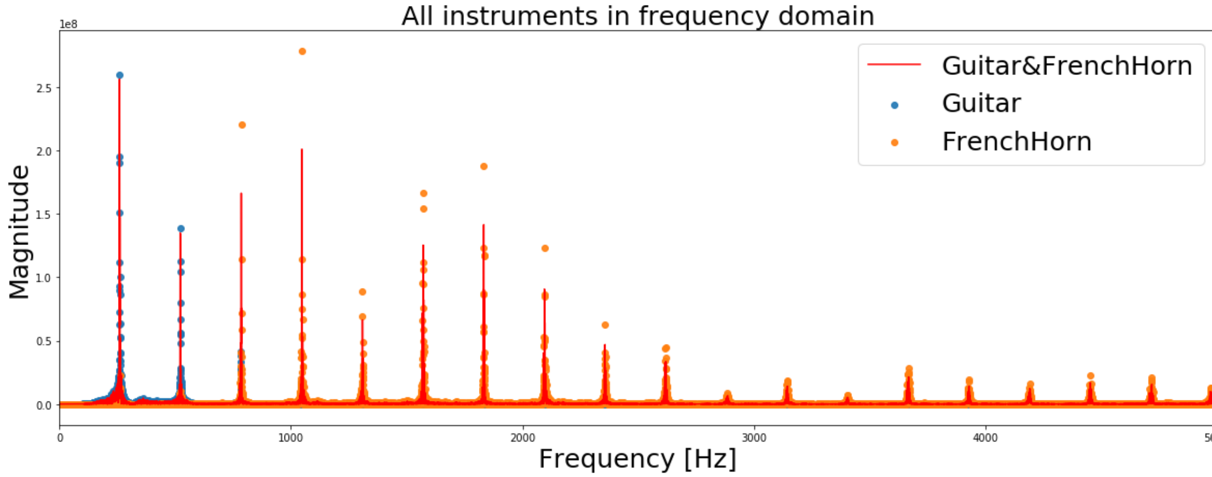

FrenchHorn in frequency domain
1e8
A2
B2
C2
Magnitude
Frequency [Hz]

In [13]:
```
plt.figure(figsize = (7,4))

sample = samples['AcousticGuitar_C']
#b=[(ele/2**8.)*2-1 for ele in sample] # this is 8-bit track, b is now norm
c_guit = rfft(sample) # calculate fourier transform (complex numbers list)
fs = freqs['AcousticGuitar_C']

fig, ax = plt.subplots(1,2, figsize=(20,7))
ax[0].plot(np.linspace(0,fs, len(c)), abs(c), alpha = 0.8)
ax[0].set_title('Acoustic Guitar in frequency domain', fontsize = 25)
ax[0].set_xlabel('Frequency [Hz]', fontsize = 25)
ax[0].set_ylabel('Magnitude', fontsize = 25)
ax[0].set_xlim(0,5000)

sample = samples['FrenchHorn_C']
c_fh = rfft(sample) # calculate fourier transform (complex numbers list)
fs = freqs['FrenchHorn_C']
ax[1].plot(np.linspace(0,fs, len(c)), abs(c), alpha = 0.8)

ax[1].set_title('French horn in frequency domain', fontsize = 25)
ax[1].set_xlabel('Frequency [Hz]', fontsize = 25)
ax[1].set_ylabel('Magnitude', fontsize = 25)
ax[1].set_xlim(0,5000)

fig, ax = plt.subplots(figsize=(20,7))

sample = mix_samples['Guitar&FrenchHorn_C']
c = rfft(sample) # calculate fourier transform (complex numbers list)

ax.plot(np.linspace(0,fs, len(c)), abs(c), c= 'red',label = 'Guitar&FrenchH
ax.scatter(np.linspace(0,fs, len(c)), abs(c_guit), label = 'Guitar',alpha =
ax.scatter(np.linspace(0,fs, len(c)), abs(c_fh), label = 'FrenchHorn',alpha
ax.set_title('All instruments in frequency domain', fontsize = 25)
ax.set_xlabel('Frequency [Hz]', fontsize = 25)
ax.set_ylabel('Magnitude', fontsize = 25)
ax.set_xlim(0,5000)
ax.legend(fontsize= 25)
```

Out[13]:
```
<matplotlib.legend.Legend at 0x2208ae9b5f8>
```

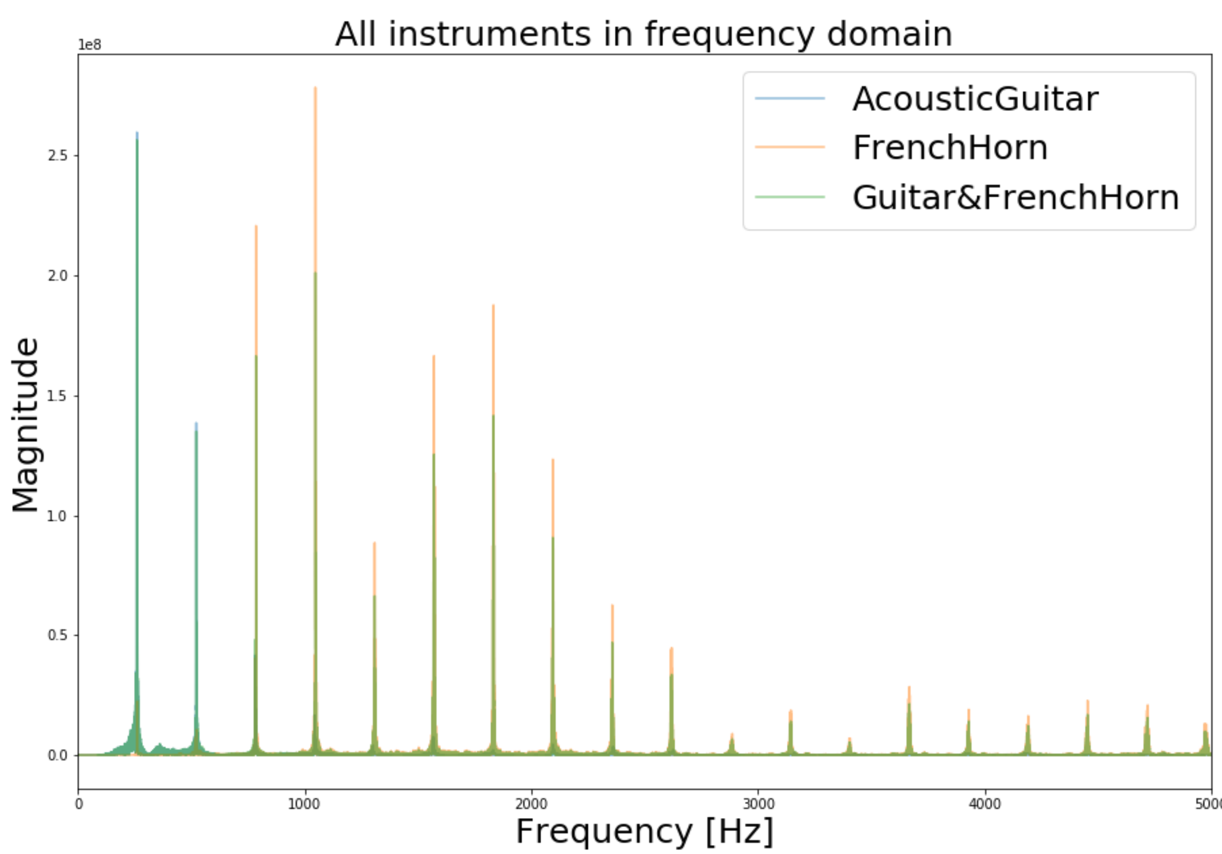
```
<Figure size 504x288 with 0 Axes>
```




All instruments in frequency domain
1e8
Magnitude
Frequency [Hz]
Guitar&FrenchHorn
Guitar
FrenchHorn
0.0
0.5
1.0
1.5
2.0
2.5
0
1000
2000
3000
4000
5000

In [14]:
```
plt.figure(figsize = (15,10))
sample = samples['AcousticGuitar_C']
#b=[(ele/2**8.)*2-1 for ele in sample] # this is 8-bit track, b is now norm
c = rfft(sample) # calculate fourier transform (complex numbers list)
fs = freqs['AcousticGuitar_C']

plt.plot(np.linspace(0,fs, len(c)), abs(c), label = 'AcousticGuitar', alpha

sample = samples['FrenchHorn_C']
#b=[(ele/2**8.)*2-1 for ele in sample] # this is 8-bit track, b is now norm
c = rfft(sample) # calculate fourier transform (complex numbers list)
fs = freqs['FrenchHorn_C']
plt.plot(np.linspace(0,fs, len(c)), abs(c), label = 'FrenchHorn', alpha = 0

sample = mix_samples['Guitar&FrenchHorn_C']
#b=[(ele/2**8.)*2-1 for ele in sample] # this is 8-bit track, b is now norm
c = rfft(sample) # calculate fourier transform (complex numbers list)


plt.plot(np.linspace(0,fs, len(c)), abs(c), label = 'Guitar&FrenchHorn', al

plt.title('All instruments in frequency domain', fontsize = 25)
plt.xlabel('Frequency [Hz]', fontsize = 25)
plt.ylabel('Magnitude', fontsize = 25)
plt.legend(fontsize= 25)
plt.xlim(0,5000)
```

Out[14]: (0, 5000)



2. Peak Finding Model

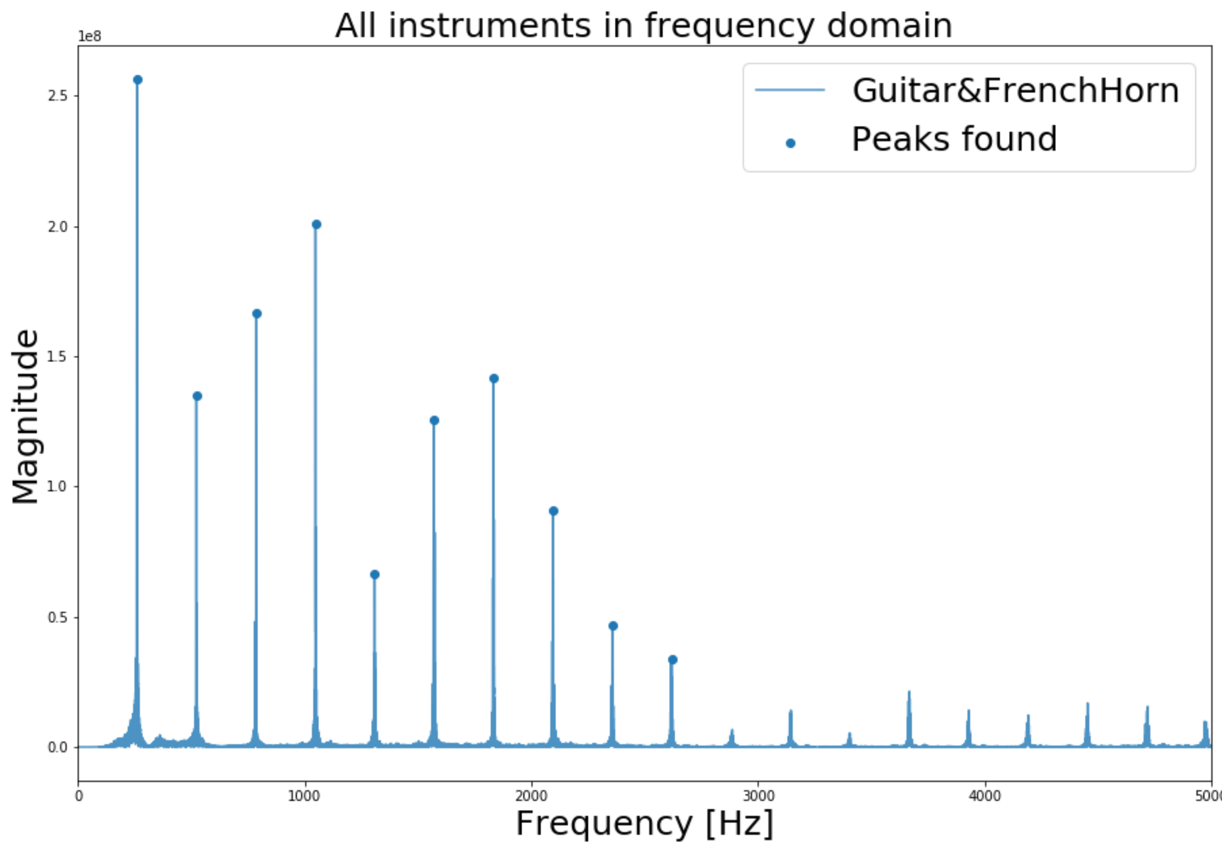
In [15]:
```
def peak_finding(fs, magnitudes, n = 10):
    # find the top freq for each note, lets say higher than 10e7
    threshold = 1e4
    freqs = np.linspace(0,fs, len(magnitudes))
    top_freq = []
    top_magn = []
    for i, magn in enumerate(magnitudes):
        if magn >= threshold:
            freq = freqs[i]
            if len(top_freq) >= 1 and (abs(freq - top_freq[-1])<=30):
                if magn > top_magn[-1]:
                    top_magn[-1] = magn
                    top_freq[-1] = freq
            else:
                top_freq.append(freq)
                top_magn.append(magn)

    idx = np.array(top_magn).argsort()[::-1][:n]
    top_freq_sorted = np.array(top_freq)[idx]
    top_magn_sorted = np.array(top_magn)[idx]

    # convert back to increasing f
    idx = np.array(top_freq_sorted).argsort()[:n]
    top_freq_sorted = np.array(top_freq_sorted)[idx]
    top_magn_sorted = np.array(top_magn_sorted)[idx]

    return top_freq_sorted, top_magn_sorted
```

In [16]:
```
top_freq, top_magn = peak_finding(fs, abs(c))
print(top_freq)
```

```
[ 260.25147535  522.0029592   785.50445297 1047.50593824 1309.25742209
 1571.25890736 1833.01039122 2095.01187648 2356.76336034 2619.01484702]
```

In [17]:
```
plt.figure(figsize = (15,10))
plt.scatter(top_freq, top_magn, label = 'Peaks found')
plt.plot(np.linspace(0,fs, len(c)), abs(c), label = 'Guitar&FrenchHorn', al
plt.title('All instruments in frequency domain', fontsize = 25)
plt.xlabel('Frequency [Hz]', fontsize = 25)
plt.ylabel('Magnitude', fontsize = 25)
plt.legend(fontsize= 25)
plt.xlim(0,5000)
```

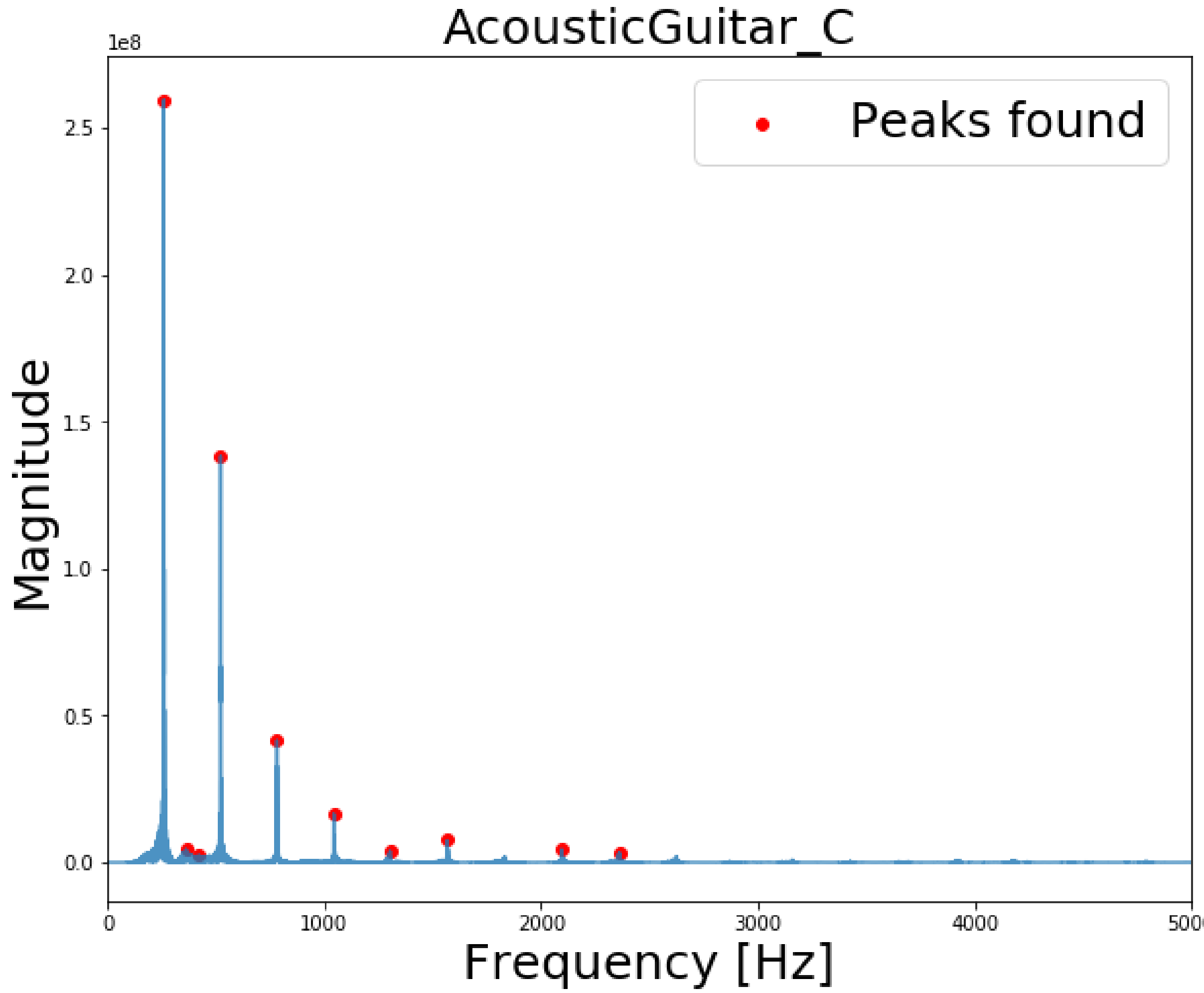
Out[17]:
```
(0, 5000)
```

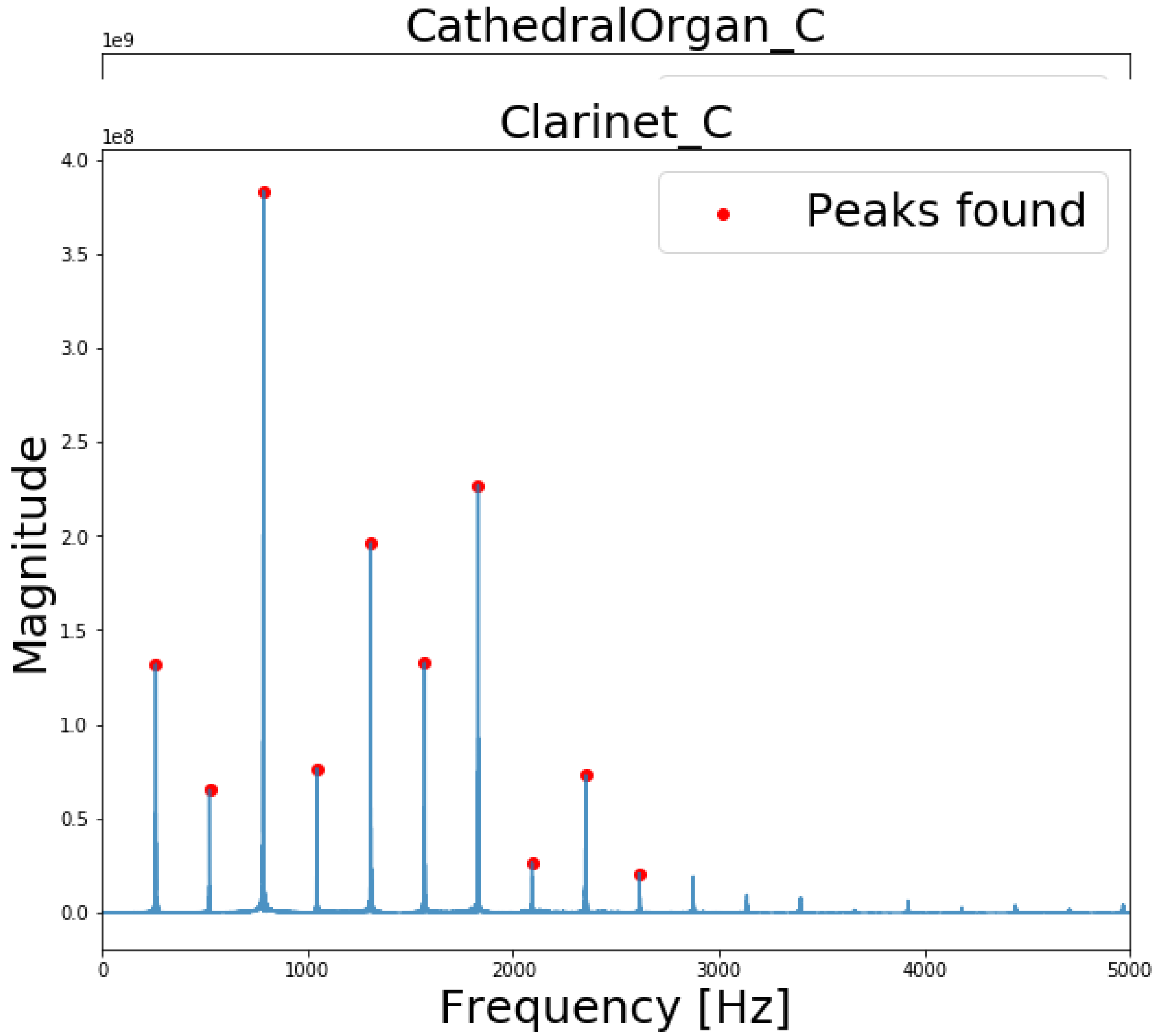


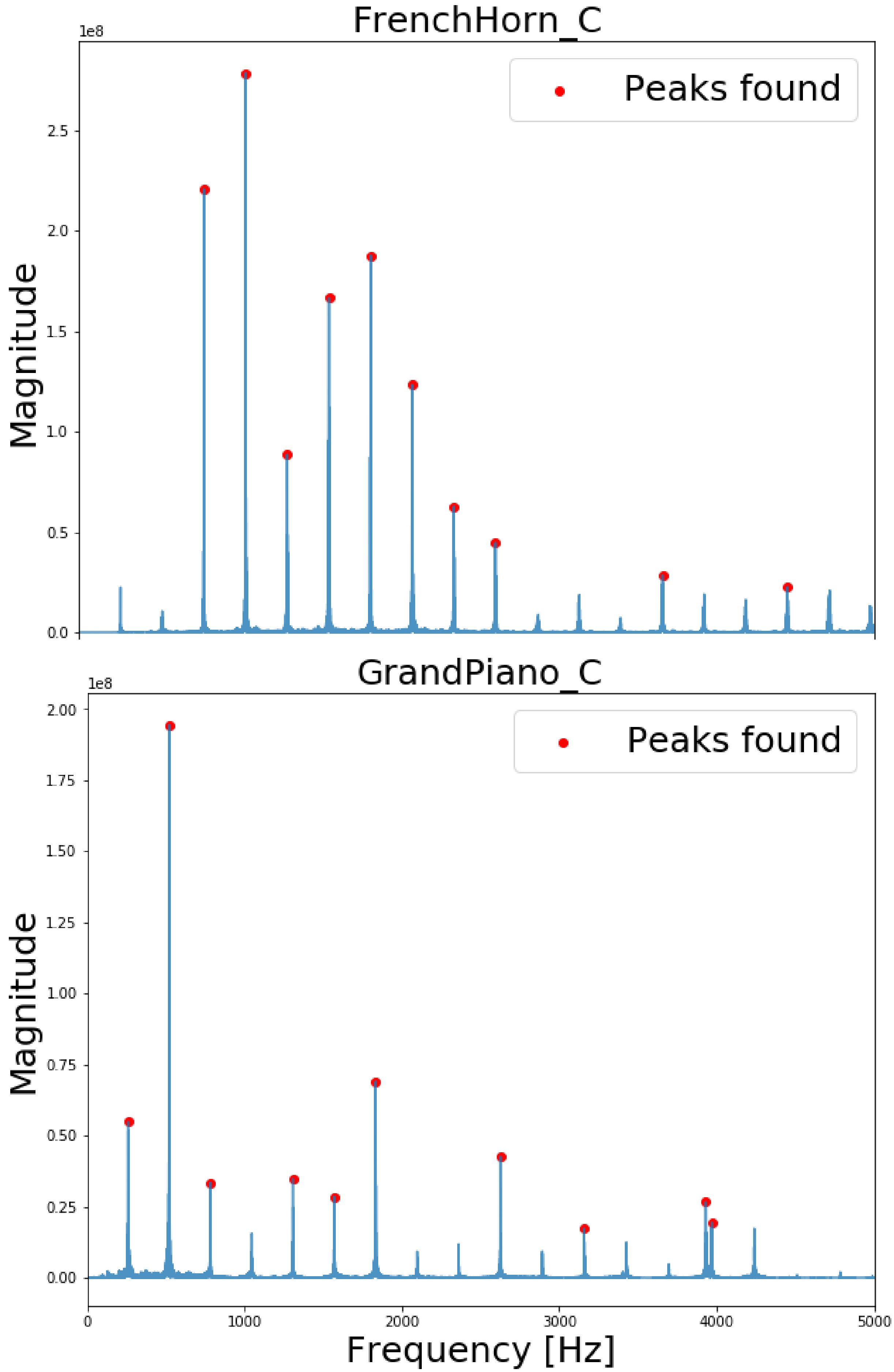
Let's find the peak freqs of all instruments

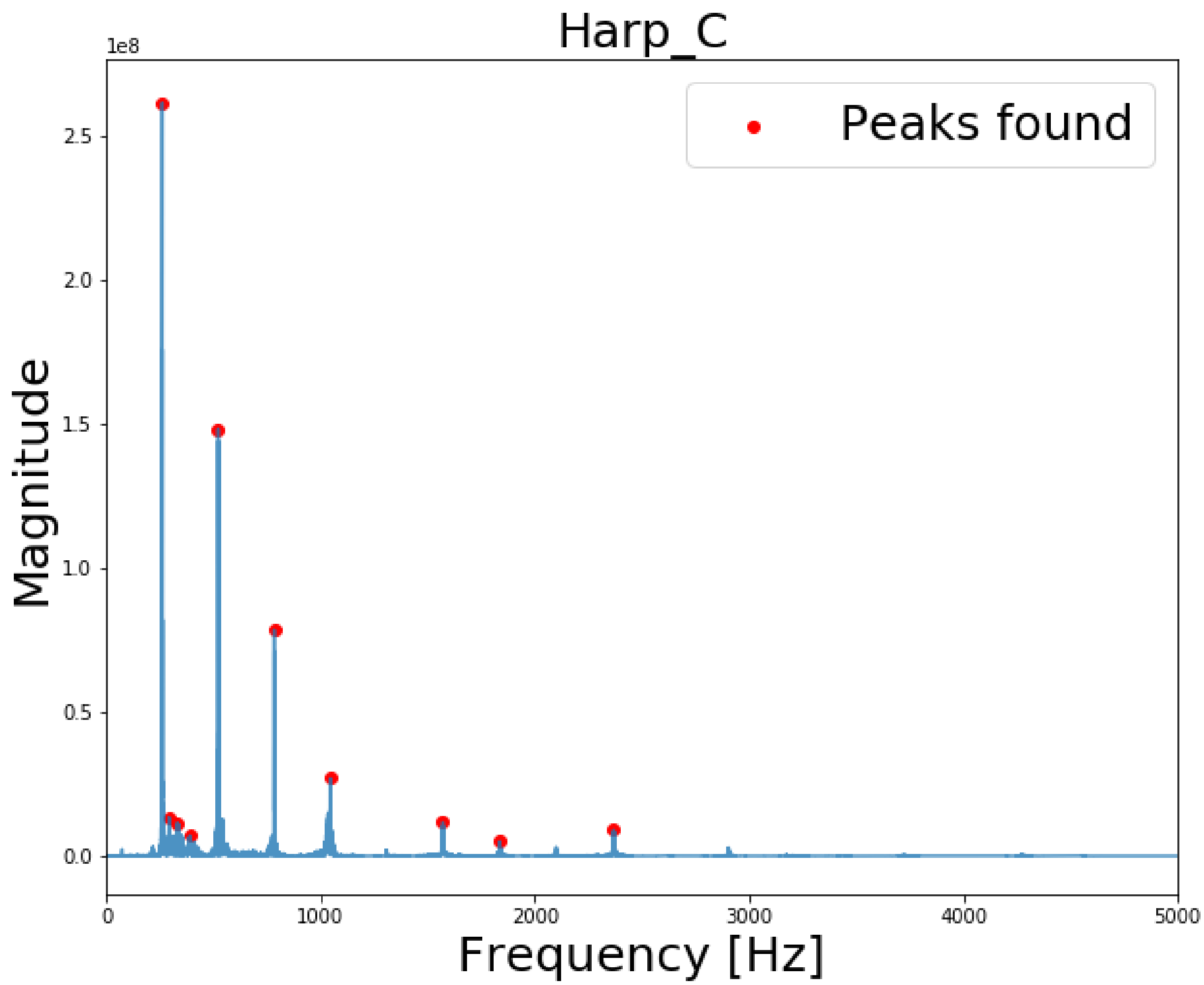
In [19]:

```
dict_peaks = dict()

for instrument in instruments:
    sample = samples[instrument]
    c = rfft(sample) # calculate fourier transform (complex numbers list)
    fs = freqs[instrument]
    plt.figure(figsize = (10,8))
    plt.plot(np.linspace(0,fs, len(c)), abs(c), alpha = 0.8)
    top_freq, top_magn = peak_finding(fs, abs(c))
    dict_peaks[instrument] = top_freq
    plt.scatter(top_freq, top_magn, label = 'Peaks found', c = 'red')
    plt.title(instrument, fontsize = 25)
    plt.xlabel('Frequency [Hz]', fontsize = 25)
    plt.ylabel('Magnitude', fontsize = 25)
    plt.legend(fontsize= 25)
    plt.xlim(0,5000)
    plt.show()

for instrument in mix_samples.keys():
    sample = mix_samples[instrument]
    c = rfft(sample) # calculate fourier transform (complex numbers list)
    plt.figure(figsize = (10,8))
    plt.plot(np.linspace(0,fs, len(c)), abs(c), alpha = 0.8)
    top_freq, top_magn = peak_finding(fs, abs(c))
    dict_peaks[instrument] = top_freq
    plt.scatter(top_freq, top_magn, label = 'Peaks found', c = 'red')
    plt.title(instrument, fontsize = 25)
    plt.xlabel('Frequency [Hz]', fontsize = 25)
    plt.ylabel('Magnitude', fontsize = 25)
    plt.legend(fontsize= 25)
    plt.xlim(0,5000)
    plt.show()
```

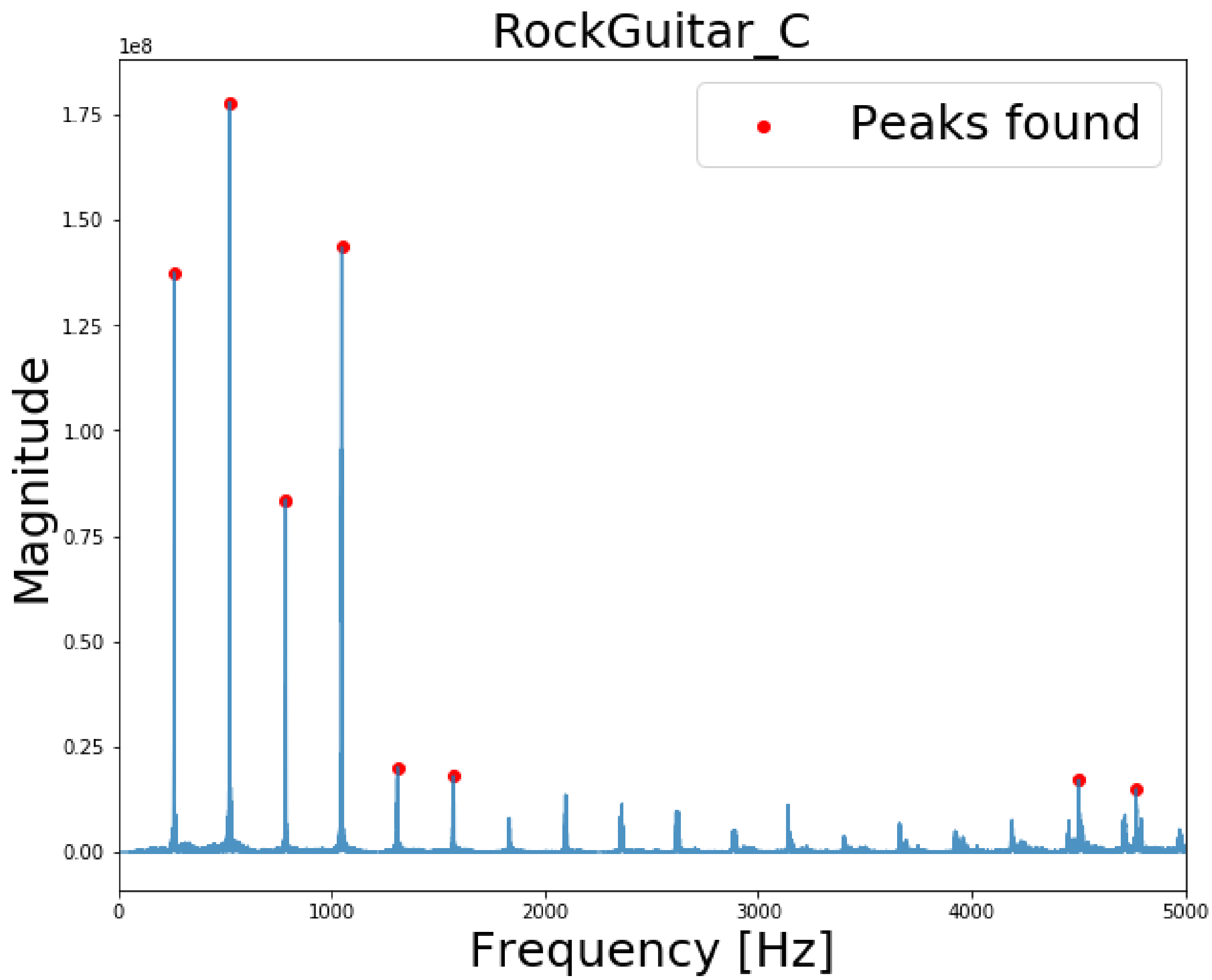

AcousticGuitar_C
1e8
2.5
2.0
1.5
1.0
0.5
0.0
Magnitude
Peaks found
0
1000
2000
3000
4000
5000
Frequency [Hz]


CathedralOrgan_C
1e9
Clarinet_C
1e8
Peaks found
Magnitude
Frequency [Hz]
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0
1000
2000
3000
4000
5000

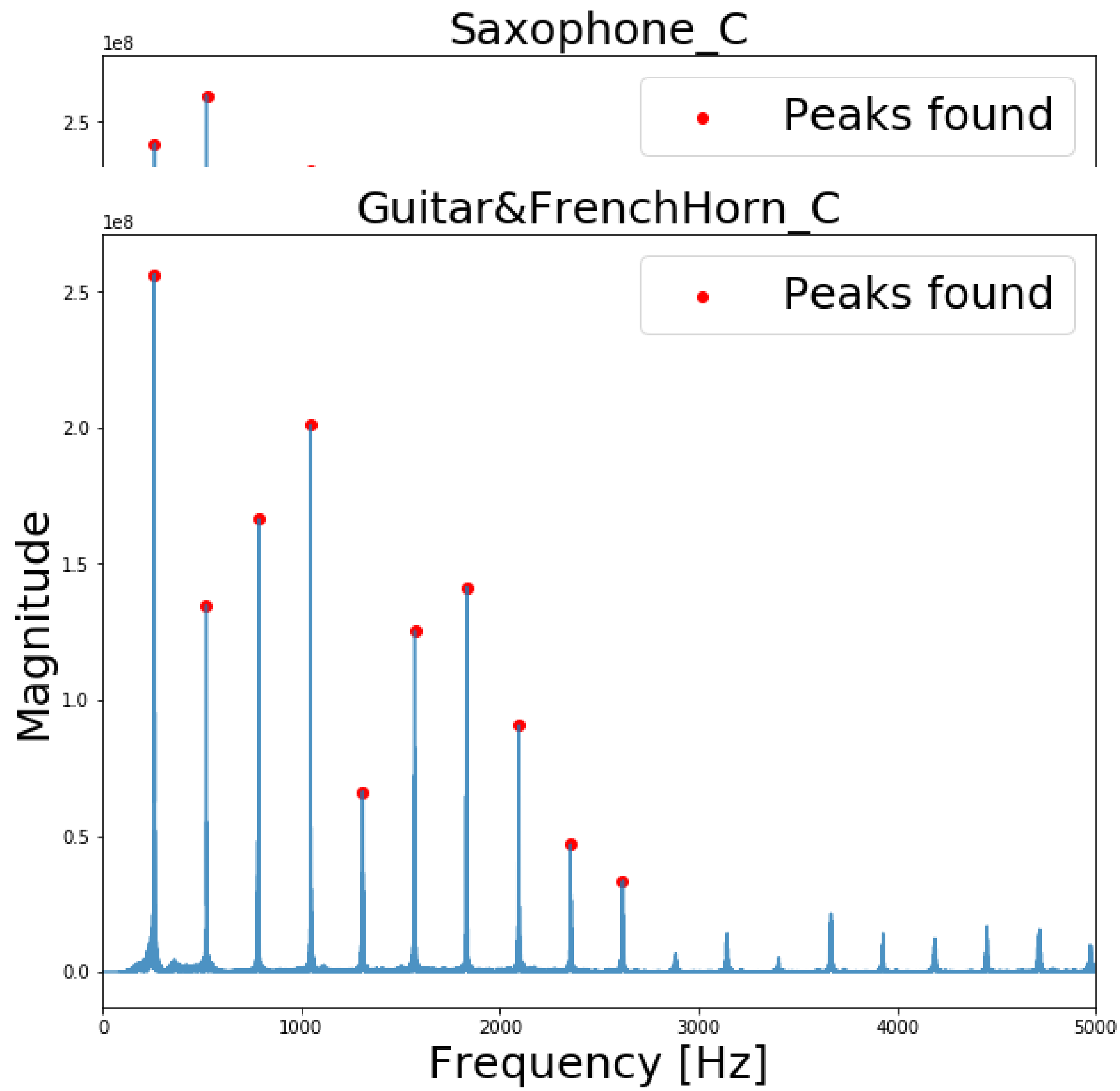

FrenchHorn_C
1e8
Peaks found
Magnitude
0.0
0.5
1.0
1.5
2.0
2.5
GrandPiano_C
1e8
Peaks found
Magnitude
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
0
1000
2000
3000
4000
5000
Frequency [Hz]

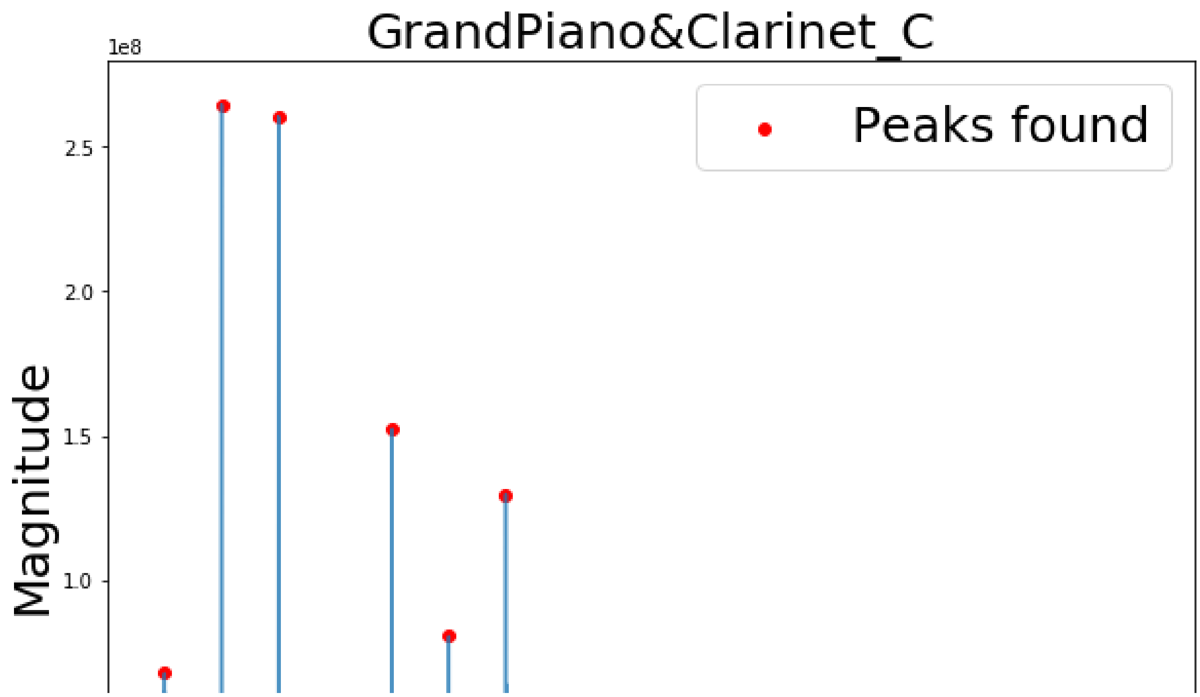


Harp_C
1e8
2.5
2.0
1.5
1.0
0.5
0.0
Magnitude
Peaks found
0
1000
2000
3000
4000
5000
Frequency [Hz]



RockGuitar_C
1e8
Peaks found
Magnitude
Frequency [Hz]
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
0
1000
2000
3000
4000
5000


Saxophone_C
1e8
2.5
Peaks found
Guitar&FrenchHorn_C
1e8
2.5
2.0
1.5
1.0
0.5
0.0
Magnitude
Peaks found
0
1000
2000
3000
4000
5000
Frequency [Hz]



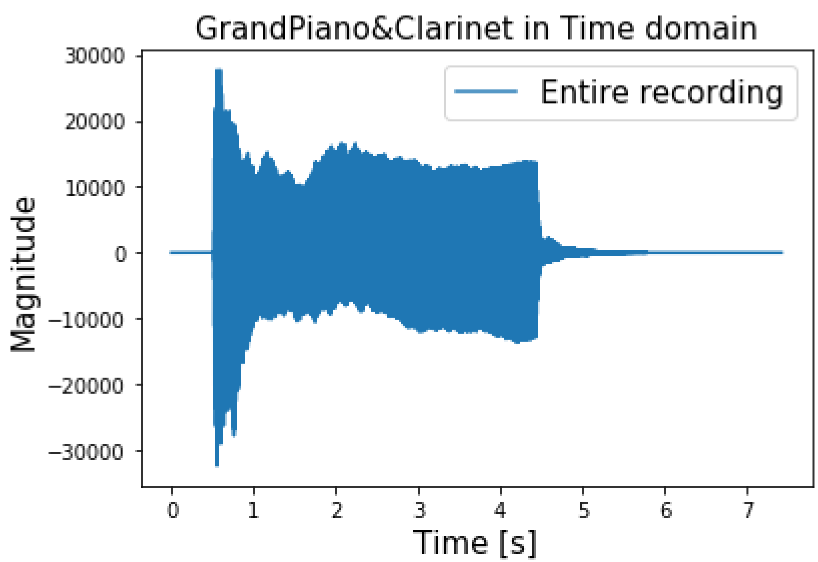
In [20]:

```
# let's now find the peaks of the combo and loop through the instruments
peaks_combo = dict_peaks['Guitar&FrenchHorn_C']
for instrument in instruments:
    peaks = dict_peaks[instrument]
    lst_ones_zeros = np.zeros(len(peaks))
    for i, peak_i in enumerate(peaks):
        for peak_c in peaks_combo:
            if abs(peak_c - peak_i) <= 1:
                lst_ones_zeros[i] = 1
    print('For {}, we found {} peak frequencies in the combo'.format(instru
```

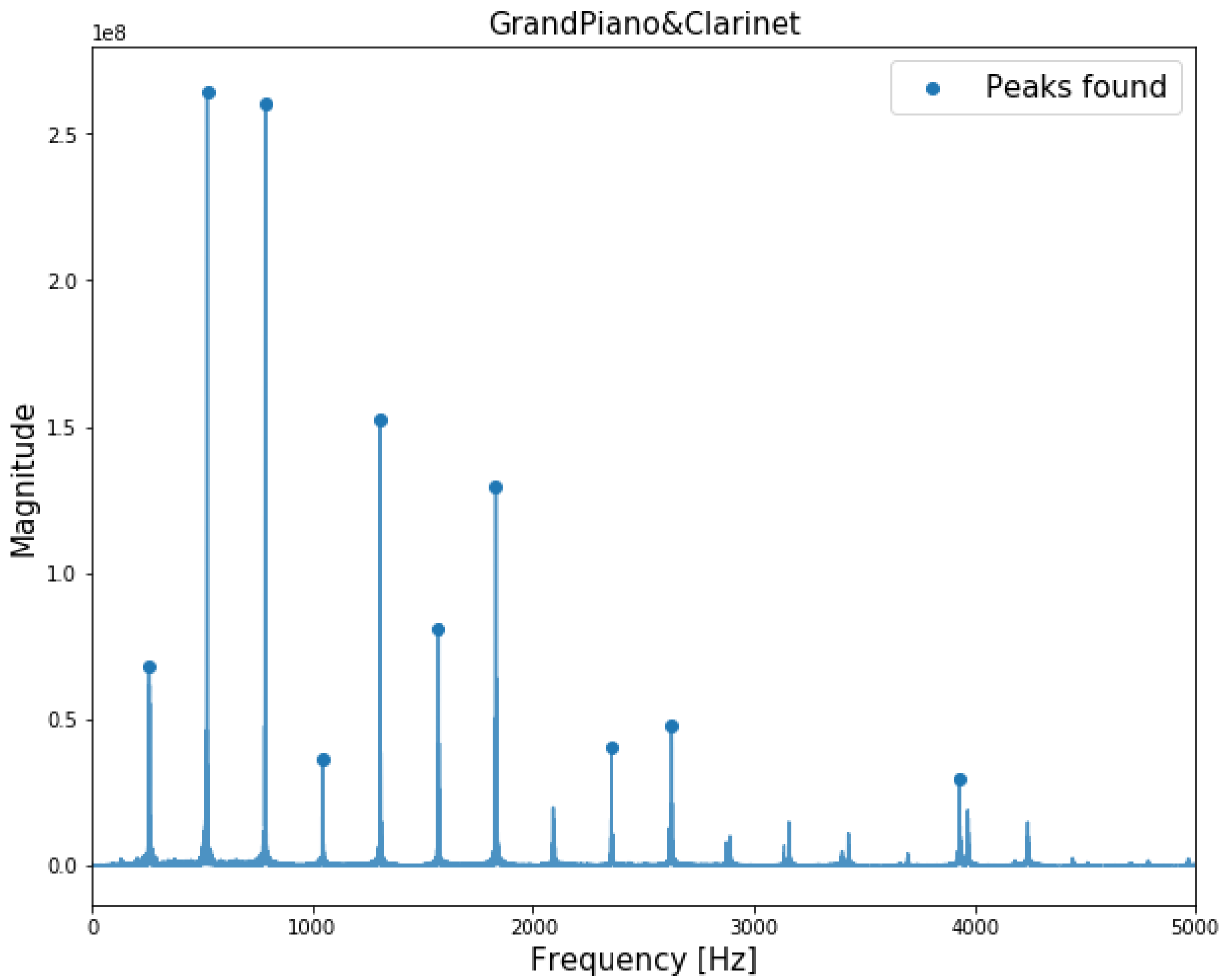
```
For AcousticGuitar_C, we found 3.0 peak frequencies in the combo
For CathedralOrgan_C, we found 0.0 peak frequencies in the combo
For Clarinet_C, we found 0.0 peak frequencies in the combo
For FrenchHorn_C, we found 8.0 peak frequencies in the combo
For GrandPiano_C, we found 2.0 peak frequencies in the combo
For Harp_C, we found 2.0 peak frequencies in the combo
For RockGuitar_C, we found 1.0 peak frequencies in the combo
For Saxophone_C, we found 2.0 peak frequencies in the combo
```

In [23]:
```
plt.figure()
instrument = 'GrandPiano&Clarinet'
print(instrument)
fs, data = wavfile.read('sounds/'+instrument+'.wav') # load the data
freqs[instrument] = fs

# sample selected = 1 to 2 sec
start = 0*fs
end = 4*fs
samples[instrument] = data.T[0][start:end]
t_length = data.shape[0]
plt.plot(np.linspace(0,t_length/fs, t_length), data.T[0], label = 'Entire r
#plt.plot(np.linspace(start/fs, end/fs, (end-start)), samples[instrument],
plt.title(instrument + ' in Time domain', fontsize = 15)
plt.xlabel('Time [s]', fontsize = 15)
plt.ylabel('Magnitude', fontsize = 15)
plt.legend(fontsize = 15)
plt.show()
```

```
GrandPiano&Clarinet
```



In [24]:
```
c = rfft(sample) # calculate fourier transform (complex numbers list)
fs = freqs[instrument]
plt.figure(figsize = (10,8))
plt.plot(np.linspace(0,fs, len(c)), abs(c), alpha = 0.8)
top_freq, top_magn = peak_finding(fs, abs(c))
dict_peaks[instrument] = top_freq
plt.scatter(top_freq, top_magn, label = 'Peaks found')
plt.title(instrument, fontsize = 15)
plt.xlabel('Frequency [Hz]', fontsize = 15)
plt.ylabel('Magnitude', fontsize = 15)
plt.legend(fontsize= 15)
plt.xlim(0,5000)
plt.show()
```



In [25]:
```
# let's now find the peaks of the combo and loop through the instruments
peaks_combo = dict_peaks['GrandPiano&Clarinet']
for instrument in instruments[:]:
    peaks = dict_peaks[instrument]
    lst_ones_zeros = np.zeros(len(peaks))
    for i, peak_i in enumerate(peaks):
        for peak_c in peaks_combo:
            if abs(peak_c - peak_i) <= 1:
                lst_ones_zeros[i] = 1
    print('For {}, we found {} peak frequencies in the combo'.format(instru
```

```
For AcousticGuitar_C, we found 1.0 peak frequencies in the combo
For CathedralOrgan_C, we found 1.0 peak frequencies in the combo
For Clarinet_C, we found 8.0 peak frequencies in the combo
For FrenchHorn_C, we found 0.0 peak frequencies in the combo
For GrandPiano_C, we found 6.0 peak frequencies in the combo
For Harp_C, we found 2.0 peak frequencies in the combo
For RockGuitar_C, we found 2.0 peak frequencies in the combo
For Saxophone_C, we found 6.0 peak frequencies in the combo
```

3. Linear least squares Model

We now want to formulate the problem as a linear least square problem:

$$Ax = b$$

and

$$x = argmin_x(||Ax - b||^2)$$

In [26]:
```
print(instruments)
```

```
['AcousticGuitar_C', 'CathedralOrgan_C', 'Clarinet_C', 'FrenchHorn_C', 'G
randPiano_C', 'Harp_C', 'RockGuitar_C', 'Saxophone_C']
```

In [35]:
```
# compute A
A = np.zeros((len(c), len(instruments)))
for i, instrument in enumerate(instruments):
    sample = samples[instrument]
    c = rfft(sample)
    fs = freqs[instrument]
    A[:,i] = abs(c)

print('A has shape: {}'.format(A.shape))
```

```
A has shape: (176400, 8)
```

In [36]:
```
# compute b
sample = samples['GrandPiano&Clarinet']
c = rfft(sample) # calculate fourier transform (complex numbers list)
fs = freqs[instrument]
b = abs(c)
print('b has shape: {}'.format(b.shape))
```

```
b has shape: (176400,)
```

In [39]:
```
# solve the system
x = np.linalg.lstsq(A,b, rcond = None)[0]
print(x)
```

```
[ 0.0174321  -0.0209994   0.49020096 -0.04278293  0.95801986 -0.01027909
 -0.00741543 -0.01227416]
```

In [40]:
```
# let's see if it predicts correctly
print(instruments[2])
print(instruments[4])
```

```
Clarinet_C
GrandPiano_C
```

In [42]:
```
# let's try another sample
sample = mix_samples['Guitar&FrenchHorn_C']
c = rfft(sample) # calculate fourier transform (complex numbers list)
fs = freqs[instrument]
b = abs(c)
x = np.linalg.lstsq(A,b, rcond = None)[0]
print(x)
```

```
[ 0.96862827  0.01268244 -0.008542    0.71469271  0.02168073 -0.03536389
 -0.03074802  0.00145718]
```

In [43]:
```
print(instruments[0])
print(instruments[3])
```

```
AcousticGuitar_C
FrenchHorn_C
```

In [44]:
```python
# let's formalize this approach
def lls_instruments(mix, instruments, n_instr, return_coef = False):
    '''
    Input:
    - mix : sound sample
    - instruments : dictionary of the available data of instruments
    - n_instr : number of instruments expected
                when None, the algorithm will come up with its best guess
    '''
    # convert sound sampel to freq domain
    c_mix = rfft(mix)
    b = abs(c_mix)

    # compose A
    A = np.zeros((len(c_mix), len(instruments)))
    instr_list = []
    for i, instrument in enumerate(instruments):
        instr_list.append(instrument)
        sample = samples[instrument]
        c = rfft(sample)
        A[:,i] = abs(c)

    x = np.linalg.lstsq(A,b,rcond=None)[0]

    # sort from large to small
    idx = x.argsort()[::-1]
    coef_sorted = x[idx]
    instr_sorted = np.array(instr_list)[idx]

    if return_coef == True:
        return instr_sorted, coef_sorted
    else:
        return instr_sorted[:n_instr]
```

In [46]:
```python
print(lls_instruments(mix_samples['Guitar&FrenchHorn_C'], instruments, n_in
print(lls_instruments(mix_samples['GrandPiano&Clarinet_C'], instruments, n_
```

```
['AcousticGuitar_C' 'FrenchHorn_C']
['GrandPiano_C' 'Clarinet_C']
```

4. Sparse Regression Model

If we know that not all features play a role in the model, we can use the mathematics of sparse regression to drive specific coefficients - of non relevant information - to zero. Recall we can write the linear least square problem as:

$$Ax = b$$

and

$$x = argmin_x(||Ax - b||^2)$$

For this equation, a closed form solution named the 'normal equations' was available to compute the best x:

$$\hat{x} = (AA^T)^{-1}A^T b$$

From literature we know that l_1 regularization can drive the non-relevant coefficients to zero. Or mathematically, the equation now becomes:

$$x = argmin_x(||Ax - b||^2 + \lambda||x||_1)$$

The inclusion of $\lambda||x||_1$ in the expression to be minimized leads to a penalization of large coefficient values. The larger λ, the harder this penalization and thus the more the not-contributing feature coefficients will be driven towards zero. Note that this expression is not straight-froward anymore to optimize, as the l_1 norm makes it not differentiable.

The expression can be rewritten as:

$$x = argmin_x(x^T x - 2x^T A^T b + b^T b + \lambda||x||_1)$$

The solution of this corresponds to the one x for which the gradient of the expression between brackets is equal to zero. The gradient of $||x||_1$ is defined at all points except for $x_i = 0$. Therefore, the partial derivatives with respect to x_i can be written as:

$$\begin{pmatrix} 2x_i - 2A_i^T b + \lambda <=> x_i > 0 \\ [-\lambda, \lambda] - 2A_i^T b <=> x_i = 0 \\ 2x_i - 2A_i^T b - \lambda <=> x_i < 0 \end{pmatrix}$$

Note that $[-\lambda, \lambda]$ means that the value can be any value inside the interval. Setting this partial derivative equal to zero, while keeping in mind the constraint on x_i, we get the following expression for all x_i values:

$$\begin{pmatrix} 0 <=> \lambda > 2|A_i^T b| \\ A_i^T b - sign(A_i^T b)\frac{\lambda}{2} <=> \lambda \leq 2|A_i^T b| \end{pmatrix}$$

Intuitively, this makes sense. The value $A_i^T b$ is related with the correlation of feature i and response variable b. If this correlation is below a certain threshold λ, its coefficient is set to 0. This sparse regression can now be implemented for our instrument case.

In [47]:
```
from sklearn.linear_model import LassoCV

# let's formalize this
def lasso_instruments_skl(mix, instruments, n_instr, return_coef = False):
    '''
    Input:
    - mix : sound sample
    - instruments : dictionary of the available data of instruments
    - n_instr : number of instruments expected
                when None, the algorithm will come up with its best guess
    - lam_list : list of regularization params
    '''
    # convert sound sampel to freq domain
    c_mix = rfft(mix)
    b = abs(c_mix)

    # compose A
    A = np.zeros((len(c_mix), len(instruments)))
    instr_list = []
    for i, instrument in enumerate(instruments):
        instr_list.append(instrument)
        sample = samples[instrument]
        c = rfft(sample)
        A[:,i] = abs(c)

    # run through lambda values until only n_instr
    model = LassoCV(cv = 3)
    model.fit(A,b)
    coefs = model.coef_
    idx = np.array(coefs).argsort()[::-1]
    coef_sorted = np.array(coefs)[idx]
    instr_sorted = np.array(instr_list)[idx]

    if return_coef == True:
        return instr_sorted, coef_sorted
    else:
        return coef_sorted[:n_instr], instr_sorted[:n_instr]
```

In [48]:
```
print(lasso_instruments_skl(mix_samples['Guitar&FrenchHorn_C'], instruments
print(lasso_instruments_skl(mix_samples['GrandPiano&Clarinet_C'], instrumen
```

```
(array([0.8530474 , 0.64991455]), array(['AcousticGuitar_C', 'FrenchHorn_
C'], dtype='<U16'))
(array([0.78664833, 0.45110001]), array(['GrandPiano_C', 'Clarinet_C'], d
type='<U16'))
```

Add more notes for piano

In [49]:
```
notes = ['A', 'Asharp','B', 'C','Csharp','D', 'Dsharp','E','F','Fsharp','G'
```

In [51]:
```
samples = dict()
instr_c = ['AcousticGuitar_C', 'CathedralOrgan_C', 'Clarinet_C', 'FrenchHor
           'RockGuitar_C', 'Saxophone_C']
for instrument in instr_c:
    fs, data = wavfile.read('sounds/'+instrument[:-2]+'.wav') # load the da
    start = 0*fs
    end = 4*fs
    samples[instrument] = data.T[0][start:end]

for note in notes:
    fs, data = wavfile.read('sounds/PianoOctave/'+note+'.wav') # load the d
    start = 0*fs
    end = 4*fs
    samples['GrandPiano_'+ note] = data.T[0][start:end]
    fs, data = wavfile.read('sounds/AcousticGuitarOctave/'+note+'.wav') # l
    samples['AcousticGuitar_'+ note] = data.T[0][start:end]
    fs, data = wavfile.read('sounds/FrenchHornOctave/'+note+'.wav') # load
    samples['FrenchHorn_'+ note] = data.T[0][start:end]
```

```
C:\Users\Matthieu\Pictures\Anaconda3_2\lib\site-packages\scipy\io\wavfil
e.py:273: WavFileWarning: Chunk (non-data) not understood, skipping it.
  WavFileWarning)
```

In [52]:
```
fs, data = wavfile.read('sounds/GuitarFrenchHornPiano'+'.wav')
mix_samples['GuitarFrenchHornPiano'] = data.T[0][start:end]
fs, data = wavfile.read('sounds/AcousticGuitarDFrenchHornFSharpGrandPianoA.
mix_samples['AcousticGuitarDFrenchHornFSharpGrandPianoA'] = data.T[0][start
fs, data = wavfile.read('sounds/Youtube/Mary-FH-MjpAvGl_xM0.wav')
mix_samples['Mary_FrenchHorn'] = data[start:end]
fs, data = wavfile.read('sounds/Youtube/Mary-cD6sq7TjZAw.wav')
mix_samples['Mary_piano'] = data.T[0][start:end]
fs, data = wavfile.read('sounds/2Guitar2FrenchHorn2GrandPiano.wav')
mix_samples['Mixof6'] = data.T[0][start:end]
mix_samples.keys()
```

Out[52]:
```
dict_keys(['Guitar&FrenchHorn_C', 'GrandPiano&Clarinet_C', 'GuitarFrenchH
ornPiano', 'AcousticGuitarDFrenchHornFSharpGrandPianoA', 'Mary_FrenchHor
n', 'Mary_piano', 'Mixof6'])
```

In [53]:
```
samples.keys()
```

Out[53]:
```
dict_keys(['AcousticGuitar_C', 'CathedralOrgan_C', 'Clarinet_C', 'FrenchH
orn_C', 'GrandPiano_C', 'Harp_C', 'RockGuitar_C', 'Saxophone_C', 'GrandPi
ano_A', 'AcousticGuitar_A', 'FrenchHorn_A', 'GrandPiano_Asharp', 'Acousti
cGuitar_Asharp', 'FrenchHorn_Asharp', 'GrandPiano_B', 'AcousticGuitar_B',
'FrenchHorn_B', 'GrandPiano_Csharp', 'AcousticGuitar_Csharp', 'FrenchHorn
_Csharp', 'GrandPiano_D', 'AcousticGuitar_D', 'FrenchHorn_D', 'GrandPiano
_Dsharp', 'AcousticGuitar_Dsharp', 'FrenchHorn_Dsharp', 'GrandPiano_E',
'AcousticGuitar_E', 'FrenchHorn_E', 'GrandPiano_F', 'AcousticGuitar_F',
'FrenchHorn_F', 'GrandPiano_Fsharp', 'AcousticGuitar_Fsharp', 'FrenchHorn
_Fsharp', 'GrandPiano_G', 'AcousticGuitar_G', 'FrenchHorn_G', 'GrandPiano
_Gsharp', 'AcousticGuitar_Gsharp', 'FrenchHorn_Gsharp'])
```

In [54]:
```
print(lls_instruments(mix_samples['Guitar&FrenchHorn_C'], samples.keys(), n
print(lasso_instruments_skl(mix_samples['Guitar&FrenchHorn_C'], samples.key
```

```
['AcousticGuitar_C' 'FrenchHorn_C']
(array([0.74302396, 0.54665561]), array(['AcousticGuitar_C', 'FrenchHorn_
C'], dtype='<U21'))
```

In [55]:
```
print(lls_instruments(mix_samples['GrandPiano&Clarinet_C'], samples.keys(),
print(lasso_instruments_skl(mix_samples['GrandPiano&Clarinet_C'], samples.k
```

```
['Clarinet_C' 'GrandPiano_C']
(array([0.5169564 , 0.22847113]), array(['Clarinet_C', 'GrandPiano_C'], d
type='<U21'))
```

In [56]:
```
print(lls_instruments(mix_samples['GuitarFrenchHornPiano'], samples.keys(),
print(lasso_instruments_skl(mix_samples['GuitarFrenchHornPiano'], samples.k
```

```
['GrandPiano_G' 'AcousticGuitar_Csharp' 'FrenchHorn_E']
(array([0.46713801, 0.42970508, 0.36608022]), array(['GrandPiano_G', 'Fre
nchHorn_E', 'AcousticGuitar_Csharp'],
      dtype='<U21'))
```

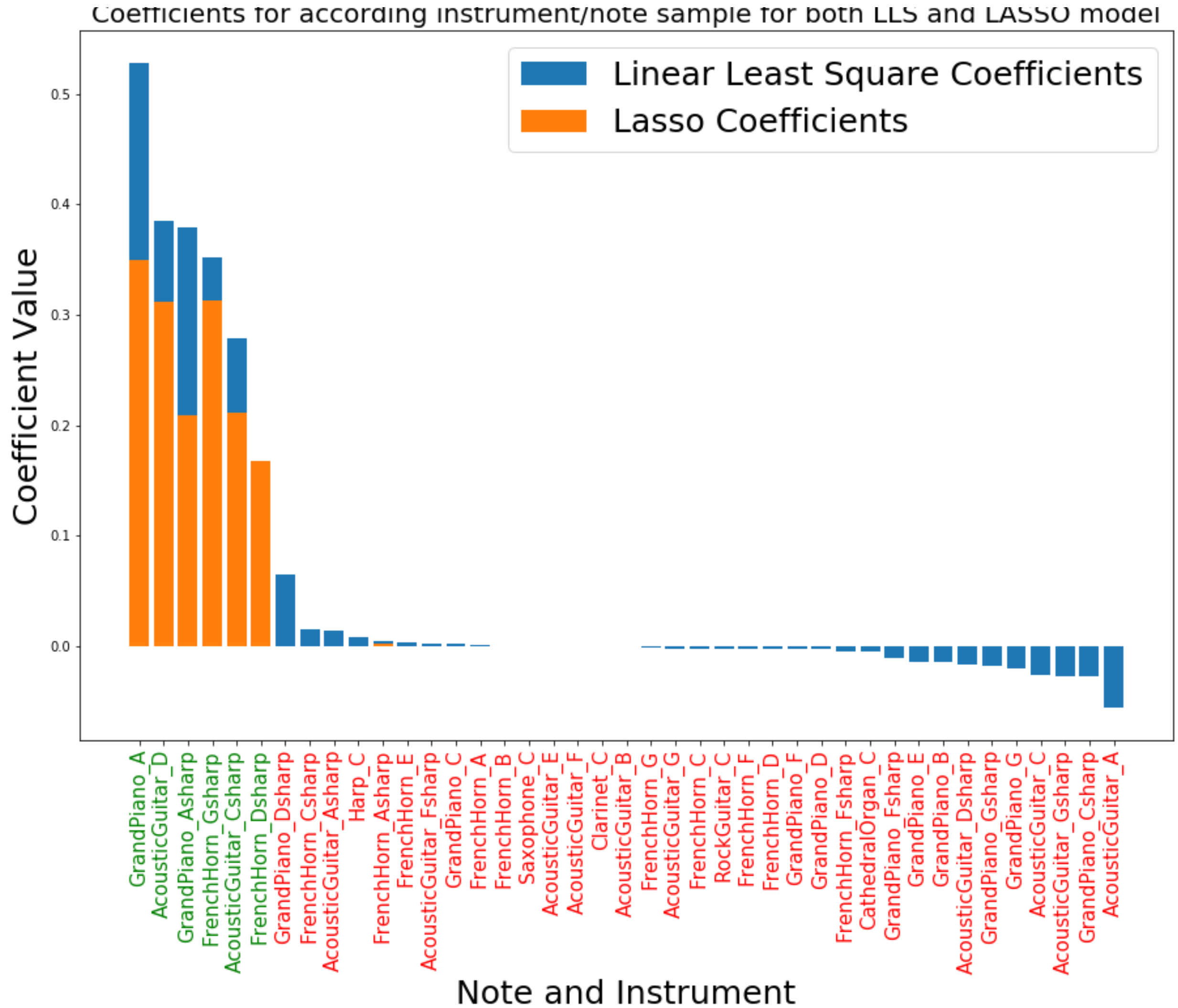
In [57]:
```
print(lls_instruments(mix_samples['AcousticGuitarDFrenchHornFSharpGrandPian
print(lasso_instruments_skl(mix_samples['AcousticGuitarDFrenchHornFSharpGra
```

```
['GrandPiano_A' 'AcousticGuitar_D' 'FrenchHorn_Fsharp']
(array([0.61894003, 0.47049882, 0.27581609]), array(['GrandPiano_A', 'Aco
usticGuitar_D', 'FrenchHorn_Fsharp'],
      dtype='<U21'))
```

In [58]:
```
instr_sorted_ll, coef_sorted_ll = lls_instruments(mix_samples['Mixof6'], sa
instr_sorted_lasso, coef_sorted_lasso = lasso_instruments_skl(mix_samples['
```

In [59]:

```python
# create one nice plot of the coefficients
# make dict of coefs
coef_ll = dict()
coef_lasso = dict()
for i, instr_ll in enumerate(instr_sorted_ll):
    coef_ll[instr_ll] = coef_sorted_ll[i]

for i, instr_lasso in enumerate(instr_sorted_lasso):
    coef_lasso[instr_lasso] = coef_sorted_lasso[i]

# get the colors
my_colors = []
for i in range(len(instr_sorted_lasso)):
    if i < 6:
        my_colors.append('g')
    else:
        my_colors.append('r')

plt.figure(figsize = (15,10))
instrs = coef_ll.keys()
coefs_ll_list = [coef_ll[k] for k in instrs]
coefs_lasso_list = [coef_lasso[k] for k in instrs]
plt.bar(instrs, coefs_ll_list, label = 'Linear Least Square Coefficients')
plt.bar(instrs, coefs_lasso_list, label = 'Lasso Coefficients')
plt.xticks(list(instrs),list(instrs), rotation='vertical', fontsize = 15)
plt.xlabel('Note and Instrument', fontsize = 25)
plt.ylabel('Coefficient Value', fontsize = 25)
plt.title('Coefficients for according instrument/note sample for both LLS a
plt.legend(fontsize = 25)
for ticklabel, tickcolor in zip(plt.gca().get_xticklabels(), my_colors):
    ticklabel.set_color(tickcolor)
plt.show()
```



5. Testing to real world data

In [60]:
```
# real world
# we expect French Horn C, D and E
print(lls_instruments(mix_samples['Mary_FrenchHorn'], samples.keys(), n_ins
print(lasso_instruments_skl(mix_samples['Mary_FrenchHorn'], samples.keys(),
```

```
['GrandPiano_D' 'FrenchHorn_D' 'GrandPiano_G' 'GrandPiano_C']
(array([0.34626176, 0.32173365, 0.24069142, 0.19240348]), array(['GrandPi
ano_D', 'FrenchHorn_D', 'GrandPiano_G', 'GrandPiano_C'],
      dtype='<U21'))
```

In [62]:
```
# we expect the piano D, C, H-sharp and F
print(lls_instruments(mix_samples['Mary_piano'], samples.keys(), n_instr =
print(lasso_instruments_skl(mix_samples['Mary_piano'], samples.keys(), n_in
```

```
['GrandPiano_D' 'GrandPiano_E' 'Harp_C']
(array([0.13463647, 0.09313905, 0.06991269]), array(['GrandPiano_D', 'Gra
ndPiano_E', 'Harp_C'], dtype='<U21'))
```

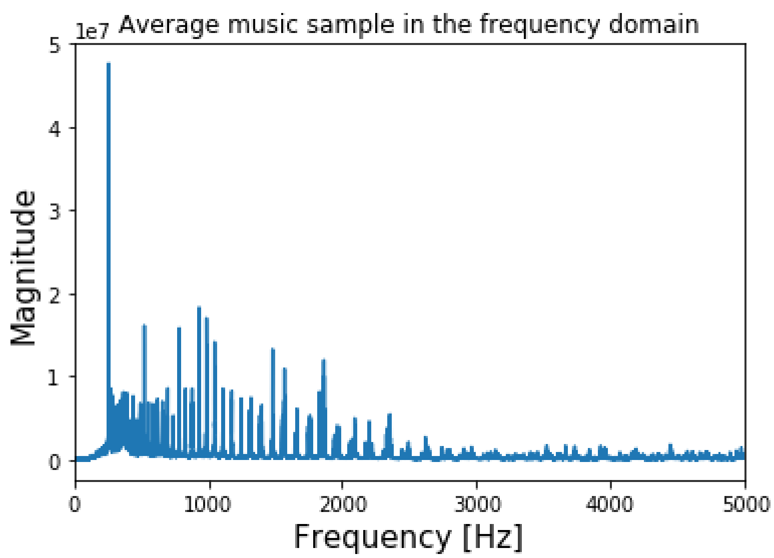
6. Principle Component Analysis

In [63]:
```
from sklearn.decomposition import PCA
```

In [64]:
```
# compose A
options = samples.keys()
half = int(len(c))
A = np.zeros((half, len(options)))
for i, instrument in enumerate(options):
    sample = samples[instrument]
    c = rfft(sample)
    half = int(len(c))
    A[:,i] = abs(c[:half])

mean = A.mean(axis = 1)
print(A.shape)

for i in range(A.shape[1]):
    A[:,i] = A[:,i] - mean
```

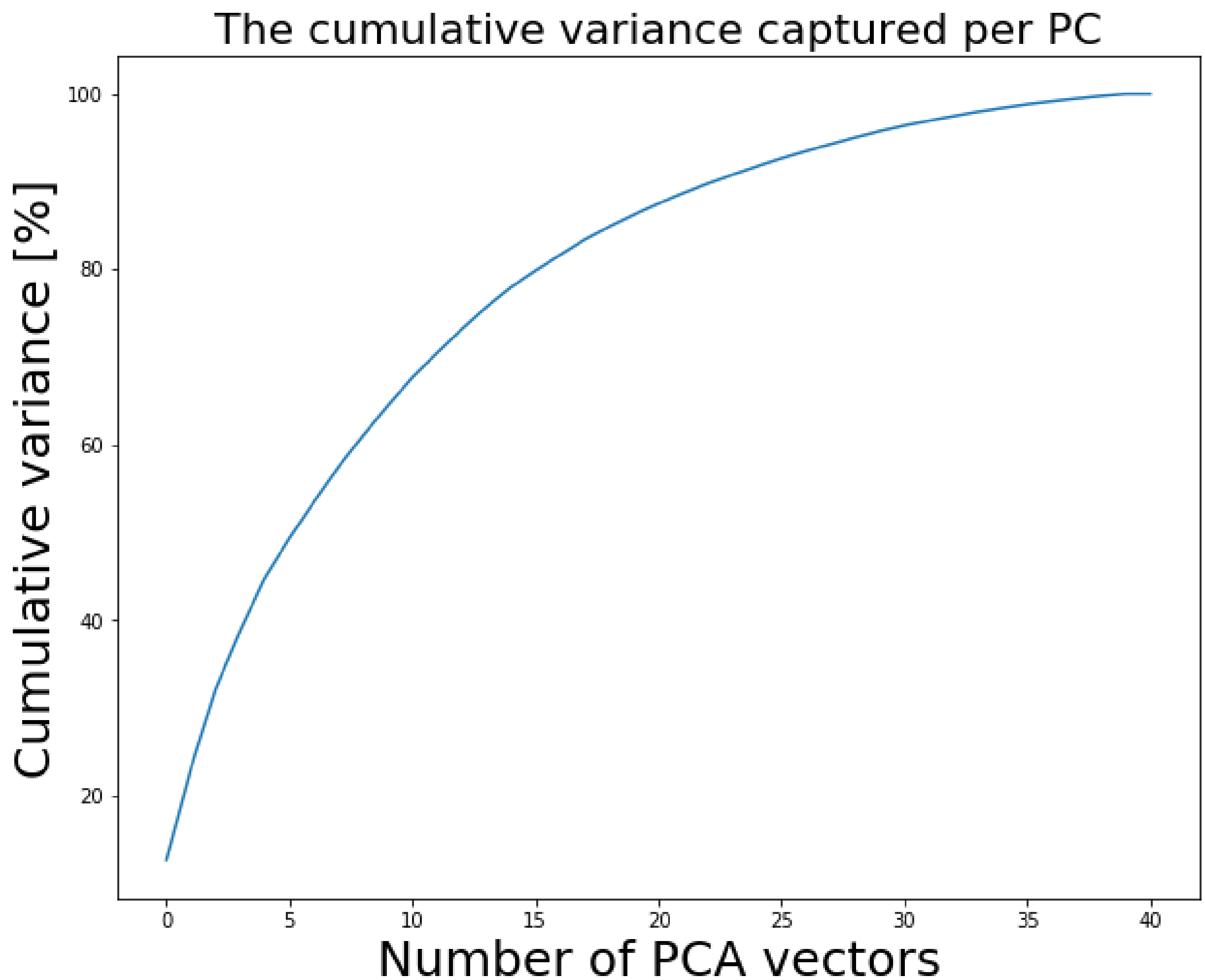
```
(176400, 41)
```

In [65]:
```
plt.figure()
plt.plot(np.linspace(0,fs,A.shape[0]), mean)
plt.xlabel('Frequency [Hz]', fontsize = 15)
plt.ylabel('Magnitude', fontsize = 15)
plt.title('Average music sample in the frequency domain', fontsize = 12)
plt.xlim(0,5000)
plt.show()
```



In [67]:
```
PCA_ = PCA().fit(A)
```

In [70]:
```
var_per_comp = PCA_.explained_variance_ratio_
cum_var = 100*np.array([sum(var_per_comp[:k+1]) for k in range(len(var_per_

plt.figure(figsize = (10, 8))
plt.plot(range(len(var_per_comp)), cum_var)
plt.title('The cumulative variance captured per PC', fontsize = 22)
plt.xlabel('Number of PCA vectors', fontsize = 25)
plt.ylabel('Cumulative variance [%]', fontsize = 25)
plt.show()
```



In [71]:
```
PCA_trans = PCA_.transform(A)
PCA_trans.shape
```

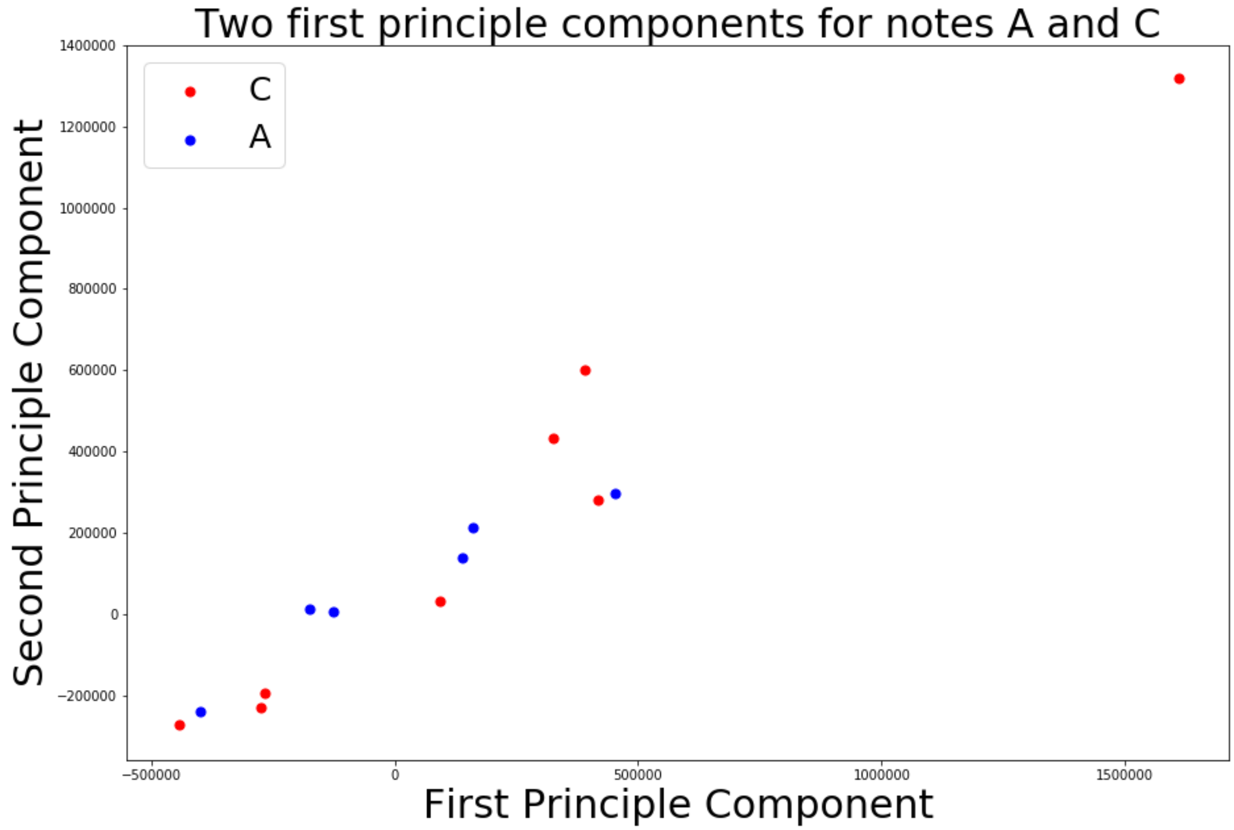
Out[71]: (176400, 41)

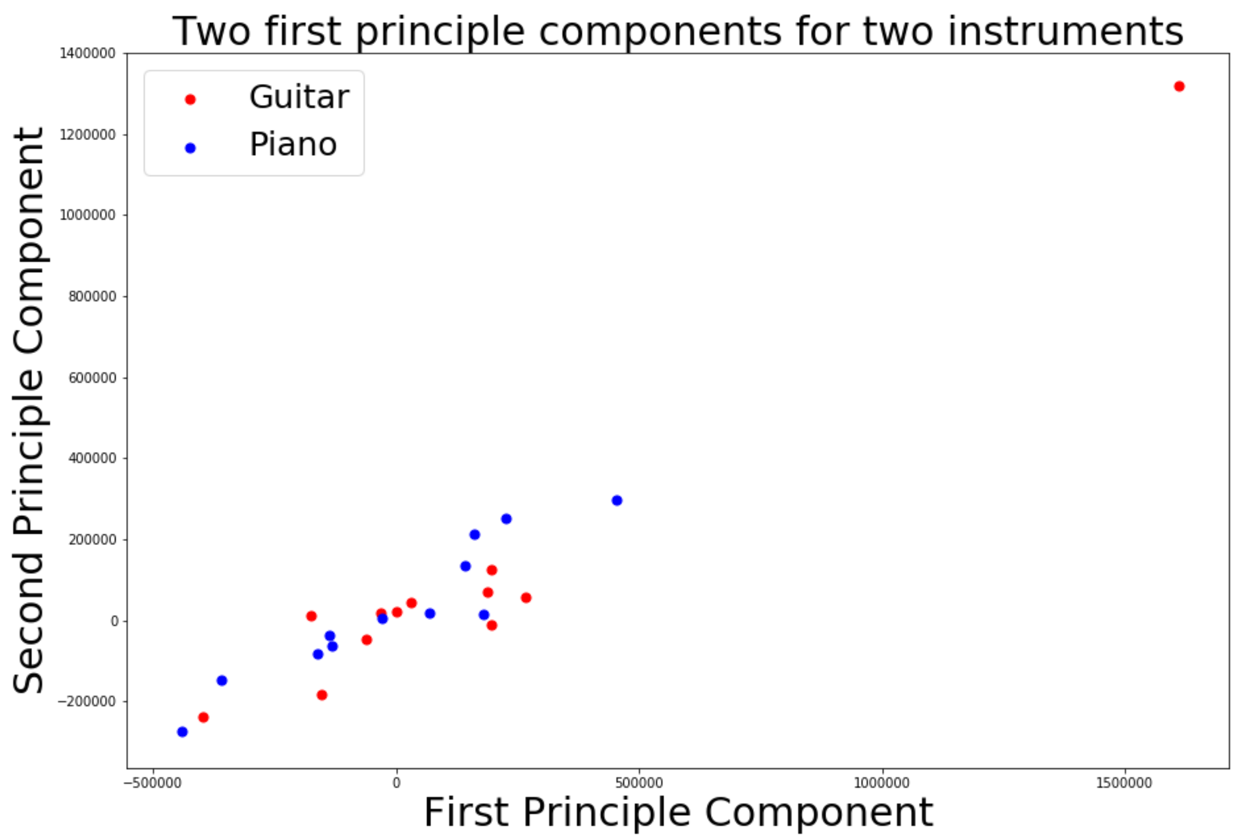
In [72]:
```
samples.keys()
```

Out[72]:
```
dict_keys(['AcousticGuitar_C', 'CathedralOrgan_C', 'Clarinet_C', 'FrenchH
orn_C', 'GrandPiano_C', 'Harp_C', 'RockGuitar_C', 'Saxophone_C', 'GrandPi
ano_A', 'AcousticGuitar_A', 'FrenchHorn_A', 'GrandPiano_Asharp', 'Acousti
cGuitar_Asharp', 'FrenchHorn_Asharp', 'GrandPiano_B', 'AcousticGuitar_B',
'FrenchHorn_B', 'GrandPiano_Csharp', 'AcousticGuitar_Csharp', 'FrenchHorn
_Csharp', 'GrandPiano_D', 'AcousticGuitar_D', 'FrenchHorn_D', 'GrandPiano
_Dsharp', 'AcousticGuitar_Dsharp', 'FrenchHorn_Dsharp', 'GrandPiano_E',
'AcousticGuitar_E', 'FrenchHorn_E', 'GrandPiano_F', 'AcousticGuitar_F',
'FrenchHorn_F', 'GrandPiano_Fsharp', 'AcousticGuitar_Fsharp', 'FrenchHorn
_Fsharp', 'GrandPiano_G', 'AcousticGuitar_G', 'FrenchHorn_G', 'GrandPiano
_Gsharp', 'AcousticGuitar_Gsharp', 'FrenchHorn_Gsharp'])
```

In [74]:

```python
# scatter two instruments and two notes in space defined by two PC's
Cs = list(samples.keys())[:8]
As = list(samples.keys())[8:14]
idx_guit = [0,9,12,15,18,21,24,27,30,33,36,39]
idx_piano = [4,8,11,14,17,20,23,26,29,32,35,38]

plt.figure(figsize = (15, 10))
for i in range(8):
    if i == 0:
        plt.scatter(PCA_trans[0,i], PCA_trans[1,i], label = 'C', c = 'red',
    else:
        plt.scatter(PCA_trans[0,i], PCA_trans[1,i], c = 'red', alpha = 1, s
for i in range(8,14):
    if i == 8:
        plt.scatter(PCA_trans[0,i], PCA_trans[1,i], label = 'A', c = 'blue'
    else:
        plt.scatter(PCA_trans[0,i], PCA_trans[1,i], c = 'blue',alpha = 1, s

plt.title('Two first principle components for notes A and C', fontsize = 30
plt.xlabel('First Principle Component', fontsize = 30)
plt.ylabel('Second Principle Component', fontsize = 30)
plt.legend(fontsize=25)
plt.show()

plt.figure(figsize = (15, 10))
for i in idx_guit:
    if i == 0:
        plt.scatter(PCA_trans[0,i], PCA_trans[1,i], label = 'Guitar', c = '
    else:
        plt.scatter(PCA_trans[0,i], PCA_trans[1,i], c = 'red', alpha = 1, s
for i in idx_piano:
    if i == 8:
        plt.scatter(PCA_trans[0,i], PCA_trans[1,i], label = 'Piano', c = 'b
    else:
        plt.scatter(PCA_trans[0,i], PCA_trans[1,i], c = 'blue',alpha = 1, s

plt.title('Two first principle components for two instruments', fontsize =
plt.xlabel('First Principle Component', fontsize = 30)
plt.ylabel('Second Principle Component', fontsize = 30)
plt.legend(fontsize=25)
plt.show()
```


Two first principle components for notes A and C
C
A
Second Principle Component
First Principle Component
1400000
1200000
1000000
800000
600000
400000
200000
0
−200000
−500000
0
500000
1000000
1500000


Two first principle components for two instruments
Guitar
Piano
Second Principle Component
First Principle Component
1400000
1200000
1000000
800000
600000
400000
200000
0
−200000
−500000
0
500000
1000000
1500000

In [76]:

```
# plot the two first PC's
plt.figure()
for i in range(2):
    plt.plot(PCA_trans[:,i], alpha = 0.8, label = 'PC {}'.format(i))
plt.xlim(0,5000)
plt.xlabel('Frequency [Hz]', fontsize = 15)
plt.ylabel('Magnitude', fontsize = 15)
plt.title('First two PCs in the frequency domain', fontsize = 12)
plt.legend()
plt.xlim(0,5000)
plt.show()
```