NOC: Introduction to Airplane Performance
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Lecture - 16
A Closer Look: Point Mass Model Dimensional Analysis

Dear friends, now come out of the class room. It is enough, we have seen lot many algebraic expressions, lot many mathematics, I am afraid that, we should not get lost and we should not forget. The finally, we have to build a relationship with an aircraft. So, let us try to understand, whatever we have been writing on the black board, what exactly we are doing. Remember, when we are writing equation of motion, the trust equal to drag, lift equal to weight, we have given a statement that we assuming it to be point mass model.

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What is that point mass model physically means? Let us see this is our Hansa 3 aircraft, I always say this is flied of India in particular NAL, Bangalore. They have manufactured it, designed it. Now, let us see, what is the meaning of point mass model? If I ask you a question, where is the CG of this airplane, how can you guess, one first sight common sense tells, the CG should be somewhere near the landing gear. This landing gear, but the next question is, whether the CG ahead of landing gear or behind landing gear.

Now, if you think carefully, if the CG was behind the landing gear, then if there is a slide disturbance in the tail, the airplane will hit the ground, even while on the ground. So, you will not like to keep the CG behind the landing gear. Although, the CG will be close to landing gear; that is why, if you see an aircraft the landing gear, once it is located, you know CG will be somewhere very close to landing gear, but ahead of landing gear.

So, we say the CG is somewhere here and second part, how do I distribute the mass so that CG is at a point, where I want, one is okay, I have seen the landing gear. So, I will put it little ahead, of landing gear. Second thing you see here, this is a the thrust line and if the vertical CG is very away, very far away from the thrust line, then this trust will be giving you a moment about CG and the pilot will have to make some corrections before he goes for a flight.

So, natural tendency would be, try to keep the CG in the central line, but it would not be possible all the times, because sometimes wing is lowing like here, sometime wing could be high wing. So, you try to ensure that the CG, in the vertical height is close to the central line. But, what we are now talking about, once we have identified where is the CG, then I will try to take the advantage of Newton's law of motion.

If we recall, F equal to m a, for a distributed mass I can show that F equal to mass into acceleration of the center of mass; that is if there are different particles and the forces acting on the each particle, then if I want to measure the acceleration, predict the acceleration, I can easily do that prediction,. If I assume that all the masses are at center of mass and that acceleration will be the acceleration of the center of the mass.

For our case, center of gravity and center of mass is the same; that is what exactly the point mass model will be doing that assume that, no more mass is distributed, because we are only interested in the acceleration for rectilinear motion. We are not talking about rotational motion, if rotational motion is there, then I am more bother about moment of inertia, because distribution of mass; that is important.

When, I am talking about acceleration for rectilinear motion, I am interested in the acceleration of the center of mass. So, it is fair that I assume the whole mass is concentrated at the centre of gravity. Now, once this is done that center of mass I have identified and I have agreed that I will be applying F equal to m a to analyse rectilinear motion.

So, what are the forces? Now, let us also have a closer look. If I separate out fuselage, wing, tail and other components, I can fairly do an approximation. Let us say the forces on the fuselage is acting somewhere here as the lift, I am talking about the lift part. Similarly, drag will be acting like this, depending upon velocity direction, do not forget lift will be perpendicular to the velocity and drag will be opposing the velocity.

Let us take example of a lift, for a small angle of attack. If this is the lift directions, but I am writing equation of motion, assuming the whole mass is at center of gravity, somehow here. So, then I need to transfer this force from here to the center of gravity and once, I transfer a force from here to a distance, I know that I can transfer this with the force as well as a moment.

So, I am putting, because of this fuselage, there is one force, which is lift and a moment. Similarly, take case of a wing, so there will be a force, lift force, again transfer it with a moment to the CG, similarly from the horizontal tail, all those forces which were acting at different, different point of locations, I am transferring them to CG with force and moment. So, now, when I apply point mass model, I have a assumption that all these moments are balanced.

In practice, how they are balanced? They are balanced by design, by a configuration design and addition, you can use elevator, trimmer, etcetera, etcetera, to make all those things moments are balanced. So, now, when I am analysing the cruise, I am very clear that whole mass of the aircraft is at center of gravity and net force, I have calculated by transferring them from different components.

And then, I am now resolving those forces and finding accelerations and if it is non accelerated or unaccelerated flight, I am putting that 0 and going step by step whatever is required. So, that is what is typically the point mass model, but when I say this, this typically, when I am talking about forces separately on fuselage, on wing, on horizontal tail and go and try to add them with a lot of care that I know, there is an interference effect. This method is called component build up method; that is you find out the component and then add.

But, in actual practice what will happen, you will like to find the overall pressure distribution over the whole airplane and then, find out the resultant forces and find out where they actually acting and then, try to transfer them to CG. Typically, we use CFD Computational Fluid Dynamics or second stage, what we do, we try to go for a wind tunnel testing and find out the pressure distribution or find the forces acting through the balances. There is a balance which will measure forces and moment; that is matter of detail.

This is the basic background, when we were talking about equation of motion and point mass model, right. Before, I stop discussion on this, please also understand, we have also shown you that, over the wing if I try to represent why should the lift will act or CL will act, then I know that there should be at the quarter cord point, I take this is chord and at 25 present from leading edge. So, I say this is my CL location is here. So, these are the things, which we need to be kept in mind.

The similar thing will happen on other side and we will define something called mean aerodynamic chord we will see as we develop this course. But, at this point it is important to understand, what is the meaning of point mass model and how the forces are transferred and what is the effect of that? Up to this point, this is fine. Let us go back to George Cayley, when George Cayley told us that yes and he was pioneer in making us understand that, if there is angle between the plate and the velocity vector, there will be lift. This is a plate and there is a velocity vector, there is a reaction and component of that is lift, one is drag.

This is fine, but when we try to go deep into it, how this lift depends on the environment or the geometric dimensions of the wing or the flat surface. Then, we have to understand few things a little more deeper. Let us see what happens, if a flow is passing over this wing surface, because air has viscosity. So, what will happen? The layer, the particle, which are just on the surface, they will get stuck to this surface, because of property of viscosity. That means, they will not have any motion, they will have the motion of the aircraft. They are now part of the aircraft.

So, relative sense they have no motion with the aircraft, however, as I go higher and higher, the influence of this viscosity via the wing surface get diminished. So, at some point it regains its, let say, regains its velocity. So, at that boundary layer thickness could be is not there huge boundary layer for the wing, but it depends upon something which I called Reynolds number. That is please understand when a fluid is flowing over this, it has an inertia, it is I will use, it has an inertia associate with the momentum of the fluid. In text book, they called inertia force. So, that will try to ensure that, it remained in the state of motion.

However, because of viscosity, it will try to drag it, it will try to discourage that motion. So, now, who will dominate that will decided by the ratio of inertia of force and the viscous force and ratio of inertia force and viscous force is only defined as Reynolds number. So, we say, if Reynolds number is high; that means, relative sense the inertia force is more than viscous force.

So, the flow the pressure distribution over it is less influence by the viscous effect, but if the flow is low Reynolds number, then we say the viscous force is more dominative the inertia force and the pressure distribution other characteristic are dictated by primarily by the viscous effect. So, whether it is high or low, one thing we agree that, the lift part has some implicit relationship with Reynolds number, drag has some relationship with Reynolds number.

What is that Reynolds number? How do I use that? Those are questions were answered through dimensional analysis; that is why we did dimensional analysis in last class. So, what will be doing today, we will once you understand this, we will again revisit that dimensional analysis which is typically Pi theorem, Buckingham pi theorem, very important, very, very important theorem, which is used many, many designs.

So, let us now from here go back to the classroom and we continue. We are just again came back to the class room after seeing our Hansa 3 aircraft and try to correlate, what we are teaching in the class room to what actually happens in the air craft.

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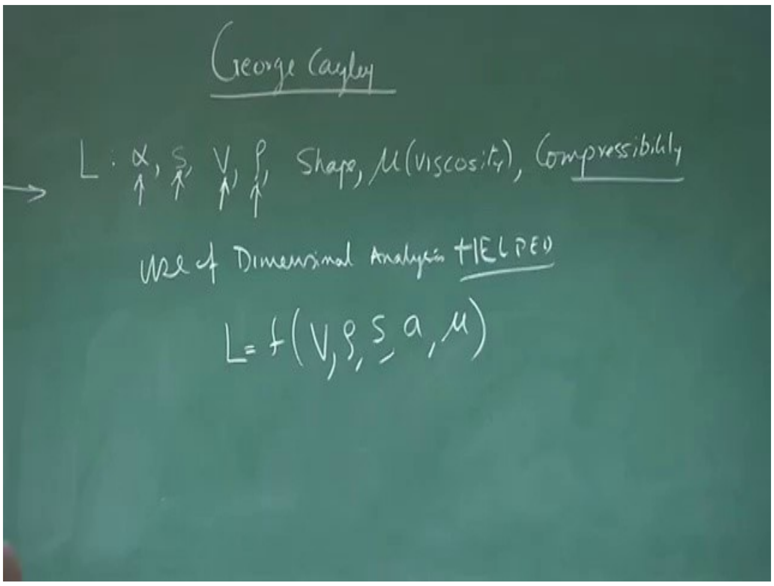

Let us see, we again go back to George Cayley. Why we are again and again going to George Cayley, try to understand that. Remember, this was contribution of George Cayley that there will be reaction by now, you are very clear. The flat plate moving like this, there is a reaction and one component is lift one component is drag, absolutely fine.

So, one thing was very clear, this lift will be proportional to the angle alpha, it will be proportional to area S, it will be proportional to velocity, density, then shape, then mu viscosity and I am also writing something compressibility. Because, air is compressible if the vehicle moves little faster, then the air particle gets compressed and accordingly we have defined three types of flow.

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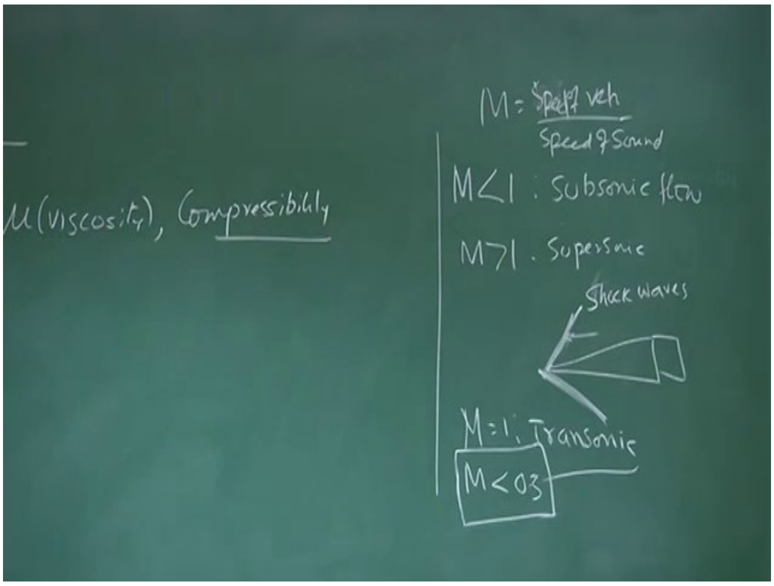

Remember, you can check if it is less than 1, we call subsonic flow and what was mach number, mach number was velocity of vehicles or speed of vehicle, its more important, it does not matter for attaining a (Refer Time:14:50)) flight by speed of sound. Why this was important? Conceptual wise, I know that disturbance propagates with the speed equal to velocity of sound or call speed of sound.

So, suppose a vehicle is moving like this ((Refer Time: 15:10)), then the air particle get disturbed. So, any other particle ahead of the vehicle is already displaced, because of disturbance. So, when it moves, it get less resistance, but imagine if it is moving with speed higher than the speed of sound, then the next particle before they get displaced the vehicle will crash on it. That is how if flight particle get compressed and we call it, it is a supersonic flow; that is mach greater than 1, which is supersonic and there, we have shock waves.

Shock waves are like, this is the vehicle moving in this direction, fluid get compressed and there are shock waves high density and they get compressed like this, so the shock wave there is form. This is not a part of this lecture, but you need to know a few things, you know to make your understanding better. It goes without say, if mach number equal to 1, it is transonic.

And we are talking about incompressible flow, when that means, we are talking about typically mach number less than 0.3, most of all lecture is governed by this condition to the Mach number is less than 0.3. So, keep that back of your mind, now see here, if the lift is function of alpha, area, velocity or speed, density, shape, shape means, you know by now aerofoil what type of aerofoil, what is the contour of the aerofoil, then viscosity and compressibility.

Now, imagine if I want to create a database, where I need to know what is the lift for given area S, what I have to do, I have to go to your wind tunnel and I will conduct a test put that wing conduct a test and keep alpha V rho all this things same. But, I go on changing S. Similarly, if you want to know what is the effect with V, I will keep another variable same and go on changing; its speed inside the tunnel.

Like that, I will be doing similar thing for all the parameters. So, it becomes a Herculean in task correct very difficult to scale it up or scale it down; that is I am very much restricted by the amount of testing required to recapture the model; that is where this the use of the dimensional analysis helped, this is typically using Buckingham's, Pi theorem. If you get a chance go, do internet surfing, and read something more about it.

In there, what we did, you said, let us identify lift as function of it is a V rho S, a and mu, mu is the coefficient of viscosity; that is we are trying to address a question it is fine, lift is proportional angular of drag fine. But, at a given angle of a drag, it is also function of so many other things, that model, I am try to capture. In doing that, we use the benefit of the dimensional analysis, what we did we know, the dimension of lift, dimension of speed, density, area, etcetera, etcetera.

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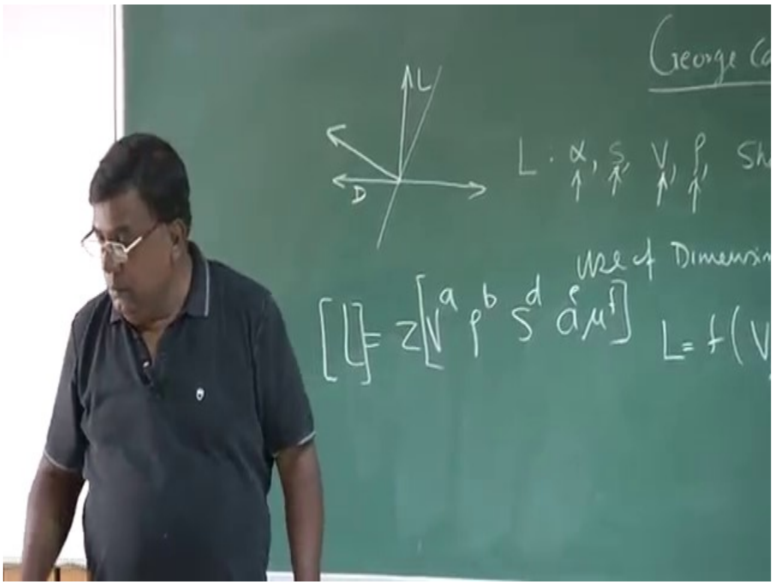

And you compare them, before doing that, we wrote L equal to Z, V to the power a, row to the power b, S to the power d, a to power e and mu to the power f. This exponent a, b, d, e, f, there all constants, Z is also a constant. Then, what we did we compare the dimension of left hand side and dimension of right hand side, form some equation to get finally, very important result.

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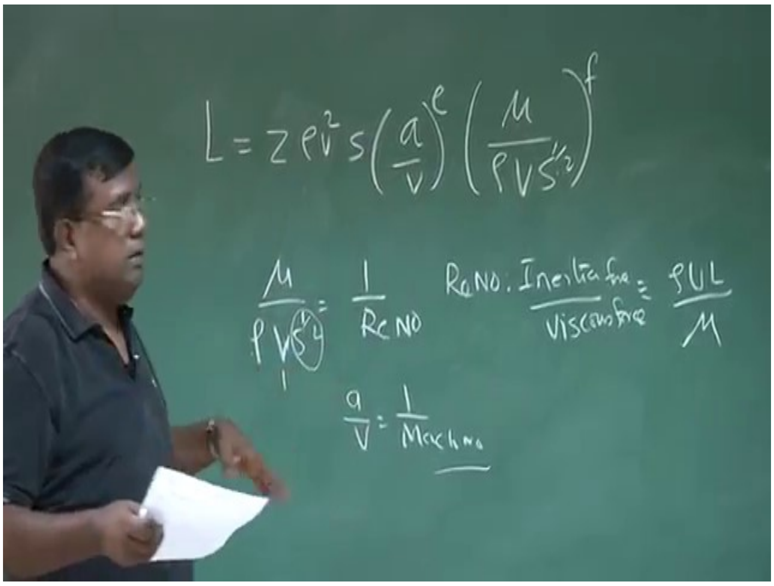

As lift equal to Z, rho V^2 S, a by V to the power e and mu by row V, S to the power half to the power f. And we identified that mu by row V S to the half is by Reynolds number and Reynolds number, we know that it is a ratio of inertia to viscous force and that is one can show a derivation a rho U L by mu, L is characteristiCLength. If you see here, this is row U, U is the velocity here; that is say V here and S to the power half has a unit of length. So, this is L is here and mu is the coefficient of viscosity for air.

$$L = Z\rho V^2 S \left(\frac{a}{V}\right)^e \left(\frac{\mu}{\rho V S^{\frac{1}{2}}}\right)^f$$

$$L = Z\rho V^2 S \left(\frac{1}{M}\right)^e \left(\frac{1}{Re}\right)^f$$

$$\frac{\mu}{\rho V S^{\frac{1}{2}}} = \frac{1}{Re}\,; Re = \frac{Inertia\ force}{Viscous\ force} = \frac{\rho UL}{\mu}$$

Also, we identified here a by V is 1 Mach number.

$$\frac{a}{V} = \frac{1}{Mach\ no}$$

So, message here is, there are two non-dimensional number, which are to be used to reduce your number of testing and adding better values in the interpretation of whole of this modelling. That is, when I talked about this as a Reynolds number, I understand this is inertia to viscous force, when I talk about a by V understand, it is with the Mach number.

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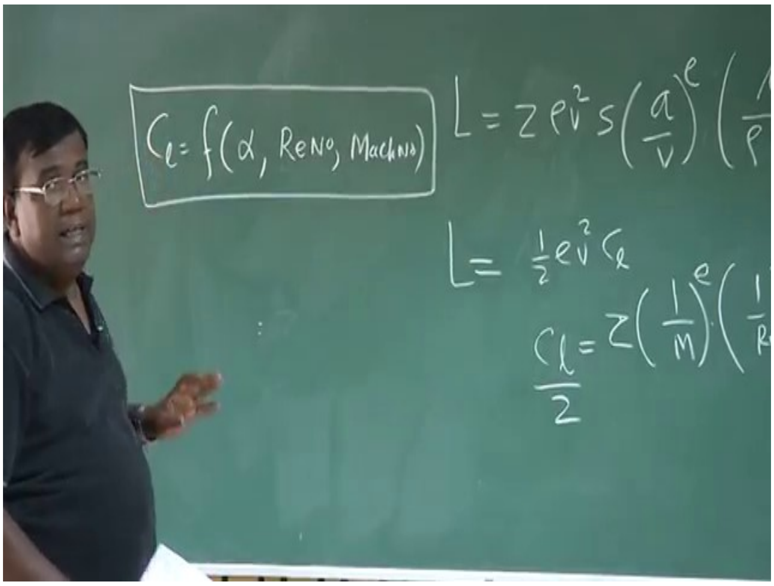

So, we now express this as half row V^2 CL, where CL by 2, I have written as Z, 1 by m to the power e and 1 by Re to the power f. By doing that, what has happened to see now, we know this CL is function of alpha, you are knowing also Reynolds number and Mach number our life has become so simple. How do I utilize this, question is this and what is this CL, CL is the coefficient, lift coefficient here.

$$\frac{C_L}{2} = Z\left(\frac{1}{M}\right)^e \left(\frac{1}{Re}\right)^f$$
$$C_L = f(\alpha, Re, M)$$

My question is do I work is L or work with CL, what are the advantage I get, if I try to give a physical interpretation to this, it says, if I am having two geometrically similar configuration, geometrically similar you understand, there lengths are proportional. If you are geometrically similar configuration, then to know the variation of CL with alpha, Reynolds number and Mach number, I need to do vey a limited number of experiments.

And be careful, first identify what is the speed regime and which one is important Reynolds number or Mach number. As we are doing here for Mach number less than 0.3, it is incompressible. So, if I want to model CL, I will not give you a weightage to this, I will know that CL will be function of alpha, yes and Reynolds number; that is important.

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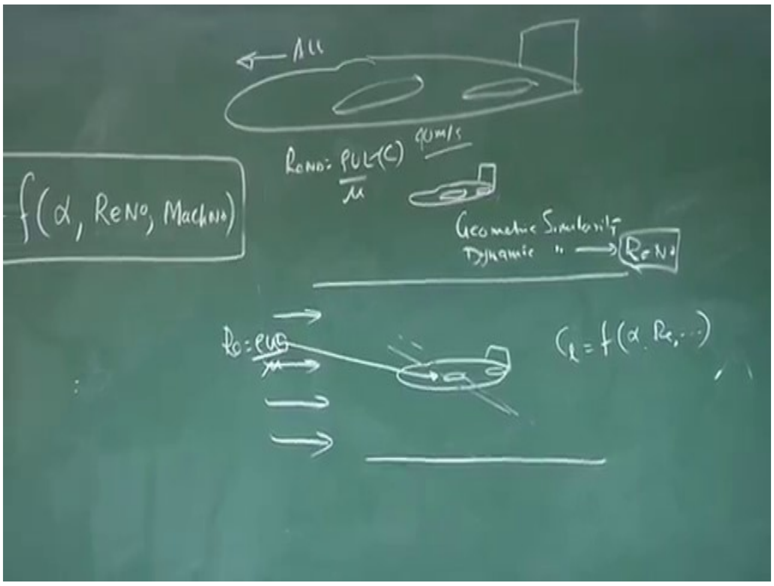

So, what I do I, suppose there is a big airplane and this is flying at let say 90 meter per second or less than 0.3 Mach number, altitude is chosen like that. Now, what do I do to help of it, I make a smaller similar configuration exactly geometrically similar, I will called a scale zone model, when 125th, 1 by 50th, 1 by 10th its dimensions are reduce like that. Then I put this inside this tunnel; I want to generate CL verses function of alpha.

So, what I can do now, I can go on generating different CL by giving different, different, orientation and ensure you one thing; that is very important. When, it was flying at a particular altitude, what was the Reynold number it was flying, so I calculate rho U L by mu, L in this case will be the chord of the wing. So, I know what is the Reynold number, the aircraft was flying, I will duplicate the same Reynolds number here, because here I know Reynolds number again is rho U L by mu or L will be the chord of this scale down model.

If I can ensure that, this flow has got such a speed density combination that Reynolds number experienced by this body is exactly same as, what is the actual airplane was seeing, then if I model CL, I can generate CL as function of different, different Reynolds number. And depending upon what altitude the airplane is flying, I can use that CL value for actual computation.

However, when I do wind tunnel testing, there are some limitations, there are some error introduced, because the wall effect ((Refer Time: 26:05)) because of the model, may not be the exactly the same, but those are secondary things, physics is this. If you can do that, then I am saving so much of testing and my life become simpler and this is the concept that, if you have a geometrically similar at geometrical similar similarity and the flow dynamic similarity; that is I am duplicating the Reynolds number.

So, that makes our life very simple. So, will stop it here with a final statement is through this Pi theorem or dimensional analysis, I now know that CL not only the model in terms of alpha, but Reynolds number and Mach number.

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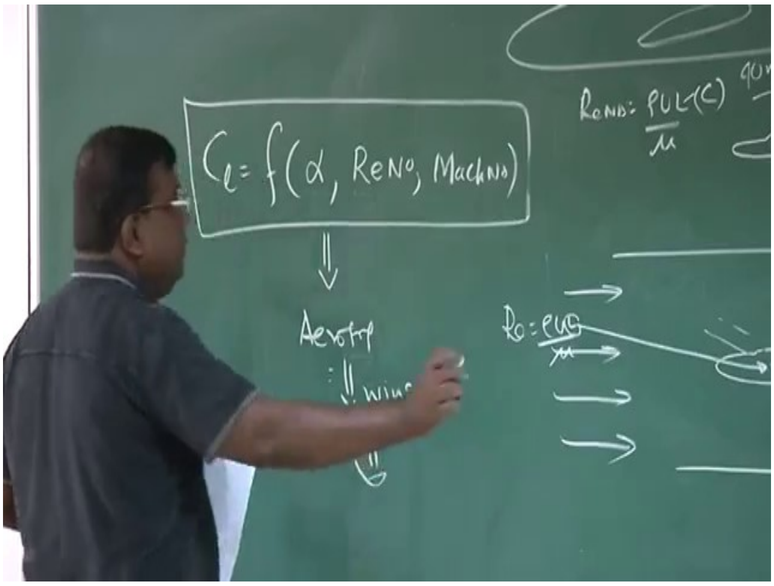

So, our approach will be, first I will use it, I now from here, I will go to aerofoil, then aerofoil to wing and wing to flying.