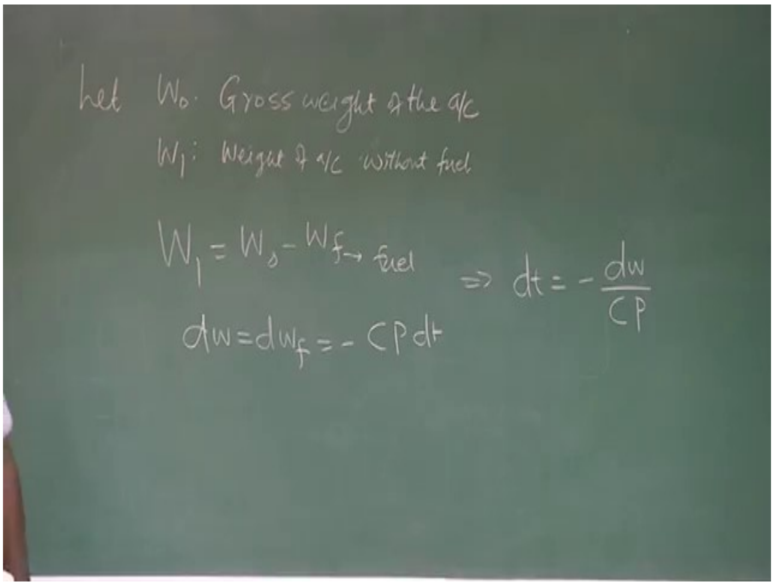
NOC: Introduction to Airplane Performance

Prof. A. K. Ghosh
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture - 21
Range and Endurance: Continued

(Refer Slide Time: 00:11)



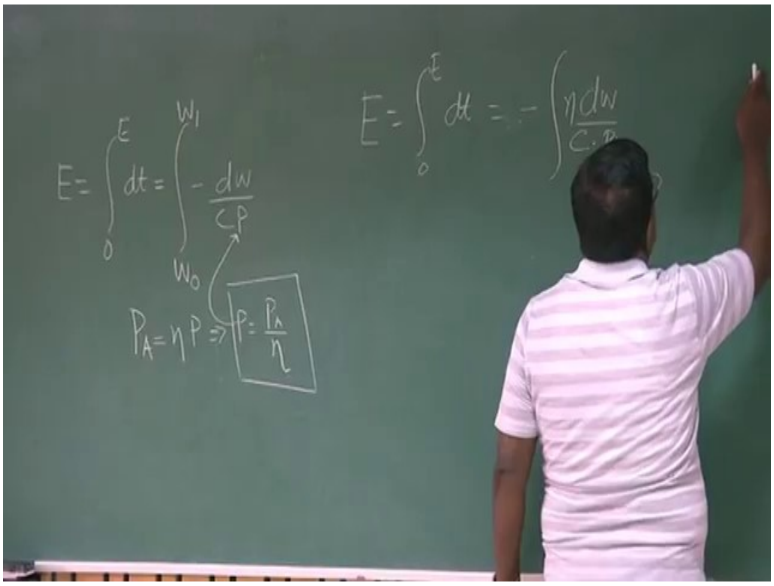
Let us say, W0 is the gross weight of the airplane or the aircraft; that means, it has structure weight, payload, fuel, everything and let W 1 is the weight of aircraft without fuel. Please understand, in the gross weight the payload weight, structure weight, fuel weight everything is included. In W 1 structure weight, payload weight, any other weight is included, except the fuel weight. So, I can write W 1 is equal to W0 minus W f, where W f means fuel weight.

$$W_1 = W_0 - W_f$$

So, from here I can write d w is equal to d w f, because I am pretty sure that I am not dropping anything out of the aircraft. The change in the weight of the aircraft is because of consumption of fuel, fuel is burnt, so it has reduced, so this is equal to minus C p d t. From here I can simply write d t equal to minus d w by C p, again I want to restress this p is the power available at the brake. Not the power available that is, not the power extracted by the propeller.

$$dw = dw_f = -CPdt \Rightarrow dt = -\frac{dw}{CP}$$

(Refer Slide Time: 02:00)



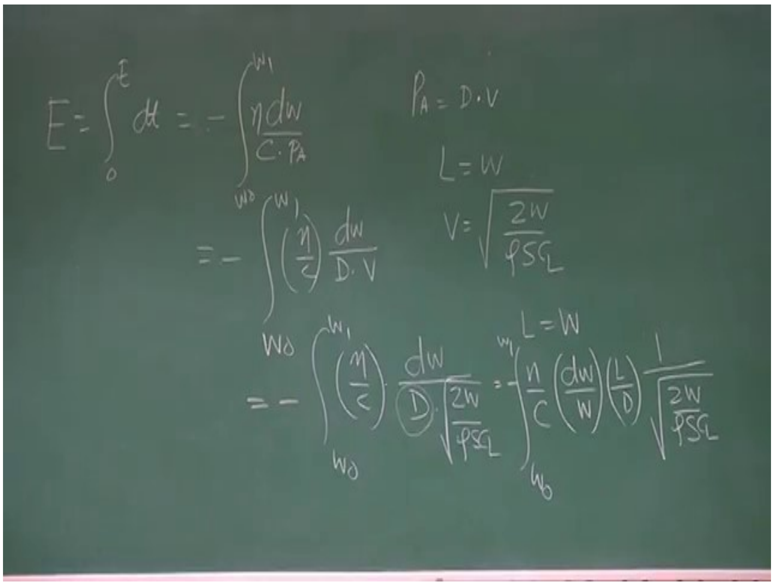
So, if I want to now find out endurance, endurance will be 0 to E, total time d t and during the total time, the gross weight which is W0 and when the fuel weight is consumed that is, W 1. W 1 is the weight of the aircraft when no fuel; that means, whole fuel is consumed, so I write minus d w by C p and we know that p is the power available at the brake. But, power available for the airplane is eta into P, so I can write P is equal to P available by eta, this P I will be using here.

$$E = \int_0^E dt = \int_{W_0}^{W_1} -\frac{dW}{CP}$$

$$P_A = \eta P \Rightarrow P = \frac{P_A}{\eta}$$

So, now my expression for E becomes 0 to E d t equal to E, E is already written, is equal to minus d w by C into power available and the eta goes on top, eta here. Now, we know since we are talking about cruise and we know that power available equal to drag.

(Refer Slide Time: 03:14)



So, this I can into V, power available is drag into v, so I can substitute power available here, then I get an expression minus. This is W0 to W 1, again I put here W0 to W 1 eta by CD w by D into V.

$$E = \int_{0}^{E} dt = -\int_{W_0}^{W_1} \frac{\eta dw}{C.P_A} : P_A = D.V$$

$$= -\int_{W_0}^{W_1} \left(\frac{\eta}{C}\right) \frac{dW}{D.V}$$

$$L = W; V = \sqrt{\frac{2W}{\rho S C_L}}$$

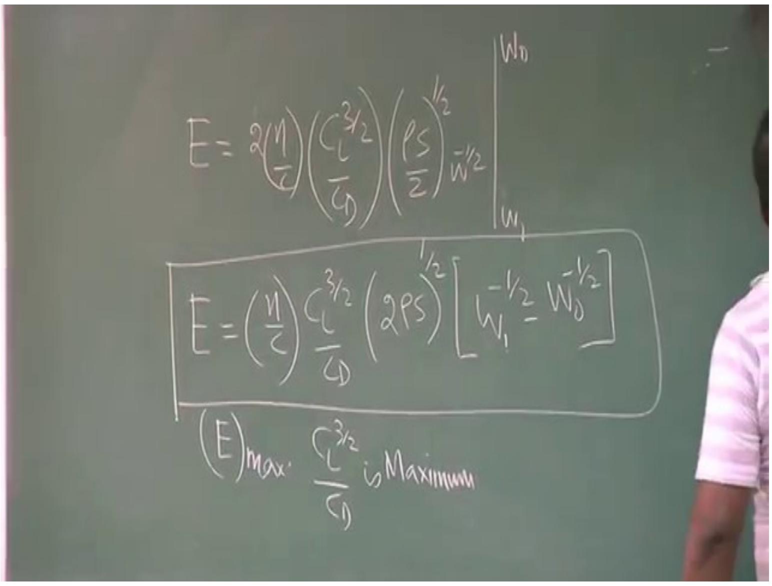
Now, what is V? From lift equal to weight, I know V equal to 2 W by rho S CL. What is rho? The altitude at which the airplane is flying, CL is the CL cruise. Typical value of CL cruise for most efficient airplane will be around 0.2, 0.3, but if you are flying at a different other altitude for different conditions, the CL may change.

So, now I will substitute here value of V, so I will get minus W0 W 1 eta by C into d w by D into 2 W by rho S CL under root. For cruise, lift equal to weight, so what I will do? I will write this as minus eta by C W0 to w 1, I will write d w by W, since W equal to lift, I compensate this W by putting L and already D is here. So, and then, I have under root 2 W by rho S CL. What I have done?

$$= -\int_{W_0}^{W_1} \left(\frac{\eta}{C}\right) \frac{dW}{D.\sqrt{\frac{2W}{\rho S C_L}}} = -\int_{W_0}^{W_1} \left(\frac{\eta}{C}\right) \frac{dW}{W} \left(\frac{L}{D}\right) \frac{1}{\sqrt{\frac{2W}{\rho S C_L}}}$$

Since, lift equal to weight, what I have done from here I have added one W in the denominator and since lift equal to weight I have multiplied by lift. So, expression remains same, but this has given me an advantage, I have got an expression for L by D and here, also there is a CL sitting here.

(Refer Slide Time: 05:42)



Now, if I write further if I further simplify, I can write E as 2 eta by C, CL 3 by 2 by CD rho S by 2 to the power half W minus half W 1 to W0. Please note, that the order of integration has been changed to absorb this minus sign, it was W0 to W 1, now minus sign is absorbed, it is W 1 to W0. So, if I expand it I get an expression E is equal to eta by C, CL 3 by 2 by CD 2 rho S to the power half W minus half W 1 minus half W0 minus half.

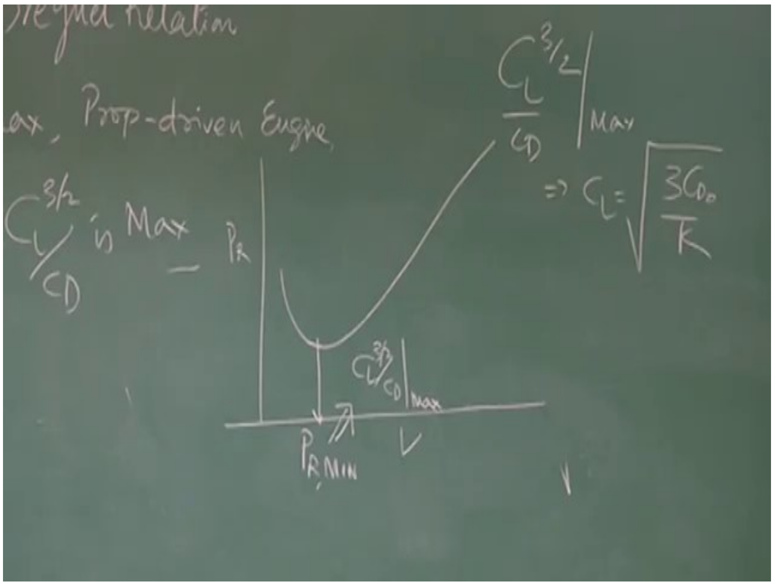
$$E = 2\frac{\eta}{C}\left(\frac{C_L^{3/2}}{C_D}\right)\left(\frac{\rho S}{2}\right)^{1/2} W^{-1/2}\Bigg|_{W_1}^{W_0}$$

$$E = \frac{\eta}{C}\left(\frac{C_L^{3/2}}{C_D}\right)(2\rho S)^{1/2}\left[W_1^{-1/2} - W_0^{-1/2}\right]$$

Please note that, when we arrived at this expression we have taken out eta by C outside this integration. We have assumed that there is no effect on C, because of change in weight.

Also you should remember that C does change with the altitude, but here for simplicity we have assume it to be constant. What is more important here? See, what we are seeing here, seeing that E will be maximum for a given altitude, given initial weight and final weight or fuel weight. E will be maximum, when CL 3 by 2 by CD is maximum.

(Refer Slide Time: 07:56)



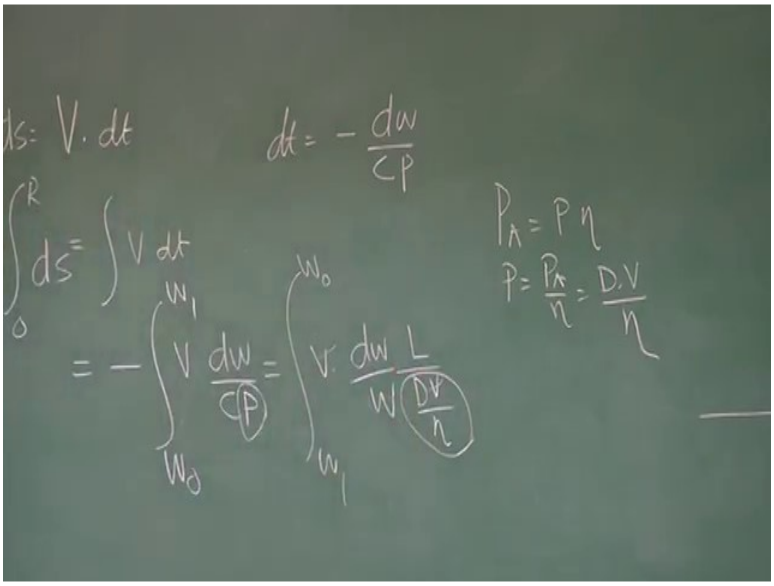
Please understand, it is very important result which is also popularly known as Breguet relation. It is very handy in the design process of an aircraft. What we are seeing here, that E is maximum for propeller driven engine provided. You fly the airplane, such that CL 3 by 2 by CD is maximum. Are you familiar with this term, CL 3 by 2? Are you familiar with this term? if CL 3 by 2 by CD is maximum and this is propeller driven engine.

Let us go back to power required versus speed graph, this was the point, power required minimum and what was the condition. Condition was CL 3 by 2 by CD is maximum. So, what is the understanding, that if I want to fly a propeller driven IC engine aircraft and if I want to have larger time in air or maximum endurance, I should fly such that CL 3 by 2 CD is minimum. This CL by 3 by 2 by CD is maximum or in turn, it means CL 3 by 2 by CD maximum, it means you are flying at CL equal to 3 CD0 by K.

$$\left.\frac{C_L^{3/2}}{C_D}\right|_{Max} \Rightarrow C_L = \sqrt{\frac{3C_{D_0}}{K}}$$

So, for a pilot he will be now flying at a CL, which is a fixed number, which depends upon the CD0 value of the airplane and K of the airplane. This is clear, again I come back here. If I want to have E max, I need to fly such that CL 3 by 2 by CD is maximum that in turn mean, I am flying at CL equal 3 CD0 by K and this is strictly for propeller driven IC type of engine aircraft.

(Refer Slide Time: 10:15)



Now, from endurance now we go to range. This is again range for which type of aircraft? This is piston driven propeller driven IC engine aircraft. Fundamentally, what is range? Please understand, typically range means the total range from takeoff to landing, but here we are focused mostly on the range during the cruise. So, this is that way approximate, but very handy for designing an airplane. So, for if it is a cruise, then range I can find out small distance travelled will be V into d t as a distance.

So, if I integrate this d s 0 to range, it is equal to V d t. Now, question is what is d t? Now, we have already seen d t is minus d w by C P, so I put it here, so this becomes minus V d w by C into P, again the weight from W0 to W 1. Here I can do little manipulation; first I will change the order of integration to absorb this minus sign.

So, I write W 1 to W0, then V for d w I will divide by W and multiply by lift, because lift equal to weight and then, here the power that is d v by eta correct.

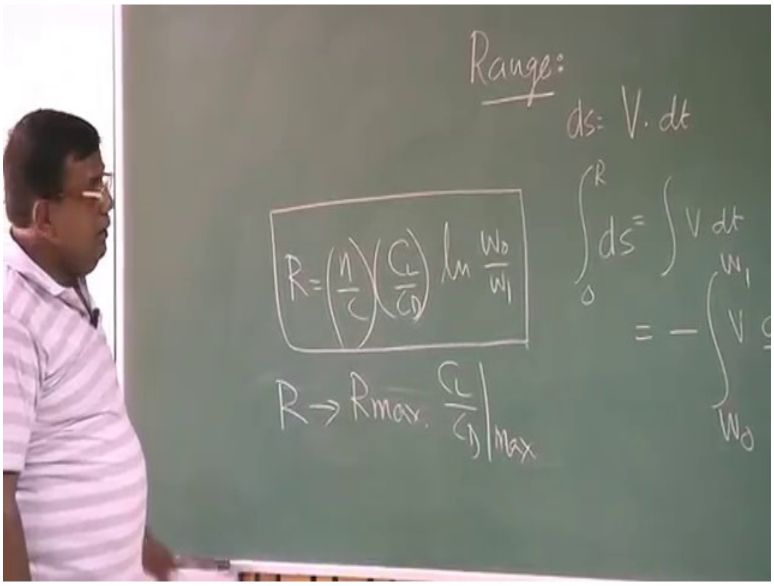
$$ds = V.dt\,; dt = -\frac{dw}{CP}$$

$$\int_0^R ds = \int_0^R Vdt = -\int_{W_0}^{W_1} V\frac{dw}{CP} = \int_{W_1}^{W_0} \frac{V}{C}\frac{dw}{w}\frac{L}{\frac{DV}{\eta}}$$

$$P_A = P\eta\,; P = \frac{P_A}{\eta} = \frac{DV}{\eta}$$

Why this term? We know that P available is equal to P into eta and here this term, this is the P which is at the brake. So, this P is equal to P A by eta and P A is nothing but, D into V, so D A, D into V by eta. These V, V gets cancelled. So, what we are getting?

(Refer Slide Time: 12:39)



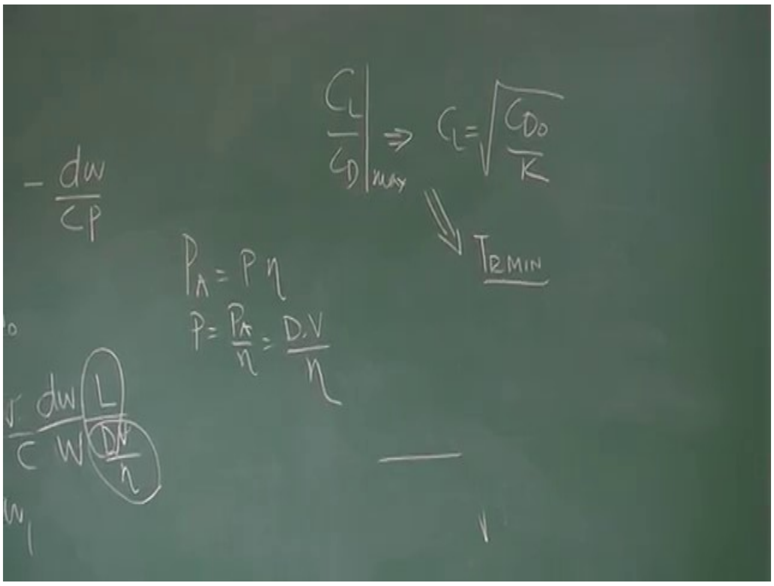
We get range equal to eta by C, see there will be a C here, there is another term C here, it is C into P. So, C was missing here, so I put this C, so eta goes on the top, so eta by C integral n CL by CD and ln W0 by W 1. Let us repeat here, d s is minus V d w by C P that is equal to W 1 to W0. We have changed the order of integration and V and C V by CD w by W, I have put one W multiplied by L, because lift equal to weight.

It does not change, P we know d v by eta and this V and this V gets cancelled, so this eta goes on the numerator. So, I have got eta by C and here I have got L by D, which I can write CL by CD. So, this is d w by W and the integration of W by W is ln, when I put the limit it comes to ln W0 by W 1, clear. Now, see for range, what is the message. For range message is, for a given eta and C, range will be maximum, when CL by CD is maximum.

$$R = \left(\frac{\eta}{C}\right)\left(\frac{C_L}{C_D}\right)\ln\frac{W_0}{W_1}$$

$$R \to R_{max}; \frac{C_L}{C_D}\bigg|_{max}$$

(Refer Slide Time: 14:18)



What is the meaning of that? We have already seen CL by CD maximum means, it is CL equal to under root CD0 by K or this is also a condition for thrust required minimum. So, if I want to fly for higher range for a given tank of fuel, message is very clear, that for maximum range I should fly, such that CL by CD is maximum or in turn I should fly at a CL which is fixed given by CD0 by K under root.

$$\frac{C_L}{C_D}\bigg|_{max} \Rightarrow C_L = \sqrt{\frac{C_{D_0}}{K}} \;\; ; T_{R_{Min}}$$

And the pilot will be flying at a speed at this CL, so that lift equal to weight and he is having a cruise flight. So, this covers our discussion on range and endurance for propeller driven IC engine. Next, we will be talking about jet engine.