NOC: Introduction to Airplane Performance

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Lecture - 12
Excess Thrust Power: Climb Angle and Rate of Climb

Yes, good morning friends, in the last lecture, we were discussing about the Thrust required, Power required for a cruise flight and you also found out a condition. If I want to fly the machine with thrust required minimum, then what should be the CL, which I should fly. And we also found out an expression, if I want to fly the airplane in cruise mode, in power required minimum mode, then what should be the CL, which I should fly.

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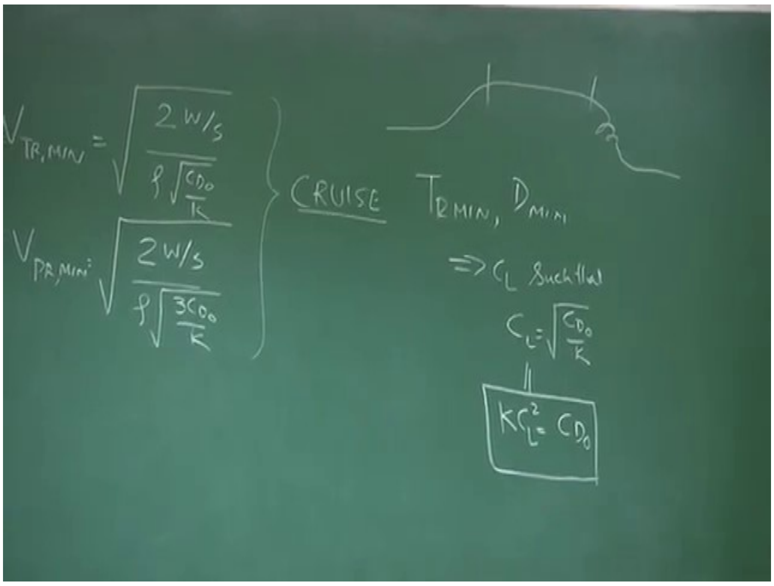

If I write those things V for thrust required minimum, the expression we got was something like this, 2 W by S and rho CD0 by K and V for power required minimum was 2 W by S rho under root 3 CD0 by K.

$$V_{T_R,MIN} = \sqrt{\frac{2W/S}{\rho\sqrt{\frac{C_{D_0}}{K}}}}$$

$$V_{P_R,MIN} = \sqrt{\frac{2W/S}{\rho\sqrt{\frac{3C_{D_0}}{K}}}}$$

And please remember, these are all pertaining to cruise flight, this we should not miss cruise. And more pictorially, we know that these are the basically schematic of different flight path and airplane will be taking and we are concentrating here, which is the cruise phase.

So, the message here is very clear, for a designer if I want to fly the machine for a cruise and I want to ensure that, thrust required is minimum that means drag is minimum. Then, I need to fly at a CL, thrust required minimum, which is basically drag minimum, I need to fly at a CL, such that CL equal to CD0 by K, which actually means, K CL^2 is equal to CD0. This is K CL^2; this is the induced drag component, why this induced drag is there; that is because of lift.

So, you call it lift induced drag and at a speed when this induced drag component and parasite drag component in terms of coefficients, if they are same and at that point, if I flying the machine, which actually means I flying at a CL equal to CD0 by K, which also means I am flying at a velocity 2 W by S by rho CD0 by K under root, I will be getting thrust required minimum. If I have a closer look here, I could see that, if W by S if the wing loading is less, then velocity or speed required for thrust required minimum also will be less.

Similarly, for speed for power required minimum, if wing loading is less, the speed for power required minimum also will be less. And when I say speed will be less, it has a direct impact on the engine; that means I will be requiring a lesser rated engine. So, the engine weight will be less and engine complexities will be less. So, designer will depending upon the different conflicting requirement he has, he will actually select W by S.

And at the end of this course, you will realise that how do I select a wing loading for a particular airplane, which should satisfy best of all those flights phases or sometimes or many a times, the designers has to give a weightage, what is the primary role of this machine, if it is for a cruise for a transport plane, he may give more weightage here. If it is a fighter airplane, he may give more weightage towards the manoeuvre or take of distance or landing distance. So, all these things will come. But before we take educate ourselves to take a decision on this, we are trying to build the basic understanding, right.

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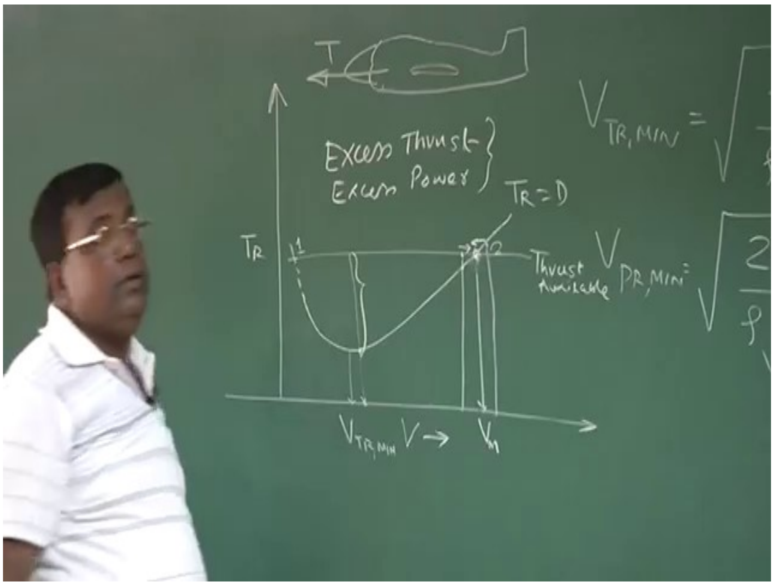

Now, let us see, we have also seen this thrust required versus speed or velocity, the graph typically goes like this and this is the point, where the V for thrust required minimum. Please remember, thrust required is equal to drag. So, when I say thrust requirement minimum, I am implicitly say this is telling that, the drag required minimum. Now, come back to the engine, the airplane has an engine, it will be delivering some thrust.

Typically, let us say I am drawing typical representation of thrust from a jet engine, turbo jet type, I can fairly assume, it is constant and it remain almost constant. Though in practice, you will find, there are secondary influences on the speed as well. Now, see there is a two interesting point, suppose I allow me to extend it like this. So, I have got one point here, I have got one second point here.

Let us try to understand, what are these points, what is this point, at this point, you could see that, the drag required which is given by this graph, because this is thrust required and we know thrust required equal to drag. So, at this point, whatever engine is applying; that is we call it thrust available, at this point, they are same; that means, if I am flying at this speed, this is a maximum speed and I will be going with without any acceleration, constant speed.

But, if I want to fly at some point here, you could see here, here I see that, if I am flying at this speed, there is an excess thrust, this excess thrust can accelerate the airplane, it can lift of the airplane. So, many things it could do. So, this is a one concept of excess thrust.

Similarly, we will see excess power, these two are very important concept and we see how this concept can be utilized to analyze an airplane performance in terms of rate of climb, in terms of acceleration, without changing the altitude, all those things will be discussing.

But, before you go to that let us ask a question, let us say the airplane was flying at this point 2 and because of certain reason, the velocity has increased. Is this is clear? Suppose it was flying at speed, let us say I call it VM (V_M) and because of certain reason, the velocity has suddenly increased. What this graph tells you? The moment the velocity increases, you could see now the drag is more than the thrust, available from the engine. So, it will automatically decelerate.

So, it will have a tendency to come back to this point and suppose for some reason, it is now flying at a lower speed, then what happens, here the thrust is more than the drag. So, it will accelerate the airplane towards this point 2. So, we say as far as velocity stability is concerned; that is statically stable at point 2. I repeat again, suppose I am flying at point 2, if for some reason the velocity has increased. From this graph you could see, now at this point, this drag experienced by the airplane would be more than the thrust given by the engine.

So, it will start decelerating, so it will have a tendency to come back to this point. Similarly, if it is flying at a speed less than VM, the thrust will be more than the drag experience by the airplane. So, it will have a tendency to go back to this point 2. So, we say at point 2, the aircraft is statically stable in terms of speed is concerned. But, if I try to see here at point 1, suppose if I am flying at point 1, here also thrust and drag are equal.

If for some reason, there is an increment in the speed, what will happen, see any point beyond this, the moment there is an increment, the trust is always more than the drag. So, the airplane will go on accelerating and this point will never come back to this point 1. So, at this point, it is not statically stable in relation to the speed. Also, there should be another point, it should come to our mind, for any airplane there is something called a speed called Vstall which we have discussed.

Vstall there is a speed, the lowest speed with which, it can manage lift equal to weight; that means, he is actually putting CL to CL max, he has put all the flaps down and angle of attack maximum.

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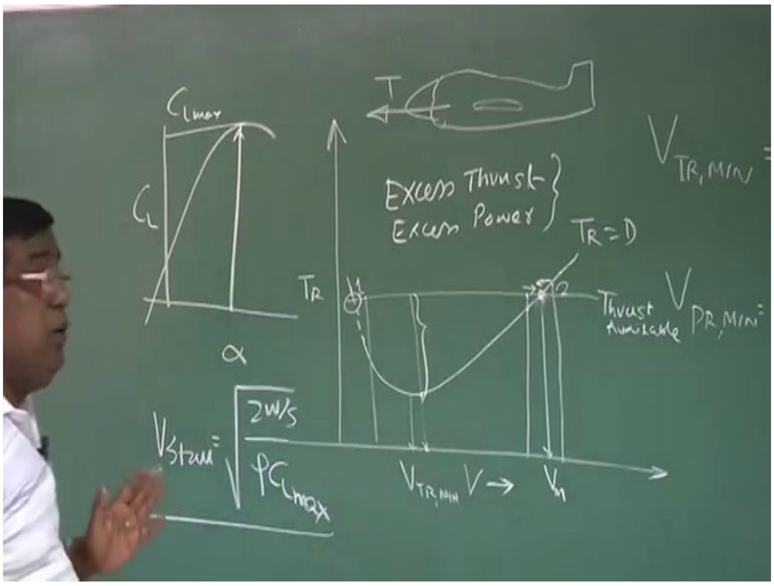

If we recall, this is CL versus alpha if something like this, if you have flying somewhere here. So, this value CL max $\left(C_{L_{max}}\right)$ and Vstall (V_{stall}) will be equal to 2 W by S, rho CL max.

$$V_{stall} = \sqrt{\frac{2W/S}{\rho C_{L_{max}}}}$$

So, this decides your minimum speed limit that, if the airplane goes below this speed, it is of no worthy as far as flying is concerned. That means, for all practical purpose, for us the relevant speed or velocity will be which are greater than Vstall or greater than equal to Vstall.

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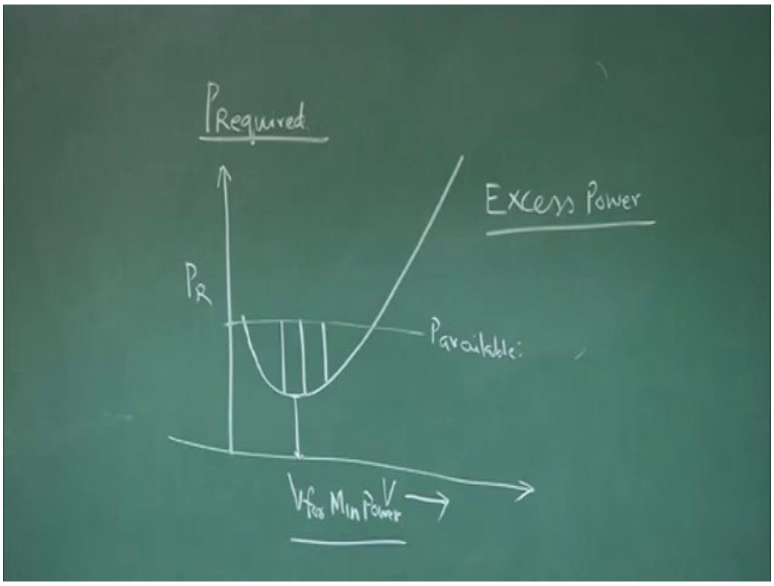

Now, if you again revisit power required versus speed or velocity, graph is also looks like this. And let us say, this is the power available, typically, a propeller IC engine combination, where the power available almost remain constant for practical purpose, as far as understanding the basic physics behind it. In actual practice, please remember, there are secondary effects, we will be talking later.

Now, let see here, here this gap, by now we know that, this is the V for minimum power. But, what are these lines? This gap is as I was mentioning this is excess power. And let us see, how can we utilise this excess power in adding value to the performance of an airplane, when I say power available, please also understand. When, I say power available or thrust available, they all will go on changing with altitude.

So, whenever I draw a diagram it is for a fixed altitude, for all our convenience in your discussion, you can take this as a sea level condition and as a lecture progresses, we will give you a correction to it as we gain the altitude, the physics does not change.

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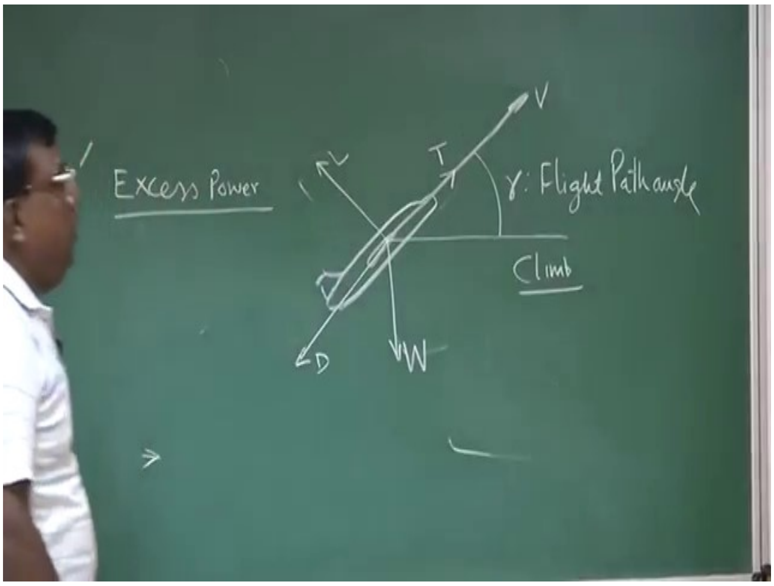

Let us draw this diagram, this is a typical schematic representation of an airplane, which is not cursing, it is now climbing, you say it is climbing or flight phrase is climb. Let us see here, if I now try to write the force balance along the velocity direction, along this direction, what do I get? I get of course, there is a thrust.

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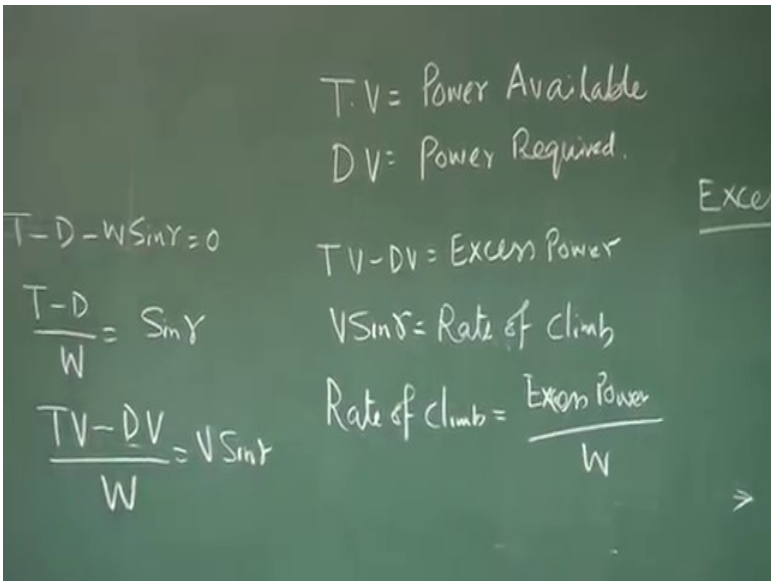

So, I get T minus D minus W sin gamma, is equal to m dv by dt.

$$T - D - W\sin\gamma = \frac{mdv}{dt}$$

What are the assumptions here, when I written this equation? First of all, I am assuming that thrust and velocity in same direction. Number 2, I am assuming that, the mass is remaining constant and of course, alpha is very small. So, if this is the equation and if I am discussing only static performance, which actually means that steady state performance, so we are not taking any acceleration. Then, I can simplify this equation from T minus D minus W sin gamma is equal to 0.

$$T - D - W \sin \gamma = 0$$

There is no acceleration, this is a static performance. So, what happens now? T minus D by W is sin gamma. What is gamma? Gamma is a, we know very well, gamma is a flight path angle; that is the velocity vector mixed with the horizontal plane. So, how it is climbing?

$$\frac{T - D}{W} = \sin \gamma$$

$$\gamma : Flight\ path\ angle$$

The velocity vector with the horizontal is the flight path angle or climb angle. So, what is a message from here? If I want to fly the airplane at a particular climb angle, I should ensure that it has enough thrust to compress a drag and this relation should be valid.

If I now multiply both sides by V, what happens? This will be T into V minus D into V by W is equal to V sin gamma, I multiplied both side by V. What is this T V by D V by W, let us understand this. T into V is what is basically thrust into velocity or speed, so this is the power available; that is power available from the engine at a given altitude. What is D into V; that is the power required. So, what is T V minus D V? So, T V minus D V is an excess power.

Remember, we are talking about excess power. So, now, I and what is V sin gamma? See here, if this is V, I can resolve this into components, V sin gamma, it is a vertical component. It is the speed at which the airplane is going up and that is called rate of climb. So, now if I, use this equation, I can write rate of climb equal to excess power divided by W.

$$\frac{TV - DV}{W} = V \sin\gamma$$

$$TV : Power\ available$$

$$DV: \ Power\ Required$$

$$TV - DV : Excess\ Power$$

$$V \sin\gamma : Rate\ of\ Climb$$

$$\therefore Rate\ of\ climb = Excess \frac{Power}{W}$$

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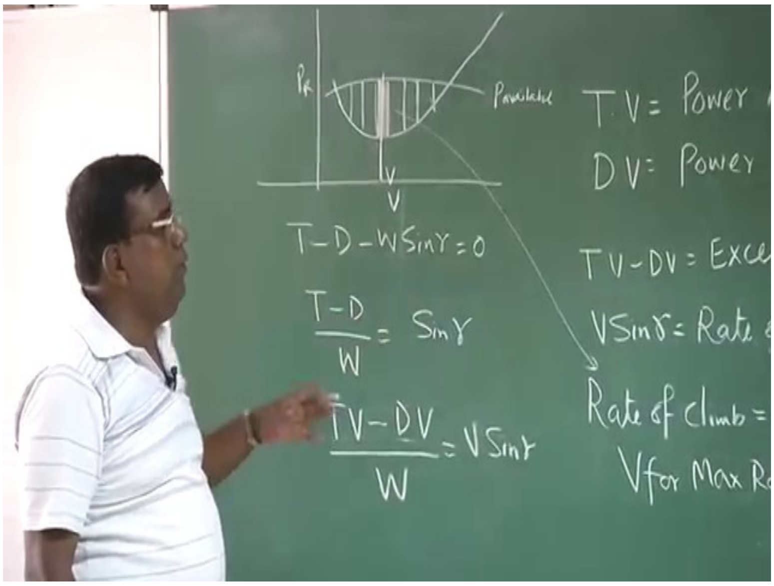

Now, if you again go back and recapitulate, what was the diagram, this was power required. So, speed and let’s say, this was the power available, then this gap, they are the excess power, this is what is this and this excess power is linked to rate of climb. So, now, understanding, this understand this diagram, can you answer a question at what speed should I fly to get maximum rate of climb, you could see here the excess power goes on increasing and comes to a maximum value here.

And then, again goes on decreasing, that as you know rate of climb is linked to an excess power. So, I know that V, where the excess power is maximum is that speed, I should fly to get rate of climb maximum. So, could you understand the linking of excess power to rate of climb, this concept will be utilized in defining a few more parameters, which are very, very important for aircraft performance.

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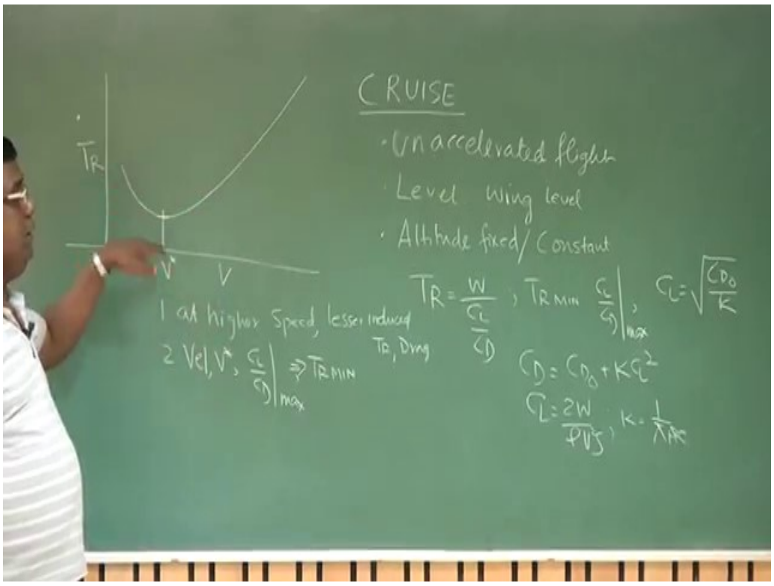

Yes, welcome back to this additional session. Let us revised, what we have covered so far, we were focusing on cruise flight. And what do you understand by cruise flight, cruise is, it is a unaccelerated flight. Number 1; unaccelerated flight and when you say level cruise, level means wings level; ok, so typically it is like this. If I am doing a cruise, I am going the constant speed, wings level not like this, wings level like this and also I am ensuring that, there is no change in the altitude.

So, I say third point the altitude fixed or remain constant, this we have understood and once we understood, what is a cruise flight, we try to understand, what is the thrust required to maintain a cruise flight. And in that, we realise that, thrust required can be model as CL by CD, W divided by CL by CD and we also understood that to have thrust required minimum CL by CD should be maximum. And for that, we realise that to have CL by CD maximum, to have thrust requirement minimum, we need to fly a particular CL, which is given by CD0 by K.

$$T_R = \frac{W}{\frac{C_L}{C_D}};T_{R,MIN} \Rightarrow \frac{C_L}{C_D}\Big|_{max};C_L = \sqrt{\frac{C_{D_0}}{K}}$$

Then, we try to understand, how can I plot the variation of thrust required verses speed and we realise that thrust required will follow this trend and we also understood at higher speed. Since, CL required for maintaining lift equal to weight is less, it will have lesser induced thrust, thrust required or drag, which are same. And typically there is a point, which is if I fly at a velocity as given V* or which CL by CD is maximum, then I have thrust required minimum. I also understood how to calculate thrust, because we know thrust is W by CL by CD.

And from drag polar, I know CD equal to CD0 plus K CL^2 and for level flight CL is nothing, but 2 W by rho V square S. So, given weight, given speed, given density; that is that what altitude is flying. Even, the wing area, I know CL, K I know because K is 1 by pi aspect ratio e. So, I can calculate CD as I also know the CD0 is fixed for an airplane for at low speed. So, once I know CD, once I know CL, I know CL by CD, I can find thrust required at different speeds and I have plotted the thrust required different speed in this figure.

$$C_D = C_{D_0} + KC_L^2$$
$$C_L = \frac{2W}{\rho V^2 S}; K = \frac{1}{\pi ARe}$$

And in that we found out, if there is a particular speed at which thrust required is minimum and that corresponds to a case, where CL by CD is maximum and for that CL is CD0 by K. Once, we are through with thrust required, we try to also understand, if this is the engine, which is supplying thrust from the engine say for typically a jet engine, then the point of intersection here will give me the maximum velocity.

That is a time, when I have put the throttle of the engine full and at that point, the thrust available and drag, or thrust required same. So, this is my Vmax, this also we have understood.

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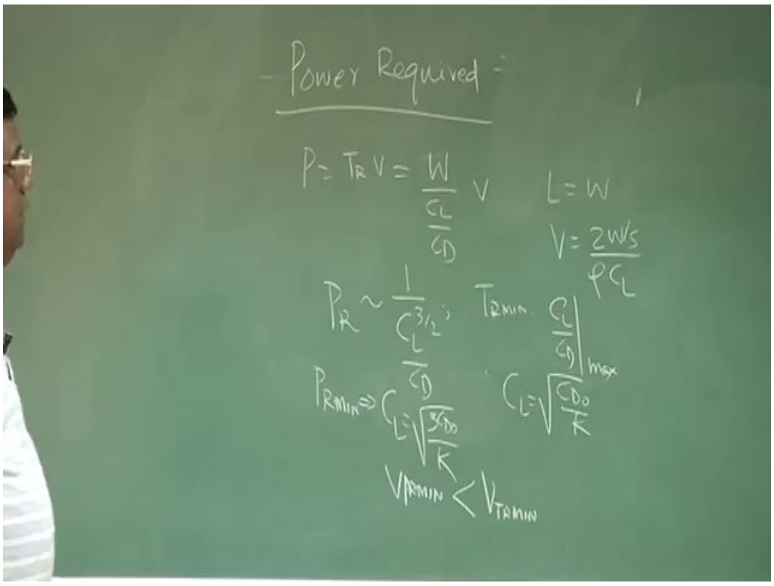

After that we went for power required, it was straight forward, because this is a power required at a cruise; that means, power required with the constant velocity for that power required is thrust required into V. And we already know thrust required as W by CL by CD into V and V, I also know, because it's a cruise. So, lift equal to weight. So, V is nothing but, 2 W by S by rho CL.

So, once I know V, once I know CL by CD, I can find the power required and in this, when we substituted V here, we realise power required will be proportional to CL 3 by 2 by CD and for power required minimum, we realize that CL 3 by 2 by CD should be maximum. Unlike, for thrust required minimum, it was CL by CD maximum and like for thrust requirement minimum, this translated into CL equal to CD0 by K under root for power required minimum, we found this translated into CL equal to under root 3 CD0 by K.

$$P_R \sim \frac{1}{\frac{C_L^{3/2}}{C_D}}\,;\left.\frac{C_L^{3/2}}{C_D}\right|_{max}\,;T_{R_{MIN}} = \left.\frac{C_L}{C_D}\right|_{max}$$

$$P_{R_{MIN}} \Rightarrow C_L = \sqrt{\frac{3C_{D_0}}{K}};\;\; T_{R_{MIN}} \Rightarrow C_L = \sqrt{\frac{C_{D_0}}{K}}$$

We also realised that, if I want to fly at CL equal to CD0 by K; that is thrust required minimum. Then, the CL require to fly a thrust require minimum is lesser compare to CL required to fly at power require minimum. Therefore, the velocity require to fly at some thrust required minimum will be higher compare to velocity require to fly a power required minimum. So, V power required minimum is less than V thrust required minimum.

$$V_{P_R,MIN} < V_{T_R,MIN}$$

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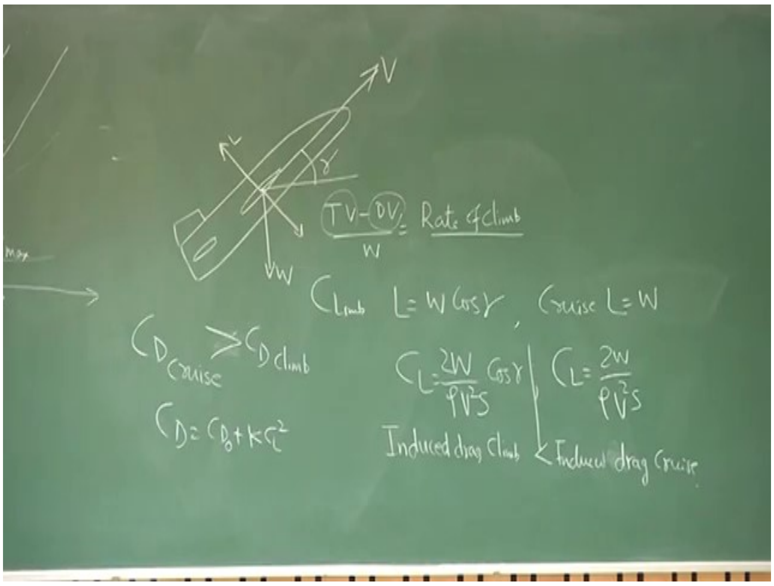

Once, we understood this, we also try to see, what happens if an airplane goes on climb and there, we also realize that, that TV minus DV by W we got as rate of climb. This TV minus DV is nothing but, the excess power. What is TV, TV is the thrust available into V to power available and what is DV, DV is the power required to overcome the drag. But, there is a catch, when I am doing a climb, let us understand, when I am doing a climb, then lift is W cos of gamma or gamma is the fight path angle, when it is the cruise then lift equal to weight.

So, you could see that CL is 2 W by rho V^2 S into cos gamma and for CL in this case is 2 W by rho V^2 S. So, which one is greater, since here is the cos gamma, you could see clearly that CL during climb is less compare to CL during cruise. That means, the induced drag for climb is less than induced drag cruise; that in totality means CD cruise is less than is greater than CD climb.

$$Climb : L = W \cos\gamma \quad ; Cruise : L = W$$

$$C_L = \frac{2W}{\rho V^2 S}\cos\gamma \ ; \qquad C_L = \frac{2W}{\rho V^2 S}$$

$$Induced\ drag\ climb < Induced\ drag\ cruise$$

$$C_{D_{cruise}} > C_{D_{climb}}$$

So, when I interpret this, this you understand, because CD is nothing but, CD0 plus K CL square. So, that means, if I come back here, I need to understand that D into V in this case is not the power required for cruise, it is power required for climb. This distinction we must have, although for gamma less than 15 degrees, for all practical purpose, they are same or comparable.

So, you do not make such of a mistake, but fundamentally, we need to understand that while we climb D into V is the power required in climb, which is less compared to power require in cruise, keeping other thing same.

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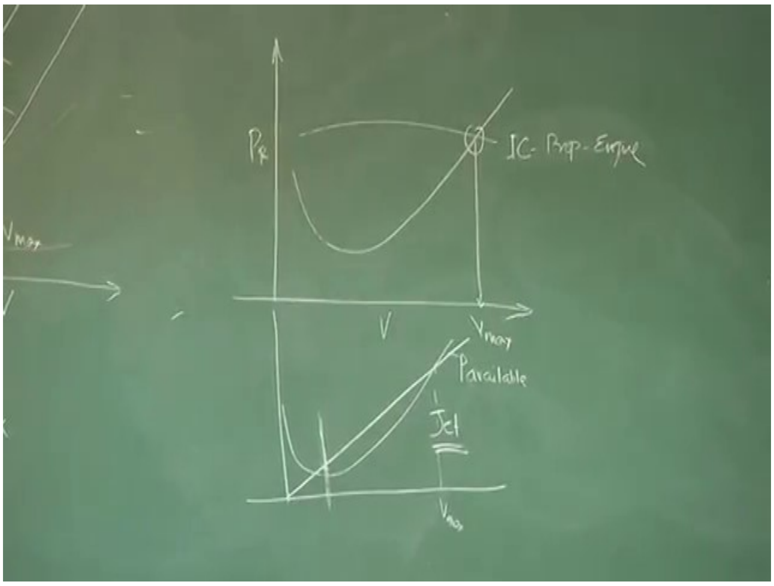

We also realized that, this power required verses V, if I draw like this and if this is typically for a IC driven propeller combination engine, this is the Vmax and for jet engine, if you want to draw it like this. Again, this is the power required, the jet engine, this is the power available; this is for jet engine to see this point is the Vmax. But, as a designer, we need to also that if I am coming for a landing, I will be operating somewhere here. So, I have got enough excess power.

So, in case of any eventuality, I can use that excess power to come out of it. However, when I am using a jet engine, you could see that this excess power near the stall or nearby landing that time the excess power relatively is less compare to for such engine. So, this is one distinction a designer need to keep in mind.

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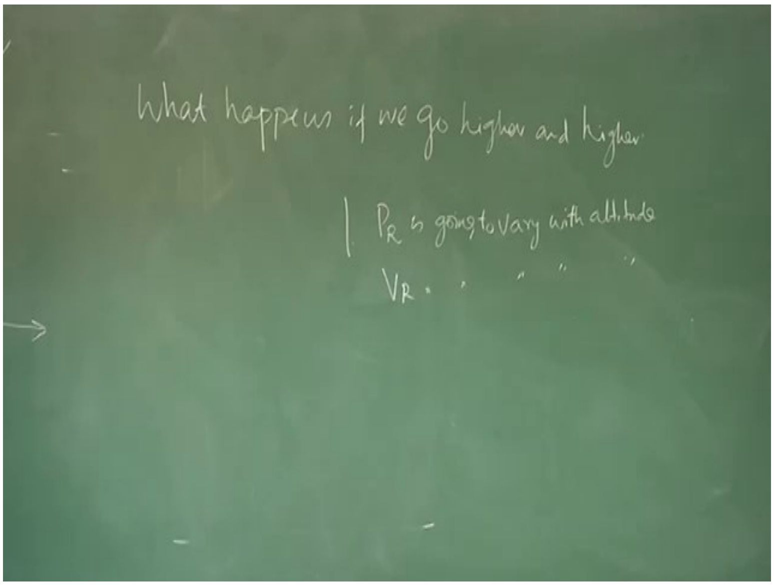

What happens, if we go higher and higher; that is more specifically, how the power required is going to vary with altitude, how the velocity required is going to vary with altitude.

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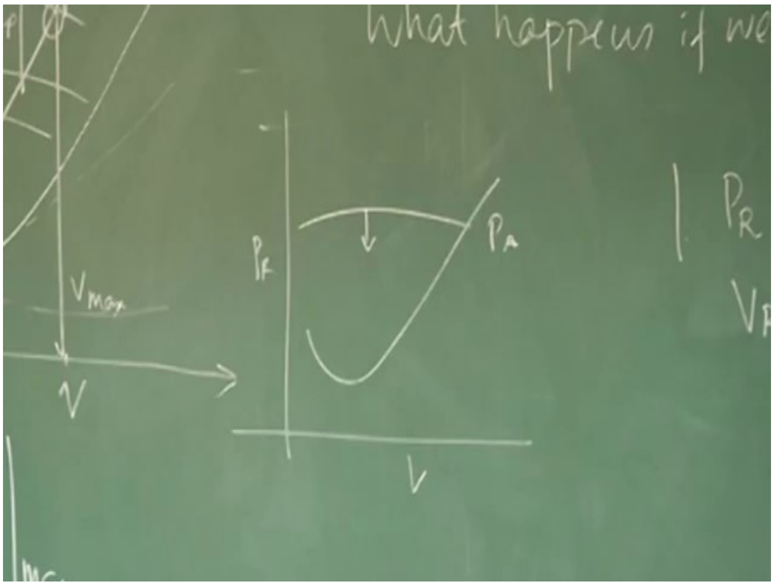

So, this is important, because please understand, we are writing power required verses V plotting it like this, this is fixed, one altitude. As I am going higher and higher, the density of air is reducing, so that, will have an effect on this. Second thing power available again as we going higher and higher density is going to reduce. So, this also will have an effect on this, this will try to come like this and this will also try to shift. Let us see how I can we can guess, we can estimate how this variation will, change with altitude; that is exactly will be doing.

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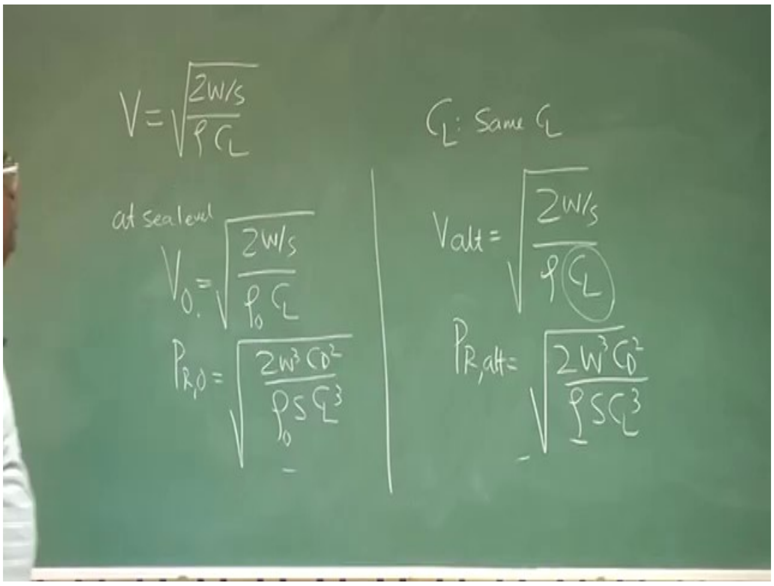

V, we know for cruise is 2 W by S by rho CL. So, let us say at sea level, we denote it as V0 as 2 W by S rho0 CL. So, this V0 this represent sea level condition, rho0 is sea level density. Similarly, power required at sea level will be, we know this expression 2 W^3 CD^2 by rho0 S CL^3.

$$V = \sqrt{\frac{2W/S}{\rho C_L}}$$

$$At\ sealevel: V_0 = \sqrt{\frac{2W/S}{\rho_0 C_L}}; P_{(R,0)} = \sqrt{\frac{2W^3 C_D^2}{\rho_0 S C_L^3}}$$

Now, let us assume that we are still flying at same CL, but where going higher. So, different altitude, so what does it mean physically, also if I am coming same CL, let say 1 kilometre, I am having lift equal to weight. Now, I want to fly at 2 kilometre, so lift still equal to weight. So, for that since density has reduce, I must compensate that loss of lift through velocity increasing the speed if I am keeping the CL same. Because, CL is achieved by the orientation of airplane, whatever CL is required, I have orient the airplane in such a way, I have to get such an angle of attack. So, that CL is what CL desired.

Now, we are keeping the same CL. So, we have compensate through velocity. So, now we can do for same CL, we can write V altitude is equal to 2 W by S by rho CL. So, I am not changing CL, because we are trying to see that, what is the effect if you keep the CL same, but you can notice that, now rho is not rho0, rho is rho at some altitude. Similarly, power required at altitude will become 2 W^3 CD^2 by rho S CL^3., again note that it's rho not rho0.

$$At\ altitude : V_{alt} = \sqrt{\frac{2W/S}{\rho C_L}}; P_{(R,alt)} = \sqrt{\frac{2W^3 C_D^2}{\rho S C_L^3}}$$

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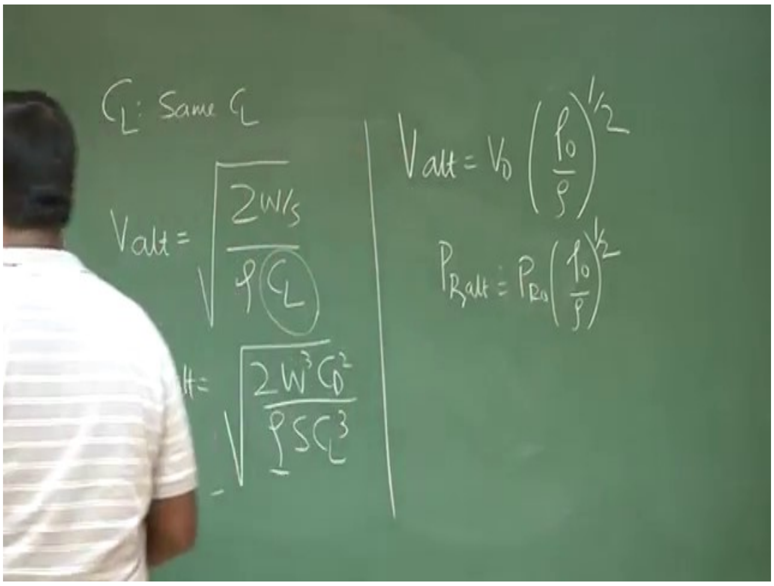

So, using these two equations, I can write in a combined way V altitude equal to V naught, rho0 by rho to the power half and power required at altitude is equal to PR0 into rho0 by rho to the power half.

$$V_{alt} = V_0 \left(\frac{\rho_0}{\rho}\right)^{1/2}$$

$$P_{R_{alt}} = P_{R_0} \left(\frac{\rho_0}{\rho}\right)^{1/2}$$

What is the meaning of that, it is very clear as I am going higher and higher, this rho value is going become lesser and lesser. So, this ratio will go on increasing I need higher velocity to fly at same CL to maintained lift equal to weight, similar story also here.

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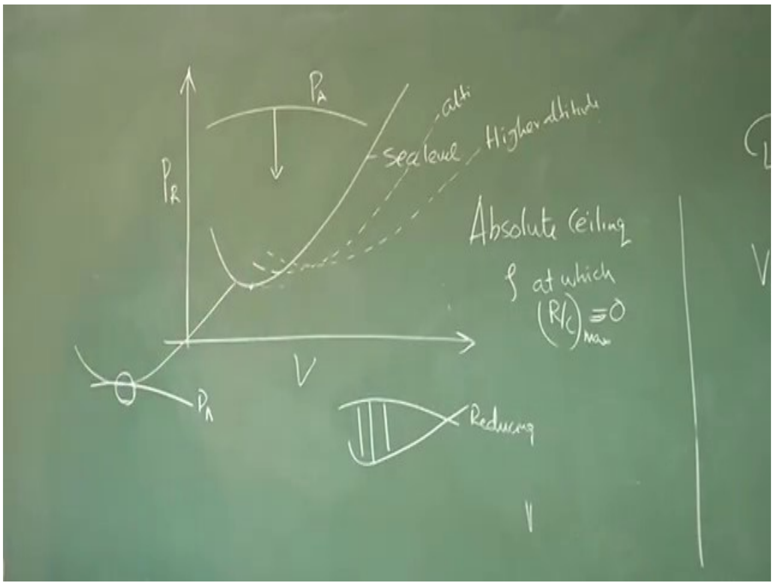

So, the power required suppose this was sea level as I write sea level condition, I can see from there, as I am going to fly at a higher altitude keeping same CL, the velocity which was here, now I require higher velocity. So, you will find the graph, this is altitude, then further, this is higher altitude. What happens to thrust or power available, the power available let us say here, typically for IC engine, propeller combination.

As I go higher and higher, this man goes down, this man rotate upwards. So, there is as more and more, you go this gap will go reducing. So, there will be an altitude, where will find if this is a power required the power available is just matching here and this is the point, where you have absolutely no excess power for rate of climb. And that is defined in our aerospace industry as well as absolute ceiling; that is we call absolute ceiling that is reach that altitude at which the rate of climb, maximum rate of climb is 0.

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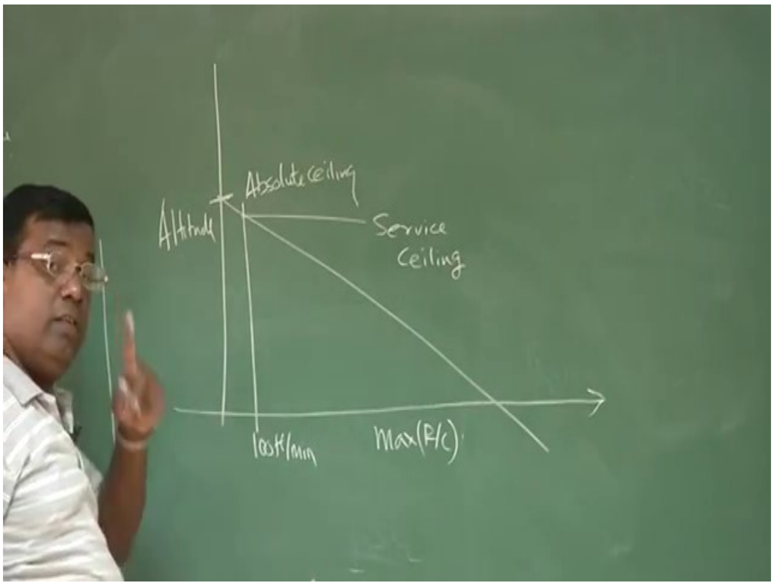

So, how it is generated, if I draw a graph, this is altitude variation can be and this is maximum rate of climb, graph is something like this, here this altitude is called absolute ceiling and we for practical purpose. We define an altitude, which is called service ceiling is called service ceiling, service ceiling is an altitude at which the maximum rate of climb is not 0. By it is 100 feet per minute for all operation purpose, we use service ceiling not absolute ceiling.