NOC: Introduction to Airplane Performance
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Lecture - 14
Thrust Required: A Closer Look

Good morning friends, we have been talking about Thrust required, Power required. We have talked about for thrust required CL by CD should be maximum, for power required CL 3 by 2 by CD should be maximum. Now, by now aware, what will be the speed? I will be going little one step ahead in understanding, what is the meaning of thrust required minimum via CL by CD maximum.

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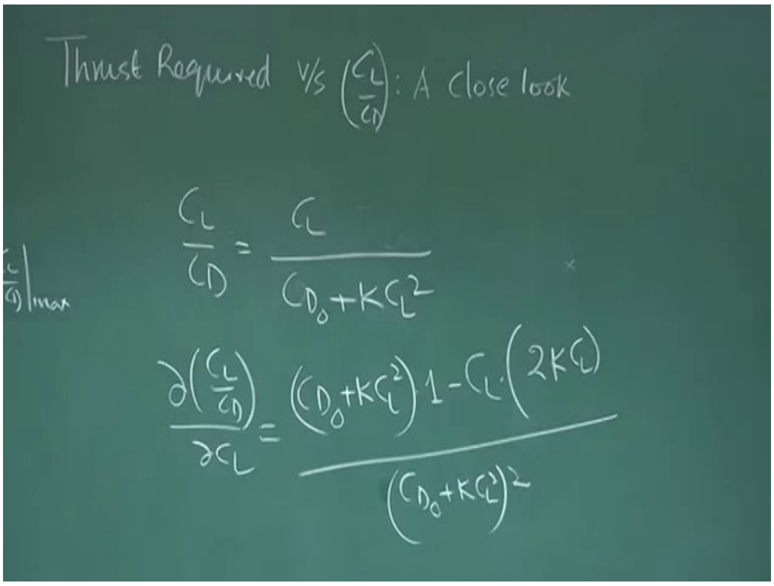

So, this is thrust required versus CL by CD a closer look. Let us see, we are very comfortable writing this W by CL by CD and this means, thrust required minimum implies, I must fly at CL by CD maximum. Let us do little bit of algebra or some sort of a and little mathematical insight into this CL by CD max. See, you know CD is CD0 plus K CL square and we know that, this is parasite drag and K CL square is the induced drag, all these things are very clear to us.

$$C_D = C_{D_0} + KC_L^2$$

Now, if I ask a question, what is that, that CL I should fly, so that, CL by CD is maximum. If I want to find out mathematically, I can follow this few steps and get an answer. This I will write CL by CD as CL by CD0 plus K CL square and I will differentiate this CL by CD by CL and if I do that, I find CD0 plus K CL square into 1 minus CL into 2 K CL divided by CD0 plus K CL square, whole square.

$$\frac{C_L}{C_D} = \frac{C_L}{C_{D_0} + KC_L^2}$$

$$\frac{\partial\left(\frac{C_L}{C_D}\right)}{\partial C_L} = \frac{(C_{D_0} + KC_L^2)\,.1 - C_L.(2KC_L)}{\left(C_{D_0} + KC_L^2\right)^2}$$

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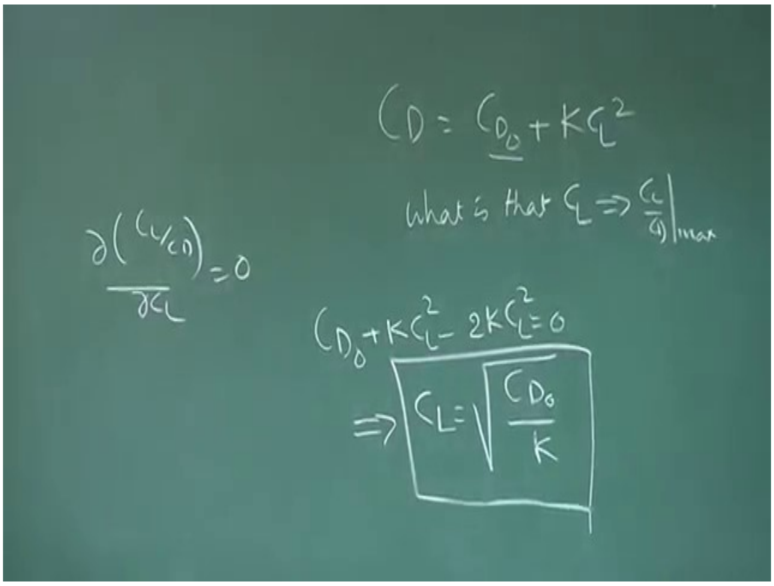

Now, next step is, I find the turning point, so I put this equal to 0. If I put d CL by CD by CL equal to 0, then I get an expression CD0 plus K CL square minus 2 K CL square equal to 0 or I get the popular expression CL equal to under root CD0 by K.

$$\frac{\partial\left(\frac{C_L}{C_D}\right)}{\partial C_L} = 0$$

$$C_{D_0} + KC_L^2 - 2KC_L^2 = 0$$

$$\Rightarrow C_L = \sqrt{\frac{C_{D_0}}{K}}$$

As simple as that, you can check the second derivative to see is it is indeed a coordination of maxima; the second derivative should be less than 0.

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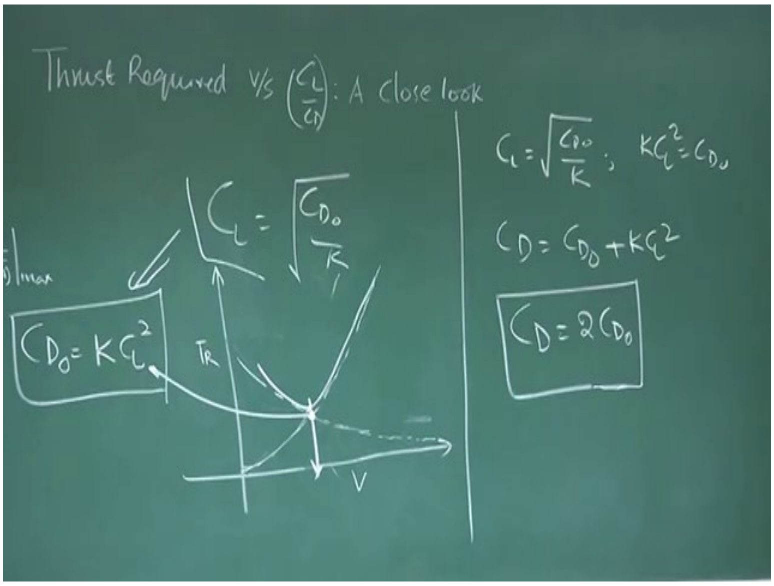

Now, the interpretation demands CL equal to CD0 by K, when I write like this, you know once I solve this what did I get, I get CD0 equal to K CL square. How do I give an interpretation? Let us draw thrust required versus speed and we by now know that, this is the induced drag or induced thrust component and another was parasite and this was the net thrust required. And this was the point where thrust required was minimum and thrust required was minimum as far we understand, this is the point corresponds to CL by CD maximum.

But, from this analysis or this one, what did I get that is the point where the parasite drag equal to induced drag, this is in terms of coefficient. So, I call CD0 equal to K CL square and you could see that, this is indeed the point of intersection of parasite and induced drag. So, this point and this point are same point. So, we are now trying to add little bit of mathematical value to our understanding.

Second thing, so if I am flying at CL equal to CD0 by K; that means, what is the CD I am flying actually, I know CD equal to CD0 plus K CL square. Since, at this point CL is nothing but, CD0 by K or K CL square is nothing but, CD0. So, my overall CD becomes twice CD0, this is very important.

$$C_L = \sqrt{\frac{C_{D_0}}{K}}\ ; KC_L^2 = C_{D_0}$$

$$C_D = C_{D_0} + KC_L^2$$

$$C_D = 2C_{D_0}$$

So, you can understand, if we are flying at thrust required minimum; that means the airplane designer should ensure that CD0 should be as low as possible.

That means, it calls for aerodynamics to work more and make sure the drag component at alpha equal to 0, which is CD0 or CD at CL equal to 0, CD0 should be as low as possible. It should have a lesser skin friction drag, less lesser pressure drag and that is what is important for our interpretations; that one should be very, very careful and translate this to designers mind.

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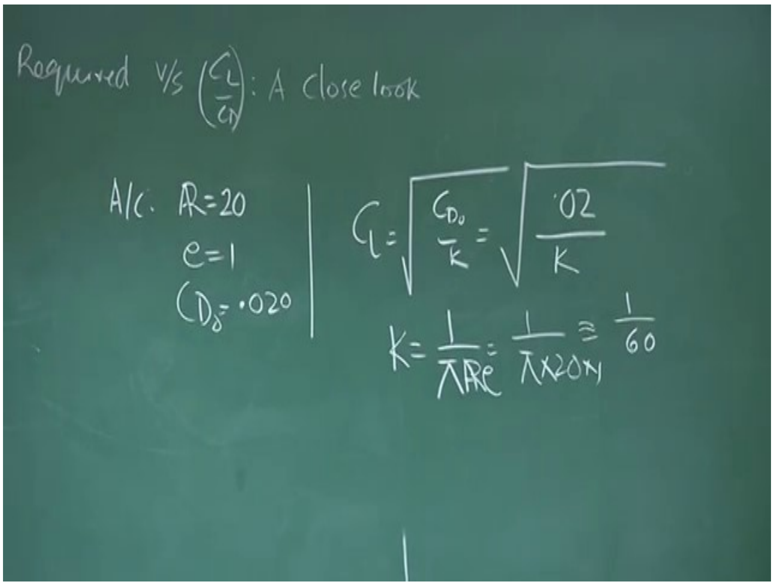

Now, the second part we will see, see for flying with thrust required minimum, we have this flying with power required minimum, we have already seen that, it is 3 CD0 by K.

$$T_{R,MIN} : C_L = \sqrt{\frac{C_{D_0}}{K}}$$

$$P_{R,MIN} : C_L = \sqrt{\frac{3C_{D_0}}{K}}$$

Let us take an example, let us take an airplane, whose aspect ratio is let us say 20, large aspect ratio like a glider and large aspect ratio, why we are tempted to a large aspect ratio.

Because, if we are thinking one dimensional that induced drag should be low, induced drag should be low, then one of the option is make the aspect as high as possible.

So, it is aspect ratio is 20 and let us take e is 1, this is elliptic distribution and CD0 realistic value, I will take as 0.020, which is little oversight, but a good airplane will have this range of CD0. If this is the data I have or what does this mean as far a CL is concerned, CL is CD0 by K; that is for thrust required minimum. So, that will be 0.02 divided by K. What is the value of K? K is 1 by pi aspect ratio e and that is 1 by pi into 20 and into 1, I have taken the value of e assuming perfect elliptic distribution.

So, K is roughly equal to 1 by 60, the approximation with pi I have taken just 3, not 3.1416. So, if I put that value of K in this expression, what the CL I get, this 0.02 into 60 and that is, if I calculate, it will be around 1.1, because this is around 1.2, under root of 1.2, 1.1. So, what is the message, message is, if I want to fly at thrust required minimum, the CL should be 1.1.

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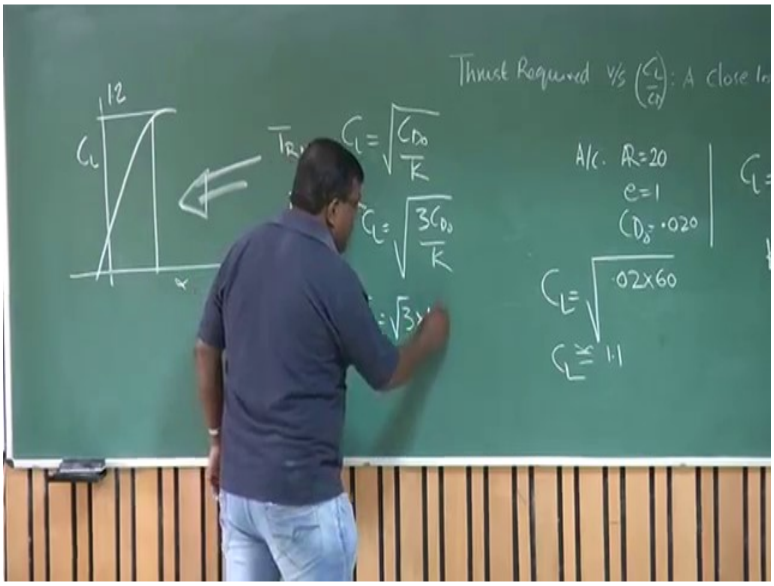

Now, the moment CL is 1.1, I get an alarm, I immediately see, what is the CL versus alpha for my aircraft and we know that, this maximum value is for normal case without any high level devices, etcetera. This value is typically for most of the business aircraft of six seaters, seven seaters, eight seaters, they are around 1.2. Even, without any high lift devices most of the airplane will have CL maximum value of around 1.2.

Now, you are demanding the pilot to fly at CL 1.1, see the problem slight, already he is operating somewhere here. So, he is near this region. So, just putting up a condition that CL you need 1.1 is not sufficient. So, your design should ensure that, whatever CL you are demanding to get thrust required minimum, it should be consistent with this back of mind, are you able to get this understanding.

Because, you see the seriousness, if I want to fly at CL power required minimum, then this CL value will be furthermore root 3 times 1.1, where from you will get that CL from normal aircraft. So, the question come, should we go for aspect ratio 20, although induced drag was reducing, but there are so many other issues. So, as a designer you have to be multidimensional, you cannot go on giving maximum weightage to only one parameter; that is why I wanted to share this experience with you.

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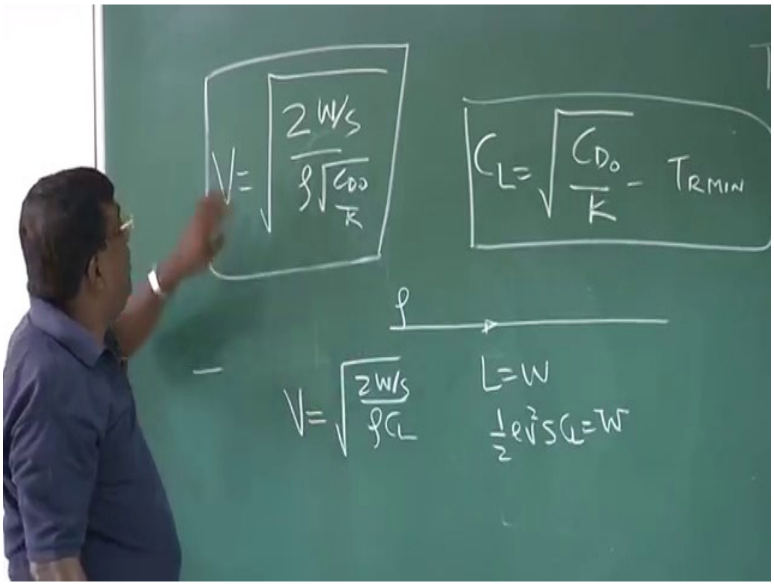

We should also be careful to know, what are the other salient features, which is to be known before I talk about fly at CD0 by K, etcetera, this is for thrust required minimum. This is fine that you need to fly at CL equal to CD0 by K to ensure thrust required minimum, nobody is contesting that. But, what is our main purpose? Main purpose is, I want to fly at a particular altitude. Let say that altitude is rho, I am talking about density.

My purpose is, I have to ensure that lift equal to weight; that is I have to ensure that half rho V^2 S CL is equal to weight and that means, I need to fly at 2 W by S by rho CL and this CL is nothing but, dictated by this for thrust required minimum. So, this is 2 W by S, rho CD0 by K. Now, see interesting part, if we are flying at a particular altitude, the velocity should be this much.

$$V = \sqrt{\frac{2W/S}{\rho C_L}}$$

$$V = \sqrt{\frac{2W/S}{\rho\sqrt{\frac{C_{D_0}}{K}}}}$$

To ensure that thrust required is minimum, when you are flying means you are cruising at altitude where density is rho, but this will give some value of V. Now, where from I will get the V, the moment there is the V, I know that I need to have an engine, which should be able to give me enough power or thrust to generate that V. That is thrust equal to half rho V^2 S CD, I need to have enough thrust, so that I get this much of V, because as the V is increased or for a particular V, this airplane will experience a drag.

So, I have to ensure that, I have my engine is enough powerful to give that much of thrust. So, there again I go back to the engine site, whether really this velocity is achievable or not. If it is achievable fine, if it is not, then I will be, if we want to reduce this V, I will try to play around with the wing loading W by S, I will try to reduce the wing loading.

And that is how this iteration will go on happening and finally, you get a configuration, where you are giving enough bandwidth to your pilot, so that, he can get those solution by looking at the instrument getting those numbers and manoeuvring the airplane to that conditions. Because, if your design does not have those conditions, which can be expected by a pilot, then he will not be able to fly at that conditions. So, it is a primary role of a designer to make sure, you have given all those solution in the design itself and the pilot starts gaining into those domain and try to fly the way it should be flown.