NOC: Introduction to Airplane Performance

Prof. A. K. Ghosh
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture - 15
Modelling of CL: Dimensional Analysis

Good morning friends, we have been seeing that the major friend for aerospace or for an aircraft is the lift, right. So, we need to go little step more into this lift formulation, when I say lift equal to half rho V^2 S CL, how people, how the researcher got that insight. And I personally believe that we need to use dimensional analysis approach to get the insight the whole physical problem. It will not give a complete solution, but it definitely helps in forming the mathematical model or the structure of a physical phenomena, if your intuitive power is great.

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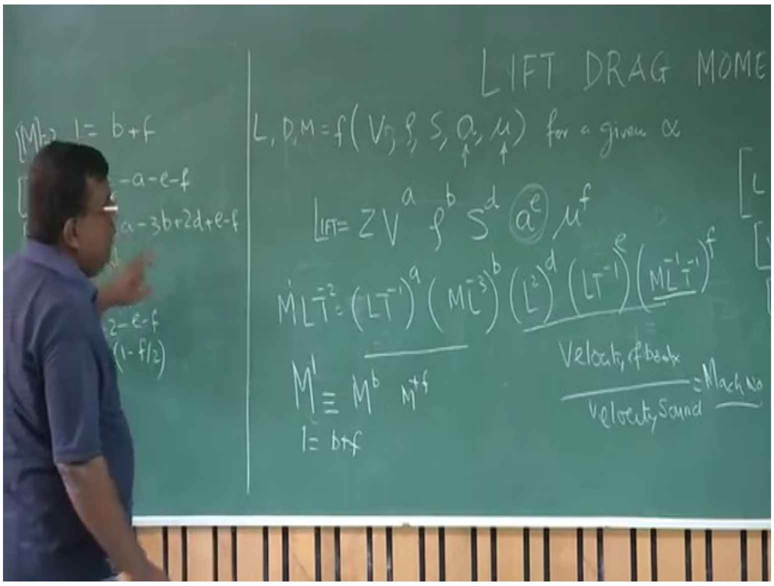

So, let us appreciate those who have those insights and helped us. Now, we use in many places to get better feel to develop a model structure, I will be using dimensional analysis assumption is this, if we have something on the left hand side and right hand side. If we are trying to develop a model, one of the necessary condition is that, their dimensions should be same; it is a necessary condition not a sufficient condition.

So, we will see that, how that can be utilized through dimensional analysis to answer this question L or CL. Question is, I should work with L or I should work in non dimensional, lift coefficient, non dimensional drag coefficient, non dimensional pitching moment coefficients. So, many others are there and why should I do that, what is the advantage I get and how do I ensure that, what are the parameters which have a physical significance on which the CL or CD or CM depends. For example, so far we have been talking about CL and alpha and we are talking about CL as a function of alpha.

The question I have asked is, for a given alpha taking into the consideration about the flow and the body interaction, what is that so called complete description, which will ensure that CL at a given alpha has been modelled accurately. Keeping in mind, there is the flow flowing over the surface, there is a boundary layer, there is a viscous force, the viscous boundary layer will be distributing, redistributing the pressure distribution, all these things.

And if it is a supersonic speed, then there is a shockwave; that shockwave will also you know alter the pressure distribution. In a holistic manner, can I think of modelling CL for a given alpha; that is the basic question. As I told you, we will be talking in terms of dimensional analysis. So, first is this lift drag and moment, we know that it will be function of velocity or speed definitely with dynamic pressure, density, if it is density is more, this resistance interaction will be more.

Then, area yes, we all agree, then this is the velocity of sound. We will come back to this and this is the kinematic viscosity or coefficient of viscosity; that is that will decide, what is the sticking property of the fluids with the body, how dense it is, it’s a coefficient of viscosity. Because, why this is important, because depending on mu the value, the interaction near the boundary layer will be decided and because of this boundary layer, the pressure distribution also will get reconfigured.

$$L, D, M = f(V, \rho, S, a, \mu) \ : for\ a\ given\ \alpha$$

So, this is also important, why velocity of sound let us think, see how do define velocity of sound, I will not go into detail on this, but I will try to build a platform. How do define velocity of sound? If there is a disturbance and it gets propagated with a speed of sound. So, any disturbance or small disturbance in a medium, they get propagated with the speed, which is equal to velocity of sound.

Now, think off, suppose there is a body moving like this and the speed of this body is less than the velocity of sound. So, what will happen, before it reaches the another particle, this disturbance will reach there earlier than the body reaches, because body speed is less than the velocity of sound. So, when it moves, the particles will arrange themselves based on the disturbances, they have received already before the body as it there.

However, think our situation, when the speed of the body is more than the velocity of sound, speed of the body is more than the velocity of sound, so what happens as it starts moving, the particle here will be restricted by the speed of sound. But, the vehicle speed is more than speed of sound, so before this particles are arranged, the vehicle will reach there, so what will happen there will be a compression of air particle, which was not happening when the velocity of the body was less than the velocity of sound.

So, because now the velocity of the body is more than the velocity of sound, so there will be a compression and that compression means lot of energy, it has absorbed from the vehicle. And hence, we say, it will increase drag or we say there will be formation of a shockwave. And the shockwave again will interact with the body and it will again redistribute the pressure distribution. So, it will have different effect on lift and drag.

So, what is important is, I need to know the what is the ratio velocity of body and velocity of sound and that is typically called Mach number. Since, this course is designed for low speed; I am not going a much detail into it. But, once you are modelling it; that is why it is important to incorporate, a velocity of sound also in lift drag moment postulation.

As we are following dimensional analysis approach, so I will now first, since this is function of this, I take an example of lift. So, I write lift is equal to Z a constant and I say V to the power a, rho to the power b, S to the power d, a to the power e and mu to the power f, where a, b, d, e, f, z are constants, they are not having any dimension.

$$L(lift) = Z\,V^a \rho^b S^d a^e \mu^f$$

Now, let us see what is the dimension of L, L is a lift here, L is the lift.

What is the dimension of lift; that is force, so mass into acceleration, so M L T minus 2, it is mass into acceleration, acceleration is V by T, V is L T minus 1, T is by T, so L T minus 2, no issues. What is the dimension of this? L T minus 1, dimension of density, this is mass per volume. So, if I write density is mass per volume and mass is M, volume is L cube dimension by. So, density becomes M L minus 3. Similarly, S is the area, it becomes L square, a is the velocity of sound, it become L T minus 1.

$$[L] = MLT^{-2}$$
$$[V] = LT^{-1}$$
$$[\rho] = ML^{-3}$$
$$[S] = L^2$$
$$[a] = LT^{-1}$$
$$[\mu] = ML^{-1}T^{-1}$$

Now, come to the coefficient of viscosity, if you recall and I am sure, if you might have seen this expression. In a fluid, the shear stress, which is developed, because of viscosity, what happens is a plate, if the fluid is flow like this, flowing like this, then on the surface, the fluid particle will get stuck and as you go upward and upward, their effect will be neutralized. So, at a certain distance, the fluid will again regain almost 90 to 95 percent of a speed or velocity.

So, there is a differential in terms of velocity as I am going perpendicular to the surface and that gradient is d u by d y and this is a shear stress.

$$\tau = \frac{\mu du}{dy}$$

So, from this, I can find out the dimension of coefficient of viscosity as M L minus 1, T minus 1 or otherwise also from basic definition, you can check the dimension of coefficient of viscosity to be this. Once, I know this, then I will compare their dimension, what I will do, I will compare their dimension.

So, for lift, I will write M L T minus 2, for V, the Z having no dimensionless, but I am comparing the dimensions, because I understand, if it is consistent, then the dimension of left hand side and right hand side should be same. So, L T minus 1 is V to the power a, then rho is M L minus 3 to the power b, S is L square to the power d, a is velocity of sound or speed of sound to be more precise. It is a speed of sound and that is L T minus 1 to the power e and mu we have discussed M L minus 1, T minus 1 to the power f.

$$MLT^{-2} = (LT^{-1})^a(ML^{-3})^b(L^2)^d(LT^{-1})^e(ML^{-1}T^{-1})^f$$

Now, we will compare the exponent, the powers of M L and T, left hand side and right-hand side. Like, we will compare the exponent of L M and T; both the sides and try to find the equation. If I see the dimension M, then what is the exponent here M to the power 1. So, one is here, here if I go, no, M is here, M to the power b, so b is the exponent. So, b here and again further I go, I go, I go, yes, here M to the power f, so f.

So, 1 equal to b plus f. Is it clear? I repeat again, here it is M to the power 1 and if you see here M has, here it is b and also from here it has minus f. So, plus f, this is M plus f. So, I write 1 equal to b plus f. Similarly, I do for T, again let us check for T, T here is minus 2, so minus 2 is here, I go here in this minus a, so minus a is here, I further go here, here, here, yes, minus e is here. Further, I go here, I find it is minus f, so minus f is here, if I check the dimension of L in the left hand side, the exponent part, I am comparing the exponent, this is L to the power 1, so 1 is here.

If I come here L to the power a, so a is here, if I come here, it is minus 3 b, you could see minus 3 b is here, if I come here 2 d, you could see 2 d is here, if I come here, it is a L to the power e. So, e is there and here, L to the power minus f, so minus f is here, this is by comparing the exponent of dimension M T and L. Now, if I do some rearrangement, I can show by doing some rearrangement that b equal to 1 minus f, a equal to 2 minus e minus f and d equal to 1 minus f by 2.

$$[M] \Rightarrow 1 = b + f$$
$$[T] \Rightarrow -2 = -a - e - f$$
$$[L] \Rightarrow 1 = a - 3b + 2d + e - f$$
$$\Rightarrow b = 1 - f$$
$$a = 2 - e - f$$
$$d = 1 - \frac{f}{2}$$

We were discussing about the utility of dimensional analysis approach to understand the CL more particularly about the form of CL; that is CL for a given angle depends upon what and try to add physics to it. By doing the earlier part of algebra, we have seen algebra a little bit of manipulation here, we have seen that, b is 1 minus f, a is 2 minus e minus f and d is 1 minus f 2.

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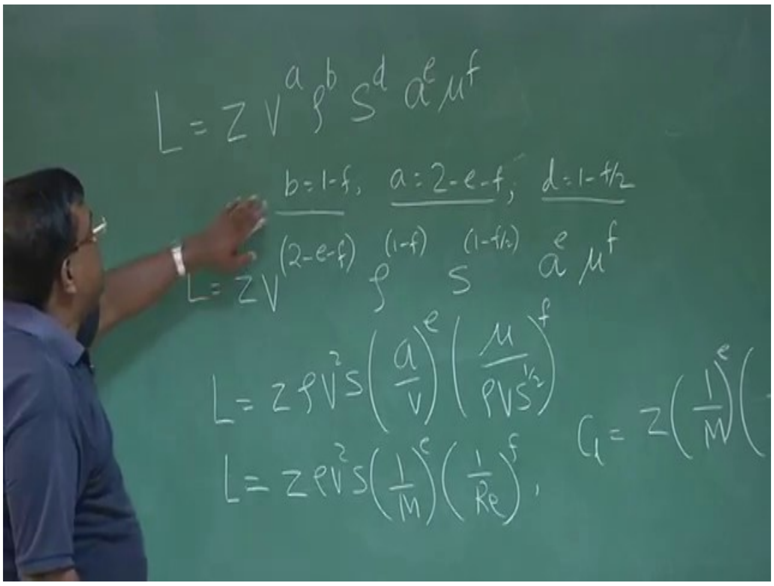

And we have already modelled lift as Z, which is a constant V to the power a, rho to the power b, S to the power d, a to the power e and mu to the power f and we know the value of or expression for a, b and d. So, what I do, I substitute this in this equation, so I get lift equal to Z, V to the power a, a means this 2 minus e minus f. So, I put it here, rho to the power b and b means, 1 minus f, I am putting here 1 minus f, S to the power d, this 1 minus f, S to the power 1 minus f 2 and a to the power e and mu to the power f.

I hope you will be able to get this expression, if we need more help, you can simply see as I explained here V square 2 has come from here, a to the power e was already here and V to the power minus e was here. So, a by V, e, similarly we get mu rho V S half to the power f. now, this I now write as Z rho V square, we have decided and discussed agreed the importance of Mach number, the speed of sound and the speed of vehicle which is V.

So, what is a by V, a by V is 1 by Mach number, because we define Mach number as V by a. So, this I am writing 1 by M to the power e and this term see, this is 1 by Reynolds number.

$$L = Z\, V^{(2-e-f)} \rho^{(1-f)} S^{(1-f/2)} a^{e} \mu^{f}$$

$$L = Z\rho V^2 S \left(\frac{a}{V}\right)^{e} \left(\frac{\mu}{\rho V S^{\frac{1}{2}}}\right)^{f}$$

$$L = Z\rho V^2 S \left(\frac{1}{M}\right)^e \left(\frac{1}{Re}\right)^f$$

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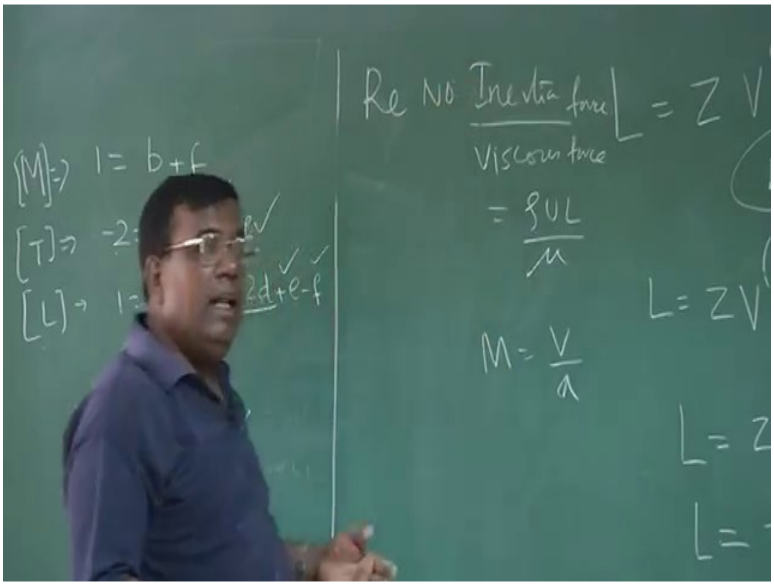

Let us go Reynolds number, typically it is ratio of inertia versus viscous force, inertia force or viscous force, it can be shown that it is rho u, L is the characteristic length by mu the coefficient of viscosity.

$$Re = \frac{Inertia\ force}{Viscous\ force} = \frac{\rho UL}{\mu}$$

Physically, if you want to understand Reynolds number, you can see from here, if the flow is having a high Reynolds number, then in a relative sense, the viscous force is less compared to inertia force. That means, the flow when I am analysing, the viscous effects will be minimal or lesser.

If Reynolds numbers is low, then very clear that, the viscous force effect will be more dominating. And is true, if I want to get the pressure distribution over the aerofoil or the wing, it is important, whether it is a viscous, how viscous it is, if it is highly viscous that will have a larger effect on the whole redistribution of pressure through shear stress and all. So, very important physical non dimensional parameter, we got called Reynolds number and Mach number. This is important for compressible flow, when the speed is near Mach 1 to more than 1.

In fact, beyond 0.3, this effect starts playing role, because the flow becomes compressible. For all our lectures in this course, we will be talking about incompressible flow; that is the speed of the aircraft is low and the Mach number is less than 0.3. Let us understand, what is the advantage, what is the benefit we are having from here, we could see now that at a given angle of attack, the CL is function of Mach number and Reynolds number.

We also know that, if Mach number is less than 0.3, then we are really not bothering about the Mach number effect and since, most of our lectures are low speed. So, we are talking more about a Reynolds number, but yes, this tells us that, if I want to know how CL is changing at given alpha, I have to be very, very clear; that the variation will depend upon, what is the Mach number, what is the Reynolds number.

So, if I want to generate result or database, what I will do, I will put fix angle and then, go on changing Reynolds number and Mach number or Reynolds number, only if Mach number is not that important for that fly regime and then, create a database.

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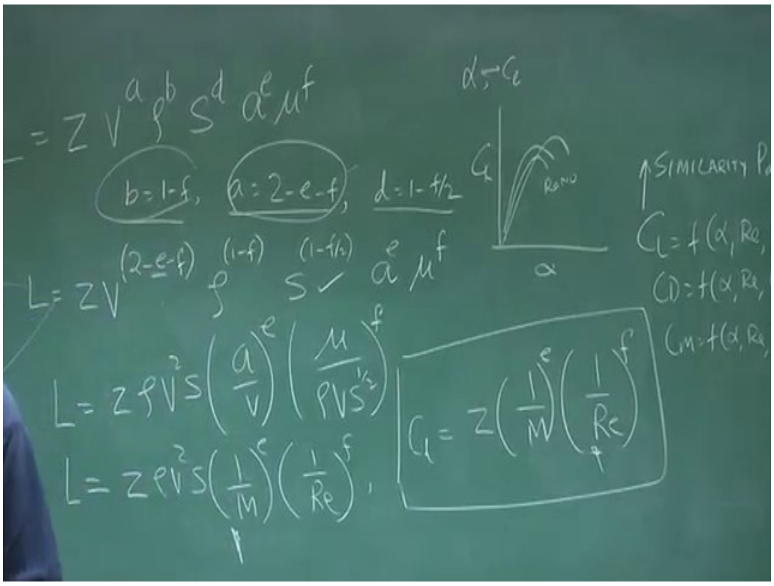

So, I will know that, yes CL versus alpha for different Reynolds number, the characteristics is like this, if I were not knowing this, please understand the problem. Then, I have to do so much of testing, I will try to see CL, how it is changing with viscosity, I will check how CL is changing with the speed of sound, I will check how the CL is changing with density, area, mu. So, all I have to generate a separating testing, it is impossible to even simulate those things.

So, this really is very handy that I will be testing generating the data for different Reynolds number, because I know that the CL is function of Reynolds number for a given angle of attack. Similarly, if it is a high speed flow, I make give more importance to Mach number. So, my testing will be for different angle different Mach number I will test and I will create the database. This is one of the very important understanding of this CL through dimensional analysis.

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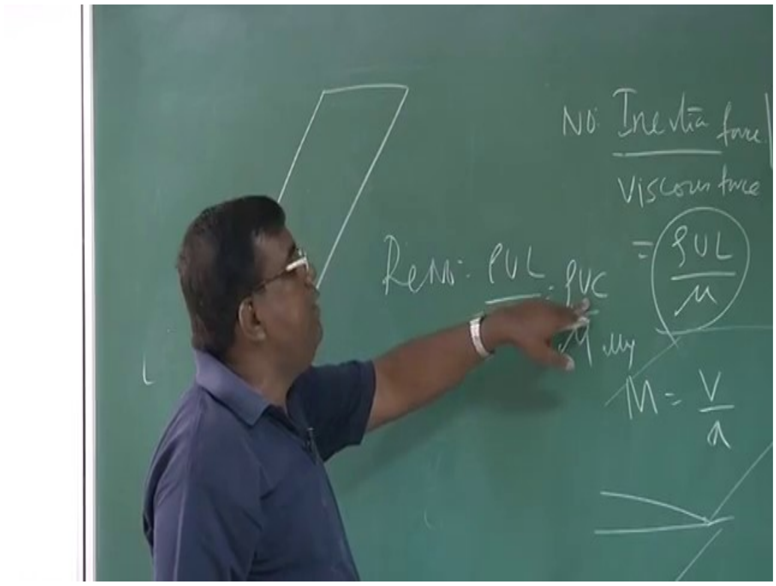

Now, it has helped us further, let us say, I want to generate CL characteristic for huge aircraft. But, that means, if I want to generate the CL for the whole aircraft, I have to take the whole aircraft in a tunnel and a huge tunnel, but see this, this expression tells me that, if I can test a smaller vehicle, which are geometrical similar to the vehicle of interest.

Then, if I generate the flows such that, the Reynolds number are same; that is when the larger aircraft was flying, it was experiencing a particular Reynolds number, let us say Reynolds number I know as rho u, L by mu, for aircraft, it is say, chord I take at a different altitude, if I flying at a different speed, then I have different Reynolds number.

$$Re = \frac{\rho UL}{\mu} = \frac{\rho UC}{\mu}$$

Let us say at 11 kilometre at the speed of 100 metre per second, the aircraft was experiencing a Reynolds number, which is given by this.

Now, what I do I create a geometrically similar model, smaller model and test that in a tunnel and ensure that, the flow is such, it is having same Reynolds number as the actual airplane flying. That means, I can I have to select the model I have to select the speed, the density or the tunnel air, such that, whatever actual Reynolds number the airplane was experiencing the smaller model is also experiencing same Reynolds number.

So, now whatever CL comes, because I know CL depends upon Reynolds number for a given angle of attack. So, I am least bothered about any other parameters. So, that CL, I use for CL of my aircraft. So, how much advantage, it has given and once, I say, I am ensuring that flow in a tunnel is having same Reynolds number, I am actually talking about dynamic similarity.

If I am testing for a high speed, then I must ensure that Mach number is same; that is actual airplane, whatever Mach number. It was experiencing same Mach number should the small model in a tunnel should also experience by you have take their speed of the tunnel, you know, what speeds density, etcetera. Temperature, because velocity of sound is function of temperature for supersonic high speed these are the issues.

But message is, I have to duplicate the dynamic similarity, I need to ensure that, the models are geometrical similar and we are able to take advantage of this analysis and that is why, this CL CD CM all these things are also sometimes referred to as similarity parameter. So, dynamic similarity in terms of flow and similarity parameter means, the geometrically similar airplane and if you have ensured that, it is dynamically similar, I can get a meaningful value of CL through testing by doing scaling.

Thank you.