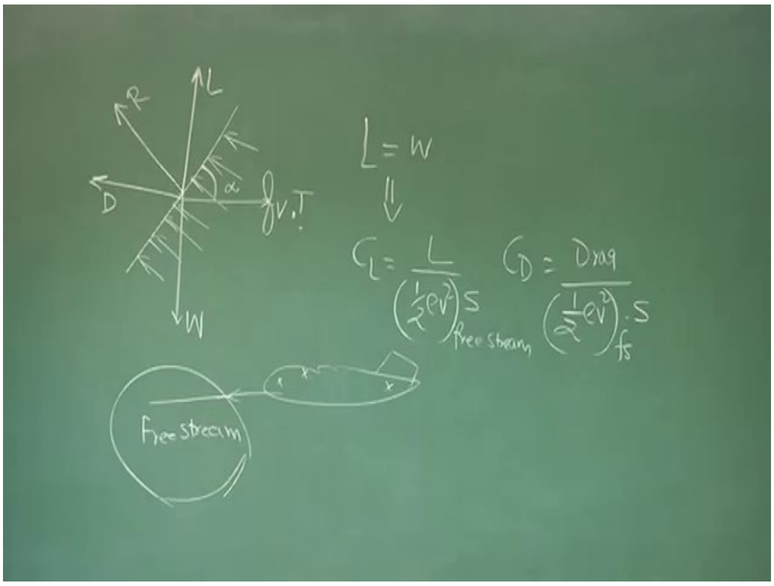
NOC: Introduction to Airplane Performance

Prof. A. K. Ghosh
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture - 06
Drag Polar

See, we were discussing about George Cayley's explanation for generation of lift.

(Refer Slide Time: 00:18)



We just quickly revise, so that we can understand few things better. Remember, this is the plate and if I move this plate at an angle with respect to the speed. And this angle, because of this angel that there will be a pressure from the air outside as we moves like this and that will get translated into force, because of this area and there will be normal force acting on this body.

So, I represent this as, this is the pressure, because of air as it moves through the air and resultant of this I can write this is R, the reaction and one component perpendicular to velocity, we call it lift. Another component which is opposing the velocity called drag and this lift is supposed to balance the weight and if I put an engine which gives thrust.

So, this thrust is supposed to balance the drag, so that I can move like this which is typically we will see soon, we will be defining this flight as a cruise flight. But, let us understand first that if lift has to balance the weight. Then, what is this lift? This lift is generated, because there is an angle between the velocity vector and the surface and this angle we call let say alpha, this lift definitely will depend upon the speed, it will depend upon the area of this plate.

When it comes to the area, we also now let us investigate one thing. If I move this plate like this or I move the plate like this, both are having the same angle, but which case you think more lift will be there, this way or this way. You see this way or this way? It is obvious that lift will be more when I moving it like this; that means, it is not only the area, but also how the area is laid out with respect to the velocity vector.

If you see, soon we will be defining some term of that, of this piece in terms of aspect ratio. Watch out for understanding, what is the meaning of aspect ratio and before we go for aspect ratio, we will have a relook here and try to see, how do I translate this to a pilot in terms of flying the machine. Because, I always said whatever aerodynamics, whatever flight mechanics we do finally, I have to translate this in terms of a language which pilot can understand, right.

Then, from here we realize that we will be operating in terms of CL which is nothing but, lift getting down dimensionalized by dynamic pressure into S reference. And when we say the dynamic pressure, we talked about freestream dynamic pressure, because we realised that the velocity at different points will be different. If it is the aircraft, if it is moving like this, velocity here, velocity here, velocity here or the dynamic pressure here, here, here all will be different.

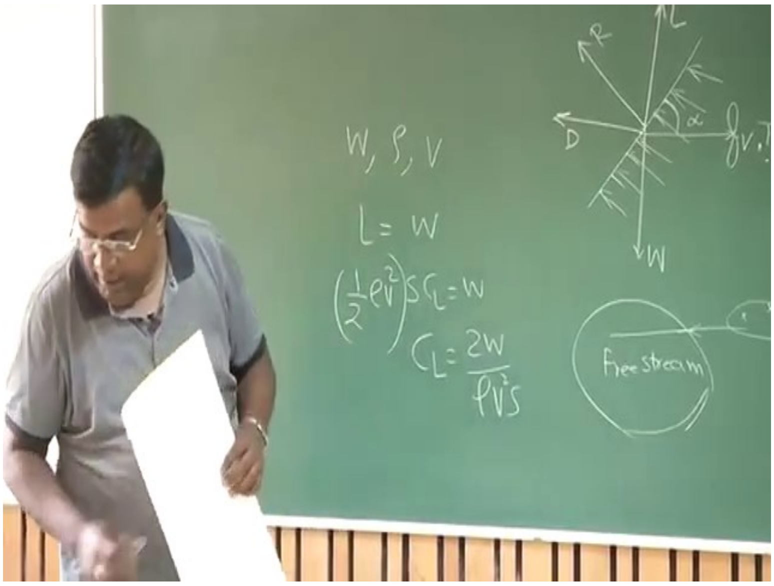
So, when a non-dimensionalise this, which dynamic pressure I take. So, for that we choose freestream condition, the meaning thereby that at this condition, is not affected by the presence of the body and that is what is half rho V^2 freestream dynamic pressure and S is the reference area.

$$C_L = \frac{L}{\left(\frac{1}{2}\rho V^2\right)_{freestream} S}$$

By now, you know that S is the wing area for aircraft. Similarly, CD we have drag, non dimensionalized with dynamic pressure, again freestream and wing area.

$$C_L = \frac{Drag}{\left(\frac{1}{2}\rho V^2\right)_{freestream} S}$$

(Refer Slide Time: 04:17)



Now, let us see if I want to fly a machine and if I decided to operate in terms of CL (C_L), then I will say for a given weight (W), given rho or given altitude (ρ) and given speed (V). This lift equal to weight $(L = W)$and lift is nothing but, from that expression I see it is half rho V^2 S C L equal to weight or C L equal to 2 W by rho V^2 S.

$$\left(\frac{1}{2}\rho V^2\right) SC_L = W$$

$$C_L = \frac{2W}{\rho V^2 S}$$

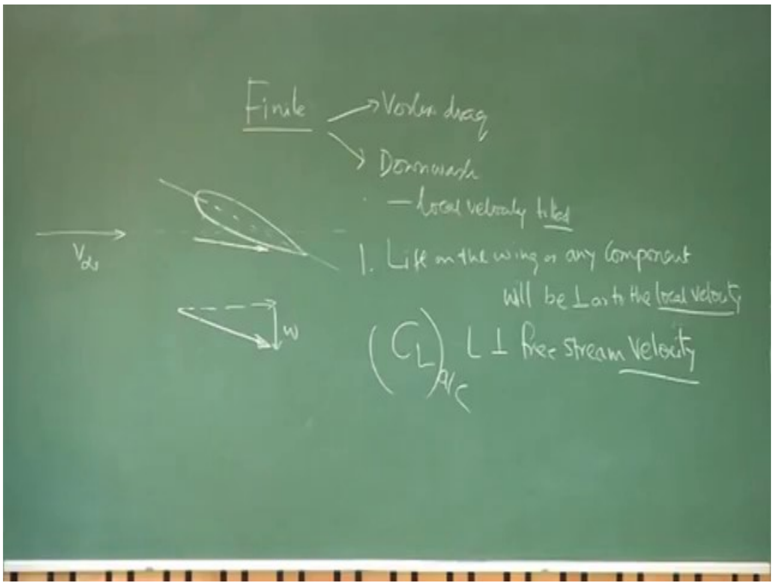
So, what I know, for a given weight, if I am to maintain lift equal to weight, I must fly at this CL. But, then how do I generate this CL?

I will be turning this plate at an angle or turning the airplane at an angle, so that total CL of the aircraft is, what is dictated by this CL and how do I do that. Because, I know the airplane have wing, has tail and at angle of attack, they produce lift, the fuselage produce lift, the total lift and total CL should be equal to the CL required. This part we have already covered, in covering this we also have spoken about the finite wing and quickly, I go through that finite wing we understand.

This is typically a finite wing, where this is the span, this is the chord here, we have already discussed that. If I joining a line, the leading edge and the trailing edge, the straight line is the chord and this is the span and if it is a finite wing, then what is happening that, at an angle of attack when there is a lift this bottom portion is high pressure compared to the top portion. So, there are vortices will be formed and that has two effect.

One is, because it is carrying out rotational kinetic energy and that energy is come from the energy of the machine. So, the machine will lose energy or say increase drag and that is termed as vortex drag or lift induced drag and second thing, as the vortices form like this, they induce a downwash in the downward direction. The meaning thereby, if this is the freestream velocity, as it comes close to the aircraft it's velocity vector gets tilted, because of downwash component. So, if I try to draw that…

(Refer Slide Time: 06:45)



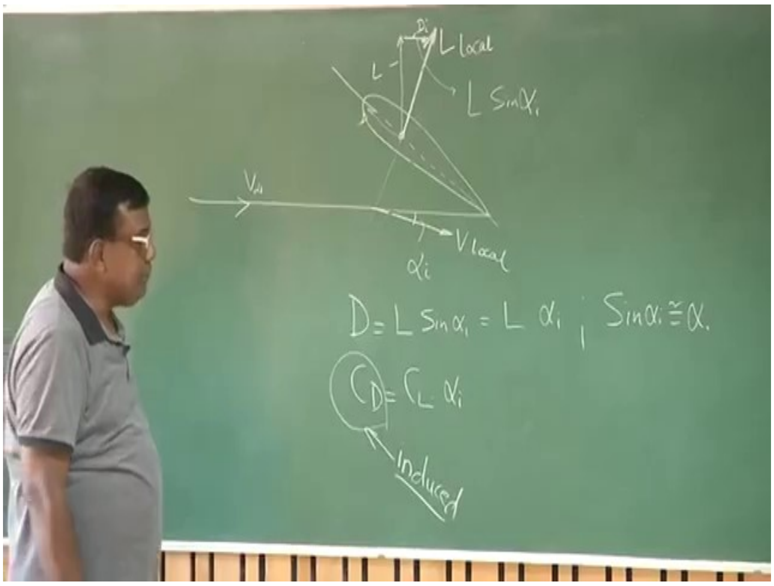
So, finite wing before I draw anything, quickly I should have my thinking clear. One is vortex drag or lift induced drag, another component is it induce downwash. So, so the local velocity is tilted, right ok, just to draw a diagram on this. Let us see, this is the wing and this is the free stream speed or velocity, as it comes close there is a downwash. So, the velocity vector no more remains straight like this. They also get tilted like this and this tilt if I try to draw that is nothing but, this is the freestream and this is the downwash denoted by w, then this is the resultant velocity vector which comes because of the downwash.

Please note that, this is the freestream velocity and this freestream velocity gets super impose with the downwash, because of wing to vortices and the velocity vector get tilted. So, local velocity vector is different than the freestream velocity vector. Now, the question is, when I try to find lift on this wing, the lift will be perpendicular to the local flow condition. But, we define drag and lift for the whole airplane based on freestream condition, this is very important.

This is to be noted down, the lift on the wing or any component will be perpendicular to the local velocity. However, CL of the whole aircraft when I try to write, the CL or the lift when I try to model it, I should ensure that, that lift is perpendicular to freestream velocity. And by now you know, the local velocity and freestream velocity they are not in same direction, because of downwash near the wing or near any other component.

In this case we are discussing downwash, because of the wing to vortices that is, as these vortices go like this, they induced the downwash, right. So, let us try to use this understanding and try to get an approximate expression for the induced drag.

(Refer Slide Time: 09:49)



Let’s say, this is the lift of the… This is the wing and this is your freestream velocity direction and because of downwash, this direction has changed the local, v local. Now, here if I try to draw lift, I know this will be perpendicular to the local flow condition. This lift seen by the wing will perpendicular to the local flow condition, not to v infinity and this angle is called… This angle that tilt in the velocity vector which is called induced angle of attack.

Who has induced this? It is induced by the downwash. Why downwash has come? Because of the vortices. Why vortices have come? Because of pressure difference in the bottom and the top position and why do we need a pressure difference, because I want lift, right. So, it is alpha induced. Now, since I want to define lift and drag, see if I want to write lift I have to resolve this lift which I will call it local, for clarity that should be perpendicular to the freestream direction for the overall aircraft.

So, if I take the component, so this is the lift and this is the drag, ok, so if I now try to write the drag, you see this will be L sin alpha i, ($D = L\ sin\alpha_i$) this component. I write Di this is nothing but, L sin alpha i ($D_i = L \sin \alpha_i$) and for a small angle, I can write this as L alpha i that is of course, for small angle you know sin alpha i approximately equal to alpha i.

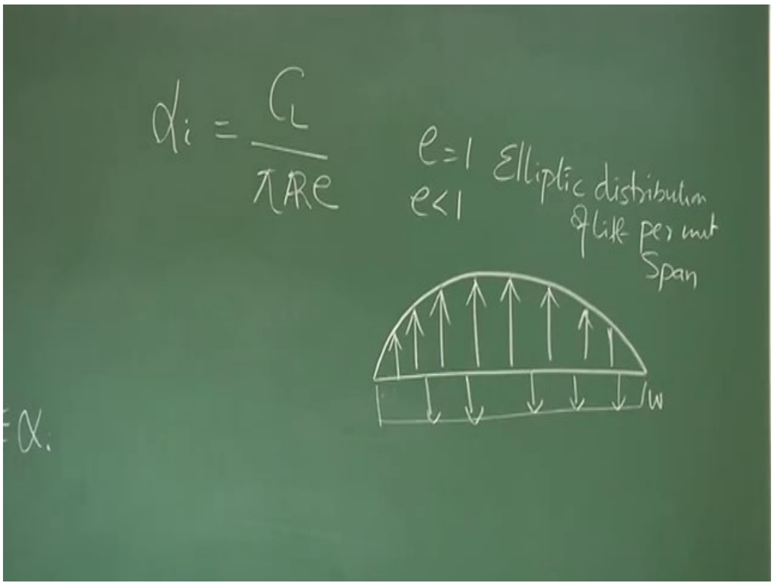
$$D = L \sin \alpha_i = L\ \alpha_i\ ; \sin \alpha_i = \alpha_i\ (for\ small\ angle)$$

Similarly, if I, I can write this equivalently CD equal to CL into alpha i, where C D when I am writing like this, please understand I am talking about induced drag coefficient, because it is coming from the lift.

$$C_D = C_L \alpha_i$$

Where $C_D\ :\ induced\ drag$

(Refer Slide Time: 12:29)

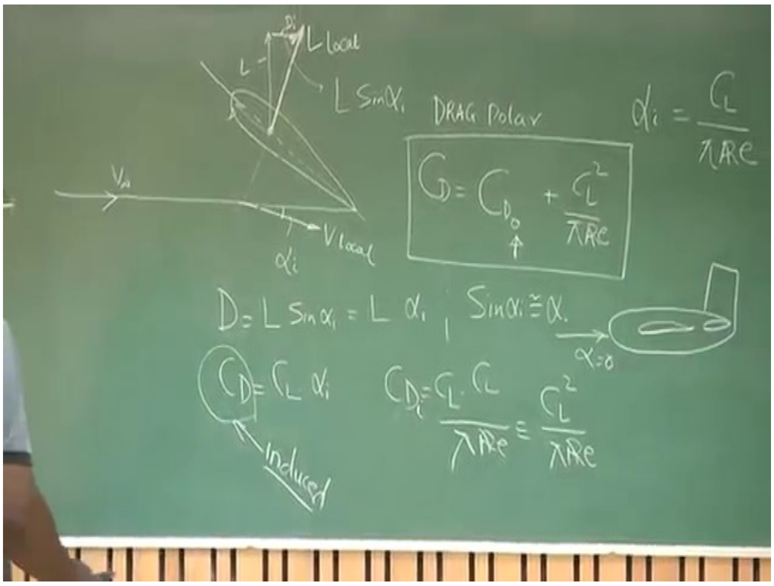


And from theory of incompressible flow, for elliptic distribution one can show that alpha i can be approximated as CL by pi aspect ratio e. Of course, e is 1 for elliptic distribution of lift per unit span that is, if I draw it, you'll find This is typically elliptic distribution of lift, any standard textbook you can get this information in detail and the bottom I am writing that downwash component and that denoted by w, right. And of course, e is less than 1 for anything which is not non elliptic.

$$\alpha_i = \frac{C_L}{\pi ARe}\ ; \begin{array}{c} e = 1(for\ elliptical\ distribution\ of\ lift\ per\ unit\ span) \\ e < 1(non\ elliptical\ distribution) \end{array}$$

Mostly, we have non elliptic distribution.

(Refer Slide Time: 13:49)



Now, if I substitute this here, so what I get, CD equal to CL into CL by pi aspect ratio e and that is equal to CL square by pi aspect ratio e, ok.

$$C_D = \frac{C_L . C_L}{\pi ARe} = \frac{C_L^2}{\pi ARe}$$

So, this is nothing but, CDi, C D induced. So, now, for a wing what we see. For a wing if I want to find total CD, I know total CD will be what. One part is induced because of lift, which is given by CL^2 by pi aspect ratio e and what about the other part. Suppose, this is the wing or further we might have, this is the airplane.

As I am along the air to flow over the wing at alpha equal to zero, so there will be lot of resistance because of skin friction. Depending upon how stable it is, the skin friction value will change. There could be flow separation at different points, so there will be, pressure drag because of flow separation, so a typical resistance which will be at alpha equal to zero and typical resistance because of lift and the typical resistance for alpha equal to zero.

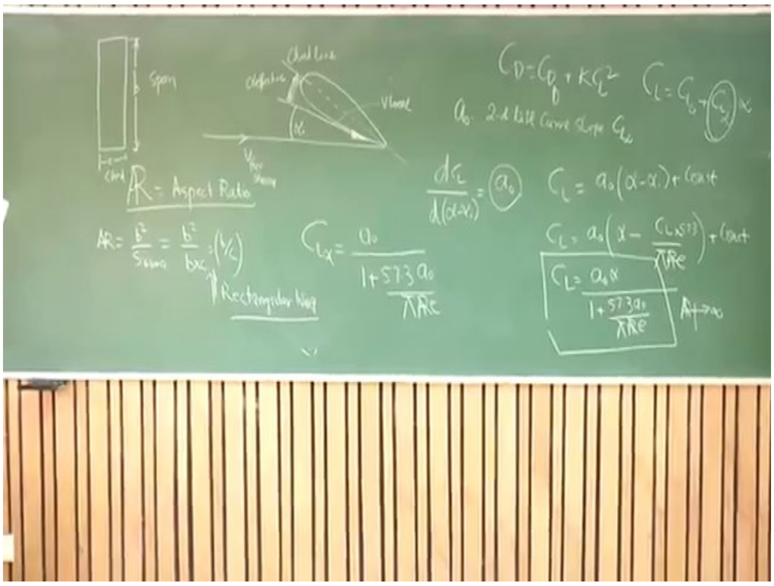
We call parasite drag, which are dependent on the flow condition, which is a subsonic, low subsonic, supersonic, very high speed depends upon, what is the geometry of the airplane. You will know, the aerodynamic something called Reynolds number, which talks about ratio of inertia of the viscous force, that is if it is a low Reynolds number, then we expect to have more viscous effect, right. So, accordingly to that, this CD0 will be decided and once we say CD0, this ‘0’ means that CD at zero lift.

For a symmetric aerofoil, it is good enough to say CD at alpha equal to zero. For cambered aerofoil, we define this you write CD at CL equal to zero and as we progress you'll see that, some time we try to represent this drag coefficient using CD minimum also. Those are matter of detail, at this present only understand that the drag coefficient will have two component. One because of parasite drag, zero lift drag and another lift induced drag and this representation is known as drag polar.

$$C_D = C_{D_0} + \frac{C_L^2}{\pi ARe} \quad : DRAG\ POLAR$$

Whenever you finalize the design, when aircraft is flight tested, everything is done, then this has to be estimated and every aircraft is bench marked with a given drag polar under different, different fight conditions, right? So, this is one of the contribution of finite wing, that because of finite wing there is vortices, because of vortices there are drag, induced drag and because of vortices, they downwash which will also change the lift curve slope or CL alpha of the wing. Let us see, how that happens. Let me draw this diagram.

(Refer Slide Time: 17:20)



So, this is typically let us say this is a wing and this is a freestream velocity. As it comes near the wing there is a downwash, because of wing tilt vortices, the velocity vector gets tilted downward. So, this is the local velocity direction and this is the chord line, all these thing you are now familiar and this is the angle, which I call it alpha effective $(\alpha_{effective})$. Because, as far as this wing or the body is concerned, all the lift generation will happen because of this angle, because come back to the John George Cayley, that the lift will depend upon the angle between the velocity vector, local velocity vector and the surface or the chord line in this case.

So, this is alpha effective and alpha i is the induced angle of attack. So, now, we understand one thing if I denote a0 (a_0) as 2D lift curve slope, that is lift curve slope which is typically C l alpha represented by C l alpha $\left(C_{l_\alpha}\right)$. What is the meaning of this 2D lift curve slope? That we are assuming that span is infinite, so there are no alpha i, because if it is infinite; that means, there are no wing tip, it is not a finite wing. So, there won't be any wing to vortices, so there won't be any downwash, so there are no alpha i, ok, so that is what is a0.

So, if I now write dCL by dalpha minus alpha i, this should give me a0.

$$\frac{\partial C_L}{\partial(\alpha - \alpha_i)} = a_0$$

Because, I have taken out alpha i from the angle, this slope when I am writing CL versus alpha effective, which does not take alpha i, or alpha here has been taken out, the effect is taken out. We are talking about this, so this should be the a0, if that is a0, then I can write CL equal to a0 alpha minus alpha i plus constant.

$$C_L = a0\ (\alpha - \alpha_i) + constant$$

So, CL I can write as a0 alpha, for alpha i I put the expression which I have written earlier which is CL by pi aspect ratio e $\left(\frac{C_L}{\pi ARe}\right)$. I multiplied by 57.3 to convert this value from radian to degree, right, then plus constant.

$$C_L = a_0\left(\alpha - \frac{C_L * 57.3}{\pi ARe}\right) + constant$$

So, if I now manipulate this, I do some algebraic adjustment, I can find that CL equal to a0 alpha by 1 plus 57.3 a0 by pi aspect ratio e, ok or I can write dCL by dalpha equal to a 0 divided by 1 plus 57.3 a naught by pi aspect ratio e.

$$C_L = \frac{a_0\alpha}{1 + \left(\frac{57.3a_0}{\pi ARe}\right)}$$

$$C_{L_\alpha} = \frac{a_0}{1 + \left(\frac{57.3a_0}{\pi ARe}\right)}$$

Let us see, again you come back here. What was a0? A0 is the lift curve slope for an aerofoil, that is 2D value; that means, the 2D value does not have any alpha i. So, when I try to find the slope CL and angle of attack, I am eliminating or taking out the contribution of alpha i. So, if I take this slope, this should a0, right. Now, from here I write CL equal to this and I get an expression CL equal to this a0 not alpha plus 1 plus 57.3 a naught by pi aspect ratio is e, 57.3 I put to convert radian to degree.

What is the message here? If I have got aerofoil lift curve slope, let us say 0.1 per degree, it is degree that is per degree I am talking about. If I want to really calculate CL, what I have to do, I have to use that value, put alpha in degree and add such value a0 here, so from 2D value I will get 3D value. In this whole expression, there is one term which I am being writing AR that is the aspect ratio. We need to know, what is this aspect ratio?

Remember, we will go back again, if this is the plate ((Refer Time: 23:08)) and I am flying like this, as per as George Cayley the lift will be… Because, there is a angle between the velocity vector and the surface and the lift depends upon the reaction which depends upon velocity, area, density, etcetera, etcetera. If it depends upon area, then whether it goes like this or it goes like this they should be same, but they are not same.

We know that lift, for generating lift I should fly like this. Very simple way to get an understanding of it, remember when I am flying like this I can assume that it is composed of so many aerofoils like this. But, if I am flying like this, aerofoil which are not as large as aerofoil like this, because why aerofoil, because the pressure difference will come because of contour of the aerofoil. and here, if I am flying like this, so many aerofoils, so the contour will get, effect will get added and we get lift.

To make sure that you are flying like this, we also define some term call aspect ratio and that gives us a feel, which way I am orienting this way or this way. For example, aspect ratio if I draw a wing, if I call this b as span and this c as chord, aspect ratio will define as b square by S wing $\left(AR = b^2/S_{wing}\right)$. For a rectangular wing, it automatically becomes b square by b into c, so it becomes b by c, for rectangular planform wing, rectangular wing which I mean planform.

$$AR = \frac{b^2}{S_{wing}} = \frac{b^2}{b\ c} = \frac{b}{c} \ \ (for\ rectangular\ wing)$$

Now, see if this is the wing, if the span is increased to infinity what will happen to the aspect ratio. So, aspect ratio also will become infinite. If aspect ratio becomes infinite here, what happens here? If aspect ratio becomes infinite very large, then this term will become 0, so in that case the CL will be same as CL what you get from 2D or coming here, the CL alpha of the whole wing will be same as CL alpha of the aerofoil, that is it will now behave like a 2D.

So, if you want to get larger CL alpha, it is better to have larger aspect ratio. By there is a problem of larger aspect ratio, one of the problem is if it is very large aspect ratio wing, how do you balance the weight of the wing and there are other issues will come, as we progress we will understand. In the nutshell, whole effort here is towards making one thing very clear that, if you have a finite wing it will give induced drag and there, because of the induced drag your drag efficient will have CD parasite or zero lift drag plus K CL square.

This typically follows a parabolic form and also the lift curve slope or CL alpha of the wing will also reduced as compared to CL alpha of the aerofoil. If you understand this, then we have understood, what is minimum required for and using this concept for performance analysis. Why do I need the relationship between CL and alpha? Remember, if we written CL equal to CL0 plus CL alpha into alpha, why do you require this, because to maintain lift equal to weight, we need some CL which will be given by the lift equal to weight equation.

But, for a pilot he has to generate that CL, so he has to turn the airplane by some angle of attack, which will be dictated by this relationship. And finite wing, we were trying to understand with aspect ratio, how CL alpha of the wing is going to change, right? So, this is good enough of understanding for us to use this first performance analysis. We may most likely we have going to start with the cruise performance next.

Thank you.