**NOC: Introduction to Airplane Performance**
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**Lecture - 11**
**Thrust Required, Power Required: Cruise.**

Hello dear friends, now today we will be discussing on one of the important phase, flight phase of airplane the cruise. When I say cruise, so airplane will be going this straight lines like this and it will be unaccelerated, all the forces and moments will be balanced and wings level that is, ((Refer Time: 00:32)) if the wings will not have any orientation like this, this straight and level and this is a straight flight, that is typically a cruise flight.

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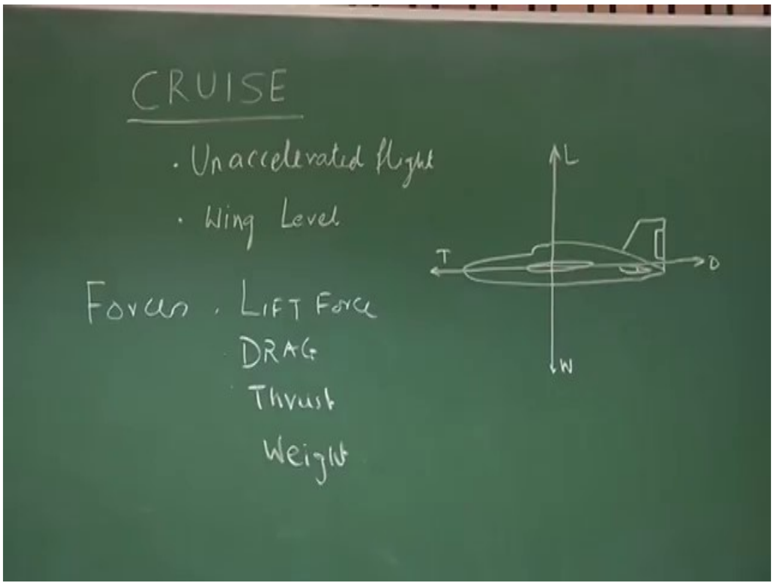


And, when I say the accelerated and unaccelerated flight, we mean that, the net forces are balanced and what are the forces acting on it, the forces acting are one is lift force, then drag, then the thrust and the weight, whole performance exercise. Understanding the performance of a airplane is focused towards this four forces lift, drag, thrust and weight.

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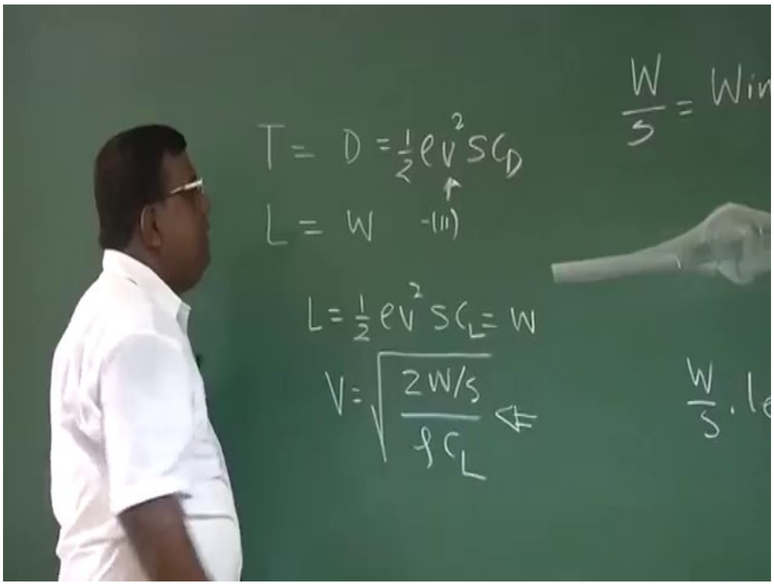


If I now, come back to this diagram if really this aircraft is going in an unaccelerated flight, then I can write thrust equal to drag and lift equal to weight.

$$T = D$$
$$L = W$$

Let us, also remember one thing a designer will design an aircraft, but who will be flying it, a human being a pilot and the pilot may not be an engineer and may not be a person, who understands lift, drag all the theory that has gone into designing an airplane.

So, it is a role of a designer to ensure that he develops a language, which pilot can understand and mostly this is done through instrumentation, we will be… unfolding this understanding step by step. Let us, come back to first on this force balance, which is thrust equal to drag and lift equal to weight. If I take the second equation, the lift is we know by now, lift is half rho $V^2$ S CL, so I can write V is equal to under root 2 W by S by rho CL.

$$V = \sqrt{\frac{2W/S}{\rho S}}$$

Because, the lift is half rho $V^2$ CL, which is equal to the weight, as per the equation number 2. So, let us focus on this equation, the two very important parameters here one is CL another is W by S and W by S we will be referring very frequently as wing loading, this W by S wing loading.

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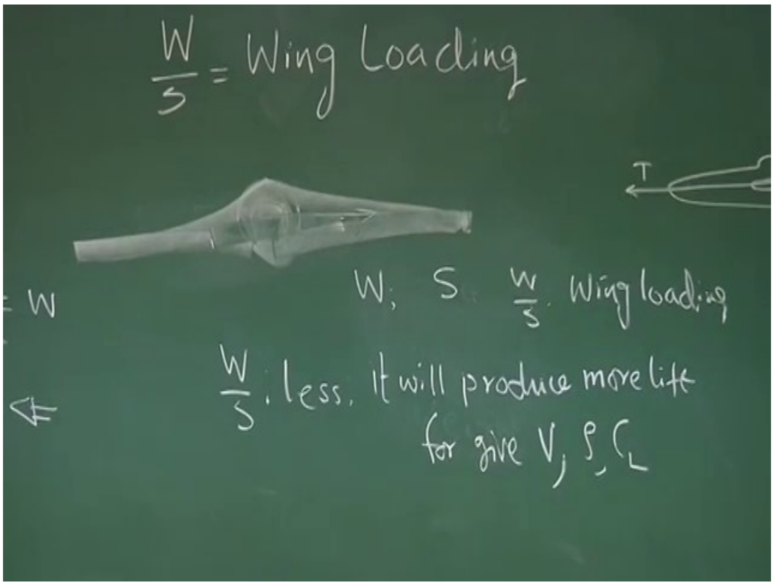


Let us, develop some feel for this wing loading, suppose we want to lift this mass m by giving a forward velocity. By now, we have understood that I can generate a lift using a wing a fixed wing, so I translate this to a diagram like this, where I call this part wing. And, let us say for a given weight of a airplane the area is S is the area of the wing, then the ratio W by S is called wing loading. What is the physical significance of it?

What is our aim? Our aim is to lift the airplane, lift the airplane how, by generating the lift through the wing, primarily from the wing. So, if the wing area is large, then I will be able to produce more lift, you can come here this equation half rho $V^2$ S, more area means I will be able to produce lift for a given velocity, if area is more I will have more lift. So, in a sense if I now translate to W by S, which wing loading if S is large it will have more lifting characteristics it will produce more lift.

So, the wing loading will be less, so W by S less means produce more lift for a given V rho and CL. What is the more lift for a given, yes that is right. So, what is the utility of this wing loading ratio? Think typically of a glider, the gliders will have a larger wing, larger wing area. So, if larger wing area, a .., larger wing area, so smaller wing loading I repeat the glider will have larger wing area for a given weight it is wing loading will be less; that means, the velocity required to balance lift equal to weight at a given altitude also will be less.

So, what is the implication of that? If velocity is less; that means ((Refer Time: 06:10)), your drag required come to this equation half rho $V^2$ S CD. If, velocity is less your drag experience by the airplane will be less, in turn the thrust required also will be less and in turn the engine size will be less and the weight also will be less. This is very, very important parameter and we must always try to see these two equations together.

So, that is about the wing loading, some first exposure. This another important turn, if you see in this equation is CL. What is this CL? This is not just CL of the wing, it is CL of the whole airplane.

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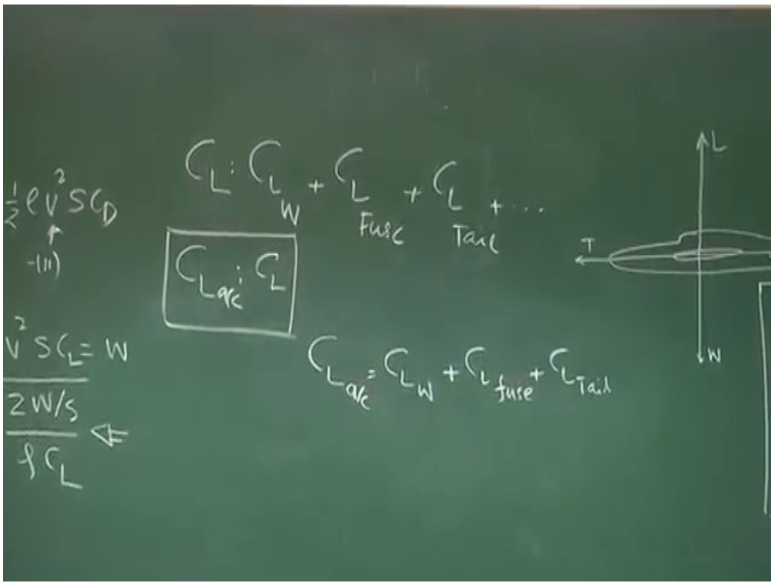


So, I will write, I will be very clear in my mind this is CL, because of wing this is CL, because of fuselage this is CL, because of tail and if any other components are there, there could be a canard there could be any other surfaces. So, CL is for the CL aircraft. But, CL we are talking about the CL of the whole aircraft, not just CL of the wing that should be very, very clear in mind.

$$C_L: C_{L_W} + C_{L_{fuselage}} + C_{L_{tail}} + \cdots$$

Now, there is a word of equation, when I write CL of the aircraft as CL of the wing plus CL of the fuselage plus CL of the tail, it should not give a wrong impression that CL of the wing.

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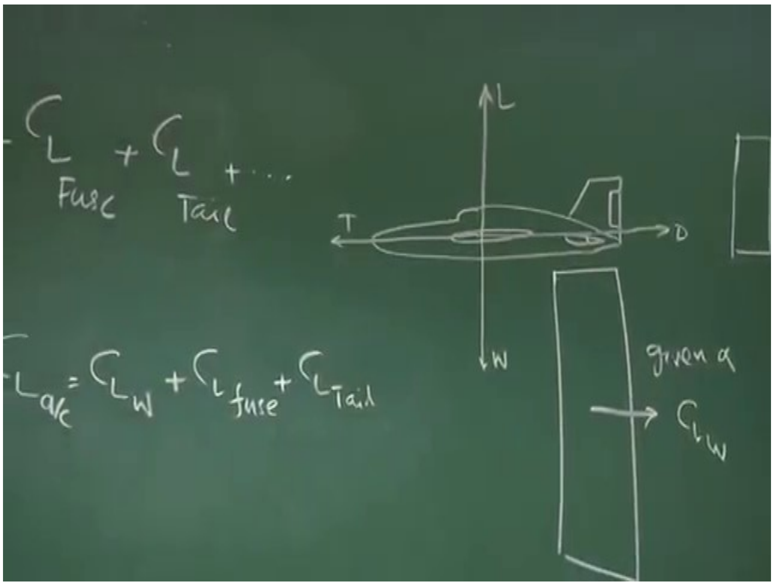


If, I want to estimate CL of the wing I take the wings separately and find for a given alpha, I find CL of the wing in isolation. Similarly, I find CL of the fuselage in isolation, so will I find CL tail in isolation and then add it up, that will be a mistake that is not correct. Because, see the point, point is CL of the wing in isolation and CL of the wing, when it is attached to the body have a different geometry conditions.

For example, the moment I put this wing over this aircraft there will be an influence of body on the wing and the wing on the body. So, there will be an interference on body to wing and wing to body, so CL get modified, similarly when I put the tail, if I take tail in isolation, I should be very careful that the value of CL computed in isolation will not be the exact, what airplane will be generating. Because, this tail will be near the vicinity of the body, fuselage near the empennage and there will be interference.

So, when you find out CL of the aircraft through a wind tunnel, or through a analytical method all these things are taken into account and finally, we say it is the CL of the whole aircraft. It is true that, if I see the numbers the CL of the aircraft is predominantly decided by CL of the wing, because and that is true also, because the wing has a unique role, wings role is to generate lift, and what is the role of tail the role of tail is to generate moment.

So, lift is not that important for tail, because the tail moment arm, the distance between aerodynamic center of the tail to CG of the airplane, that distance into the small force of the tail gives the moment, which is required to control the airplane. Ok, now, this V when I use this equation ((Refer Time: 10:03)), this V is actually is V cruise that is, what is the interpretation of this, it is the cruise velocity at a given attitude, where you are maintaining lift equal to weight, right? And this is another subtle point about this V cruise, if I ask, is it really the velocity of the airplane with respect to the ground, the answer is no.

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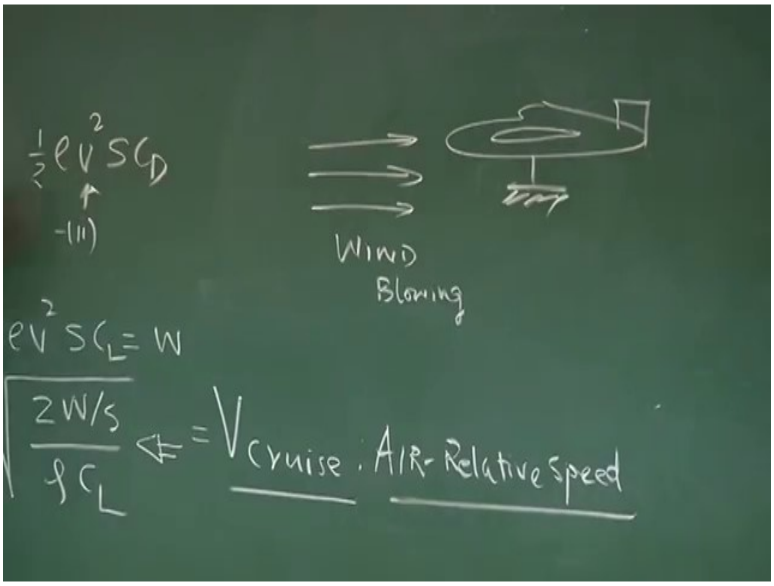


Because, the lift depends upon the velocity, which is air relative velocity, that is meaning thereby, suppose the airplane is stationary on the ground. Suppose, I have fixed it on the ground and there is a wing and there is a wind blowing, it is possible there is a wind blowing. So, what is the speed of the airplane with respect to the ground? It is zero, it is fixed, how it will produce lift, because there is an air relative speed. So, V cruise we should very clear this is AIR relative speed.

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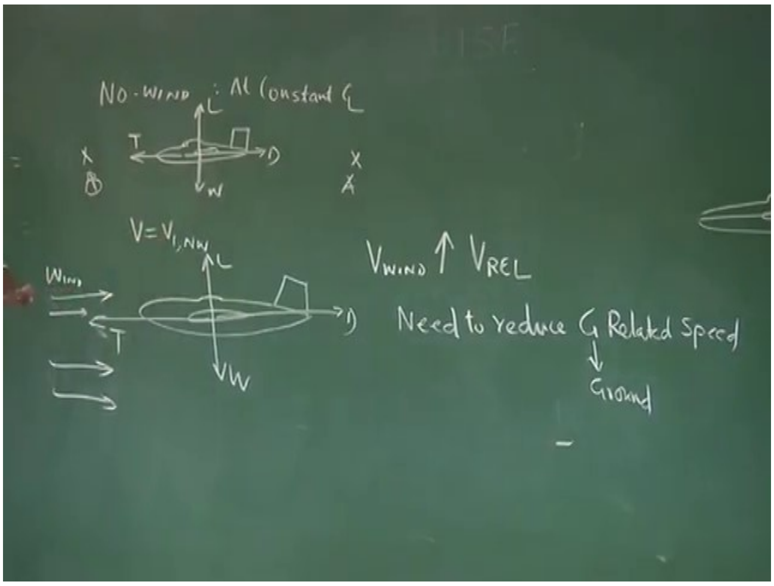


Ok, now, let us go little bit deep into this, let us do an example, suppose I am reaching I want to reach from point A to point B and I am flying like this. I am assuming that there is no wind and also I am assuming that, I will be flying at a constant CL, no wind and at constant CL. So, let us say for this configuration the velocity is V1($V_1$) or I say V1 no wind $(V_{1,NW})$. What will happen? If I, if suddenly similar situation everything remaining same and suddenly, there is a head wind blowing like this say wind and, still I want to fly at this constant CL, ok.

To maintain lift equal to weight my question is should I increase the speed or decrease the speed. The answer is obvious, because the lift depends upon the air relative speed since the wind is coming towards the airplane, the air relative speed will increase. So, actually I have to reduce the speed with respect to the ground, so as V wind increases V relative. So, I need to reduce ground related speed, this is ground related speed.

So, that the air relative speed remains same as in the first case and lift is equally balanced with the weight, otherwise the airplane will lift up. So, this concept will be used, you will see that, when we are moving in a airplane into the wind or there is a tail wind, how the work performance of the airplane will change, sometimes to advantage, sometimes to disadvantage.

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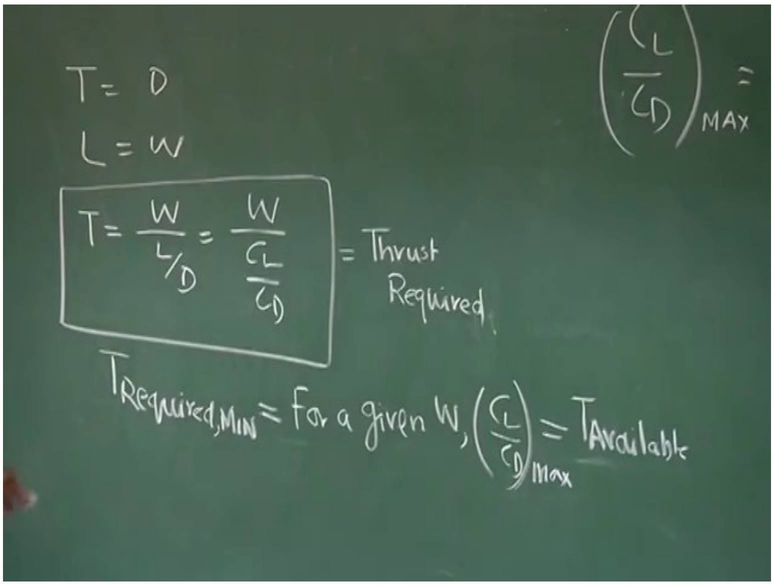


So, with this understanding let us come back to the cruise, we say thrust equal to drag and lift equal to weight, so I can manipulate this two equation and write thrust equal to W by L by D or this is W by CL by CD,

$$T = \frac{W}{L/D} = \frac{W}{C_L \;/\; C_D} : (Thrust\ Required)$$

what is this thrust, this thrust is thrust required, this is required, required for what this thrust, this thrust required for ensuring it is equal to the drag and there is a sufficient appropriate value of CL, at that given speed, which maintain lift equal to the weight, what is our aim our aim should be I should fly the machine in such a way, that thrust required is always minimum most of the time the cruise will try to look for thrust required minimum.

So, I will look for thrust required minimum and if I see here, what does mean, it means for a given weight, CL by CD should be maximum. It means for a given weight CL by CD should be maximum. Now, what is the meaning of CL by CD to maximum, that also should now, ask our self, what is the meaning of CL by CD maximum or I put the question differently, I want the pilot to fly the machine.

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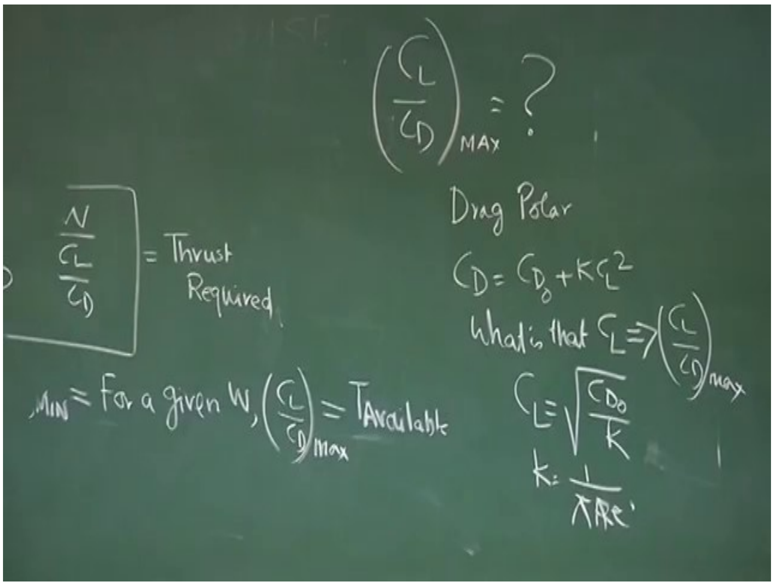


So, the thrust required is minimum and this thrust required minimum will be compensated by thrust available, which comes through the engine. So, if thrust required is minimum, then thrust to be delivered by the engine also minimum, that is an efficient way of flying. But, if you tell the pilot please fly the machine such that CL like CD is maximum pilot will not understand, what the meaning of this. So, let us try to decode information, what is the meaning of CL by CD maximum for a pilot.

So, we have to go back to drag polar a typical drag polar form CD equal to CD0, plus K CL$^2$ for low speed almost, what we say 100 meter per second less, then 0.3 and smooth surface mostly you'll find this will be a, almost exactly representation. The question is, what is that CL, what is that CL, which will ensure CL by CD is maximum, very straightforward this will be one of the assignment problems. You have to find out the condition through differentiation and you can show that answer is CL should be equal to CD0 by K, where K is K you know is roughly it is 1 by pi aspect ratio into e.

$$C_L = \sqrt{\frac{C_{D_0}}{K}}$$

$$K = \frac{1}{\pi ARe}$$

So, what is a message? Messages is if we want to fly at thrust required minimum ((Refer Time: 18:36)), then I need to fly at CL by CD maximum, which means I need to fly at a fix CL. Because, CD0 values is fixed typically value will be 0.02, to 0.023, K for a given aspect ratio K is fix, so CL is fixed.

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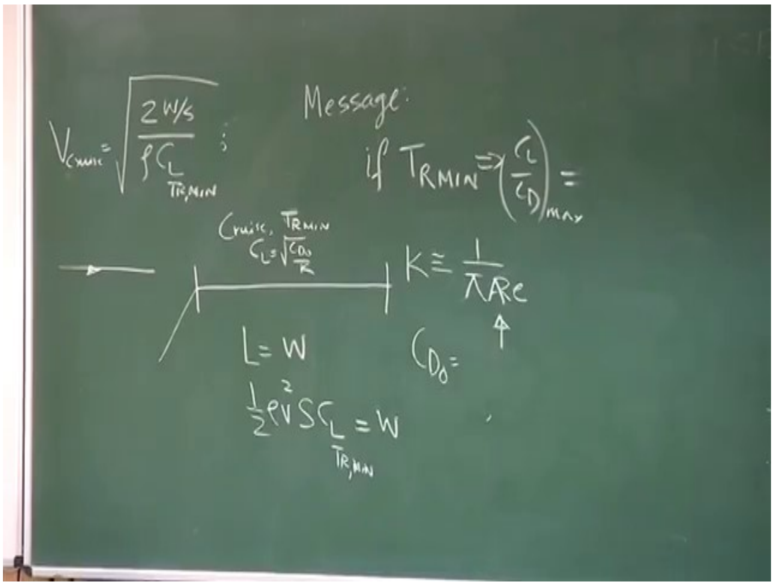


So, what is a message, message to pilot is message is, if I want to fly as thrust require minimum, then fly at CL by CD maximum and that is fly at CL is equal to CL fixed, which is given by CD0 by K for a different airplane this value will be different. Because, you know the K is 1 by pi aspect ratio e, so depending upon an aspect ratio depending upon the wing layout value of e will change aspect ratio will change. Then, CD0 also will change depending, what is the geometric contour, what are the protrusions, what are the profile of the wing, so for a different airplane this value CL fixed will change.

However, if I want to fly at thrust required minimum, where your engine be minimum thrust, which I call thrust available I need to fly to fix CL, which is CD0 by K. Let us say, this value is around 0.2 let us say, so what is the meaning suppose you are telling pilot you cruise, you cruise here, please cruise such that thrust required is minimum, that is a statements you are given or a designer as given to the pilot it means he is to fly CL equal to CD0 by K again pilot will not be comfortable.

If, you want to calculate CD0 value and K he is not aware, what is CD0, what is K mostly. So, what is the way to handle this situation, see lift equal to weight. So, you know that half rho $V^2$ S CL for thrust required minimum, will be equal to weight. So, from here I can write V cruise will be equal to 2 W by S by rho CL for thrust required meaning.

$$V_{cruise} = \sqrt{\frac{(2W/S)}{\rho C_{L_{T_R,MIN}}}}$$

Let us see here, W is fixed if I assume I am assuming that there's not much difference in weight, because of fuel consumption S is fixed CL thrust required minimum is fixed, because it is given by CD0 by K, which are fixed.

So, for a given altitude for a given value of a rho there is again fixed value of V cruise, so, pilot will be flying around that value. If, you can generate a table for him for at this altitude maintain this velocity at this altitude by this velocity and that is all the guide guideline for him to do the flying a such required minimum and of course, with the feel he may not be recording explicit those numbers, but he has an idea, then through instruments he ensures that he is flying at a thrust requirement minimum, so this part clear.

So, I will just repeat this and decode it point by point, so that it goes into your mind I always remember, whenever you are talking about an airplane finally, it will be the pilot in command flying this machine, he is not the designer he need not be aerospace engineer. So, we need to translate always understanding into a language with pilot should to be able to easily understand. And, that is what makes the difference between a good engineer and bad engineer.

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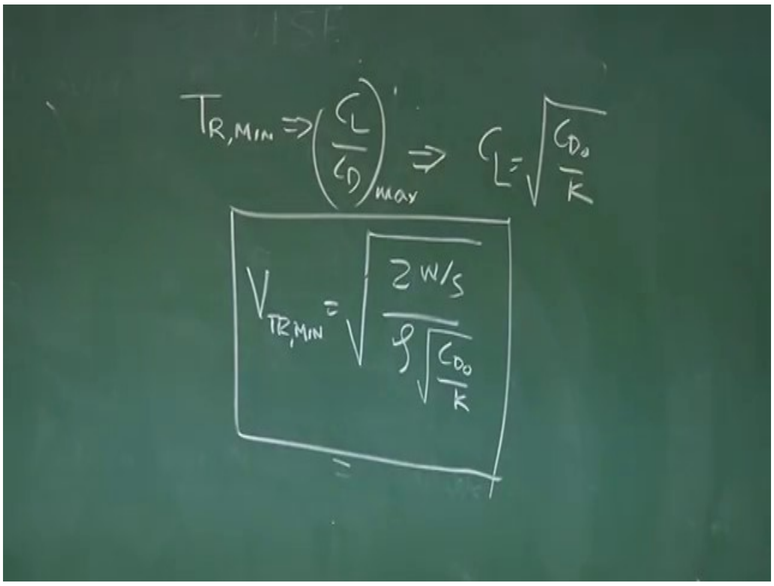

So, what we say, we know that for thrust required minimum I must fly at CL by CD maximum; that means, I have to fly at a CL, which is CD0 by K; that means, the velocity for thrust required minimum will be nothing but, 2 W by S by rho into CD0 by K. So, that is what the velocity for thrust required minimum, should be maintained by the pilot at a given altitude,

$$T_{R,MIN} \Rightarrow \left(\frac{C_L}{C_D}\right)_{max} \Rightarrow C_L = \sqrt{\frac{C_{D_0}}{K}}$$

$$V_{T_{R,MIN}} = \sqrt{\frac{2W/S}{\rho\sqrt{\frac{C_{D_0}}{K}}}}$$

please understand also here this value will change depending upon the altitude. As I go higher and higher the value of rho is decreases, so this value also will change it will increase, so far we are discussing about the thrust required for cruise.

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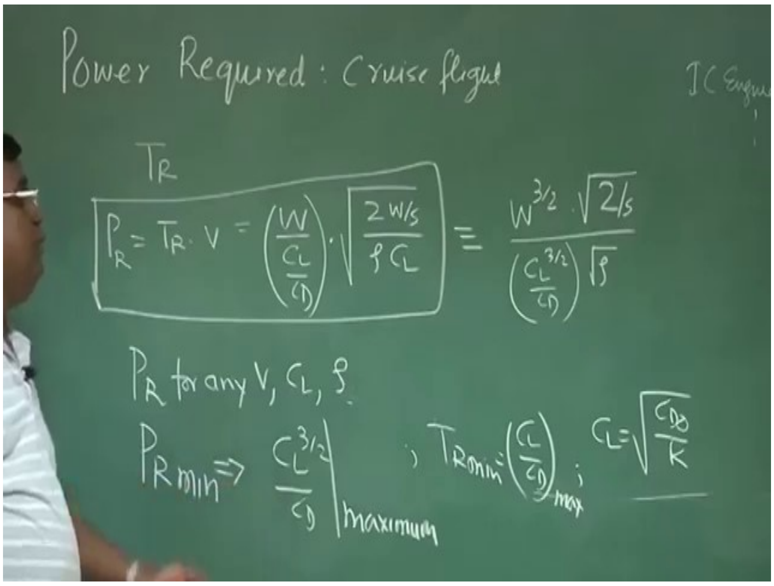


Now, will be discussing about power required for cruise before, I start talking about power required and trust required. Let us also understand we are talking about two types of engine one is IC engine with propeller combination with this engine are rated terms of power and another one is jet engine this engines are rated in terms of thrust newtons of thrust. So, far we are talking about thrust required, now we are talking about power required and power required for what, power required for cruise flight.

Remember, power required for cruise flight is one important mission, then power required to climb is another important mission. Similarly, thrust required for climb is another important mission. So, we are only parts of a segment of whole flight envelope you talking about, which is cruise flight, and cruise is, what, we know the airplane goes unaccelerated with the constant speed like this in a almost in a straight line. So, very simple if I know thrust required was TR, then power required will be TR into V force into velocity, what is this V, V is nothing, but velocity required at that altitude to maintain lift equal to weight.

So, I can write this as W by CL by CD, which I know thrust required expression and for V, I can write 2 W by S by rho CL.

$$P = T_R.V = \left(\frac{W}{\frac{C_L}{C_D}}\right).\sqrt{\frac{2W/S}{\rho S}}$$

So, this is now my power required expression. So, what we can see from here, this I can finally, also write like this W 1 plus half 3 by 2, then this is roots 2, here I write CL 3 by 2 by CD here, again I Wright root rho, somewhere this root of S also will come. So, I write club it here 2 by S is it ok? let us check W and W 1 by 2 is W 3 by 2 root 2 is here, and S, I put it here and CL 1 and CL half CL 3 by 2 CD is here and rho.

$$P = T_R.V = \left(\frac{W}{\frac{C_L}{C_D}}\right).\sqrt{\frac{2W/S}{\rho S}} = \frac{W^{3/2}.\sqrt{2/S}}{\frac{C_L^{3/2}}{C_D}\sqrt{\rho}}$$

This simple expression tells us one thing that for power required for any V or CL at a given altitude can be computed, if I know, what is the weight during the cruise, what is the CL 3 by 2 by CD and what is the density of air at that altitude. Also, it tells me one more information that, if I want if I am looking power required minimum, for a given weight given altitude there in the condition is CL, 3 by 2 by CD should be maximum,

$$P_{R_{min}} \Rightarrow \left.\frac{C_L^{3/2}}{C_D}\right|_{maximum}$$

$$T_{R_{min}} = \left(\frac{C_L}{C_D}\right)_{max} ; C_L = \sqrt{\frac{C_{D_0}}{K}}$$

what was thrust required minimum condition. For, trust required minimum, we call the condition was CL by CD should be maximum and we have demonstrated that it means to fly the machine at fixed CL, which is given by CD0 by K.

Similar question I will be asking here, what is that CL I should fly, so that CL 3 by 2 by CD is maximum or in return the power required will be minimum. So, that is a question we are going to address, also in addressing this question, we also try to see two components of powers like, we have thrust required zero lift thrust plus induced thrust. Here, also will try to see, what is zero lift power and induced power that is our objective, let us go step by step.

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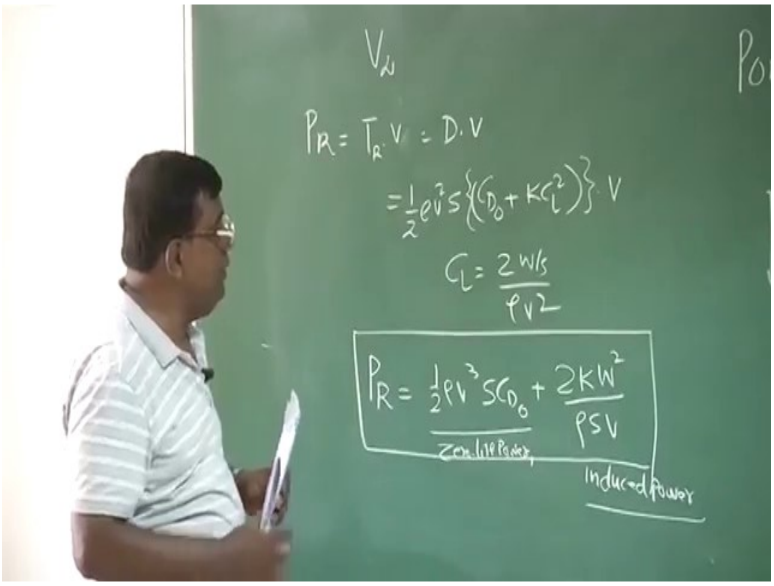


So, now I do a little bit of algebra or simple expression T into V infinity is equal to D I do not write infinity V into V many text book, they follow the nomenclature for V it is V infinity it is called free stream velocity. That is the velocity for away from their aircraft, what is the significance of V infinity, will be talking in as the course develops, but I am using V.

So, this is equal to what half rho $V^2$ S, then CD0 plus K $CL^2$, into V and I can write CL equals to 2 W by S by rho $V^2$. And finally, I can write an expression for pop power required as off rho $V^3$ S CD0 plus 2 K $W^2$ by rho S V this is straight forward.

$$P_R = T_R.V = D.V$$

$$P_R = \frac{1}{2}\rho V^2 S\left\{\left(C_{D_0} + KC_L^2\right)\right\}.V$$

$$C_L = \frac{2W/S}{\rho V^2}$$

$$P_R = \frac{1}{2}\rho V^3 S\, C_{D_0} + \frac{2KW^2}{\rho SV}$$

Message you have to just plugging this expression here and do some arrangements to get this expression, what is important for us is, what is this term this is as the speed increases power required also will increase as far as this term is concerned and that is called zero lift power zero lift power.

The physical interpretation is if you make the airplane aerodynamically efficient in a sense that the shape is such its parasite drag is minimal, then you have this value also less. And, second one goes inversely with V this is the induced power, it is very clear as I am increasing the speed the CL required is less CL require less means drag will be less drag require less means your induced power, also will reduce. This is basically, coming from the CL requirement as CL reduces your induced drag reduces, so induced power also go on reducing with speed.

$$P_R = \underbrace{\frac{1}{2}\rho V^3 S\, C_{D_0}}_{Zero\ lift\ power} + \underbrace{\frac{2KW^2}{\rho SV}}_{induced\ power}$$

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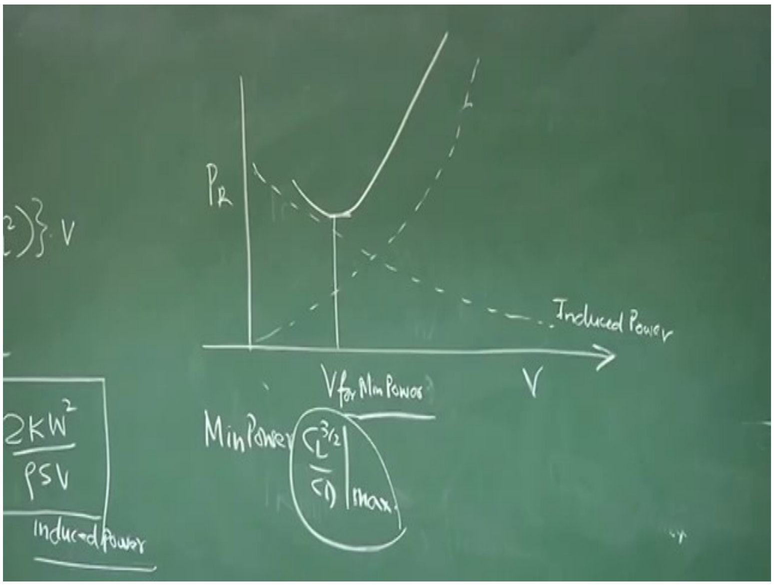


So, if I now plot this, this is the induced power and this is the zero lift power and you find that net power will be something like this. And, this is the point, where this is V for minimum power, ok, please note that this point is not here, this is less than this point and I will explain this.

Let us, analytically find out, what will be this point, but one thing we understand for minimum power, we have seen that CL 3 by 2 by CD should be maximum, or the meaning, if I want to fly at minimum power I should fly corresponding to that CL, for which this is maximum and naturally for the that CL there is a fixed velocity, because it is a cruise for a given altitude. So, we know find out analytically what is that CL, for which I should fly, so that I get minimum power.

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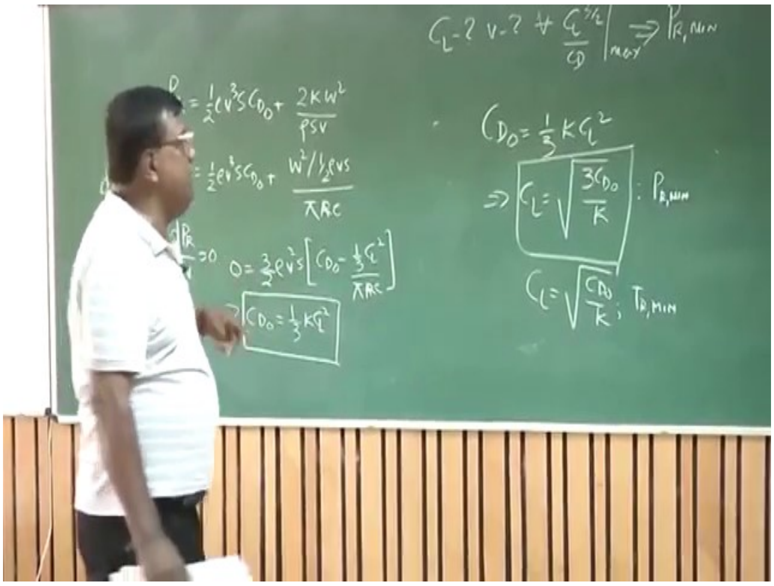

So, the question I am asking is, what is that CL or V, for which CL 3 by 2 by CD is maximum, that will ensure I am flying at power required minimum.

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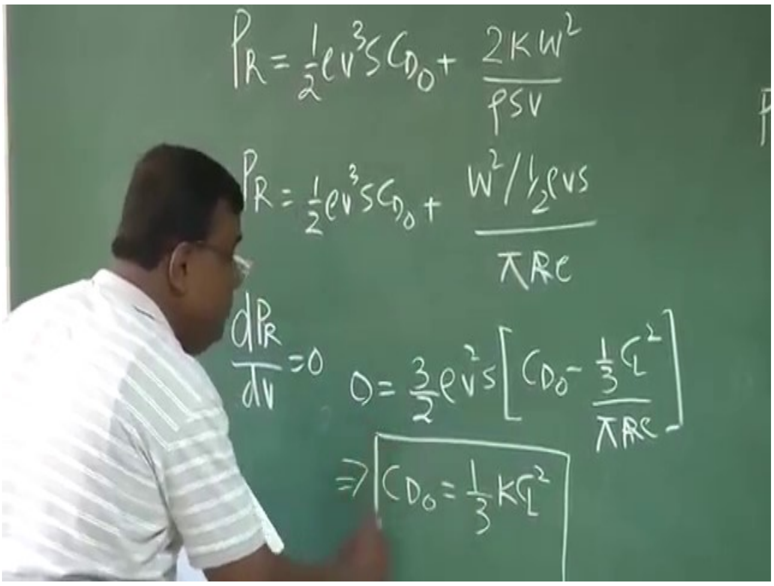


So, I will now take derivative power required by d V and V equal to 0 and that will tell me will give me expression, which you can check 0 is equal to 3 by 2 rho V square S, CD0 minus 1 by 3, $CL^2$ by pi aspect ratio e, which implies CD0 equal to 1 by 3, K $CL^2$, ((Refer Time: 34:20)) CD0 is equal to 1/3rd K $CL^2$ it means CL equal to under root 3 CD0 by K.

$$P_R = \frac{1}{2}\rho V^3 S\, C_{D_0} + \frac{2KW^2}{\rho S V}$$

$$P_R = \frac{1}{2}\rho V^3 S\, C_{D_0} + \frac{W^2/\left(\frac{1}{2}\rho V S\right)}{\pi A R e}$$

$$\frac{dP_R}{dV} = 0 \Rightarrow 0 = \frac{3}{2}\rho V^2 S\left[C_{D_0} - \frac{\frac{1}{3}C_L^2}{\pi A R e}\right]$$

$$\Rightarrow C_{D_0} = \frac{1}{3}KC_L^2 \ or\ C_L = \sqrt{\frac{3C_{D_0}}{K}}$$

Now, I have to fly the machine at a CL, which is given by 3 CD0 by K to get power required, power required minimum, what was for thrust required minimum for thrust required minimum CL was under root CD0 by K for thrust required minimum.

$$C_L = \sqrt{\frac{C_{D_0}}{K}}\ ; T_{R,MIN}$$

Now, for cruise flight if I am having same altitude, then which case I need more velocity that will be decided by, if I am required flying at a power required minimum my CL will be higher. Then, for CL thrust required minimum since velocity is higher the velocity will be less in this case, V will be V for power required minimum, will be less than V for thrust required minimum.

$$V_{P_R,MIN} < V_{T_R,MIN}\ ; since\ C_{L_{P_R,MIN}} > C_{L_{T_R,MIN}}$$

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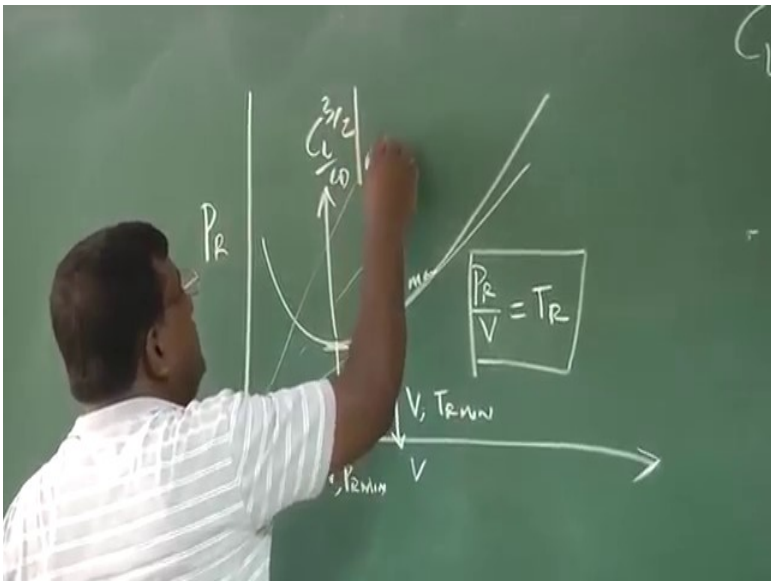


There is another interesting thing we can see from power required verses velocity graph, if I draw a tangent, then this point will correspond point V for thrust required minimum and of course, this point is V for PR minimum, why this point for thrust required minimum, you know the slope is P R by V, which is nothing, but thrust required and that is a minimum thrust required, ok if you go on drawing this slope. So, from power required versus velocity graph I can find out what is the velocity I should fly. So, the thrust required is minimum, and this will corresponds to CL by CD maximum this will corresponds CL 3 by 2 by CD maximum, ok.