NOC: Introduction to Airplane Performance
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Lecture - 19
Revision

Good morning friends, I hope you are able to follow the lectures, I understand the limitations and this is our Mann Ki Baat session. ((Refer Time: 00:22))As I am being telling, I also spend some time with these lectures, these videos and then I try to assume thing that, I am doing my B. Tech course that is 77, 82 in IIT, Kanpur and what sort of feeling I use to have when professor is to come and write lot many equations, lot many theories.

At some point of time, I used to tell myself [FL], enough it is enough. Then, what is the best way we were handled or we are given assignments and also in the lecture, whatever theory has been explained some numerical examples were given. Why this is important? Please understand, if you want to be really successful and you are sure that you have understood, there is a better way of checking it, whatever I understand is, I believe there are three steps are there, one is "know how".

So, I have learnt from theory, one is "do how" I should know how to implement it and finally, it is just "show how", when you use this "know how" and "do how" to produce a product, in this case an aircraft. So, now we will believing understanding, know how do how and show how, we will be attacking the do how part, that is we will try to understand how this theory to be implemented for practical purpose and I have taken the example of standard atmosphere.

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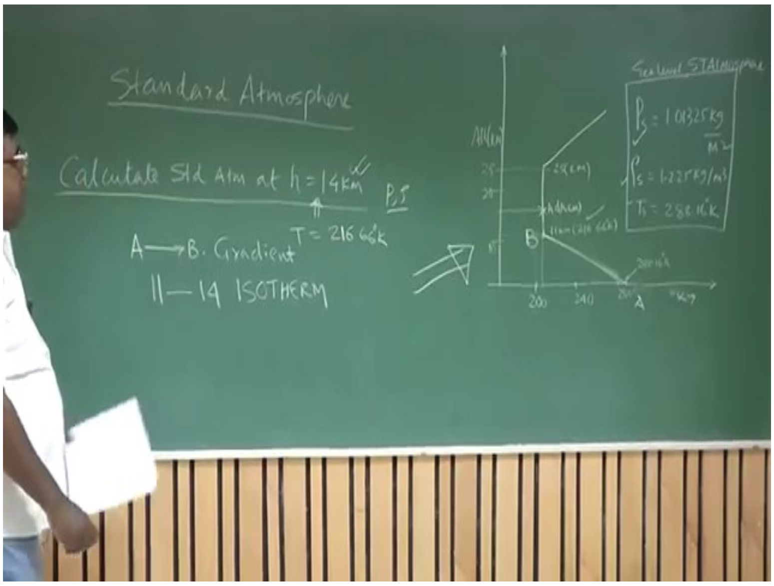

I will be solving one practical problem, let us say that I want to calculate standard atmosphere at h equal to 14 kilometres, this is my question. And what do I mean by standard atmosphere? I am asking for do I know pressure, density and temperature. The same time I know that temperature data versus altitude has been generated experimentally by firing sounding rocket launching; weather balloons for different places and then, some average value have been taken.

This average values were used to represent the variation of temperature with altitude, which is here which we have already seen. Please, understand this is experimentally generated, this is the starting point. Now, what is our question? Our question is at h equal to 14 kilometres, what are our pressure and density. Temperature anyway we know from this variation which is agreed by every team, every organization working in aerospace.

So, let us see, when I see this is to be evaluated at h equal to 14 kilometres, I should immediately see the temperature variation where that is the starting point and in that we know, at sea level the, in standard atmosphere condition these are the values. Pressure is 1.01325 kg per metre square, density is 1.225 kg per meter cube and temperature is 288.16 degree k and of course, this Ps standard the pressure we are meaning static pressure.

$$P_s = 1.01325\, kg/m^2$$
$$\rho_s = 1.225\, Kg/m^3$$
$$T_s = 288.16^{\circ}K$$

Whenever we say pressure and we do not mention anything associated with it, we are actually meaning static pressure.

So, if I want to find the 14 kilometre which will be somewhere here, let say this is a point which is h equal to 14 kilometres. Its very simple, from here I have to start and find what is a property here, I know from here to here there is something called gradient slope and then, there is an isotherm. So, there are two regions and we know by now, how to calculate the properties using p equal to rho R T that is I am assuming air to be a perfect gas.

Since, it is h equal to 14 kilometres, I immediately see here, I see here and I find 14 kilometre means, the temperature is 216.66 degree Kelvin, which I have written here. This I am directly getting from this chart, in practice you will find this is standard atmosphere table generated. You can do a Google search, you will find and I am trying to give you an example, how that table was generated. A specific case, I have taken h equal to 14 kilometres, what are the value of pressure, densities and your temperature we know.

So, first what I do, I mark it point B and let me call it point A. I will first see up to 11 kilometres is point A to B, because there it is a gradient part. Because, the temperature has a lapse rate and then, from 11 to 14 it is isotherm that is temperature is constant. We have already developed a relationship for the gradient region as well as for isotherm, we have to simply apply that, let us do that.

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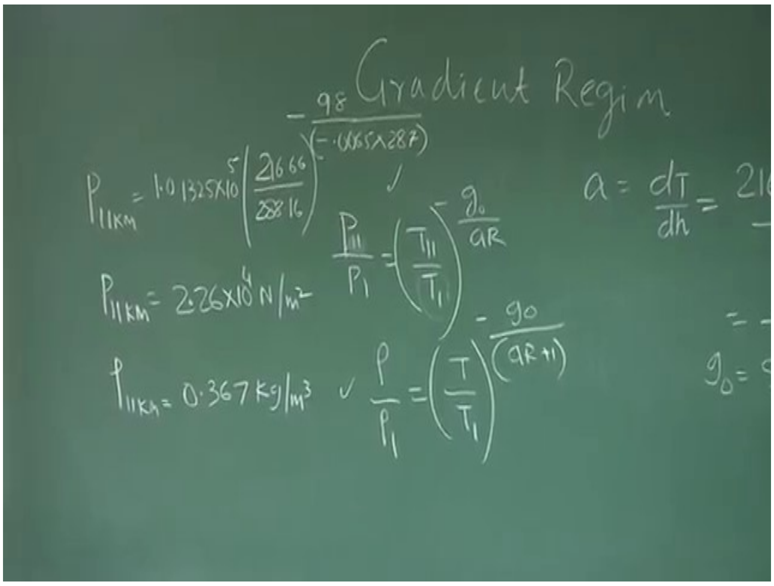

If you see, in the gradient region the lapse rate which is d T by d h is, how much. This point is 216.66, so this will be 216.66 minus 288.16 by 11.0 minus 0. This is the standard condition 288.16, this minus this and height is 0 and 11 kilometre. So, I get as minus 0.0065 Kelvin per meter which is you know that, lapse rate is this.

$$a = \frac{dT}{dh} = \frac{216.66 - 2.88.16}{11.0 - 0} = -0.0065\ K\ /\ m$$

This is nothing we are calculating, because it is the way we have plotted this altitude and temperature variation through experimental data.

Now, in the gradient region, what are the relations? Let me check the notes. It is p by p1 is equal to T by T1 into minus g 0 by a R, R is the gas constant and rho by rho 1 is T by T1 minus g0 by a R plus 1. What was g0? g0 is the sea level value of g, g0 concept we use for geopotential height, so that is 9.8 meter per second square.

$$\frac{P}{P_1} = \left(\frac{T}{T_1}\right)^{-\frac{g_0}{aR}}$$

$$\frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-\frac{g_0}{aR+1}}$$

$$g_0 : 9.8\ m\ /\ s^2$$

Now, what should I plug? I want to find pressure at 14 kilometres, pressure at 14 kilometres we want to find out, but we are now first finding pressure up to B, 11 kilometres. Because, up to B it follows the gradient law that is up to B, this relationship are available and valid.

So, first I find P11 by P1. What is P1? P1 is the standard atmosphere here which I know, T and T1, T1 is the standard atmosphere and T is T at 11 kilometres, which also I know as 216.66. So, if I substitute this what I get, P11 kilometres is 1.01325 into 10 to the power 5, that is the p at sea level into T is 216.66 divided by 288.16 to the power minus g 0 is 9.8 and what is a, a is minus 0.0065 into R is 287 in SI unit.

$$\frac{P_{11}}{P_1} = \left(\frac{T_{11}}{T_1}\right)^{-\frac{g_0}{aR}}$$

$$P_{11} = 1.01325 \times 10^5 \left(\frac{216.66}{288.16}\right)^{\frac{-9.8}{-.0065 \times 287}}$$

So, this will give me P at 11 kilometres equal to 2.26 into 10 to the power 4 Newton per metre square. Is it clear? I have just substituted the values. Similarly, if I substitute for rho I get rho 11 kilometres is equal to 0.367 kg per meter cube, try to develop a feel for this number. If we are flying at 11 kilometres, the pressure is 2.26 into 10 to the power 4 and what is the atmosphere, atmospheric pressure at sea level. It is 1.01325 into 10 to the power 5 kg per, this is Newton per metre square, this is correct it.

$$P_s = 1.01325 \times 10^5 N/m^2 : P_{11} = 2.26 \times 10^4 N\ /\ m^2$$
$$\rho_s = 1.225\ kg/m^3 : \rho_{11} = 0.367\ kg\ /\ m^3$$

This is 1.01325 into 10 to the power 5 Newton per metre square. It was 1.01 into 10 to the power 5, where here it is 10 to the power 4 into 5, so pressure has reduced so much. So, what will happen? Suppose, if the cabin is pressurised with, let us say with atmospheric pressure or little less than atmospheric pressure. Generally, calibrated 6000 feet, 5000 feet, 8000 feet, let say for assume it is just atmospheric pressure.

Then the difference is so much, if there is a crack on the windshield things inside will be thrown out and also if we do not take proper care with an understanding, that there is so much of difference in pressure, our structural design also will not be appropriate. So, we have to be very careful when we are designing an airplane, we must see at that altitude, what is the pressure difference. So, that way this is and most of the airplane, the jet engines and all you will find they will be flying at 11 kilometres which is tropopause.

So, 11 kilometres values we now know, we have to go to 14 kilometres. So, what is the way? Now, you know that you have to apply relationship, which were used for tropopause. So, let us go back and check, what was the relationship for tropopause.

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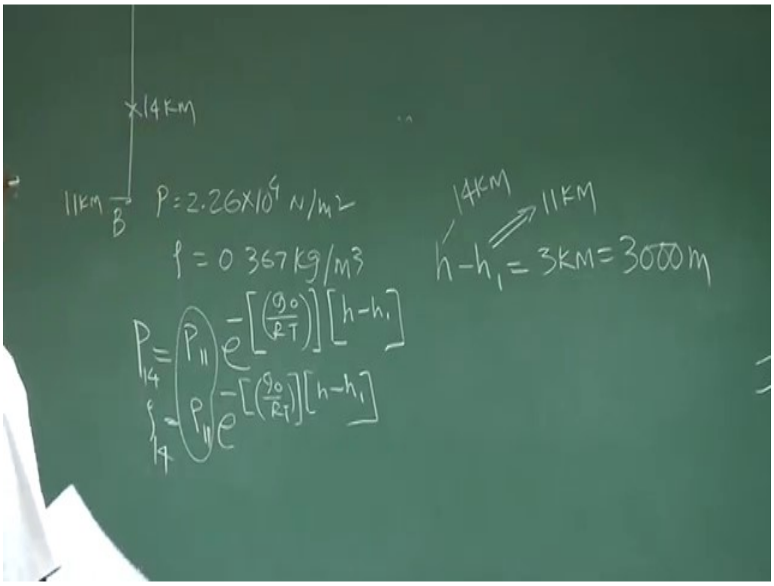

So, our problem is now this is the point B, now I know P is 2.26 into 10 to the power 4 Newton per metre square and rho is 0.367 kg per metre cube. I have to find at 14 kilometres and this is 11 kilometres. So, we will check what was the relationship for tropopause. If we check your notes, you will find P was given as P1 e to the power g0 by R T into h minus h1. Similarly, rho was given as rho1 e to the power, same expression g0 by R T into h minus h1.

What is P1 here now? I am going from 11 to 14, so basically I want to find at 14 given P1 which is at P11 kilometres. Again I want to find rho at 14, given rho at 11 kilometres, these two values we have already calculated, I have to just substitute that. And for h minus h1, what should I put? H minus h1 means, from here to here h is 14 kilometres. What is h1? H1 is 11 kilometres, so this difference is 3 kilometres and to be consistent this unit is 3000 metre, now simply substitute here.

What about temperature? You know it is tropopause, it is tropopause, so whatever temperature was here, same will remain here. So, this is 216.66 degree Kelvin and if I substitute this, then I get value I get P at 14 kilometres as, you please do it yourself. It is very explicit, now Newton per metre square and rho at 14 kilometres is 0.23 kg per metre cube. Place the calculation yourself, I may commit some error, although this example I have taken from a book by Introduction to Flight by Anderson.

There are plenty of books you can use, you can refer, but mostly you will find the examples I am picking from Anderson. Then, there is a book for Stability and Control by Nelson, to do a Google search you will find so many books. So, what is the message that once this temperature profile is given which is measured and agreed, based on experimental result, you ask me any altitude I can find out the pressure, density and temperature, temperature anyway is known.

That is how whole atmospheric table is created and the table will be given H, what is the pressure, what is the density, what is the temperature. So, you can look through the table and find out. So, this is the basis of generation of standard atmosphere table, this is clear.

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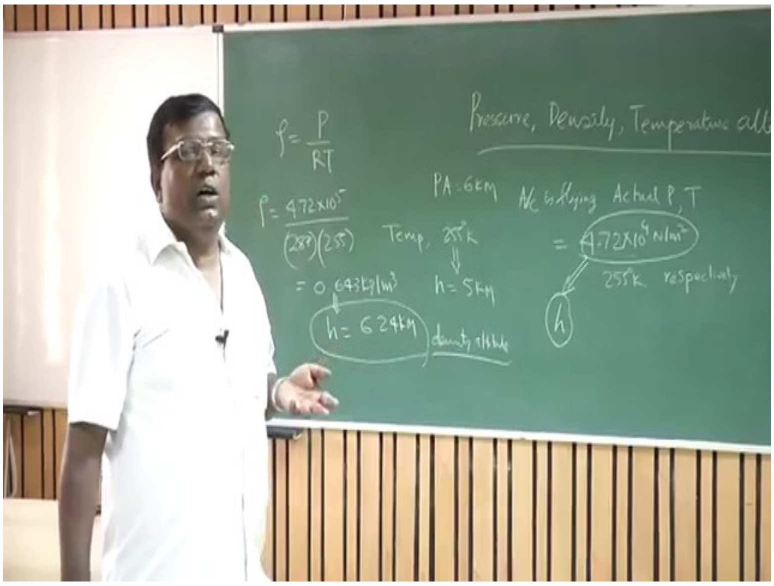

Now, second part which also I want to solve the problem, so that you are clear about in your mind, what you are doing. We talked about pressure, density and temperature altitudes. Let us quickly recap, what is the pressure altitude, what we were telling that suppose an airplane is flying at 11 kilometres and it is sensing some static pressure. Now, pressure altitude define like that, whatever static pressure it is sensing in reality, you now come back and see the standard atmospheric table and see, what is that altitude in standard atmosphere which will record this much of static pressure.

That is, if I am flying at an altitude with some static pressure I have some value, then I check the standard atmosphere and look for static pressure, this static pressure and what is the corresponding altitude in standard atmosphere. So, I call that pressure altitude.

Similarly, temperature altitude and then I know density altitude, because P equal to rho R T, I find density altitude. From there I again find out, what is that density, then I check this atmosphere, what is that altitude that gives that much of density in standard atmosphere as simple as that. It will be clearer once I solve a problem.

Let us say suppose an aircraft is flying, where actual pressure and temperature are 4.72 into 10 to the power 4 Newton per metre square and temperature 255 degree Kelvin respectively. Question is, what is the pressure altitude, what is the temperature altitude and what is the density altitude? What I will do now, I will take this number, this much of pressure then I will go to the standard atmosphere table, I will find out where is this pressure and check what is the altitude that correspond to that pressure, that is I see standard atmosphere table and look for this pressure and corresponding value of h that will be the pressure altitude.

In this case, it will be you find that will be pressure altitude will be 6 kilometres, then for temperature altitude again same thing, so mechanical. What is the temperature recorded 255 degree Kelvin, then from there go to standard table and see what is the corresponding altitude in standard atmosphere as far as h is concerned. So, that h you will find is around 5 kilometres. So, it is now to find the density altitude, what we do is, simply first find out the density. How do we find density? I write rho equal to P by R T from the perfect Gauss law.

So, I will find rho equal to 4.72 into 10 to the power 5 which was the pressure recorded, 4.72 10 to the power 5, then divided by R we know 287 and T we have recorded as 255. So, I get a density value 0.643 kg per metre cube, now it is very simple, I go to standard atmospheric table and see this density corresponds to what value of h and that h will be, in this case it will be 6.24 kilometres. So, this becomes my density altitude. So, you could see although the aircraft is flying at a fixed geometric altitude, but their pressure altitude, temperature altitude and density altitudes are different, that should be borne in my mind. Now, it is obvious you should be able to answer this question, at what altitude the pressure, density and temperature altitudes are same. The answer is at sea level.

Thank you very much.