NOC: Introduction to Airplane Performance
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Lecture – 08
Standard Atmosphere: Description and Modeling

See, you were trying to understand how the lift is generated through a reaction between body and the medium. So, we have been talking about body in terms of CL, CD in terms of shape, in terms the area. But, since it is the interaction between body and medium, we must also know characterize the medium through which the airplane is flying.

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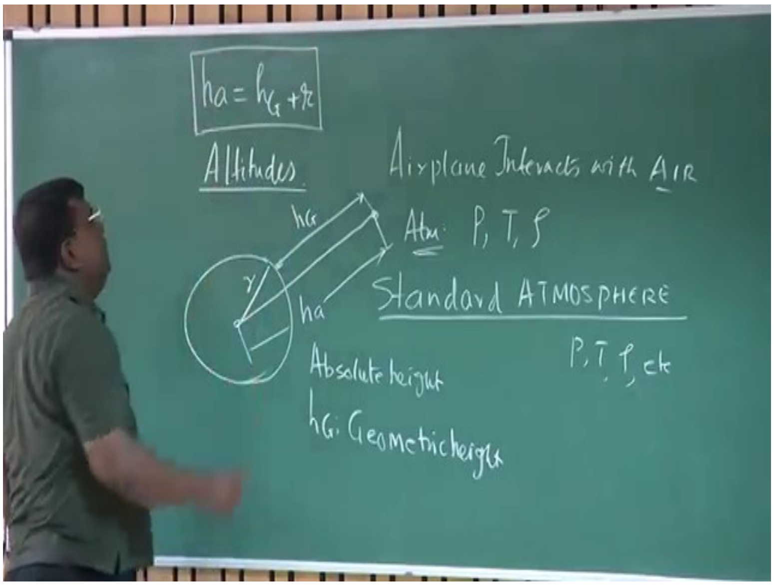

In this case, the airplane interacts, interacts with air and we call that the airplane is flying in atmosphere and that is where they are called atmospheric vehicle unlike space vehicle, which travel most of his path in non atmosphere, in a space. But, we will be restricting ourselves to atmospheric vehicle and we are trying to know little bit about the atmosphere.

You know this atmosphere is characterized for the purpose of interaction between airplane wing or airplane body with atmosphere through three thermo dynamic variables; that is pressure (P), temperature (T) and density (ρ). After all, it is the force which comes because of the pressure. The temperature decides the density and also density by itself will decide, how much force actually the airplane will be experiencing for a given a speed or for a given angle of attack.

So, we are trying to understand atmosphere through pressure, temperature and density and we also know that, we need additional information's like, what are the wind conditions there or the wind is 20 knots or 40 knots, heavy wind, low wind. What sort of immunity is there or any other atmosphere, if attributes are there are not before you design and airplane for that flight corridor, ok.

And why we need to define something called standard atmosphere, it is also very simple and relevant. When we design an airplane, we go for a wind tunnel test, we go for flight test. Then, suppose we have designed two airplanes, we want to compare the performances. But, suppose I am evaluating the performance of an airplane in Delhi, atmosphere, suppose I am same airplane I am using to evaluate its performance in Srinagar.

So, I cannot expect they will generate, although will experience a same force for other conditions of speed, area being same. So, when I want to compare two airplanes performance, I need to compare them on a common platform or common atmospheric conditions and that is why it is important that we define standard atmosphere through pressure, temperature, density, etcetera.

So, that whatever design you have made, whatever airplane we have configured, if I want to compare them, I will compare them assuming all their airplane flying in standard atmosphere or what are their performance's is, when I try to see them in the standard atmosphere, ok. Let us again come back, I need to know little bit on the altitudes, when I use the term altitude, we always think about the height from the surface of earth. For all practical purpose; that is the height we refer to all the time.

We also know by now that there is a height which is relevant from the center of earth, for a space vehicle, etcetera, etcetera. We are more bothered about the height for the center of the earth, because we need to capture the gravitational force. So, we will define two types of height, one is ha (h_a), which we call absolute height and we will also define the height with respect to the earth surface hG and hG, we say geometric height (h_G). And if I call this r, small r as the radius of the earth, then it is very simple to see, ha is equal to hG plus radius of earth, right.

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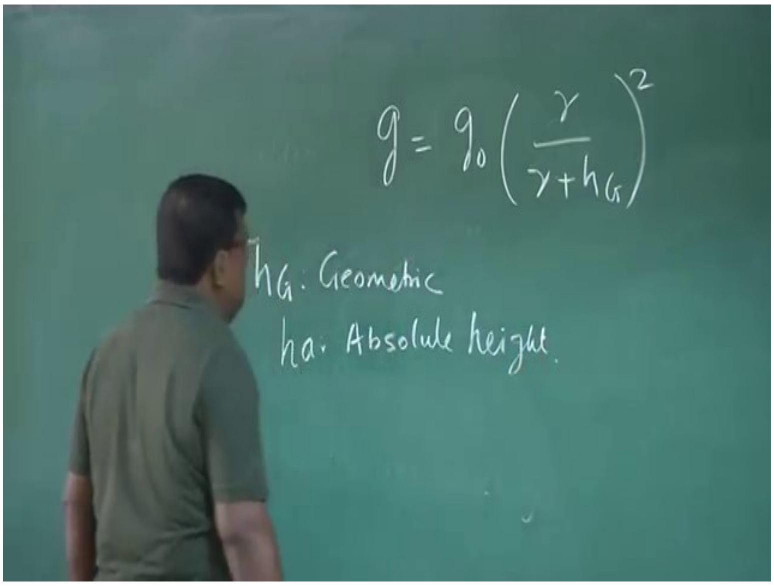

And we know that g at any station can be written as r by r plus hG squared.

$$g = g_0\left(\frac{r}{r+h_G}\right)^2$$

The message is, if I know radius of earth, if I know what geometric height I have, then if I know the local value of the g at the surface of the earth, which is g0 or which technically we say sea level g0. Then, depending upon the hG, I can find out what is the value of g at a certain height, certain geometric height hG. So, that is not a problem.

So, we have define two altitudes, one is hG which is geometric, another is ha, which is absolute.

$$h_G\ : Geometric$$
$$h_a\ : Absolute\ height$$

Now, you see the problem, the atmospheric properties like pressure, temperature, densities, they are not only function of altitudes at what height you are or also function of, whether it is day, whether it is night, with seasonalities. And so many other factors may be sun activity, right, so it is a very dynamic in nature.

So, very difficult to define the standard atmosphere based on local measurements. So, what is done is, take measurements of different, different points, different, different places and then, make an average value, take an average value and internationally everybody will agree that yes, this is the standard atmosphere we will be the referring to when we are comparing two flight tests or two performances test of two different airplane.

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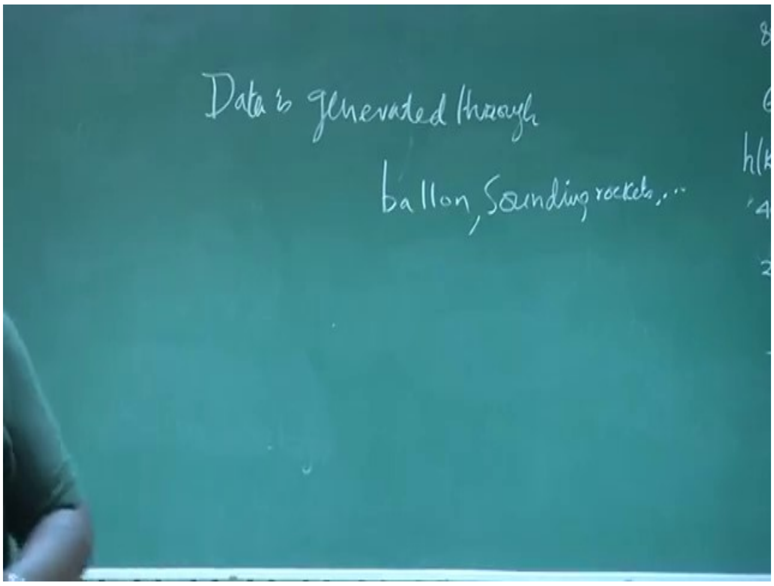

So, the process is like this, what is done, that this data is generated through balloon. You know, there are weather balloon they are launched and it goes on sensing thermodynamic variables, primarily temperature. Then, sounding rockets, etcetera.

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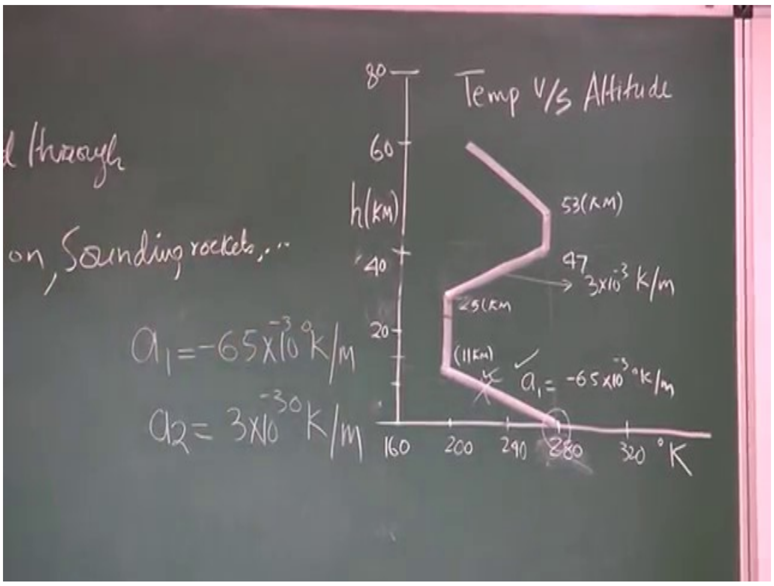

What do we see classically, the temperature versus altitude. Typically, if I take the average values of all parts, most of the parts, relevant parts and take the average, we will find the temperature with altitude, it will vary like this. If I come here, see up to 11 kilometer, which is typically the tropopause, and most of us civilian airplane, jet engine airplane, they will fly at around at 11 kilo meter tropopause.

If I see here up to 11 kilometer, the variation is linear and the lapse rate a1 (a_1) is approximately 6.5 into 10 to the power minus 3 degree Kelvin per meter.

$$a_1 = -6.5\ X\ 10^{-3}\ °K\ /\ m$$

Similarly, the experimental observation shows that from 11 kilometer to around 25 kilometers, there is no change in temperature and we call it as isotherms. And again beyond 25 kilometers to around 47 kilometers, there is having the gradient.

But, unlike in this region, these are the positive slope and positive lapse rate or we call it a2 (a_2) is plus 3 into 10 to the power minus 3 degree Kelvin per meter.

$$a_2 = 3\ X\ 10^{-3}\ °K\ /\ m$$

So, this data or this trim is purely generated through experiments, through balloon, weather balloon, your sounding rockets at different places, different times and some average value has been agreed upon by international aviation society, ok.

But, please come back to our original question once, once I want to characterize the atmosphere, I need to know it is thermodynamic variables, the pressure, temperature, densities, temperature profile is here.

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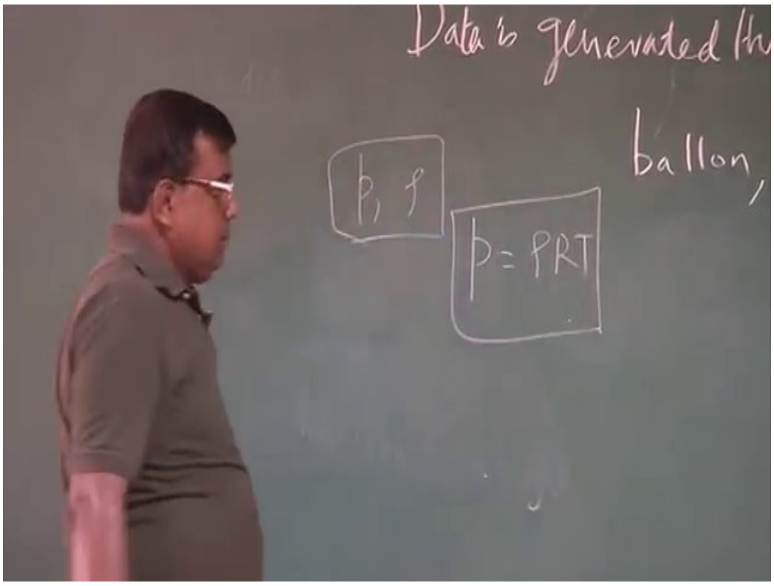

So, I need to know pressure and density that is extremely important, if I want to find out lift, where if I want to find out drag, these two thermodynamic variables will play important role. So, I need to create a standard atmospheric model by using this experimental data ((Refer Time: 10:57)) and how do I do that. That is done simply assuming that, the air follows perfect Gas law; that is P equal to rho R T.

$$P = \rho RT$$

That is we will be treating here following the perfect Gas laws and then, we will be try to find out, what will if I know temperature is here, pressure here, density here, then what is the pressure.., temperature density at that point by using this in mind that, they are perfect gas.

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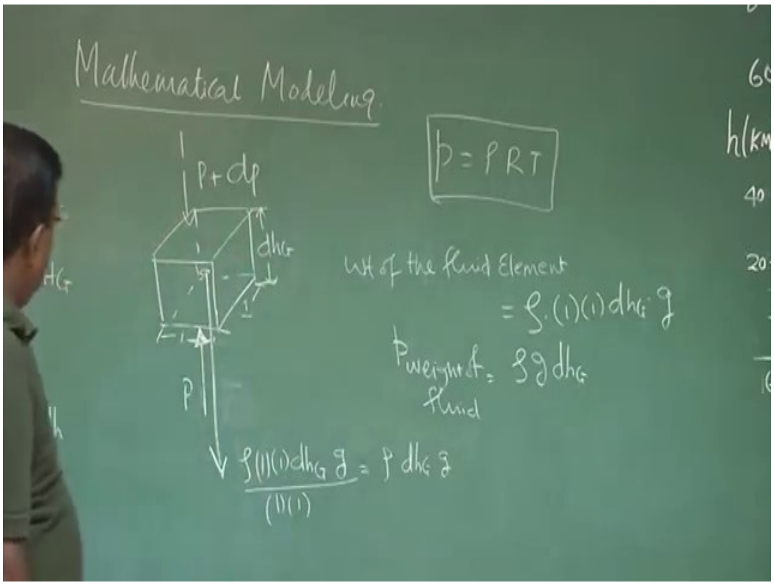

Here, we are trying to develop a mathematical model and we are assuming that, this is the element of air, which is cubical shape with length 1 unit, breadth 1 unit and the height dhG (dh_G), why hG, because hG is relevant as well as atmospheric presence is concerned, right? So, if I try to do a static balance, we know there will be a pressure acting upward the bottom surface let say that is P (P).

At the top, it will be P plus dP ($P + dP$) and also note here, the weight of this fluid element. What will the weight of the fluid element? Weight of the fluid element will be mass into local value of g, mass is density into volumes. So, density is rho, volume is 1, length, breadth, dhG and into g, this is the weight of the fluid element.

$$wt. of\ the\ fluid\ element = \rho.(1).(1).dh_G.g$$

And this weight is acting on unit area, because this weight is acting on unit area 1 into 1. So, pressure because of this weight of fluid, in this case it is here that will be rho g d h G, because area is 1.

$$P_{weight\ of\ fluid} = \rho g\ dh_G$$

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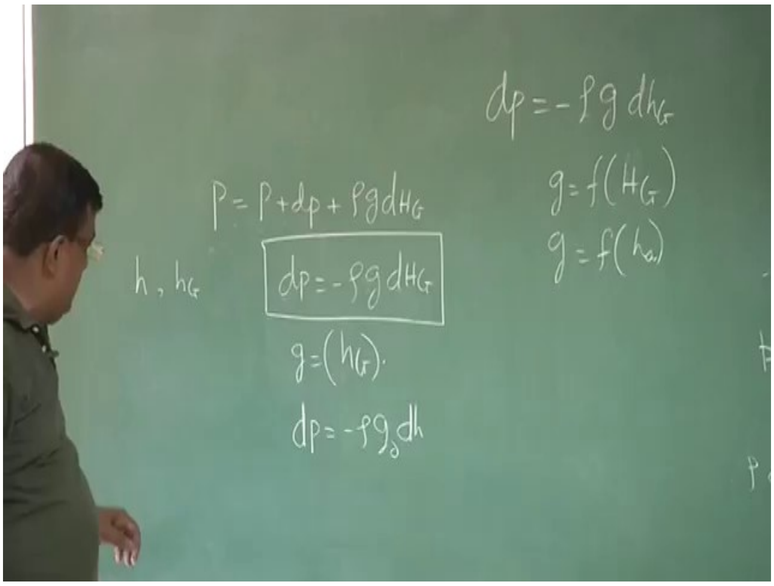

So, now if I simplify this I can write P is equal to P plus d P, P here, P plus dP acting down and pressure because of the way weight acting down. So, I can required them and find out that dP equal to minus rho g delta h G.

$$P = P + dp + \rho g dh_G$$
$$dp = -\rho g dh_G$$

Now, see here, there is a catch as for as operational part is concerned. If I clearly see that, d P is minus rho g delta hG, g is function of hG. That is as I am going up and up; the hG that is the geometric height also increasing and g also will become function of hG.

$$g = f(h_G)$$
$$g = f(h_a)$$

So, if I want to integrate this to find pressure and an all, so I have to know explicitly, one thing that g is not going to remain constant. In fact to be more precise, the g should be also function of you know h, ha, which is the absolute height, because as for as g is concern, it is the references from the center of earth. But, since here dhG is there, it was g become function of hG, then integration become it is complex.

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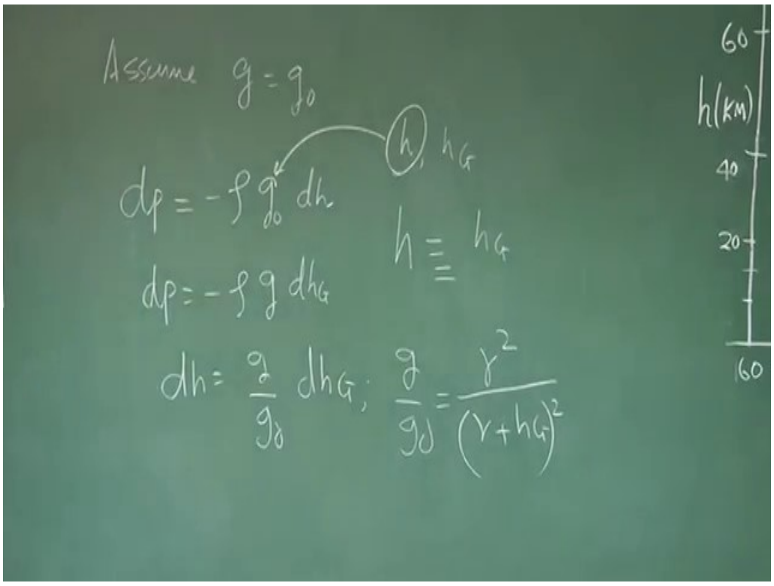

So, what is done for simplicity, assume g is equal to g0; that is, we are assuming that value of g is, what the value of g at sea level; that is constant. So, now, it could see for same differential pressure, I am writing minus rho g0 dh, for same differential pressure, I am writing minus rho g dhG. So, definitely from this relation I can see that, h and hG are not same and by geometry also, we know h and hG should not be same, hG because of the consideration, because of the atmosphere.

if this g potential by this h is defined as a fictitious altitude, where I assume the value of g is not changing and you're taking as g0 it is also at fictitious altitude and we it call geopotential altitude. As long as I know, what is the relationship between h and hG, I have no problem, I can revert it back, right , ok. So, do to that you could see for, if I use this two relationship, I find that dh equal to g by g0 and it dhG and since I know that g by g0 is nothing but, r squared by r plus hG square, where r the radius of earth.

$$dh = \frac{g}{g_0} dh_G; \frac{g}{g_0} = \frac{r^2}{(r + h_G)^2}$$

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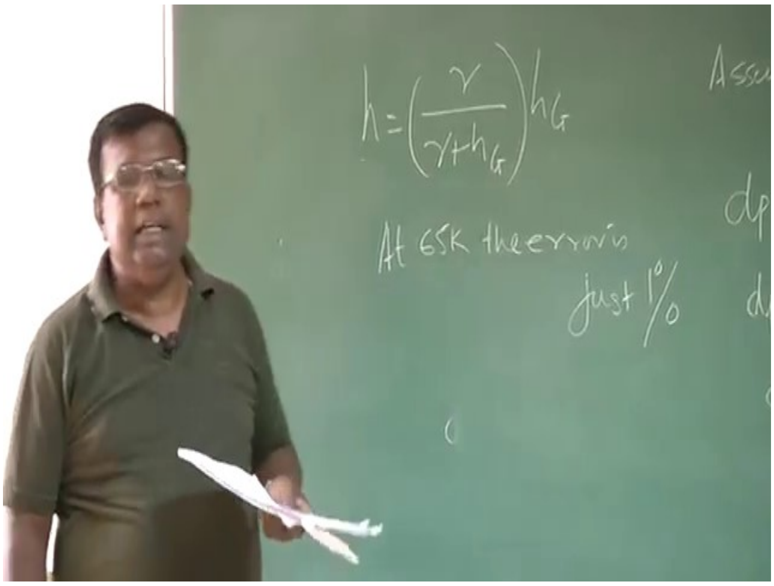

So, using this two relationship, I can write the h equal to r plus hG to hG.

$$h = \left(\frac{r}{r + h_G}\right) h_G$$

Now, what is the error I am taking when I assuming hG and h, are same. Just to have an idea, if you plug in the value you will find at 65 kilometer, the error. The error is just 1 percent; so you could understand assumption that h and hG are same that helps in making the analysis so simple. Still, you are getting the physics of the situation, ok.

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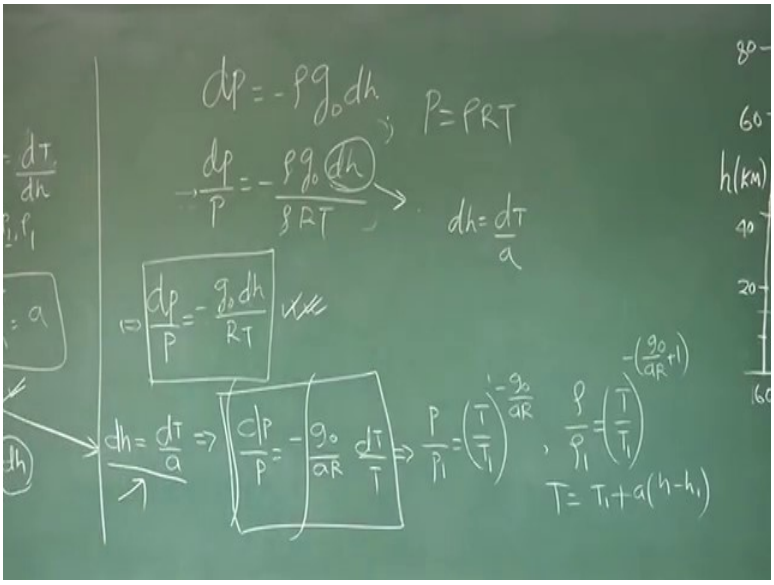

So, what is that important thing we have found out is, which will be rigorously use to model atmosphere is minus rho g0 d h, $(dp = -\rho g_0 dh)$ now come back to this. What is our aim, the aim is the temperature profile is available, I want to find out, what is the temperature which I know from here, what is the pressure, what are the density at this point or some other point here or some other point here.

And what is given to us, I know what is the condition here, right? from the static balance, what important relationship we got, we got d P equal to minus rho g0 dh and h, we have define a geopotential height, which is a fictitious height. We are assuming that, acceleration due to gravities constant and we have also seen that h and hG hardly, they are differ. There are either 1 percent, again at 65 kilometers.

So, we carry forward with this relationship and also now, we use the perfect gas relationship P equal to rho R T. So, divide dP by P. So, I get rho g0 dh by rho R T. So, I get a very important relationship d P by P is equal to minus g0 by R T in to d h and you see, this will be used for getting the values for pressure and temperature at different points.

$$\frac{dP}{P} = -\frac{\rho g_0 dh}{\rho RT}$$

$$\frac{dP}{P} = -\frac{g_0 dh}{RT}$$

Now, we are taking the first case, where this is the gradient region; that is the temperature is not constant. For example, here we are saying it isotherm; that is temperature is constant, but here temperature is changing. So, we will take how to find pressure and temperature and density for gradient region, right.

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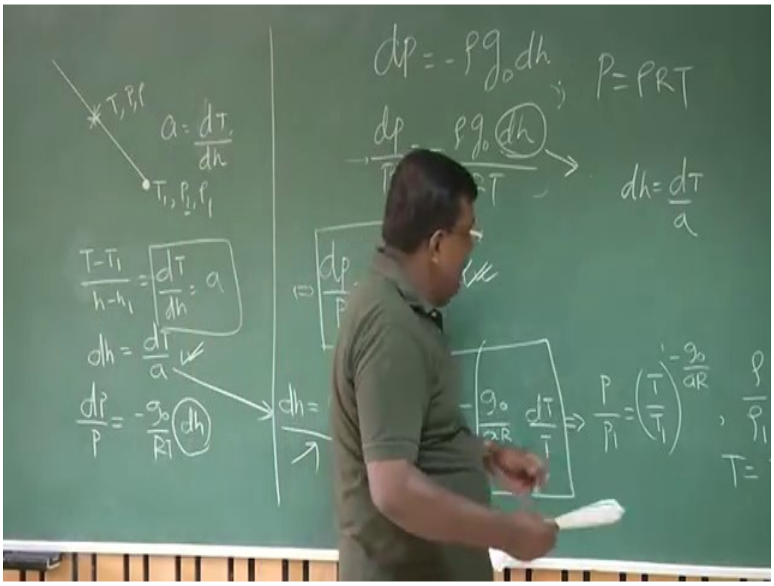

For example, is this condition is T1, P1, rho1, if I know this and I know a as the lapse rate dh by dt, it could see for this region there’s a lapse rate a1. So, I can write T minus T1 divided by h minus h1, which is nothing but, dT by dh is equal to a which is lapse rate.

$$\frac{T - T_1}{h - h_1} = \frac{dT}{dh} = a$$

So, dh I can write as dT divide by a, from this relationship, once a write that I can write dP by P, see here, dP by P is what, dP by P is rho g0 d h by rho R T.

$$dh = \frac{dT}{a}$$

$$\frac{dP}{P} = -\frac{\rho g_0 dh}{\rho RT}$$

So, now, I substitute this dh by dh equal to dT by a, by write that, then I get dP by P is equal to minus g0 R T in to dh, right.

$$dh = \frac{dT}{a}$$

$$\frac{dP}{P} = -\frac{g_0}{RT}.dh$$

So, dP by P is minus g0 dh by R T, here I will be using this relationship that is dh is, from here I know by definition dh is dT by a, $\left(dh = \frac{dT}{a}\right)$ I will replace this dh by this value to get dP by P is equal to minus g0 a R dT by T. Now, you see very neat expression, I can integrate both side and I know g0 is constant.

$$\int \frac{dP}{P} = -\int \frac{g_0}{aT}\frac{dT}{T}$$

So, I can write expression of P by P1 is equal to T by T1 to the minus g0 by a R, and similarly rho by rho1 is equal to T by T1 minus g0 a R plus 1 and of course, we know T is T1 plus a h minus h1 full lace rate. So, simple I repeat again, I know this condition here, I use this relationship and I just substitute dh equal to dT by a, I get an expression like this. I integrate I get the pressure density and temperature for that point in the gradient region, right?

$$\frac{P}{P_1} = \left(\frac{T}{T_1}\right)^{-\frac{g0}{aR}}, \frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-\left(\frac{g_0}{aR}+1\right)}, T = T_1 + a(h - h_1)$$

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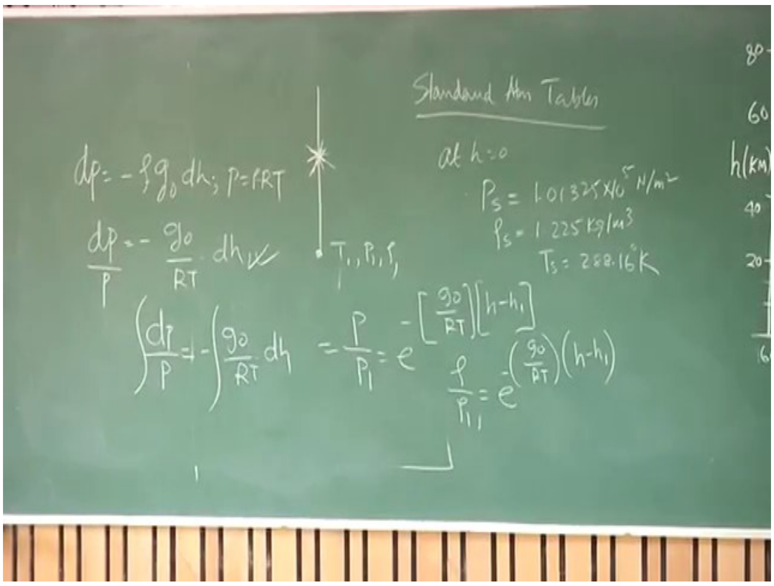

Now, if I want to find out for region whether temperature remain constant, which call isotherm; that is this region, let say this is the region I know here again T1, P1, rho1 condition I know and find out this condition. What is the value of temperature, pressure and densities, we are working towards the method to find out, the thermodynamic variable, temperature, pressure and density.

The pressure and densities is at different attitude, ok, given the initial condition T1, P1 rho1. Again, use like this dP is equal to minus rho g0 d h, this we have derived from the static balance, assuming fluid element in a static condition. And we have also derived this relation dp equal to minus g0 R T into dh, because we have assume that, here is we have a perfect gas. So, p equal to rho R T and I divided dP by that. So, I got this relationship dP by P, P is rho R T that as be P equal to rho R T and I divide by dP by P to get this is expression, this is done.

$$dp = -\rho g_0 dh \, ; p = \rho RT$$

$$\frac{dP}{P} = -\frac{g_0}{RT} dh$$

Now, if you see in the isotherm, I can simply integrate this, if I integrate both sides, because temperature is constant is isotherm it comes out. So, what do I get, I get P by P1 is equal to e to the power minus g0 by R T in to h minus h1. Similarly, I can find rho by rho1 is equal to e to the power minus g0 by R T, h minus h1. So, you could see in isotherm is a very straight forward.

$$\int \frac{dP}{P} = -\int \frac{g_0}{RT} dh$$

$$\frac{P}{P_1} = e^{-\left[\frac{g_0}{RT}\right][h-h_1]}$$

$$\frac{\rho}{\rho_1} = e^{-\left[\frac{g_0}{RT}\right][h-h_1]}$$

So, what is the message, message is, if I want to create the atmospheric description is in terms of the thermodynamic variable, I need to identify whether we are looking in a region, which is governed by the gradient or by the isotherm. And then I have applied this relationship and in practices will find, there will be standard atmosphere tables available, they are generated using this relationship, right.

And of course, the standard value for a pressure, temperature and density at sea level is a fix number and we all take that as the standard condition; that is at h equal to 0; that is P standard at is 1.01325 into 10 to the power 5 Newton per meter square. Then, density standard is 1.225 kg per meter cube and temperature standard is 288.16 degree K.

$$at\ h = 0$$
$$P_s = 1.01325\ X\ 10^5\ N/m^2$$
$$\rho_s = 1.225\ Kg/m^3$$
$$T_s = 288.16^{\circ}K$$

So, this initial condition the standard condition at h is equal to 0 along with this relationship in gradient region and isotherms are sufficient enough to generate the whole standard atmosphere table, ok. Why you have generated all these things, please understand that should be clear, that this standard atmosphere is required primarily to relate the performance two aircraft. So, that I can check their performance is in a common reference, for all this, we are doing to address a question, which aircraft has got better performance.