MATH 362—Work Sheet 07

Dr. Justin M. Curry

Due on Friday, February 26th, 2021

Name: _____

1. (5 points) An urn contains 4 red balls and 3 green balls.
   (a) (2 points) Nine draws are made with replacement. Let X be the number of times a green ball appears. Identify by name the probability distribution of X. Find the probabilities $P(X \geq 1)$ and $P(X \leq 5)$.

   (b) (2 points) Draws with replacement are made until the first green ball appears. Let N be the number of draws that are needed. Identify by name the probability distribution of N. Find the probability $P(N \leq 5)$.

   (c) (1 point) Compare $P(X \geq 1)$ and $P(N \leq 9)$. Is there a reason why these should be the same?

2. (2 points) The population of a small town is 500. 20% of the population has red hair. One morning I go to a diner and see 15 customers are there.
   (a) (1 point) What is the probability that 10 of the 15 customers have red hair?

(b) (1 point) What is the probability that at most 2 of the 15 customers has red hair?

3. (2 points) An urn contains 4 red balls and 3 green balls. Two balls are sampled at random.

   (a) (1 point) Let Z denote the number of green balls in the sample when the draws are done without replacement. Give the possible value of Z and its probability mass function (PMF).

   (b) (1 point) Let W denote the number of green balls in the sample when the draws are done with replacement. Give the possible values and the PMF of W.

4. (5 points) Choose one of the words in the following sentence uniformly at random and then choose one of the letters of that word, again uniformly at random:

   `SOME DOGS ARE BROWN`

   (a) (1 point) Find the probability that the chosen letter is `R`.

   (b) (1 point) Let X denote the length of the chosen word. Determine the PMF of X.

(c) (1 point) For each possible value k of X determine the conditional probability $P(X = k \mid X > 3)$.

(d) (1 point) Determine the conditional probability $P(\text{the chosen letter is } \texttt{R} \mid X > 3)$.

(e) (1 point) Given that the chosen letter is `R`, what is the probability that the chosen word was `BROWN`?

5. (2 points) *This is a challenge question and will not be on the exam.* You are given a fair die. You must decide ahead of time how many times to roll. If you roll exactly 2 sixes, you win a new Tesla. How many rolls should you take to maximize your chances and what are the chances of winning? Note: there are two equally good choices for the best number of rolls.