Math 362 Midterm 1

PRACTICE

Name: _____

- This exam contains 9 pages (including this cover page) and 7 problems.

- Calculators and phones are **not** permitted.

- You are allowed a single 8.5x11 inch cheat sheet.

- Please show your work, simplify compound fractions, and *box your final answer.*

Problem	Points	Score
1	5	
2	10	
3	15	
4	5	
5	15	
6	15	
7	10	
Total:	75	

1. (5 points) A monkey sits in front of a typewriter with 36 keys, which includes the 26 letters of the English alphabet (all lower case) and 10 additional characters for punctuation (spaces, periods, commas and the like). According to `wordcounter.net`, Shakespeare's play Hamlet has 169,541 characters. Assuming the monkey hits keys independently and at random, what's the probability that the first 169,541 characters that the monkey types is exactly Shakespeare's Hamlet (again, not counting capitalization)?

2. (10 points) The New York Times (NYT) recently conducted a survey of Albany residents, which showed that 70% of people subscribe to their newspaper. The Times Union (TU) also recently surveyed Albany residents and calculated that 60% of people subscribe to their newspaper.

   (a) (5 points) For this part of the question, assume that a third independent survey concluded that 50% of Albany residents subscribe to both the Times Union and the New York Times. What percentage of Albany residents subscribe to **neither** the Times Union **nor** the New York Times?

   (b) (5 points) For this part of the question, assume that the third survey was never conducted so *we don't know* how many Albany residents subscribe to both newspapers. What's the **minimum percent** of Albany residents that must subscribe to both newspapers, assuming that the surveys conducted by the NYT and TU are accurate?

3. (15 points) On one of my shelves in my office there are 59 books. 27 are yellow, 8 are green, 8 are red, 6 are white, and 10 are blue. I tend to shuffle my books as I take them off the shelf and I randomly put them back on the shelf. Consequently, any permutation of books on my shelf is equally likely.

   (a) (3 points) One morning, I look up at my shelf and I see that miraculously all my books on the shelf are in alphabetical order by author. Assuming no two books have the same author, what's the probability of such an event?

   (b) (5 points) The first author alphabetically is Adams and the last author is Zeeman. Let A be the event that Adams is in the left most position on my shelf and Zeeman is in the right most position on my shelf. What is the probability of A?

   (c) (5 points) On another day I notice that all my yellow books occupy the left most 27 positions, my green books occupy the next 8 positions, my red books occupy the next 8 positions, my white books occupy the next 6 positions, and my blue books occupy the next 10 positions. What's the probability of that?

   (d) (2 points) How does the answer in part (a) change if three of my books have the same author?

4. (5 points) You are dealt a 5 card hand from a 52 card deck, which has 26 black cards (13 spades and 13 clubs) and 26 red cards (13 hearts and 13 diamonds).

   What's the probability that you are dealt 3 black cards and 2 red cards?

5. (15 points) The United States has a fairly even split of men and women, meaning roughly 50% of the population identify as men and 50% identify as women[1]. However, when it comes to yoga practitioners, the split is vastly different: 72% of yoga practitioners are women and 28% are men. Of course this is possible because yoga practitioners make up only 10% of the US total population.

   (a) (5 points) Given the above information, what's the probability that a randomly selected woman in the US practices yoga?

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   [1]This is a simplification. In actuality 50.8% of the population identify as female and 0.6% of the US population identify as transgender.

(b) (2 points) Are the events of being a woman and doing yoga independent?

(c) (8 points) The other day I went to a yoga class and to my surprise 7 of the 8 people in class were men. What's the probability of observing such an event?

6. (15 points) In front of you are two identical urns. In one of those urns (call it U_1) there are 8 tiny brown goats and 2 tiny white goats. In the other urn (call it U_2) there is only 1 tiny brown goat and 4 tiny white goats.

   (a) (5 points) If you choose one of the urns uniformly at random and reach inside to pull out a tiny goat, what is the probability that the goat you selected is brown?

   (b) (5 points) Assume that you selected a brown goat, what is the probability that you initially selected the urn U_1?

(c) (5 points) Now we consider a variation on the urn-selecting goat-selecting experiment described above. Suppose you pick an urn at random, reach inside and pull out a goat, observe its color, then put it back in the urn, swirl all the tiny goats inside that same urn so that the next selection is independent of the first and you pull out another tiny goat. This means that in our modified experiment, you choose an urn uniformly at random and then select two goats, with replacement.

Suppose you do this modified experiment and you get a tiny brown goat both times, what's the probability that you selected from the urn U_1?

7. (10 points) The organization People for the Ethical Treatment of Animals (PETA) discovers that I've been doing cruel experiments on tiny goats for my probability class. They demand that I release the 15 goats from the two urns in the previous problem, give them individual names and feed the entire trip[2] peanut butter and jelly (PB&J) sandwiches.

   (a) (5 points) As per PETA's request, every day I buy two loaves of bread and I make 18 PB&J sandwiches. In how many ways can I distribute these sandwiches to the 15 tiny goats?

   (b) (5 points) PETA subsequently discovers that I haven't been allocating sandwiches fairly and demands that each tiny goat gets at least one sandwich. Assuming I adhere to PETA's request, in how many ways can I distribute the sandwiches?

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[2]A group of goats is called a "trip."