

AMAT 362—Work Sheet 22

Dr. Justin M. Curry

Due: April 27, 2022. Worth 22 points.

Name: _____

1. (4 points) Suppose X and Y is uniformly distributed on $\{(x, y) \mid 0 < |y| < x < 1\}$. Find
 - (a) (1 point) the joint density function of X and Y ;

- (b) (1 point) that marginal densities of X and Y ;

- (c) (1 point) Are X and Y independent?

- (d) (1 point) Find $E(X)$ and $E(Y)$.

2. (3 points) A random point (X, Y) from the unit square has JDF $f(x, y) = c(x^2 + 4xy)$ for $0 < x < 1$ and $0 < y < 1$ for some constant c .

(a) (1 point) Find c .

(b) (1 point) Find $P(X \leq a)$ for $0 < a < 1$.

(c) (1 point) Find $P(Y \leq b)$ for $0 < b < 1$.

3. (4 points) My washer's lifetime is an exponential RV X with an expected life time of 10 years. My dryer's lifetime is an exponential RV with an expected lifetime of 12 years. Suppose both lifetimes are independent of each other.

(a) (2 points) Find the probability that my dryer lasts longer than my washer.

(b) (2 points) Find the distribution of the difference $D = Y - X$.

4. (2 points) Suppose I throw a dart uniformly at random at a dart board with radius R . Find the marginal distribution for X .

5. (4 points) Suppose a marksman shoots at a round target. The distance of their shots from the center follows the Rayleigh distribution $f_R(r) = re^{-r^2/2}$ for $r > 0$, zero otherwise.

(a) (1 point) At what radius circle about the center includes 50% of all shots?

(b) (1 point) What percent of shots can be found in twice the radius you found in part (a)?

- (c) (2 points) Find the approximate average absolute distance of the shots measured from a horizontal line that passes through the center of the target?

6. (5 points) For X and Y with JDF $f(x, y) = 6e^{-2x-3y}$ for $x, y > 0$ and zero otherwise, find:

(a) (2 points) $P(X \leq x, Y \leq y)$

(b) (2 points) $f_X(x)$ and $f_Y(y)$

(c) (1 point) Are X and Y independent? Say why or why not.