MATH 362—Work Sheet 04

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Due on Tuesday (!) February 16, 2021

Name: _____

1. (4 points) An urn contains 15 red balls, 20 green balls and 12 yellow balls. Consider the experiment where we draw two balls without replacement. Consider the event A where exactly one red ball *or* one yellow ball is drawn.

   (a) (2 points) Decompose the event A into three mutually exclusive events and use this decomposition to compute the probability of A.

   Mutually exclusive events: RG, YG, RY

   $$P(A) = P(RG) + P(YG) + P(RY)$$

   $$= 2\cdot\frac{15}{47}\cdot\frac{20}{46} + 2\cdot\frac{12}{47}\cdot\frac{20}{46} + 2\cdot\frac{12}{47}\cdot\frac{15}{46}$$

   (b) (2 points) Notice that A is the union of the events

   $$R = \{\text{exactly one red ball is drawn}\} \cup Y = \{\text{exactly one yellow ball is drawn}\}$$

   Why are these events not disjoint? Compute $P(A)$ using the formula

   $$P(A) = P(R) + P(Y) - P(R \cap Y).$$

   same as a)

   Not disjoint because $RY \in R$ and $RY \in Y$,

   i.e. $R \cap Y = \{RY\}$

   $$P(A) = P(R) + P(Y) - P(R\cap Y) = 2\cdot\frac{15}{47}\cdot\frac{32}{46} + 2\cdot\frac{12}{47}\cdot\frac{35}{46} - 2\cdot\frac{12}{47}\cdot\frac{15}{46}$$

2. (2 points) Suppose I deal a regular 52 card deck to 4 people, by giving each person 13 cards uniformly at random. What's the probability that each person gets exactly one Ace?

   $$\frac{\binom{13}{1}^4}{\binom{52}{4}} = \frac{13^4\, 4!\, 48!}{52!}$$

   choose where in each 13 card hand the ace is placed

   choose what positions aces are in 52 card deck

3. (2 points) An urn contains 10 red balls, 7 green balls and 3 yellow balls. Draw 5 balls.
   (a) (1 point) What's the probability that you draw 2 red, 2 green and 1 yellow?
   (b) (1 point) Same experiment as above. What's the probability that you draw 2 red, 1 green and 2 yellow?

a) $\dfrac{\binom{10}{2}\binom{7}{2}\binom{3}{1}}{\binom{20}{5}}$ b) $\dfrac{\binom{10}{2}\binom{7}{1}\binom{3}{2}}{\binom{20}{5}}$

4. (1 point) How many ways are there of rearranging the letters in WAAHOOO?

$$\binom{7}{3\,2\,1\,1} = \frac{7!}{3!\,2!\,1!\,1!} = \frac{7!}{3!\,2}$$

5. (1 point) 6 people in a house are charged with completing one of three different chores: two people sweep, two people clean bathrooms, and two people do the dishes. How many ways are there of assigning people to do these tasks?

$$\binom{6}{2}\binom{4}{2}\binom{2}{2}$$

6. (1 point) How many ways are there of splitting 6 people into 3 pairs?

$$\frac{\binom{6}{2}\binom{4}{2}\binom{2}{2}}{3!}$$

pair labels matter in 5 but not in 6