

AMAT 362—Work Sheet 10

Dr. Justin M. Curry

Due: March 7th, 2022. Worth 20 points.

Name: _____

1. (3 points) Consider the following game. You toss a fair coin 3 times. Every time the coin lands heads, you win \$3, every time it lands tails, you win nothing. You have to pay \$5 to play. Let X denote the number of heads out of 3 tosses of the coin. Let W denote the winnings from playing this game, accounting for the cost to play.
 - (a) (1 point) Write W as a function of the random variable X .
 - (b) (1 point) What's the expectation of W ? Should you play this game?
 - (c) (1 point) Find the distribution of $Y = |X - 1|$.
2. (6 points) Let X and Y denote the numbers obtained from two draws from a hat, which contains four tickets 1, 2, 3 and 4, written on them.
 - (a) (2 points) Write down the joint distribution table for X and Y , assuming the draws are made *without* replacement. Calculate the probability $P(X \leq Y)$.
 - (b) (1 point) Assuming the draws are made without replacement, what is the probability $P(X = Y)$?

(c) (2 points) Write down the joint distribution table for X and Y , assuming the draws are made *with* replacement. Calculate the probability $P(X \leq Y)$.

(d) (1 point) Compute the expectation of $E(X + Y)$ for both part (a) and part (c). How do they compare?

3. (6 points) Let X_1 and X_2 be the numbers associated to two rolls of a fair 6-sided die. Let $Y_1 = \max\{X_1, X_2\}$ denote the maximum value and let $Y_2 = \min\{X_1, X_2\}$ denote the minimum value of these two rolls.

(a) (2 points) Write down the joint distribution table for Y_1 and Y_2 .

(b) (2 points) Write down the PMF of $Z = X_1 \cdot X_2$ the multiplication of the values of the two rolls.

(c) (2 points) Compute the expectation $E(Z)$ and the expectation $E(Y_1 + Y_2)$.

4. (3 points) A fair coin is tossed 3 times. Let X be the number of heads in the first two tosses. Let Y be the number of heads in the last two tosses.
- (a) (1 point) Write down the joint distribution table for X and Y .

(b) (1 point) Are X and Y independent?

(c) (1 point) Find the distribution of $X + Y$.

5. (2 points) A box contains 8 tickets. Two are marked 1, two are marked 2, two are marked 3, and two are marked 4. Tickets are drawn uniformly at random from the box without replacement until a number appears that has appeared before. Let X be the number of draws that are made. Make a table to display the probability distribution of X .