AMAT 362—Work Sheet 06

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Due: February 16th, 2022. Worth 16 points.

Name: ______________________________

1. (5 points) Assume the following (overly simplified) statements are true:

   - In the US, 50% of the population are men and 50% of the population are women.
   - 10% of the population in the US practices yoga.
   - Of yoga practitioners, 72% are women and 28% are men.

   (a) (1 point) Suppose that I'm told a person practices yoga, what is the probability that they are a woman?

   (b) (1 point) Are the events of being a woman and practicing yoga independent?

   (c) (2 points) Suppose I randomly select a woman from the US population. What is the probability that they practice yoga?

   (d) (1 point) What's the probability of a yoga class with five people in it consisting of all men?

2. (4 points) *Conditional Independence and Testing Twice for a Disease:* For two events E_1 and E_2 we sometimes write

$$P(E_1 \cap E_2) = P(E_1 \text{ and } E_2) \qquad \text{as} \qquad P(E_1 E_2)$$

in order to save space. E_1 and E_2 are independent if $P(E_1 E_2) = P(E_1)P(E_2)$. Furthermore, we say E_1 and E_2 are *conditionally independent given* B if

$$P(E_1 E_2 \mid B) = P(E_1 \mid B)P(E_2 \mid B).$$

An example of conditional independence occurs when we re-run a test for a disease D. The probability of two positive tests assuming you have the disease can be computed as

$$P(++ \mid D) = P(+ \mid D)P(+ \mid D) = P(+ \mid D)^2$$

because each test is performed independently, even assuming you have the disease D.

For this question, assume the following is true:

- The probability that a randomly selected person has disease D is 0.5%.
- $P(+ \mid D) = 96\%$
- $P(+ \mid D^c) = 2\%$

(a) (2 points) Assuming someone tests positive for the disease D, what's the probability that they actually have disease D?

(b) (2 points) Assuming someone tests positive *twice* for a disease, what's the probability that they actually have disease D?

3. (2 points) Suppose I have two urns, U_1 has two red balls and one white ball and U_2 has two red balls and two white balls. I select an urn uniformly at random, and draw out a red ball. What's the probability that I selected U_1?

4. (1 point) Suppose that $P(A) = 1/3$ and $P(B) = 1/3$ and $P(AB^c) = 2/9$. Are A and B independent?

Figure 1: Two Components in Series

5. (2 points) Suppose that in an electrical device with two components in *series*, the device works only if both components C_1 and C_2 work. Say the probability of each component failing on a given day is 10% and 5% respectively. What's the probability that the device works on a given day?

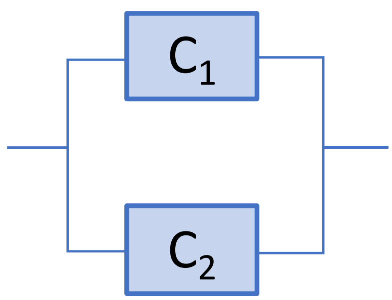

Figure 2: Two Components in Parallel

6. (2 points) Suppose that in an electrical device with two components in *parallel*, the device works only if either component C_1 or C_2 works. Say the probability of each component failing on a given day is 10% and 5% respectively. What's the probability that the device works on a given day?