## MATH 362—Work Sheet 07

## Dr. Justin M. Curry

Due on Friday, February 26th, 2021

1. (5 points) An urn contains 4 red balls and 3 green balls.

(a) (2 points) Nine draws are made with replacement. Let X be the number of times a green ball appears. Identify by name the probability distribution of X. Find the probabilities P(X > 1) and P(X < 5).

$$X \sim \text{BINOMIAL DISTRIBUTION} P = 3/7$$

$$P(X3/1) = 1 - P(X=0) = 1 - (4/4)^{9} \sim 99.396$$

$$P(X \le 6) = (4/4)^{9} + (9/4)(3/4)(4/4)^{8} + (9/3/4)(4/4)^{8} + (9/3/4)(4/4)^{8} + (9/3/4)(4/4)^{9} + (9/3/4)(4/4)$$

the number of draws that are needed. Identify by name the probability distribution of N. Find the probability  $P(N \leq 5)$ .

Find the probability 
$$P(N \le 5)$$
.

N = Greenetric D: Stribution  $P = 3/7$ 

$$\frac{P(N \le 5) = (3/7) + (4/7)(3/7) + (4/7)^2(3/7) + (4/7)(3/7)}{(3/7) + (4/7)^2(3/7) + (4/7)^2(3/7)}$$

$$= 1 - P(N > 5) = 1 - P(1s+5 \text{ an Red bells})$$

$$= 1 - (4/7)^5 = 93.9\%$$
(c) (1 point) Compare  $P(X \ge 1)$  and  $P(N \le 9)$ . Is there a reason why these should be the

Is there a reason why these should be the

$$P(x_3)=1-P(x=0)=1-P(q Redballs)$$
 These  $P(N=q)=1-P(N>q)=1-P(q Redballs)$  and the Same  $T$ 

2. (2 points) The population of a small town is 500. 20% of the population has red hair. One morning I go to a diner and see 15 customers are there.

(a) (1 point) What is the probability that 10 of the 15 customers have red hair?

$$\binom{15}{10}(.2)^{10}(.8)^{5} \propto \frac{1}{10,100}$$

(b) (1 point) What is the probability that at most 2 of the 15 customers has red hair?

$$P(X \le 2) = (.8)^{15} + (.5)(.2)(.8)^{14} + (.5)(.2)(.8)^{15}$$

$$\sim 39.8\%$$

- 3. (2 points) An urn contains 4 red balls and 3 green balls. Two balls are sampled at random.
  - (a) (1 point) Let Z denote the number of green balls in the sample when the draws are done without replacement. Give the possible value of Z and its probability mass function (PMF).

$$\frac{7}{4} \in \{0, 1, 2\}, \quad P(2=0) = \left(\frac{4}{7}\right) \left(\frac{3}{2}\right) = \left(\frac{2}{7}\right) \left(\frac{4}{7}\right) = \left(\frac{4}{7}\right) \left(\frac{3}{7}\right) = \left(\frac{4}{7}\right) \left(\frac{4}{7}\right) = \left(\frac{4}{7}$$

4. (5 points) Choose one of the words in the following sentence uniformly at random and then choose one of the letters of that word, again uniformly at random:

## SOME DOGS ARE BROWN

(a) (1 point) Find the probability that the chosen letter is R.

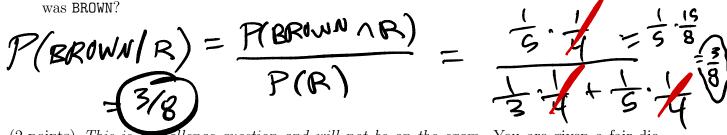
$$P(R') = P(R'/ARE') P('ARE') + P('R'/'BROWN') P('BROWN') P('B'') P('B'')$$

(b) (1 point) Let X denote the length of the chosen word. Determine the PMF of X.

- (c) (1 point) For each possible value k of X determine the conditional probability  $P(X = k \mid X > 3)$ . 3 WIROS w | X > 3: SOME, DOGS, BRIWN | X > 3:  $| Y = Y \mid X > 3$  | Y = 3  $| Y \mid X = 3$   $| Y \mid X = 3$
- (d) (1 point) Determine the conditional probability  $P(\text{the chosen letter is } \mathbb{R} \mid X > 3)$ .  $P(X = X)^3$



(e) (1 point) Given that the chosen letter is R, what is the probability that the chosen word was BROWN?



5. (2 points) This is a challenge question and will not be on the exam. You are given a fair die. You must decide ahead of time how many times to roll. If you roll exactly 2 sixes, you win a new Tesla. How many rolls should you take to maximize your chances and what are the chances of winning? Note: there are two equally good choices for the best number of rolls.

Question asks to maximize 
$$P(x=2) = \binom{n}{2} \binom{1}{6}^2 \binom{5}{6}^{n-2}$$
 a cross all possible  $n$ .

Use a calculator to find out  $p$