

AMAT 362—Work Sheet 12

Dr. Justin M. Curry

Due: March 21st, 2022. Worth 20 points.

Name: _____

1. (5 points) *This problem will be done in class, but make sure to write the answers down!* A fair coin is flipped 3 times. Let X be the indicator random variable that indicates if the first flip was a tails. Let Y be the total number of tails among the 3 flips.
 - (a) (1 point) Write down the joint PMF/distribution table for (X, Y) .
 - (b) (1 point) Are X and Y independent? Say why or why not.
 - (c) (1 point) Suppose you win \$3 for every tails that comes up, moreover, if the first flip is tails, you get \$6 for every tails, including the first one. Write down your winnings W as a function of X and Y .
 - (d) (1 point) Compute the expected winnings $E[W]$.
 - (e) (1 point) Compute the variance and standard deviation of your winnings $\text{Var}(W)$ and $SD(W)$.

2. (2 points) A card is drawn at random from a deck consisting of cards numbered 2 through 10. A player wins 1 dollar if the number on the card is odd and loses 1 dollar if the number is even. What is the expected value of their winnings?
3. (3 points) Suppose a multiple choice test is given where every question has 4 choices for the answer. There are 60 questions on the test. If someone has not studied at all and guesses uniformly at random on each question what is the expectation and variance of the number of correct answers?
4. (4 points) Ten thousand people each buy a lottery ticket. Each lottery ticket costs \$1. 80 people will win \$2, 19 will win \$100, 1 lucky person will win \$7000. Let X denote profit from the game, i.e. winnings minus cost of the ticket.
- (a) (1 point) Write down the probability mass function (PMF)/distribution table for X .
- (b) (1 point) Compute $E[X]$.
- (c) (2 points) Compute $\text{Var}(X)$ and $SD(X)$.

5. (6 points) There are 5 closed boxes in a table. In 3 of the boxes there are nice prizes. In the other 2, there are smelly socks. You open boxes one at a time until you open one with a prize, after which you stop. Let X denote the number of boxes you open
- (a) (1 point) Find the PMF for X
 - (b) (1 point) Compute $E[X]$.
 - (c) (2 points) Compute $\text{Var}(X)$ and $SD(X)$.
 - (d) (2 points) Suppose the “nice prize” in the three boxes is \$100, but every time you open a box without a prize, you lose \$100. Compute your *expected profit* from playing this game, as well as the *standard deviation in your profit*.