MATH 362—Work Sheet 15

Dr. Justin M. Curry

Due on Saturday, April 3rd, 2021

Name: ______________________________

1. (1 point) A random number generator produces numbers between 1 and 10,000 on each day. What's the probability that after one year no numbers have been repeated?

2. (3 points) A different random number generator produces 10,000 digits selected uniformly at random from the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

   (a) (1 point) What's the probability that the digit 3 is produced less than 932 times.

   (b) (1 point) What's the probability that the digit 3 is produced 1000 times exactly?

   (c) (1 point) What's the probability that the digit 3 is produced exactly 932 times?

3. (2 points) The PMF for a Poisson random variable is

$$\text{Poisson}(\lambda, k) = \frac{\lambda^k}{k!}e^{-\lambda} \qquad \text{for} \qquad k \geq 0$$

and zero otherwise. Verify that this PMF satisfies the two properties every PMF must satisfy:

(i) **Non-negativity** $p_X(k) \geq 0$ for all k

(ii) **Normalization:** $\sum_k p_X(k) = 1$

4. (4 points) Suppose I'm making a batch of 50 cookies with 400 chocolate chips.

(a) (1 point) What's the average number of chocolate chips each cookie will get?

(b) (1 point) What's the probability that a particular chocolate chip makes it into a particular cookie?

(c) (1 point) What's the approximate probability that a particular cookie gets no chocolate chips? *Hint:* Your answer should have e in it.

(d) (1 point) What's the approximate probability that a particular cookie has 5 chocolate chips?

5. (4 points) On average, only 1 in 1000 people have a particular rare blood type.
   (a) (2 points) Find the probability that in a city of 10,000 people, no has this blood type.

   (b) (2 points) How many people would have to be tested to give a probability greater than 1/2 of finding at least one person with this blood type.

6. (2 points) Back in the old days, phone calls were connected through a switchboard. If I wanted to call someone in Albany, I would dial the operator and tell them who I wanted to talk to and they would then connect my phone call to that specific person by unplugging and plugging cables that patched me into that person's landline. Let's say that a switchboard receives one call per second with probability .01. Give a good approximation that the operator will miss at most two phone calls if she takes a 5 minute coffee break.

7. (4 points) A dormitory has n students, all of whom like to gossip. One of the students hears a rumor and tells it to one of the other $n-1$ students. Subsequently each student who hears the rumor tells it to a student picked at random from the dormitory (excluding, of course, themself and the person they heard the rumor from). Let p_r be the probability that the rumor is told r times without coming back to a student who has already heard the rumor before. For example $p_1 = 1$ and $p_2 = 1$ because neither the first or second student to first hear the rumor is going to repeat it back to them. Also note that $p_n = 0$ by the pigeonhole principle.

   (a) (2 points) Find a formula for p_r for r between 3 and $n-1$.

   (b) (2 points) Estimate this probability for $n = 300$ and $r = 30$.