WS 15

LEC 15 - LIVE

Q1)

Prob(not repeat first)

$$\boxed{1 \cdot \left(1 - \frac{1}{10{,}000}\right)\left(1 - \frac{2}{10{,}000}\right) \cdots \left(1 - \frac{365}{10{,}000}\right)}$$

Prob of not repeating

$$e^{-\frac{365^2}{2 \cdot 10{,}000}} \approx .00127$$

99.8%

$$e^{-N^2/2 \cdot D} \qquad D > N$$

Q2)

$$X_i = \begin{cases} 1 & \text{if } i\text{th digit is } 3 \\ 0 & \text{o.w.} \end{cases} \qquad p = \frac{1}{10}$$

$$S_n = X_1 + \cdots + X_{10000}$$

$$np = \frac{10000}{10} = 1000$$

$$P(0 \leq S_n \leq 931) = \Phi\left(\frac{931.5 - 1000}{30}\right) - \underbrace{\Phi\left(\frac{0 - 1000}{30}\right)}_{0}$$

$$\sqrt{10000 \cdot pq} = 30$$

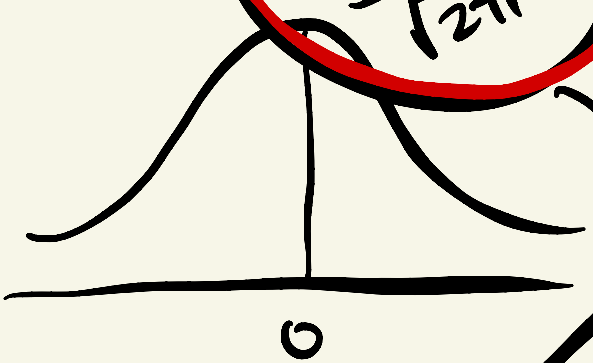
$$= \Phi(-2.28)$$

.0113

$\approx 1.13\%$

$$P(S_n = 1000) = \frac{\varphi\left(\frac{1000 - np}{\sqrt{npq}}\right)}{\sqrt{npq}}$$

$$\frac{e^{-t^2/2}}{\sqrt{2\pi}} = f_Z(z)$$



$$= \frac{\varphi(0)}{30} = \frac{1}{30\sqrt{2\pi}}$$

$$\frac{e^{-\left(-\frac{68}{30}\right)^2/2}}{30\sqrt{2\pi}} = \frac{\varphi\left(\overbrace{\frac{932 - 1000}{30}}^{z}\right)}{30}$$

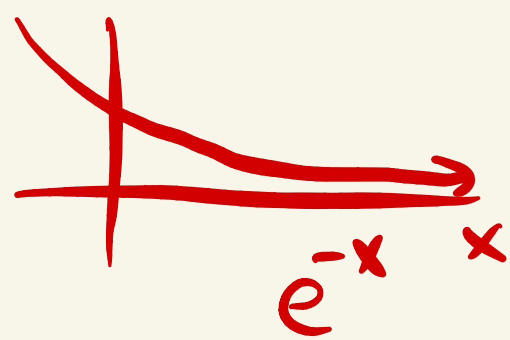
Q3] Check that

1) $\text{Poisson}(\lambda, k) = \boxed{\frac{\lambda^k}{k!}} e^{-x} \overset{?}{\geq} 0$

≥ 0 ?

$np = \lambda$

$p \geq 0 \quad n \geq 0$

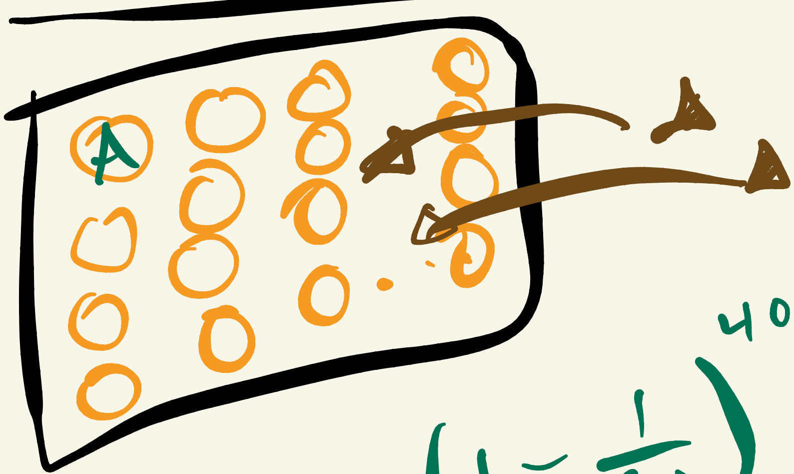


2) $\displaystyle\sum_{k=0}^{\infty} \frac{\lambda^k}{k!} e^{-\lambda} \stackrel{?}{=} 1$

equiv

$$\left[e^{-\lambda} \underbrace{\sum_{k=0}^{\infty} \frac{\lambda^k}{k!}}_{\left(\frac{\lambda^0 = 1}{0! = 1} = 1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots\right) = e^{\lambda}}\right] \stackrel{?}{=} 1$$

$$e^{-\lambda} \cdot e^{\lambda} = e^{0} = 1$$



$$\left(1 - \frac{1}{50}\right)^{400} \approx \left(e^{-\frac{1}{50}}\right)^{400} = e^{-\frac{400}{50}} = e^{-8}$$

Poisson$(\lambda=8, k=0)$ = .00033

$-x \approx e^{-x}$

Poiss$(\lambda=8, k=8)$ $\approx$ 13.9%

Poisson$(\lambda=8, k=5)$ $\displaystyle\frac{\lambda^5}{5!} e^{-\lambda} \approx$ **9.16%**

Q7

$$P_r = \underbrace{\left(\frac{n-1}{n-1}\right)}_{p_1}\left(\frac{n-2}{n-2}\right)\left(\frac{n-3}{n-2}\right)\cdots\left(\frac{n-r}{n-2}\right)$$

p_1

p_2

p_3

$$\prod_{i=2}^{r}\left(\frac{n-i}{n-2}\right) = \prod_{i=2}^{30}\left(1-\frac{i-2}{n-2}\right)$$

$$\frac{n-2-(i-2)}{n-2}$$

$$e^{-\frac{1}{n-2}}\, e^{-\frac{2}{n-2}} \cdots e^{-\frac{28}{n-2}}$$

$n = 300$

$r = 30$

$$\approx e^{-\frac{28\cdot 29}{2\cdot 298}} \approx 25.6\%$$

5(b)

$P\left(\text{no one w/ blood type found}\right) < 1/2$

$\left(1-\frac{1}{1000}\right)\left(1-\frac{1}{1000}\right)\cdots\left(1-\frac{1}{1000}\right)$

$= \left(\frac{999}{1000}\right)$

$= \left(1-\frac{1}{1000}\right)^n < \frac{1}{2}$

$1-x \approx e^{-x}$

$\left(e^{-\frac{1}{1000}}\right)^n < \frac{1}{2}$

$e^{-\frac{n}{1000}} < \frac{1}{2}$

$\ln(-)$

$-\frac{n}{1000} < \ln(1/2)$

$= \cancel{\ln 1}^{\,0} - \ln 2$

$-\frac{n}{1000} < -\ln 2$

$n/1000 > \ln 2$

$\boxed{n > 1000 \cdot \ln 2 \quad 693.14}$

Q1) 30 people

$$P(\text{at least 2 birthdays}) = 1 - \text{Prob}(\text{no common})$$

$$= 1 - 1\cdot\left(1-\frac{1}{365}\right)\left(1-\frac{2}{365}\right)(\cdots)\left(1-\frac{29}{365}\right)$$

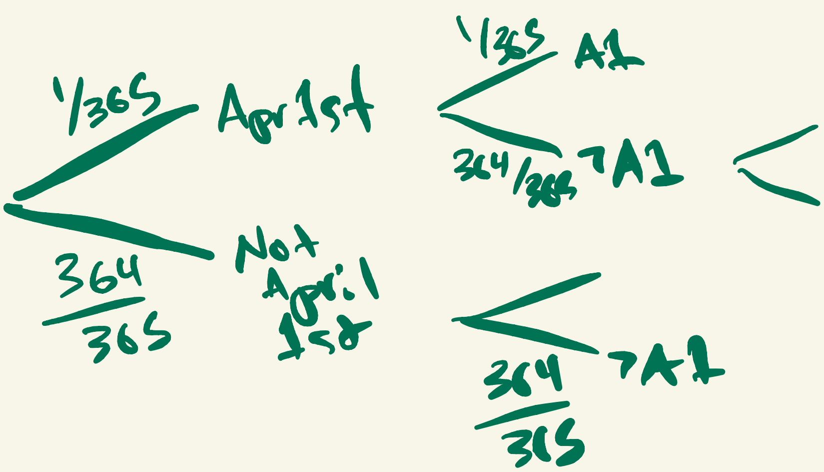
Q2) What's the probability that in a room of 30 at least one person has April 1st as their birthday?

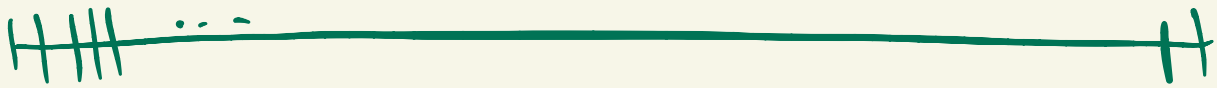
$$1 - \text{Prob}(\text{no one has April 1st})$$

$$1 - \underbrace{\left(1-\frac{1}{365}\right)^{30}}_{} \approx 1 - e^{-\frac{30}{365}}$$

7.89%

$\boxed{92.11\%}$





$n = 60 \cdot 5 = 300$ secs in 5mins

$p = \frac{1}{100}$ $\qquad \lambda = np = \frac{300}{100} = 3$

$$P\left(\begin{matrix}\text{No}\\ \text{calls}\end{matrix}\right)_{\text{in 5mins}} = \left(1 - \frac{1}{100}\right)^{300} \approx \left(e^{-\frac{1}{100}}\right)^{300} = e^{-3}$$

$$P(1\text{ call}) \approx \frac{3}{1!} e^{-3} \xleftarrow{k=1} \frac{\lambda^k}{k!} e^{-\lambda}$$

$$P(2\text{ calls}) \approx \frac{3^2}{2!} e^{-3}$$