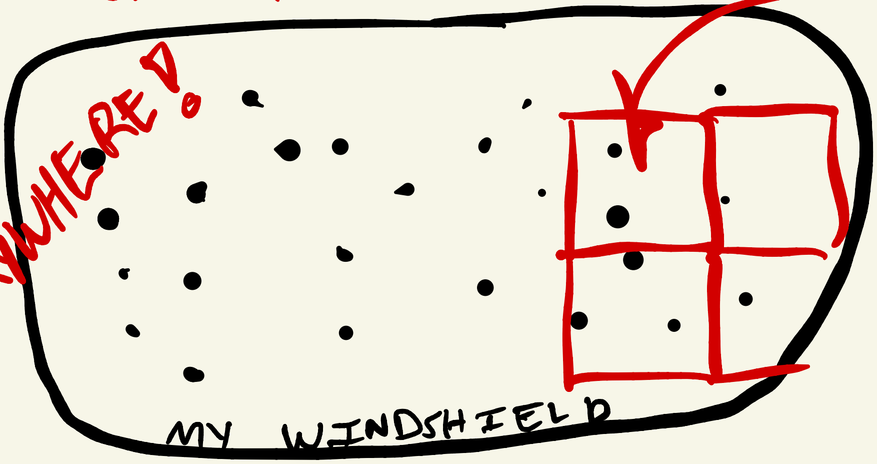
LECTURE 18 LIVE

SPOTS ON MY WINDSHIELD FROM TREES POLLEN

POISSON PROCESSES ARE EVERYWHERE!

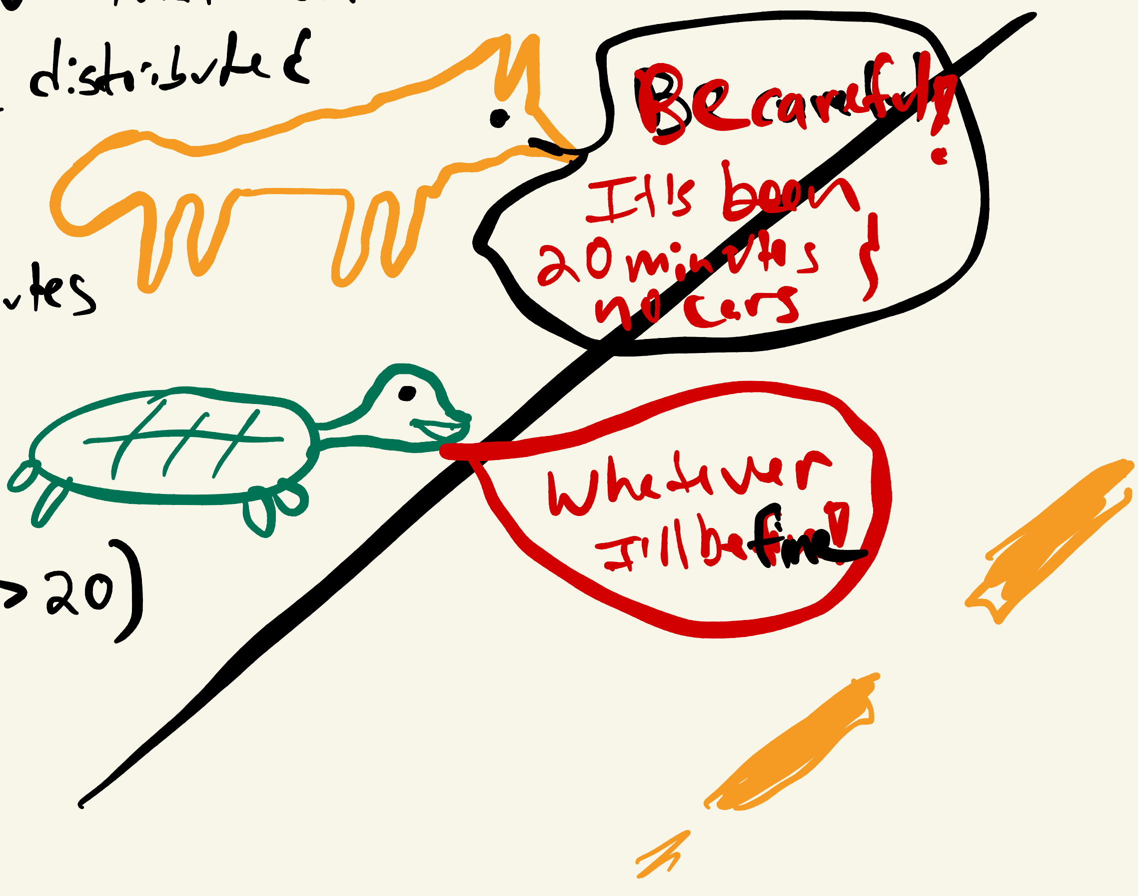


$P(k^{\#}_{\text{spots}}) \approx \frac{\lambda^k}{k!} e^{-\lambda}$

THE TURTLE & THE FOX

Animals know that arrival of cars is exponentially distributed w/ param λ

Avg = 30 minutes



$P(T > 30 \mid T > 20)$

$= ?$

$$P(T>30 \mid T>20) = \frac{P(T>30 \text{ AND } T>20)}{P(T>20)} = \frac{P(T>30)}{P(T>20)}$$

$$P(T>a) = \int_a^\infty \lambda e^{-\lambda t}\,dt = -e^{-\lambda t}\Big|_a^\infty = e^{-\lambda a}$$

$\Rightarrow$ So... $P(T>30 \mid T>20) = \dfrac{e^{-30\lambda}}{e^{-20\lambda}} = \dfrac{\cancel{e^{-20\lambda}}\, e^{-10\lambda}}{\cancel{e^{-20\lambda}}}$

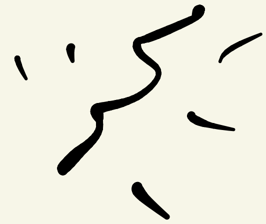
$$\boxed{P(T>10) = e^{-10\lambda}}$$

$$P(T>a+b \mid T>a) = \frac{e^{-\lambda(a+b)}}{e^{-\lambda a}} = e^{-\lambda b} = P(T>b)$$

Proof of Memoryless Property

Geometric RV also has this prop-erty!

EARTHQUAKES



$\lambda = 1 \frac{1}{\text{year}}$ Earthquake E

(a) $P(0 \le E \le 1) = \int_0^1 \lambda e^{-\lambda t}\, dt$

$= -e^{-\lambda t}\Big|_0^1 = 1 - e^{-\lambda}$

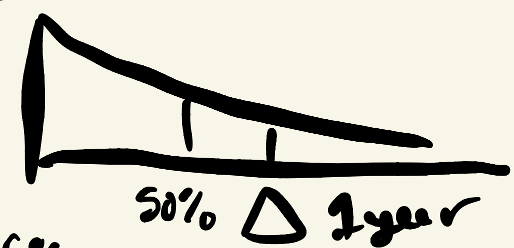
$= 1 - e^{-1}$

$\boxed{= 63\%}$

$t_{1/2} = \frac{\ln(2)}{\lambda} = .69$ of a year

(b) $P(0 \le E \le \frac{1}{2}) = 1 - e^{-\lambda \cdot \frac{1}{2}} = 39\%$

(c) $P(T > 2) = e^{-2\lambda} = e^{-2}$ $= 13.5\%$



(d) $P(T > 2 \mid T > 1) = P(T > 1) = 37\% = \frac{1}{e}$

Q3] Avg = mean = 10 hrs $\quad \lambda = \frac{1}{10}$

(a) $P(T > 20) = e^{-20\lambda} = e^{-2}$
$= 13.5\%$

(b) Median lifetime

$$\frac{\ln 2}{(1/10)} = 10 \cdot \ln 2 \approx 6.9 \text{ hours}$$

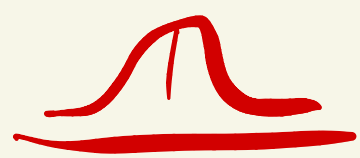
(c) $SD = \frac{1}{\lambda} = 10$ hours

(d) Q: What is the probability that the average lifetime of 100 IID components each of which $X_i \sim Exp(\frac{1}{10})$ exceeds 11 hours?

$S_n = X_1 + \cdots + X_n$ where each $X_i \sim Exp(\lambda)$

$$P(a < S_n < b) \approx \Phi\left(\frac{b - n\mu}{\sigma\sqrt{n}}\right) - \Phi\left(\frac{a - n\mu}{\sigma\sqrt{n}}\right)$$

GROWN UP CLT



NORMAL

CAN USE For "large" n

Define $A_n = S_n/n$

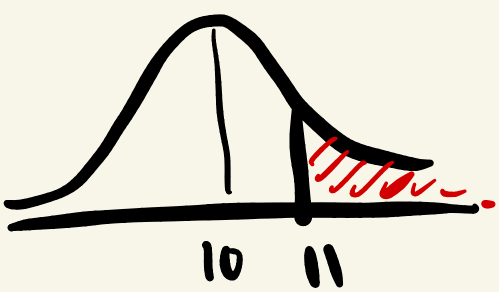
$$E(A_n) = E\left(\frac{S_n}{n}\right) = \frac{1}{n}\cdot E(\underbrace{S_n}_{X_1+\cdots+X_n})$$

$$= \frac{1}{\cancel{n}}\cdot \cancel{n}\cdot \underbrace{E(X_i)}_{=1/\lambda}$$

For our example $= 10$ hrs

$$SD(A_n) = \sqrt{Var(A_n)} = \sqrt{\frac{1}{n^2}\cdot Var(X_1+\cdots+X_n)}$$

$$= \sqrt{\frac{1}{n^2}\cdot n \cdot \underbrace{Var(X_i)}_{=1/\lambda^2}}$$



$$= \frac{1}{\lambda}\,\frac{1}{\sqrt{100}}$$

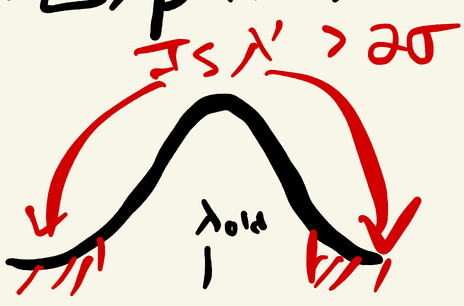
$1 - \Phi(1) = 16\%$

$$= 10\cdot\frac{1}{10} = 1$$

An aside on statistics: Trying to establish whether fracking causes earthquakes

$T \sim Exp(\lambda_{old})$

Error in Measuring $\lambda \sim Normal(\mu, \sigma^2)$



New λ' = fracking time between Earthquakes

If $|\lambda' - \lambda_{old}| > 2\sigma$ then w/ 95% yes fracking does!

(e) What's the probability that the average lifetime of 2 indep components > 11 hours

Can't use CLT b/c $n=2$ is TOO SMALL

Lifetime of Part 1 $= T_1$

" " $2 = T_2$

$$P\left(\frac{\overbrace{T_1 + T_2}^{S_2}}{2} > 11\right) = P(\underbrace{T_1 + T_2}_{\text{Gamma}(=\Gamma)\ \text{Distribution}} > 22)$$

$=$ Prob(0 or 1 failures in first 22 hours)

POISSON POINT PROCESS

$=$ # of ARRIVALS / FAILURES

$$\sim \text{Poisson}(\lambda \cdot (\underset{22-0}{b-a})) \simeq \text{Poisson}_{2.2}(k)$$

$$= \text{Poisson}_{2.2}(0) + \text{Poisson}_{2.2}(1) \quad \boxed{= e^{-2.2} + 2.2e^{-2.2} = 35.45\%}$$

Q3(e) = intro to Poissont Point Process

Q4 = Fully Poisson Point Process

A store 9am – 6pm avgs 45 customers a day

Compute λ = Avg rate of arrival per hour

$$= \frac{45}{9} = \frac{\cancel{9}\cdot 5}{\cancel{9}} = \boxed{5 = \lambda}$$

(a) A_1 = No customers arrive between 9 & 10am

$$P(A_1) = \text{Poisson}_{\lambda(1-0)=\lambda}(k=0)$$

$$e^{-\lambda} = e^{-5} \doteq \boxed{.67\%}$$

(b) A_2 = 3 customers arrive between 10 & 10:30 am

$$P(A_2) = \text{Poisson}_{\lambda(1.5-1)}(k=3)$$ 21.4%

$$\frac{(.5\lambda)^3}{3!}e^{-.5\lambda} = \frac{(2.5)^3}{3!}e^{-2.5}$$

(c) $P(A_1 \cap A_2) \underset{\uparrow}{=} P(A_1)P(A_2)$

Indep $\quad = \frac{(2.5)^3}{3!} e^{-7.5}$ $\quad e^{-7.5} = e^{-2.5} \cdot e^{-5}$

b/c $[9 \text{ to } 10] \cap [10 \text{ to } 10{:}30]$

Mateen's Q

$$P\left(\begin{matrix}3 \text{ customers}\\ 10-10{:}30\end{matrix}\right) \overset{?}{\longleftrightarrow} P\left(\begin{matrix}6 \text{ customers}\\ 10-11\end{matrix}\right) = \frac{\lambda^6}{6!} e^{-\lambda}$$

$$P\left(\begin{matrix}6 \text{ customers}\\ 10-11\end{matrix}\right) \neq P\left(\begin{matrix}3 \text{ customers}\\ 10-10{:}30\end{matrix}\right) P\left(\begin{matrix}3 \text{ customers}\\ 10{:}30-11\end{matrix}\right) = \left[\frac{(\lambda/2)^3}{3!} e^{-\lambda/2}\right]^2$$

The difference here is that 6 customers could be distributed between 10-10:30 & 10:30-11 as

0	6
1	5
2	4

But $P(3)P(3)$ requires exactly 3 in each half hour!