MATH 362—Work Sheet 17

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Due Saturday, April 17, 2021

Name: ______________________________

1. (1 point) Suppose a number is generated uniformly at random from the unit interval $(0,1)$, i.e. $X \sim \text{Unif}(0,1)$. What's the probability of X being within 2 decimal places of .35, after rounding? For example, .349 would round up to .35 and .353 would round down.

2. (1 point) Repeat the question above except under the assumption that $X \sim \text{Norm}(0,1)$ distribution, i.e. normally distributed with mean 0 and variance 1.

3. (5 points) Suppose X is a random variable whose density is $f(x) = cx(1-x)$ for $0 < x < 1$ and $f(x) = 0$ otherwise.

   (a) (1 point) Find the value of c in order for this to be a valid PDF.

   (b) (1 point) $P(X \leq 1/2)$

   (c) (1 point) $P(X \leq 1/3)$

(d) (2 points) Make a drawing of the function $F(s) := \int_{-\infty}^{s} f(x)dx$, which is called the *cumulative distribution function (CDF)* of X.

4. (4 points) *We'll do this in class, but please copy down the derivations and understand them!* Recall that X is exponentially distributed with parameter λ, written $X \sim \text{Exp}(\lambda)$, if its PDF is $f_X(x) = \lambda e^{-\lambda x}$ for $x \geq 0$ and is zero otherwise.

   (a) (1 point) Derive a formula for $E(X)$ in terms of λ.

   (b) (1 point) Derive a formula for $V(X)$ in terms of λ.

   (c) (1 point) Derive and draw the CDF for X.

(d) (1 point) Derive a formula for the median of X in terms of λ. This is also called the *half-life*, which measures the amount of time needed for a sample of radioactive material to degrade by half or for some chemical to degrade or be absorbed by half. Bonus question: what is the half-life of caffeine?

5. (2 points) Suppose a particular kind of radioactive element has a half-life of 1 year. Find
   (a) (1 point) The probability that an atom of this type survives for at least 5 years.

   (b) (1 point) The time at which a sample of this element decays to 10% of its original purity.

6. (2 points) Suppose the length L of a phone call is exponentially distributed with $\mu = 10$ minutes.
   (a) (1 point) Compute $P(L \geq 20)$

   (b) (1 point) Compute $P(8 \leq L \leq 22)$

7. (3 points) Measurements on the mass of a metal part produced at a factory are IID with $\mu = 12$ grams and $\sigma = 1.1$ grams.

   (a) (1 point) Find the chance that a single measurement is between 11.8 and 12.2 grams, assuming the mass varies according to a normal distribution.

   (b) (1 point) Estimate the chance that the average of 100 measurements is between 11.8 and 12.2 grams. Answer this is as well: Is it necessary to assume that each measurement is normally distributed?