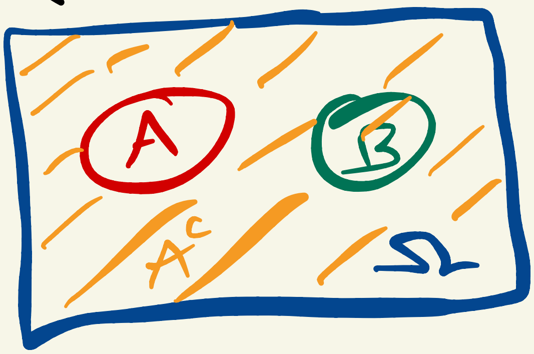
AMAT 362 - PROBABILITY for STATISTICS - LEC 3



If A & B are disjoint (= mutually exclusive)

then $\boxed{P(A \cup B) = P(A) + P(B)}$

In general $\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$

A & $B = A^c$

$P(A \cup A^c) = P(\Omega) = 1$

$= P(A) + P(A^c) \rightsquigarrow \boxed{P(A) = 1 - P(A^c)}$

LAW OF COMPLEMENTS

BIRTHDAY PROBLEM or "PARADOX"

Q: What is the probability of at least $A = ($2 people in a class of size N having the same birthday$)$?

Here Ω = space of possible birthday assignments

$s_1, s_2, \ldots, s_N \rightsquigarrow b(s_1) \in \{\text{Jan } 1, \ldots \text{Dec } 31\}$ (365)

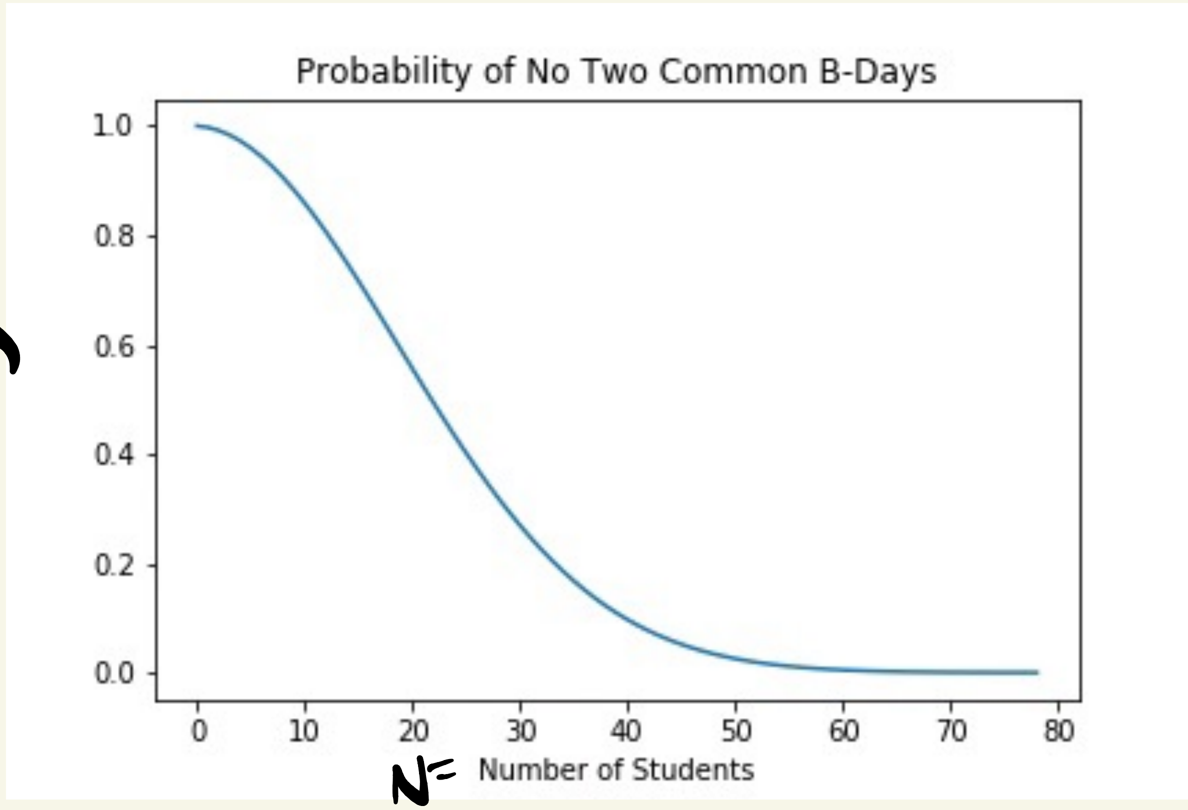
$\vdots$

$b(s_N) \in \{\text{Jan } 1, \ldots \text{Dec } 31\}$

$$\Rightarrow |\Omega| = \underbrace{365 \times \cdots \times 365}_{N \text{ times}}$$

$= 365^N$

$P(A^c)$



A^c = NO TWO STUDENTS HAVE SAME BDAY

$$|A^c| = 365 \times 364 \times \cdots \times (365 - N + 1) = \frac{365!}{(365-N)!}$$

CURVE ABOVE $\rightsquigarrow$ Graph of

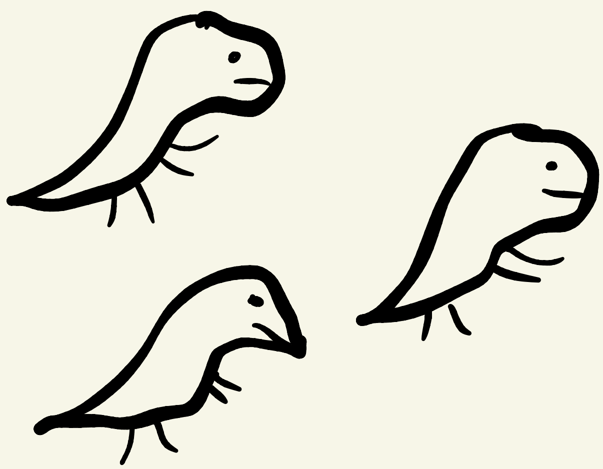
$$P(A^c) = \frac{|A^c|}{|\Omega|} = \frac{\left(\frac{365!}{(365-N)!}\right)}{365^N}$$

Q: what happens when $N > 365$?

A: $P(A^c) = 0$ for $N > 365$

PIGEONHOLE PRINCIPLE

(Dinosaur)



If # of pigeons > # of holes then at least two pigeons <u>must</u> go through the same hole.

Generalized Pigeonhole Principle

If n pigeons & m holes then

$\left\lceil \frac{n}{m} \right\rceil$ = ceiling of n/m pigeons must go through the same hole!

<u>Ex</u> 800 students

then at least 3 students would have to have the same BDAY $\left\lceil \frac{800}{365} \right\rceil$

BACK TO THE URN

SAMPLING w/o REPLACEMENT where ORDER **DOES NOT** MATTER

EX Urn w/ 5 unique balls (all different colors)

EXPERIMENT Draw 3 balls at once

EVENT "A" Drawing the GREEN BALL

Q: $P(A) = ?$

A: Identify the space of possibilities Ω (SAMPLE SPACE)

Ω = Set of size 3 subsets from a 5 element set

$\{1,2,3,4,5\} = \{$Green, Blue, Red, Orange, Purple$\}$

THE CHOOSE FORMULA $= \boxed{\dfrac{n!}{(n-k)!\,k!}}$

$|\Omega|$ = # of size 3 subsets of 5 elt. set $= \binom{5}{3} = \dfrac{5!}{3!\,2!}$

$= \dfrac{5 \cdot 4 \cdot \cancel{3!}}{\cancel{3!}\,2 \cdot 1} = \underline{\underline{10}}$

WRONG WAY $5 \cdot 4 \cdot 3 = 60$ ✓ $\dfrac{5 \cdot 4 \cdot 3}{3!}$ $\dfrac{60}{3!} = 10$

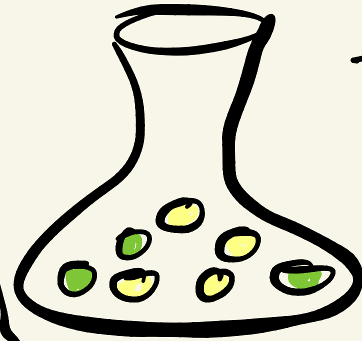
A = Green ball is in my size 3 subset

$\{1\} \cup \{$Any size 2 subset of $\{2,3,4,5\}\}$

$|A|$ = # of size 2 subsets of $\{2,3,4,5\}$ $\dfrac{4!}{2!\,2!} = \dfrac{4 \cdot 3}{2} = 6$

A $P(A) = \frac{6}{10} = 60\%$

EXAMPLE 2



3 Green
& 4 Yellow balls

Draw 2
w/o replacement

Q: Describe possible $\underbrace{\text{outcomes of this}}_{\Omega}$ experiment

A: $S = \{G_1, G_2, G_3, Y_1, Y_2, Y_3, Y_4\}$

$$\Omega = \{B \subseteq S \mid |B| = 2\}$$

$$|\Omega| = \binom{7}{2} = \frac{7!}{(7-2)!\,2!} = \frac{7 \times 6}{2} = 21$$

Q: A = event that the two drawn balls have different colors

$P(A) = ?$

A $P(A) = \frac{|A|}{|\Omega|} \rightsquigarrow |A| = \binom{3}{1} \cdot \binom{4}{1} = 12$

#ways of drawing 1 green from 3 green

ways of drawing 1 yellow from 4 yellow

$$\Rightarrow P(A) = \frac{12}{21}$$

ORDERED vs. UN-ORDERED SAMPLES

Samples w/o replacement

k objects drawn from n objects

$P_{n,k}$

$$\frac{n!}{(n-k)!} = (n)_k$$

CHOOSE FORMULA

$$\frac{n!}{(n-k)!\,k!} = \binom{n}{k}$$

$$\underbrace{n}_{1st} \times \underbrace{(n-1)}_{2nd} \times \cdots \underbrace{(n-k+1)}_{k^{th}}$$

$$\underbrace{n}_{1st} \times \underbrace{(n-1)}_{2nd} \times \cdots \times \underbrace{(n-k+1)}_{k^{th}} \Big/ k!$$

ways of rearranging the k drawn elements

Ex Draw 5 cards from a 52 card deck

$$\underbrace{52}_{1st} \times \underbrace{51}_{2nd} \times \underbrace{50}_{3rd} \times \underbrace{49}_{4th} \times \underbrace{48}_{5th} \Big/ 5! = 60$$

permutations of 5 objects

Why is the choose formula true?

n objects $x_1, x_2, \ldots, x_n \to \boxed{n!}$ # permutations

How to choose k

$$\boxed{C_{n,k} = \frac{n!}{k!(n-k)!}}$$

$$\underbrace{x_{\sigma(1)}, x_{\sigma(2)}, \ldots}_{k} \;\Big|\; \underbrace{\ldots x_{\sigma(n)}}_{n-k}$$

$\boxed{k!}$ perms give same subset

$\boxed{(n-k)!}$ perms give same complement

$$\boxed{\frac{n!}{k!(n-k)!}}$$