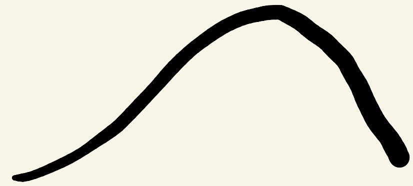
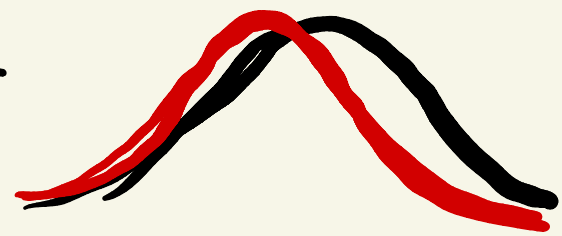
LEC 13 - LIVE

Q1 (Pt a) No! Mean + Median In table 6

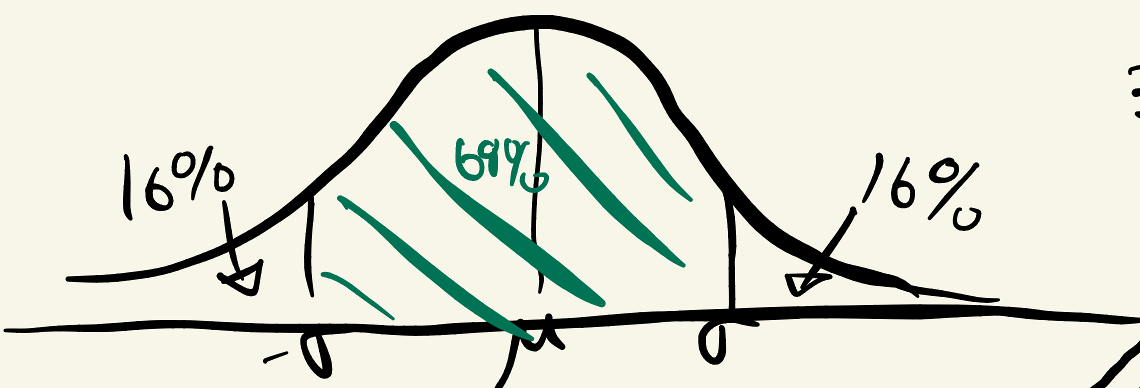
Also, violates





68 - 95 - 99.7 Rule

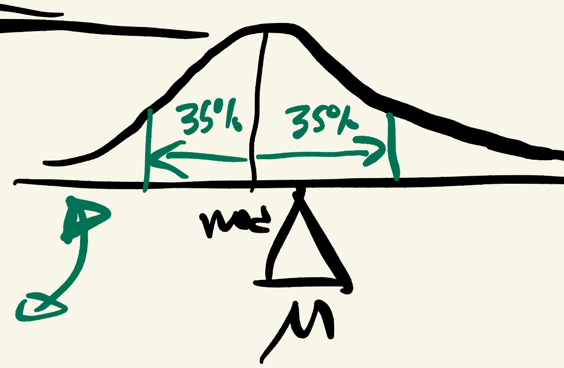
$\pm\sigma$ $\pm 2\sigma$ $\pm 3\sigma$ of μ



$32 = 100 - 68$

$192 - 154 = 38$ lbs

$244 - 192 = 52$ lbs



Assume $\mu = 200$ $\sigma = 45$ $Y = \dfrac{X}{2.2}$

X = Weight in lbs

Y = wt in kg

Z = wt in stones $Z = \dfrac{X}{14}$

$E(X) = 200$ $\quad SD(X) = 45$

$Y = \frac{X}{2.2}$ kg $\quad E(Y) = E\left(\frac{X}{2.2}\right) = \frac{1}{2.2}E(X) \approx 90.9$

$SD(Y) = \frac{1}{|2.2|} SD(X) \approx 20.45$

$Z = \frac{X}{14} \quad E(Z) = \frac{1}{14}E(X) \approx 14.28$

$SD(Z) = \frac{1}{|14|} SD(X) \approx 3.21$

Q2 Avg family income is 20k RV $X \geq 0$

(a) Give upper bound on $P(X \geq 100k)$

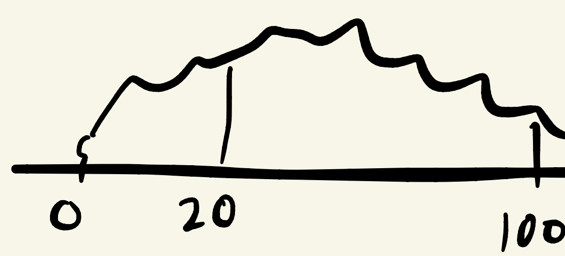
$\rightarrow$ Markov's Inequality $P(X \geq c) \leq \frac{E(X)}{c}$

$$\frac{20k}{100k} = \frac{1}{5} \approx 20\%$$

(b) Sps $\sigma = 16k$ Give a better bound.

$\rightarrow$ Chebyshev's Inequality

$$P(|X - \mu| \geq c) \leq \frac{\sigma^2}{c^2}$$



$\mu = 20k$ $\quad c = 80k \quad$ Percent of families $\leq \frac{16^2}{80^2} = \left(\frac{16}{80}\right)^2 \approx 4\%$

$\sigma = 16k$

Mean $= 30{,}550$ $\quad 28{,}573$
σ

Q3

(b) $E(x) \le \dfrac{30{,}550}{100{,}000} = \cancel{38.5\%}$
30%

$8\% \le 30\%$

(c)

$\ge 64{,}450$ $\quad \le \left(\dfrac{28{,}573}{64{,}450}\right)^2$

$8 \le 16.9\%$

Q6!

$10 = np$

$5 = \sqrt{npq}$ $\quad 25 = npq$

$E(X) = \frac{n+1}{2}$

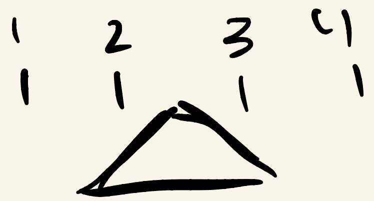
1st moment

$E(X^2) = \frac{(n+1)(2n+1)}{6}$

2nd Moment

$n=4 \quad \frac{5 \cdot 9}{6} = \frac{45}{6}$

$n=4 \quad \mu = \frac{5}{2} = 2.5$



$Var(X) = E(X^2) - E(X)^2$

$\sigma^2 = \left(\frac{45}{6}\right) - \left(\frac{5}{2}\right)^2$

$Var(X) = \frac{n^2 - 1}{12}$

$n=4$ $\quad = \frac{\cancel{256}-1}{12} = \frac{\cancel{255}}{12}$ (with $n=4$ above, and 15 written over the struck 255) $= 1.25$

y

$P(|A_n - \mu| \geq \varepsilon) \leq \frac{\sigma^2}{n\varepsilon^2}$

$\varepsilon = .5$

$\frac{1}{20} \geq \frac{\sigma^2}{n \cdot (1/2)^2} \qquad \frac{n}{4} \geq 20\sigma^2 \Rightarrow n \geq 80\sigma^2$

$P(|A_n - \mu| \geq \varepsilon) \leq .05 \geq \frac{\sigma^2}{n\varepsilon^2}$

7 rolls

$\cancel{n \cdot \varepsilon} \quad n \cdot (.05) \geq \frac{\frac{255}{12}}{(1/4)} = 25$

$\frac{}{1/20}$

$\frac{15 \cdot 5}{12} = 6.25$

$n \geq (1.25)^2 80$

125

$n \geq \frac{\cancel{255}}{12} \cdot 5 = \cancel{106.25}$ (with 15 written over the struck 255)

$$\mathrm{Var}(X) = E[X^2] - E[X]^2$$

x	1	2	3	4
$P(X=x)$	$1/4$	$1/4$	$1/4$	$1/4$

$$E[X^2] = 1^2 \cdot \frac{1}{4} + 2^2 \cdot \frac{1}{4} + 3^2 \cdot \frac{1}{4} + 4^2 \cdot \frac{1}{4}$$

$$= \frac{1}{4}\left(\frac{5 \cdot 9}{6}\right)$$

$$1^2 + 2^2 + \cdots + n^2 = (n+1)(2n+1)/6$$