AMAT 362—Work Sheet 18

Dr. Justin M. Curry

Due: April 18, 2022. Worth 20 points.

Name: ____

1. (1 point) If a part has a lifetime modeled by $T \sim \text{Exp}(\lambda)$, prove the **memoryless property**, which says that

$$P(T > a + b \mid T > a) = P(T > b)$$

According to Pitman (p. 281) this is like saying

"As long as a part is working, it's as good as new!"

2. (4 points) One of the reasons exponential RVs are important is that they model the time *between* earthquakes. Suppose the time to the next earthquake is exponentially distributed with rate 1 per year. Find the probability that the next earthquake happens

   (a) (1 point) within one year;

   (b) (1 point) within six months;

   (c) (1 point) after two years;

(d) (1 point) after two years, given that one year has already gone by without an earthquake.

3. (5 points) Suppose component lifetimes are exponentially distributed with mean 10 hours. Find
   (a) (1 point) the probability that a component survives 20 hours;

   (b) (1 point) the median component lifetime;

   (c) (1 point) the SD of component lifetime;

   (d) (1 point) The probability that the average lifetime of 100 independent components exceeds 11 hours;

(e) (1 point) The probability that the average lifetime of 2 independent components exceeds 11 hours;

4. (3 points) A store is open from 9am-6pm and averages 45 customers a day.

(a) (1 point) Compute the probability of no customers arriving between 9 and 10am. Call this event A_1.

(b) (1 point) Compute the probability of 3 customers arriving between 10 and 10:30am. Call this event A_2.

(c) (1 point) Compute the probability $P(A_1 \cap A_2)$.

5. (3 points) For this problem you'll want to know that the probability distribution for T_r, which is the time of the r^{th} arrival in a Poisson Point Process with rate λ, or, alternatively, the distribution of $W_1 + \cdots + W_r$ the sum of r IID exponentials, has PDF

$$f_{T_r}(t) = \frac{\lambda^r t^{r-1}}{(r-1)!} e^{-\lambda t} \qquad \text{for } t \geq 0$$

and has mean r/λ and standard deviation $\sqrt{r}/\lambda$.

Suppose calls are arriving at a call center with an average rate of 1 call per second. Find:

(a) (1 point) the probability that the fourth call after $t = 0$ arrives within 2 seconds of the third call;

(b) (1 point) the probability that the fourth call arrives by times $t = 5$ seconds;

(c) (1 point) the expected time at which the fourth call arrives.

6. (4 points) Transistors are produced by one machine have a lifetimes that is exponentially distributed with mean 100 hours. Transistors produced by a second machine have lifetime with exponential distribution and mean 200 hours. A package of 12 transistors has 4 produced by the first machine and 8 produced by the second machine. Let X be the lifetime of a randomly selected transistor from this package of 12. Find:

   (a) (1 point) $P(X \geq 200 \text{ hours})$

   (b) (1 point) $E(X)$

   (c) (2 points) $Var(X)$