

# LECTURE 12 LIVE



ST. PATRICK'S  
DAY

$$X = \begin{cases} 1 & \text{if 1st flip tails} \\ 0 & \text{o.w.} \end{cases}$$

$$Y = \{\# \text{ of tails in 3 flips}\}$$

	Y	0	1	2	3
X	0	HHH	HTH or HHT	HTT	$\emptyset$
	$\frac{4}{8}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
	$\frac{4}{8}$	$\emptyset$	THH	THT or TTH	TTT
(a) $\Rightarrow$	0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$

(b) Indep? If  $X \{ Y$  were indep  $\Rightarrow P(x,y) = P_x(x) \times P_y(y)$

$$P \Rightarrow Q \Leftrightarrow \neg Q \Rightarrow \neg P$$

$$P_X(0) P_Y(3) \neq P_{X,Y}(0,3)$$

$$\frac{4}{8} \cdot \frac{1}{8} = \frac{1}{64} \neq 0$$

$$(c) W = g(x,y) = \begin{cases} 3y & x=0, y=0,1,2 \\ 6y & x=1 \end{cases}$$

$$(x+1)3y \Rightarrow W = 3(x+1)y$$

Recall

$$E[g(x, y)] = \sum_{x, y} g(x, y) \frac{P(X=x, Y=y)}{p(x, y)}$$

$$E[W] = E[3(x+1)y]$$

		Y	0	1	2	3	
			1/8	3/8	3/8	1/8	
		X	0	1	2	3	
$\frac{4}{8}$	0	HHH	$\frac{1}{8}$				
	1	$w=0$	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{6}{8}$	$\frac{9}{8}$	
$\frac{4}{8}$	0	$\emptyset$	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{12}{8}$	$\frac{18}{8}$	
	1	$w=0$					

$$E[W] = 3 \cdot \left(\frac{2}{8}\right) + 6 \cdot \left(\frac{2}{8}\right) + 12 \cdot \left(\frac{2}{8}\right) + 18 \cdot \left(\frac{1}{8}\right)$$

$$= \frac{6 + 12 + 24 + 18}{8} = \frac{60}{8} = \frac{15}{2}$$

Erick's Way

$$E[W] = \left\{ \begin{array}{l} \frac{1}{2} \cdot E[W | X=0] \\ \end{array} \right.$$

$$E[W] = E[W | X=0] P(X=0) + E[W | X=1] P(X=1)$$

$$\text{Var}(w) = E[w^2] - E[w]^2$$

$$= E[(w-\mu)^2]$$

	0	1	2	3
0	$\frac{1}{8}$ HHH $w=0$	$\frac{3}{8}$ HTH or HHT	$\frac{3}{8}$ HTT	$\frac{1}{8}$ $\emptyset$
1	$\frac{1}{8}$ $w=0$	$\frac{1}{8}$ THH	$\frac{1}{8}$ THT or TTH	$\frac{1}{8}$ TTT
2	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

$$E[w^2] = 6^2 \left[ \frac{2}{8} \right] + 9 \left[ \frac{2}{8} \right] + 12^2 \left[ \frac{2}{8} \right] + 18^2 \left[ \frac{1}{8} \right]$$

$$\frac{2}{8} \left[ 36 \quad 9 + 144 \right] + \frac{324}{8}$$

$$\text{VAR} = 87.75 - \left( \frac{15}{2} \right)^2 \quad 87 \quad \frac{3}{4}$$

$$= 31.5$$

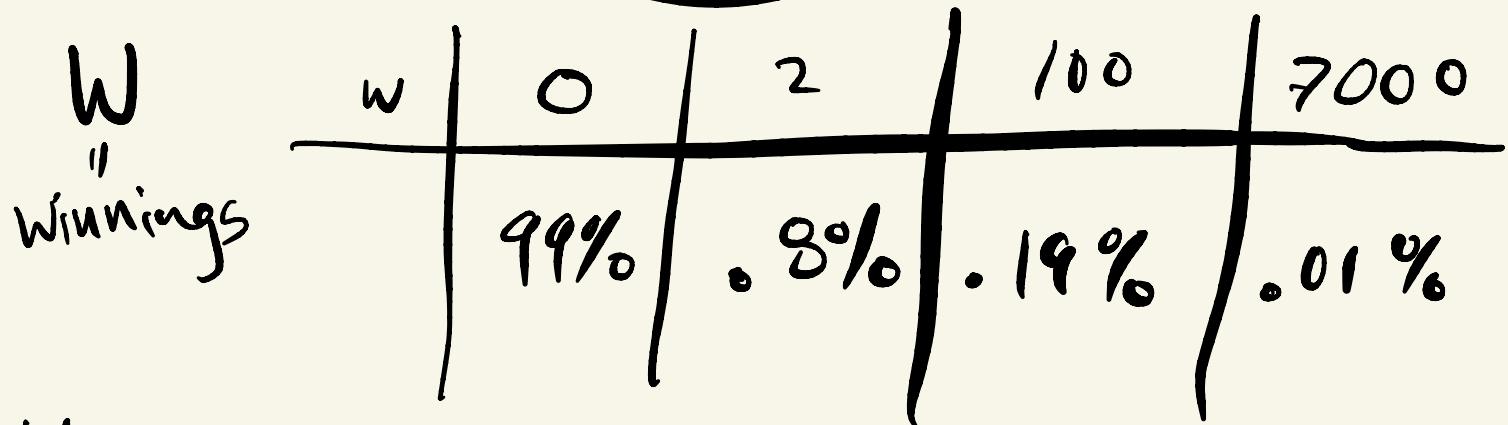
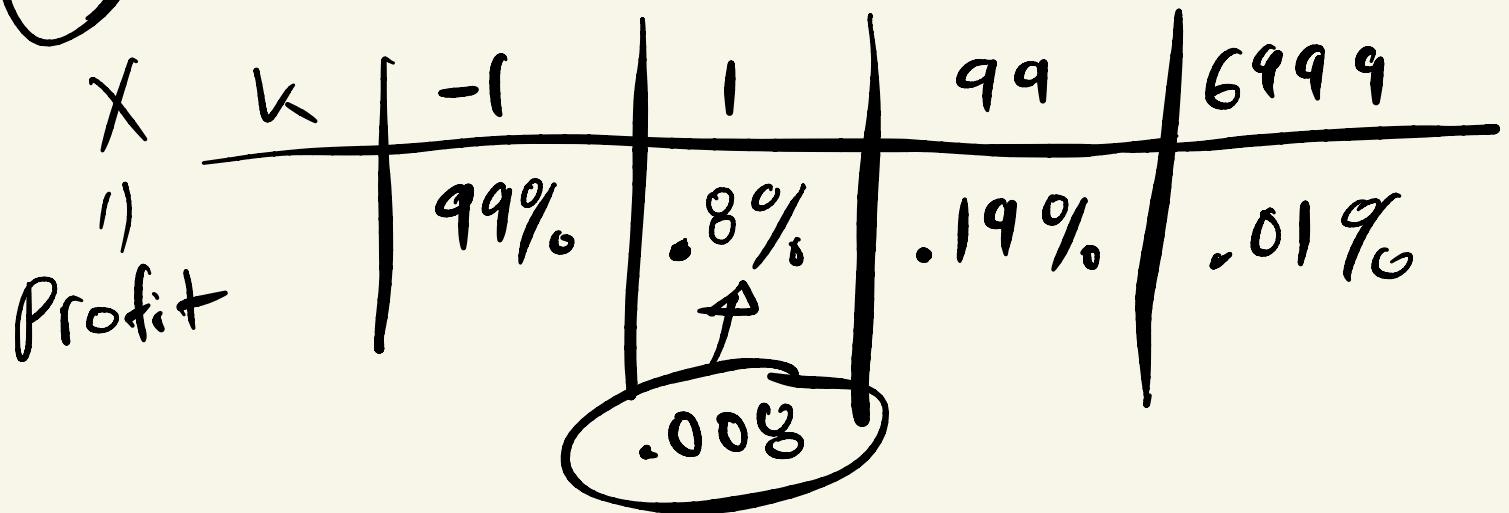
$$SD = 5.612$$

→ 1 SD window

$$\$ [1.9, 13.1] \quad \rightarrow \quad \left[ \frac{15}{2} - 5.6, \frac{15}{2} + 5.6 \right]$$

~~→ \\$1, 12 \\$~~

(4)



$$X = W - 1$$

$$E[W] = 2(.8\%) + 100(.0019) \\ (.008) + 7000(.0001)$$

$$E[X] = E[W-1] = E[W] - 1$$

$$V(X) = V(W-1)$$

$$\hookrightarrow V(W) = E[W^2] - E[W]$$

$$V(ax+b) = a^2 V(x)$$