AMAT 362—Work Sheet 24

Dr. Justin M. Curry

NOT DUE BUT RECOMMENDED FOR THE FINAL

Name: _____

1. Suppose (X, Y) have a JDF that is uniform on the triangle bounded above by the line $y = 1 - x$ and $y \geq 0$ and $x \geq 0$.
   (a) Suppose a point is sampled from this distribution and a rectangle is formed using the point and the origin. Compute the expected area.

   (b) Compute the covariance of X and Y.

   (c) Compute the probability that the rectangle from Part (a) has area less than $1/16$.

2. Suppose that you're working a shift at Nikos. The first customer that arrives on your shift according to an exponential distribution whose average arrival time is 10 minutes. Your process of taking and filling their order is uniformly distributed between 1 and 6 minutes. Suppose you start your shift at 10am. Compute the PDF of the time when you finish serving your first customer. What's the expected time that you'll finish serving your first customer?

3. Suppose X has a uniform $(0, 1)$ distribution and $P(A \mid X = x) = x^2$. What is $P(A)$?

4. Suppose X, Y are RVs with JDF $f(x, y) = \lambda^3 x e^{-\lambda y}$ for $0 < x < y$ and 0 otherwise.
   (a) Find the PDF of Y.

   (b) Compute $E(Y)$

(c) (2 points) Compute $E(X \mid Y = 1)$.

5. (8 points) The RV X has a uniform distribution on $(0, 1)$. Given that $X = x$, the RV Y is binomial with parameters $n = 5$ and $p = x$.

   (a) Find $E(Y)$ and $E(Y^2)$

   (b) Find $P(Y = y \text{ and } x < X < x + dx)$

   (c) Find the density of X given $Y = y$. Do you recognize it? If yes, as what?