MATH 362—Work Sheet 05

Dr. Justin M. Curry

Due on Saturday February 20th, 2021

Name: ★ SOLUTIONS ★

1. (2 points) You are dealt two cards, one at a time, from a standard 52 card deck.
   (a) (1 point) What's the probability that the second card is black?

$$P(2\text{nd } B) = P(2\text{nd } B \mid 1\text{st } B)P(1\text{st } B) + P(2\text{nd } B \mid 1\text{st NOT } B)P(1\text{st NOT } B)$$
$$= \frac{25}{51}\cdot\frac{26}{52} + \frac{26}{51}\cdot\frac{26}{52} = \frac{1}{2}\left(\frac{25+26}{51}\right) = \frac{1}{2}$$

   (b) (1 point) What's the probability that the second card is black, given that the first card is black?

$$P(2\text{nd } B \mid 1\text{st } B) = \frac{25}{51}$$

2. (4 points) *The Two Queens Problem:* In Texas Hold-em, every player is dealt two cards from a standard 52 card deck.
   (a) (1 point) What's the probability of being dealt two Queens?

$$\frac{\binom{4}{2}}{\binom{52}{2}} = \frac{\frac{4!}{2!2!}}{\frac{52!}{2!50!}} = \frac{4\cdot 3}{52\cdot 51} = \frac{1}{13}\cdot\frac{1}{17}$$

   (b) (1 point) What's the probability of having two Queens assuming you know you have at least one queen?

$$P(2Q \mid \geq 1Q) = \frac{P(2Q)}{P(\geq 1Q)} = \frac{\frac{1}{13}\cdot\frac{1}{17}}{\frac{33}{13\cdot 17}} = \frac{1}{33}$$

$$1 - P(\text{NO QUEENS}) = 1 - \frac{48}{52}\cdot\frac{47}{51}$$

   (c) (1 point) What's the probability of having two Queens assuming you know you have the Queen of Hearts?

$$P(2Q \mid Q\heartsuit) = \frac{P(2Q \cap Q\heartsuit)}{P(Q\heartsuit)} = \frac{2\cdot 3/52\cdot 51}{2\cdot 51/52\cdot 51} = \frac{3}{51}$$

↳ Intuitively need 1 of 3 remaining Q's $= \frac{3}{51} = \frac{1}{17}$

   (d) (1 point) How do these probabilities compare?

1/17 is nearly 2x $\frac{1}{33}$ so prob goes up!

3. (2 points) Describe *The Monty Hall Problem*. Should you switch doors?

3 doors have 3 different prizes behind them. 2 are goats, 1 is a new car. You choose a door @ random and another door w/ a goat behind it is opened. You're offered the opportunity to switch to the remaining closed door.

⟹ You should switch! Only way to lose is if you choose the right door to begin w/ = 1/3 chance of losing

4. (3 points) *Electrical Components:* Suppose there are two electrical components in a device. The chance that the first component fails is 10%. If the first component fails then the chance that the second component fails is 20%. If the first component works, the chance the second component fails is only 5%.

   (a) (1 point) What's the probability at least one component works.

   1 − Prob both fail = 1 − (.10)(.20) = 1 − .02 = 98%

   (b) (1 point) What's the probability exactly one component works?

   P(1st fails ∩ 2nd Works) = P(2nd Works | 1st fails) P(1st fails)
   ↳ (.8)(.1) = .08

   + P(1st works ∩ 2nd fails) = P(2nd fails | 1st works) P(1st works)
   ↳ (.05)(.9) = .045

   (c) (1 point) What's the probability the second component works?

   P(2nd Works | 1st fails) P(1st fails) = .08

   + P(2nd Works | 1st works) P(1st Works) = (.95)(.9) = .855

   ⟹ .08 + .855 = .935 ≈ 93.5%

   12.5%

5. (1 point) Suppose

   - $P(\text{snow today}) = 40\%$, and
   - $P(\text{snow tomorrow}) = 50\%$, and
   - $P(\text{snow today and tomorrow}) = 30\%$.

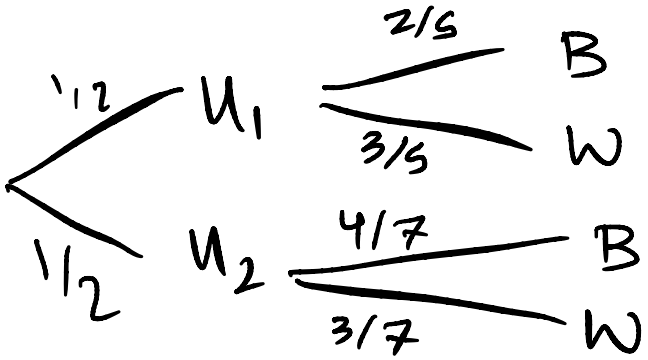
   What is $P(\text{snow tomorrow} \mid \text{snow today})$ =?

   $$\frac{P(\text{Snow Tom} \cap \text{Snow Tod})}{P(\text{snow today})}$$

   $$\frac{.3}{.4} = 75\%$$

6. (3 points) *Two Urns:* One urn contains 2 black and 3 white balls. The other urn contains 4 black balls and 3 white balls. An urn is chosen uniformly at random and a ball is chosen uniformly at random from that urn.

   (a) (1 point) Draw a tree diagram illustrating the possible outcomes of this experiment.

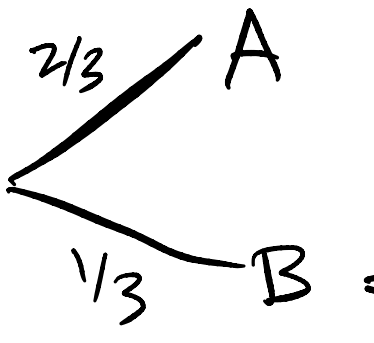


   (b) (1 point) Assign probabilities and conditional probabilities to branches in the tree.

   (c) (1 point) Calculate the total probability that ~~the urn~~ the ball is black.

$$\frac{1}{2}\cdot\frac{2}{5}+\frac{1}{2}\cdot\frac{4}{7}=\frac{1}{2}\left(\frac{14+20}{35}\right)=\frac{17}{35}$$

7. (2 points) A light bulb company has factories in two cities. The factory in city A produces two thirds of the company's light bulbs. The remainder are produced in city B, and of these, 1% are defective. Among all bulbs manufactured by the company, what proportion are not defective and made in city B?



$$P(\text{Not Def AND City B}) = \frac{1}{3}\cdot 99\% = 33\%$$

8. (4 points) *True and False:* In a particular population of men and women. 92% of women are right handed and 88% of men are right handed. Indicate whether each of the following statements are (i) true, (ii) false, or (iii) can't be determined using the information given.

   (a) (1 point) The overall proportion of right handers in the population is exactly 90%.

   (b) (1 point) The overall proportion of right handers in the population is between 88% and 92%.

   (c) (1 point) If the sex ratio in the population is 1-to-1 then Part A is true.

   (d) (1 point) If Part A is true, then the sex ratio is 1-to-1.