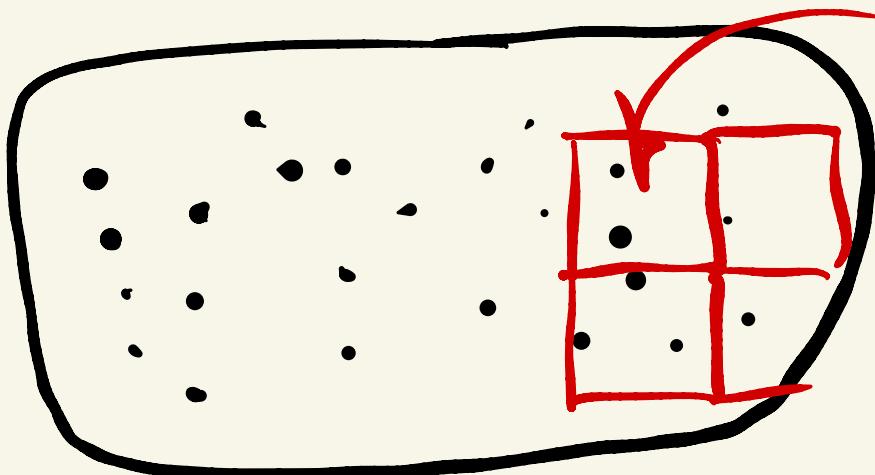


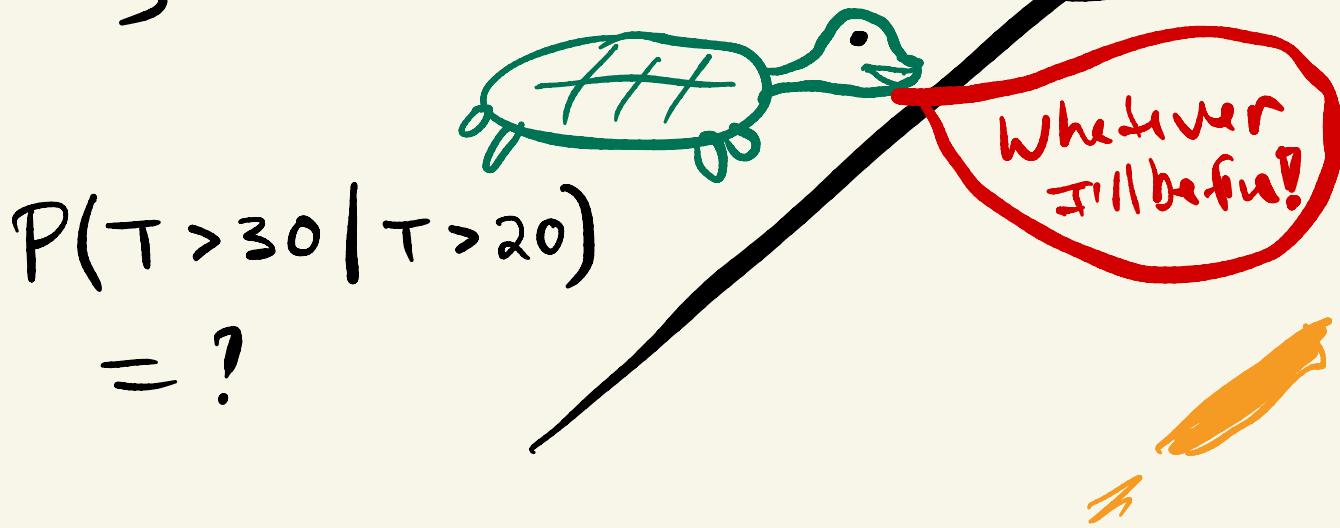
LECTURE 18 LIVE



$$P(k \text{ spot}) \approx \frac{\lambda^k}{k!} e^{-\lambda}$$

THE TURTLE { THE FOX

Animals know that arrival of cars
is exponentially distributed
w/ param λ
Avg = 30 minutes



$$P(T > 30 | T > 20) = \frac{P(T > 30 \text{ AND } T > 20)}{P(T > 20)} = \frac{P(T > 30)}{P(T > 20)}$$

$$P(T > a) = \int_a^\infty \lambda e^{-\lambda t} dt = -e^{-\lambda t} \Big|_a^\infty = e^{-\lambda a}$$

$$\Rightarrow \text{So... } P(T > 30 | T > 20) = \frac{e^{-30\lambda}}{e^{-20\lambda}} = \frac{\cancel{e^{-20\lambda}}}{\cancel{e^{-30\lambda}}} e^{-10\lambda}$$

$$P(T > 10) = e^{-10\lambda}$$

$$P(T > a+b | T > a) = \frac{e^{-\lambda(a+b)}}{e^{-\lambda a}} = e^{-\lambda b} = P(T > b)$$

Proof of Memoryless

Property

Geometric RV also has this prop!

EARTHQUAKES

1.5 -

$$\lambda = 1 \text{ year} \quad \text{Earthquake rate} E$$

$$(a) P(0 \leq E \leq 1) = \int_0^1 \lambda e^{-\lambda t} dt$$

$$= -e^{-\lambda t} \Big|_0^1 = 1 - e^{-1}$$

$$= 1 - e^{-1}$$

$$t_{1/2} = \frac{\ln(2)}{\lambda} = .69 \text{ of a year}$$

$= 63\%$

$$(b) P(0 \leq E \leq \frac{1}{2}) = 1 - e^{-\lambda \cdot \frac{1}{2}} = 39\%$$

$$(c) P(T > 2) = e^{-2\lambda} = e^{-2}$$

= 13.5%

50% Δ 2 years

$$(d) P(T > 2 | T > 1) = P(T > 1) = 37\% = \frac{1}{e}$$

Q3) Avg = med = 10 hrs $\lambda = \frac{1}{10}$

(a) $P(T > 20) = e^{-20\lambda} = e^{-2}$
 $= 13.5\%$

(b) Median lifetime

$$\frac{\ln 2}{(\lambda)} = 10 \cdot \ln 2 \approx 6.9 \text{ hours}$$

(c) $SD = \sqrt{\lambda} = 10 \text{ hours}$

(d) Q: What is the probability that the average lifetime of 100 IID components each of which $X_i \sim \text{Exp}(\frac{1}{10})$ exceeds 11 hours?

$S_n = X_1 + \dots + X_n$ where each $X_i \sim \text{Exp}(\lambda)$

$$P(a < S_n < b) \approx \Phi\left(\frac{a - n\mu}{\sigma\sqrt{n}}\right) - \Phi\left(\frac{b - n\mu}{\sigma\sqrt{n}}\right)$$

GROWN UP CLT

CAN USE For "large" n

NORMAL

Define $A_n = \frac{S_n}{n}$

$$E(A_n) = E\left(\frac{S_n}{n}\right) = \frac{1}{n} \cdot E(S_n)$$

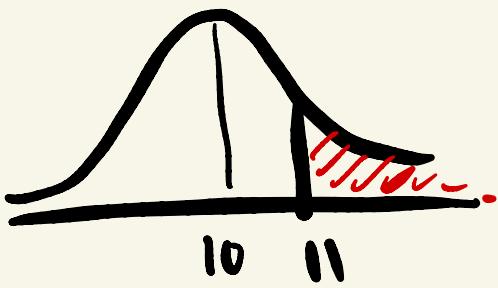
$x_1 + \dots + x_n$

$$= \frac{1}{n} \cdot \underbrace{n \cdot E(x_i)}_{\text{"/}/\lambda}$$

For our example $= 10 \text{ hrs}$

$$SD(A_n) = \sqrt{\text{Var}(A_n)} = \sqrt{\frac{1}{n^2} \cdot \text{Var}(x_1 + \dots + x_n)}$$

$$= \sqrt{\frac{1}{n^2} \cdot n \cdot \text{Var}(x_i)}$$

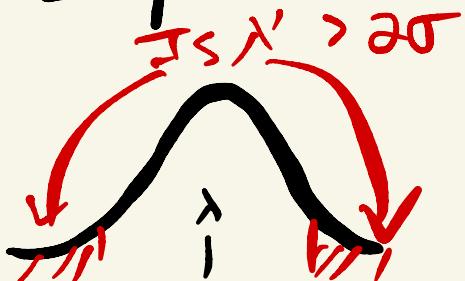


$$= \frac{1}{\lambda} \cdot \frac{1}{\sqrt{100}}$$

$$1 - \underline{\Phi}(1) = 16\% \quad = 10 \cdot \frac{1}{10} = 1$$

An aside on statistics

$T \sim Exp(\lambda)$



Error in Measuring $\lambda \sim Normal(\mu, \sigma^2)$

New $\lambda' = \text{fracking time between Park dates}$

(e) What's the probability that
the average lifetime of 2
indep components > 11 hours

Can't use CLT b/c $n=2$
is TOO SMALL

Lifetime of Part 1 = T_1

" " 2 = T_2

$$P\left(\frac{T_1 + T_2}{2} > 11\right) = P(T_1 + T_2 > 22)$$

Gamma (= Γ)
Distribution

= Prob (0 or 1 failures in $\overset{\text{first}}{22}$ hours)

POISSON POINT PROCESS

= # of ARRIVALS / FAILURES

$\sim \text{Poisson}(\lambda \cdot (b - a)) \approx \text{Poisson}_{2.2}(k)$

= $\text{Poisson}_{2.2}(0) + \text{Poisson}_{2.2}(1)$

$$\begin{aligned} &= e^{-2.2} + 2.2 e^{-2.2} \\ &= 35.45\% \end{aligned}$$

(Q3/e) = intro to Poisson Point Process

Q4 = Fully Poisson Point Process

A store 9am - 6pm avg's 45 customers
a day

Compute λ = Avg rate of arrival per hour

$$= \frac{45}{9} = \frac{5}{1} = 5 = \lambda$$

(a) A_1 = No customers arrive between 9 { 10 am

$$P(A_1) \cong \text{Poisson } \lambda(1-0) = \lambda \quad (k=0)$$

$$e^{-\lambda} = e^{-5} = .67\%$$

(b) A_2 = 3 customers arrive between 10 { 10:30 am

$$P(A_2) = \text{Poisson } \lambda(1.5-1) \quad (k=3) \quad 21.4\%$$

$$\frac{(.5\lambda)^3}{3!} e^{-5\lambda} = \frac{(2.5)^3}{3!} e^{-2.5}$$

$$(c) P(A_1 \cap A_2) = P(A_1)P(A_2)$$

↑
Indep

$$= \frac{(2 \cdot 5)^3}{3!} e^{-7 \cdot 5}$$

b/c $[9 \text{ to } 10] \cap [10 \text{ to } 10:30]$

$$\bar{e}^{2 \cdot 5} \cdot \bar{e}^{-5}$$

Mateen's Q

$$P(3 \text{ customers}_{10-10:30}) \xleftarrow{?} P(6 \text{ customers}_{10-11})$$

$\neq P(3 \text{ customer}_{10-10:30})P(3 \text{ customer}_{10:30-11})$

$$\left[\frac{\lambda^3}{3!} e^{-\lambda/2} \right]^2$$