Q1 (BIG PICTURE) LEC 14-LIVE

For fixed p & large n

$S_n \sim Bin(n,p)$ Approx Normally Distributed

Flip coin
$100 = n$
$1/2 = p$

$$\frac{S_n - np}{\sqrt{npq}} \sim Norm(0,1)$$

(a) $np = 50$

$\sqrt{npq} = \sqrt{100 \cdot \frac{1}{2} \cdot \frac{1}{2}} = \sqrt{25} = 5$

$$P(S_{100} \leq 45) = \Phi\left(\frac{45-50}{5}\right)$$

$$= \Phi\left(-\frac{5}{5}\right) = \Phi(-1) = .1587$$

$\sim 15.87\%$

CONTINUITY CORRECTION ($\leq$)

$$\Phi\left(\frac{45.5-50}{5}\right) = \Phi(-.9) = 18.41\%$$

(b)

$$P(45 < S_{100} < 55) = \Phi\left(\frac{55-50}{5}\right) - \Phi\left(\frac{45-50}{5}\right)$$

$\Phi(-1,1)$

$$= \Phi(1) - \Phi(-1)$$

$$= 2\Phi(1) - 1$$

$$.8413 - .1587 = \boxed{68.26\%}$$

68-95-99.7
$\pm 1\sigma$ $\pm 2\sigma$ $\pm 3\sigma$

(c)
(d) PRACTICE!

(2) Flip fair coin 400 times

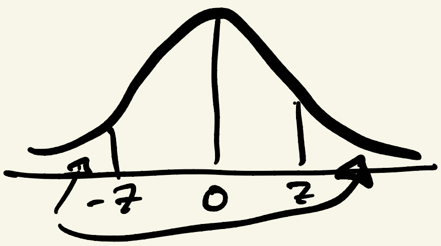
Find x s.t.

$$P(200 - x < S_{400} < 200 + x) \approx .8$$

Recognize $200 = np = \mu$

Problem Reduces to finding

z s.t. $\Phi(z) - \Phi(-z) = .8$



$$\Phi(z) - (1 - \Phi(z))$$

$$= 2\Phi(z) - 1 = .8$$

$$\Rightarrow \Phi(z) = \frac{1.8}{2} = .9$$

REVERSE Z-TABLE LOOKUP

$z = 1.28 \rightsquigarrow \Phi(z) = .9$

What is x?

$$\frac{x - np}{\sqrt{npq}} = z = 1.28$$

$$\sqrt{400 \cdot \frac{1}{2} \cdot \frac{1}{2}} = \sqrt{100} = 10$$

$$x = 200 + 12.8$$

$$\approx \boxed{x = 12.8}$$

Q4 400 customers per day
20% order apple pie

(a) $S_{400} \rightsquigarrow np = \frac{1}{5} \cdot 400 = 80$

$\sqrt{npq}$

$P(a \le S_{400} \le b) \approx .95$

$\sqrt{400 \cdot \frac{1}{5} \cdot \frac{4}{5}}$

$= P\left(a^* \le \frac{S_{400} - 80}{8} \le b^*\right)$ $\quad \sqrt{64} = 8$

$\Phi(-z, z) = .95$

$2\Phi(z) - 1 = .95$

$\Rightarrow \Phi(z) = \frac{1.95}{2} = .975$

$\hat{p} \pm \frac{2\sigma_{\text{percent}}}{\sqrt{n}}$

$= .2 \pm \frac{2 \times .4}{20}$ $.04$

Reverse z-table lookup $\to \boxed{z = 1.96}$ $\rightsquigarrow$ $z = 2$ OR for 68-95-99.7

$\frac{x - 80}{8} = 1.96 \sim 2$

$\boxed{4\%}$

$x = 80 + 16 = 96$

95% confidence interval $\longrightarrow 80 \pm 2\sigma$ $\boxed{64 - 96}$ slices of pie

(b) $n \geq \left(\frac{1.96\, \sigma_{\text{percent}}}{\varepsilon}\right)^2$ $\quad \sigma_{\text{percent}} = .4$, $\varepsilon = .01$

$\sigma_{\text{percent}} = \sqrt{pq}$

$\left(\frac{2 \times .4}{.01}\right)^2$ $= \sqrt{\frac{1}{5} \cdot \frac{4}{5}} = \frac{2}{5}$

2

$= .4$

$\left(\frac{.8}{.01}\right)^2$ // $\boxed{n \geq (80)^2 \quad 6400}$

What value of z gives

.68

$$\underbrace{\Phi(-z,z) = .68}$$

$$\Downarrow$$

$$2\Phi(z) - 1 = .68$$

$$\Phi(z) = \frac{1.68}{2}$$

$$= .84$$

$$\Rightarrow z = .99 \text{ or } 1.0$$

Q5 $\quad z = 2.33\,\sigma$

$$\sigma = \sqrt{n \cdot \tfrac{1}{4}} = \sqrt{250} = 15.8$$

36.84

537 seats

~~Q6~~
Q3 $\quad$ 98.21%

$$\mu = 95 = .05 \times 1900$$

$$\sigma = \sqrt{1900 \cdot .05 \cdot .95} = 9.5$$

$$\frac{115 - 95}{9.5} = 2.105$$

1.79% $\quad 1 - .9821$

6 $X \sim \text{Unif}\{1,\dots,6\}$ fair 6-sided die

$S_n = X_1 + \dots + X_n$ where $X_i \sim \text{Unif}\{1,\dots,6\}$

$$\boxed{V(X_i) = \frac{35}{12}}$$

$V(S_n) = V(X_1 + \dots + X_n)$

(indep) $= V(X_1) + \dots + V(X_n)$

$= n\,V(X_i)$

$\mu = \frac{7}{2}$ $n\mu = 24 \cdot \frac{7}{2}$

$V(S_{24}) = 24 \cdot \frac{35}{12} = 70$ $\sqrt{70} = SD$

$P(S_{24} > 84) = 1 - \Phi\left(\frac{84 - 12 \cdot 7}{\sqrt{70}}\right)$

$\frac{}{0}$

$= 50\%$

$P(S_{24} = 84) \approx \Phi\left(\frac{84.5 - 84}{\sqrt{70}}\right) - \Phi\left(\frac{83.5 - 84}{}\right.$

CONTINUITY CORRECTION $2 \cdot \Phi\left(\cdot \frac{5}{\sqrt{70}}\right) - 1$ $.04$

$\Phi(.059)$

$2 \cdot 52\%$ 4%

$\frac{1}{70}$

$\frac{S_n - n\mu}{\sigma\sqrt{n}} \sim \text{Norm}(0,1)$ $\varphi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$ $.005$

$P(S_n = n\mu) \approx \frac{\varphi(0)}{\sqrt{\sigma^2 n}}$ $\varphi(0) = \frac{1}{\sqrt{2\pi}}$