Lecture 10 - AMAT 362

PROB. for Statistics

Last time: Mean, Median, Mode

$$\text{Mean} = E(X) = \text{Expectation of } X$$

or Expected value of X

TODAY

LINEARITY of EXPECTATION

↳ 1) $E(aX+b) = aE(X)+b$

↳ 2) $E(X+Y) = E(X)+E(Y)$

Motivates two more questions

Q1 When is it true that $E(g(X)) = g(E(X))$

NOT ALWAYS!

Q2 What does it mean to sum to RV's? $Z = X+Y$???

Ex

Suppose you're told that the average temperature in April is 10 degrees Celsius. What is the average temp in Fahrenheit?

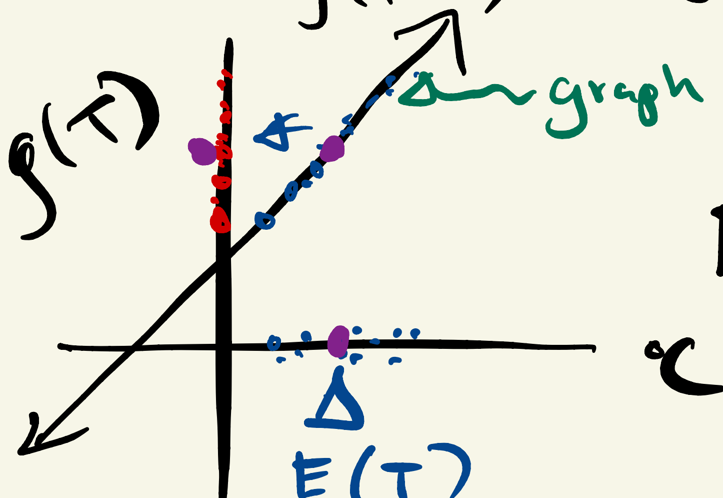
$$\{{}^\circ C\} \longrightarrow \boxed{g} \longrightarrow \{{}^\circ F\}$$

$$g(x) = \frac{9}{5}\cdot x + 32$$

$$\boxed{g(10) = 50^\circ F}$$

$$g(0) = 32^\circ F$$

$$g(100) = 180 + 32 = 212^\circ F$$



$$E(g(T)) = E\left(\frac{9}{5}\cdot T + 32\right)$$

$$= \frac{9}{5}E(T) + 32$$

$$= 50^\circ F$$

* R.V. measuring hi temp in Albany NY on days in April

Ex Ordinary 6-sided die

$X \rightarrow \{1,2,3,4,5,6\}$

Relabel my die $\Delta\, E(X)$

$Y \rightarrow \{0,10,20,30,40,50\}$

Recognize $\Delta\, E(Y)$

$Y = 10X - 10$

Q: $E(Y) = ?$ $\quad \underline{\Delta}\;\; E(X) = 3.5$

$$\Rightarrow E(Y) = 10 \cdot E(X) - 10$$
$$= 35 - 10$$
$$= 25$$

Def If X is a discrete RV and g is a function, then $Y = g(X)$

$$E(g(X)) = E(Y) = \sum_{\substack{\text{all possible values } x \\ \text{for } X}} g(x) \underbrace{P(X = x)}_{\text{PMF for } X}$$

Example where $E(g(X)) \neq g(E(X))$

X is uniform on $\{-1, 0, 1\}$

x	-1	0	1
$P(X=x)$	$1/3$	$1/3$	$1/3$

$$\boxed{E(X) = -\frac{1}{3} + 0 + \frac{1}{3} = 0}$$

Consider $g(x) = x^2$ Define $Y = g(X)$

y	0	1
$Y=g(X)$	$1/3$	$2/3$

??

$$P(Y=1) = \sum_{\substack{x \text{ s.t.} \\ g(x)=1}} P(X=x)$$

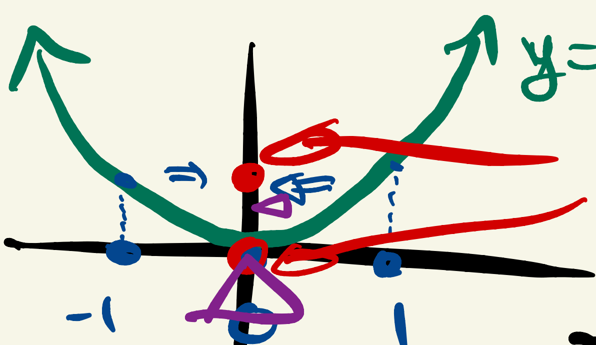
$$= \frac{1}{3} + \frac{1}{3}$$

$P(X=-1) + P(X=1)$

$$E(Y) = \sum_x g(x) P(X=x)$$

$$= (-1)^2 \cdot P(X=-1) + 0^2 \cdot P(X=0) + (1)^2 \cdot P(X=1)$$

$$\boxed{2/3 = E(X^2) = E(g(X))}$$



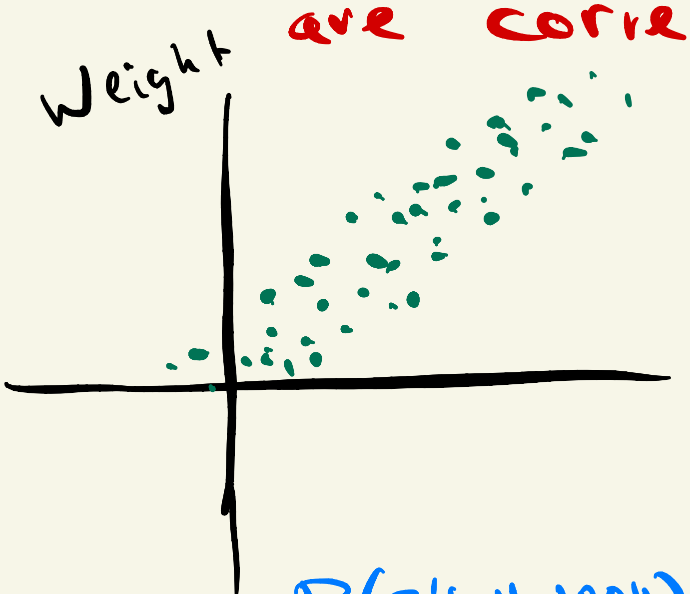
$$g(E(X)) = g(0) = 0^2 = 0 \neq E(g(X))$$

What does it mean to sum two R.V.s? $X + Y = Z$

Why study 2 (or more) RV's?

↳ Two or more properties associated to an experiment

⟹ Sometimes these properties are correlated & sometimes not!



• 7' tall weighs 100lbs?

NOT LIKELY say = 0

$$\underset{\text{AND}}{P(7'\text{tall}, 100\text{lb})} \neq \underbrace{P(7'\text{tall})\,P(100\text{lbs})}_{\text{NON ZERO}}$$

$$P(7'\text{tall}, 100\text{lb}) = 0$$

Def The joint probability distribution

$$P_{X,Y}(x, y) = P(X = x \text{ AND } Y = y)$$

Def X, Y indep $\iff P_{X,Y}(x,y) = P_X(x)\,P_Y(y)$

Ex

X = # on first draw

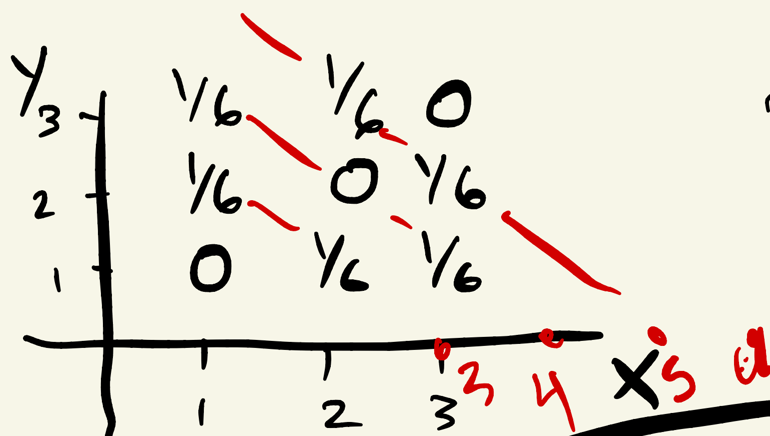
Y = # on second draw

Draws w/o replacement

↳ B/c we record order we need to distinguish ordered pairs

$(1, 3) \neq (3, 1)$ $\Rightarrow$ 6 distinct outcomes all equally likely

x → 1, y → 3; x → 3, y → 1



Prob(1 1st, then 2)

$\left(\frac{1}{3}\right) \cdot \frac{1}{2} = 1/6$

Joint PMF $P(x, y) = \begin{cases} 1/6 & \text{if } x \neq y \\ 0 & \text{if } x = y \end{cases}$

Consider

$$Z = X + Y$$

Q: What are the possible values for Z?

A: $Z \in \{3, 4, 5\}$

$$P(Z=4) = P(X=1, Y=3) + P(X=3, Y=1)$$

$$\tfrac{1}{6} + \tfrac{1}{6} = \tfrac{1}{3}$$

$$P(Z=5) = P(X=2, Y=3) + P(X=3, Y=2)$$

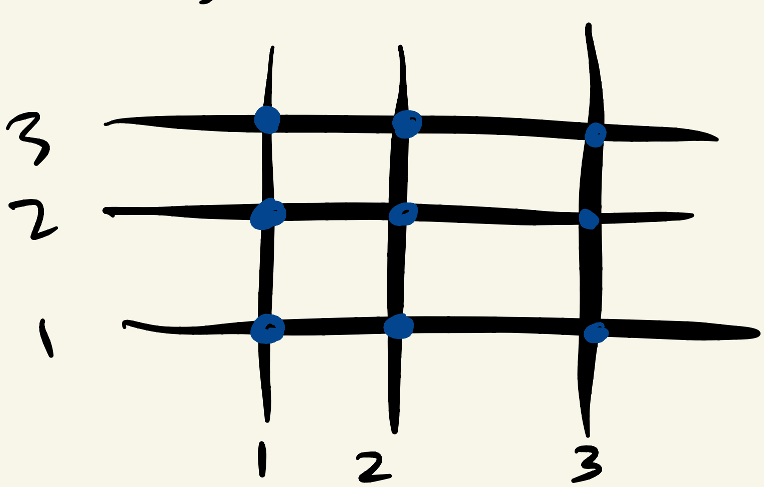
$\Rightarrow$

z	3	4	5
	$\tfrac{1}{3}$	$\tfrac{1}{3}$	$\tfrac{1}{3}$

Uniform?

Draws w/ replacement?

(i, j) where $i \in \{1, 2, 3\}$ $j \in \{1, 2, 3\}$



Different joint PMF

$$P(x, y) = \frac{1}{9}$$

$$P(X+Y=3) = P(1,2) + P(2,1)$$
$$= 1/9 + 1/9 = 2/9$$

$$P(X+Y=4) = P(1,3) + P(2,2) + P(3,1)$$
$$1/9 + 1/9 + 1/9$$
$$= 3/9$$

$$P(X+Y=5) = P(2,3) + P(3,2)$$
$$= 2/9$$

$$P(X+Y=6) = P(3,3)$$
$$= 1/9$$