# Math 362 FINAL Practice

May 12-15, 2021

Name: ______________________________

- You may use a calculator and notes from our class as well as worksheet solutions on Blackboard, but nothing else. If you don't understand a word, you can use an online dictionary. In general, please show your work, simplify as completely as possible, and *box your final answer*.
- *Collaboration is strictly prohibited* on this exam. Use of Chegg or similar sites carries steep penalties that are described in the syllabus.

| Problem | Points | Score |
|---|---|---|
| 1 | 2 | |
| 2 | 9 | |
| 3 | 6 | |
| 4 | 14 | |
| 5 | 6 | |
| 6 | 13 | |
| 7 | 10 | |
| 8 | 10 | |
| Total: | 70 | |

1. (2 points) Each human's genome can be thought of as a three billion ($3 \times 10^9$) letter long word made using only the letters $A, T, G, C$, which correspond to the nucleic acids adenine, thymine, guanine and cytosine, respectively.

   **How many** three billion letter long words, which only use the letters $A, T, G, C$, are possible?

2. (9 points) Let $A$ and $B$ be subsets of a sample space $\Omega$. Recall that $A^c = \Omega - A$ is the set of events where $A$ does not occur.

   Suppose we are told the following probabilities: $P(A \cap B) = 1/4$, $P(A^c) = 1/3$, and $P(B) = 1/2$.

   (a) (2 points) **What is $P(B^c)$ equal to?**

   $P(B^c) =$

   (b) (2 points) **What is $P(A)$ equal to?**

   $P(A) =$

   (c) (2 points) **What is $P(A \cup B)$ equal to?**

   $P(A \cup B) =$

   (d) (3 points) Does $A \cup B = \Omega$? **Explain why or why not in a few words.**

3. (6 points) In front of you are two identical urns. In $U_1$ there are 3 red balls and 2 green balls. In $U_2$ there are 4 red balls and 6 green balls.

   (a) (3 points) You choose an urn uniformly at random. Let $R$ be the event of drawing a red ball. **Compute $P(R)$.**

   $P(R) =$

   (b) (3 points) Let $G$ be the event of drawing a green ball. **Compute $P(U_2 \mid G)$.**

   $P(U_2 \mid G) =$

4. (14 points) A Sacagawea[1] gold coin is biased to come up heads 60% of the time.

   (a) (3 points) Suppose I flip a Sacagawea gold coin 10 times. Let $S_{10}$ be the random variable that counts the number of heads out of 10 flips of a Sacagawea gold coin. **Compute the probability $P(S_{10} = 5)$.**

   $P(S_{10} = 5) =$

   (b) (2 points) **What's the expectation of $S_{10}$?**

   $E(S_{10}) =$

   (c) (2 points) **What's the variance of $S_{10}$?**

   $V(S_{10}) =$

[1] Sacagawea was a 16 year old Native American woman who helped Captains Meriwether Lewis and William Clark survey Thomas Jefferson's Louisiana Purchase from 1804-1806, which involved traveling from Missouri to the Pacific Ocean. In her honor a special issue dollar coin was minted in 2000.

(d) (3 points) Consider a new experiment: I flip a Sacagawea gold coin until I get a heads. Let $T$ be the random variable that counts the flip number of the first success. **Compute the probability** $P(T = 42)$

$P(T = 42) =$

(e) (2 points) **What's the expectation of $T$?**

$E(T) =$

(f) (2 points) **What's the variance of $T$?**

$V(T) =$

5. (6 points) Suppose $X$ is uniformly distributed on $[-1, 2]$. Let $Y = X^4$.

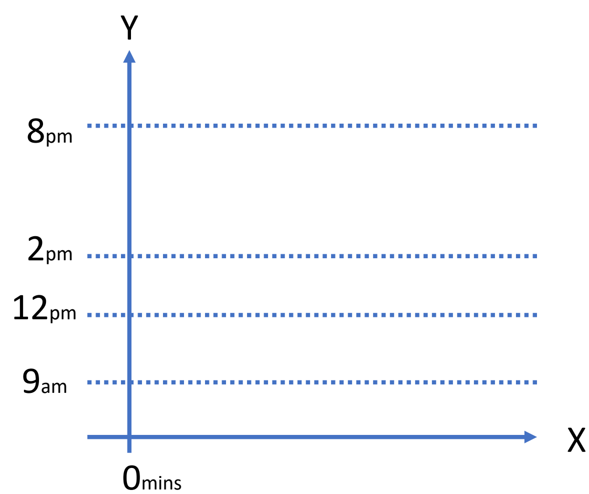
   (a) (3 points) Compute the probability that $P(0 < Y < 1)$.

   (b) (3 points) Compute the probability that $P(1 < Y < 16)$.

6. (13 points) (*Joint PDFs and Marginals*) In the UAlbany Student Center there is a restaurant called Nikos. Let $X$ be the random variable of how much time you need to wait to get your food. This depends on another random variable $Y$ which is when you arrive at Nikos. The joint PDF for $X$ and $Y$ is as follows:

$$f_{X,Y}(x,y) = \begin{cases} \frac{c_1}{7}e^{-x/7} & \text{for } 0 \le x \le \infty \text{ minutes} \quad \text{and} \quad 9\text{am} \le y < 12\text{pm} \\ \frac{c_2}{15}e^{-x/15} & \text{for } 0 \le x \le \infty \text{ minutes} \quad \text{and} \quad 12\text{pm} \le y \le 2\text{pm} \\ \frac{c_3}{10}e^{-x/10} & \text{for } 0 \le x \le \infty \text{ minutes} \quad \text{and} \quad 2\text{pm} \le y \le 8\text{pm} \\ 0 & \text{otherwise} \end{cases}$$

(a) (2 points) Using your pen or pencil, outline and shade in the support of the PDF in the figure below.




(b) (3 points) Using the joint PDF $f_{X,Y}(x,y)$ described above, compute the PDF for $Y$, and put your answer into the equation below.

$$f_Y(y) = \begin{cases} & \text{for } 9\text{am} \le y < 12\text{pm} \\ & \text{for } 12\text{pm} \le y \le 2\text{pm} \\ & \text{for } 2\text{pm} \le y \le 8\text{pm} \\ & \text{otherwise} \end{cases}$$

(c) (3 points) Suppose 20% of customers arrive at Nikos between 9am and 12pm, 40% of customers arrive between 12pm and 2pm, and 40% of customers arrive between 2pm and 8pm. Calculate the constants $c_1$, $c_2$, and $c_3$.

(d) (2 points) Derive the PDF for $X$ using the constants you obtained in part (c). You have to plug in the values for $c_1, c_2, c_3$ to get full credit.

(e) (3 points) (*1 point for each case below*) What's the expected wait time if I arrive at...

10am?

1pm?

7pm?

7. (10 points) Suppose you stay up late to watch a meteor shower from 11am to 3am. Suppose meteors arrive as a Poisson point process with $\lambda = 4$, which denotes the average number of meteors seen per hour.

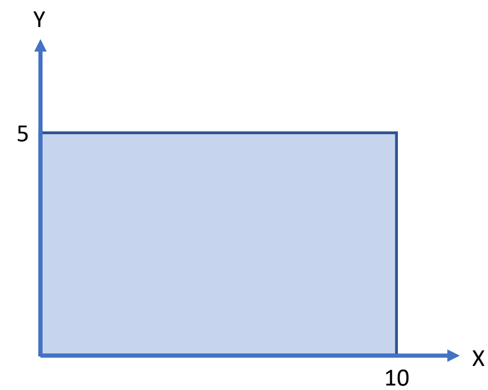
   (a) (3 points) Compute the probability that you see 3 or more meteors in the first hour.

   (b) (3 points) Compute the probability that you see no meteors in the first hour but at least 5 meteors from midnight to 3am.

   (c) (4 points) Given the information that there were 13 meteors seen between 11pm and 3am, what's the probability that no meteors were seen in the first hour?

8. (10 points) (*Independence and Continuous Probability*)

Suppose I select a point uniformly at random from the rectangle below.




(a) (2 points) Derive the PDFs for the $x$-coordinate $X$ and the $y$-coordinate $Y$.

(b) (2 points) Are $X$ and $Y$ independent? explain why or why not.

(c) (3 points) Suppose I pick a point from the rectangle above and form a right triangle with height $y$ and base $x$. Compute the expected area of the resulting triangle.

(d) (3 points) Compute the probability that the area of a random triangle constructed in part (c) is less than 10.