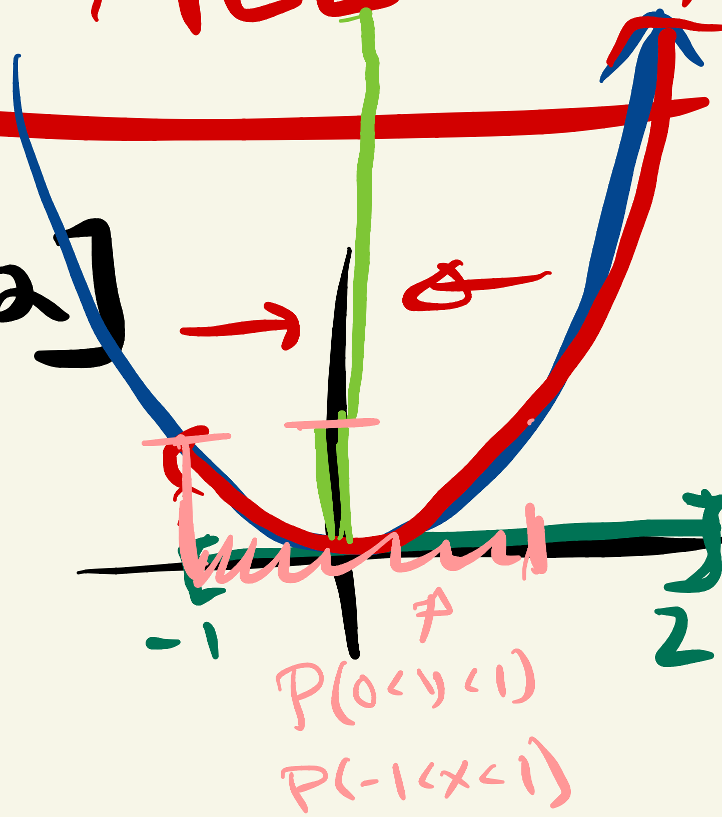
LAST CLASS MEETING



Q5 $X \sim \text{Unif}[-1, 2]$

$Y = X^4$

(a) $P(0 < Y < 1) =$

$P(0 < X^4 < 1)$

$= P\left(-\sqrt[4]{1} < X < +\sqrt[4]{1}\right)$

$= P(-1 < X < 1) = \int_{-1}^{1} f_X(x)\,dx$

$$f_X(x) = \begin{cases} \frac{1}{3} & -1 < x < 2 \\ 0 & \text{o.w.} \end{cases} \qquad = \int_{-1}^{1} \frac{1}{3}\,dx = 2/3$$

(b) $P(1 < Y < 16) = 1/3$

(b') $P(0 < Y < 2) = \underbrace{P(0 < Y < 1)}_{= 2/3} + P(1 < Y < 2)$?

$$P(1 \le Y < 2) = P(1 \le X^4 < 2)$$

$$= P(1 \le X \le 2^{1/4})$$

$$= \frac{2^{1/4} - 1}{3} + \frac{2}{3}$$

$$\boxed{P(0 < Y < 2) = \frac{2^{1/4} + 1}{3}}$$

Draw the CDF of Y!

Bonus

Q6 Nikos

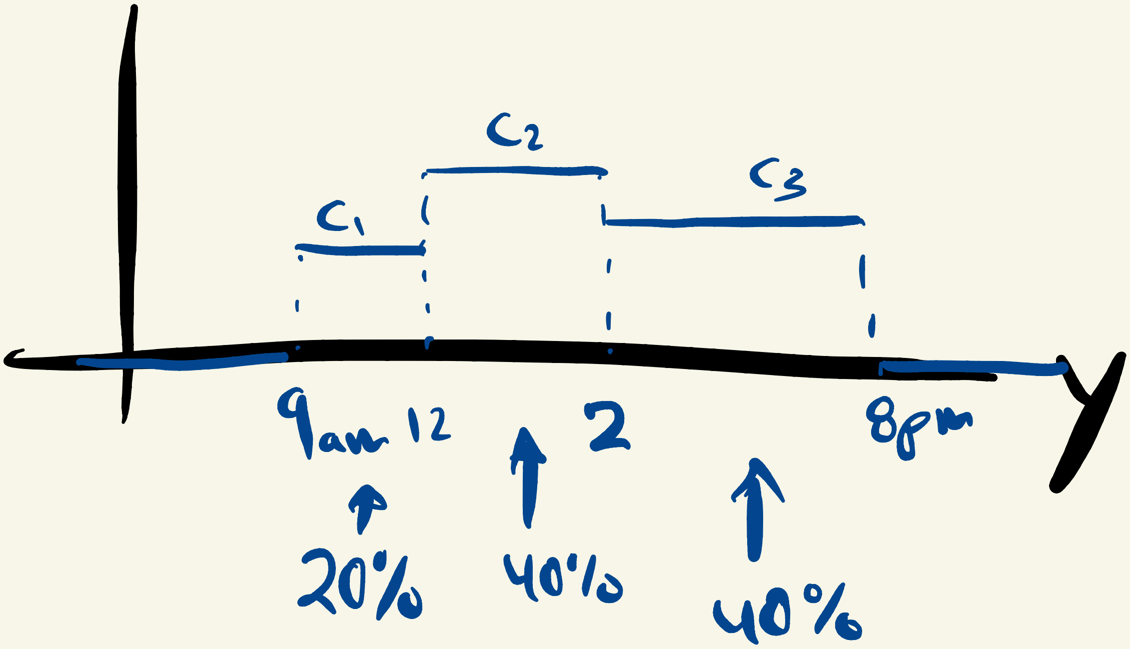
oint PDF for X and Y is as follows:

$$f_{X,Y}(x,y) = \begin{cases} \frac{c_1}{7}e^{-x/7} & \text{for } 0 \le x \le \infty \text{ minutes} \quad \text{and} \quad 9\text{am} \le y < 12\text{pm} \\ \frac{c_2}{15}e^{-x/15} & \text{for } 0 \le x \le \infty \text{ minutes} \quad \text{and} \quad 12\text{pm} \le y \le 2\text{pm} \\ \frac{c_3}{10}e^{-x/10} & \text{for } 0 \le x \le \infty \text{ minutes} \quad \text{and} \quad 2\text{pm} \le y \le 8\text{pm} \\ 0 & \text{otherwise} \end{cases}$$

$$f(x, Y=y) = c\lambda e^{-\lambda x}$$

$$\int_0^\infty c\lambda e^{-\lambda x}\,dx = c$$

$$f_Y(y) = \begin{cases} c_1 & 9 \le y \le 12 \\ c_2 & 12 \le y \le 2 \\ c_3 & 2 \le y \le 8 \\ 0 & \text{o.w.} \end{cases}$$



$$P(9 < y < 12) = .2$$

$$\int_9^{12} f_Y(y)\,dy = .2 \Rightarrow 3c_1 = .2$$

$$= \int_9^{12} c_1\,dy = c_1(12-9)$$
$$= c_1 \cdot 3$$

$$\boxed{c_1 = \frac{.2}{3}}$$

$$P(12 < y < 2)$$

$$\int_{12}^{2} c_2 = .4 \Rightarrow 2c_2 = .4$$

$$\boxed{c_2 = \frac{.4}{2}}$$

$$P(2 < y < 8) = \int_2^8 c_3 = 6c_3 = .4$$

$$\boxed{c_3 = .4/6}$$

(d)

$$f_X(x) = \int_9^{12} c_1 \lambda_1 e^{-\lambda_1 x}\,dy + \int_{12}^{2} c_2 \lambda_2 e^{-\lambda_2 x}\,dy$$

$$+ \int_2^8 c_3 \lambda_3 e^{-\lambda_3 x}\,dy$$

$$= \lambda_1 e^{-\lambda_1 x} \underbrace{\int_9^{12} c_1\,dy}_{.2} + \lambda_2 e^{-\lambda_2 x} \underbrace{\int_{12}^{2} c_2\,dy}_{.4} + \lambda_3 e^{-\lambda_3 x} \underbrace{\int_2^8 c_3\,dy}_{.4}$$

$$\boxed{f_X(x) = .2\,\lambda_1 e^{-\lambda_1 x} + .4\,\lambda_2 e^{-\lambda_2 x} + .4\,\lambda_3 e^{-\lambda_3 x}}$$

Where → $\boxed{\lambda_1 = 1/7 \qquad \lambda_2 = 1/15 \qquad \lambda_3 = 1/10}$

<u>10 am</u> $E[X \mid Y = 10\text{ am}] = \int_0^\infty x\, c_1 \lambda_1 e^{-\lambda_1 x}\,dx$

$$= c_1 E[\text{Exp}(\lambda_1)]$$

$$= c_1 \frac{1}{\lambda} = \frac{.27}{3}$$

$$= .4\bar{6} \text{ mins}$$

<u>1pm</u> $E[X \mid Y = 1\text{pm}]$

$$= c_2 E[\text{Exp}(\lambda_2)]$$

$$= \frac{.4}{2}[15] = 3 \text{ mins}$$

$$E[X \mid Y - 7pm] = c_3 E[\text{Exp}(\tfrac{1}{10})]$$
$$= 10\, c_3$$
$$= 10\left(\frac{.4}{6}\right) = \frac{2}{3} \text{ min}$$

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(7) Poisson Point Process $\lambda = 4 \frac{\text{meteors}}{\text{hr}}$

(a) Let $N_1 = \#$ of arrivals (meteors) between 11 & 12

$N_1 \sim \text{Poisson}(K)$, $1 \cdot \lambda = 4$

$$P(N_1 \geq 3)$$
$$= 1 - P(N_1 = 0 \text{ or } 1 \text{ or } 2)$$
$$= 1 - \left(e^{-4} + 4e^{-4} + \frac{4^2}{2} e^{-4}\right)$$
$$= .7618$$

(b) $N_1 = \#$ of meteors in 1st hr

$N_2 = \#$ of meteors b/w 12 & 3am

$N_2 \sim \text{Poisson}(K)$, $(3am - 0hr) \cdot 4 = 12$

$$P(N_1 = 0 \text{ AND } N_2 \geq \cancel{5}) = P(N_1 = 0) P(N_2 \geq \cancel{5})$$

v, 10 $P(N_2 \geq 10) = .75$ v, 10

☆ N.B.

In v1 of Practice Asks for ≥ 10 meteors $\rightarrow P(N_2 \geq 10) = .75$

In v2 of " " Asks for ≥ 5 $\rightarrow P(N_2 \geq 5) = .992$

In v1 (b) $= e^{-4} \cdot .75 = .0137$

In v2 (b) $= e^{-4} .992 = .018$

(c) N_3 = # Meteors b/w 11 & 3 am $\sim$ Poisson$_{4 \cdot 4 = 16}$

$$P(N_1 = 0 \mid N_3 = 13) = \frac{P(N_1 = 0 \text{ AND } N_3 = 13)}{P(N_3 = 13)}$$

$$\frac{\lambda_2^{13}/13!\, e^{-\lambda_2}}{\lambda_3^{13}/13!\, e^{-\lambda_3}} \; \propto \; = \frac{P(N_2 = 13)\boxed{P(N_1 = 0)}}{P(N_3 = 13)}$$

$\lambda_2 = 3 \cdot 4 = 12$

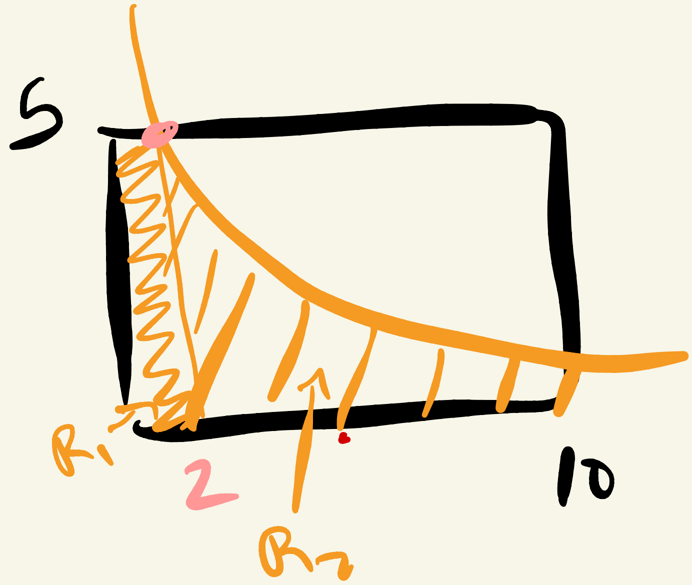
$\lambda_3 = 4 \cdot 4 = 16$

$$\frac{12^{13}\, e^{-12}}{16^{13}\, e^{-16}} = \boxed{\left(\frac{12}{16}\right)^{13} e^{4} \cancel{e^{-4}} = \cancel{96.\%}}$$

$\left(\frac{12}{16}\right)^{13} = .0238$

2.3%

8(d)



$xy = \cancel{10}$ 20

$y = \frac{10}{x}$ $\frac{20}{x}$

$y=5 \left\{ y = \frac{10}{x} \right. \Rightarrow 5 = \frac{10}{x} \Rightarrow x = 2$

Rel Area of $R_1 = \frac{2 \cdot 5}{50} = \frac{1}{5} = .2$

$$\int_2^{10} \frac{10}{x}\,dx = 10 \cdot \ln x \Big|_2^{10} = 10 \cdot \ln\left(\frac{10}{2}\right) = 10 \cdot \ln(5)$$

Rel Area of $R_2 = \frac{10 \cdot \ln(5)}{50} = .32$

$= .54$ Total

$\rightsquigarrow$ Err...

Area of Rectangle