AMAT 362—Work Sheet 14

Dr. Justin M. Curry

Due: March 28th, 2022. Worth 14 points.

Name: _____

Most of these questions are drawn from Grinstead and Snell's Introduction to Probability:
`https://math.dartmouth.edu/~prob/prob/prob.pdf`

1. *Not Graded. We'll do this in class.* Let S_{100} be the number of heads that turn up in 100 tosses of a fair coin. Use the Central Limit Theorem to estimate

    (a) $P(S_{100} \leq 45)$

    (b) $P(45 < S_{100} < 55)$

    (c) $P(S_{100} > 63)$

    (d) $P(S_{100} < 57)$

2. (2 points) A fair coin is flipped 400 times. Determine the number x such that the probability that the number of heads is between $200 - x$ and $200 + x$ is approximately .80.

3. (2 points) A noodle machine in Spumoni's spaghetti factory makes about 5 percent defective noodles even when properly adjusted. The noodles are then packed in crates containing 1900 noodles each. A crate is examined and found to contain 115 defective noodles. What is the approximate probability of finding at least this many defective noodles if the machine is properly adjusted?

4. (4 points) A restaurant feeds 400 customers per day. On the average 20 percent of the customers order apple pie.

  (a) (2 points) Give a range (called a 95 percent confidence interval) for the number of pieces of apple pie ordered on a given day such that you can be 95 percent sure that the actual number will fall in this range.

  (b) (2 points) How many customers must the restaurant have, on the average, to be at least 95 percent sure that the number of customers ordering pie on that day falls in the 19 to 21 percent range?

5. (2 points) Once upon a time, there were two railway trains competing for the passenger traffic of 1000 people leaving from Chicago at the same hour and going to Los Angeles. Assume that passengers are equally likely to choose each train. How many seats must a train have to assure a probability of .99 or better of having a seat for each passenger?

6. (2 points) A die is rolled 24 times. Use the Central Limit Theorem to estimate the probability that

   (a) (1 point) the sum is greater than 84.

   (b) (1 point) the sum is equal to 84.

7. (2 points) A random walker starts at 0 on the x-axis and at each time unit moves 1 step to the right or 1 step to the left with probability 1/2. Estimate the probability that, after 100 steps, the walker is more than 10 steps from the starting position.

8. (4 points) A piece of rope is made up of 100 strands. Assume that the breaking strength of the rope is the sum of the breaking strengths of the individual strands. Assume further that this sum may be considered to be the sum of an independent trials process with 100 experiments each having expected value of 10 pounds and standard deviation 1. Find the approximate probability that the rope will support a weight

   (a) (2 points) of 1000 pounds.

   (b) (2 points) of 970 pounds.

9. (4 points) This is a question where we compare bounds produced by Chebyshev's inequality and the central limit theorem. You may find the following information useful: for the uniform distribution on $\{1, 2, \ldots, n\}$ the first and second moments are $(n+1)/2$ and $(n+1)(2n+1)/6$.

(a) (2 points) Let A_n be the average face value after rolling my 12-sided die n times. Using Chebyshev's inequality, how large does n need to be to make $Pr(|A_n - \mu| \geq 1) \leq .05$?

(b) (2 points) Repeat Part A, but by using the central limit theorem.