

AMAT 362—Work Sheet 07

Dr. Justin M. Curry

Due: February 21st, 2022. Worth 16 points.

Name: _____

1. (5 points) An urn contains 4 red balls and 3 green balls.
 - (a) (2 points) Nine draws are made with replacement. Let X be the number of times a green ball appears. Identify by name the probability distribution of X . Find the probabilities $P(X \geq 1)$ and $P(X \leq 5)$.
 - (b) (2 points) Draws with replacement are made until the first green ball appears. Let N be the number of draws that are needed. Identify by name the probability distribution of N . Find the probability $P(N \leq 5)$.
 - (c) (1 point) Compare $P(X \geq 1)$ and $P(N \leq 9)$. Is there a reason why these should be the same?
2. (2 points) The population of a small town is 500. 20% of the population has red hair. One morning I go to a diner and see 15 customers are there.
 - (a) (1 point) What is the probability that 10 of the 15 customers have red hair?

(b) (1 point) What is the probability that at most 2 of the 15 customers has red hair?

3. (2 points) An urn contains 4 red balls and 3 green balls. Two balls are sampled at random.

(a) (1 point) Let Z denote the number of green balls in the sample when the draws are done without replacement. Give the possible value of Z and its probability mass function (PMF).

(b) (1 point) Let W denote the number of green balls in the sample when the draws are done with replacement. Give the possible values and the PMF of W .

4. (5 points) Choose one of the words in the following sentence uniformly at random and then choose one of the letters of that word, again uniformly at random:

SOME DOGS ARE BROWN

(a) (1 point) Find the probability that the chosen letter is R.

(b) (1 point) Let X denote the length of the chosen word. Determine the PMF of X .

(c) (1 point) For each possible value k of X determine the conditional probability $P(X = k \mid X > 3)$.

(d) (1 point) Determine the conditional probability $P(\text{the chosen letter is R} \mid X > 3)$.

(e) (1 point) Given that the chosen letter is R, what is the probability that the chosen word was BROWN?

5. (2 points) *This is a challenge question and will not be on the exam.* You are given a fair die. You must decide ahead of time how many times to roll. If you roll exactly 2 sixes, you win a new Tesla. How many rolls should you take to maximize your chances and what are the chances of winning? Note: there are two equally good choices for the best number of rolls.