MATH 362—Work Sheet 21

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Saturday May 1, 2021

Name: _____

1. (3 points) Suppose X and Y are independent and uniformly distributed on $[0, 1]$.
   (a) (1 point) Find the PDF of $X + Y$

   (b) (1 point) What's the probability $P(X + Y \leq 1)$?

   (c) (1 point) Find the PDF of $X - Y$.

2. (3 points) Suppose again that X and Y are independent and uniformly distributed on $[0, 1]$.
   (a) (1 point) Find $P(X^2 + Y^2 \leq 1)$

   (b) (1 point) Find $P(X^2 + Y^2 \leq 1) \mid X + Y \geq 1)$

   (c) (1 point) Find $P(Y \leq X^2)$.

3. (2 points) Two people try to meet at a certain place between 5pm and 5:30pm. Suppose each person arrives independently and uniformly at random in this window in time and then waits 5 minutes to see if the other person is there. What's the probability of them meeting?

4. (7 points) Suppose (X, Y) is uniformly distributed over the region $\{(x, y) \mid 0 < x < y < 1\}$.

   (a) (2 points) Compute the PDF of $Y - X$.

   (b) (1 point) Are X and Y independent?

   (c) (1 point) Find $E(Y - X)$.

   (d) (3 points) Find the *covariance* of X and Y.