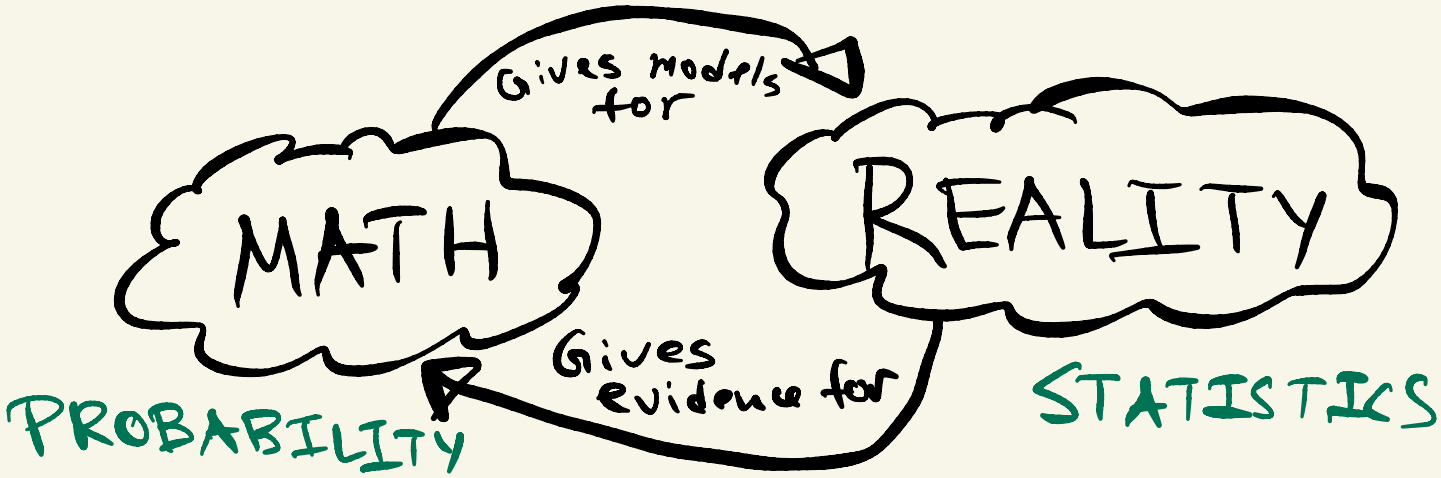
AMAT 362 – Prob. for Statistics

LEC 1

Def Probability is the mathematical study of models for uncertainty.

Def Statistics is the study of data that aims to infer the underlying probabilistic models.



Consider the following

1) What's the probability of a 4 of a kind?
2) How much money do you need to make to be in the top 1%? For New Yorkers?
3) What's the probability of life on Mars?
4) How many people should you date before settling on "the one"?

1) 4 of a kind

↳ Be more concrete! ← STRATEGY

One way: $\underline{K}\ \underline{K}\ \underline{K}\ \underline{K}\ \underline{?}$

$$\left(\frac{4}{52}\right)\left(\frac{3}{51}\right)\left(\frac{2}{50}\right)\left(\frac{1}{49}\right)\left(\frac{48}{48}\right) = \frac{4!\,48!}{52!}$$

SAME

Another way: $\underline{K}\ \underline{K}\ \underline{?}\ \underline{K}\ \underline{K}$

$$\left(\frac{4}{52}\right)\left(\frac{3}{51}\right)\left(\frac{48}{50}\right)\left(\frac{2}{49}\right)\left(\frac{1}{48}\right) = \frac{4!\,48!}{52!}$$

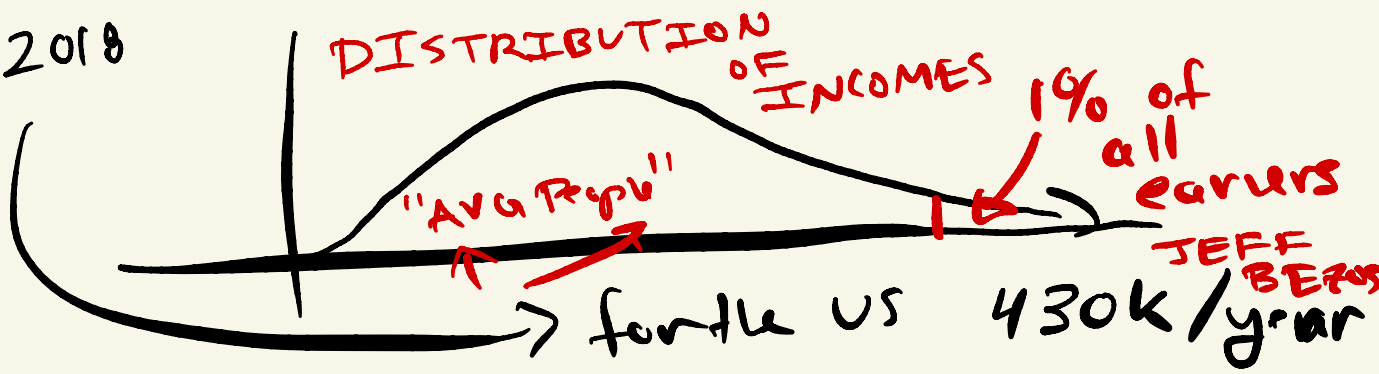
Total 5 possible ways $\Rightarrow$ Prob of 4 Kings out of 5 cards $\quad 5 \cdot \frac{4!\,48!}{52!}$

13 different face cards

A K Q J ... 2

$\Rightarrow \frac{13 \cdot 5 \cdot 4!\,48!}{52!}$

2)



3) Life on Mars

Differing Schools of thought on what "Probability" means

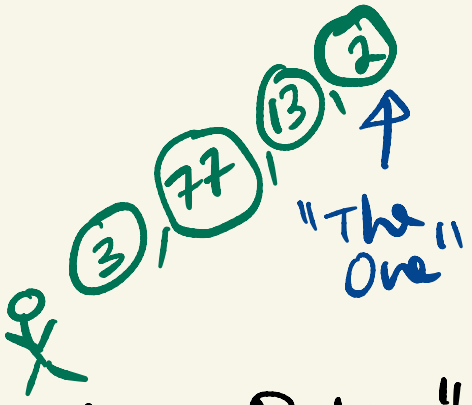
↳ Frequentist: Run an experiment a bunch of times and count "successes" e.g. flip a coin & count heads

↳ Bayesian:

Probability represents degree of belief in a statement "What's the probability that it's going to rain today?"

4) How many to date?

12



↳ "Candidate Problem" / "Secretary Problem"

Strategy: Interview 12 people & reject them all.

Random Permutations

The first person better than the "trial 12" you should accept. → 25% chance of ☺

Random Variables

FUNDAMENTAL MODEL OF PROBABILITY

SAMPLE // SPACE

1) DRAW A CARD ⟶ one of 52 possibilities

"size"

Ω = Deck of cards

$|\Omega| = 52$

2) Ask a person their income ⟶ $\Omega = [0, 2\,Trillion)$

continuously varying

3) Go to Mars look for life ⟶ {Yes!, NO ∴}

4) Marry 1st person you date ⟶ $\Omega = \{1, \ldots, N\}$

rank order of best fits for you

In Probabibility you always need to clearly identify the space of possibilities Ω

R Set consider subsets!

Notation

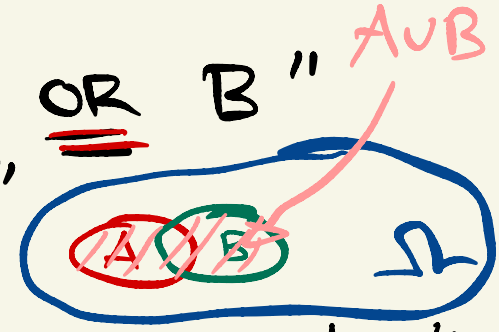
- $x \in \Omega$ "x is an element of Ω"
- $A \subseteq B$ "A is a subset of B"
  = "every element of A is an element of B"

Probability Interp.

- A = "Event of some outcome in A occurring"
- $A^c = \Omega - A$ = "set of elements NOT in A"

(Probability) = Event where A does NOT occur

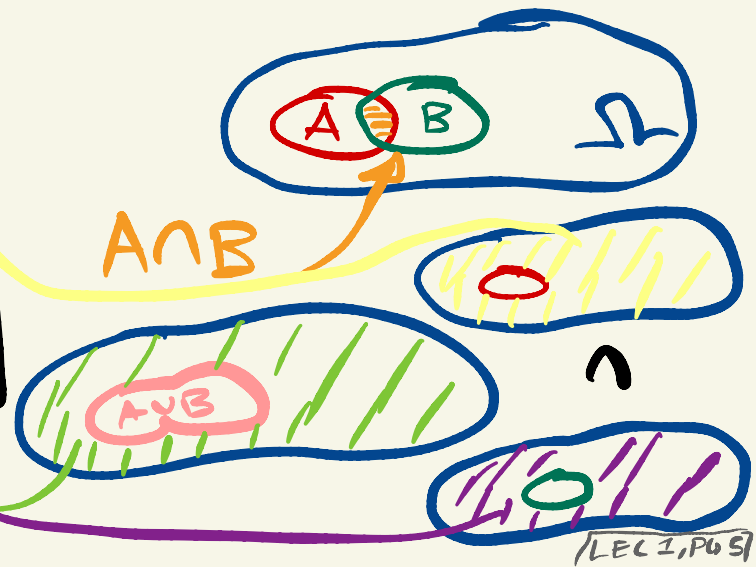
- $A \cup B$ = "A union B"
  = "elements of A OR B"
  (Prob) = "A or B occurs"

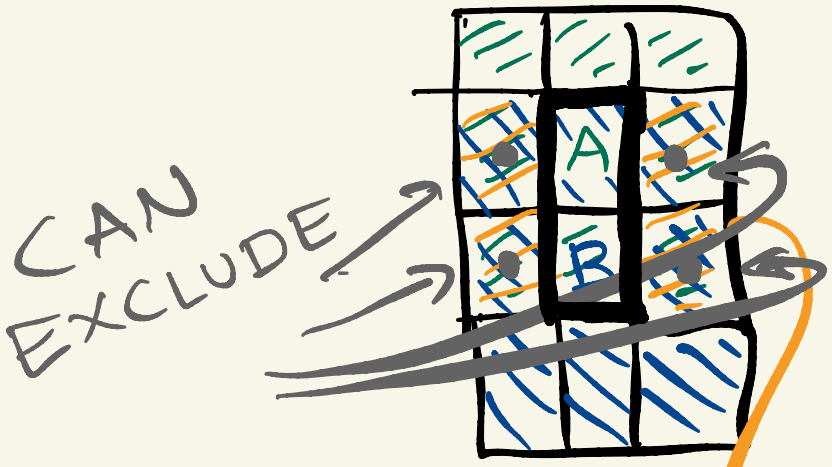


$A \cap B$ = "A intersect B"
= "elements that are both in A & B"
(Prob) = "A and B occurs"

De Morgan's Laws

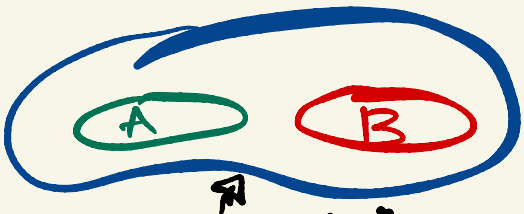
$$(A \cup B)^c = A^c \cap B^c$$





★ could be in A OR B $= (A \cup B)$

NOT a ★ in $A^c \cap B^c$



- $A \cap B = \emptyset$ ← Empty set = set w/ no elements

"A and B are disjoint"

- $A_1 \cap A_2 \cap \cdots \cap A_k = \emptyset$ "$A_1, \ldots, A_k$ are pairwise disjoint" "mutually exclusive"

Cartesian Product

Useful where any element of Ω can be specified w/ two features

e.g. Ω = Deck of cards = Rank × Suit

$= \{2, 3, \ldots, 10, J, Q, K\} \times \{\heartsuit, \diamondsuit, \clubsuit, \spadesuit\}$

$\Omega = R \times S$

$\Rightarrow |\Omega| = |R||S| = 13 \cdot 4 = 52$ ✓

$\cup$ and $\cap$ as operations defined using two sets

$(A, B) \xrightarrow{\cup} A \cup B$

$(A, B) \xrightarrow{\cap} A \cap B$

Symmetry $A \cup B = B \cup A$

Associative $(A \cup B) \cup C = A \cup (B \cup C)$

Distributive $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Draw pictures! ↗