AMAT 362 Lecture 1 Worksheet

Dr. Justin M. Curry

Due: February 6, 2021

Name: _____

Questions marked with a ⋆ have subjective answers and will only be graded for completeness.

1. (2 points) What's the probability of a 4 of a kind, assuming you're dealt a 5 card hand, uniformly at random, from a standard 52 card deck?

$$13 \cdot \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} \cdot \frac{1}{49}$$

2. (1 point) ⋆ How much money do you need to make to be in the top 1% of income earners in the United States? What about the top 1% of New Yorkers? Do you think there is such a thing as a "fair" distribution of incomes? What does that look like?

3. (1 point) ⋆ What's the probability of life on Mars? What does this question illustrate about the different meanings of the term "probability"?

4. (1 point) ⋆ How many people should you date before settling down on "the one," assuming that's something you want to do?

5. (6 points) Translate each of the following symbolic expressions into English statements:
   (a) (1 point) $x \in A$

   x is in A

   (b) (1 point) $A \subseteq B$

   A is a subset of B

(c) (1 point) A^c The set of elements not in A

(d) (1 point) $R \times S$ The set of pairs from R & S.

(e) (1 point) $|A|$ The size of A

(f) (1 point) How do you interpret $\varnothing$ and Ω in probability?

$\varnothing$ - "null event" Ω - all events

6. (2 points) State De Morgan's Laws. What does this have to do with the star battle problem? `https://krazydad.com/tablet/starbattle/`

$(A \cup B)^c = A^c \cap B^c$ De Morgan's laws help you rule out star placements

7. (2 points) Consider the set Ω, the union operation $A \cup B$ of subsets of Ω and the intersection operation $A \cap B$ on Ω. What does it mean to say that $\cup$ and $\cap$ are associative and symmetric? How does the union operation "distribute over" the intersection operation?

Symmetry: $A \cap B = B \cap A$, $A \cup B = B \cup A$

Associative: $A \cup (B \cup C) = (A \cup B) \cup C$, $A \cap (B \cap C) = (A \cap B) \cap C$

Distributes: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

8. (5 points) Suppose we have a deck of 20 cards, 10 are red and 10 are blue. Each of the blue cards has a unique number between 1 and 10. Each of the red cards has a unique number also between 1 and 10.

(a) (1 point) Describe the sample space Ω as a Cartesian product.

$\{red, blue\} \times \{1, 2, \ldots, 10\}$

(b) (1 point) Consider the following events:

- Let A be the event that a card drawn has an even number on it.
- Let B be the event that a blue card is drawn.
- Let C be the event that a card with a number (strictly) less than 5 is drawn.

What are the sizes of A, B, and C?

$|A| = 10$

$|B| = 10$

$|C| = 8$

(c) (2 points) Describe the events $A \cup B \cup C$ and $A^c \cap B^c \cap C^c$.

$A \cup B \cup C$ contains all blue cards, even red cards and red 1, 3

$A^c \cup B^c \cup C^c$ contains red 5, 6, 7

(d) (1 point) What are the number of outcomes in each of the events in part (c)?

$|A \cup B \cup C| = 7$

$|A^c \cup B^c \cup C^c) = 3$