GIRR vega. (option on rates)

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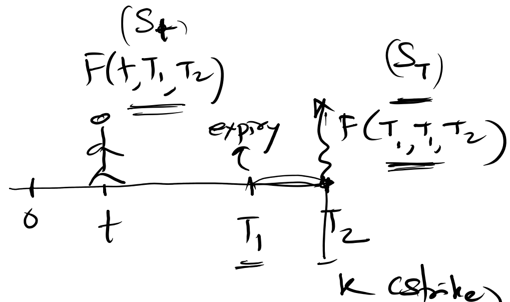
→ caps/caplets

⇒ swaptions

risk factors are 2 dimensional
- → option maturity
- → underlier maturity
  - ↳ forward rate (caplets/caps)
  - ↳ swap rate (swaptions)

① Caplet



$\tau = T_2 - T_1$

$dF = \sigma F dW$

$$V^{Bl76}_{caplet} = N \tau P(t, T_2) \left(F(t, T_1, T_2) N(d_1) - K N(d_2) \right)$$

$$d_1 = \frac{\ln(F_t/K) + \frac{1}{2}\sigma^2 (T_1 - t)}{\sigma\sqrt{T_1 - t}}, \quad d_2 = d_1 - \sigma\sqrt{T_1 - t}$$

risk factor → expiry $(T_1 - t)$ | tenor $T_2 - T_1 = \tau$

$$\delta = N \tau P(t, T_2) \cdot N(d_1), \quad \frac{\partial V}{\partial F}$$

$F \phi(d_1) = K \phi(d_2)$

$S_0 \phi(d_1) = K e^{-r(T-t)} \phi(d_2)$

$$\nu = \left(F_t \phi(d_1) \sqrt{T-t} \right) N \tau P(t, T_2)$$

$S_t \phi(d_1) \cdot \sqrt{T-t}$

Cap.

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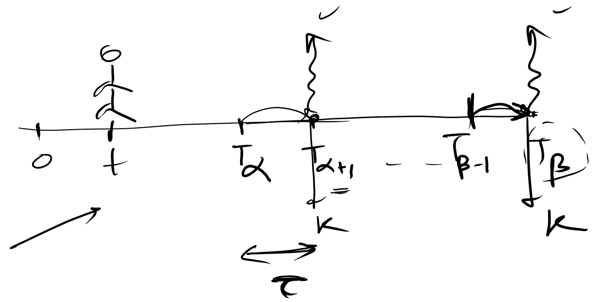
$$\text{Cap} = \sum \text{Caplets}$$



$$V_{cap} = N\tau \sum_{i=\alpha+1}^{\beta} P(t, T_i)\, Bl\left(K, F(t, T_{i-1}, T_i), \sigma\sqrt{T_{i-1}-t}\right)$$

$$d_1 = \frac{\ln\left(\frac{F(t, T_{i-1}, T_i)}{K}\right) + \frac{1}{2}\sigma^2 (T_{i-1}-t)}{\sigma\sqrt{T_{i-1}-t}}$$

RF	expiry	tenor
1	$(T_\alpha - t)$	τ
2	$(T_{\alpha+1} - t)$	τ
—	—	—
n	$(T_{\beta-1} - t)$	τ

Swaption.

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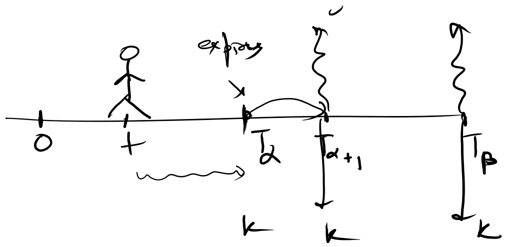


RF	expiry	tenor
1	$(T_\alpha - t)$	$(T_\beta - T_\alpha)$

$$S_{\alpha\beta}(t) = \frac{P(t,T_\alpha) - P(t,T_\beta)}{\tau \cdot \sum_{i=\alpha+1}^{\beta} P(t,T_i)}$$

$$dS_{\alpha\beta}(t) = \sigma S_{\alpha\beta}(t)\, dW$$

$$V_{swaption} = N\tau \underbrace{\left(\sum_{i=\alpha+1}^{\beta} P(t,T_i)\right)}_{A \text{ (annuity)}} \left[Bl\left(K, S_{\alpha\beta}(t), \sigma\sqrt{T_\alpha - t}\right)\right]$$

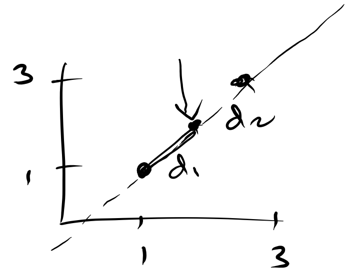
$$\delta = N\tau A \cdot N(d_1)$$

$$\nu = N\tau A\, S_{\alpha\beta}(t)\, \phi(d_1) \sqrt{T_\alpha - t}$$

$$(1.5,\ 1.5) \leftarrow S$$

$$\begin{array}{ccc} & (1.5,\ 1.5) & \\ \swarrow d_1 & & \searrow d_2 \\ (1,\ 1) & & (3,\ 3) \end{array}$$

$$S \cdot \frac{d_2}{d_1+d_2} \qquad S \cdot \left(\frac{d_1}{d_1+d_2}\right)$$



$$d_1 = \sqrt{(1.5-1)^2 + (1.5-1)^2}$$