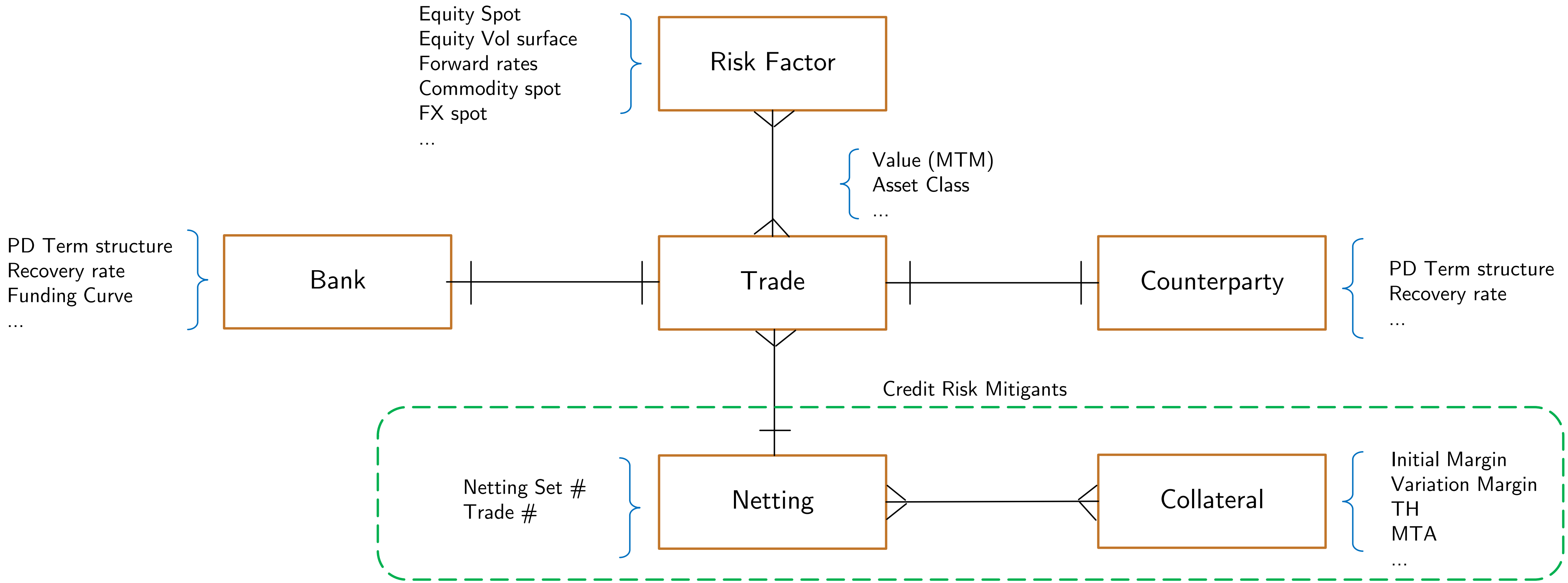
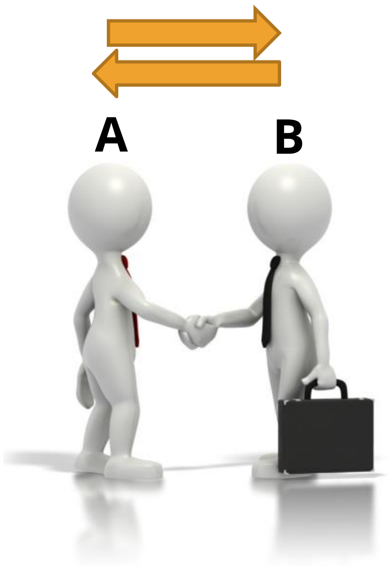
Counter Party Credit Risk

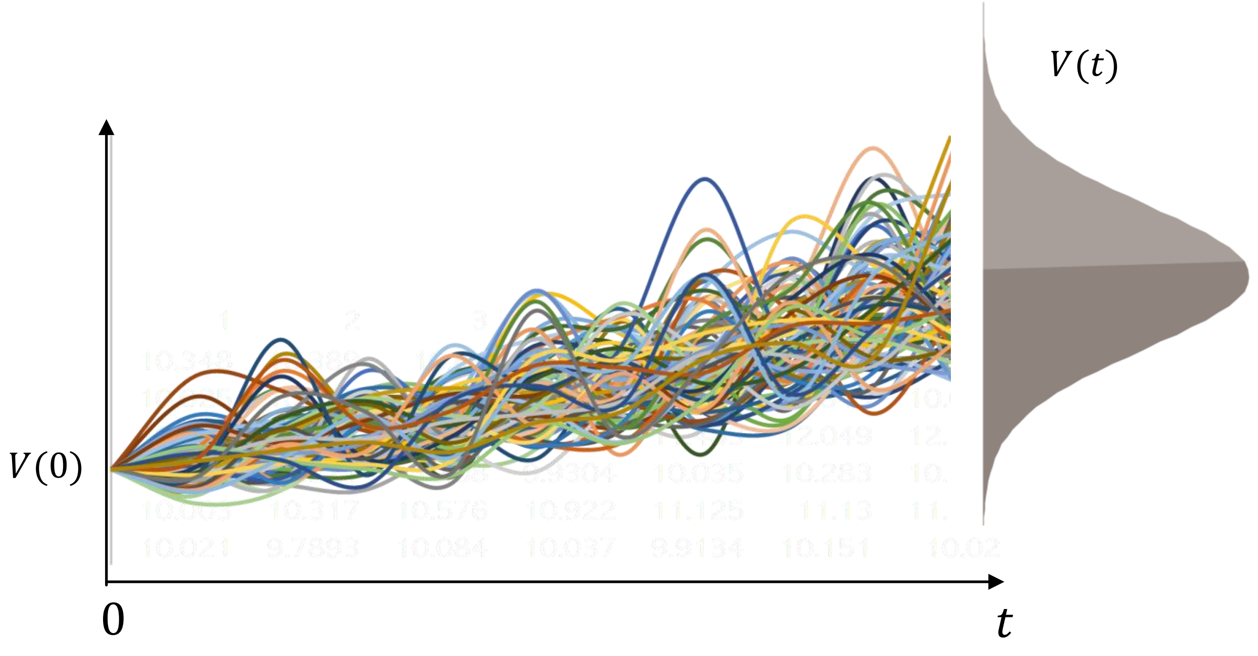
Derivative Portfolio - Entity Relationship



Exposure Metrics

Bilateral Exposure





V_0 = MTM of the trade or CE (current exposure)

V_t = Future value of the trade at time t (random)

$EFV(t)$ = (Expected Future Value) $= \mathbb{E}[V_t]$

Positive Exposure $= {V_t}^+ = \max\{V_t, 0\}$

Negative Exposure $= {V_t}^- = \min\{V_t, 0\}$

$EE(t)$= (Expected Exposure) $= \mathbb{E}[{V_t}^+]$

$ENE(t)$= (Expected Negative Exposure) $= \mathbb{E}[{V_t}^-]$

$PFE(t)$ = (Potential Future Exposure) $= q_\alpha(V_t)$

EPE= (Expected Positive Exposure) $= \underset{t \in (0,T)}{\text{Avg}} \mathbb{E}[{V_t}^+]$

$EEE(t)$= (Effective Expected Exposure) $= Non\ decreasing\ EE(t)$

$EEPE$= (Effective Expected Positive Exposure) $= \underset{t \in (0,T)}{\text{Avg}} EEE(t)$

Equity Class

Modelling Exposure Metrics

Generate simulated risk factors $S(t)$ for each time $t \in [0, T]$

↓

Generate simulated exposures $V(t)$ by plugging simulated $S(t)$ in the valuation formula for each time $t \in [0, T]$

↓

Calculate Exposure profiles from simulated $V(t)$

stochastics.py

- Map risk factor to a stochastic process
- calibrate to market data
- Perform Monte Carlo Simulation (steps, paths, discretization method)
- Calculate and plot density at t

Equity.py

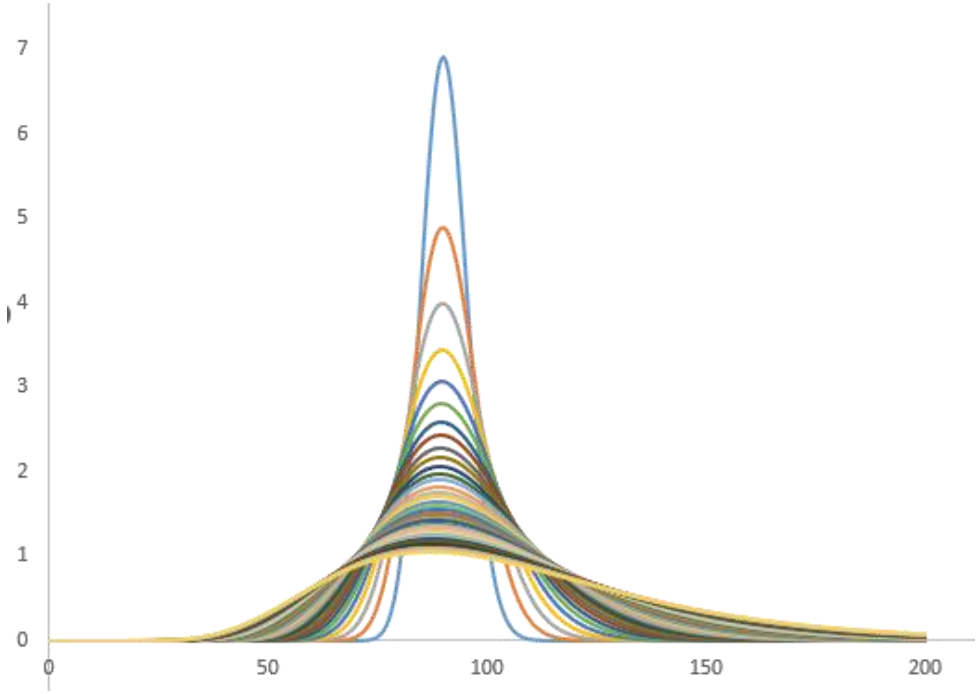
- Map risk factor to an Equity Derivative
- Generate V(t) using analytical / semi-analytical formula (don't use Monte Carlo within Monte Carlo)
- Calculate and plot Exposure metric

Equity Models - GBM

SDE $$dS(t) = rS(t)dt + \sigma S(t)dW$$ Risk neutral dynamics

Solution $$S(T) = S_t \exp\left(\left(r - \frac{1}{2}\sigma^2\right)(T-t) + \sigma Z\sqrt{T-t}\right)$$

$$\mathbb{E}[S(T)] = S_t \exp\big(r(T-t)\big)$$



Density of $S(t)$ is lognormal

Time Discretization for Monte Carlo

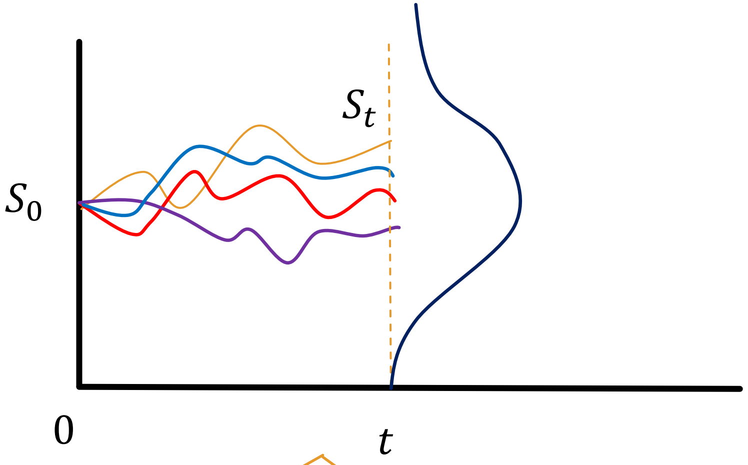
Explicit $$S(t+\delta t) = S(t)\exp\left(\left(r - \frac{1}{2}\sigma^2\right)(\delta t) + \sigma Z\sqrt{\delta t}\right)$$

Euler Maruyama $$S(t+\delta t) = S(t)\big(1 + r\delta t + \sigma Z\sqrt{\delta t}\big)$$

Milstein $$S(t+\delta t) = S_t\left(1 + r\delta t + \frac{1}{2}\sigma^2(z^2-1)\delta t + \sigma Z\sqrt{\delta t}\right)$$

Simulate Exposure for Equity Products

1. Simulate equity dynamics

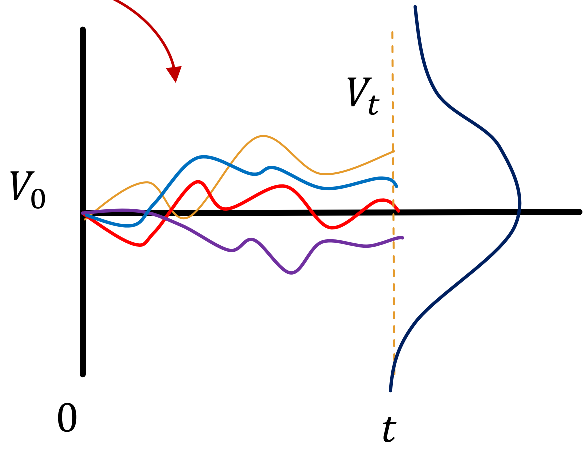


2. Pass simulated risk factors into the valuation functions

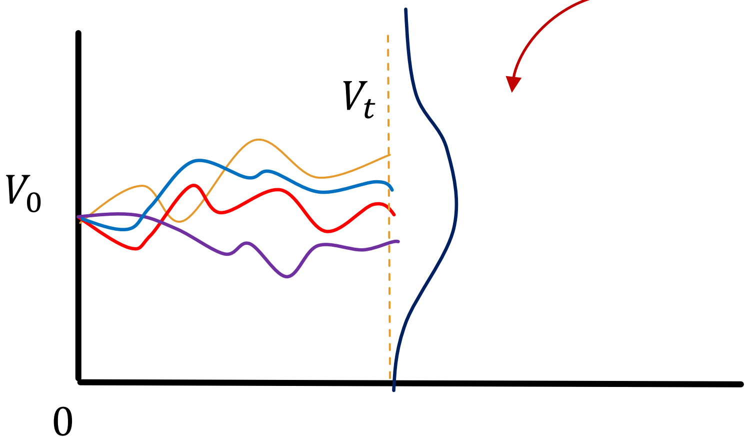
$\boxed{V_t^{Forward} = S_t - Ke^{-r(T-t)}}$

$\boxed{V_t^{Option} = \omega S_t N(\omega d_1) - \omega K e^{-r(T-t)} N(\omega d_2)}$

$\omega = 1$ for call and -1 for put



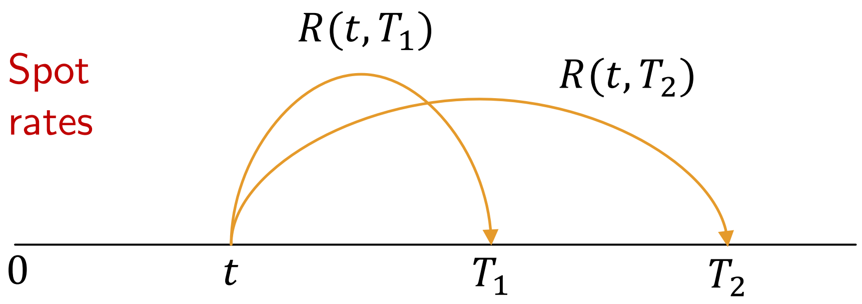
3. Obtain the Exposure distribution $V(t)$



Rate Class

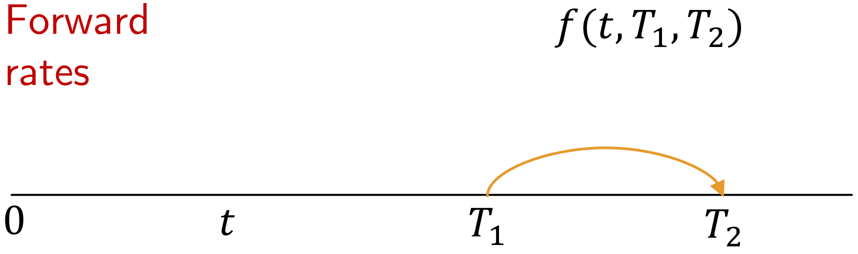
Rates – Spot, forward, ZCB

Spot rates



(rate agreed date (t) = period start date (T_1))

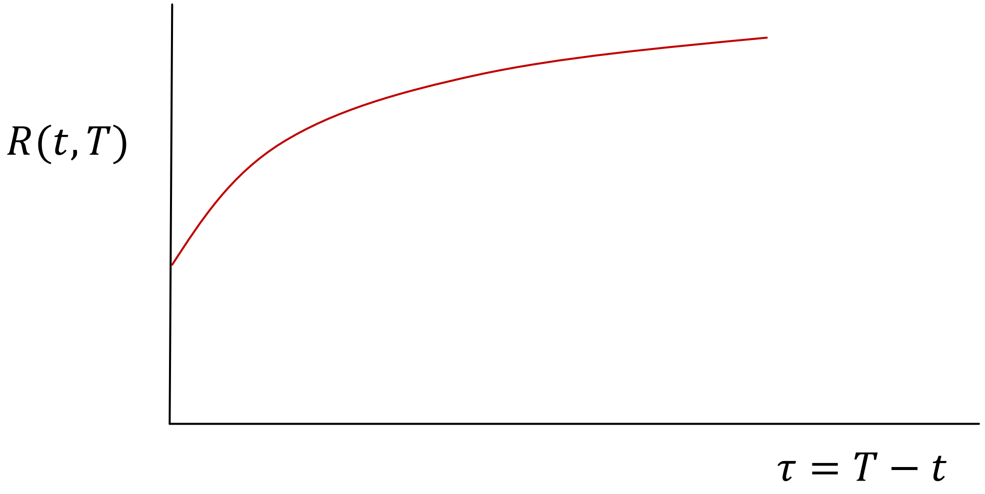
Forward rates



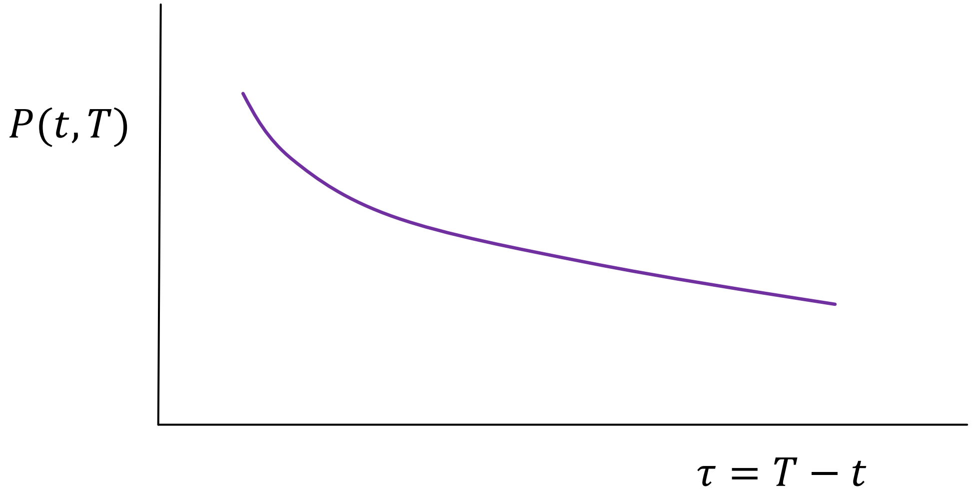
(rate agreed date (t) < period start date (T_1))

- ✓ If t = today, everything is known
- ✓ If t is in future, everything is random and needs to be modelled

Yield curve at (t)



ZCB $\quad P(t, T) = e^{-R(t,T)(T-t)}$



Rates – Modelling choices

Short rate = instantaneous spot rate

$$r(t) \coloneqq f(t,t,t+\epsilon)$$

$0 \qquad t \qquad t+\epsilon \qquad T$

ZCB $\quad P(t,T) = \mathbb{E}\left[\exp\left(-\int_t^T r(s)ds\right)\right]$

Short rate models

- Vasicek, CIR, Ho-Lee, Hull-White

Instantaneous forward rate

$$f(t,T) \coloneqq f(t,T,T+\epsilon)$$

$0 \qquad t \qquad T \qquad T+\epsilon$

ZCB $\quad P(t,T) = e^{-\int_t^T f(t,s)ds}$

Instantaneous forward rate modelling framework

- HJM

market rate $\quad L(t,T)$

$0 \qquad t \qquad T$

ZCB $\quad P(t,T) = e^{-R(t,T)(T-t)}$

Market rate models

- BGM (Black 76 – a special case)

$$r(t) = \lim_{T \to t} f(t,T)$$

Short rate $:= \ r(t)$

$$P(t,T) = \mathbb{E}\left[\exp\left(-\int_t^T r(s)ds\right)\right]$$

Instantaneous forward rate $:= \ f(t,T)$

$$P(t,T) = e^{-\int_t^T f(t,s)ds}$$

$$f(t,T) = -\frac{\partial}{\partial T}\ln P(t,T)$$

$$R(t,T) = \frac{1}{T-t}\int_t^T f(t,s)ds$$

$$f(t,T) = R(t,T) + \frac{\partial R(t,T)}{\partial T}(T-t)$$

$$f(t,T_1,T_2) = \frac{R(t,T_2)(T_2-t) - R(t,T_1)(T_1-t)}{T_2-T_1}$$

Market rate $:= R(t,T)$

$$P(t,T) = e^{-R(t,T)(T-t)}$$

$$R(t,T) = -\frac{1}{T-t}\ln\big(P(t,T)\big)$$

Term Structure Models - Vasicek Model Dynamics

SDE $$dr(t) = k\big(\theta - r(t)\big)dt + \sigma dW$$

Solution $$r(t) = \underbrace{r(s)e^{-k(t-s)} + \theta\big(1 - e^{-k(t-s)}\big)}_{\text{deterministic drift}} + \underbrace{\sigma \int_s^t e^{-k(t-u)} dW_u}_{\text{diffusion}}$$

$r(s)$ $r(t)$

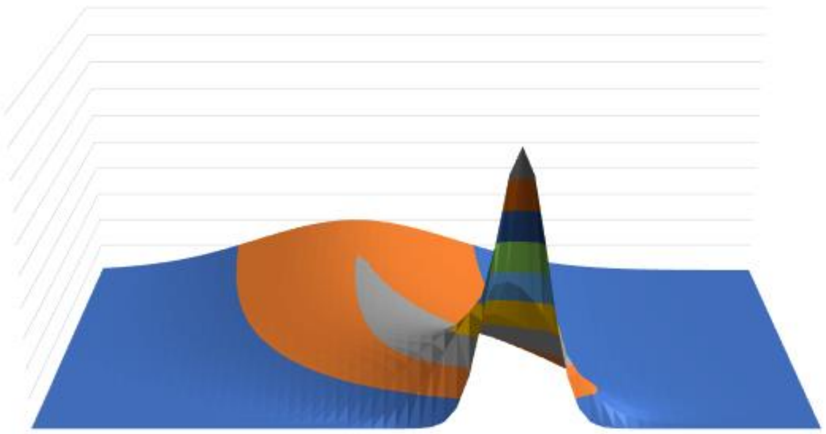
0 s $s+\epsilon$ t $t+\epsilon$

$$\mathbb{E}[r(t)] = r(s)e^{-k(t-s)} + \theta\big(1 - e^{-k(t-s)}\big)$$

$$t \to \infty, \mathbb{E}[r(t)] = \theta$$

$$\mathbb{V}[r(t)] = \left(\frac{\sigma^2}{2k}\right)\big(1 - e^{-2k(t-s)}\big)$$

$$t \to \infty, \mathbb{V}[r(t)] = \frac{\sigma^2}{2k}$$

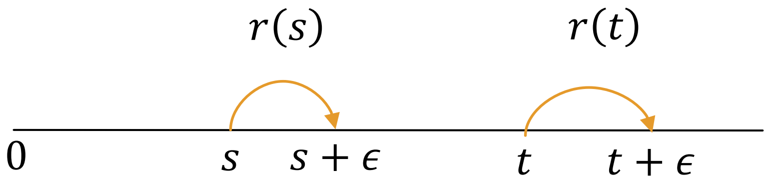


Density of $r(t)$ is normal

- ✓ Equilibrium model
- ✓ Mean Reverting
- ✓ Rates are Gaussian
- ✓ Negative interest rates are allowed

Term Structure Models - CIR Model Dynamics

SDE $$dr(t) = k\big(\theta - r(t)\big)dt + \sigma\sqrt{r}\,dW$$



Solution $$r(t) = r(s)e^{-k(t-s)} + \theta\big(1 - e^{-k(t-s)}\big) + \sigma\int_s^t e^{-k(t-u)}\sqrt{r(u)}\,dW_u$$

$$\mathbb{E}[r(t)] = r(s)e^{-k(t-s)} + \theta\big(1 - e^{-k(t-s)}\big)$$

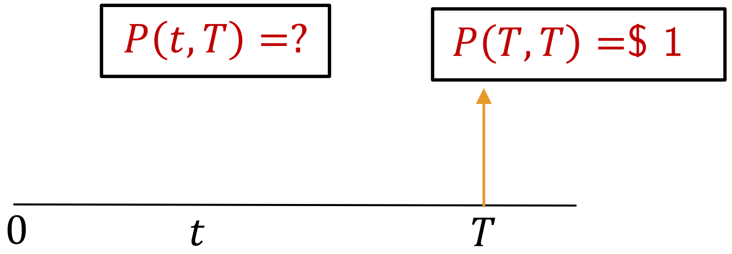
$$t \to \infty\,,\ \mathbb{E}[r(t)] = \theta$$

$$\mathbb{V}[r(t)] = \left(\frac{\sigma^2 r(s)}{k}\right)\big(e^{-k(t-s)} - e^{-2k(t-s)}\big) + \left(\frac{\sigma^2\theta}{2k}\right)\big(1 - e^{-k(t-s)}\big)^2$$

$$t \to \infty\,,\ \mathbb{V}[r(t)] = \left(\frac{\sigma^2\theta}{2k}\right)$$

- ✓ Equilibrium model
- ✓ Mean Reverting
- ✓ Rates are not Gaussian (non-central Chi squared)
- ✓ Negative interest rates are not allowed

Term Structure Models – Bond Price under Vasicek & CIR



$$P(t,T) = A(t,T)e^{-r(t)B(t,T)}$$

(Affine Solution)

- ✓ Analytical (closed form)
- ✓ Value depends only on the short rate at valuation date

Vasicek Model

$$B(t,T) = \frac{1 - e^{-k(T-t)}}{k}$$

$$A(t,T) = \exp\left\{\left(\theta - \frac{\sigma^2}{2k^2}\right)\left(B(t,T) - (T-t)\right) - \frac{\sigma^2}{4k}B(t,T)^2\right\}$$

CIR Model

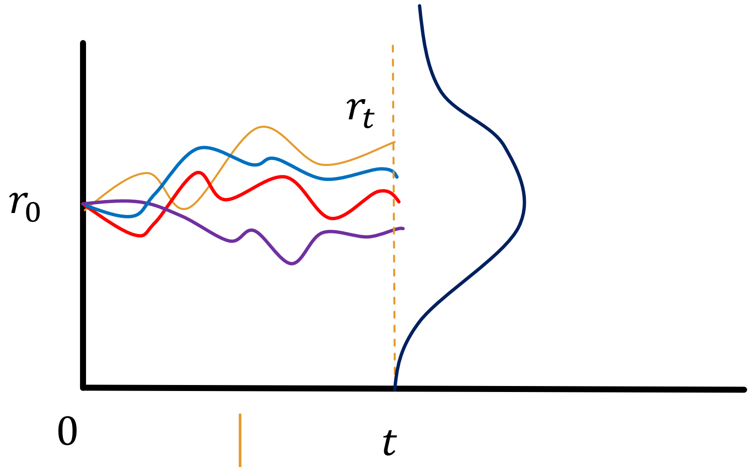
$$\gamma = \sqrt{k^2 + 2\sigma^2}$$

$$B(t,T) = \frac{2\left(e^{\gamma(T-t)} - 1\right)}{2\gamma + (k+\gamma)(e^{\gamma(T-t)} - 1)}$$

$$A(t,T) = \left(\frac{2\gamma e^{\frac{(\gamma+k)(T-t)}{2}}}{2\gamma + (k+\gamma)(e^{\gamma(T-t)} - 1)}\right)$$

Simulate Exposure of rate products

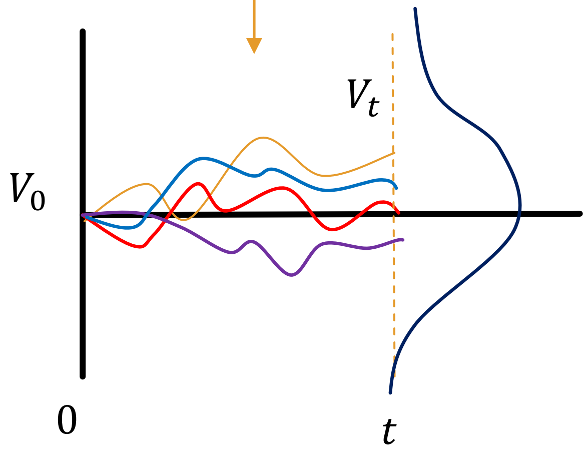
1. Simulate short rate dynamics



2. Pass simulated risk factors into closed form affine solution

$$V(t) = V(t, r(t), T_1, T_2, \dots)$$

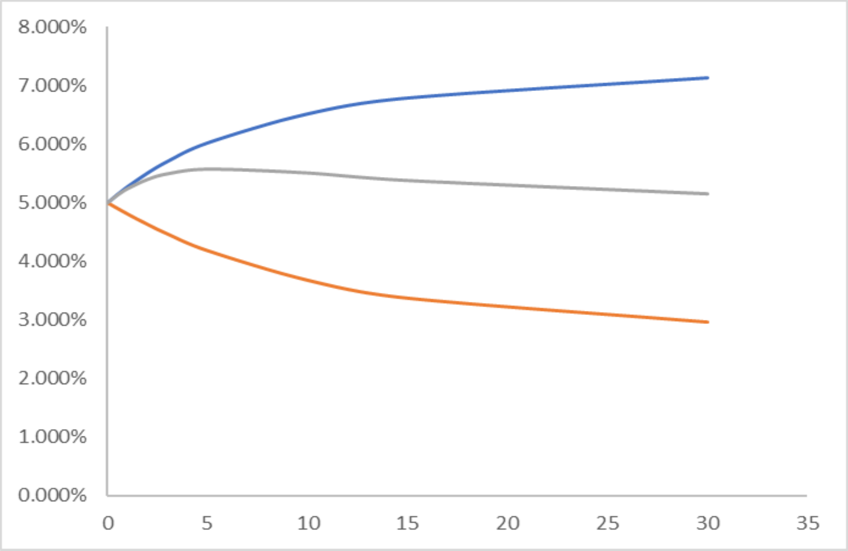
3. Obtain the Exposure distribution $V(t)$



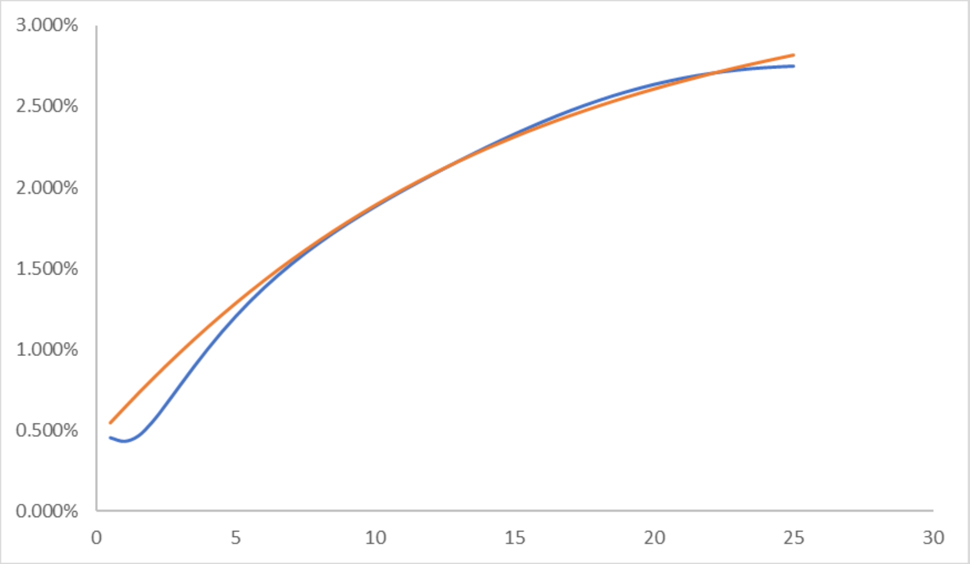
Issue with Equilibrium Models

- Equilibrium models are used to produce theoretical term structures based on economic arguments
- The theoretical term structures generated from Vasicek / CIR model are too simplistic and no where resembles the complex dynamics of term structures observed in practice
- The limited number of parameters are not adequate to fit the term structure
- We therefore resort to no arbitrage models which have one or more time-varying parameters, and these parameters are calibrated from the observed yield curve
- These models therefore match the yield curve by design i.e., model generated ZCB prices match the market observed ZCB prices
- In practice, the yield curves are constructed from liquid instruments such as futures , IRS etc. and the volatility surfaces / cubes of those yields are constructed from caplets / swaption prices
- Next, we will see two popular short rate no arbitrage models (Ho-Lee , Hull-White) which have affine solutions

Simplistic Term structures



Calibration results



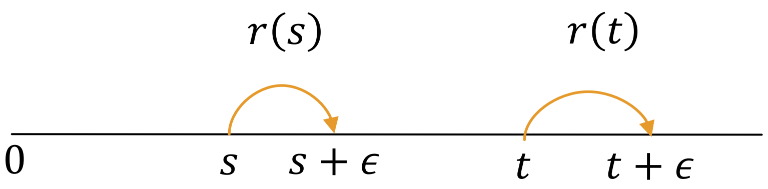
Term Structure Models – Ho Lee Dynamics & Calibration

SDE $$dr(t) = \theta(t)dt + \sigma dW$$

Solution $$r(t) = r(s) + \int_s^t \theta(u)du + \sigma Z\sqrt{t-s}$$

$$\mathbb{E}[r(t)] = r(s) + \int_s^t \theta(u)du$$

$$\mathbb{V}[r(t)] = \sigma^2(t-s)$$

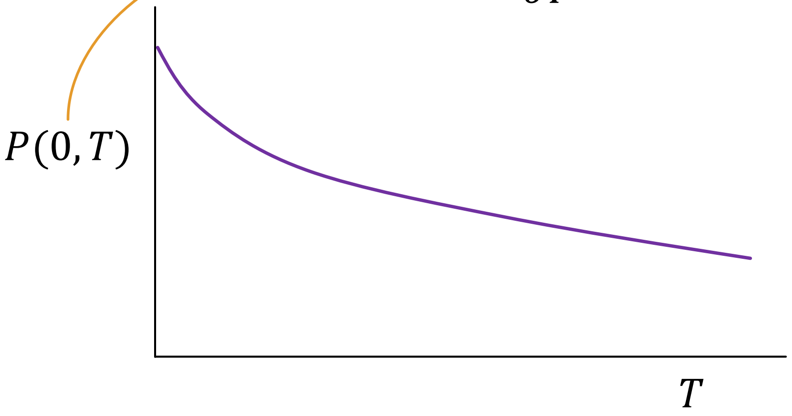


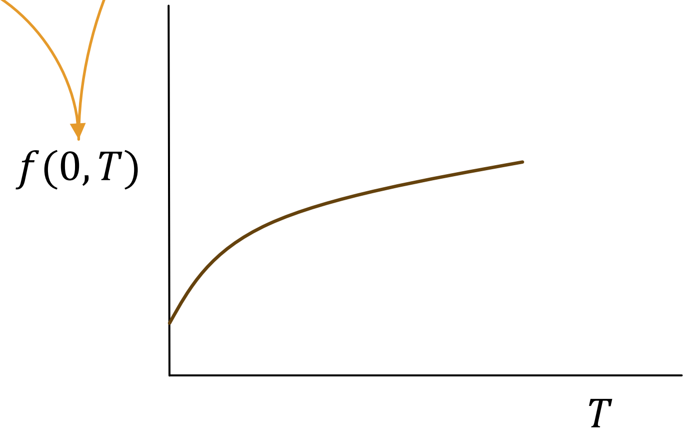
- ✓ No arbitrage model
- ✓ Not mean reverting
- ✓ Rates are Gaussian
- ✓ Negative interest rates are allowed

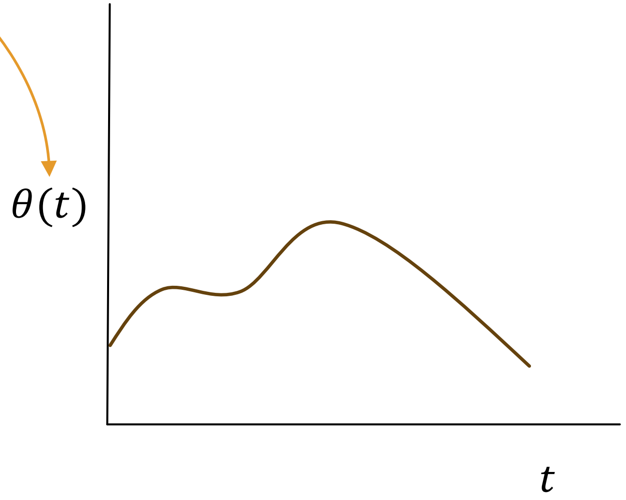
For any valuation date 0

$$f(0,T) = -\frac{\partial}{\partial T}\ln P(0,T)$$

$$\theta(T) = \frac{\partial f(0,T)}{\partial T} + \sigma^2 T$$







Term Structure Models – Bond Price under Ho-Lee

$\boxed{P(t,T) = ?}$ $\boxed{P(T,T) = \$\ 1}$

0 t T

$$P(t,T) = A(t,T)e^{-r(t)B(t,T)}$$ (Affine Solution)

$$B(t,T) = T - t$$

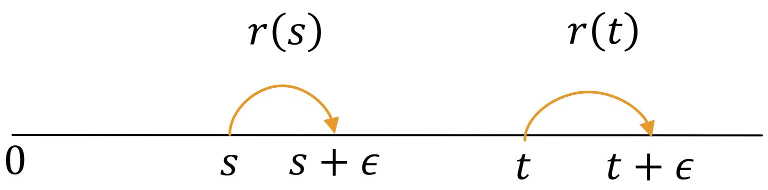
$$A(t,T) = \exp\left\{-\int_t^T \theta(s)(T-s)\,ds + \frac{1}{6}\sigma^2(T-t)^3\right\}$$

SDE $$dr(t) = k\big(\theta(t) - r(t)\big)dt + \sigma dW$$

Solution $$r(t) = r(s)e^{-k(t-s)} + \int_s^t k\theta(u)e^{-k(t-u)}du + \sigma\int_s^t e^{-k(t-u)}dW_u$$

$$\mathbb{E}[r(t)] = r(s)e^{-k(t-s)} + \int_s^t k\theta(u)e^{-k(t-u)}du$$

$$\mathbb{V}[r(t)] = \left(\frac{\sigma^2}{2k}\right)\left(1 - e^{-2k(t-s)}\right)$$

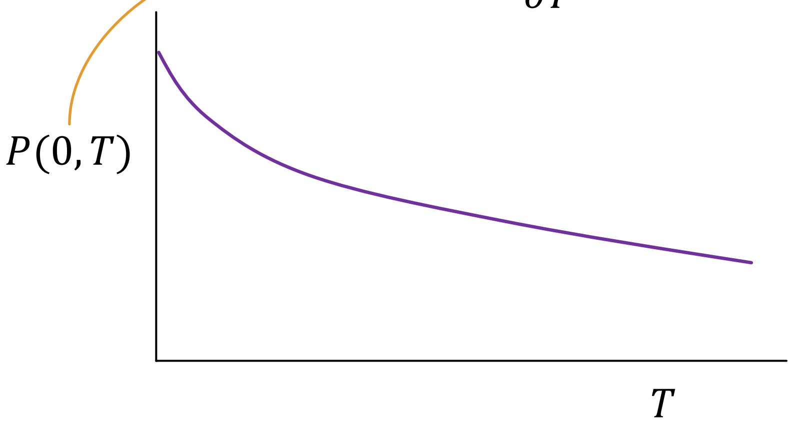


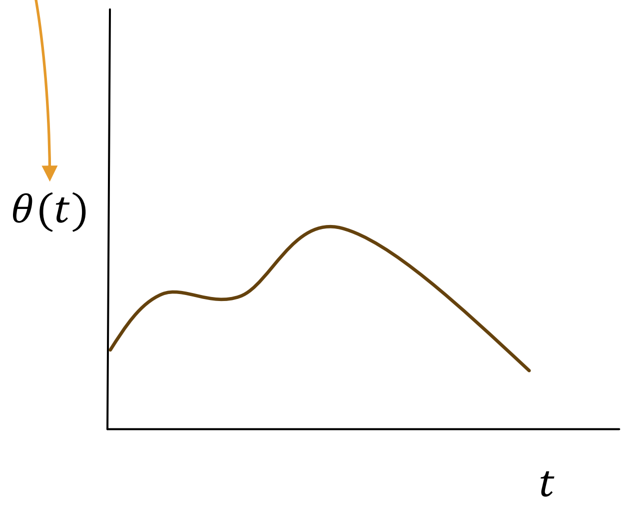
- ✓ No arbitrage model
- ✓ Mean Reverting
- ✓ Rates are Gaussian
- ✓ Negative interest rates are allowed

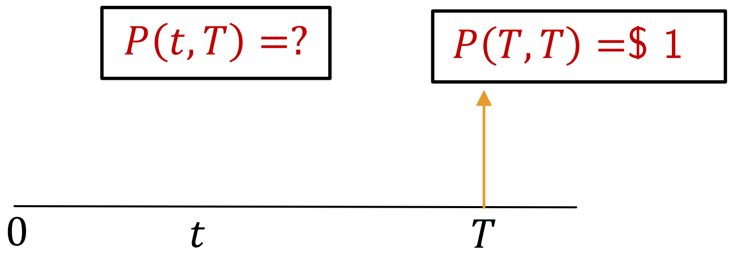
For any valuation date 0

$$f(0,T) = -\frac{\partial}{\partial T}\ln P(0,T)$$

$$\theta(T) = f(0,T) + \frac{1}{k}\frac{\partial}{\partial T}f(0,T) + \frac{\sigma^2}{2k^2}\left(1 - e^{-2kT}\right)$$





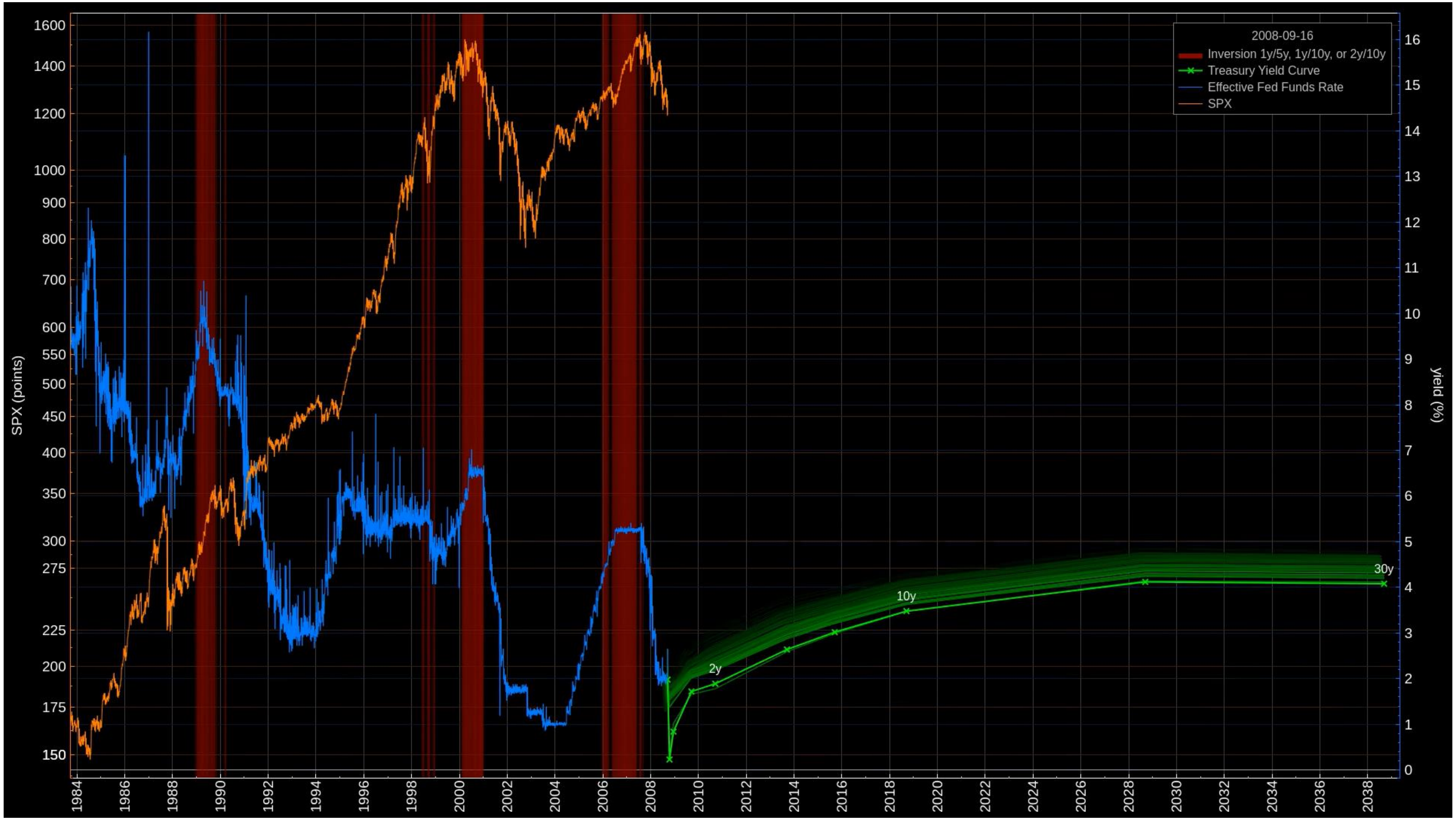


$$P(t,T) = A(t,T)e^{-r(t)B(t,T)} \qquad \text{(Affine Solution)}$$

$$B(t,T) = \frac{1 - e^{-k(T-t)}}{k}$$

$$A(t,T) = \exp\left\{-\int_t^T k\theta(s)B(s,T)\,ds + \frac{\sigma^2}{2k^2}\left(T - t + \frac{2}{k}e^{-k(T-t)} - \frac{1}{2k}e^{-2k(T-t)} - \frac{3}{2k}\right)\right\}$$

https://panthema.net/2019/0403-US-Treasury-Yield-Curve-Inversions-Animation/



Exposure Modeling for Rate Derivatives

Parameter Calibration

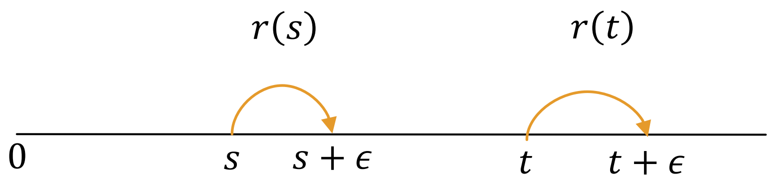
1. Understand the Model Dynamics

SDE $\quad dr(t) = \theta(t)dt + \sigma dW$

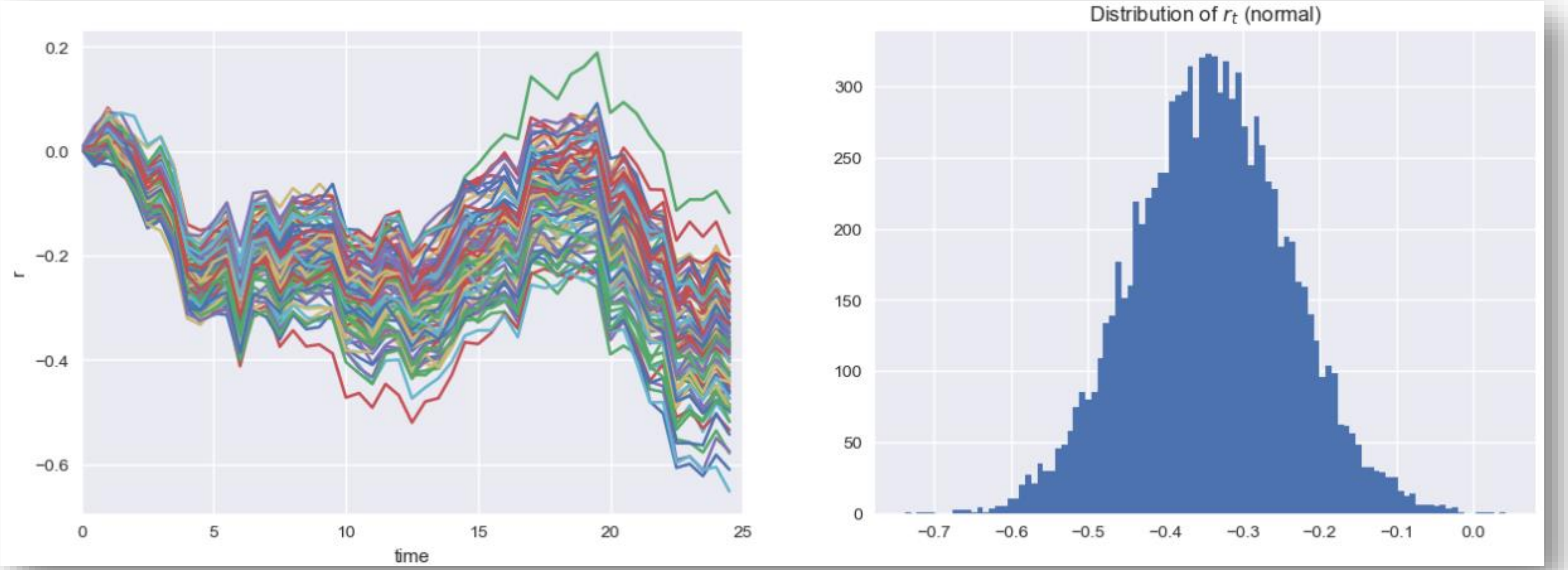
Solution $\quad r(t) = r(s) + \int_s^t \theta(u)du + \sigma Z\sqrt{t-s}$

$$\mathbb{E}[r(t)] = r(s) + \int_s^t \theta(u)du$$

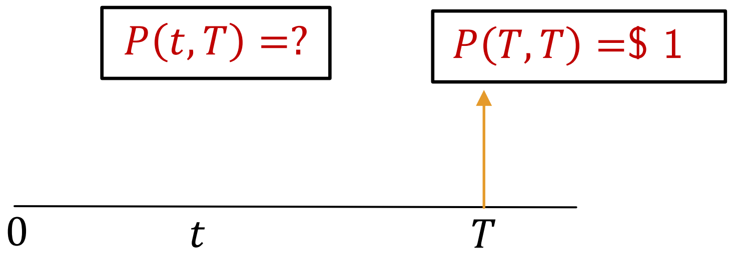
$$\mathbb{V}[r(t)] = \sigma^2(t-s)$$



- ✓ No arbitrage model
- ✓ Not mean reverting
- ✓ Rates are Gaussian
- ✓ Negative interest rates are allowed



2. ZCB Price under Ho-Lee



$$P(t,T) = A(t,T)e^{-r(t)B(t,T)} \qquad \text{(Affine Solution)}$$

$$B(t,T) = T - t$$

$$A(t,T) = \exp\left\{-\int_t^T \theta(s)(T-s)\,ds + \frac{1}{6}\sigma^2(T-t)^3\right\}$$

- ✓ Affine solutions only need the short rate $r(t)$ at the valuation date (t) for ZCB price. Everything else is deterministic

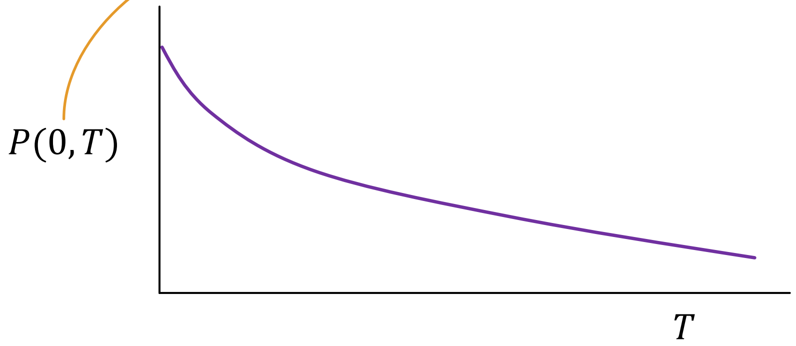
3. Calibrate Ho-Lee parameters $(\sigma, \theta(t))$ to current market prices

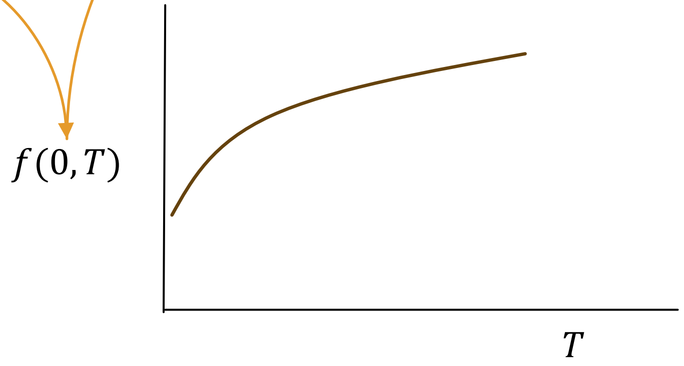
- ✓ Calibrate σ to caplet / swaption prices
- ✓ Calibrate $\theta(t)$ to ZCB prices : $P(0,T)$

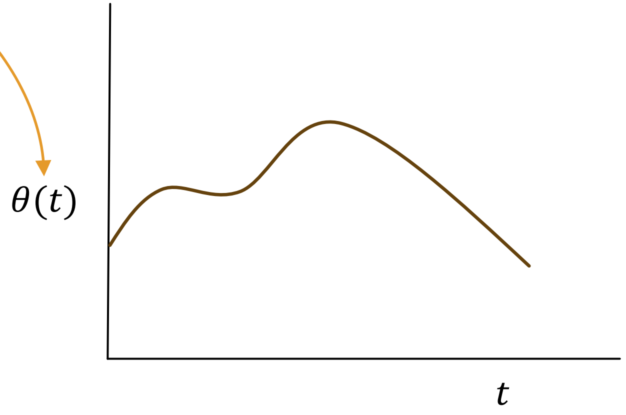
(Analytical Solution)

$$f(0,T) = -\frac{\partial}{\partial T}\ln P(0,T)$$

$$\theta(T) = \frac{\partial f(0,T)}{\partial T} + \sigma^2 T$$





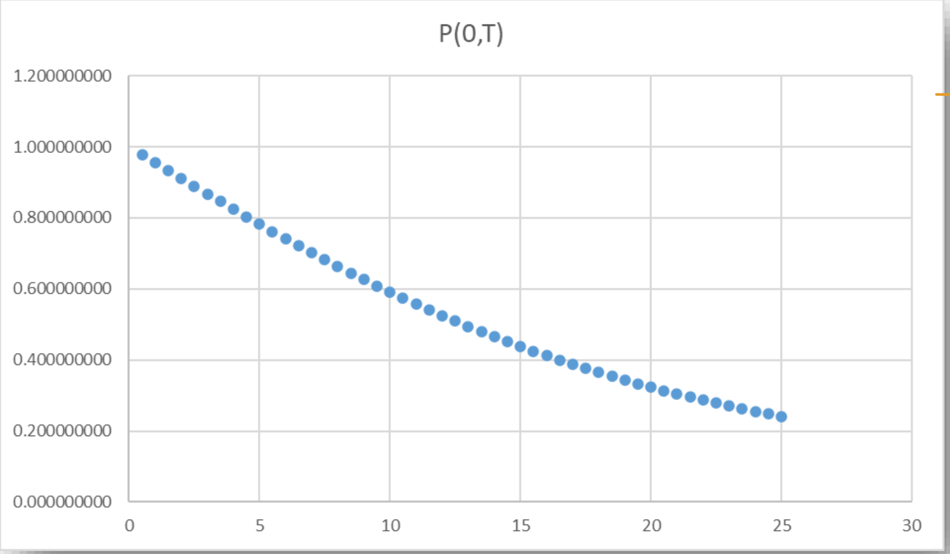


(Piecewise constant $\theta(t)$)

$$\int_0^T \theta(s)(T-s)ds = \frac{1}{2}\theta_j\left(T_j - T_{j-1}\right)^2 + \frac{1}{2}\sum_{i=1}^{j-1}\theta_i\left[\left(T_j - T_{i-1}\right)^2 - \left(T_j - T_i\right)^2\right]$$

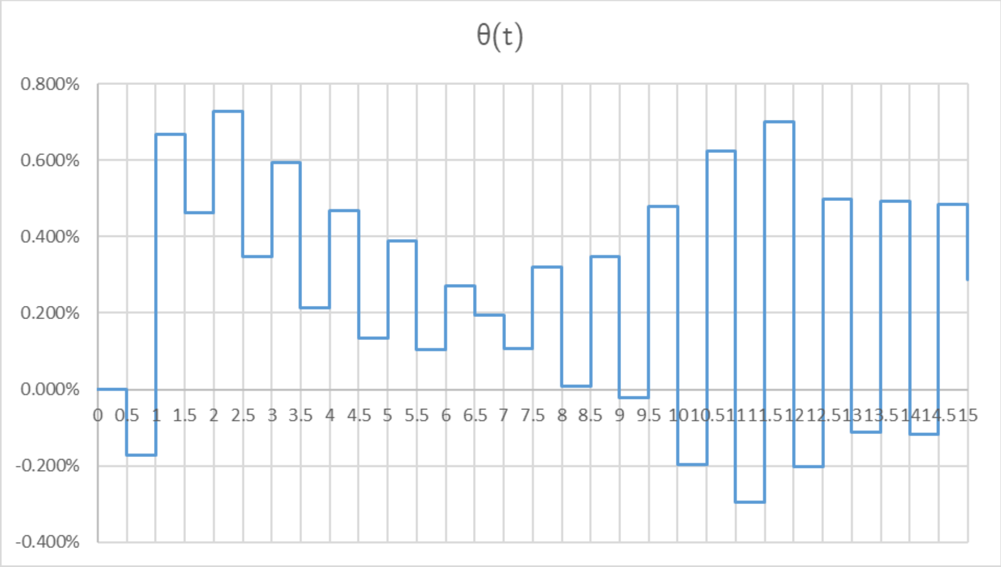
$$\theta_j = \frac{2}{\left(T_j - T_{j-1}\right)^2}\left(-\ln P\left(0,T_j\right) - r(0)T_j + \frac{1}{6}\sigma^2 T_j^{\,3} - \frac{1}{2}\sum_{i=1}^{j-1}\theta_i\left[\left(T_j - T_{i-1}\right)^2 - \left(T_j - T_i\right)^2\right]\right)$$

Excel Snapshot



(Today's ZCB prices)

$$\theta_j = \frac{2}{\left(T_j - T_{j-1}\right)^2}\left(-\ln P(0,T_j) - r(0)T_j + \frac{1}{6}\sigma^2 {T_j}^3 - \frac{1}{2}\sum_{i=1}^{j-1}\theta_i\left[\left(T_j - T_{i-1}\right)^2 - \left(T_j - T_i\right)^2\right]\right)$$



(Calibrated drift parameter)

Risk Neutral Calibration

σ	0.0070	(calibrated from CAP prices)
r(0)	4.520%	(slope of the short end)
dt	0.5	

	Today							(Sanity Check)
T0	**0.00**	**spot rate**	**P (0,T)^{obs}**	f(0, T)	θ	A(0,T)	B(0,T)	**P(0,T)**
T1	0.5	4.5200%	0.977653467	4.520%	0.0008%	1.000000	0.50	0.977653467
T2	1	4.5680%	0.955347625	4.616%	0.388%	0.999520	1.00	0.9553476245
T3	1.5	4.6150%	0.933116705	4.709%	-0.006%	0.998576	1.50	0.933116705
T4	2	4.6610%	0.910993060	4.799%	0.381%	0.997184	2.00	0.910993060
T5	2.5	4.7060%	0.889007151	4.886%	-0.013%	0.995361	2.50	0.889007151
T6	3	4.7500%	0.867187554	4.970%	0.374%	0.993124	3.00	0.867187554
T7	3.5	4.7930%	0.845560972	5.051%	-0.020%	0.990491	3.50	0.845560972
T8	4	4.8350%	0.824152247	5.129%	0.367%	0.987479	4.00	0.824152247
T9	4.5	4.8760%	0.802984394	5.204%	-0.028%	0.984108	4.50	0.802984394
T10	5	4.9160%	0.782078625	5.276%	0.360%	0.980395	5.00	0.782078625

4. Calculate $A(t,T)$ and $B(t,T)$ grid

A(t,T) and B(t,T) under time varying drift

σ	0.007
r(0)	0.0452
dt	0.5

T →

t ↓

θ(T)		0.0008%	0.3881%	-0.0063%	0.3810%	-0.0134%	0.3739%	-0.0205%	0.3668%
A(t,T)	0	0.5	1	1.5	2	2.5	3	3.5	4
0		1.0	0.999520115	0.99858	0.99718	0.99536	0.99312	0.99049	0.98748
0.5			0.999516034	0.99856	0.99715	0.99531	0.99304	0.99038	0.98732
1				1.00001	0.99956	0.99866	0.99733	0.99558	0.99344
1.5					0.99952	0.9986	0.99723	0.99545	0.99326
2						1.00002	0.99959	0.99874	0.99747
2.5							0.99953	0.99863	0.99731
3								1.00003	0.99963
3.5									0.99954
4									

$$A(t,T) = \exp\left\{-\int_t^T \theta(s)(T-s)\,ds + \frac{1}{6}\sigma^2(T-t)^3\right\}$$

T →

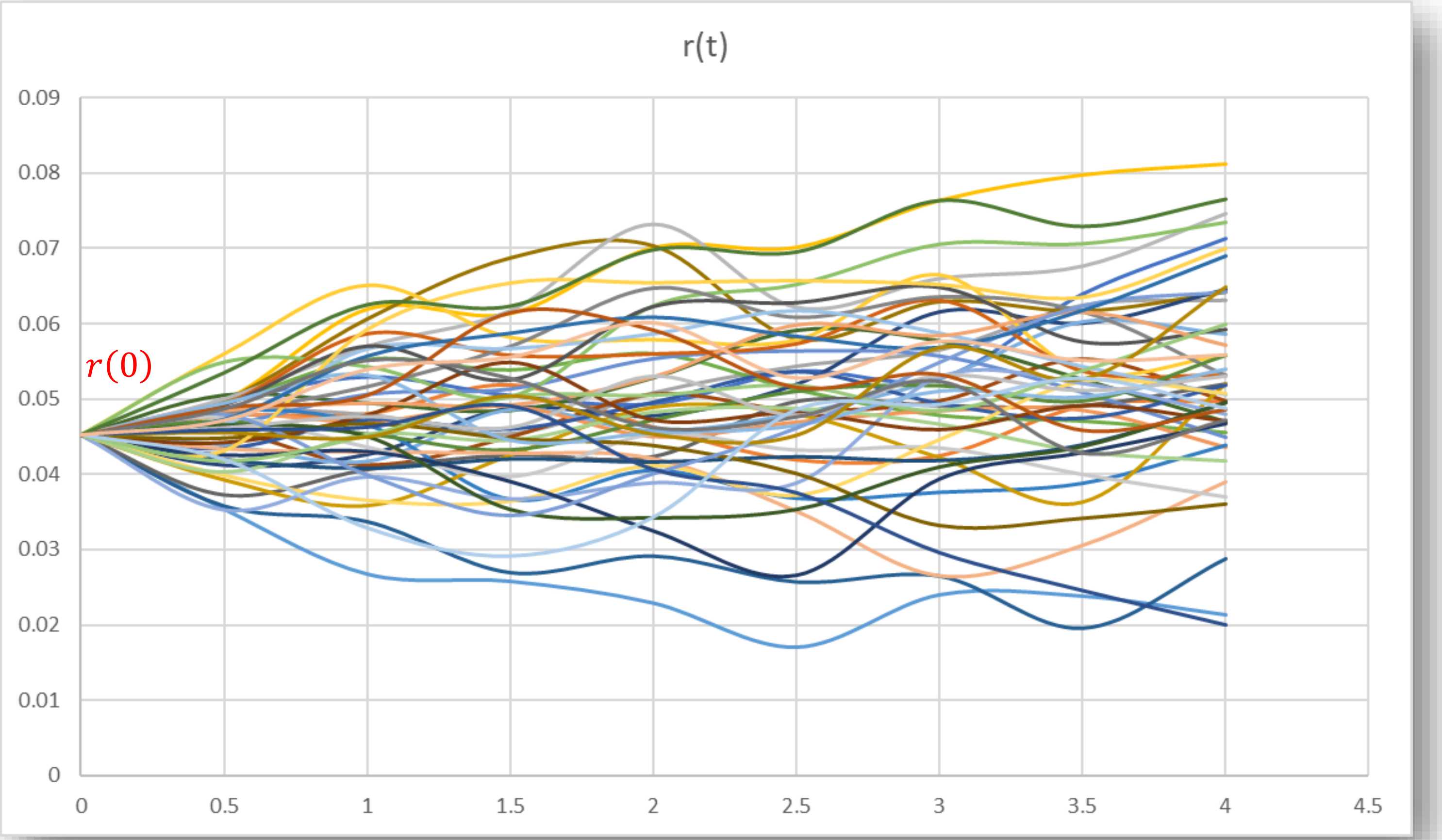
t ↓

B(t,T)	0	0.5	1	1.5	2	2.5	3	3.5	4
0		0.5	1	1.5	2	2.5	3	3.5	4
0.5			0.5	1	1.5	2	2.5	3	3.5
1				0.5	1	1.5	2	2.5	3
1.5					0.5	1	1.5	2	2.5
2						0.5	1	1.5	2
2.5							0.5	1	1.5
3								0.5	1
3.5									0.5
4									

$$B(t,T) = T - t$$

Simulation

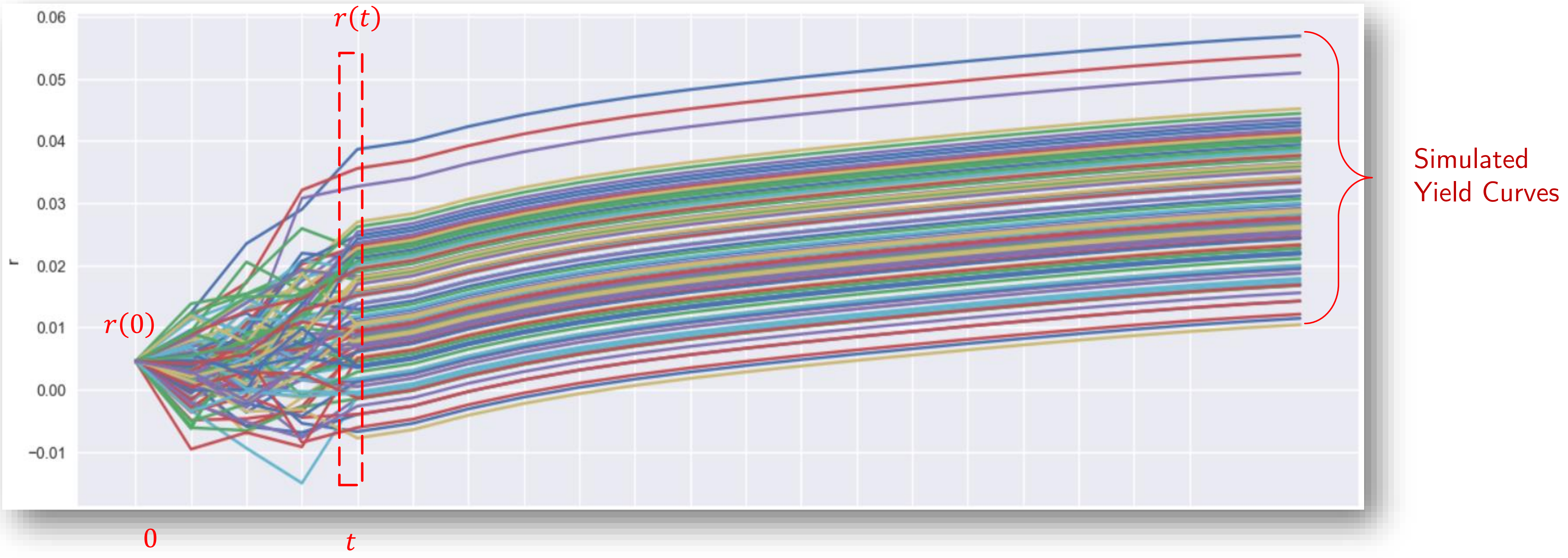
5. Simulate r(t)



$$r(t + \delta t) = r(t) + \theta(t)\delta t + \sigma Z\sqrt{\delta t}$$

6. Simulate ZCB prices $P(t,T)$ / yield curve $R(t,T)$

Plugin simulated $r(t)$ into Affine solution to obtain simulated prices of $P(t,T)$



And you are ready to model exposure of derivatives !!

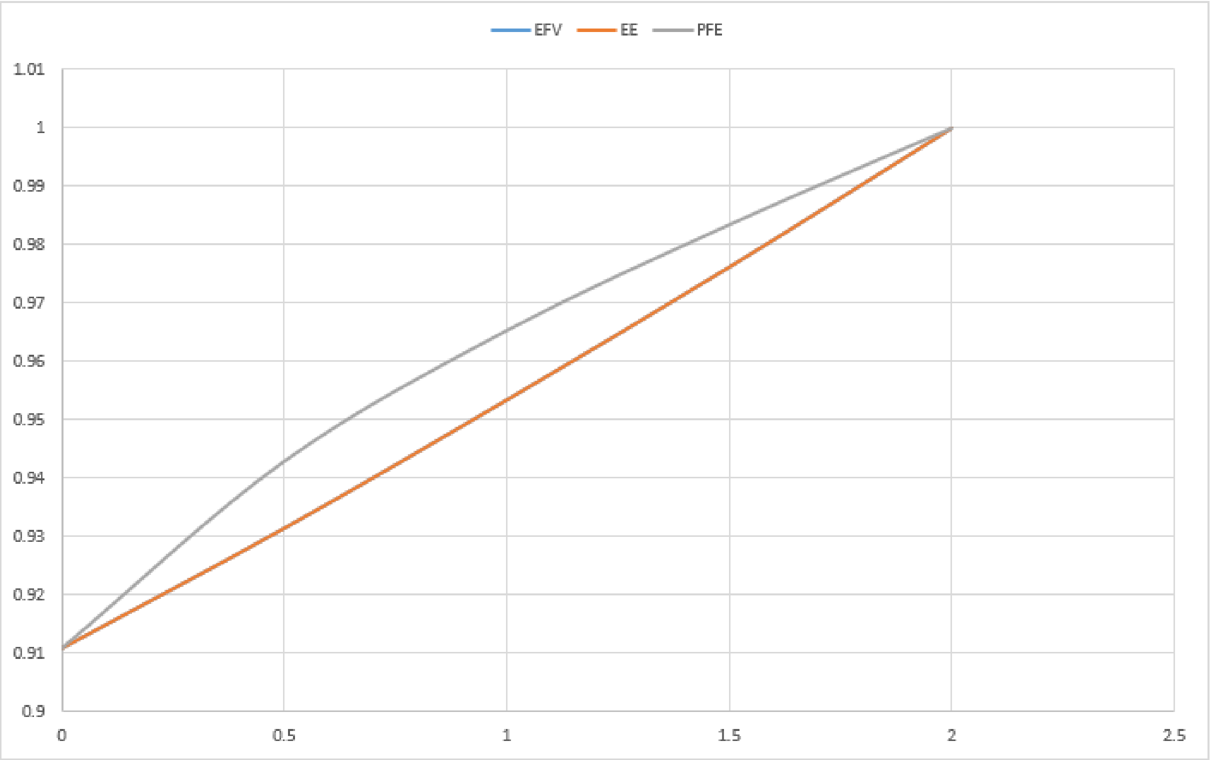
Exposure Modelling (EFV, EE, ENE, PFE)

ZCB

Exposure profile of a ZCB

Find the Exposure profile of a zero coupon bond maturing at T = 2 years

PFE	0.91099	0.943	0.96537	0.98348	1
EE	0.91099	0.93154	0.95349	0.97623	1
EFV	0.91099	0.93154	0.95349	0.97623	1
t →	0	0.5	1	1.5	2



Coupon Bond

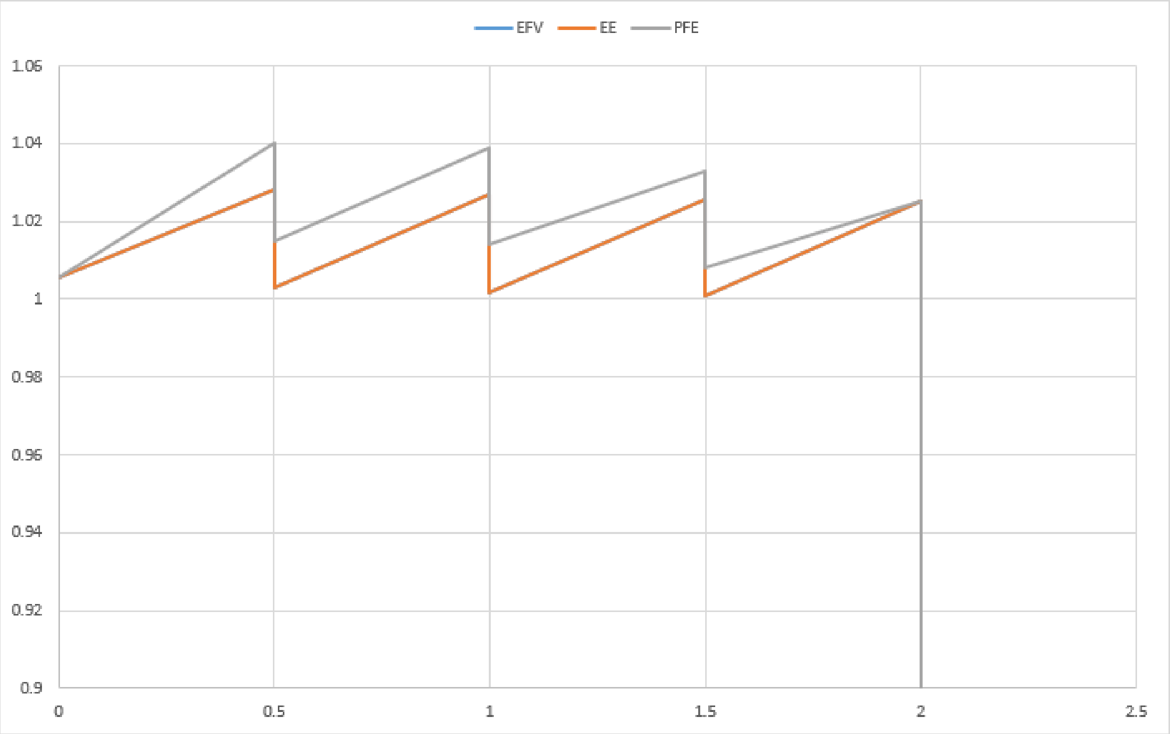
Exposure profile of a Coupon Bond

Find the Exposure profile of a coupon bond maturing at T = 2 years

Coupon = 5.00% (semi-annual)

PFE	1.00542	1.04015	1.01515	1.03908	1.01408	1.03306	1.00806	1.025	0
EE	1.00542	1.02811	1.00311	1.02674	1.00174	1.02563	1.00063	1.025	0
EFV	1.00542	1.02811	1.00311	1.02674	1.00174	1.02563	1.00063	1.025	0
t →	0	0.5	1	1.5	2				
CF		0.025	0.025	0.025	1.025				

#	0	0.5	0.5	1	1	1.5	1.5	2	2
1	1.00542	1.02624	1.00124	1.02654	1.00154	1.02773	1.00273	1.025	0
2	1.00542	1.02613	1.00113	1.01989	0.99489	1.02207	0.99707	1.025	0
3	1.00542	1.03151	1.00651	1.02397	0.99897	1.02179	0.99679	1.025	0
4	1.00542	1.03425	1.00925	1.03502	1.01002	1.02896	1.00396	1.025	0
5	1.00542	1.02554	1.00054	1.02712	1.00212	1.02476	0.99976	1.025	0
6	1.00542	1.03395	1.00895	1.04119	1.01619	1.0347	1.0097	1.025	0
7	1.00542	1.0305	1.0055	1.02773	1.00273	1.02619	1.00119	1.025	0
8	1.00542	1.02108	0.99608	1.02139	0.99639	1.02187	0.99687	1.025	0
9	1.00542	1.03558	1.01058	1.03401	1.00901	1.02851	1.00351	1.025	0
10	1.00542	1.02625	1.00125	1.01953	0.99453	1.02213	0.99713	1.025	0
11	1.00542	1.02061	0.99561	1.01918	0.99418	1.0238	0.9988	1.025	0
12	1.00542	1.01863	0.99363	1.01551	0.99051	1.01675	0.99175	1.025	0
13	1.00542	1.03999	1.01499	1.0345	1.0095	1.02827	1.00327	1.025	0
14	1.00542	1.02342	0.99842	1.00876	0.98376	1.0196	0.9946	1.025	0

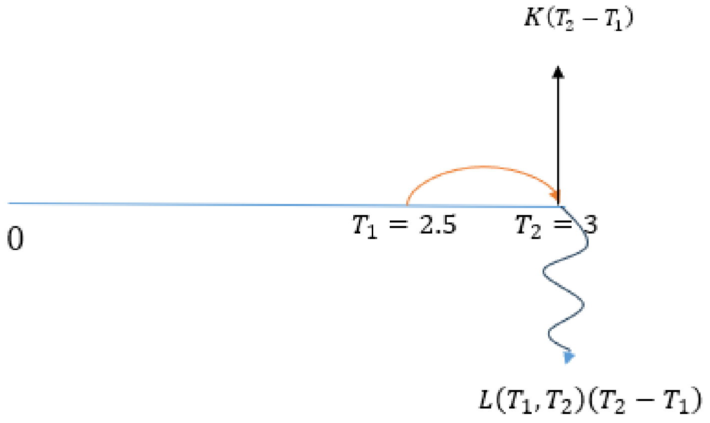


FRA

Exposure profile of an FRA

Find the Exposure profile of an FRA (30 x 36)
K(cont) = 4.970%
K(semi) = 5.032%

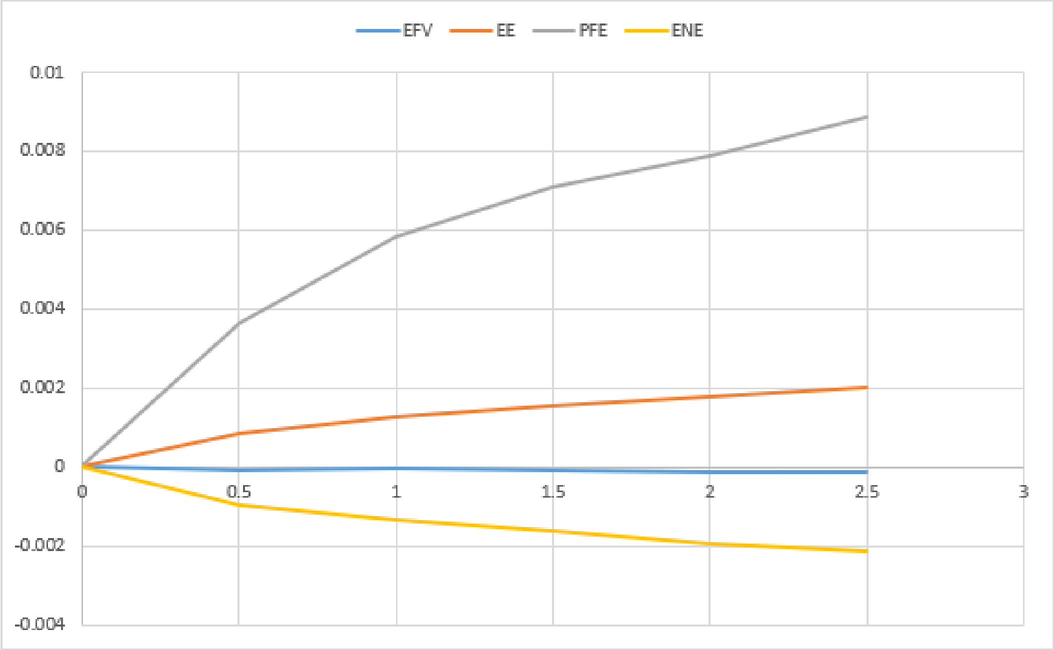
Tenor	Spot	ZCB
0.5	4.5200%	0.97765347
1	4.5680%	0.95534762
1.5	4.6150%	0.93311671
2	4.6610%	0.91099306
2.5	4.7060%	0.88900715
3	4.7500%	0.86718755



FRA

$$FRA(t) = N[P(t,T_2)K(T_2 - T_1) + P(t,T_2) - P(t,T_1)]$$

PFE	0	0.00366	0.00582	0.00708	0.00789	0.00887
ENE	0	-0.00095	-0.00132	-0.00162	-0.00193	-0.00215
EE	0	0.00085	0.00128	0.00156	0.00179	0.00201
EFV	0	-9.1E-05	-4E-05	-6.4E-05	-0.00013	-0.00014
#	0	0.5	1	1.5	2	2.5
1	0.0000000	-0.000683	-0.000163	0.001906	0.002266	0.002143
2	0.000000	-0.000717	-0.003260	-0.003464	-0.004437	-0.006791
3	0.000000	0.000947	-0.001366	-0.003728	-0.002704	-0.003818
4	0.000000	0.001803	0.003862	0.003086	0.003672	0.003786
5	0.000000	-0.000899	0.000109	-0.000923	0.000999	0.001931
6	0.000000	0.001710	0.006844	0.008689	0.007981	0.004564
7	0.000000	0.000635	0.000395	0.000439	0.003299	0.006480
8	0.000000	-0.002265	-0.002566	-0.003650	-0.004712	-0.002171
9	0.000000	0.002220	0.003378	0.002654	-0.000005	-0.003273
10	0.000000	-0.000681	-0.003428	-0.003410	-0.004149	-0.004494
11	0.000000	-0.002409	-0.003589	-0.001830	-0.005517	-0.002269
12	0.000000	-0.003012	-0.005275	-0.008400	-0.007191	-0.006952
13	0.000000	0.003612	0.003611	0.002431	-0.001690	-0.002321
14	0.000000	-0.001552	-0.008329	-0.005765	-0.007906	-0.007901
15	0.000000	-0.000886	0.000393	-0.000165	-0.001458	-0.009989
16	0.000000	-0.000891	-0.003825	-0.008330	-0.010405	-0.013851



IRS

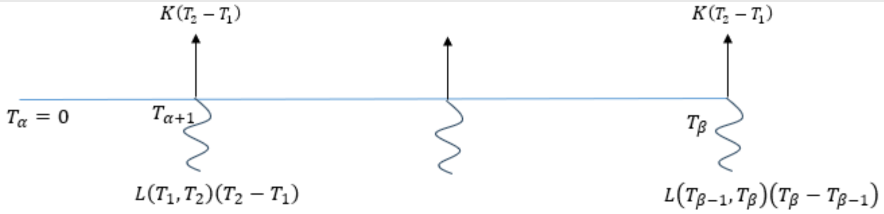
Exposure profile of an IRS

Find the Exposure profile of an IRS (T_alpha = 0, T_beta = 3)

s(0) = 4.800%

Work with 6 FRAs

Tenor	Spot	DF
0.5	4.5200%	0.9776535
1	4.5680%	0.9553476
1.5	4.6150%	0.9331167
2	4.6610%	0.9109931
2.5	0.04706	0.8890072
3	0.0475	0.8671876



$$FRA(t) = N[P(t,T_2)K(T_2-T_1) + P(t,T_2) - P(t,T_1)]$$

FRA 1 T1 = 0, T2 = 0.5

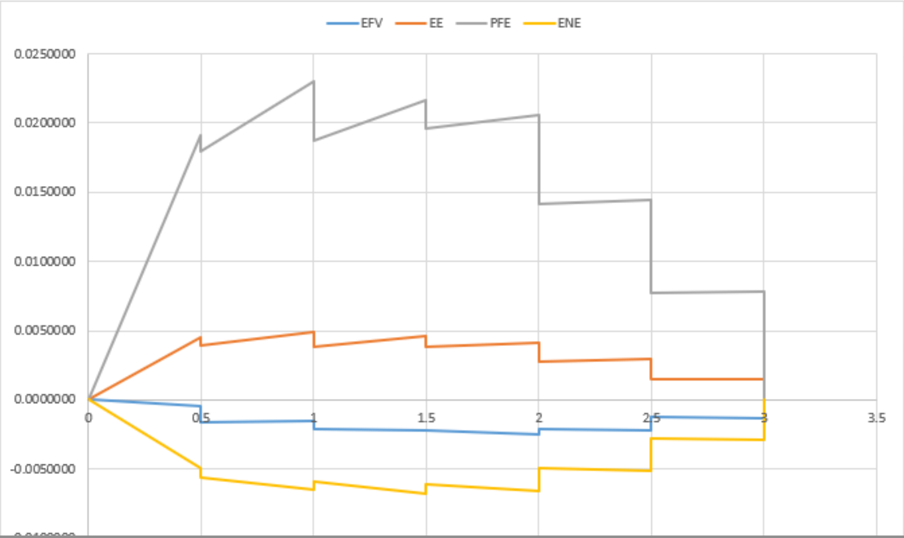
#	0	0.5
1	0.001119467	0.00114505
2	0.001119467	0.00114505
3	0.001119467	0.00114505
4	0.001119467	0.00114505
5	0.001119467	0.00114505
6	0.001119467	0.00114505
7	0.001119467	0.00114505
8	0.001119467	0.00114505
9	0.001119467	0.00114505
10	0.001119467	0.00114505
11	0.001119467	0.00114505
12	0.001119467	0.00114505
13	0.001119467	0.00114505
14	0.001119467	0.00114505
15	0.001119467	0.00114505
16	0.001119467	0.00114505
17	0.001119467	0.00114505
18	0.001119467	0.00114505
19	0.001119467	0.00114505
20	0.001119467	0.00114505

FRA 2 T1 = 0.5, T2 = 1

#	0	0.5	1
1	0.00062	-9E-05	-9.3E-05
2	0.00062	-0.00013	-0.00013
3	0.00062	0.001702	0.00174
4	0.00062	0.002632	0.002688
5	0.00062	-0.00033	-0.00034
6	0.00062	0.002531	0.002585
7	0.00062	0.00136	0.001391
8	0.00062	-0.00185	-0.0019
9	0.00062	0.003083	0.003147
10	0.00062	-8.8E-05	-9.1E-05
11	0.00062	-0.00202	-0.00207
12	0.00062	-0.0027	-0.00277
13	0.00062	0.004577	0.004666
14	0.00062	-0.00106	-0.00108
15	0.00062	-0.00032	-0.00032
16	0.00062	-0.00032	-0.00033
17	0.00062	-0.00192	-0.00197
18	0.00062	0.001704	0.001742
19	0.00062	-0.00085	-0.00087
20	0.00062	-0.00105	-0.00108

FRA 3 T1 = 1, T2 = 1.5

FRA 4 T1 = 1.5, T2 = 2



1.5	2
0175	0.00179
0039	-0.004
0042	-0.0043
0298	0.00304
0012	-0.0012
0872	0.00885
0022	0.00023
0041	-0.0042
0253	0.00258
0038	-0.004
0022	-0.0022
0092	-0.0095
0023	0.00235
0064	-0.0066
0004	-0.0004
0091	-0.0094
0002	-0.0002
0014	-0.0015
0033	-0.0034
0017	-0.0018

FRA 5 T1 = 2, T2 = 2.5

#	0	0.5	1	1.5	2	2.5
1	-0.0006	-0.0014	-0.0008	0.00127	0.00163	0.00167
2	-0.0006	-0.0014	-0.004	-0.0042	-0.0053	-0.0054
3	-0.0006	0.00031	-0.0021	-0.0045	-0.0035	-0.0036
4	-0.0006	0.00118	0.00327	0.00247	0.00306	0.00312
5	-0.0006	-0.0016	-0.0006	-0.0016	0.00034	0.00034
6	-0.0006	0.00109	0.00628	0.00814	0.00742	0.00754
7	-0.0006	-9E-06	-0.0003	-0.0002	0.00268	0.00274
8	-0.0006	-0.003	-0.0033	-0.0044	-0.0055	-0.0057
9	-0.0006	0.00161	0.00278	0.00203	-0.0007	-0.0007
10	-0.0006	-0.0014	-0.0042	-0.0042	-0.005	-0.0051
11	-0.0006	-0.0031	-0.0044	-0.0026	-0.0064	-0.0066
12	-0.0006	-0.0037	-0.0061	-0.0094	-0.0081	-0.0084
13	-0.0006	0.00302	0.00301	0.0018	-0.0024	-0.0025
14	-0.0006	-0.0022	-0.0093	-0.0066	-0.0088	-0.0091
15	-0.0006	-0.0016	-0.0003	-0.0008	-0.0022	-0.0022
16	-0.0006	-0.0016	-0.0046	-0.0093	-0.0115	-0.0119
17	-0.0006	-0.003	-0.0059	-0.0007	-0.003	-0.0031
18	-0.0006	0.00031	0.00236	-0.0018	0.00391	0.00399
19	-0.0006	-0.0021	-0.0045	-0.0037	-0.0014	-0.0015
20	-0.0006	-0.0022	-0.0023	-0.0021	-0.0047	-0.0048

Forward Swap

Exposure profile of a forward IRS

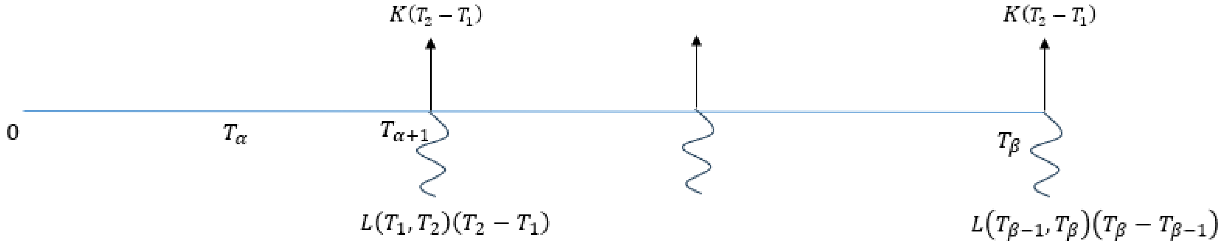
Find the Exposure profile of an IRS (T_alpha = 1, T_beta = 3)

s(0) = 4.897%

Work with 4 FRAs

$$FRA(t) = N[P(t,T_2)K(T_2 - T_1) + P(t,T_2) - P(t,T_1)]$$

Tenor	Spot	DF
0.5	4.5200%	0.9776535
1	4.5680%	0.9553476
1.5	4.6150%	0.9331167
2	4.6610%	0.9109931
2.5	0.04706	0.8890072
3	0.0475	0.8671876

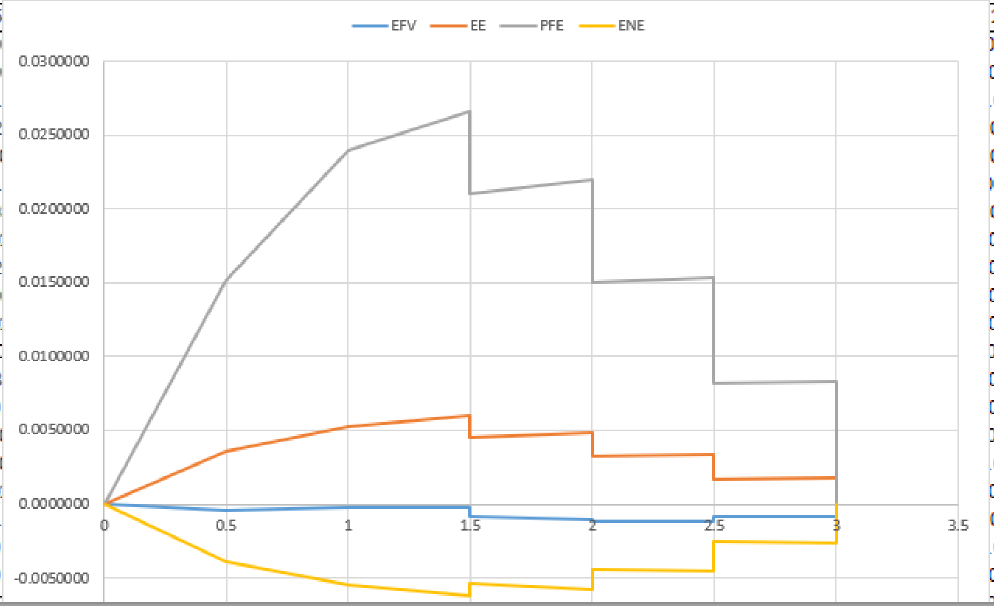


FRA 1 T1 = 1, T2 = 1.5

#	0	0.5	1	1.5
1	0.000618158	-8.652E-05	0.00051	0.00052
2	0.000618158	-0.0001232	-0.0029	-0.0029
3	0.000618158	0.00166647	-0.0008	-0.0008
4	0.000618158	0.00257898	0.00479	0.00488
5	0.000618158	-0.0003201	0.0008	0.00082
6	0.000618158	0.00247948	0.00789	0.00802
7	0.000618158	0.00133168	0.00111	0.00113
8	0.000618158	-0.001805	-0.0021	-0.0022
9	0.000618158	0.00302156	0.00428	0.00436
10	0.000618158	-8.453E-05	-0.003	-0.0031
11	0.000618158	-0.0019631	-0.0032	-0.0033
12	0.000618158	-0.0026234	-0.0051	-0.0052
13	0.000618158	0.00449202	0.00452	0.00461
14	0.000618158	-0.0010286	-0.0085	-0.0088
15	0.000618158	-0.000306	0.00111	0.00113
16	0.000618158	-0.0003111	-0.0035	-0.0036
17	0.000618158	-0.0018715	-0.0049	-0.0051
18	0.000618158	0.00166852	0.00385	0.00393
19	0.000618158	-0.0008239	-0.0034	-0.0034
20	0.000618158	-0.0010232	-0.001	-0.001

FRA 2 T1 = 1.5, T2 = 2

#	0	0.5
1	0.000184	-0.00
2	0.000184	-0.00
3	0.000184	0.001
4	0.000184	0.002
5	0.000184	-0.00
6	0.000184	0.001
7	0.000184	0.000
8	0.000184	-0.00
9	0.000184	0.002
10	0.000184	-0.00
11	0.000184	-0.00
12	0.000184	-0.00
13	0.000184	0.003
14	0.000184	-0.00
15	0.000184	-0.00
16	0.000184	-0.00
17	0.000184	-0.00
18	0.000184	0.001
19	0.000184	-0.00
20	0.000184	-0.00



FRA 3 T1 = 2, T2 = 2.5

2	2.5
0021	0.00215
0048	-0.0049
.003	-0.0031
0353	0.00361
0081	0.00083
079	0.00803
0315	0.00322
0051	-0.0052
0002	-0.0002
0045	-0.0046
0059	-0.0061
076	-0.0079
0019	-0.002
0084	-0.0087
017	-0.0018
.011	-0.0114
0026	-0.0026
0438	0.00447
.001	-0.001
0042	-0.0043

FRA 4 T1 = 2.5, T2 = 3

#	0	0.5	1
1	-0.0006	-0.0013	-0.0008
2	-0.0006	-0.0013	-0.0039
3	-0.0006	0.00035	-0.002
4	-0.0006	0.0012	0.00324
5	-0.0006	-0.0015	-0.0005
6	-0.0006	0.00111	0.00621
7	-0.0006	3.4E-05	-0.0002
8	-0.0006	-0.0029	-0.0032
9	-0.0006	0.00161	0.00276
10	-0.0006	-0.0013	-0.004
11	-0.0006	-0.003	-0.0042
12	-0.0006	-0.0036	-0.0059
13	-0.0006	0.003	0.00299
14	-0.0006	-0.0021	-0.0089
15	-0.0006	-0.0015	-0.0002
16	-0.0006	-0.0015	-0.0044
17	-0.0006	-0.0029	-0.0057
18	-0.0006	0.00035	0.00235
19	-0.0006	-0.002	-0.0043
20	-0.0006	-0.0021	-0.0022