Term Structure Modelling

- FRE(6411): Fixed Income Securities -

Project Report

by

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0.1 Introduction

The aim of this project is to calibrate the daily yield curves based on the benchmark bonds of the Australian market for the month of December 2018. The yield curve will be generated using the non parametric Cubic Spline Method and the parametric Nelson Siegel Method for each day throughout the month based on the daily reference data of the "Constant Maturity Treasury Rates" or CMTs sourced from Bloomberg(BBG).

0.2 Cubic Spline

As mentioned earlier, the Cubic Spline is an non parametric model which relies on the third order polynomial equations for the interpolation of the existing market reference data with a smooth curve which is usually prone to over-fitting of the data. The model constructed is based on the assumption that the CMTs are on zero coupon bonds with the mentioned expiration and hence have no intermediate payments.

0.3 Nelson Siegel

The Nelson Siegel is a parametric model with parameters $\beta 0, \beta 1, \beta 2$ which are optimized to reduced the mean square error between the actual and estimated prices. Though the curve generated from this method doesn't always pass through the existing points, it is considered a better approximation taking in to regard the additional information it conveys through the parameter values. This model defines the yield curve by the function-

$$y = \beta 0 + \beta 1 * \left(\frac{1-e^{-\lambda t}}{\lambda t}\right) + \beta 2 * \left(\frac{1-e^{-\lambda t}}{\lambda t} - e^{-\lambda t}\right)$$

where $\beta 0$ = Level/ Inflation $\beta 1$ = Slope/
Business Cycle $\beta 2$ = Curvature/ Interest
Rate Volatility λ = Time Constant t =
Maturity

0.4 Project Execution

1. Step 1 - Data Collection: We have downloaded the Australian Bonds data for month of December 2018 from Bloomberg terminal. Our program reads the excel files downloaded using pandas library. For bonds having tenor as 12, 20 and 30 years, we have same maturity date for different yields as tenor 10, 15 and 15 years respectively. So, we removed these data points to reduce complexity in yield curve fitting.

2. Step 2 - Import Data to Python: A function to retrieve all the necessary input data (yields, time to maturity in months, bond prices, coupons, schedule of all cash flows) from the .xlsx files to our python code was written.

3. Step 3 - Cubic Spline Generation: Using the *CubicSpline* library function *scipy.interpolate* from module to get (n-1) piece-wise cubic equations between (n) points and plot the yield curve obtained. In addition to this, an extension of the program which runs on the entire data for the month of December 2018 has also been inculcated.

4. Step 4 - Nelson Siegel Optimization: Initialize parameters with a starting value of $\beta 0 = 1, \beta 1 = 1, \beta 2 = 1$ and the Time constant $\lambda = 0.0605$ for all calculations and the optimum value of βs are estimated. All calculations for factor-loadings are done with time to maturity taken in units of months. The Optimization is done in order to reduce the error difference between the Actual and Predicted values of the Bond prices which are calculated taking in to account every cash flow payment (unlike Cubic Spline) discounted with its corresponding yield value derived from the NS equation. In addition, similar extensions of the program to run on the entire data and also calculate the progression of the βs throughout the 31 Day period have also been inculcated.

0.5 Results

Cubic Spline:

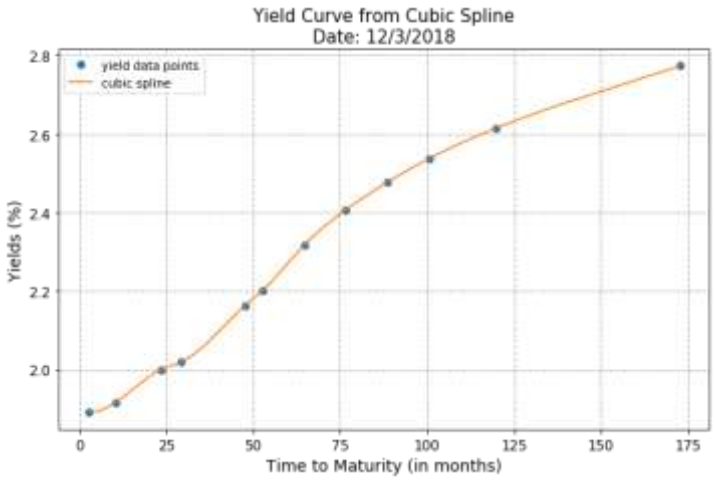


Figure 1: Yield Curve generated using Cubic Spline Model

	Tenors (months)	Month Yields	Mid Tenors (months)	Mid Tenor Yields
0	3.0	1.891737	7.5	1.900260
1	12.0	1.924958	18.0	1.967484
2	24.0	2.001415	30.0	2.021643
3	36.0	2.060700	42.0	2.113232
4	48.0	2.164824	54.0	2.211983
5	60.0	2.270528	66.0	2.328199
6	72.0	2.375010	78.0	2.414657
7	84.0	2.450936	90.0	2.484349
8	96.0	2.515029	102.0	2.543194
9	108.0	2.569083	114.0	2.592951
10	120.0	2.615053	150.0	2.708074
11	180.0	2.795478	NaN	NaN

Figure 2: Values predicted using the Cubic Spine model at and in between Tenor positions

Nelson Siegel:

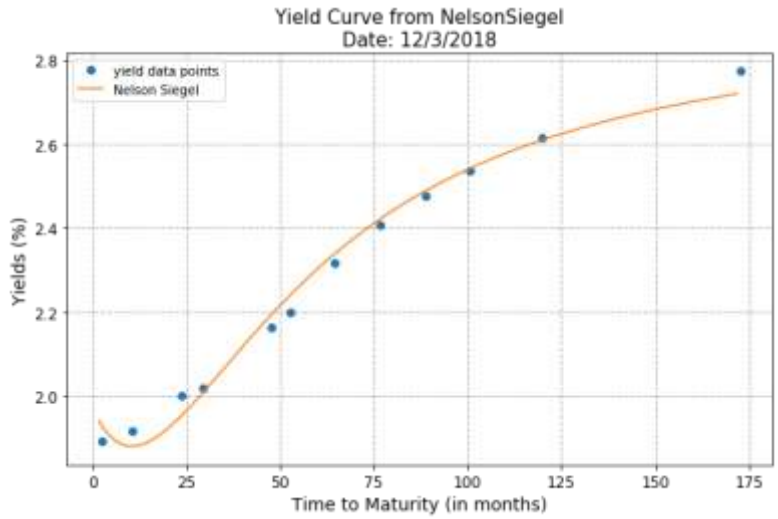


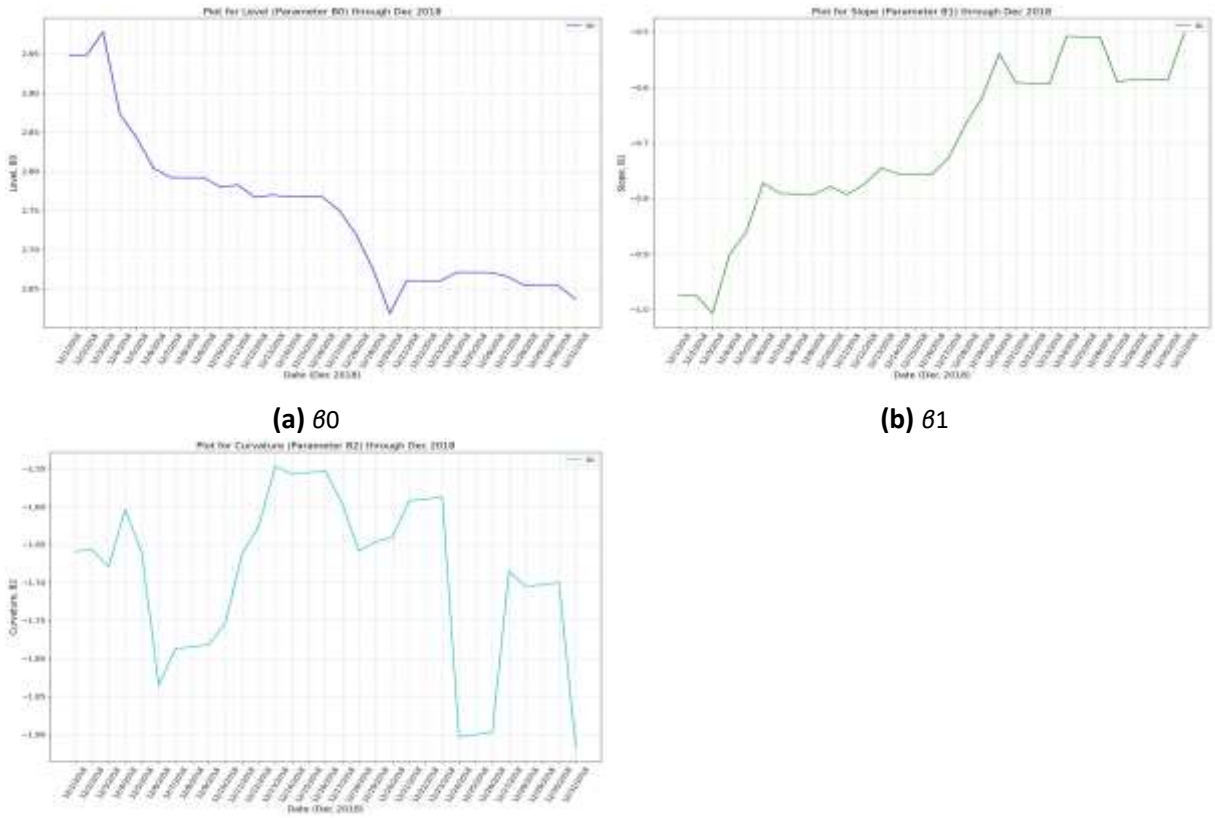
Figure 3: Yield Curve generated using Nelson Siegel Method

Bonds, 12/03/18	BBG Prices	Estimated Prices(NS)	BBG Yields	NS Yields	Price Error
ACTB 0 02/22/19 Corp	99.587	99.577474	1.892	1.926340	-0.009526
ACGB 2 ¾ 10/21/19 Corp	100.720	101.088627	1.916	1.879386	0.368627
ACGB 1 ¾ 11/21/20 Corp	99.521	99.685740	2.000	1.950953	0.164740
ACGB 5 ¾ 05/15/21 Corp	108.856	109.231113	2.019	2.006300	0.375113
ACGB 2 ¼ 11/21/22 Corp	100.331	100.356209	2.162	2.192303	0.025209
ACGB 5 ½ 04/21/23 Corp	113.695	114.319828	2.200	2.238678	0.624828
ACGB 2 ¾ 04/21/24 Corp	102.180	102.522257	2.316	2.338550	0.342257
ACGB 3 ¼ 04/21/25 Corp	104.958	105.460832	2.406	2.420616	0.502832
ACGB 4 ¼ 04/21/26 Corp	111.886	112.638835	2.477	2.487396	0.752835
ACGB 4 ¾ 04/21/27 Corp	116.603	117.569664	2.537	2.541721	0.966664
ACGB 2 ¾ 11/21/28 Corp	101.182	101.700384	2.614	2.608665	0.518384
ACGB 4 ½ 04/21/33 Corp	120.353	122.492752	2.773	2.721091	2.139752

Figure 4: Comparison of Actual vs Predicted Bond Prices using the Nelson Siegel Method

	Tenors (months)	Month Yields	Mid Tenors (months)	Mid Tenor Yields
0	3.0	1.921942	7.5	1.885321
1	12.0	1.880675	18.0	1.906647
2	24.0	1.954212	30.0	2.012469
3	36.0	2.074604	42.0	2.136498
4	48.0	2.195779	54.0	2.251200
5	60.0	2.302222	66.0	2.348732
6	72.0	2.390874	78.0	2.428925
7	84.0	2.463228	90.0	2.494146
8	96.0	2.522034	102.0	2.547228
9	108.0	2.570034	114.0	2.590728
10	120.0	2.609554	150.0	2.682348
11	180.0	2.731497	NaN	NaN

Figure 5: Values predicted using the Nelson Siegel Method at and in between Tenor positions



(a) *β*0 **(b)** *β*1

(c) *β*2

Figure 6: Evolution of *β*s throughout the month of December 2018