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PreAssignment 2016

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February 15, 2016

Contents

1 **Introduction** 1

2 **Materials and Methods** 1

2.1 Programming Tools 2

2.2 Programming Logic 2

2.2.1 Creating a Random Binary Tree 2

2.2.2 Traversing a Binary Tree in inorder 3

3 **Results** 4

4 **Discussion** 5

Preassignment - Task 1

1 Introduction

The first task of the preassignment consists in experimentally test that the amortized complexity of $(n-1)$ calls to **inorder_next** in a binary tree is $2(x-1)$.

To achieve this task, the following strategy was followed:

1. First, we created a program that generates random Binary Trees. This allows us to had a test set in which we prove the current hypothesis.
2. After creating the random Binary Tree generator, we create a second program that allows us to transverse the binary tree in inorder *(inorder means, first visiting the left node, then the root and finally the right node)*. The program also returns the amount of steps involved in this operation.
3. By using both programs, we made a table that compare the amount of steps taken to transverse every binary tree in comparison with the number of nodes of each tree.
4. Finally, we made a plot with the data of the table and by using linear regression, we get the equation corresponding to that line plot. We expect that the slope of that line plot should be something bigger or equal to $2(x-1)$ *(it should be bigger or equal because the amortized complexity correspond to an upper bound of the actual complexity of an algorithm)*.

   The results obtained in the analysis are described in the results section of this paper.

2 Materials and Methods

In this section, we will describe the tools and the logic used to solve every single step that composed the task 1 of this preassignment. This section include topics like the programming framework used to solve this task and also what was the logic applied to develop those programs.

2.1 Programming Tools

For developing the required programs we choose to use the following tools:

- Python 3.5.1 as the programming language
- Github as a repository tool
- Sublime Text as the code editor

In addition, the random library of python was used for accomplish the tasks related with randomness.

2.2 Programming Logic

The two programs involved in this task were programmed by using OOP paradigm. Considering the fact that the task 1 of the preassignment involves the creation of two different programs, we choose to explain the logic of each program in a different subsection.

2.2.1 Creating a Random Binary Tree

The creation of the binary tree was implemented by developing two different classes: a binary node class and a binary tree class. The binary node class contains 3 different attributes, an Id, a reference to a left child node and a reference to a right child node while the binary tree class contains only two attributes, a list of binary nodes and the total cost of traversing the tree in inorder. By using this two classes, creating a random binary tree was possible. Also when every binary tree is created, a random tree setting method is called so that every tree created has a different and also random structure.

The random tree setting method work as follows:

- First, a integer number between 50 and 100 is choose randomly. This number will represent the number of nodes of the corresponding binary tree
- Second, a left-right flag is set by choosing a random integer number between 0 and 1

- Then, after the left-right flag is set, a random node of the tree is chosen. Only the nodes that doesn't already have the corresponding child are candidates for this random choice. *For example, if the left-right flag is set to left, only nodes that doesn't have left childs are possible candidates of this random choice. Conversely, if the left-right flag is set to right, only nodes that doesn't have right childs are possible candidates*

- Finally, the corresponding child is created in the randomly chosen node of the tree

The described logic repeats until the number of nodes of the binary tree is reached.

2.2.2 Traversing a Binary Tree in inorder

The traversing of a binary tree in inorder can be made mainly in two different ways: by using recursion and by not using recursion. In this case, we choose to implement this method using recursion as this method allow to achieve a smaller and cleaner code and also allow us to avoid the implementation of a link between a child node and it's parent node.

The idea of this implementation is mainly the following:

- We start by applying this method to the root node of the tree

- If the root node has a left child, then we call again the traverse_inorder method but this time we pass the left child node as an argument to the method

- When the corresponding node has no left child node, then we ask if the node has a right child. If the node has a right child, then we call again the traverse_inorder method but this time we pass the right child node as an argument to the method

- After no more left and right child are found, the traverse_inorder method returns. Of course we should consider a logic for counting the corresponding steps in each call of the method so that when the method ends, we have the total amount of steps already calculated.

In the next section we will show and analyze the results of the above implementations.

3 Results

When the two generated programs are run, an output table is created. This table includes three columns:

1. **Binary Tree ID** Represents the Id of the binary tree
2. **Number of nodes of the Tree** Represents the number of nodes of the binary tree
3. **Total steps of inorder traverse** Represents the amounts of steps involved in the inorder traverse operation

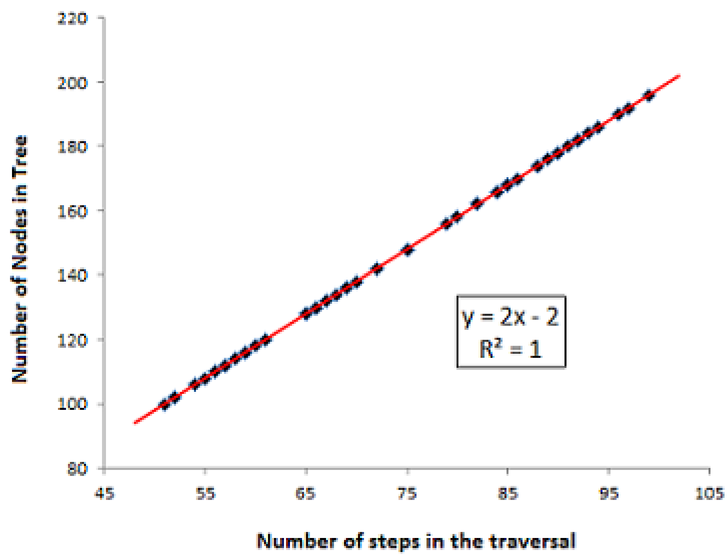
The results obtained in this computation are shown in the following table:

Table 1: Binary Tree traverse results using 50 different trees

Tree Id	Number of Nodes	Total Steps	Tree Id	Number of Nodes	Total Steps
1	83	164	26	70	138
2	94	186	27	95	188
3	69	136	28	68	134
4	99	196	29	88	174
5	53	104	30	98	194
6	63	124	31	99	196
7	72	142	32	63	124
8	61	120	33	66	130
9	55	108	34	59	116
10	90	178	35	94	186
11	81	160	36	85	168
12	74	146	37	59	116
13	92	182	38	68	134
14	85	168	39	93	184
15	75	148	40	62	122
16	66	130	41	75	148
17	95	188	42	100	198
18	64	126	43	92	182
19	71	140	44	82	162
20	95	188	45	84	166
21	52	102	46	58	114
22	77	152	47	63	124
23	53	104	48	69	136
24	84	166	49	51	100
25	80	158	50	66	130

By inspecting the results of the table we can clearly see that the amount of steps that takes to traverse a binary tree, directly depends of the amount of nodes that compose that particular binary tree.

To obtain the equation that describes the relation between the number of nodes and the amount of steps that takes to traverse a binary tree, we decided to plot the data in the table shown previously and use linear regression to obtain the equation. The result of this linear plot is shown in the following image:



As we can see, the value of the **coefficient of determination** also know as R^2 is 1. This means that the linear regression line perfectly fits the plotted data. The equation that describes this relation is correspond to $y = 2x - 2$ where x represents the number of nodes in a binary tree and y represents the amounts of steps that takes to traverse that particular tree in inorder.

We will discuss the meaning of this result in the next section of the text.

4 Discussion

As we saw in the last section, the amount of steps that takes to traverse a random binary tree in inorder is $2x - 2$ where x represents the number of nodes of that particular tree. In this section, we will use this result to test experimentally that the amortized complexity of $(n-1)$ calls of **inorder_next** correspond to $2(x-1)$.

To do this task, we need to consider the following information:

- First, the amortized complexity is an upper bound on total actual complexity. *This means that the amortized complexity is always bigger or equal than the total actual complexity*
- Second, the operation of traverse an entire binary tree with n nodes is exactly the same than call the **inorder_next** function n times
- Third, if we do n calls to a function the actual complexity of this operation will always be bigger or equal than the actual complexity of $n-1$ calls to the same function

Using the above information and also considering the results obtained in the last section, we can state the following:

- The actual complexity for n calls of **inorder_next** can be represented by the equation $2(x-1)$. *where x represents the number of nodes in a binary tree*
- Considering that $n-1$ calls to a function is les expensive than n calls in the same function (in terms of complexity), we can state than $2(x-1)$ **is an upper bound of the actual complexity of** $n-1$ **calls**. *This state is based in the fact that* $2(x-1)$ *will always be bigger or equal than the actual complexity of* $n-1$ *calls*
- Finally as $2(x-1)$ is an upper bound of the actual complexity of $n-1$ calls to the function **inorder_next**, we can also said that $2(x-1)$ **is a possible value for the amortized complexity of this function**

In conclusion, by collecting data and using the method shown in this paper, we managed to test experimentally that $2(x-1)$ or $2x-2$ is a totally possible value for the amortized complexity of $n-1$ calls of the **inorder_next** function.