T3-9.11 Short Circuit Admittance Parameter

VISVESVARAYA TECHNOLOGICAL UNIVERSITY, BELAGAVI
B.E: Electronics & Communication Engineering / B.E: Electronics & Telecommunication Engineering
NEP, Outcome Based Education (OBE) and Choice Based Credit System (CBCS)
(Effective from the academic year 2021 – 22)

IV Semester

Circuits & Controls			
Course Code	**21EC43**	CIE Marks	50
Teaching Hours/Week (L: T: P: S)	(3:0:2:0)	SEE Marks	50
Total Hours of Pedagogy	40 hours Theory + 13 Lab slots	Total Marks	100
Credits	04	Exam Hours	03

Module-1	
Basic concepts and network theorems Types of Sources, Loop analysis, Nodal analysis with independent DC and AC Excitations. (Textbook 1: 2.3, 4.1, 4.2, 4.3, 4.4, 10.6) Super position theorem, Thevenin's theorem, Norton's Theorem, Maximum Power transfer Theorem. (Textbook 2: 9.2, 9.4, 9.5, 9.7)	
Teaching-Learning Process	Chalk and Talk, YouTube videos, Demonstrate the concepts using circuits **RBT Level:** L1, L2, L3

Module-2	
Two port networks: Short- circuit Admittance parameters, Open- circuit Impedance parameters, Transmission parameters, Hybrid parameters (Textbook 3: 11.1, 11.2, 11.3, 11.4, 11.5) **Laplace transform and its Applications**: Step Ramp, Impulse, Solution of networks using Laplace transform, Initial value and final value theorem (Textbook 3: 7.1, 7.2, 7.4, 7.7, 8.4)	
Teaching-Learning Process	Chalk and Talk **RBT Level:** L1, L2, L3
Module-3	
Basic Concepts and representation: Types of control systems, effect of feedback systems, differential equation of physical systems (only electrical systems), Introduction to block diagrams, transfer functions, Signal Flow Graphs (Textbook 4: Chapter 1.1, 2.2, 2.4, 2.5, 2.6)	
Teaching-Learning Process	Chalk and Talk, YouTube videos **RBT Level:** L1, L2, L3

Module-4	
Time Response analysis: Time response of first order systems. Time response of second order systems, time response specifications of second order systems (Textbook 4: Chapter 5.3, 5.4) **Stability Analysis:** Concepts of stability necessary condition for stability, Routh stability criterion, relative stability Analysis (Textbook 4: Chapter 5.3, 5.4, 6.1, 6.2, 6.4, 6.5)	
Teaching-Learning Process	Chalk and Talk, Any software tool to show time response **RBT Level:** L1, L2, L3

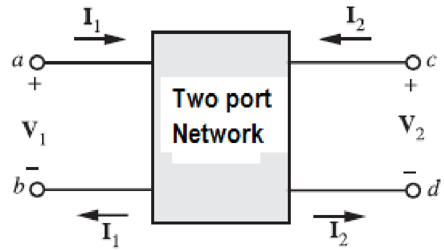
Module-5	
Root locus: Introduction the root locus concepts, construction of root loci (Textbook 4: 7.1, 7.2, 7.3) **Frequency Domain analysis and stability**: Correlation between time and frequency response and Bode plots (Textbook 4: 8.1, 8.2, 8.4) **State Variable Analysis:** Introduction to state variable analysis: Concepts of state, state variable and state models. State model for Linear continuous –Time systems, solution of state equations. (Textbook 4: 12.2, 12.3, 12.6)	
Teaching-Learning Process	Chalk and Talk, Any software tool to plot Root locus, Bode plot **RBT Level:** L1, L2, L3

Suggested Learning Resources:

Text Books

1. Engineering circuit analysis, William H Hayt, Jr, Jack E Kemmerly, Steven M Durbin, Mc Graw Hill Education, Indian Edition 8e.
2. Networks and Systems, D Roy Choudhury, New age international Publishers, second edition.
3. Network Analysis, M E Van Valkenburg, Pearson, 3e.
4. Control Systems Engineering, I J Nagrath, M. Gopal, New age international Publishers, Fifth edition.

2) Short circuit Admittance or Y- parameters



Consider the variables I_1 & I_2 as dependent variable and

V_1 & V_2 as independent variables.

The coefficients of independent variables, V_1 and V_2 are called as **Y parameters**.

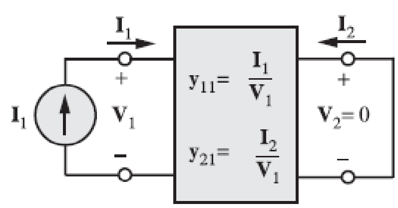
Hence, the two equations that describe the two-port network are

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

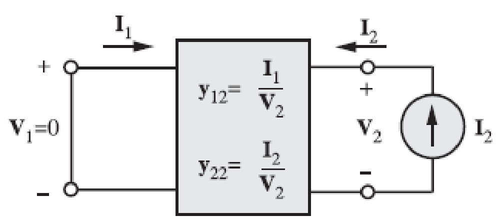
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Putting the above equations in matrix form, we get

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$



$$Y_{11} = \frac{I_1}{V_1},\ when\ V_2 = 0 \qquad Y_{21} = \frac{I_2}{V_1},\ when\ V_2 = 0$$



$$Y_{12} = \frac{I_1}{V_2},\ when\ V_1 = 0 \qquad Y_{22} = \frac{I_2}{V_2},\ when\ V_1 = 0$$

$$\underbrace{Y_{11}\quad Y_{12}\quad Y_{21}\quad Y_{22}}_{\text{Y parameters}}$$

unit - mho

The SI unit of admittance is the **SIEMENS (symbol S);** the older, synonymous unit is mho, and its symbol is ℧

Y parameters are called as **admittance parameters** because these are simply, the ratios of currents and voltages. Units of Y parameters are mho.

We can calculate two Y parameters, Y_{11} and Y_{21} by doing short circuit of port2.

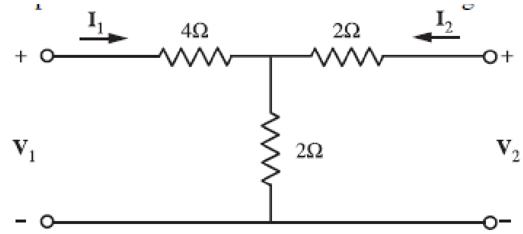
Similarly, we can calculate the other two Y parameters, Y_{12} and Y_{22} by doing short circuit of port1.

Hence, the Y parameters are also called as **short-circuit admittance parameters**.

Relationship of parameter

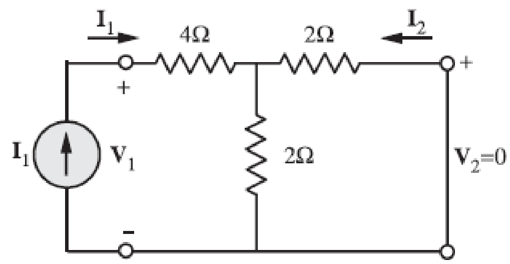
1)	Open circuit impedance (Z parameters)	→ Express V_1, V_2	→ In terms of I_1, I_2	→ $\begin{cases} V_1 = z_{11}I_1 + z_{12}I_2 \\ V_2 = z_{21}I_1 + z_{22}I_2 \end{cases}$
2)	Short circuit admittance (Y parameters)	→ Express I_1, I_2	→ In terms of V_1, V_2	→ $\begin{cases} I_1 = y_{11}V_1 + y_{12}V_2 \\ I_2 = y_{21}V_1 + y_{22}V_2 \end{cases}$
3)	Transmission (T parameters)	→ Express V_1, I_1	→ In terms of V_2, I_2	→ $\begin{cases} V_1 = AV_2 - BI_2 \\ I_1 = CV_2 - DI_2 \end{cases}$
4)	Inverse transmission (T' parameters)	→ Express V_2, I_2	→ In terms of V_1, I_1	→ $\begin{cases} V_2 = A'V_1 - B'I_1 \\ I_2 = C'V_1 - D'I_1 \end{cases}$
5)	Hybrid (h-parameters)	→ Express V_1, I_2	→ In terms of I_1, V_2	→ $\begin{cases} V_1 = h_{11}I_1 + h_{12}V_2 \\ I_2 = h_{21}I_1 + h_{22}V_2 \end{cases}$
6)	Inverse hybrid (g-parameters)	→ Express I_1, V_2	In terms of V_1, I_2	→ $\begin{cases} I_1 = g_{11}V_1 + g_{12}I_2 \\ V_2 = g_{21}V_1 + g_{22}I_2 \end{cases}$

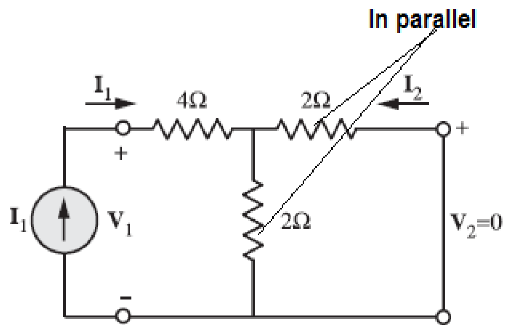
1) Determine the admittance parameters of the T network shown in Fig



<u>To find y11 and y21,</u>

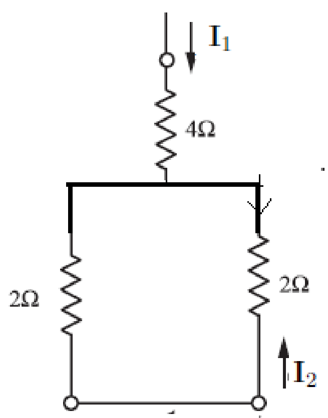
Short the output terminals and connect a current source I_1 to the input terminals.





$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{4 + \dfrac{2 \times 2}{2+2}} = \frac{\mathbf{V}_1}{5}$$

Hence, $$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} = \frac{1}{5}\mathrm{S}$$



Jsing the principle of current division,

$$-\mathbf{I}_2 = \frac{\mathbf{I}_1 \times 2}{2+2} = \frac{\mathbf{I}_1}{2}$$

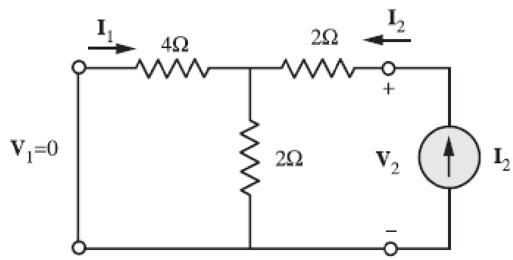
- Negative sign flowing against actual direction

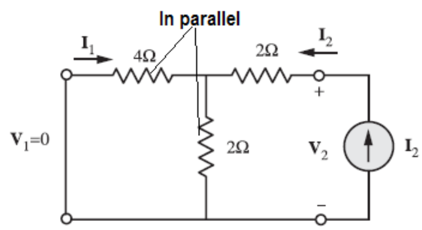
$$-\mathbf{I}_2 = \frac{1}{2}\left[\frac{\mathbf{V}_1}{5}\right]$$

Hence, $$\mathbf{y}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} = \frac{-1}{10}\mathrm{S}$$

<u>To find y12 and y22,</u>

Short-circuit the input terminals and connect a current source I2 to the output terminals. The circuit so obtained is shown in Fig.



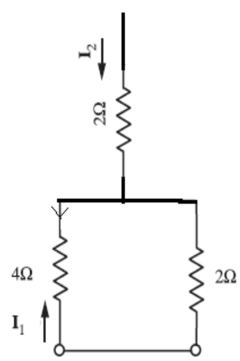


$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{2 + \frac{4 \times 2}{4 + 2}}$$

$$= \frac{\mathbf{V}_2}{2 + \frac{4}{3}}$$

$$= \frac{3\mathbf{V}_2}{10}$$

Hence, $\mathbf{y}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} = \frac{3}{10}\mathrm{S}$



Employing the principle of current division, we have

$$-\mathbf{I}_1 = \frac{\mathbf{I}_2 \times 2}{2 + 4}$$

$$-\mathbf{I}_1 = \frac{2\mathbf{I}_2}{6}$$

$$-\mathbf{I}_1 = \frac{1}{3}\left[\frac{3\mathbf{V}_2}{10}\right]$$

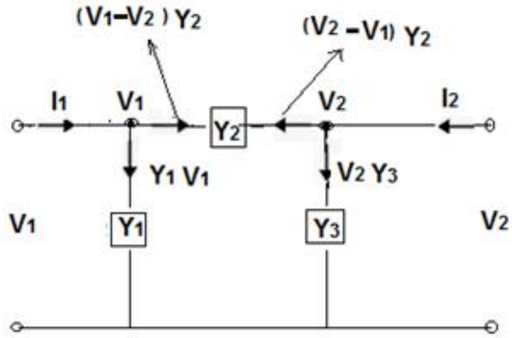
Hence, $\mathbf{y}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} = \frac{-1}{10}\mathrm{S}$

It may be noted that, $\mathbf{y}_{12} = \mathbf{y}_{21}$. Thus, in matrix form we have

$$\mathbf{I} = \mathbf{Y}\mathbf{V}$$

$$\Rightarrow \begin{bmatrix} \mathbf{I_1} \\ \mathbf{I_2} \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{-1}{10} \\ \frac{-1}{10} & \frac{3}{10} \end{bmatrix} \begin{bmatrix} \mathbf{V_1} \\ \mathbf{V_2} \end{bmatrix}$$

2)Y- Parameter shortcut



Apply KCL at the node 1

$I_1 = Y_1 V_1 + (V_1 - V_2) Y_2$

$I_1 = (Y_1 + Y_2) V_1 - Y_2 V_2$ ....(1)

Apply KCL at the node 2

$I_2 = (V_2 - V_1) Y_2 + Y_3 V_2$

$I_2 = - Y_2 V_1 + (Y_2 + Y_3) V_2$ ....(2)

$I_1 = (Y_1 + Y_2) V_1 - Y_2 V_2$ ....(1)

Compare the co efficients of V_1 & V_2'

$I_1 = Y_{11} V_1 + Y_{12} V_2$

$Y_{11} = (Y_1 + Y_2)$

$Y_{12} = Y_{21} = - Y_2$

$I_2 = - Y_2 V_1 + (Y_2 + Y_3) V_2$ ....(2)

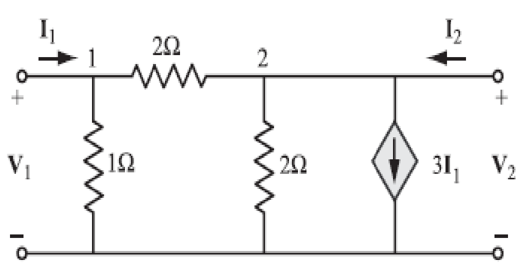
Compare the co efficients of V_1 & V_2'

$I_2 = Y_{21} V_1 + Y_{22} V_2$

$Y_{22} = (Y_2 + Y_3)$

$Y_{12} = Y_{21} = - Y_2$

3) Find y and z parameters for the network shown in Fig. which contains a current controlled source.



Apply KCL at node 1 and 2

At node 1, $1.5\mathbf{V}_1 - 0.5\mathbf{V}_2 = \mathbf{I}_1$

At node 2, $-0.5\mathbf{V}_1 + \mathbf{V}_2 = \mathbf{I}_2 - 3\mathbf{I}_1$

In matrix form,

$$\begin{bmatrix} 1.5 & -0.5 \\ -0.5 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 1.5 & -0.5 \\ -0.5 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$= \begin{bmatrix} -0.4 & 0.4 \\ -3.2 & 1.2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

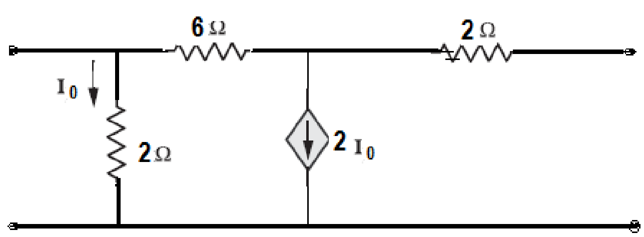
Therefore,

$$[\mathbf{z}] = \begin{bmatrix} -0.4 & 0.4 \\ -3.2 & 1.2 \end{bmatrix}$$

$$[\mathbf{y}] = [\mathbf{z}]^{-1} = \begin{bmatrix} 1.5 & -0.5 \\ 4 & -0.5 \end{bmatrix}$$

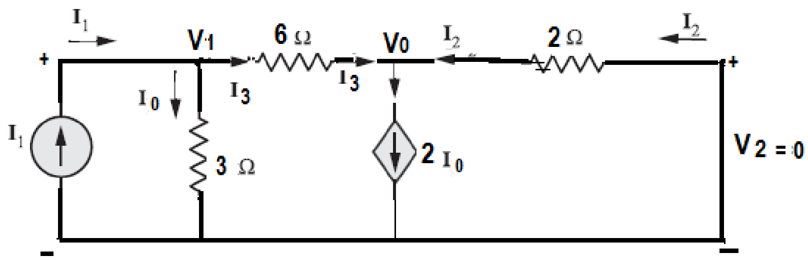
$$[\mathbf{y}] = \frac{1}{[\mathbf{z}]}$$

3)Obtain the Y- parameter of the circuit shown



Case –(1) to find Y_{11} and Y_{21}

$$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} \qquad \mathbf{y}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}$$



At node-1 Apply KCL

$$I_1 = I_0 + I_3$$

$$I_1 = \frac{V_1}{3} + \frac{V_1 - V_o}{6} \quad \text{..........(1)}$$

At Node 2 Apply KCL

$$I_3 + I_2 = 2 I_0$$

$$\frac{V1 - Vo}{6} + \frac{0 - Vo}{2} = 2x\frac{V1}{3} \qquad \Bigg| \; I_0 = \frac{V1}{3}$$

$$V1 - Vo - 3Vo = 4V1$$

$$-4Vo = 3V1$$

$$V_0 = -\frac{3}{4} V1 \quad \text{............(2)}$$

Putting the value of Vo in eq (1)

$$I_1 = \frac{V1}{3} + \frac{V1 + \frac{3}{4}V1}{6}$$

$$= \frac{V1}{3} + \frac{7V1}{24}$$

$$I_1 = \frac{16}{24} V1 \quad \text{...(3)}$$

$$Y_{11} = \frac{I_1}{V_1} = \frac{16}{24}$$

$$y_{21} = \frac{I_2}{V_1}$$

$$= \frac{\frac{-Vo}{2}}{V_1}$$

$$= \frac{\frac{\left(-\frac{3}{4} V1\right)}{2}}{V_1}$$

$$y_{21} = \frac{3}{8}$$

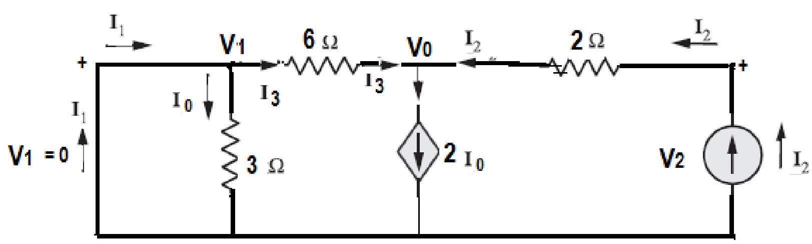
REERENCE

$$I_2 = \frac{0 - Vo}{2} = \frac{-Vo}{2}$$

$$V_0 = -\frac{3}{4} V1 \quad \text{............(2)}$$

Case –(2) to find Y_{22} and Y_{12}

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$



At node-1 Apply KCL

$$I_1 = i0 + i3$$
$$= 0 + \frac{V1-Vo}{6}$$
$$= 0 + \frac{-Vo}{6}$$
$$I_1 = \frac{-Vo}{6} \quad \ldots (1)$$

At node-2 Apply KCL

$$I_3 + I_2 = 2\,I_0$$
$$\frac{V1-Vo}{6} + I_2 = 0$$
$$\frac{0-Vo}{6} + I_2 = 0$$
$$I_2 = \frac{Vo}{6} \quad \ldots (2)$$

From the diagram

$$I_2 = \frac{V2-Vo}{2} \quad \ldots (3)$$

Comparing Eq.2 and 3

$$\frac{Vo}{6} = \frac{V2-Vo}{2}$$
$$Vo = 3V2 - 3Vo$$
$$3V2 = 4Vo$$
$$V2 = \frac{4}{3}Vo \quad \ldots (4)$$

From the diagram

$$I_2 = \frac{V2-Vo}{2} \quad \ldots (3)$$

Comparing Eq.2 and 3

$$\frac{Vo}{6} = \frac{V2-Vo}{2}$$
$$Vo = 3V2 - 3Vo$$
$$3V2 = 4Vo$$
$$V2 = \frac{4}{3}Vo \quad \ldots (4)$$

$$y_{12} = \frac{I_1}{V_2} = \frac{\frac{-Vo}{6}}{\frac{4}{3}Vo} = -\frac{1}{8} = -0.1258$$

$$y_{22} = \frac{I_2}{V_2} = \frac{\frac{Vo}{6}}{\frac{4}{3}Vo} = \frac{1}{8} = 0.125$$

Reference

$$V2 = \frac{4}{3}Vo \quad \ldots (4)$$
$$I_2 = \frac{Vo}{6} \quad \ldots (2)$$

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RELATION BETWEEN PARAMETERS

Table – Parameter relationships

	z	y	T	h
z	$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta y} & \frac{-y_{12}}{\Delta y} \\ \frac{-y_{21}}{\Delta y} & \frac{y_{11}}{\Delta y} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$
y	$\begin{bmatrix} \frac{z_{22}}{\Delta z} & \frac{-z_{12}}{\Delta z} \\ \frac{-z_{21}}{\Delta z} & \frac{z_{11}}{\Delta z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & \frac{-\Delta T}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{22}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{bmatrix}$
T	$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$
h	$\begin{bmatrix} \frac{\Delta z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$

$\Delta z = z_{11}z_{22} - z_{12}z_{21}$, $\Delta y = y_{11}y_{22} - y_{12}y_{21}$, $\Delta h = h_{11}h_{22} - h_{12}h_{21}$, $\Delta T = AD - BC$

4) Determine the y parameters for a two-port network if the z parameters are

$$\mathbf{z} = \begin{bmatrix} 10 & 5 \\ 5 & 9 \end{bmatrix}$$

$$\Delta z = \begin{vmatrix} 10 & 5 \\ 5 & 9 \end{vmatrix}$$

$$\Delta z = 10 \times 9 - 5 \times 5 = 65$$

$$y_{11} = \frac{z_{22}}{\Delta z} = \frac{9}{65}\text{S}$$

$$y_{12} = \frac{-z_{12}}{\Delta z} = \frac{-5}{65}\text{S}$$

$$y_{21} = \frac{-z_{21}}{\Delta z} = \frac{-5}{65}\text{S}$$

$$y_{22} = \frac{z_{11}}{\Delta z} = \frac{10}{65}\text{S}$$

BY THE RELATION

$$\begin{bmatrix} \frac{z_{22}}{\Delta z} & \frac{-z_{12}}{\Delta z} \\ \frac{-z_{21}}{\Delta z} & \frac{z_{11}}{\Delta z} \end{bmatrix} \quad \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

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