Model Question Paper-II with effect from 2022

USN

Fourth Semester B.E Degree Examination

Complex Analysis, Probability & Statistical Methods

All branches Except CS & ME Engg. Allied branches-21MAT41

TIME: 03 Hours **Max. Marks: 100**

Note: Answer any **FIVE** full questions, choosing at least **ONE** question from each module.

Q.No.		**Question**	**M**	**L**	**CO**
		Module -1			
01	a	Define Analytic function and hence derive C-R equations in Polar form.	**06**	**L2**	**CO1**
	b	Show that $w = f(z) = z + e^z$ is analytic and hence find its derivative.	**07**	**L3**	**CO1**
	c	Evaluate $\int_{(0,3)}^{(2,4)} (2y + x^2)\, dx + (3y - x)\, dy$ along with the parabola $x = 2t, y = t^2 + 3$.	**07**	**L2**	**CO1**
		OR			
02	a	Find analytic function $f(z) = u + iv$ where $u - v = (x - y)(x^2 + 4xy + y^2)$ by the Milne-Thomson method.	**06**	**L3**	**CO1**
	b	State and prove Cauchy's integral formula.	**07**	**L3**	**CO1**
	c	Evaluate $\int_C \frac{e^{2z}}{(z+1)\ (z+2)} dz$, where C is a circle $\lvert z \rvert = 3$.	**07**	**L2**	**CO1**
		Module-2			
03	a	Show that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$	**06**	**L2**	**CO2**
	b	If $\alpha\ and\ \beta$ are two distinct roots of $J_n(x) = 0$, then prove that $$\int_0^1 x J_n(\alpha x) J_n(\beta x)\, dx = 0.$$	**07**	**L2**	**CO2**
	c	Show that $P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$	07	**L2**	**CO2**
		OR			
4	a	Show that $J_{-\frac{1}{2}}(x) = J_{\frac{1}{2}}(x) cotx$	**06**	**L2**	**CO2**
	b	Find the series solution of the Legendre's equation $(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$, leading to Legendre polynomial of order *n*.	**07**	**L2**	**CO2**
	c	Express $4x^3 + 6x^2 + 7x + 2$ in terms of Legendre polynomials	**07**	**L2**	**CO2**
		Module-3			

<table>
<tr><td>5</td><td>a</td><td>The following table gives the heights of father(x) and sons(y). Calculate the Karl Pearson’s coefficient of correlation.
<table>
<tr><td>$x:$</td><td>65</td><td>66</td><td>67</td><td>68</td><td>68</td><td>69</td><td>70</td><td>72</td></tr>
<tr><td>$y:$</td><td>67</td><td>68</td><td>65</td><td>68</td><td>72</td><td>72</td><td>69</td><td>71</td></tr>
</table></td><td>06</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>b</td><td>Fit a straight line $y = ax + b$ for the data
<table>
<tr><td>$x:$</td><td>12</td><td>15</td><td>21</td><td>25</td></tr>
<tr><td>$y:$</td><td>50</td><td>70</td><td>100</td><td>120</td></tr>
</table></td><td>07</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>c</td><td>Using the method of least square, fit a curve $y = ax^b$ for the following data
<table>
<tr><td>$x:$</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>$y:$</td><td>2.98</td><td>4.26</td><td>5.21</td><td>6.1</td><td>6.8</td><td>7.5</td></tr>
</table></td><td>07</td><td>L2</td><td>CO3</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>6</td><td>a</td><td>The scores for 9 students in Physics (x) and Mathematics (y) are as follows
<table>
<tr><td>$x:$</td><td>35</td><td>23</td><td>47</td><td>17</td><td>10</td><td>43</td><td>9</td><td>6</td><td>28</td></tr>
<tr><td>$y:$</td><td>30</td><td>33</td><td>45</td><td>23</td><td>8</td><td>49</td><td>12</td><td>4</td><td>31</td></tr>
</table>
Compute the Ranks and Rank correlation.</td><td>06</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>b</td><td>Compute the means $\bar{x}$, $\bar{y}$ and the correlation coefficient r from the given regression lines $4x - 5y + 33 = 0$, $20x - 9y = 107$</td><td>07</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>c</td><td>Fit a second-degree polynomial $y = ax^2 + bx + c$ for the data.
<table>
<tr><td>$x:$</td><td>20</td><td>60</td><td>100</td><td>140</td><td>180</td><td>220</td></tr>
<tr><td>$y:$</td><td>0.18</td><td>0.37</td><td>0.35</td><td>0.78</td><td>0.56</td><td>0.75</td></tr>
</table></td><td>07</td><td>L2</td><td>CO3</td></tr>
<tr><td colspan="6">Module-4</td></tr>
<tr><td>7</td><td>a</td><td>A random variable X has the following probability function:
<table>
<tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>$P(X)$</td><td>0</td><td>k</td><td>$2k$</td><td>$2k$</td><td>$3k$</td><td>k^2</td><td>$2k^2$</td><td>$7k^2 + k$</td></tr>
</table>
Find k . Also find $P(X < 6)$, $P(X \geq 6)$, $P(3 < X \leq 6)$.</td><td>06</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>b</td><td>Derive the mean and variance of Poisson distribution.</td><td>07</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>c</td><td>The number of telephonic lines busy at an instant line is a binomial variate with a probability 0.1. If 10 lines are chosen at random, what is the probability that
(i) No line is busy
(ii) All lines are busy
(iii) At least one line is busy</td><td>07</td><td>L3</td><td>CO4</td></tr>
</table>

<table>
<tr><td colspan="6">OR</td></tr>
<tr><td>8</td><td>a</td><td>The diameter of a electric cable is assumed to be a continuous random variable with p.d.f $f(x) = \begin{cases} kx(1-x), & 0 \le x < 1 \\ 0, & else\ where \end{cases}$
Find the value of k and also obtain the mean and variance of the variable</td><td>06</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>b</td><td>The number of accidents in a year to taxi drivers in a city follows a Poisson distribution with mean 3. Out of 1000 taxi drivers find approximately the number of drivers with
i) No accident in a year
ii) More than three accidents in a year.</td><td>07</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>c</td><td>If the life time of a certain types electric bulbs of a particular brand was distributed normally with an average life of 2000 hours and S.D.60 hours. If a firm purchase 2500 bulbs, find the number of bulbs that are likely to last for
(i) More than 2100 hours
(ii) Less than 1950 hours
(iii) Between 1900 and 2100 hours.</td><td>07</td><td>L2</td><td>CO4</td></tr>
<tr><td colspan="5">Module-5</td><td></td></tr>
<tr><td>9</td><td>a</td><td>The joint distribution of two random variables X and Y is as follows.
<table>
<tr><td>Y
X</td><td>1</td><td>3</td><td>6</td></tr>
<tr><td>1</td><td>$\frac{1}{9}$</td><td>$\frac{1}{6}$</td><td>$\frac{1}{18}$</td></tr>
<tr><td>3</td><td>$\frac{1}{6}$</td><td>$\frac{1}{4}$</td><td>$\frac{1}{12}$</td></tr>
<tr><td>6</td><td>$\frac{1}{18}$</td><td>$\frac{1}{12}$</td><td>$\frac{1}{36}$</td></tr>
</table>
Compute the following.
i) Marginal distributions of X and Y
ii) Are X and Y stochastically independent?</td><td>06</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>b</td><td>A set of five similar coins is tossed 320 times and the result is
<table>
<tr><td>No. of heads</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>Frequency</td><td>6</td><td>27</td><td>72</td><td>112</td><td>71</td><td>32</td></tr>
</table>
Test the hypothesis that the data follows a binomial distribution at 5% significance level</td><td>07</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>c</td><td>A certain stimulus administered to each of the 12 patients resulted in the following change in the blood pressure 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4. Can it be concluded that the stimulus will increase the blood pressure? (<i>Note</i> : $t_{0.05}$ for 11 d.f. is 2.201).</td><td>07</td><td>L3</td><td>CO5</td></tr>
</table>

<table>
<tr><td colspan="3">OR</td><td></td><td></td><td></td></tr>
<tr><td>10</td><td>a</td><td>Determine (i) Marginal distributions (ii) Correlation coefficient between the variables X and Y, from the joint probability distribution given by:
<table>
<tr><td>Y
X</td><td>-2</td><td>-1</td><td>4</td><td>5</td></tr>
<tr><td>1</td><td>0.1</td><td>0.2</td><td>0</td><td>0.3</td></tr>
<tr><td>2</td><td>0.2</td><td>0.1</td><td>0.1</td><td>0</td></tr>
</table></td><td>06</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>b</td><td>The 9 item of a sample have the following values 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these differ significantly from the assumed mean of 47.5?
($t_{0.05}$ =2.306 for 8 d.f.)</td><td>07</td><td>L3</td><td>CO5</td></tr>
<tr><td></td><td>c</td><td>The theory predicts the proportion of beans in the four groups A, B, C and D should be 9:3:3:1. In an experiment among 1600 beans, the number in the Four groups were 882, 313, 287 and 118. The goodness of fit χ^2 value of above data is approximately equal to?</td><td>07</td><td>L3</td><td>CO5</td></tr>
</table>

<table>
<tr><td colspan="4"></td></tr>
<tr><td rowspan="4">Bloom's Taxonomy Levels</td><td colspan="3">Lower-order thinking skills</td></tr>
<tr><td>Remembering (knowledge): L_1</td><td>Understanding (Comprehension): L_2</td><td>Applying (Application): L_3</td></tr>
<tr><td colspan="3">Higher-order thinking skills</td></tr>
<tr><td>Analyzing (Analysis): L_4</td><td>Valuating (Evaluation): L_5</td><td>Creating (Synthesis): L_6</td></tr>
</table>