T3- 11.2 Open circuit Impedance

VISVESVARAYA TECHNOLOGICAL UNIVERSITY, BELAGAVI
B.E: Electronics & Communication Engineering / B.E: Electronics & Telecommunication Engineering
NEP, Outcome Based Education (OBE) and Choice Based Credit System (CBCS)
(Effective from the academic year 2021 – 22)

IV Semester

Circuits & Controls			
Course Code	**21EC43**	CIE Marks	50
Teaching Hours/Week (L: T: P: S)	(3:0:2:0)	SEE Marks	50
Total Hours of Pedagogy	40 hours Theory + 13 Lab slots	Total Marks	100
Credits	04	Exam Hours	03

Module-1	
Basic concepts and network theorems Types of Sources, Loop analysis, Nodal analysis with independent DC and AC Excitations. (Textbook 1: 2.3, 4.1, 4.2, 4.3, 4.4, 10.6) Super position theorem, Thevenin's theorem, Norton's Theorem, Maximum Power transfer Theorem. (Textbook 2: 9.2, 9.4, 9.5, 9.7)	
Teaching-Learning Process	Chalk and Talk, YouTube videos, Demonstrate the concepts using circuits **RBT Level:** L1, L2, L3

Module-2	
Two port networks: Short- circuit Admittance parameters, Open- circuit Impedance parameters, Transmission parameters, Hybrid parameters (Textbook 3: 11.1, 11.2, 11.3, 11.4, 11.5) **Laplace transform and its Applications**: Step Ramp, Impulse, Solution of networks using Laplace transform, Initial value and final value theorem (Textbook 3: 7.1, 7.2, 7.4, 7.7, 8.4)	
Teaching-Learning Process	Chalk and Talk **RBT Level:** L1, L2, L3
Module-3	
Basic Concepts and representation: Types of control systems, effect of feedback systems, differential equation of physical systems (only electrical systems), Introduction to block diagrams, transfer functions, Signal Flow Graphs (Textbook 4: Chapter 1.1, 2.2, 2.4, 2.5, 2.6)	
Teaching-Learning Process	Chalk and Talk, YouTube videos **RBT Level:** L1, L2, L3

Module-4	
Time Response analysis: Time response of first order systems. Time response of second order systems, time response specifications of second order systems (Textbook 4: Chapter 5.3, 5.4) **Stability Analysis:** Concepts of stability necessary condition for stability, Routh stability criterion, relative stability Analysis (Textbook 4: Chapter 5.3, 5.4, 6.1, 6.2, 6.4, 6.5)	
Teaching-Learning Process	Chalk and Talk, Any software tool to show time response **RBT Level:** L1, L2, L3
Module-5	
Root locus: Introduction the root locus concepts, construction of root loci (Textbook 4: 7.1, 7.2, 7.3) **Frequency Domain analysis and stability**: Correlation between time and frequency response and Bode plots (Textbook 4: 8.1, 8.2, 8.4) **State Variable Analysis:** Introduction to state variable analysis: Concepts of state, state variable and state models. State model for Linear continuous –Time systems, solution of state equations. (Textbook 4: 12.2, 12.3, 12.6)	
Teaching-Learning Process	Chalk and Talk, Any software tool to plot Root locus, Bode plot **RBT Level:** L1, L2, L3

Suggested Learning Resources:

Text Books

1. Engineering circuit analysis, William H Hayt, Jr, Jack E Kemmerly, Steven M Durbin, Mc Graw Hill Education, Indian Edition 8e.
2. Networks and Systems, D Roy Choudhury, New age international Publishers, second edition.
3. Network Analysis, M E Van Valkenburg, Pearson, 3e.
4. Control Systems Engineering, I J Nagrath, M. Gopal, New age international Publishers, Fifth edition.

TWO PORT NETWORK

T3- 11.2 Open circuit Impedance

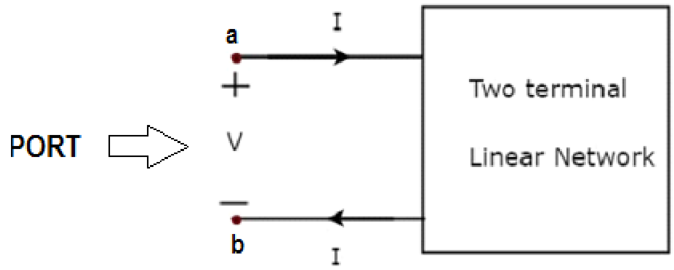
RELATIONSHIP OF TWO PORT VARIABLES

A pair of terminals through which a current may enter or leave a network is known as a port.

One port network is a two terminal electrical network in which, current enters through one terminal and leaves through another terminal.

Resistors, inductors and capacitors are the examples of one port network because each one has two terminals.

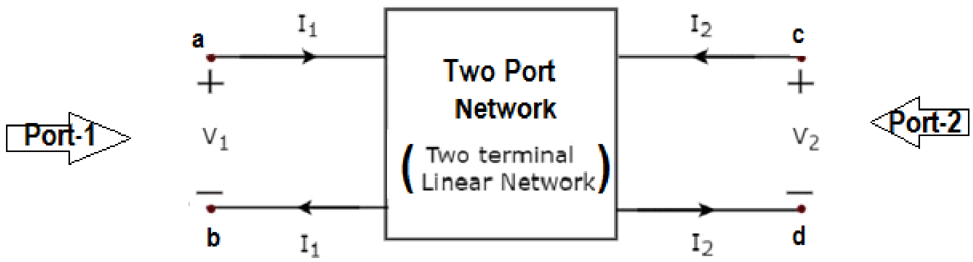
One port network representation is shown in the following figure.



Here, the pair of terminals, a & b represents a port.

Two port networks

Two port network representations is shown in the figure.



Two port networks is a pair of two terminal electrical network in which, current enters through one terminal and leaves through another terminal of each port.

A pair of terminals, a & b represents one port, which is called as **port1**

A pair of terminals, c & d represents another port, which is called as **port2**.

There are **four variables** V_1, V_2, I_1 and I_2 in a two port network as shown in the figure.

Choose two variables as independent and another two variables as dependent.

So, there will be six possible pairs of equations. These equations represent the dependent variables in terms of independent variables.

The coefficients of independent variables are called as **parameters**. So, each pair of equations will give a set of four parameters.

Two Port Network Parameters

The parameters of a two port network are called as **two port network parameters** or simply, two port parameters.

Types of two port network parameters.

- **Open circuit Impedance -Z parameters**
- **Short circuit admittance - Y parameters**
- **Transmission - T parameters**
- **Inverse Transmission -T' parameters**
- **Hybrid - h-parameters**
- **Inverse hybrid - g-parameters**

1) Open circuit Impedance or Z parameters

1) Open circuit Impedance (Z parameters) → *Express* V_1, V_2 → *In terms of* I_1, I_2 → $\left\{ \begin{array}{l} V_1 = z_{11}I_1 + z_{12}I_2 \\ V_2 = z_{21}I_1 + z_{22}I_2 \end{array} \right\}$

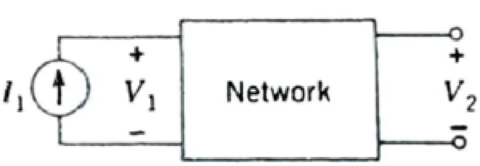
Consider V_1 & V_2 as dependent variables

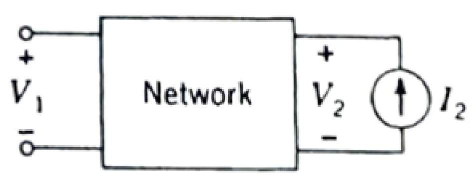
and I_1 & I_2 as independent variables

The coefficients of independent variables, I_1 and I_2 are called as **Z parameters**.

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$





$$Z_{11} = \frac{V_1}{I_1}, \ when\ I_2 = 0$$

$$Z_{12} = \frac{V_1}{I_2}, \ when\ I_1 = 0$$

$$Z_{21} = \frac{V_2}{I_1}, \ when\ I_2 = 0$$

$$Z_{22} = \frac{V_2}{I_2}, \ when\ I_1 = 0$$

z11 ⟶ **is defined as the** ***open-circuit input impedance***

z22 ⟶ **is called the** ***open-circuit output impedance,***

$\left.\begin{array}{l}\textbf{z12}\\ \textbf{z21}\end{array}\right\}$ ⟶ **are called the** ***open-circuit transfer impedances.***

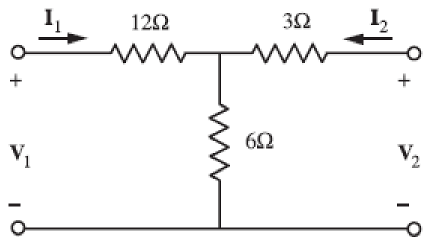
Z parameters are called as **impedance parameters** because these are simply the ratios of voltages and currents. Units of Z parameters are Ohm (Ω).

We can calculate two Z parameters, Z_{11} and Z_{21}, by doing open circuit of port2.

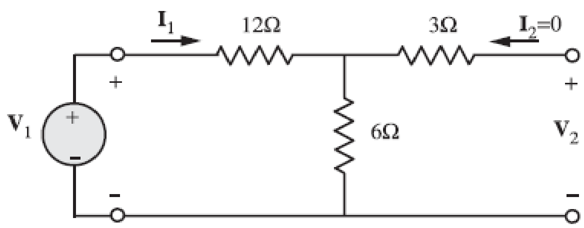
Similarly, we can calculate the other two Z parameters, Z_{12} and Z_{22} by doing open circuit of port1.

Hence, the Z parameters are also called as **open-circuit impedance parameters**.

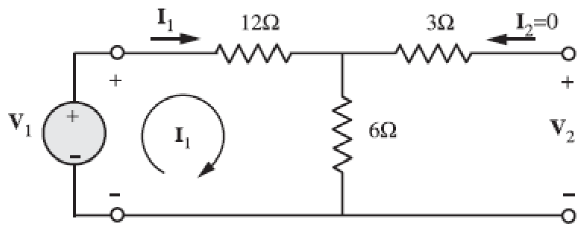
1) **Find the z parameters of this circuit. Then compute the current in a 4Ω load if a $24\angle 0$ V source is connected at the input port.**



<u>To find z_{11} and z_{21}, the output terminals are open circuited</u>.



<u>Applying KVL to the left-mesh,</u>



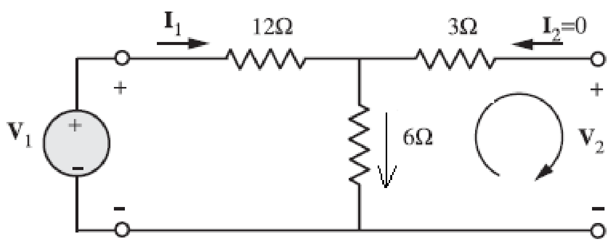
$$\mathbf{V}_1 - 12\mathbf{I}_1 - 6\mathbf{I}_1 = 0$$

$$12\mathbf{I}_1 + 6\mathbf{I}_1 = \mathbf{V}_1$$

$$\mathbf{V}_1 = 18\mathbf{I}_1$$

$$\mathbf{z}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} = 18\Omega$$

Applying KVL to the right-mesh, we get

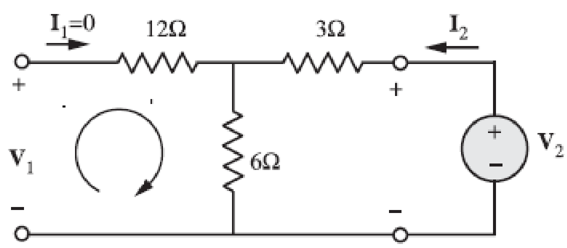


$$-\mathbf{V}_2 + 3\times 0 + 6\mathbf{I}_1 = 0$$

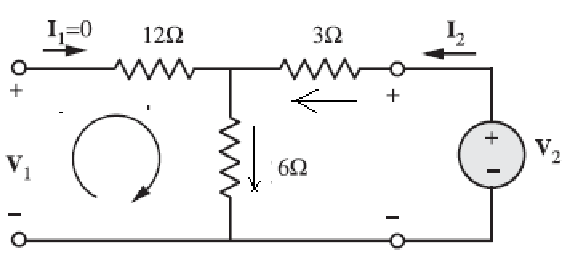
$$\mathbf{V}_2 = 6\mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = 6\Omega$$

To find z_{22} and z_{12}, the input terminals are open circuited



Applying KVL to the left-mesh, we get

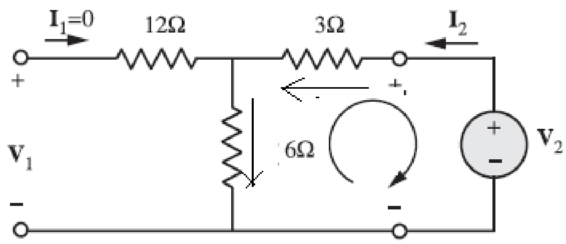


$$\mathbf{V}_1 = 12\times 0 + 6\mathbf{I}_2$$

$$\mathbf{V}_1 = 6\mathbf{I}_2$$

$$\mathbf{z}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} = 6\Omega$$

Applying KVL to the right-mesh, we get



$$-\mathbf{V}_2 + 3\mathbf{I}_2 + 6\mathbf{I}_2 = 0$$

$$\mathbf{V}_2 = 9\mathbf{I}_2$$

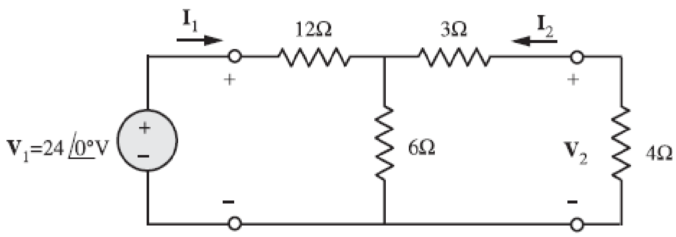
$$\mathbf{z}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} = 9\Omega$$

The equations for the two-port network are, therefore

$$\mathbf{V}_1 = 18\mathbf{I}_1 + 6\mathbf{I}_2$$
$$\mathbf{V}_2 = 6\mathbf{I}_1 + 9\mathbf{I}_2$$

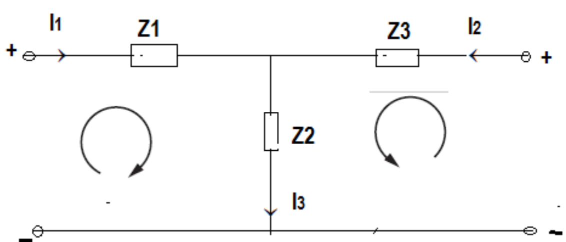
The terminal voltages for the network shown in Fig

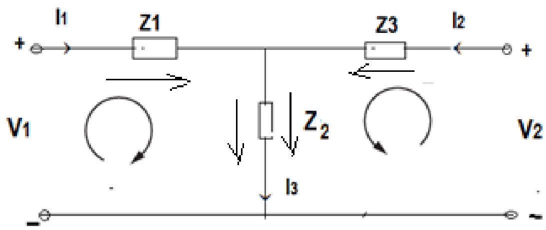
$$\mathbf{V}_1 = 24\underline{/0^\circ}$$
$$\mathbf{V}_2 = -4\mathbf{I}_2$$



$$24\underline{/0^\circ} = 18\mathbf{I}_1 + 6\mathbf{I}_2$$
$$0 = 6\mathbf{I}_1 + 13\mathbf{I}_2$$
$$\mathbf{I}_2 = -0.73\underline{/0^\circ}\ \text{A}$$

Two port network short cut





Apply KVL to left loop

$$+V_1 - Z_1 I_1 - Z_2(I_1 + I_2) = 0$$

$$V_1 = (Z_1 + Z_2) I_1 + Z_2 I_2 \quad \text{............(1)}$$

Apply KVL to Right loop

$$V_2 = Z_3 \cdot I_2 - Z_2(I_1 + I_2) = 0$$

$$V_2 = Z_2 I_1 + (Z_2 + Z_3) I_2 \quad \text{.......(2)}$$

$$V_1 = \underbrace{(Z_1 + Z_2)}_{Z_{11}} I_1 + \underbrace{Z_2}_{Z_{12}} I_2 \quad \text{............(1)}$$

$$V_1 = \underbrace{(Z_1 + Z_2)}_{Z_{11}} I_1 + \underbrace{Z_2}_{Z_{12}} I_2 \quad \text{............(1)}$$

$$V_2 = \underbrace{Z_2}_{Z_{21}} I_1 + \underbrace{(Z_2 + Z_3)}_{Z_{22}} I_2 \quad \text{.......(2)}$$

$Z_{11} = Z_1 + Z_2$ | $Z_1 = Z_{11} - Z_2$

BUT $Z_{12} = Z_{21} = Z_2$

$\therefore Z_2 = Z_{12}$

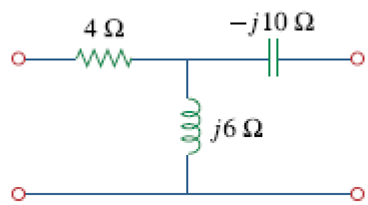
$\boxed{Z_1 = Z_{11} - Z_{12}}$

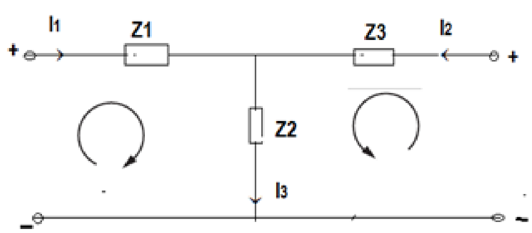
$Z_{22} = Z_2 + Z_3$ | $Z_3 = Z_{22} - Z_2$

BUT $Z_{12} = Z_{21} = Z_2$

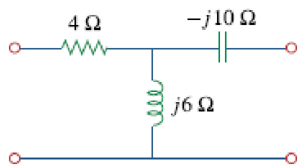
$\therefore \boxed{Z_3 = Z_{22} - Z_{12}}$

3) Find the *z* parameters of the circuit in Fig





$\boxed{Z_{11} = Z_1 + Z_2}$ $\boxed{Z_{22} = Z_2 + Z_3}$ $\boxed{Z_{12} = Z_{21} = Z_2}$



$$Z_{11} = Z_1 + Z_2 = 4 + j6\ \Omega$$

$$Z_{22} = Z_2 + Z_3 = j6 - j10 = -j4\ \Omega$$

$$Z_{12} = Z_{21} = Z_2 = j6\ \Omega$$

4) a circuit that realizes the following z parameters

$$\mathbf{z} = \begin{bmatrix} 12 & 4 \\ 4 & 8 \end{bmatrix}$$

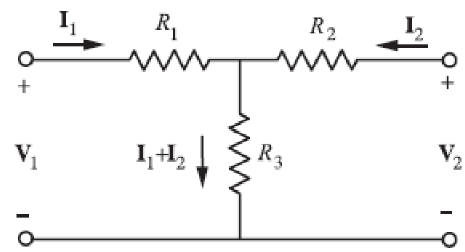
Solution

$$\mathbf{z} = \begin{bmatrix} 12 & 4 \\ 4 & 8 \end{bmatrix}$$

Comparing $\mathbf{z}$ with = $\begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix}$,

$$\mathbf{z}_{11} = 12\Omega, \quad \mathbf{z}_{12} = \mathbf{z}_{21} = 4\Omega, \quad \mathbf{z}_{22} = 8\Omega$$

consider a T network as shown in Fig.



fit in the values of R_1, R_2 and R_3 for the given $\mathbf{z}$.

Applying KVL to the input loop, we get

$$\mathbf{V}_1 = R_1\mathbf{I}_1 + R_3(\mathbf{I}_1 + \mathbf{I}_2)$$

$$= (R_1 + R_3)\mathbf{I}_1 + R_3\mathbf{I}_2$$

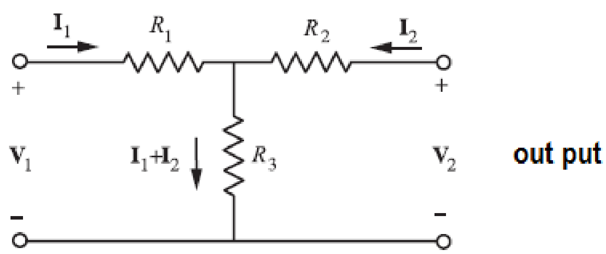
Comparing the preceeding equation with

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$

$$\mathbf{z}_{11} = R_1 + R_3 = 12\Omega$$

$$\mathbf{z}_{12} = R_3 = 4\Omega$$

$$R_1 = 12 - R_3 = 8\Omega$$



Applying *KVL to the output loop*, we get

$$\mathbf{V}_2 = R_2\mathbf{I}_2 + R_3(\mathbf{I}_1 + \mathbf{I}_2)$$
$$\mathbf{V}_2 = R_3\mathbf{I}_1 + (R_2 + R_3)\mathbf{I}_2$$

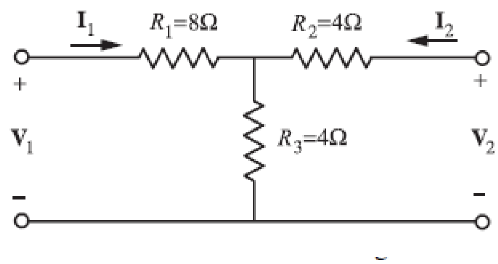
Comparing equation with

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$
$$\mathbf{z}_{21} = R_3 = 4\Omega$$

$$\mathbf{z}_{22} = R_2 + R_3 = 8\Omega$$

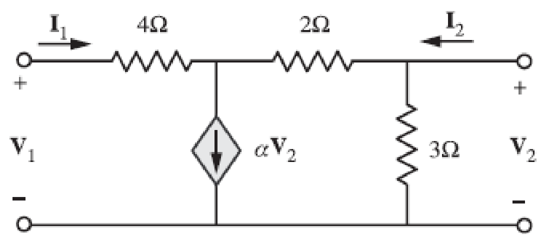
$$R_2 = 8 - R_3 = 4\Omega$$

given **z** parameter set is shown in Fig.



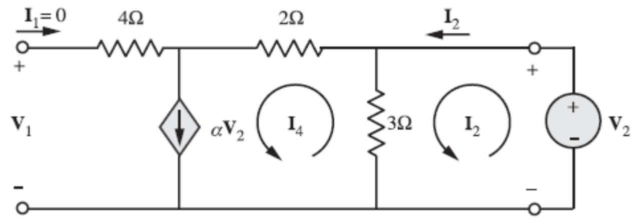
5) Find the z parameters for the network

Take $\alpha = \frac{4}{3}$



To find z11 and z21,open-circuit the output terminals as shown in Fig.

Also, connect a voltage source V2 to the output terminals.



KVL for the mesh on the left:

$$\mathbf{V}_1 + 5\mathbf{I}_4 - 3\mathbf{I}_2 = 0 \quad \text{..............(1)}$$

KVL for the mesh on the right:

$$\mathbf{V}_2 + 3\mathbf{I}_4 - 3\mathbf{I}_2 = 0 \quad \text{....................(2)}$$

Also, $\mathbf{I}_4 = \alpha\mathbf{V}_2$ ....(3)

Substitute eq. 3 in 2

$$\mathbf{V}_2 + 3\alpha\mathbf{V}_2 - 3\mathbf{I}_2 = 0$$

$$\mathbf{V}_2\,(1 + 3\alpha) = 3\mathbf{I}_2$$

$$\text{Hence,} \qquad \mathbf{z}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} = \frac{3}{1+3\alpha}$$

$$= \frac{3}{1+3\left(\frac{4}{3}\right)} = \frac{3}{5}\Omega$$

Substitute eq.3 in 1

$$\mathbf{V}_1 + 5\alpha\mathbf{V}_2 = 3\mathbf{I}_2$$

$$\text{Substituting } \mathbf{V}_2 = \frac{3}{5}\mathbf{I}_2\text{, we get}$$

$$\mathbf{V}_1 + 5\alpha\left(\frac{3}{5} \times \mathbf{I}_2\right) = 3\mathbf{I}_2$$

$$\text{Hence,} \qquad \mathbf{z}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

$$= 3 - 3\alpha$$

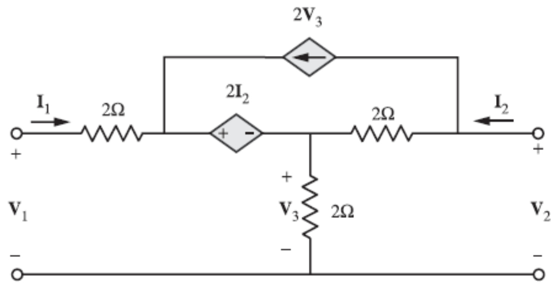
$$= 3 - 3\frac{4}{3} = -1\Omega$$

Finally, in the matrix form, we can write

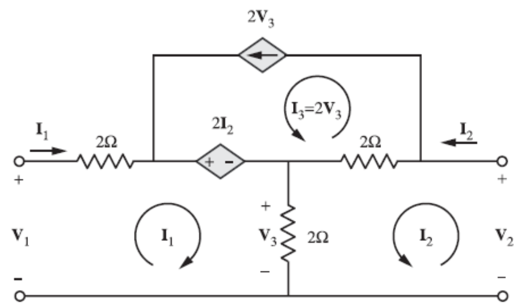
$$\mathbf{z} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ \frac{5}{3} & \frac{3}{5} \end{bmatrix}$$

Please note that $\mathbf{z}_{12} \neq \mathbf{z}_{21}$, since a dependent source is present in the circuit.

6) Find the impedance parameters of the network shown



Solution



$$\mathbf{V}_3 = 2(\mathbf{I}_1 + \mathbf{I}_2)$$

KVL for mesh 1:

$$2\mathbf{I}_1 + 2\mathbf{I}_2 + 2(\mathbf{I}_1 + \mathbf{I}_2) = \mathbf{V}_1$$

$$4\mathbf{I}_1 + 4\mathbf{I}_2 = \mathbf{V}_1$$

KVL for mesh 2:

$$2(\mathbf{I}_2 - 2\mathbf{V}_3) + 2(\mathbf{I}_1 + \mathbf{I}_2) = \mathbf{V}_2$$

$$2\mathbf{I}_2 - 4 \times 2(\mathbf{I}_1 + \mathbf{I}_2) + 2(\mathbf{I}_1 + \mathbf{I}_2) = \mathbf{V}_2$$

$$2\mathbf{I}_2 - 6(\mathbf{I}_1 + \mathbf{I}_2) = \mathbf{V}_2$$

$$-6\mathbf{I}_1 - 4\mathbf{I}_2 = \mathbf{V}_2$$

$$\mathbf{z}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} = \left.\frac{4\mathbf{I}_1 + 4\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} = 4\Omega$$

$$\mathbf{z}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} = \left.\frac{-6\mathbf{I}_1 - 4\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} = -6\Omega$$

$$\mathbf{z}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} = \left.\frac{4\mathbf{I}_1 + 4\mathbf{I}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} = 4\Omega$$

$$\mathbf{z}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} = \left.\frac{-6\mathbf{I}_1 - 4\mathbf{I}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} = -4\Omega$$

………………………………