Model Question Paper-I with effect from 2022

USN | | | | | | | | | | |

Fourth Semester B.E Degree Examination

Complex Analysis, Probability & Statistical Methods

All branches Except CS & ME Engg.Allied branches-21MAT41

TIME: 03 Hours **Max. Marks: 100**

Note: Answer any **FIVE** full questions, choosing at least **ONE** question from each module.

Q.No.		Question	M	L	CO
		Module -1			
01	a	Define analytic function and derive C-R equations in Cartesian form.	**06**	**L2**	**CO1**
	b	Show that $f(z) = logz$ is analytic and hence obtain its derivative.	**07**	**L2**	**CO1**
	c	Evaluate $\int_0^{1+i} (x^2 - iy)dz$ along the curve $y = x^2$.	**07**	**L3**	**CO1**
		OR			
02	a	Construct an analytic function, whose imaginary part is $v = e^x(xSiny + yCosy)$ by the Milne-Thomson method	**06**	**L2**	**CO1**
	b	State and prove Cauchy's integral formula.	**07**	**L2**	**CO1**
	c	Evaluate $\int_C \frac{Sin\pi z^2 + Cos\pi z^2}{(z-1)\ (z-2)}dz$, where $C: \lvert z\rvert = 3$.	**07**	**L3**	**CO1**
		Module-2			
03	a	Obtain the series solution of Bessel's differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 + n^2)y = 0$	**06**	**L2**	**CO2**
	b	Show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$.	**07**	**L2**	**CO2**
	c	Express $x^3 - 5x^2 + 6x + 1$ in terms of Legendre polynomial	07	**L2**	**CO2**
		OR			
4	a	Show that $J_{-n}(x) = (-1)^n J_n(x)$		**L2**	**CO2**
	b	Show that $P_4(\cos\theta) = \frac{1}{64}(35\cos 4\theta + 2\theta\cos\theta + 9)$.	**07**	**L2**	**CO2**
	c	Prove that $x^3 - 2x^2 - x - 3 = \frac{2}{5}P_3(x) + \frac{4}{3}P_2(x) - \frac{2}{5}P_1(x) - \frac{7}{3}P_0(x)$	**07**	**L2**	**CO2**
		Module-3			
5	a	Find Karl Pearson's coefficient of correlation.	**06**	**L2**	**CO3**

x:	1	2	3	4	5	6	7
y:	9	8	10	12	11	13	14

	b	Fit a straight line $y = ax + b$ for the data x: 5, 10, 15, 20, 25 y: 16, 19, 23, 26, 30	**07**	**L2**	**CO3**
	c	Find the regression lines of y on x and x on y for the following data x: 1, 2, 3, 4, 5 y: 2, 5, 3, 8, 7	**07**	**L2**	**CO3**
		OR			
6	a	The participants in a contest are ranked by two judges as follows. x: 1, 6, 5, 10, 3, 2, 4, 9, 7, 8 y: 6, 4, 9, 8, 1, 2, 3, 10, 5, 7 Compute the Rank correlation.	**06**	**L2**	**CO3**
	b	Compute means $\bar{x}$, $\bar{y}$ and the correlation coefficient r from the given regression lines $2x + 3y + 1 = 0, x + 6y = 4$.	**07**	**L2**	**CO3**
	c	Fit a second degree polynomial $y = ax^2 + bx + c$ for the data. x: 1, 2, 3, 4, 5 y: 10, 12, 13, 16, 19	**07**	**L2**	**CO3**
		Module-4			
7	a	A random variable X has the following probability function: X: -3, -2, -1, 0, 1, 2, 3 $P(X)$: k, $2k$, $3k$, $4k$, $3k$, $2k$, k Find k. Also find $P(X \le 1)$, $P(X > 1)$, $P(-1 < X \le 2)$,	**06**	**L2**	**CO4**
	b	Find the mean and variance of Binomial distribution.	**07**	**L2**	**CO4**
	c	In a certain factory turning out razor blades there is a small probability of $\frac{1}{500}$ for any blade to be defective. The blades are supplied in a packets of 10. Use Poisson distribution to calculate approximate number of packets containing i) No defective ii) Two defective iii) Three defective in consignment of 10000 packets.	**07**	**L3**	**CO4**
		OR			
8	a	A random variable X has density function: $f(x) = \begin{cases} kx^2 & -3 \le x \le 3 \\ 0 & Otherwise \end{cases}$, Find k. Also, find $P(X \le 2)$, $P(X \ge 2)$ and $P(X > 1)$.	**06**	**L2**	**CO4**

	b	The probability that a pen manufactured by a company will be defective is 0.1. If 12 such pens are selected at random, find the probability that i) Exactly two pens will be defective ii) At most two pens will be defective iii) None will be defective	**07**	**L2**	**CO4**
	c	The marks of 1000 students in an examination follow the normal distribution with mean 70 and standard deviation 5. Find the number students whose marks will be i) Less than 65 i) More than 75 iii) Between 65 and 75.	**07**	**L3**	**CO4**
		Module-5			
9	a	The joint distribution of two random variables X and Y is as follows. Compute the following. i) E(X) and E(Y) ii) E(XY) iii) σ_X & σ_Y	**06**	L2	**CO5**
	b	Define i) Null hypothesis ii) Type-I & Type-II errors iii) Degrees of freedom iv) Level of Significance.	**07**	L2	**CO5**
	c	Two types of batteries are tested for their length of life and the following results are obtained: Battery A: $n_1 = 10$ $\bar{x}_1 = 500\,Hrs.$ $\sigma_1^2 = 100$ Battery B: $n_2 = 10$ $\bar{x}_2 = 506\,Hrs.$ $\sigma_2^2 = 121$ Compute Student's t and test whether there is a significant difference in the two means at 5% significance level.	**07**	L3	**CO5**
		OR			
10	a	Determine (i) Marginal distributions (ii) Covariance between the variables X and Y, If the joint probability distribution is given by:	**06**	**L2**	**CO5**

Y / X	-4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

Y / X	3	4	5
2	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
5	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$
7	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

	b	Ten individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that the mean height of the universe is 66 inches at 5% significance level. ($t_{0.05}$ =2.262 for 9 d.f.)	**07**	**L3**	**CO5**
	c	In experiments on pea breeding the following frequencies of seeds were obtained: Round and Yellow: 315; Wrinkled and Yellow: 101; Round and Green: 108; Wrinkled and Green: 32; Total: 556 Theory predicts that the frequencies should be in proportions 9: 3: 3: 1. Examine the correspondence between theory and experiment	**07**	**L3**	**CO5**

Bloom's Taxonomy Levels	**Lower-order thinking skills**		
	Remembering (knowledge): L_1	Understanding (Comprehension): L_2	Applying (Application): L_3
	Higher-order thinking skills		
	Analyzing (Analysis): L_4	Valuating (Evaluation): L_5	Creating (Synthesis): L_6