4.2 • THE SUPERNODE

VISVESVARAYA TECHNOLOGICAL UNIVERSITY, BELAGAVI
B.E: Electronics & Communication Engineering / B.E: Electronics & Telecommunication Engineering
NEP, Outcome Based Education (OBE) and Choice Based Credit System (CBCS)
(Effective from the academic year 2021 – 22)

IV Semester

Circuits & Controls			
Course Code	**21EC43**	CIE Marks	50
Teaching Hours/Week (L: T: P: S)	(3:0:2:0)	SEE Marks	50
Total Hours of Pedagogy	40 hours Theory + 13 Lab slots	Total Marks	100
Credits	04	Exam Hours	03

Module-1	
Basic concepts and network theorems Types of Sources, Loop analysis, Nodal analysis with independent DC and AC Excitations. (Textbook 1: 2.3, 4.1, 4.2, 4.3, 4.4, 10.6) Super position theorem, Thevenin's theorem, Norton's Theorem, Maximum Power transfer Theorem. (Textbook 2: 9.2, 9.4, 9.5, 9.7)	
Teaching-Learning Process	Chalk and Talk, YouTube videos, Demonstrate the concepts using circuits **RBT Level:** L1, L2, L3

Module-2	
Two port networks: Short- circuit Admittance parameters, Open- circuit Impedance parameters, Transmission parameters, Hybrid parameters (Textbook 3: 11.1, 11.2, 11.3, 11.4, 11.5) **Laplace transform and its Applications**: Step Ramp, Impulse, Solution of networks using Laplace transform, Initial value and final value theorem (Textbook 3: 7.1, 7.2, 7.4, 7.7, 8.4)	
Teaching-Learning Process	Chalk and Talk **RBT Level:** L1, L2, L3
Module-3	
Basic Concepts and representation: Types of control systems, effect of feedback systems, differential equation of physical systems (only electrical systems), Introduction to block diagrams, transfer functions, Signal Flow Graphs (Textbook 4: Chapter 1.1, 2.2, 2.4, 2.5, 2.6)	
Teaching-Learning Process	Chalk and Talk, YouTube videos **RBT Level:** L1, L2, L3

Module-4	
Time Response analysis: Time response of first order systems. Time response of second order systems, time response specifications of second order systems (Textbook 4: Chapter 5.3, 5.4) **Stability Analysis:** Concepts of stability necessary condition for stability, Routh stability criterion, relative stability Analysis (Textbook 4: Chapter 5.3, 5.4, 6.1, 6.2, 6.4, 6.5)	
Teaching-Learning Process	Chalk and Talk, Any software tool to show time response **RBT Level:** L1, L2, L3
Module-5	
Root locus: Introduction the root locus concepts, construction of root loci (Textbook 4: 7.1, 7.2, 7.3) **Frequency Domain analysis and stability**: Correlation between time and frequency response and Bode plots (Textbook 4: 8.1, 8.2, 8.4) **State Variable Analysis:** Introduction to state variable analysis: Concepts of state, state variable and state models. State model for Linear continuous –Time systems, solution of state equations. (Textbook 4: 12.2, 12.3, 12.6)	
Teaching-Learning Process	Chalk and Talk, Any software tool to plot Root locus, Bode plot **RBT Level:** L1, L2, L3

Suggested Learning Resources:

Text Books

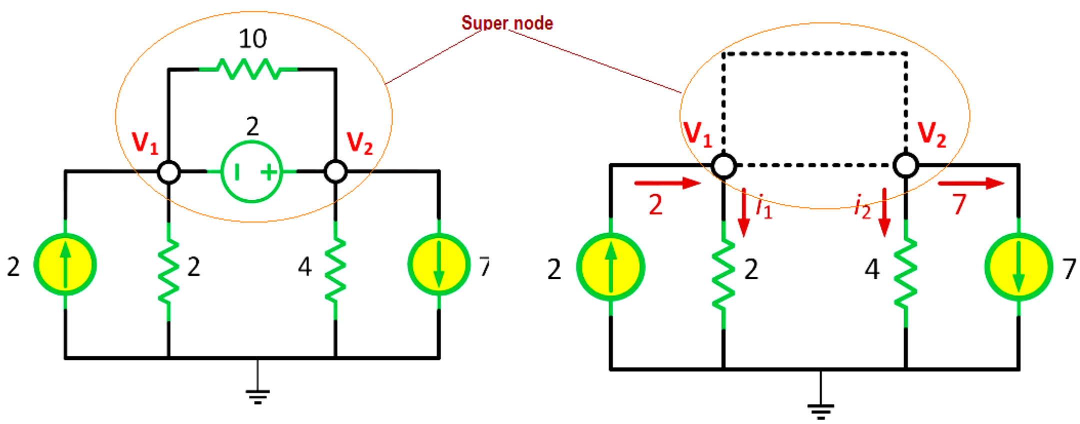
1. Engineering circuit analysis, William H Hayt, Jr, Jack E Kemmerly, Steven M Durbin, Mc Graw Hill Education, Indian Edition 8e.
2. Networks and Systems, D Roy Choudhury, New age international Publishers, second edition.
3. Network Analysis, M E Van Valkenburg, Pearson, 3e.
4. Control Systems Engineering, I J Nagrath, M. Gopal, New age international Publishers, Fifth edition.

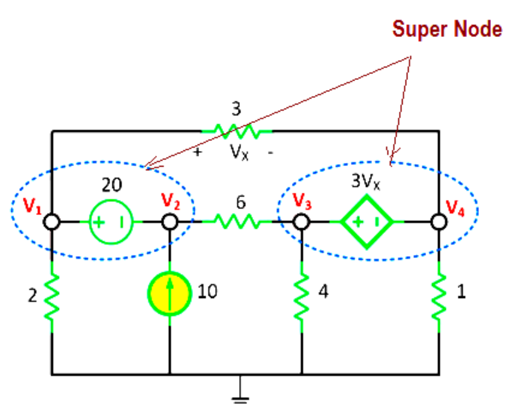
4.2 • THE SUPERNODE

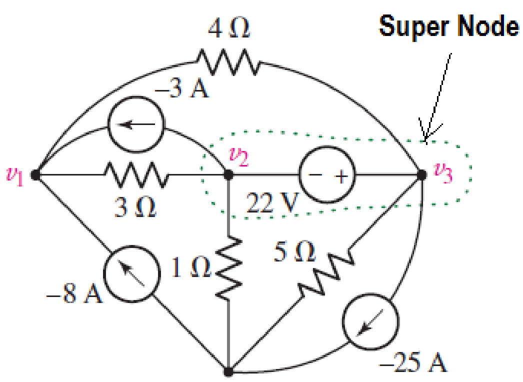
Whenever a voltage source (Independent or Dependent) is connected between the two non reference nodes then it is called the Super node. That is it is a surface enclosing the voltage source and its two nodes.

A super node is formed by enclosing a (dependent or independent) voltage source connected between two non reference nodes and any elements connected in parallel with it.

A super node requires the application of both KCL and KVL to solve it. Any element can be connected in parallel with the voltage source form to the super node

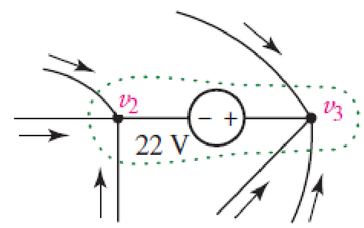






Reference node

(a)



(b)

By KCL Current leaving leaving 2=0 and Current leaving node 3 =0 therefore the total current from super node=0

It is not possible to apply KCL at the node 2 and 3 because we don't know the current flowing through the branch with the voltage source

By using super node analysis we can find the solution by applying KCL to both nodes (Nod 2 and Node 3) at once.

The total current leaving Node 2 is zero (0) and the total current leaving Node 3 is zero (0), then the total current leaving the combination of the two nodes and voltage together is zero.

This concept is shown in the following fig (b) with the super node (the area enclosed by the broken line. apply KVL (Kirchhoff's Voltage Law) which is $v_3 - v_2 = 22V$ between Node 2 and Node 3.

Steps for Super node Analysis

1. **Count the number of nodes (*N*).**

2. **Identify the reference node.**
The node with the greatest number of branches connected to it.

3. **Find the number voltage nodes= N-1**

4. **Check for super nodes and mark by dotted line**

5. **Write a KCL equation for each non reference node and for**

each super node *that does not contain the reference node.* Sum the currents flowing *into* a node/super node from current sources on one side of the equation. On the other side, sum the currents flowing *out* of the node/super node through resistors. Pay close attention to "–" signs.

6. **Relate the voltage across each voltage source to nodal voltages. This is accomplished by simple application of KVL; one such equation is needed for each super node defined.**

7. **Express any additional unknowns (i.e., currents or voltages other than nodal voltages) in terms of appropriate nodal voltages.** This situation can occur if dependent sources appear in our circuit.

8. **Organize the equations.** Group terms according to nodal voltages.

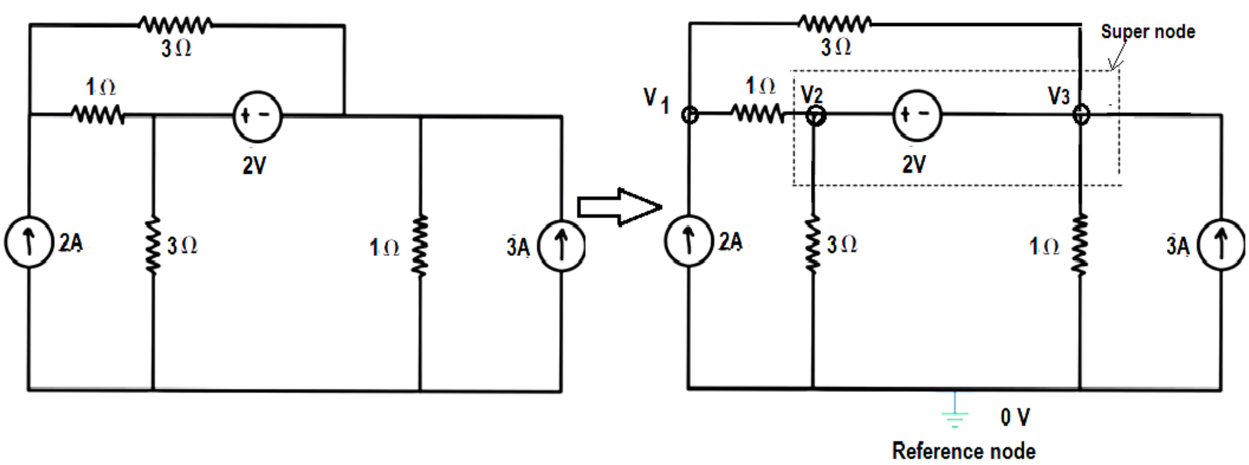
9. **Solve the system of equations for the nodal voltages** (there will be $N-1$ of them).

PROBLEMS

1) Find the nodal voltages V1, V2, and V3 of the circuit

1) Solution:

First, we redraw the circuit as shown in fig



<u>Steps for Super node Analysis</u>

1. **Count the number of nodes (*N*).**
Total number of nodes = N = 4

2. **Identify the reference node.**

The node with the greatest number of branches connected to it.

3. **Find the number voltage nodes= N-1**
Total number of voltage nodes = N -1 = 4 – 1 = 3

4. **Check for super nodes and mark by dotted line**

5. **Write a KCL equation for each non reference node and for each super node *that does not contain the reference node.***

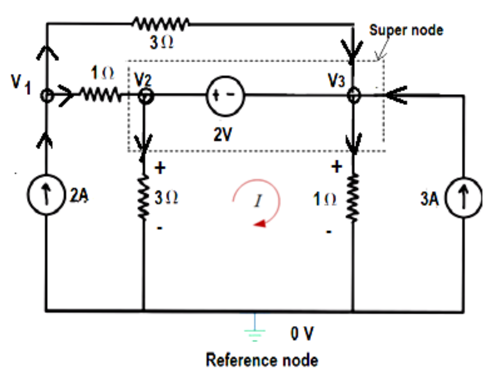
Apply KCL at node-1

$$+2 - \frac{V_1 - V_2}{1} - \frac{V_1 - V_3}{3} = 0$$

$$+2 = \frac{V_1 - V_2}{1} + \frac{V_1 - V_3}{3}$$

$$6 = 4V_1 - 3V_2 - V_3 \quad \text{———} (1)$$

Apply KCL to super nodes 2 and 3



Current entering the super node = current leaving the super node

$$\frac{V_1 - V_2}{1} + \frac{V_1 - V_3}{3} + 3 = \frac{V_2}{3} + \frac{V_3}{1}$$

$$3V_1 - 3V_2 + V_1 - V_3 + 9 = V_2 + 3V_3$$

$$4V_1 - 4V_2 - 4V_3 + 9 = 0 \quad \text{-----} (2)$$

Reference

Note the direction of current . current entering the node

$$V_1 \longrightarrow V_2 \;\therefore \longrightarrow \frac{V_1 - V_2}{1}$$

$$V_1 \longrightarrow V_3 \;\therefore \longrightarrow \frac{V_1 - V_3}{3}$$

Super node Eq: is the relationship between the two nodes

Apply KCL at super node loop

$$+V_2 - 2 - V_3 = 0$$

$$\boxed{V_2 - V_3 = 2V} \quad \text{------------------ (3)}$$

$$V_2 = V_3 + 2$$

$$6 = 4V_1 - 3V_2 - V_3 \quad \text{———— (1)}$$

$$V_2 = V_3 + 2 \quad \text{------------------ (3)}$$

Substitute V_2 in Eq.(1)

$$6 = 4V_1 - 3(V_3 + 2) - V_3$$

$$V_1 = V_3 + 3 \quad \text{--------(4)}$$

Substitute V_1 & V_2 from eq (4) & (3) in Eq (2)

$$4V_1 - 4V_2 - 4V_3 + 9 = 0 \qquad \text{----- (2)}$$

$$V_1 = V_3 + 3 \qquad\qquad V_2 = V_3 + 2$$

$$4(V_3 - 3) - 4(V_3 + 2) - 4V_3 + 9 = 0$$

$$4V_3 + 13 = 0$$

$$\boxed{V_3 = 13/4 = 3.25\text{ V}}$$

$$V_2 = V_3 + 2$$

Substitute $V_3 = 3.25$ V

$$\boxed{V_2 = 5.25\text{ V}}$$

$$V_1 = V_3 + 3$$

Substitute $V_3 = 3.25$ V

$$V_1 = 3.25 + 3$$

$$\boxed{V_1 = 6.25\text{ V}}$$

$$\boxed{V_1 = 6.25\text{ V}}$$

$$\boxed{V_2 = 5.25\text{ V}}$$

$$\boxed{V_3 = 3.25\text{ V}}$$

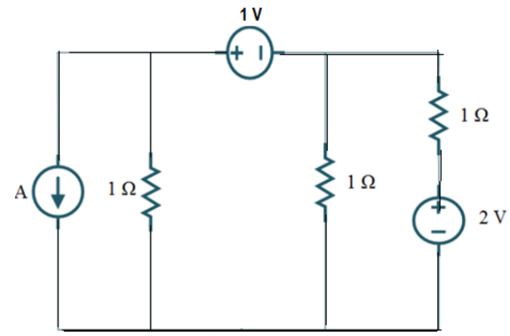
6. **Relate the voltage across each voltage source to nodal voltages. This is accomplished by simple application of KVL; one such equation is needed for each super node defined.**

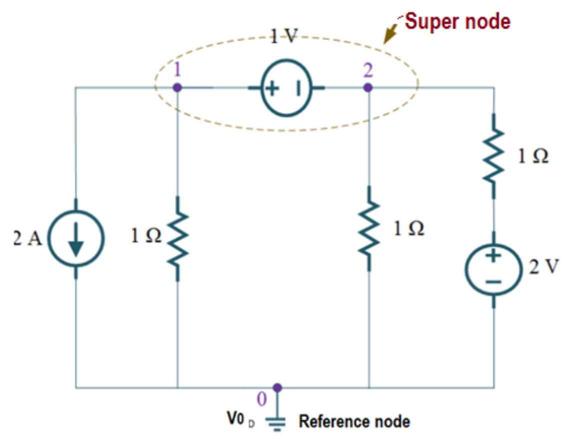
7. **Express any additional unknowns (i.e., currents or voltages other than nodal voltages) in terms of appropriate nodal voltages.** This situation can occur if dependent sources appear in our circuit.

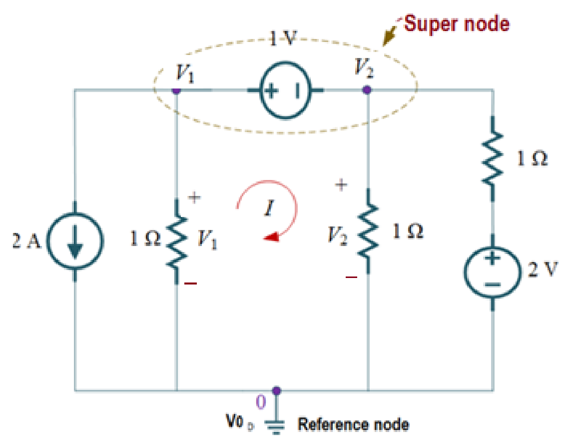
8. **Organize the equations.** Group terms according to nodal voltages.

9. **Solve the system of equations for the nodal voltages** (there will be $N - 1$ of them)

2) For the given network, find nodal voltages V_1 and V_2.







Steps for Super node Analysis

1. **Count the number of nodes (*N*).**
N = 3
2. **Identify the reference node.**
The node with the greatest number of branches connected to it.

3. **Find the number voltage nodes= N-1**

No: of voltage node = 3 -1 = 2

4. **Check for super nodes and mark by dotted line**

5. **Write a KCL equation for each non reference node and for each super node *that does not contain the reference node.***

Sum the currents flowing into a node/super node from current sources on one side of the equation. On the other side, sum the currents flowing out of the node/super node through resistors. Pay close attention to "–" signs.

Apply KCL at super node-

$$2+\frac{(V_1-0)}{1}+\frac{(V_2-0)}{1}+\frac{(V_2-2)}{1}=0$$

$$V_1+2V_2=0 \quad ......(1)$$

Super node Eq.

6. **Relate the voltage across each voltage source to nodal voltages. This is accomplished by simple application of KVL; one such equation is needed for each super node defined.**

Super node Eq.

Apply KVL super node loop

$$V_1-1-V_2=0$$

$$V_1-V_2=1 \quad ......(2)$$

$$V_2=-\frac{1}{3}\text{ V}$$

Put Eq.(2) in Eq.(1), we get

$$3V_2+1=0$$

$$\boxed{V_2=-\frac{1}{3}\text{ V}}$$

From Eq.(1),

$$V_1+2V_2=0$$

$$V_1+2\left(-\frac{1}{3}\right)=0$$

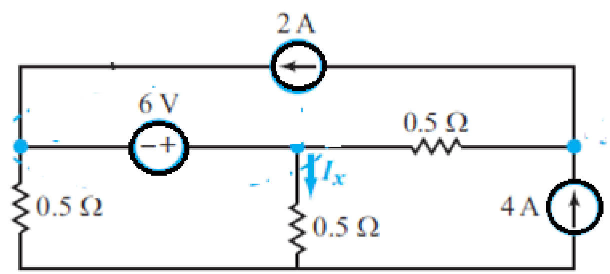
$$\boxed{V_1=\frac{2}{3}\text{ V}}$$

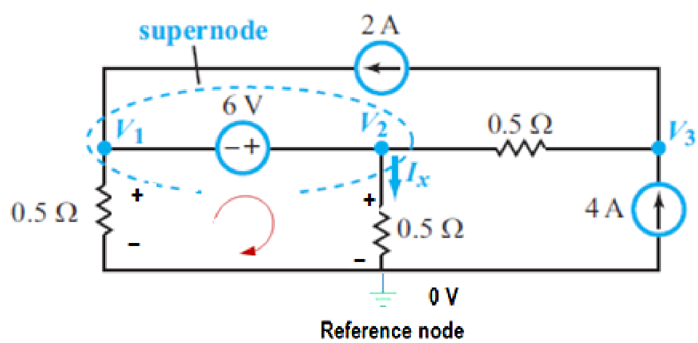
7. **Express any additional unknowns (i.e., currents or voltages other than nodal voltages) in terms of appropriate nodal voltages.** This situation can occur if dependent sources appear in our circuit.

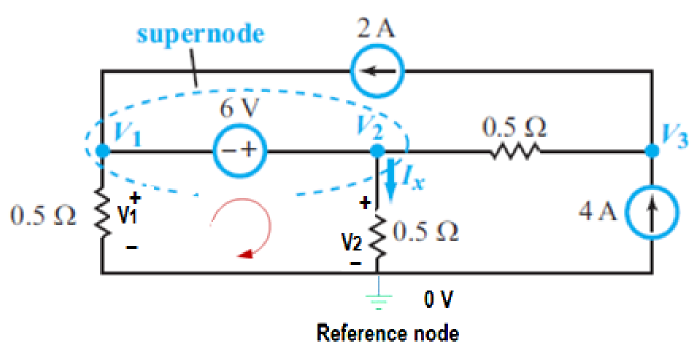
8. **Organize the equations.** Group terms according to nodal voltages.

9. **Solve the system of equations for the nodal voltages** (there will be $N - 1$ of them).

3) Use the supernode concept to find the current I_x in the circuit of **Fig.**







<u>**Steps for Super node Analysis**</u>

1. **Count the number of nodes (*N*).**

N = 3

2. **Identify the reference node.**
The node with the greatest number of branches connected to it.

3. **Find the number voltage nodes= N-1**

No: of voltage nodes= N – 1 = 3-1 = 2

4. **Check for super nodes and mark by dotted line**

5. **Write a KCL equation for each non reference node and for each super node *that does not contain the reference node*.** *Sum the currents flowing into a node/super node from current sources on one side of the equation. On the other side, sum the currents flowing out of the node/super node through resistors. Pay close attention to "−" signs.*

Apply KCL at super node-1 and 2

$$\frac{V_1}{0.5} - 2 + \frac{V_2}{0.5} + \frac{V_2 - V_3}{0.5} = 0. \qquad (1)$$

Apply KCL at super node – 3

$$\frac{V_3 - V_2}{0.5} - 4 + 2 = 0, \qquad (2)$$

6. **Relate the voltage across each voltage source to nodal voltages. This is accomplished by simple application of KVL; one such equation is needed for each super node defined.**
Super node Eq.

Apply KVL super node loop

$$V_1 - V_2 = -6. \qquad (3)$$

$V_1 = -2$ V, $V_2 = 4$ V, $V_3 = 5$ V.

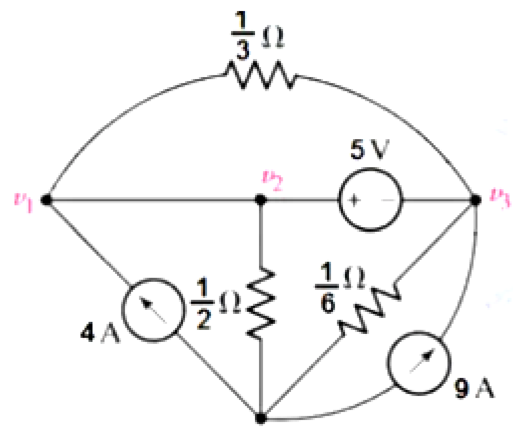
$$I_x = \frac{V_2}{0.5} = \frac{4}{0.5} = 8 \text{ A}.$$

7. **Express any additional unknowns (i.e., currents or voltages other than nodal voltages) in terms of appropriate nodal voltages.** This situation can occur if dependent sources appear in our circuit.

8. **Organize the equations.** Group terms according to nodal voltages.

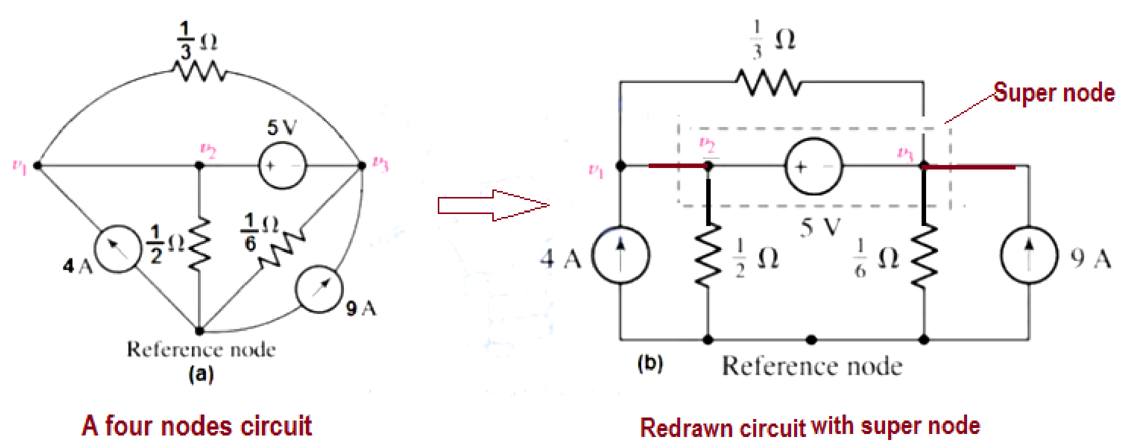
9. **Solve the system of equations for the nodal voltages** (there will be $N-1$ of them).

(4)Use Super node analysis to find voltage across each current source i.e V1 and V2 in the following fig



Solution:

First, we redraw the circuit as shown in fig



<u>**Steps for Super node Analysis**</u>

1. **Count the number of nodes (N).**

Total number of nodes N = 4

2. **Identify the reference node.**

The node with the greatest number of branches connected to it.

3. **Find the number voltage nodes= N-1**

Number of voltage nodes N – 1 = 4 -1 =

4. **Check for super nodes and mark by dotted line**

5. **Write a KCL equation for each non reference node and for each super node *that does not contain the reference node.***

Apply KCL at the node -1

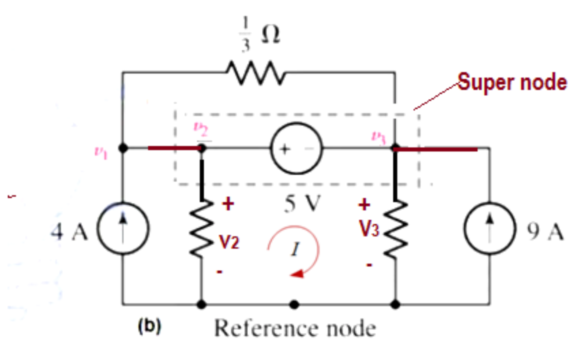
$$+4 - \frac{V1 - V2}{0} - \frac{V1 - V3}{1/3} = 0 \quad \text{-----} \quad (1)$$

Apply KCL at the super nodes -2 and 3

$$9 = \frac{V2}{1/2} + \frac{V3}{1/6} + \frac{V3 - V1}{1/3} + \frac{V3 - V2}{0} \quad \text{----------} \quad (2)$$

6. **Relate the voltage across each voltage source to nodal voltages. This is accomplished by simple application of KVL; one such equation is needed for each super node defined.**

Apply KVL



$$V_2 - 5 - V_3 = 0$$

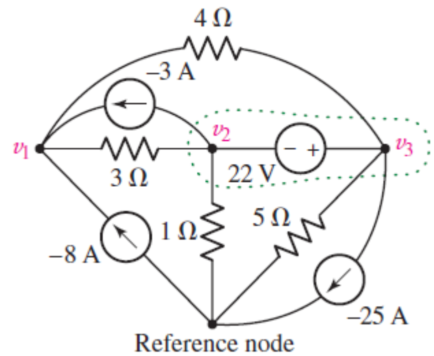
$$V_2 - V_3 = 5 \quad \text{----------} \quad (3)$$

From the Eq 1, 2, 3

The voltage across 4A current source V1 = 5.375 V

The voltage across 9A current source V3 = 375 mV.

(5) Determine the value of the unknown node voltage v_1 in the circuit of Fig.



(*a*)

Steps for Super node Analysis

1. **Count the number of nodes (*N*).**

Total number of nodes N = 4

2. **Identify the reference node.**

The node with the greatest number of branches connected to it.

3. **Find the number voltage nodes= N-1**

Number of voltage nodes N – 1 = 4-1= 3

4. **Check for super nodes and mark by dotted line**

5. **Write a KCL equation for each non reference node and for each super node *that does not contain the reference node.***

Apply KCL at the node 1

$$-8-3=\frac{v_1-v_2}{3}+\frac{v_1-v_3}{4}$$

or

$$0.5833v_1-0.3333v_2-0.2500v_3=-11 \qquad (1)$$

Apply KCL to super node 2 and 3

Next we consider the 2-3 super node.

$$3+25=\frac{v_2-v_1}{3}+\frac{v_3-v_1}{4}+\frac{v_3}{5}+\frac{v_2}{1}$$

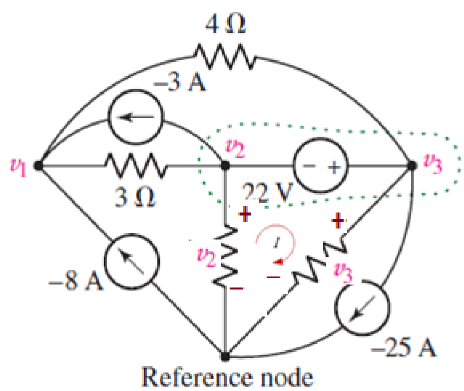
or

$$-0.5833v_1+1.3333v_2+0.45v_3=28 \qquad (2)$$

6. **<u>Relate the voltage across each voltage source to nodal voltages.</u>**
This is accomplished by simple application of KVL; one such equation is needed for each super node defined.

Since we have three unknowns, we need one additional equation, and it must utilize the fact that there is a 22 V voltage source between Nodes 2 and 3:

Apply KVL to get super node Eq



(a)

Apply KVL

V2 + 22 − V3 = 0

V2 − V3 = −22 ----------- **(3)**

From 1, 2 ,3 the solution for v_1 is 1.071 V.

7. **Express any additional unknowns (i.e., currents or voltages other than nodal voltages) in terms of appropriate nodal voltages.** This situation can occur if dependent sources appear in our circuit.

8. **Organize the equations.** Group terms according to nodal voltages.

9. **Solve the system of equations for the nodal voltages** (there will be $N-1$ of them).

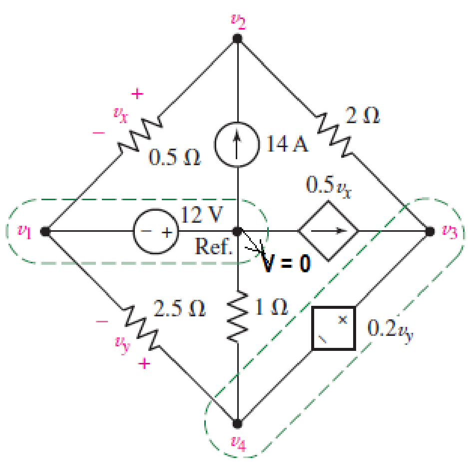
6. **Relate the voltage across each voltage source to nodal voltages.** This is accomplished by simple application of KVL; one such equation is needed for each supernode defined.

7. **Express any additional unknowns (i.e., currents or voltages other than nodal voltages) in terms of appropriate nodal voltages.** This situation can occur if dependent sources appear in our circuit.

8. **Organize the equations.** Group terms according to nodal voltages.

9. **Solve the system of equations for the nodal voltages** (there will be $N-1$ of them).

(6) Determine the node-to-reference voltages in the circuit of Fig



The circuit contains all four types of sources and has five nodes.

<u>**Steps for Super node Analysis**</u>

1. **Count the number of nodes (N).**

N = 5

2. **Identify the reference node.**

The node with the greatest number of branches connected to it.

3. **Find the number voltage nodes= N-1**

Number of voltage node- N – 1 = 5 – 1 = 4

4. **Check for super nodes and mark by dotted line**

5. **Write a KCL equation for each non reference node and for each super node** ***that does not contain the reference node***

Sum the currents flowing into a node/super node from current sources on one side of the equation. On the other side, sum the currents flowing out of the node/super node through resistors. Pay close attention to "–" signs.

KCL at node 1

At ref. voltage V= 0

Therefore $V_1 = -12$ V.

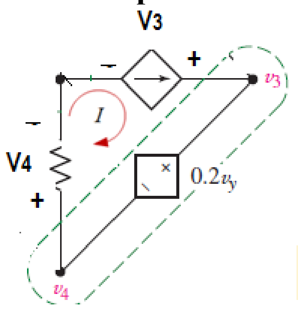
At node 2,

$$\frac{v_2 - v_1}{0.5} + \frac{v_2 - v_3}{2} = 14 \quad \text{-----------------------} \quad (1)$$

At super node 3 and 4

$$0.5v_x = \frac{v_3 - v_2}{2} + \frac{v_4}{1} + \frac{v_4 - v_1}{2.5} \quad \text{-----------------------}(2)$$

6. **Relate the voltage across each voltage source to nodal voltages. This is accomplished by simple application of KVL; one such equation is needed for each super node defined.**



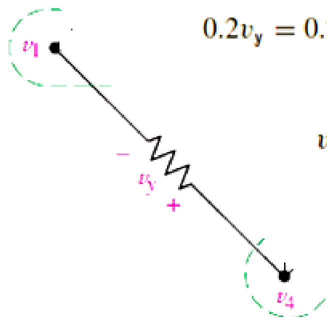
$V_3 - 2V_y - V_4 = 0$

$$v_3 - v_4 = 0.2v_y \quad \text{---------------}(3)$$

7. **Express any additional unknowns (i.e., currents or voltages other than nodal voltages) in terms of appropriate nodal voltages.** This situation can occur if dependent sources appear in our circuit.

Finally, we express the dependent current source in terms of the assigned variables:

$$0.2v_y = 0.2(v_4 - v_1)$$

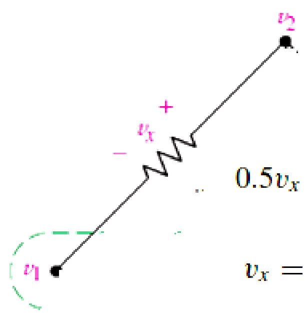


$$0.2v_y = 0.2(v_4 - v_1)$$

$$v_y = \frac{0.2(v_4 - v_1)}{0.2}$$

$$v_y = (v_4 - v_1)$$

$$0.5v_x = 0.5(v_2 - v_1)$$



$$0.5v_x = 0.5(v_2 - v_1)$$

$$v_x = \frac{0.5(v_2 - v_1)}{0.5}$$

$$v_x = (v_2 - v_1)$$

We can now eliminate *Vx* and *Vy* to obtain a set of four equations in the four node voltages:

$$\frac{v_2 - v_1}{0.5} + \frac{v_2 - v_3}{2} = 14 \quad \text{---------------------} \quad \textbf{(1)}$$

$$-2v_1 + 2.5v_2 - 0.5v_3 = 14 \quad \text{---------------------} \quad \textbf{(1)}$$

$$0.5v_x = \frac{v_3 - v_2}{2} + \frac{v_4}{1} + \frac{v_4 - v_1}{2.5} \quad \text{---------------------}\textbf{(2)}$$

$$v_x = (v_2 - v_1)$$

$$v_x = (v_2 - v_1)$$

$$v_y = (v_4 - v_1)$$

$$0.1v_1 - v_2 + 0.5v_3 + 1.4v_4 = 0 \quad \text{---------------------}\textbf{(2)}$$

$v_3 - v_4 = 0.2v_y$ ---------------------------------(3)

$v_y = (v_4 - v_1)$

$0.2v_1 + v_3 - 1.2v_4 = 0$ ------------------------------(3)

$v_1 = -12$ ------------------------------ (4)

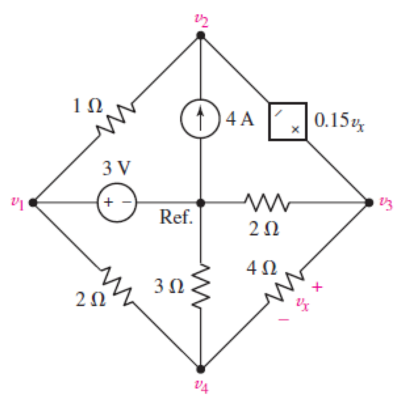
$$\text{Solving, } v_1 = -12 \text{ V}, \quad v_2 = -4 \text{ V}, \quad v_3 = 0 \text{ V}, \quad \text{and } v_4 = -2 \text{ V}.$$

8. **Organize the equations.** Group terms according to nodal voltages.

9. **Solve the system of equations for the nodal voltages** (there will be $N - 1$ of them).

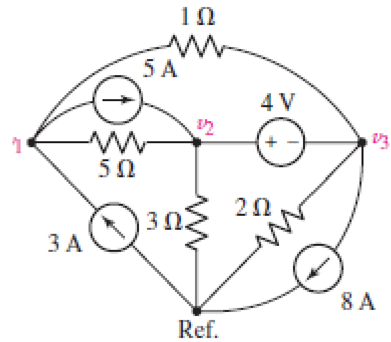
<u>PRACTICE PROBLEMS</u>

1) ; Determine the nodal voltages in the circuit of Fig

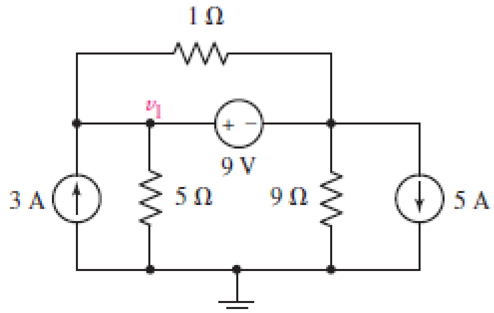


Ans: v_1 = **3** V, v_2 = **−2.33** V, v_3 = **−1.91** V, v_4 = **0.945** V.

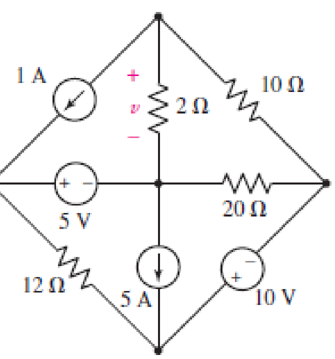
2)) Determine the nodal voltages as labeled in Fig. making use of the Super node technique as appropriate.



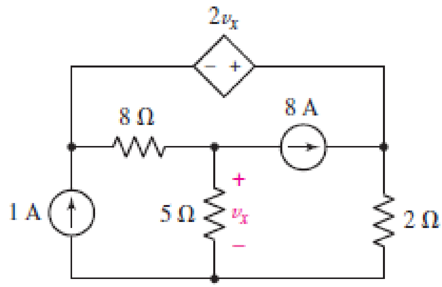
3) For the circuit shown in Fig. determine a numerical value for the voltage labeled v_1.



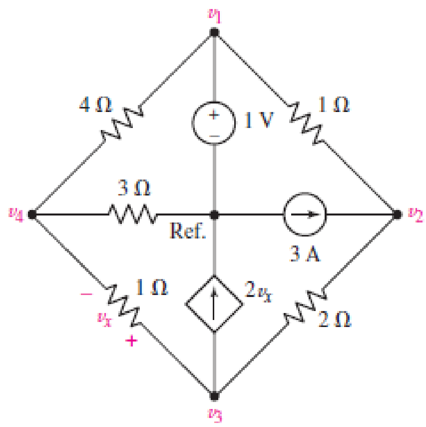
4) Determine the voltage labeled *v* in the circuit of Fig.



5) Determine the voltage V_x in the circuit of Fig. and the power supplied by the 1 A source



6) For the circuit of Fig. determine all four nodal voltages.



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