10.6 • NODAL AND MESH ANALYSIS

VISVESVARAYA TECHNOLOGICAL UNIVERSITY, BELAGAVI
B.E: Electronics & Communication Engineering / B.E: Electronics & Telecommunication Engineering
NEP, Outcome Based Education (OBE) and Choice Based Credit System (CBCS)
(Effective from the academic year 2021 – 22)

IV Semester

Circuits & Controls			
Course Code	**21EC43**	CIE Marks	50
Teaching Hours/Week (L: T: P: S)	(3:0:2:0)	SEE Marks	50
Total Hours of Pedagogy	40 hours Theory + 13 Lab slots	Total Marks	100
Credits	04	Exam Hours	03

Module-1	
Basic concepts and network theorems Types of Sources, Loop analysis, Nodal analysis with independent DC and AC Excitations. (Textbook 1: 2.3, 4.1, 4.2, 4.3, 4.4, 10.6) Super position theorem, Thevenin's theorem, Norton's Theorem, Maximum Power transfer Theorem. (Textbook 2: 9.2, 9.4, 9.5, 9.7)	
Teaching-Learning Process	Chalk and Talk, YouTube videos, Demonstrate the concepts using circuits **RBT Level:** L1, L2, L3

Module-2	
Two port networks: Short- circuit Admittance parameters, Open- circuit Impedance parameters, Transmission parameters, Hybrid parameters (Textbook 3: 11.1, 11.2, 11.3, 11.4, 11.5) **Laplace transform and its Applications**: Step Ramp, Impulse, Solution of networks using Laplace transform, Initial value and final value theorem (Textbook 3: 7.1, 7.2, 7.4, 7.7, 8.4)	
Teaching-Learning Process	Chalk and Talk **RBT Level:** L1, L2, L3
Module-3	
Basic Concepts and representation: Types of control systems, effect of feedback systems, differential equation of physical systems (only electrical systems), Introduction to block diagrams, transfer functions, Signal Flow Graphs (Textbook 4: Chapter 1.1, 2.2, 2.4, 2.5, 2.6)	
Teaching-Learning Process	Chalk and Talk, YouTube videos **RBT Level:** L1, L2, L3

Module-4	
Time Response analysis: Time response of first order systems. Time response of second order systems, time response specifications of second order systems (Textbook 4: Chapter 5.3, 5.4) **Stability Analysis:** Concepts of stability necessary condition for stability, Routh stability criterion, relative stability Analysis (Textbook 4: Chapter 5.3, 5.4, 6.1, 6.2, 6.4, 6.5)	
Teaching-Learning Process	Chalk and Talk, Any software tool to show time response **RBT Level:** L1, L2, L3
Module-5	
Root locus: Introduction the root locus concepts, construction of root loci (Textbook 4: 7.1, 7.2, 7.3) **Frequency Domain analysis and stability**: Correlation between time and frequency response and Bode plots (Textbook 4: 8.1, 8.2, 8.4) **State Variable Analysis:** Introduction to state variable analysis: Concepts of state, state variable and state models. State model for Linear continuous –Time systems, solution of state equations. (Textbook 4: 12.2, 12.3, 12.6)	
Teaching-Learning Process	Chalk and Talk, Any software tool to plot Root locus, Bode plot **RBT Level:** L1, L2, L3

Suggested Learning Resources:

Text Books

1. Engineering circuit analysis, William H Hayt, Jr, Jack E Kemmerly, Steven M Durbin, Mc Graw Hill Education, Indian Edition 8e.
2. Networks and Systems, D Roy Choudhury, New age international Publishers, second edition.
3. Network Analysis, M E Van Valkenburg, Pearson, 3e.
4. Control Systems Engineering, I J Nagrath, M. Gopal, New age international Publishers, Fifth edition.

10.6 • NODAL AND MESH ANALYSIS

Nodal and mesh analysis techniques, for AC circuits .Both Kirchhoff's laws and Ohm- law are valid for phasors.

Therefore Sinusoidal steady state circuits can be analyzed by nodal and mesh analysis techniques

Steps to analyse AC Circuits

1) Transform the circuit to the phasor or frecquency domain.

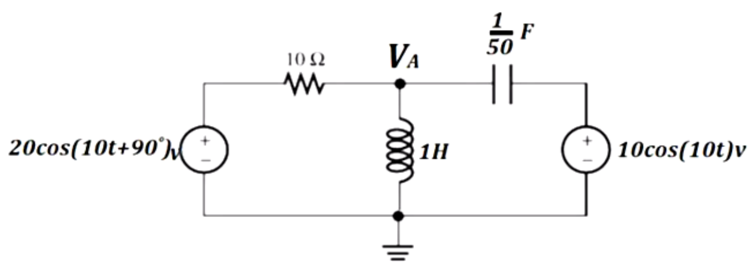
2)Solve the problem using circuit techniques Nodal analysis or

Mesh analysis.

3)Transform the resulting phasor to the time domain.

NODAL ANALYSIS IN TIME DOMAIN CIRCUITS

1)Calculate VA



$20\cos(10t+90^{\circ})$ Volts $\rightarrow$ $20\angle 90^{\circ}\,V$

$10\cos(10t)$ Volts $\rightarrow$ $10\angle 0^{\circ}\,V$

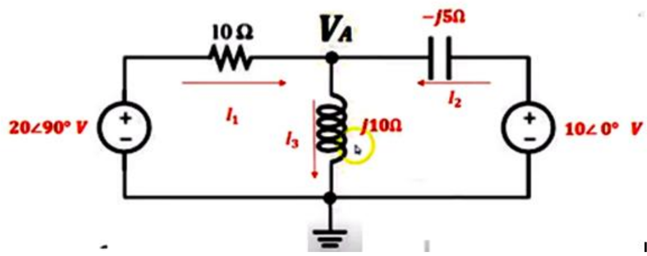
Phasor Form

$\omega = 10$

$X_C = \frac{1}{j\omega C} = -\frac{j}{\omega C} = -j5\Omega$

$X_L = j\omega L = j10\Omega$

The transformed circuit



Apply KCL at the node

$$I_1 + I_2 = I_3$$

$$\frac{20\angle 90° - V_A}{10} + \frac{10\angle 0° - V_A}{-j5} = \frac{V_A}{j10}$$

$$\frac{20\angle 90°}{10} + \frac{10\angle 0°}{-j5} = \frac{V_A}{j10} + \frac{V_A}{10} + \frac{V_A}{-j5}$$

$\angle 90° \rightarrow j$

$$\frac{20j}{10} + \frac{10}{-j5} = \frac{V_A}{j10} + \frac{V_A}{10} + \frac{V_A}{-j5}$$

$$2j - \frac{2}{j} = V_A\left(\frac{1}{10} + \frac{1}{j10} + \frac{1}{-j5}\right)$$

$$2j + 2j = V_A\left(\frac{1}{10} - \frac{j}{10} + \frac{j}{5}\right)$$

$\frac{1}{j} = -j$

$$4j = V_A\left(\frac{1}{10} + \frac{j}{10}\right)$$

Multiply both sides by 10

$$40j = V_A(1+j)$$

Convert to polar form

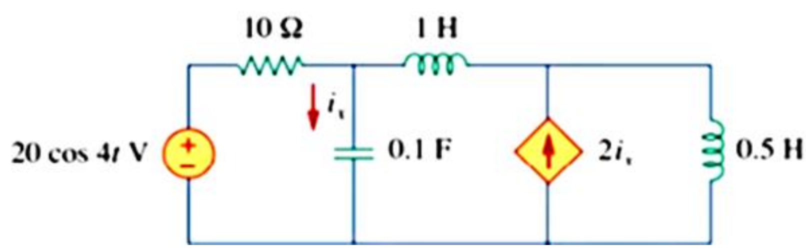
$j \rightarrow \angle 90°$

$$40\angle 90° = V_A(\sqrt{2}\angle 45°)$$

$$\therefore \quad V_A = \frac{40\angle 90°}{\sqrt{2}\angle 45°} = \quad V_A = 28.3\angle 45° V$$

Phasor to time domain $V_A = 283.3 \cos(\omega t + 45)$

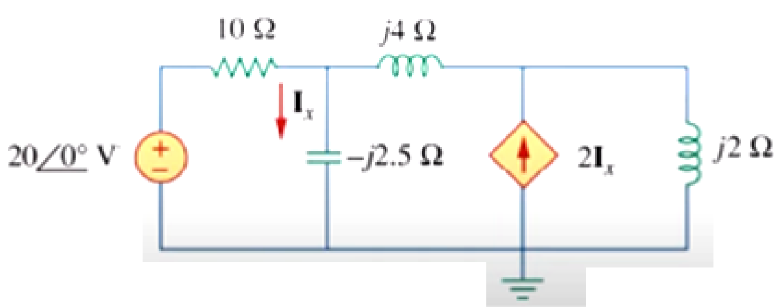
2) Find i_x in the circuit of Fig. using nodal analysis.



We first convert the circuit to the frequency domain:

Note: $\omega = 4$ rad/s

$$20\cos 4t \Rightarrow 20\angle 0^\circ.$$
$$1\text{ H} \Rightarrow j\omega L = j4$$
$$0.5\text{ H} \Rightarrow j\omega L = j2$$
$$0.1\text{ F} \Rightarrow \frac{1}{j\omega C} = \frac{-j}{\omega C} = -j2.5$$



Applying KCL at node 1,

$$\frac{20 - \mathbf{V}_1}{10} = \mathbf{ix} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} \qquad \Big| \; \mathbf{ix} = \frac{\mathbf{V}_1}{-j2.5}$$

$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

Multiplying both sides by 10

$$20 - \mathbf{V}_1 = \frac{4\,\mathbf{V}_1}{-j} + 2.5\,\frac{(\mathbf{V}_1 - \mathbf{V}_2)}{j}$$

$$20 - \mathbf{V}_1 = j4\,\mathbf{V}_1 - j\,2.5\,(\mathbf{V}_1 - \mathbf{V}_2)$$

$$20 = \mathbf{V}_1 + j4\,\mathbf{V}_1 - j\,2.5\,\mathbf{V}_1 + j2.5\mathbf{V}_2$$

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20 \quad \cdots\cdots \quad (1)$$

At node 2,

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

But $\mathbf{I}_x = \dfrac{\mathbf{V}_1}{-j2.5}$

Substituting this gives

$$\frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

By simplifying, we obtain

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0 \quad \ldots\ldots\ldots\ldots\ldots \mathbf{(2)}$$

Substituting this gives

$$\frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

By simplifying, we obtain

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0 \quad \ldots\ldots\ldots\ldots (2)$$

Reference

$-5.5\ V_1 = 7.5\ V_2$

$5.5\ V_1 + 7.5\ V_2 = 0$

Multi[ly by 2

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0$$

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20 \quad \cdots\cdots \quad (1)$$

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0 \quad \cdots \quad (2)$$

in matrix form

$$\begin{bmatrix} 1+j1.5 & j2.5 \\ 11 & 15 \end{bmatrix}\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix} \qquad \mathbf{V}_1 = \frac{\Delta_1}{\Delta} \qquad \mathbf{V}_2 = \frac{\Delta_2}{\Delta}$$

the determinants

$$\Delta = \begin{vmatrix} 1+j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = (1+j1.5)15 - 11(j2.5) = 15 - j5$$

$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300,$$

$$\Delta_2 = \begin{vmatrix} 1+j1.5 & 20 \\ 11 & 0 \end{vmatrix} = -220$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15 - j5} = 18.97\underline{/18.43^\circ} \text{ V}$$

$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{15 - j5} = 13.91\underline{/198.3^\circ} \text{ V}$$

$$\mathbf{V}_1 = 18.97\underline{/18.43^\circ} \text{ V}$$
$$\mathbf{V}_2 = 13.91\underline{/198.3^\circ} \text{ V}$$

The current $\mathbf{I}_x$ is given by

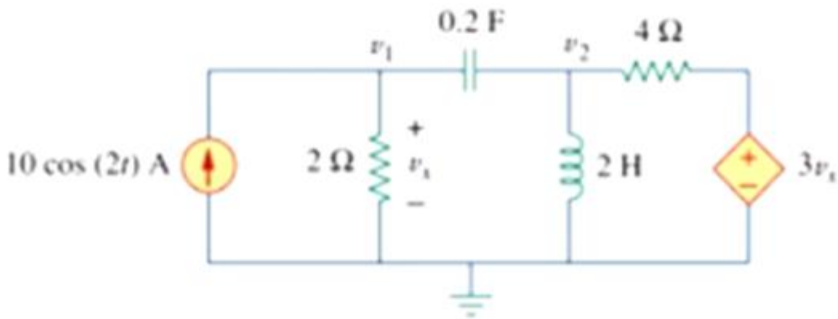
$$\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5} = \frac{18.97\underline{/18.43^\circ}}{2.5\underline{/-90^\circ}} = 7.59\underline{/108.4^\circ} \text{ A}$$

Transforming this to the time domain,

$$i_x = 7.59 \cos(4t + 108.4^\circ) \text{ A}$$

3)

Using nodal analysis, find v_1 and v_2 in the circuit of Fig.



Solution:

We first convert the circuit to the frequency domair

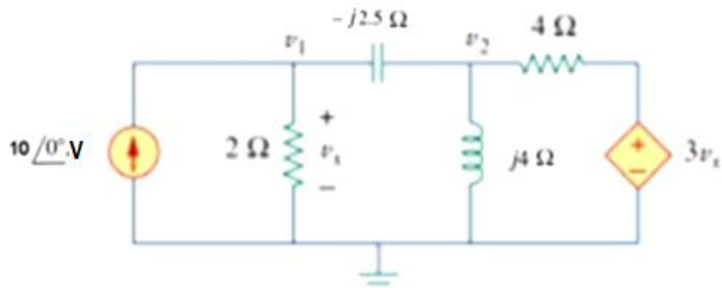
$\omega = 2$ rad/s

$10\cos(2t) \Rightarrow 10\angle 0^\circ$ V

$2\text{ H} \Rightarrow j\omega L = j4\ \Omega$ $X_L = j4\ \Omega$

$0.2\text{ F} \Rightarrow \dfrac{1}{j\omega}$; $\dfrac{-j}{\omega C} = \dfrac{-j}{0.4} = -j2.5\ \Omega$

$X_C = -j2.5\ \Omega$



Apply KCL at the node-1

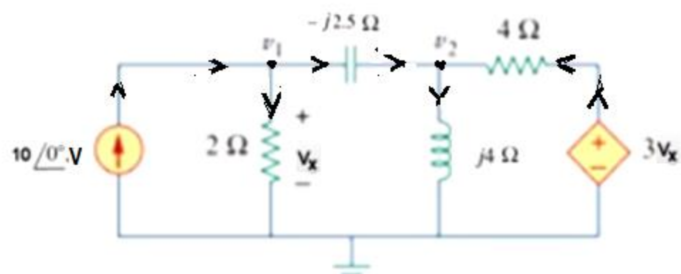
$$10 = \frac{V_1}{2} + \frac{V_1 - V_2}{-j2.5}$$

Muliply by 10 both sides

$$100 = 5V_1 + \frac{4(V_1 - V_2)}{-j}$$

$$100 = 5V_1 + j4(V_1 - V_2)$$

$$(5 + j4)V_1 - j4V_2 = 100 \quad \ldots \quad (1)$$



Apply KCL at the node- 2

$$\frac{V_1 - V_2}{-j2.5} + \frac{3V_x - V_2}{4} = \frac{V_2}{j4}$$

$$\boxed{\text{But } V_x = V_1}$$

$$\frac{V_1 - V_2}{-j2.5} + \frac{3V_1 - V_2}{4} = \frac{V_2}{j4}$$

Multiply both sides by 20

$$\frac{8(V_1 - V_2)}{-j} + 5(3V_1 - V_2) = \frac{5V_2}{j}$$

$$j8(V_1 - V_2) + 5(3V_1 - V_2) = -j5V_2$$

$$j8V_1 - j8V_2 + 15V_1 - 5V_2 = -j5V_2$$

$$(15 + j8)V_1 + (-5 - j3)V_2 = 0 \quad \dots (2)$$

$$\frac{1}{j} = -j$$

$$j = \sqrt{-1}$$

$$j^2 = -1$$

$$(5 + j4)V_1 - j4V_2 = 100 \quad \dots (1)$$

$$(15 + j8)V_1 + (-5 - j3)V_2 = 0 \quad \dots (2)$$

Matrix Eq.

$$\begin{vmatrix} (5+j4) & -j4 \\ (15+j8) & (-5-j3) \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \end{vmatrix} = \begin{vmatrix} 100 \\ 0 \end{vmatrix} \qquad V_1 = \frac{\Delta_1}{\Delta} \qquad V_2 = \frac{\Delta_2}{\Delta}$$

Determinant

$$\Delta = \begin{vmatrix} (5+j4) & -j4 \\ (15+j8) & (-5-j3) \end{vmatrix} = (5+j4)(-5-j3) - (-j4)(15+j8) = -45 + j25$$

$$\Delta_1 = \begin{vmatrix} 100 & -j4 \\ 0 & (-5-j3) \end{vmatrix} = 100(-5-j3) = -500 - j300 \qquad V_1 = \frac{\Delta_1}{\Delta} = \frac{-500 - j300}{-45 + j25} = 5.66 + j9.81$$

$$= 11.327 < 60.01$$

$$\Delta_2 = \begin{vmatrix} (5+j4) & 100 \\ (15+j8) & 0 \end{vmatrix} = -100(15+j8) = -1500 - j800 \qquad V_2 = \frac{\Delta_2}{\Delta} = \frac{-1500 - j800}{-45 + j25} = 17.92 + j27.73$$

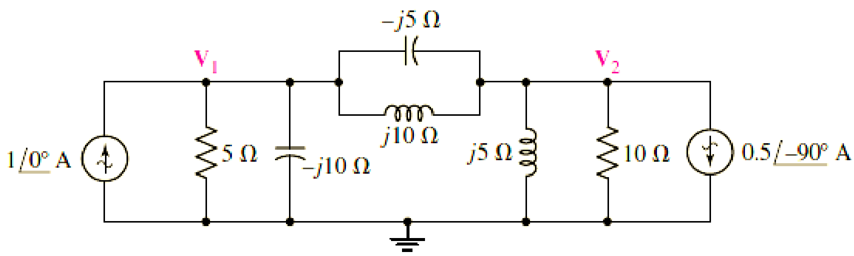
$$= 33.02 < 57.12^{o}$$

$$\mathbf{V}_1 = 11.327 < 60.01^\circ$$
$$\mathbf{V}_2 = 33.02 < 57.12^\circ$$

$$v_1(t) = 11.327\cos(2t + 60.01^\circ)$$

$$v_2(t) = 33.02\cos(2t + 57.12^\circ)$$

4) Find the time-domain node voltages $v1(t)$ and $v2(t)$ in the circuit shown in Fig.



Apply KCL at node -1

$$\frac{\mathbf{V}_1}{5} + \frac{\mathbf{V}_1}{-j10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j10} = 1\underline{/0^\circ} = 1 + j0$$

$$(0.2 + j0.2)\mathbf{V}_1 - j0.1\mathbf{V}_2 = 1 \quad \text{……..(1)}$$

Apply kcl at node-2

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{-j5} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{j10} + \frac{\mathbf{V}_2}{j5} + \frac{\mathbf{V}_2}{10} = -(0.5\underline{/-90^\circ}) = j0.5$$

$$-j0.1\mathbf{V}_1 + (0.1 - j0.1)\mathbf{V}_2 = j0.5 \quad \text{………………(2)}$$

$$(0.2 + j0.2)\mathbf{V}_1 - j0.1\mathbf{V}_2 = 1 \quad \text{……..(1)}$$

$$-j0.1\mathbf{V}_1 + (0.1 - j0.1)\mathbf{V}_2 = j0.5 \quad \text{…………(2)}$$

$$(0.2 + j0.2)\mathbf{V}_1 - j0.1\mathbf{V}_2 = 1 \quad \ldots\ldots\ldots(1)$$

$$-j0.1\mathbf{V}_1 + (0.1 - j0.1)\mathbf{V}_2 = j0.5 \quad \ldots\ldots\ldots\ldots(2)$$

Matrix form

$$\begin{bmatrix} (0.2 + j0.2) & -j0.1 \\ -j0.1 & (0.1 - j0.1) \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 1 \\ j0.5 \end{bmatrix}$$

Determinant form

$$\Delta = \begin{vmatrix} (0.2 + j0.2) & -j0.1 \\ -j0.1 & (0.1 - j0.1) \end{vmatrix} \qquad \mathbf{V}_1 = \frac{\Delta_1}{\Delta} = (1 - j2\text{ V})$$

$$\Delta_1 = \begin{vmatrix} 1 & -j0.1 \\ j0.5 & (0.1 - j0.1) \end{vmatrix} \qquad \mathbf{V}_2 = \frac{\Delta_2}{\Delta} = (-2 + j4\text{ V.})$$

$$\Delta_2 = \begin{vmatrix} (0.2 + j0.2) & 1 \\ -j0.1 & j0.5 \end{vmatrix}$$

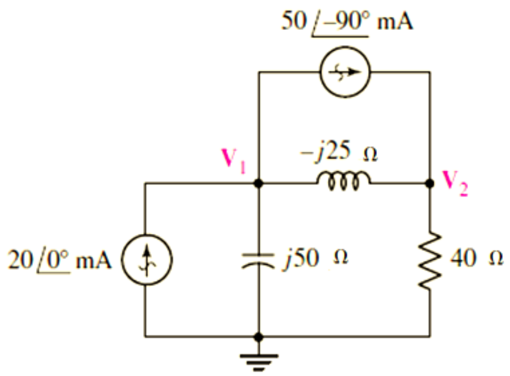
$$\boxed{\begin{aligned} \mathbf{V}_1 &= (1 - j2\text{ V})\text{ V} \\ \mathbf{V}_2 &= (-2 + j4\text{ V.})\text{ V.} \end{aligned}}$$ ⇨ **RECTANGULAR FORM**

$$\boxed{\begin{aligned} \mathbf{V}_1 &= 2.24\underline{/-63.4^\circ} \\ \mathbf{V}_2 &= 4.47\underline{/116.6^\circ} \end{aligned}}$$ ⇨ **POLAR (OR) PHASOR FORM**

$$\boxed{\begin{aligned} v_1(t) &= 2.24\cos(\omega t - 63.4^\circ)\text{ V} \\ v_2(t) &= 4.47\cos(\omega t + 116.6^\circ)\text{ V} \end{aligned}}$$ ⇨ **TIME DOMAIN FORM**

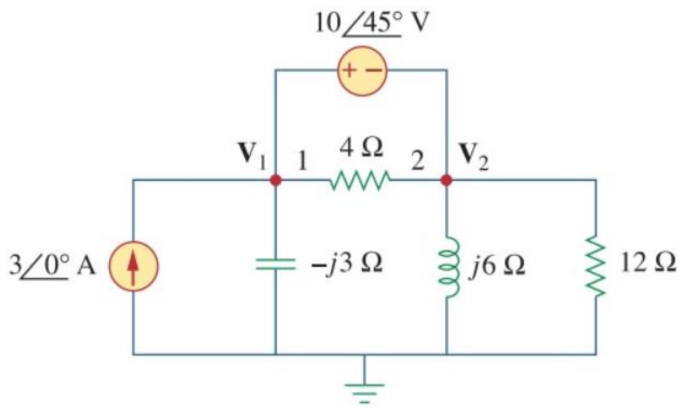
PRACTICE

Use nodal analysis on the circuit of Fig. 10.23 to find $\mathbf{V}_1$ and $\mathbf{V}_2$.



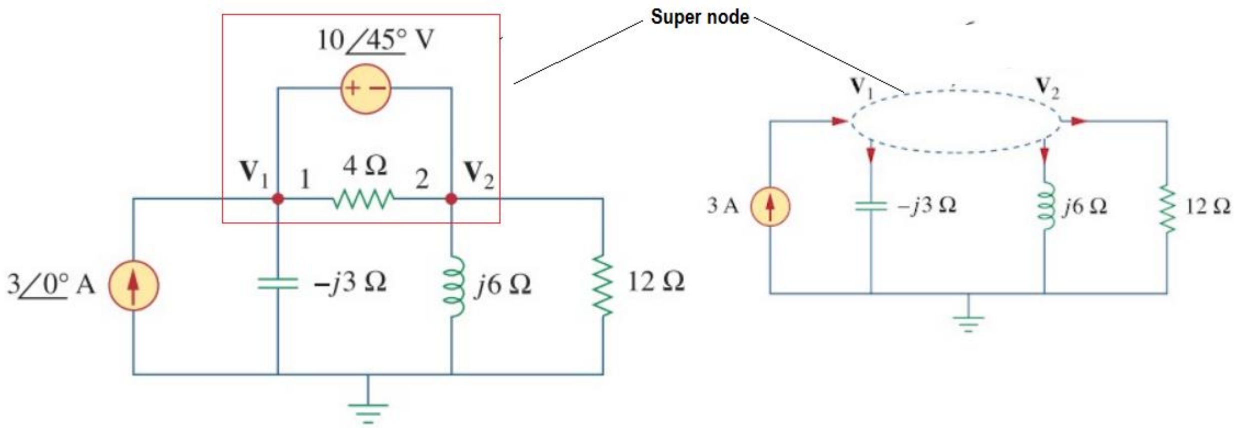
Ans: $1.062\angle 23.3^\circ$ V; $1.593\angle -50.0^\circ$ V.

4) Compute V_1 and V_2 in the circuit of Figure



Solution :

Nodes 1 and 2 form a supernode as drawn in Figure.



Applying KCL at the supernode gives

$$3 = \frac{\mathbf{V}_1}{-j3} + \frac{\mathbf{V}_2}{j6} + \frac{\mathbf{V}_2}{12}$$

$$36 = j4\mathbf{V}_1 + (1 - j2)\mathbf{V}_2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \textbf{(1)}$$

But a voltage source is connected between nodes 1 and 2, so that

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\angle 45° \qquad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Substituting Equations.(2) to (1) results

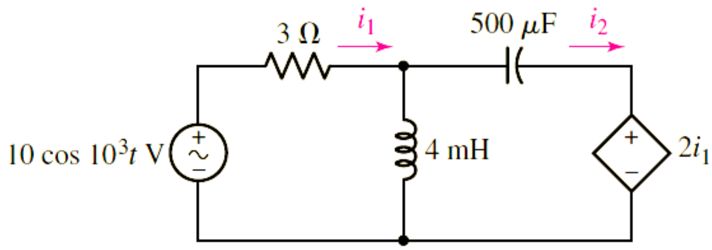
$$36 - 40\angle 135° = (1 + j2)\mathbf{V}_2 \quad \Rightarrow \quad \mathbf{V}_2 = 31.41\angle -87.18° \text{ V}$$

From Equation.(2),

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\angle 45° = 25.78\angle -70.48° \text{ V}$$

MESH ANALYSIS IN TIME DOMAIN CIRCUITS

1) Obtain expressions for the time-domain currents $i1$ and $i2$ in the circuit given as Fig.



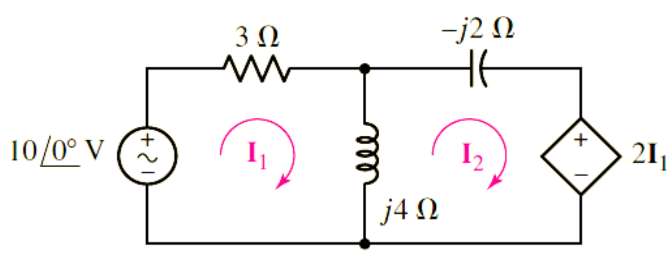
A time-domain circuit containing a dependent source should be converted to corresponding frequency-domain circuit.

$10\cos 10^3 t$ V $\Rightarrow$ $10\underline{/0^\circ}$ V

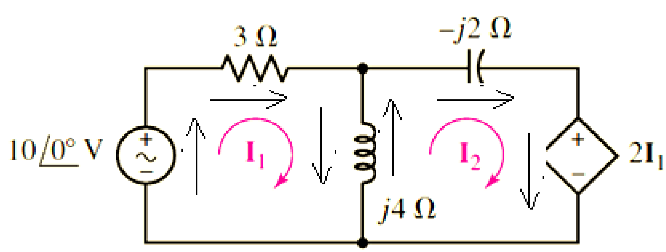
$\omega = 10^3$ rad/s

$X_L = j\omega L = j\,10^3\,(4\;10^{-3}) = j4\;\Omega$

$$X_C = \frac{1}{j\omega C} = \frac{1}{j\,10^3\,(500 \times 10^{-6})} = -j2\;\Omega$$



Apply KVL mesh -1



$$3\mathbf{I}_1 + j4(\mathbf{I}_1 - \mathbf{I}_2) = 10\underline{/0^\circ}$$

$$(3 + j4)\mathbf{I}_1 - j4\mathbf{I}_2 = 10 \quad \text{.................(1)}$$

Apply KVL mesh -2

$$j4(\mathbf{I}_2 - \mathbf{I}_1) - j2\mathbf{I}_2 + 2\mathbf{I}_1 = 0$$

$$(2 - j4)\mathbf{I}_1 + j2\mathbf{I}_2 = 0 \quad \text{.................(2)}$$

From Eq. 1 & 2

$$(3 + j4)\mathbf{I}_1 - j4\mathbf{I}_2 = 10 \quad \text{.................(1)}$$

$$(2 - j4)\mathbf{I}_1 + j2\mathbf{I}_2 = 0 \quad \text{.................(2)}$$

Matrix form

$$\begin{bmatrix} (3 + j4) & -j4 \\ (2 - j4) & +j2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

Determinant form

$$\Delta = \begin{vmatrix} (3 + j4) & -j4 \\ (2 - j4) & +j2 \end{vmatrix}$$

$$\Delta 1 = \begin{vmatrix} 10 & -j4 \\ 0 & +j2 \end{vmatrix}$$

$$\Delta 2 = \begin{vmatrix} (3 + j4) & 10 \\ (2 - j4) & 0 \end{vmatrix}$$

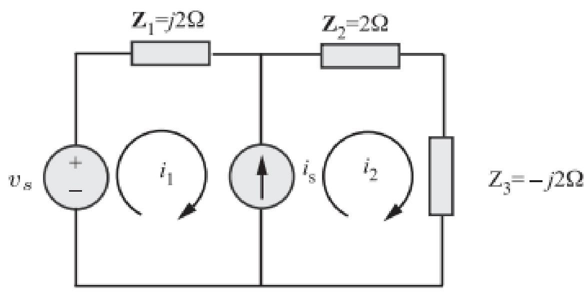
$$\mathbf{I}_1 = \frac{\Delta 1}{\Delta} = \mathbf{I}_1 = \frac{14 + j8}{13} = 1.24\underline{/29.7^\circ} \text{ A} \longrightarrow \textbf{Phasor form}$$

$$\mathbf{I}_2 = \frac{\Delta 2}{\Delta} = \mathbf{I}_2 = \frac{20 + j30}{13} = 2.77\underline{/56.3^\circ} \text{ A} \longrightarrow \textbf{Phasor form}$$

$$\left.\begin{aligned} i_1(t) &= 1.24\cos(10^3 t + 29.7^\circ) \text{ A} \\ i_2(t) &= 2.77\cos(10^3 t + 56.3^\circ) \text{ A} \end{aligned}\right| \longrightarrow \textbf{Time domain form}$$

2)

Find the steady current i_1 when the source voltage is $v_s = 10\sqrt{2}\cos(\omega t + 45^\circ)$ V **and the current source is** $i_s = 3\cos\omega t$ A **for the circuit of Fig. The circuit provides the impedence in ohms for each element at** ω



$$v_s = 10\sqrt{2}\cos(\omega t + 45^\circ) \quad \Rightarrow \quad \mathbf{V}_s = 10\sqrt{2}\,\underline{/45^\circ} = 10(1+j)$$

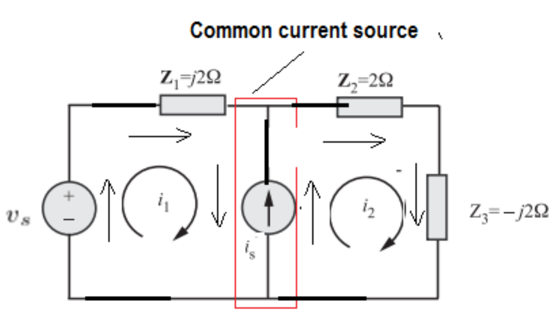
$$i_s = 3\cos\omega t \quad \Rightarrow \quad \mathbf{I}_s = 3\,\underline{/0^\circ}$$

Time domain form **Phasor form**

$$v_s = 10\sqrt{2}\cos(\omega t + 45^\circ) \quad \Rightarrow \quad \mathbf{V}_s = 10\sqrt{2}\,\underline{/45^\circ} = 10(1+j)$$

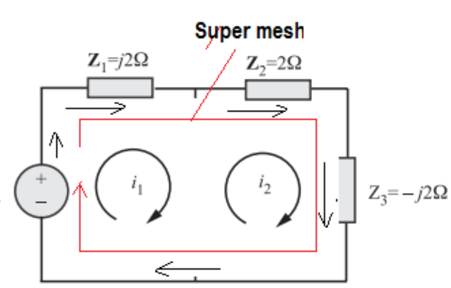
$$i_s = 3\cos\omega t \quad \Rightarrow \quad \mathbf{I}_s = 3\,\underline{/0^\circ}$$

Time domain form **Phasor form**



$\mathbf{I}_s = 3\,\underline{/0^\circ}$

$\mathbf{V}_s = 10\sqrt{2}\,\underline{/45^\circ}$



$$\mathbf{I}_2 - \mathbf{I}_1 = \mathbf{I}_s = 3\,\underline{/0^\circ}$$

Apply KVL to super mesh

$$\mathbf{V}_s - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_2\ \mathbf{Z}_2\ - \mathbf{I}_2\ \mathbf{Z}_3\ = 0$$

$$\mathbf{V}_s - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3)\ = 0$$

Substituting $\boxed{\mathbf{I}_2 = \mathbf{I}_1 + \mathbf{I}_s}$

$$\boxed{\mathbf{I}_2 = \mathbf{I}_1 + \mathbf{I}_s}$$

$$\mathbf{I}_1\mathbf{Z}_1 + (\mathbf{I}_1 + \mathbf{I}_s)(\mathbf{Z}_2 + \mathbf{Z}_3) = \mathbf{V}_s$$

$$(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3)\mathbf{I}_1 = \mathbf{V}_s - (\mathbf{Z}_2 + \mathbf{Z}_3)\mathbf{I}_s$$

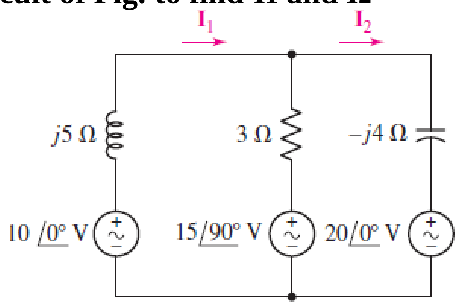
$$\mathbf{I}_1 = \frac{\mathbf{V}_s - (\mathbf{Z}_2 + \mathbf{Z}_3)\mathbf{I}_s}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3} = \frac{(10 + j10) - (2 - j2)3}{2}$$

$$= 2 + j8 = 8.25\ \underline{/76^\circ}\ \text{A} \longrightarrow \text{Phasor form}$$

$$i_1 = 8.25\cos(\omega t + 76^\circ)\ \text{A} \longrightarrow \text{Time domain form}$$

PRACTICE

Use mesh analysis on the circuit of Fig. to find I1 and I2



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