T2-9.7 MAXIMUM POWER TRANSFER THEOREM

VISVESVARAYA TECHNOLOGICAL UNIVERSITY, BELAGAVI
B.E: Electronics & Communication Engineering / B.E: Electronics & Telecommunication Engineering
NEP, Outcome Based Education (OBE) and Choice Based Credit System (CBCS)
(Effective from the academic year 2021 – 22)

IV Semester

Circuits & Controls			
Course Code	**21EC43**	CIE Marks	50
Teaching Hours/Week (L: T: P: S)	(3:0:2:0)	SEE Marks	50
Total Hours of Pedagogy	40 hours Theory + 13 Lab slots	Total Marks	100
Credits	04	Exam Hours	03

Module-1	
Basic concepts and network theorems Types of Sources, Loop analysis, Nodal analysis with independent DC and AC Excitations. (Textbook 1: 2.3, 4.1, 4.2, 4.3, 4.4, 10.6) Super position theorem, Thevenin's theorem, Norton's Theorem, Maximum Power transfer Theorem. (Textbook 2: 9.2, 9.4, 9.5, 9.7)	
Teaching-Learning Process	Chalk and Talk, YouTube videos, Demonstrate the concepts using circuits **RBT Level:** L1, L2, L3

Module-2	
Two port networks: Short- circuit Admittance parameters, Open- circuit Impedance parameters, Transmission parameters, Hybrid parameters (Textbook 3: 11.1, 11.2, 11.3, 11.4, 11.5) **Laplace transform and its Applications**: Step Ramp, Impulse, Solution of networks using Laplace transform, Initial value and final value theorem (Textbook 3: 7.1, 7.2, 7.4, 7.7, 8.4)	
Teaching-Learning Process	Chalk and Talk **RBT Level:** L1, L2, L3
Module-3	
Basic Concepts and representation: Types of control systems, effect of feedback systems, differential equation of physical systems (only electrical systems), Introduction to block diagrams, transfer functions, Signal Flow Graphs (Textbook 4: Chapter 1.1, 2.2, 2.4, 2.5, 2.6)	
Teaching-Learning Process	Chalk and Talk, YouTube videos **RBT Level:** L1, L2, L3

Module-4	
Time Response analysis: Time response of first order systems. Time response of second order systems, time response specifications of second order systems (Textbook 4: Chapter 5.3, 5.4) **Stability Analysis:** Concepts of stability necessary condition for stability, Routh stability criterion, relative stability Analysis (Textbook 4: Chapter 5.3, 5.4, 6.1, 6.2, 6.4, 6.5)	
Teaching-Learning Process	Chalk and Talk, Any software tool to show time response **RBT Level:** L1, L2, L3
Module-5	
Root locus: Introduction the root locus concepts, construction of root loci (Textbook 4: 7.1, 7.2, 7.3) **Frequency Domain analysis and stability**: Correlation between time and frequency response and Bode plots (Textbook 4: 8.1, 8.2, 8.4) **State Variable Analysis:** Introduction to state variable analysis: Concepts of state, state variable and state models. State model for Linear continuous –Time systems, solution of state equations. (Textbook 4: 12.2, 12.3, 12.6)	
Teaching-Learning Process	Chalk and Talk, Any software tool to plot Root locus, Bode plot **RBT Level:** L1, L2, L3

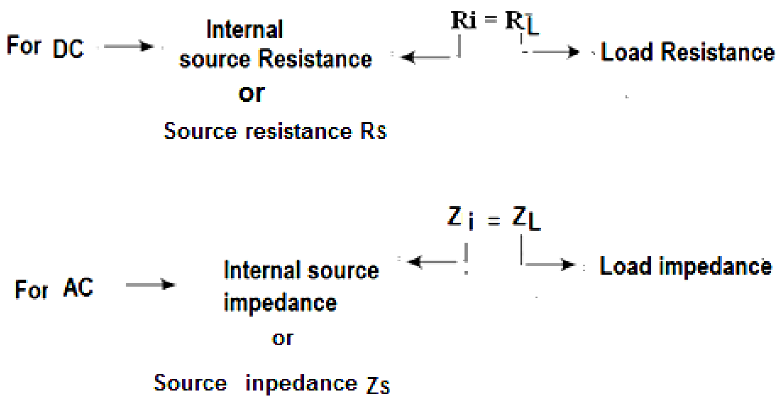
Suggested Learning Resources:

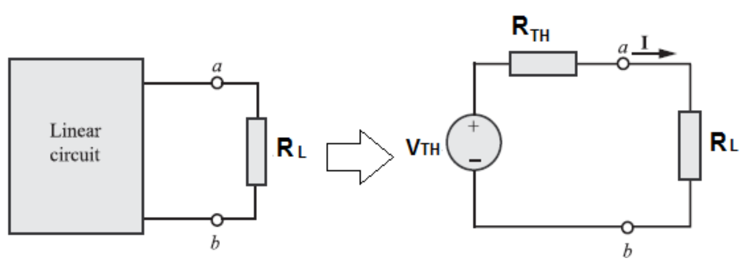
Text Books

1. Engineering circuit analysis, William H Hayt, Jr, Jack E Kemmerly, Steven M Durbin, Mc Graw Hill Education, Indian Edition 8e.
2. Networks and Systems, D Roy Choudhury, New age international Publishers, second edition.
3. Network Analysis, M E Van Valkenburg, Pearson, 3e.
4. Control Systems Engineering, I J Nagrath, M. Gopal, New age international Publishers, Fifth edition.

T2-9.7 MAXIMUM POWER TRANSFER THEOREM

Maximum power transfer theorem states that in a linear bidirectional two terminal network Maximum power will be transferred from source to the load when the internal resistance of the source is equal to the load resistance





According to Maximum Power Transfer Theorem, for maximum power transfer from the network to the load resistance, R_L must be equal to the source resistance i.e. Network's Thevenin equivalent resistance R_{TH} . I.e. $R_L = R_{TH}$

The load current **I** in the circuit shown above is given by,

$$I = \frac{V_{TH}}{R_{TH}+R_L}$$

The power delivered by the circuit to the load:

$$P = I^2R = \frac{V_{TH}^2}{(R_{TH}+R_L)^2}R_L$$

$$P = V_{TH}^2 \left\{ \frac{R_L}{(R_{TH}+R_L)^2} \right\}$$

The condition for maximum power transfer can be obtained by differentiating the above expression for power delivered with respect to the load resistance and equating it to zero

$$\frac{dP}{dR_L} = 0$$

$$= V_{TH}^2 \left[\frac{(R_{TH}+R_L)^2(1) - R_L \times 2(R_{TH}+R_L)(1)}{(R_{TH}+R_L)^4} \right] = 0$$

$$\boxed{\frac{d\,U/V}{dt} = \frac{V \cdot \frac{dU}{dt} - U \frac{dV}{dt}}{V^2}}$$

$$(R_{TH}+R_L)^2 - R_L \times 2(R_{TH}+R_L) = 0$$

$$(R_{TH}+R_L)\,[(R_{TH}+R_L) - 2R_L] = 0$$

$$(R_{TH}+R_L)(R_{TH}-R_L) = 0$$

$$(R_{TH}-R_L) = 0$$

$$\therefore \boxed{R_L = R_{TH}}$$

$$\boxed{\text{Source Resistance} = \text{Load Resistance}}$$

This is the condition for max. power transfer

The value of Maximum Power Transfer

$$P = I^2R = \frac{V_{TH}^2}{(R_{TH}+R_L)^2} R_L$$

For maximum power transfer $R_{TH} = R_L$

$$P_{MAX} = \frac{V_{TH}^2}{(R_L+R_L)^2} \times R_L = \frac{V_{TH}^2}{4R_L}$$

Under the condition of maximum power transfer, the power delivered to the load is given by

$$\boxed{P_{MAX} = \frac{V_{TH}^2}{4R_L} \text{ Watts}}$$

Efficiency at maximum power transfer

P_{OUT} = Output power at max. Power transfer = $P_{MAX} = \frac{V_{TH}^2}{4R_L}$ Watts

Input power = V I = V_{TH} X $\frac{V_{TH}}{R_{TH}+R_L}$ Watts

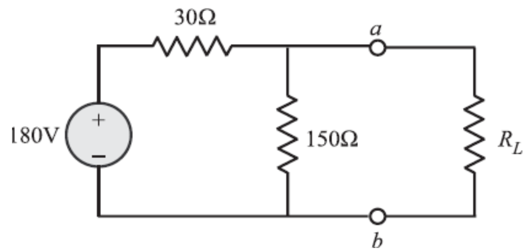
For maximum power transfer R_{TH} = R_L

$\therefore$ P_{IN} = Power Input = $\frac{V_{TH}^2}{2R_L}$ Watts

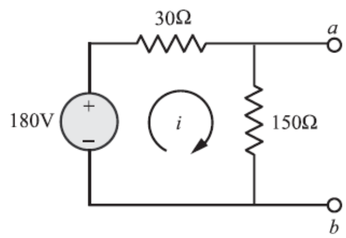
$$\text{Efficiency} = \eta = \frac{\text{output}}{\text{input}} = \frac{P_{OUT}}{P_{IN}} = \frac{V_{TH}^2}{4R_L} \Big/ \frac{V_{TH}^2}{2R_L} = 1/2 = 50\%$$

Therefore efficiency is 50% at maximum power transfer

1)Find the load RL that will result in maximum power delivered to the load for the circuit of Fig. Also determine the maximum power Pmax.



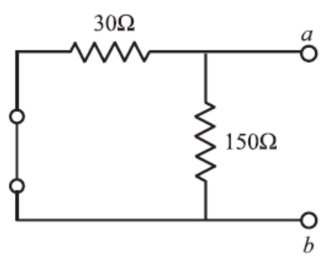
First step Disconnect the load resistor R_L and find V_{TH}

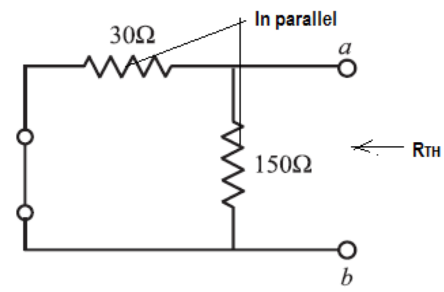


$$i = \frac{180}{150 + 30} = 1\text{A}$$

$$V_{oc} = V_{TH} = 150 \times i = 150 \text{ V}$$

<u>To find R_{TH} deactivate the independent source</u>



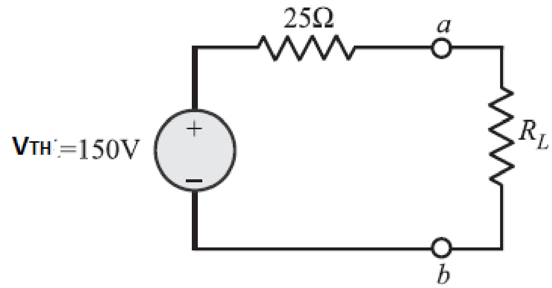


$$R_{TH} = R_{ab} = 30\ \Omega || 150\ \Omega$$

$$= \frac{30 \times 150}{30 + 150} = 25\ \Omega$$

Maximum power transfer is obtained when

$$R_L = R_{TH} = 25\ \Omega.$$

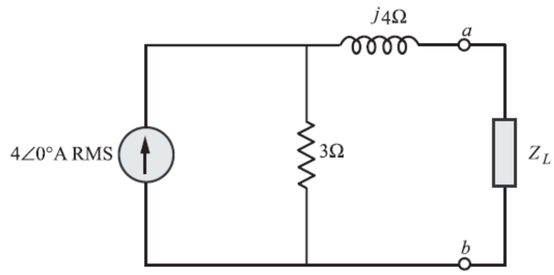


Then the maximum power is

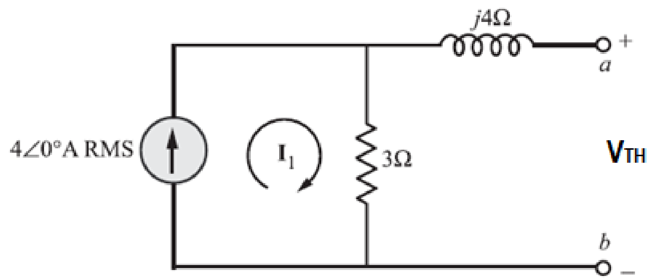
$$P_{\max} = \frac{V_{TH}^2}{4R_L} = \frac{(150)^2}{4 \times 25}$$

$$= 2.25 \text{ Watts}$$

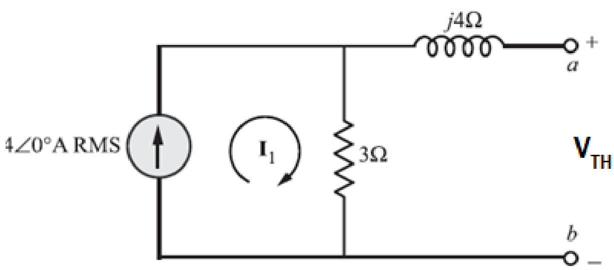
2) Find the load impedance that transfers the maximum average power to the load and determine the maximum average power transferred to the load ZL shown in Fig.



First step disconnect the load resistance



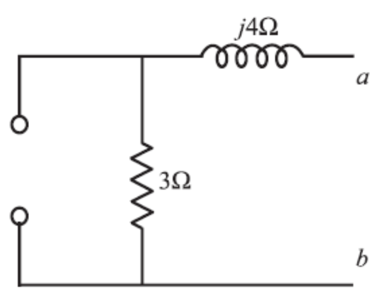
Find the V_{TH}



$$V_{TH} = V_{oc} = 4\,\underline{/0^\circ} \times 3$$

$$= 12\,\underline{/0^\circ}\ \text{Volts}$$

To find Z_{TH} , deactivate all the



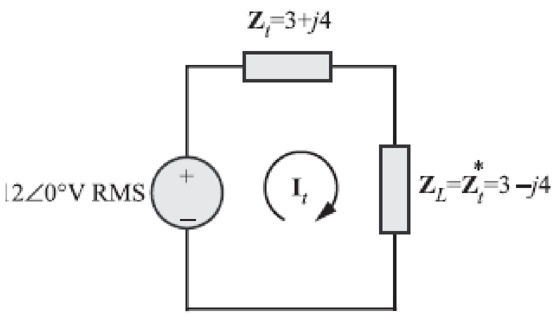
$$\mathbf{Z}_{TH} = 3 + j4\,\Omega$$

For maximum power transfer ZTH = ZL

Therefore **ZL** = $= 3 + j4\,\Omega$

For maximum average power transfer to the load, ZTH = $\mathbf{Z}_L^* = 3 - j4.$

For maximum average power transfer, the load impedance Z_L must be equal to the complex conjugate of the Thevenin impedance Z_{Th}.

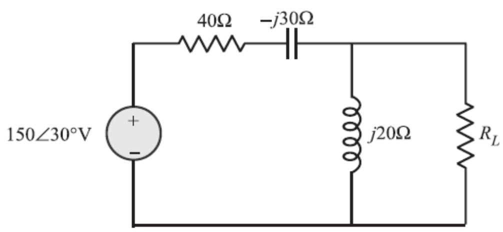


$$\therefore \quad \mathbf{I}_{TH} = \frac{12\,\underline{/0^\circ}}{3 + j4 + 3 - j4} = 2\,\underline{/0^\circ}\ \text{A}$$

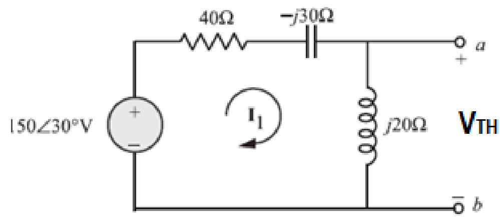
Hence, maximum average power delivered to the load is

$$P = |I_t|^2 R_L = 4(3) = 12\text{ W}$$

3) Refer the circuit given in Fig. Find the value of RL that will absorb the maximum average power.



Disconnecting the load resistor RL

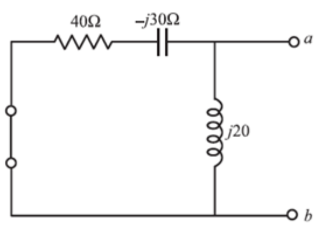


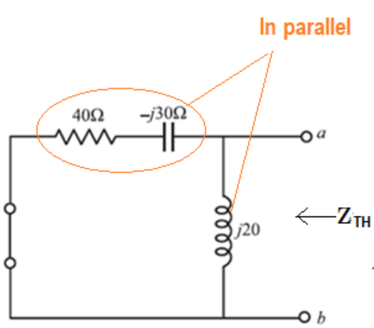
$$\mathbf{V}_{TH} = \mathbf{V}_{oc} = \mathbf{I}_1\,(j20)$$

$$= \frac{150\,\underline{/30^\circ} \times j20}{(40 - j30 + j20)}$$

$$= 72.76\,\underline{/134^\circ}\ \text{Volts.}$$

<u>To find Z_{TH} deactivate the independent source</u>



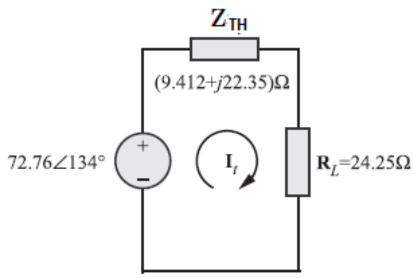


$$\mathbf{Z}_{TH} = (40 - j30)\,||\,j20$$

$$= \frac{j20\,(40 - j30)}{j20 + 40 - j30} = (9.412 + j22.35)\ \Omega$$

The Value of R_L that will absorb the maximum average power is

$$R_L = |\mathbf{Z}_t| = \sqrt{(9.412)^2 + (22.35)^2}$$
$$= 24.25\ \Omega$$



$$\mathbf{I}_{TH} = \frac{72.76\,\underline{/134^\circ}}{(9.412 + j22.35 + 24.25)} = 1.8\,\underline{/100.2^\circ}\ \mathrm{A}$$

Maximum average power absorbed by RL is

$$P_{\max} = \frac{1}{2}|I_{TH}|^2 R_L$$

$$P_{\max} = \frac{1}{2}(1.8)^2 \times 24.25$$

$$= \mathbf{39.29\ W}$$

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