

Scalable and Consistent Graph Neural Networks for Distributed Mesh-based Data-driven Modeling

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Distribute GNN

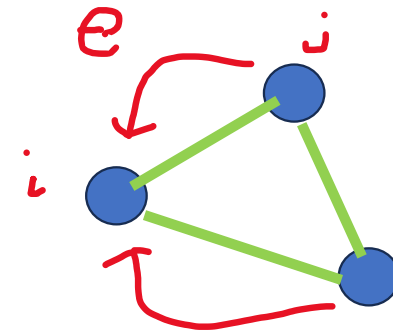
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Distribute GNN

- Scale No of nodes form $1e5$ to $1e9$
 - Finer grained mesh provide more insight
- Maintain consistence.
 - Result from 1 whole graph is the same as from distributed graph.

Graph NN notion

- Node feature, edge feature, adjacent matrix, parameter
- Message Passing
 - (1) an edge feature update conditioned on its corresponding sender and receiver nodes => msg formation
 - (2) a summation-aggregation of the updated edge features
 - (3) a node feature update conditioned on the aggregated edge features.
- Readout
 - Postprocessing layer
 - Node level -> one output each node
 - Graph level -> one output each graph



$$\text{Edge update : } \mathbf{e}_r^{ij} = \text{MLP}(\mathbf{x}_r^i, \mathbf{x}_r^j, \mathbf{e}_r^{ij}),$$

$$\text{Local edge aggr. : } \mathbf{a}_r^i = \sum_{j \in N(i)} \frac{1}{d_r^{ij}} \mathbf{e}_r^{ij},$$

$$\text{Node update : } \mathbf{x}_r^i = \text{MLP}(\mathbf{a}_r^{i,*}, \mathbf{x}_r^i).$$

Consistent Neural Message Passing Layer

- Consist message passing operator

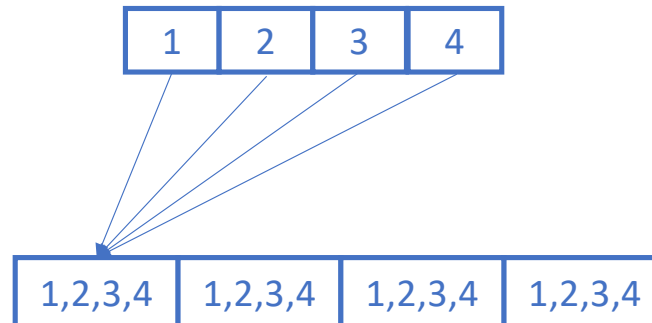
$$\mathcal{S}(\{\mathbf{Y}_1^{\text{local}}\}_{R=1}) = \mathcal{S}(\text{cat}\{\mathbf{Y}_1^{\text{local}}, \dots, \mathbf{Y}_R^{\text{local}}\}_{R>1})$$

- Consist Loss function

- AllReduce to get the aggregation of partitioned Loss on each nodes

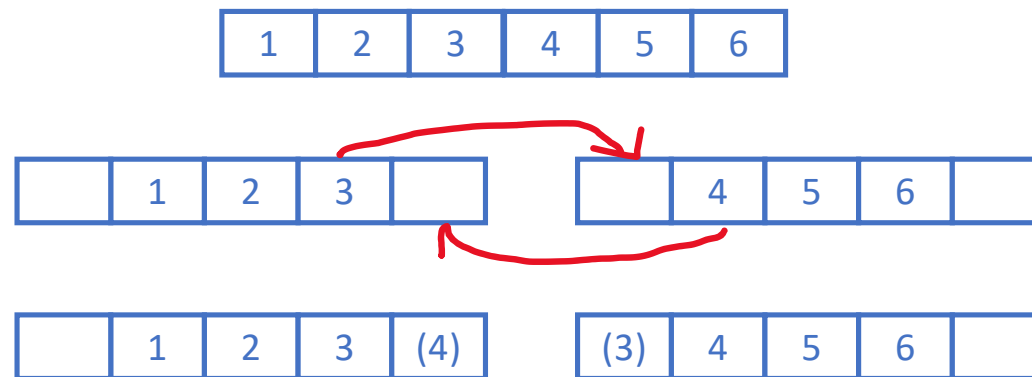
$$\left. \frac{\partial \mathcal{S}}{\partial \theta} \right|_{R=1} = \left. \frac{\partial \mathcal{S}}{\partial \theta} \right|_{R>1}.$$

$$\mathcal{L} = \frac{1}{N_{\text{eff}} F_y} \text{AllReduce}(S_r),$$

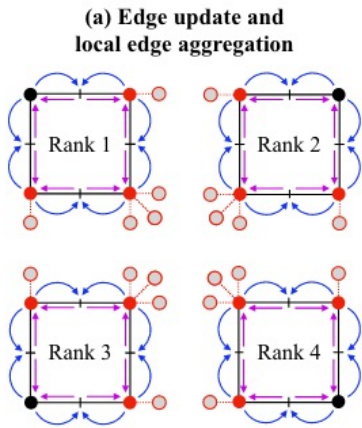


Halo swap

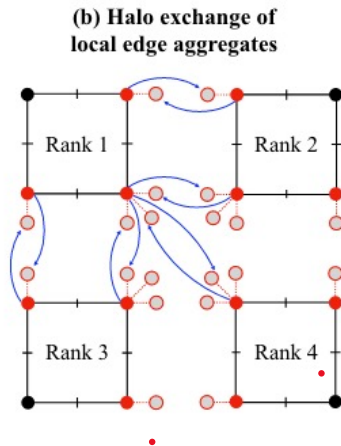
- An operation that need to know the value of its neighbors
 - such as $x[i] = \text{mean}(x[i-1], x[i+1])$



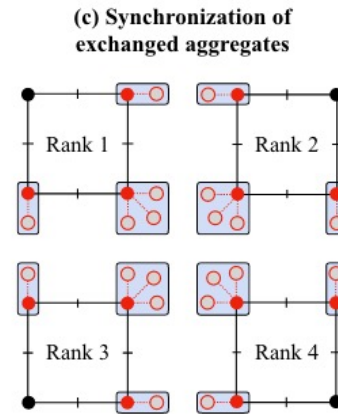
Neural halo swap



Blue array: Edge update => msg
Red array: sum of local msg



Halo swap: get msg adjacent node
which is on another rank



After sync:
summation of all aggregates
on nodes (including halo
nodes) sharing the same
global index

$$\text{Edge update : } \mathbf{e}_r^{ij} = \text{MLP}(\mathbf{x}_r^i, \mathbf{x}_r^j, \mathbf{e}_r^{ij}),$$

$$\text{Local edge aggr. : } \mathbf{a}_r^i = \sum_{j \in N(i)} \frac{1}{d_r^{ij}} \mathbf{e}_r^{ij},$$

$$\text{Halo swap : } \mathbf{a}_r^{i, \text{halo}} = \mathbf{a}_s^{k, \text{local}} \text{ if } G_r(i) = G_s(k),$$

$$\text{Synchronization : } \mathbf{a}_r^{i, *} = \sum_{\substack{j \\ G_r(j) = G_r(i)}} \mathbf{a}_r^j,$$

$$\text{Node update : } \mathbf{x}_r^i = \text{MLP}(\mathbf{a}_r^{i, *}, \mathbf{x}_r^i).$$