

First Semester B.E. Degree Examination, Dec.2019/Jan.2020
Calculus and Linear Algebra

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. With usual notations prove that $\tan \phi = r \left(\frac{d\theta}{dr} \right)$. (06 Marks)
- b. Find the angle between the curves $r = \sin\theta + \cos\theta$ and $r = 2 \sin\theta$ (06 Marks)
- c. Show that the radius of curvature for the catenary of uniform strength $y = a \log \sec \left(\frac{x}{a} \right)$ is $a \sec (x/a)$. (08 Marks)

OR

- 2 a. Show that the pairs of curves $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$ intersect each other Orthogonally. (06 Marks)
- b. Find the pedal equation of the curve $r^n = a^n \cos n\theta$. (06 Marks)
- c. Show that the evolute of $y^2 = 4ax$ is $27ay^2 = 4(x + a)^3$. (08 Marks)

Module-2

- 3 a. Find the Maclaurin's series for $\tan x$ upto the term x^4 . (06 Marks)
- b. Evaluate $\lim_{x \rightarrow 0} \left[\frac{a^x + b^x + c^x}{3} \right]^{1/x}$ (07 Marks)
- c. If $U = f(x-y, y-z, z-x)$, prove that $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z} = 0$ (07 Marks)

OR

- 4 a. Expand $\log (\sec x)$ upto the term containing x^4 using Maclaurin's series. (06 Marks)
- b. Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. (07 Marks)
- c. Find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ where $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$. (07 Marks)

Module-3

- 5 a. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dz dy dx$ (06 Marks)
- b. Evaluate $\int_2^4 \int_0^{\sqrt{4-x^2}} (2-x) dy dx$ by changing the order of integration. (07 Marks)
- c. Prove that $\beta(m, n) = \frac{\Gamma(m) \cdot \Gamma(n)}{\Gamma(m+n)}$ (07 Marks)

OR

- 6 a. Evaluate $\int \int y \, dx \, dy$ over the region bounded by the first quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (06 Marks)
- b. Find by double integration the area enclosed by the curve $r = a(1 + \cos \theta)$ between $\theta = 0$ and $\theta = \pi$. (07 Marks)
- c. Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin \theta}} \times \int_0^{\pi/2} \sqrt{\sin \theta} \, d\theta = \pi$. (07 Marks)

Module-4

- 7 a. Solve $\frac{dy}{dx} + \frac{y \cos x + \sin y + y}{\sin x + x \cos y + x} = 0$ (06 Marks)
- b. Solve $r \sin \theta - \cos \theta \frac{dr}{d\theta} = r^2$ (07 Marks)
- c. A series circuit with resistance R, inductance L and electromotive force E is governed by the differential equation $L \frac{di}{dt} + Ri = E$, where L and R are constants and initially the current i is zero. Find the current at any time t. (07 Marks)

OR

- 8 a. Solve $(4xy + 3y^2 - x)dx + x(x + 2y)dy = 0$. (06 Marks)
- b. Find the orthogonal trajectories of the family of parabolas $y^2 = 4ax$. (07 Marks)
- c. Solve $p^2 + 2py \cot x = y^2$. (07 Marks)

Module-5

- 9 a. Find the rank of $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ by elementary row transformations. (06 Marks)
- b. Apply Gauss-Jordan method to solve the system of equations
 $2x_1 + x_2 + 3x_3 = 1$,
 $4x_1 + 4x_2 + 7x_3 = 1$,
 $2x_1 + 5x_2 + 9x_3 = 3$. (07 Marks)
- c. Find the largest Eigen value and the corresponding Eigen vector of the matrix
 $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ by power method. Using initial vector $(100)^T$. (07 Marks)

OR

- 10 a. Solve by Gauss elimination method
 $x - 2y + 3z = 2$,
 $3x - y + 4z = 4$,
 $2x + y - 2z = 5$ (06 Marks)
- b. Solve the system of equations by Gauss-Seidal method
 $20x + y - 2z = 17$,
 $3x + 20y - z = -18$,
 $2x - 3y + 20z = 25$ (07 Marks)
- c. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to the diagonal form. (07 Marks)