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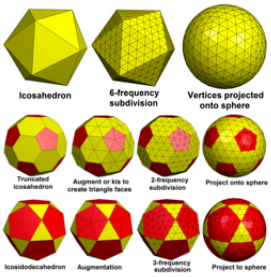
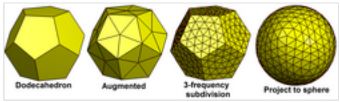
# Geodesic polyhedron



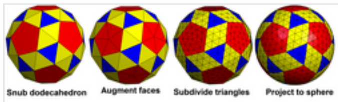

An icosahedron and related symmetry polyhedra can be used to define a high geodesic polyhedron by dividing triangular faces into smaller triangles, and projecting all the new vertices onto a sphere. Higher order polygonal faces can be divided into triangles by adding new vertices centered on each face. The new faces on the sphere are not equilateral triangles, but they are approximately equal edge length. All vertices are valence-6 except 12 vertices which are valence 5.

A **geodesic polyhedron** is a convex polyhedron made from triangles. They usually have icosahedral symmetry, such that they have 6 triangles at a vertex, except 12 vertices which have 5 triangles. They are the dual of corresponding Goldberg polyhedra, of which all but the smallest one (which is a regular dodecahedron) have mostly hexagonal faces.




Geodesic subdivisions can also be done from an augmented dodecahedron, dividing pentagons into triangles with a center point, and subdividing from that

Geodesic polyhedra are a good approximation to a sphere for many purposes, and appear in many different contexts. The most well-known may be the *geodesic domes*, hemispherical architectural structures designed by Buckminster Fuller, which geodesic polyhedra are named after. Geodesic grids used in geodesy also have the geometry of geodesic polyhedra. The capsids of some viruses have the shape of geodesic polyhedra,[1][2] and some pollen grains are based on geodesic polyhedra.[3] Fullerene molecules have the shape of Goldberg polyhedra. Geodesic polyhedra are available as geometric primitives in the Blender 3D modeling software package, which calls them *icospheres*: they are an alternative to the UV sphere, having a more regular distribution.[4][5] The Goldberg–Coxeter construction is an expansion of the concepts underlying geodesic polyhedra.




Chiral polyhedra with higher order polygonal faces can be augmented with central points and new triangle faces. Those triangles can then be further subdivided into smaller triangles for new geodesic polyhedra. All vertices are valence-6 except the 12 centered at the original vertices which are valence 5

## Notation

In Magnus Wenninger's *Spherical models*, polyhedra are given **geodesic notation** in the form $\{3,q+\}_{b,c}$, where **{3,*q*}** is the Schläfli symbol for the regular polyhedron with triangular faces, and q-valence vertices. The + symbol indicates the valence of the vertices being increased. ***b,c*** represent a subdivision description, with 1,0 representing the base form. There are 3 symmetry classes of forms: $\{3,3+\}_{1,0}$ for a tetrahedron, $\{3,4+\}_{1,0}$ for an octahedron, and $\{3,5+\}_{1,0}$ for an icosahedron.

The dual notation for Goldberg polyhedra is $\{q+,3\}_{b,c}$, with valence-3 vertices, with *q*-gonal and hexagonal faces. There are 3 symmetry classes of forms: $\{3+,3\}_{1,0}$ for a tetrahedron, $\{4+,3\}_{1,0}$ for a cube, and $\{5+,3\}_{1,0}$ for a dodecahedron.

Values for *b*,*c* are divided into three classes:

> **Class I** (b=0 or c=0): $\{3,q+\}_{b,0}$ or $\{3,q+\}_{0,b}$ represent a simple division with original edges being divided into *b* sub-edges.
> **Class II** (b=c): $\{3,q+\}_{b,b}$ are easier to see from the dual polyhedron {*q*,3} with *q*-gonal faces first divided into triangles with a central point, and then all edges are divided into *b* sub-edges.
> **Class III**: $\{3,q+\}_{b,c}$ have nonzero unequal values for *b*,*c*, and exist in chiral pairs. For $b > c$ we can define it as a right-handed form, and $c > b$ is a left-handed form.

Subdivisions in class III here do not line up simply with the original edges. The subgrids can be extracted by looking at a triangular tiling, positioning a large triangle on top of grid vertices and walking paths from one vertex *b* steps in one direction, and a turn, either clockwise or counterclockwise, and then another *c* steps to the next primary vertex.

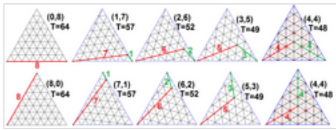
For example, the icosahedron is $\{3,5+\}_{1,0}$, and pentakis dodecahedron, $\{3,5+\}_{1,1}$ is seen as a regular dodecahedron with pentagonal faces divided into 5 triangles.

The primary face of the subdivision is called a **principal polyhedral triangle** (PPT) or the **breakdown structure**. Calculating a single PPT allows the entire figure to be created.

The **frequency** of a geodesic polyhedron is defined by the sum of $v = b + c$. A **harmonic** is a subfrequency and can be any whole divisor of *v*. Class II always have a harmonic of 2, since $v = 2b$.

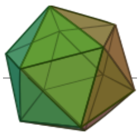
The **triangulation number** is $T = b^2 + bc + c^2$. This number times the number of original faces expresses how many triangles the new polyhedron will have.

## Elements



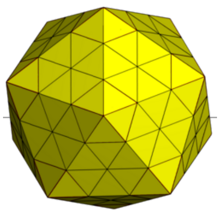
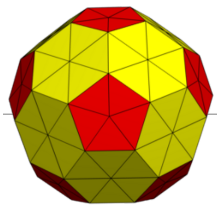
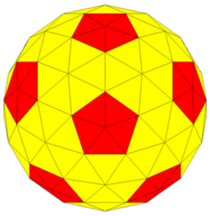
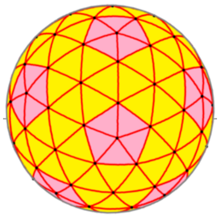
PPTs with frequency 8

The number of elements are specified by the triangulation number $T = b^2 + bc + c^2$. Two different geodesic polyhedra may have the same number of elements, for instance, $\{3,5+\}_{5,3}$ and $\{3,5+\}_{7,0}$ both have T=49.

| Symmetry | Icosahedral | Octahedral | Tetrahedral |
|---|---|---|---|
| **Base** | Icosahedron<br>$\{3,5\} = \{3,5+\}_{1,0}$ | Octahedron<br>$\{3,4\} = \{3,4+\}_{1,0}$ | Tetrahedron<br>$\{3,3\} = \{3,3+\}_{1,0}$ |
| **Image** |  |  |  |
| **Symbol** | $\{3,5+\}_{b,c}$ | $\{3,4+\}_{b,c}$ | $\{3,3+\}_{b,c}$ |
| **Vertices** | $10T + 2$ | $4T + 2$ | $2T + 2$ |
| **Faces** | $20T$ | $8T$ | $4T$ |
| **Edges** | $30T$ | $12T$ | $6T$ |

## Construction

Geodesic polyhedra are constructed by subdividing faces of simpler polyhedra, and then projecting the new vertices onto the surface of a sphere. A geodesic polyhedron has straight edges and flat faces that approximate a sphere, but it can also be made as a spherical polyhedron (a tessellation on a sphere) with true geodesic curved edges on the surface of a sphere and spherical triangle faces.

| Conway | $u_3I = (kt)I$ | (k)tI | ktI | |
|---|---|---|---|---|
| **Image** |  |  |  |  |
| **Form** | **3-frequency subdivided icosahedron** | **Kis truncated icosahedron** | **Geodesic polyhedron (3,0)** | **Spherical polyhedron** |

In this case, $\{3,5+\}_{3,0}$, with frequency $\nu = 3$ and triangulation number $T = 9$, each of the four versions of the polygon has 92 vertices (80 where six edges join, and 12 where five join), 270 edges and 180 faces.

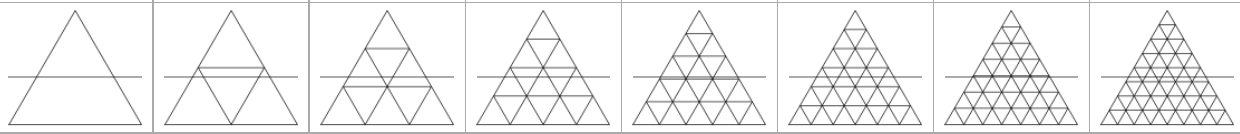
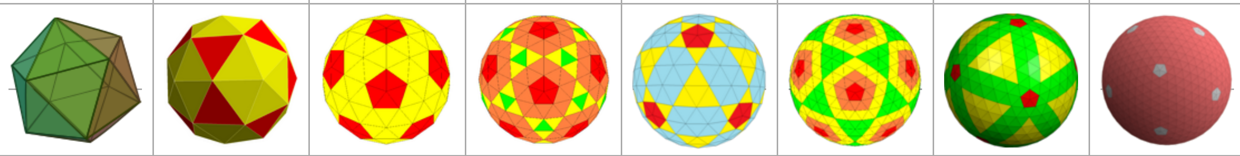
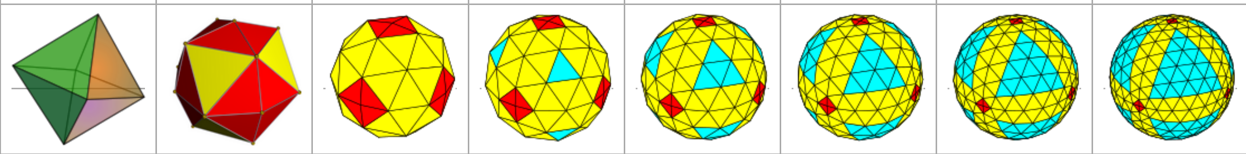
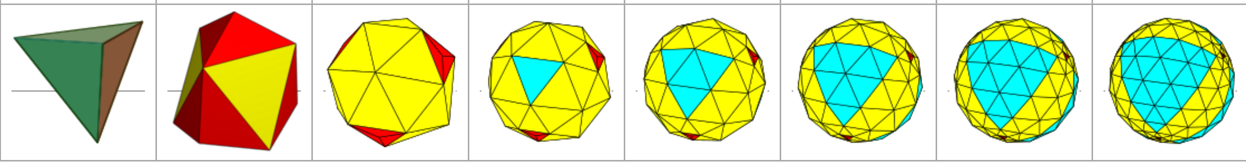
## Relation to Goldberg polyhedra

Geodesic polyhedra are the duals of Goldberg polyhedra. Goldberg polyhedra are also related in that applying a kis operator (dividing faces into triangles with a center point) creates new geodesic polyhedra, and truncating vertices of a geodesic polyhedron creates a new Goldberg polyhedron. For example, Goldberg G(2,1) kised, becomes $\{3,5+\}_{4,1}$, and truncating that becomes G(6,3). And similarly $\{3,5+\}_{2,1}$ truncated becomes G(4,1), and that kised becomes $\{3,5+\}_{6,3}$.

# Examples

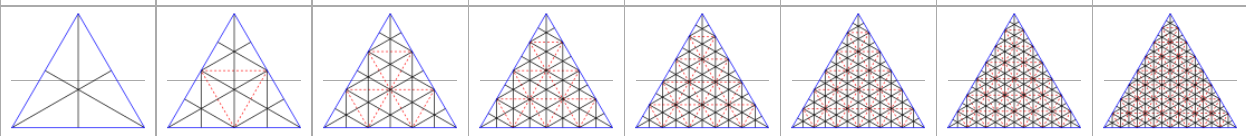
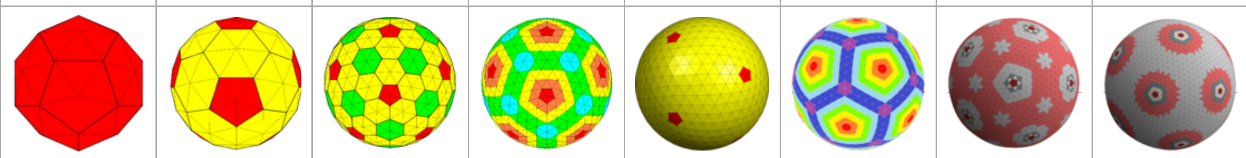
## Class I

Class I geodesic polyhedra

| Frequency | (1,0) | (2,0) | (3,0) | (4,0) | (5,0) | (6,0) | (7,0) | (8,0) | ( |
|---|---|---|---|---|---|---|---|---|---|
| *T* | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | |
| Face triangle |  | | | | | | | | |
| Icosahedral |  | | | | | | | | |
| Octahedral |  | | | | | | | | |
| Tetrahedral |  | | | | | | | | |

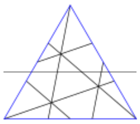
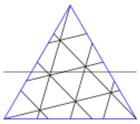
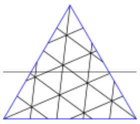
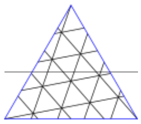
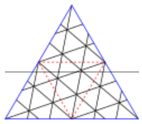
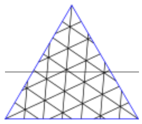
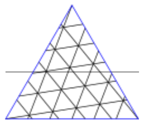
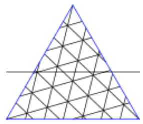
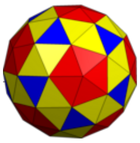
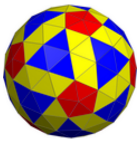
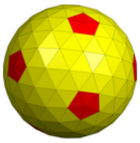
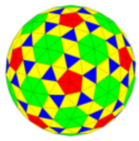
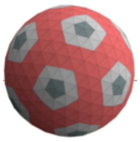
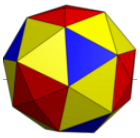
## Class II

Class II geodesic polyhedra

| Frequency | (1,1) | (2,2) | (3,3) | (4,4) | (5,5) | (6,6) | (7,7) | (8,8) | ( |
|---|---|---|---|---|---|---|---|---|---|
| *T* | 3 | 12 | 27 | 48 | 75 | 108 | 147 | 192 | |
| Face triangle |  | | | | | | | | |
| Icosahedral |  | | | | | | | | |
| Octahedral |  | | | | | | | | |
| Tetrahedral |  | | | | | | | | |

### Class III

Class III geodesic polyhedra

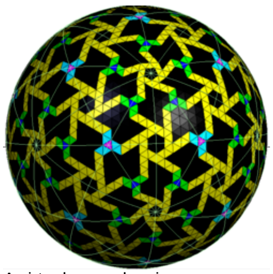
| Frequency | (2,1) | (3,1) | (3,2) | (4,1) | (4,2) | (4,3) | (5,1) | (5,2) |
|---|---|---|---|---|---|---|---|---|
| *T* | 7 | 13 | 19 | 21 | 28 | 37 | 31 | 39 |
| Face triangle |  |  |  |  |  |  |  |  |
| Icosahedral |  |  |  |  |  | | | |
| Octahedral |  | | | | | | | |
| Tetrahedral |  | | | | | | | |

## Spherical models

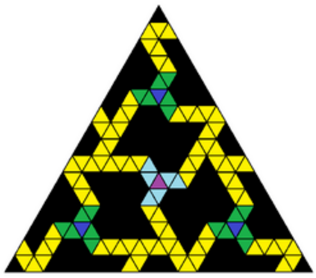
Magnus Wenninger's book *Spherical Models* explores these subdivisions in building polyhedron models. After explaining the construction of these models, he explained his usage of triangular grids to mark out patterns, with triangles colored or excluded in the models.[6]

Example model

An artistic model created by Father Magnus Wenninger called *Order in Chaos*, representing a chiral subset of triangles of a 16-frequency icosahedral geodesic sphere, $\{3,5+\}_{16,0}$



A virtual copy showing icosahedral symmetry great circles. The 6-fold rotational symmetry is illusionary, not existing on the icosahedron itself.



A single icosahedral triangle with a 16-frequency subdivision

## See also

- Conway polyhedron notation – Method of describing higher-order polyhedra

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