# University of Vavuniya, Sri Lanka

## First Examination in Information and Communication Technology - 2020

## Second Semester - December/January 2022

## TICT1212: Discrete Structures

- ⊙ Answer **four questions** only.
- ⊙ This paper has **five questions** on **six pages**.
- ⊙ Time allowed: **Two Hours**.

1. (a) Let p, q and r be three propositions:

   p: John is healthy

   q: John is wealthy

   s: John is wise

   Write each of the following statements in symbolic form:

   i. John is healthy and wealthy but not wise ;

   ii. John is not wealthy but he is healthy and wise ;

   iii. John is neither wealthy nor wise, but is healthy . [20%]

   (b) Using the theorem verify the following logical equivalence:

   i. $\sim(\sim p \wedge q) \wedge (p \vee q) \equiv p$ ;

   ii. $\sim((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (p \wedge q) \equiv p$ . [20%]

   (c) Show that the compound proposition $(p \vee q) \wedge (\sim p \vee r) \rightarrow (q \vee r)$ is a tautology. [10%]

   (d) Write down the negation for the statement "Hal is a mathematics major and Hal's sister is a computer science major". [10%]

   (e) State the converse, inverse and contrapositive of the following conditional statement: "A postive integer is a prime only if it has no divisors other than 1 and itself" [15%]

   (f) Prove whether the argument is valid or not?

   "If the auditorium was not available or there were examinations, then the music program was postponed. If the music program gets postponed then new date was announced. No new date was announced. Therefore, auditorium was available" [25%]

2. (a) Let sets $R$ and $S$ be defined as follows:

   $R = \{x \in \mathbb{Z} \mid x \text{ is divisible by } 2\}$ ;

   $S = \{y \in \mathbb{Z} \mid x \text{ is divisible by } 6\}$ .

   Is $R$ a subset of $S$? Justify your answer. [15%]

[Question 2 continues on the next page]

(b) Write down the elements in each of the following sets:

i. $A = \{x \in \mathbb{N} \mid x \text{ is an odd number}\}$

ii. $B = \{x \in \mathbb{Z} \mid x^2 \leq 4\}$

iii. $C = \{x \in \mathbb{N} \mid 4 + x = 3\}$

iv. $D = \{x \in \mathbb{N} \mid x^2 = 2 \quad \text{or} \quad x^2 = 9\}$ [20%]

(c) Draw Venn diagrams for each of the following expressions of sets $A$, $B$ and $C$.

i. $A \cup B \cup C^c$

ii. $(A \cap B)^c \cap C$

iii. $A - (B \cap C)$

iv. $A \cup (B \cap C)^c$ [20%]

(d) Suppose $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{0, 1, 2, 3, 5, 8\}$, $B = \{0, 2, 4, 6\}$ and $C = \{1, 3, 5, 7\}$, find the equivalent set for each of the following expressions:

i. $A \cap B^c$ ;

ii. $(A^c \cap C) \cup B$ ;

iii. $P(A)$ ;

iv. $A$–$B$ ;

v. $A \times B \times C$ ;

vi. $A \cup B^c$ . [30%]

(e) Let $A$, $B$ and $C$ be sets. Show that

i. $(A \cup B)$–$C = (A$–$C) \cup (B$–$C)$ ;

ii. $A$–$(A \cap B) = A$–$B$ ;

iii. $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ . [15%]

3. (a) Consider the following relations on {0,1,2,3} :

$R_1 = \{(0,0), (0,1), (0,3), (1,1), (1,0), (2,3), (3,3)\}$ ;

$R_2 = \{(0,0), (0,1), (1,1), (1,2), (2,2), (2,3)\}$ ;

$R_3 = \{(2,3), (3,2)\}$ ;

$R_4 = \{(1,2), (2,1), (1,3), (3,1)\}$ ;

$R_5 = \{(0,0), (0,1), (0,2), (1,2)\}$ .

Identify the reflexive, symmetric, antisymmetric and transitive relations from the above set of relations. [20%]

(b) Consider the relation $R = \{(1,3), (1,4), (3,2), (3,3), (3,4)\}$ on $A = \{1,2,3,4\}$. Find $R^{-1}$. [15%]

[Question 3 continues on the next page]

(c) Find the domain and range of the relation $R$ on the set $X = \{1,2,3,4,5\}$ is defined by $(x, y) \in \mathbb{R}$ if $x = y - 1, (x, y) \in \mathbb{R}$ [15%]

(d) Consider a relation $R$ from the set $A = \{0, 1, 2, 3, 4\}$ to the set $B = \{0, 1, 2, 3\}$. Relations $R_1, R_2$ and $R_3$ are defined as follows:

$R_1 = \{(a, b) \in \mathbb{R}, a \in A, b \in B |\ a = b\}$

$R_2 = \{(a, b) \in \mathbb{R}, a \in A, b \in B |\ a > b\}$

$R_3 = \{(a, b) \in \mathbb{R}, a \in A, b \in B |\ a + b = 4\}$

i. Identify the ordered pairs of each of the relations $R_1, R_2$ and $R_3$.

ii. Find the ordered pairs of each of the following relations:

i. $R_1 \cup R_2$ ;

ii. $R_2 - R_3$ ;

iii. $R_3 o R_1$ . [30%]

(e) Let $f$ and $g$ be two functions from $\mathbb{R}$ to $\mathbb{R}$ defined by $f(x) = x^2 + 1$ and $g(x) = x + 2$ respectively for all $x \in \mathbb{R}$. Find the following:

i. $f(-5)$

ii. $g(0)$

iii. $fog(3)$

iv. $gof(2)$ [20%]

4. (a) Consider the following graph shown in Figure 1 below:

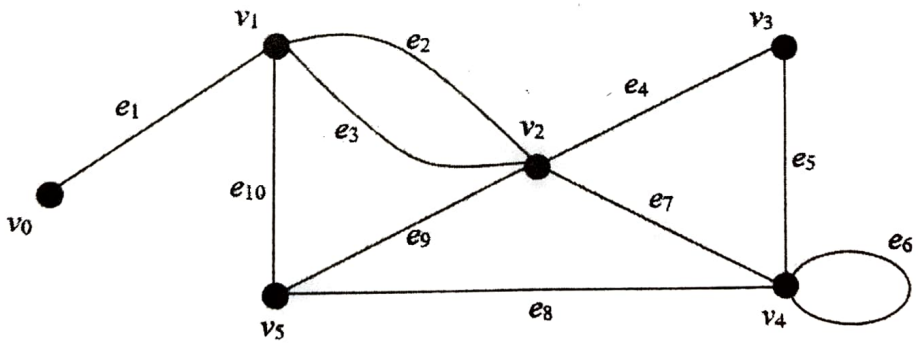


Figure 1:

i. Find the degree of each vertex of the graph. [05%]

ii. Represent the graph in the following matrix:

i. Incidence matrix ;

ii. Adjacency matrix . [20%]

[Question 4 continues on the next page]

(b) i. Define the each of the following terms:

i. Multi Graph ;

ii. Path ;

iii. Euler circuit .

iv. Consider the graph shown in Figure 2 below:

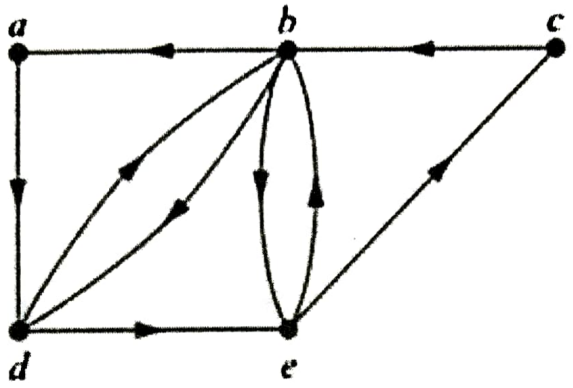


Figure 2:

i. Find the in-degree and out-degree of each vertex. [10%]

ii. Determine whether the directed graph shown above has an Euler circuit. Construct an Euler circuit if one exists. If no Euler circuit exists, determine whether the directed graph has an Euler path. Construct an Euler path if one exists. [15%]

(c) Determine whether the given pair of graphs shown in Figure 3 is isomorphic or not. [20%]

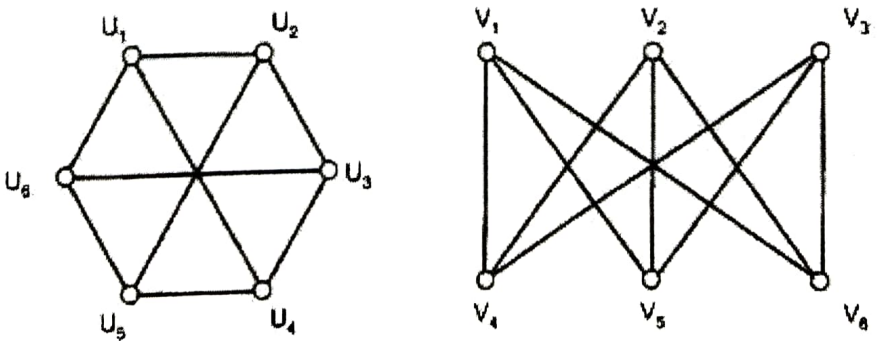


Figure 3:

[Question 4 continues on the next page]

(d) Construct a precedence graph for the following program:

$S1 : x := 0$ ;

$S2 : x := x + 1$ ;

$S3 : y := 2$ ;

$S4 : z := y$ ;

$S5 : x := x + 2$ ;

$S6 : y := x + z$ ;

$S7 : z := 4$ . [15%]

(e) Use Karnaugh map to simplify the following Boolean expression:

$F(x, y, z, t) = xyzt + xyz\bar{t} + \bar{x}\bar{y}zt + \bar{x}\bar{y}z\bar{t} + x\bar{y}zt + xy\bar{z}t + x\bar{y}\bar{z}t + \bar{x}yzt$ [15%]

5. (a) Consider the following tree shown in Figure 4 and find each of the following attributes:

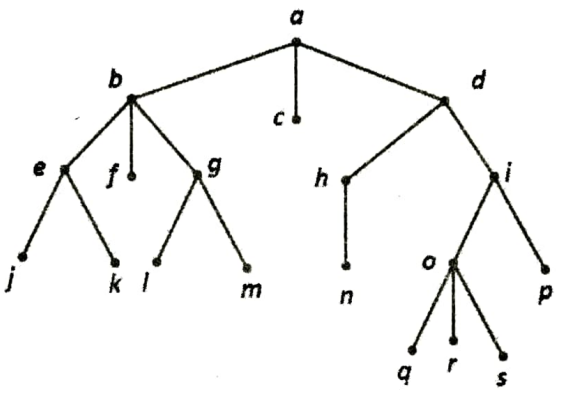


Figure 4:

i. Root ;

ii. Internal vertices ;

iii. Leaves vertices ;

iv. Children of 'g' ;

v. Parent of 'o' ;

vi. Siblings of 'e' ;

vii. Ancestors of 'm' ;

viii. Descendants of 'd' ;

ix. Subgraph contains 'i' as root node ;

x. Level and Height ;

xi. Determine whether the above tree is balanced tree or not.

[Question 5 continues on the next page]

xii. Determine the following traversals visits the vertices of the above ordered rooted tree

i. Preorder ;

ii. Inorder ;

iii. Postorder . [40%]

(b) Construct a Binary Search Tree by insert the following numbers, in the order given: 45, 67, 22, 100, 75, 13, 11, 64, 30, 3, 14, 15, 20, 25, 30, 33, 62, 200 [20%]

(c) Consider the following graph shown in Figure 5:

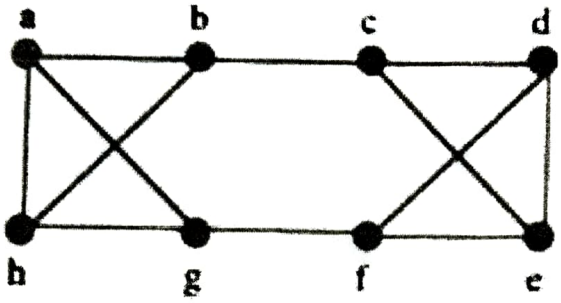


Figure 5:

i. How many number of spanning tree possible for the above graph. [05%]

ii. Find three different spanning trees of the above graph. [15%]

(d) Find a minimum spanning tree of the following weighted graph. [20%]

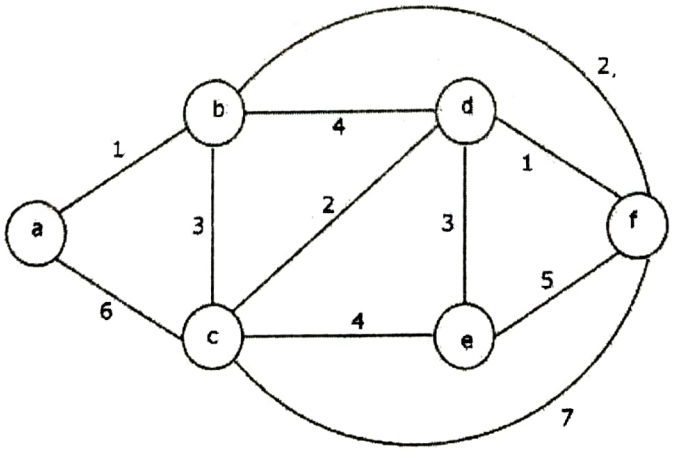


Figure 6:

End