University of Vavuniya

First Examination in Information Communication Technology - 2020

First Semester - May/June 2022 (held in August 2022)

TICT1123 Mathematics for Technology

Answer Five Questions Only

Time Allowed: Three hours

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1. (a) A square matrix A of order 3 is defined in terms of the scalar constant k by

$$A = \begin{pmatrix} 2 & -1 & 3 \\ k & 2 & 4 \\ k-2 & 3 & k+7 \end{pmatrix}.$$

If determinant of A is $|A| = 8$, then find the possible values of k .   [20%]

(b) A square matrix of order 3 is defined by

$$M = \begin{pmatrix} 5 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 3 & 1 \end{pmatrix}.$$

Compute the following:

   i. Determinant of M;

   ii. Co-factor of M;

   iii. Adjoint of M;

   iv. Inverse of M.   [50%]

[This Question continues on next page.]

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(c) Solve the following system of equations using Gaussian Elimination.

$$2x - 2y + 3z = 2,$$

$$x + 2y - z = 3,$$

$$3x - y + 2z = 1.$$

[30%]

2. (a) If $A = \begin{pmatrix} 2 & 9 \\ 1 & 7 \end{pmatrix}$, then verify that $(A^T)^{-1} = (A^{-1})^T$. [10%]

(b) Let the matrices A, B and C are defined by,

$$A = \begin{pmatrix} 0 & 2 & -3 \\ 0 & -1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} a & b & 0 \\ c & d & 0 \end{pmatrix} \quad \text{and} \quad C = \begin{pmatrix} 3 & 4 \\ 2 & 3 \\ 1 & 2 \end{pmatrix}.$$

i. Show that $AC = I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Also find CA. [20%]

ii. Find the values of a, b, c and d such that $BC = I_2$. [20%]

(c) Solve the following system of linear equations using Gauss-Jordan elimination:

$$2x + 7y + 11z = 11,$$

$$x + 2y + 8z = 14,$$

$$x + 3y + 6z = 8.$$

[50%]

3. (a) Find the derivative of $y = x^2 + 2$ using the first principle of derivatives. [10%]

(b) Evaluate each of the following limits:

i. $\lim\limits_{x \to 3} \dfrac{x^2 - 6x + 5}{x^3 - 8x + 7}$;

ii. $\lim\limits_{x \to 1} \dfrac{\sqrt{x} - 1}{x - 1}$;

iii. $\lim\limits_{x \to 2} \dfrac{x^4 - 16}{x - 2}$;

[45%]

[This Question continues on next page.]

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(c) Find the slope of the tangent to the curve $y = -5x^2 + 7x$ at the point $x = 1$. [15%]

(d) Let f(x) be a function defined by

$$f(x) = \begin{cases} \dfrac{(6+x)^2 - 36}{x} & \text{if } x < 1 \\[4mm] \dfrac{2x^2 - 17x + 8}{8 - x} & \text{if } x \geq 1 \end{cases}$$

Evaluate each of the following limits:

i. $\displaystyle\lim_{x \to (-6)} f(x)$;

ii. $\displaystyle\lim_{x \to 8} f(x)$. [30%]

4. (a) Find $\dfrac{dy}{dx}$ if $x^2 + y^2 = 1$. [15%]

(b) If $x = at^2$ and $y = 2at$, find $\dfrac{dy}{dx}$ where $t \neq 0$. [15%]

(c) Find all derivatives of the function $y = x^3 - 6x^2 - 5x + 3$. [15%]

(d) Differentiate each of the followings with respect to x:

i. $y = e^{\sqrt{x}}$

ii. $y = 5(2x + 4)(x^2 + 4x + 6)^5$

iii. $y = \dfrac{x}{\sqrt{7 - 3x}}$ [25%]

(e) Let $y = 2xe^{-3x}$, then show that

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0.$$

[30%]

5. (a) Evaluate each of the following integrals:

i. $\sqrt{3x + 2}$

ii. $\dfrac{x^2 - x + 1}{x^3}$

iii. $5x^2 - 4 + \dfrac{7}{x} + \dfrac{2}{\sqrt{x}}$ [45%]

(b) Show that $\displaystyle\int_2^7 \frac{8}{4x - 3}dx = \ln 25$ [15%]

[This Question continues on next page.]

(c) How many ways a 4 digit number can be formed among from the numbers 4,5,6,7,8,9 :

    i. No digit is repeated ;

    ii. The digits may be repeated.   [20%]

(d) A local school board with 8 people needs to form a committee with three people. If 5 of the members are girls and 3 of the members are boys, How many ways can the committee be formed :

    i. If two must be girls and one must be a boy?

    ii. If the committee wants at least 2 girls?   [20%]

6. (a) If the mid-point (x,y) of the line joining (3,4) and (p,7) lies on $2x + 2y + 1 = 0$, then find the value of p?   [20%]

(b) Write the equation of the lines through the point (1,-1)

    i. Parallel to the line $x + 3y - 4 = 0$;

    ii. Perpendicular to the line $3x + 4y = 6$.   [20%]

(c) If a straight line $2x - 3y + 5 = 0$ is perpendicular to the line $3x + ky - 1 = 0$, then find the value of the constant k.   [20%]

(d) Find the equation of a line which has slope (-5/4) and passing through the point (1,2).   [20%]

(e) The points P and Q have coordinates (-1,6) and (9,0) respectively. The line L is perpendicular to PQ and passes through the midpoint of PQ. Find an equation for line L.   [20%]