# ID5059 - Imputation

# Python and R Hybrid

How can we use both? The reticulate library (2020) enables us to use both Python and R functions and coding in one place, a R Markdown file.

More information on the library specifications and functionalities can be found here

Source all functions from the python script Imputation\_Simulation\_Study()

All other requirements for this document:

# References

### References for this document

- 1 : Flexible Imputation of Missing Data Second Edition Stef Van Buuren
- 2 : Inference and Missing Data Rubin
- 3: MICE: Multivariate Imputation by Chained Equations in R
- 4: MissForest—non-parametric missing value imputation for mixed-type data
- 5: Imputing Missing Data with R MICE Package
- 6: MissForest: Non-Parametric imputation technique

# Imputation

Imputation is the act of converting an incomplete sample into a complete sample. [1]

The idea behind this analysis is to construct a simulation study. This traditionally can be split into three key steps:

- 1. Simulation Step
- 2. Application of Methods
- 3. Assessing Performance of Methods

# 1. Removing Data At Random [Simulation Step]

## Types of Missing Data

Rubin [2] classified missing data problems into three categories, which are all driven by the so-called **response mechanism**. This mechanism can be modelled, as long as an assumption is made as to which response mechanism drove the data to be missing in the first place. The categories are **Missing Completely at Random** (MCAR), **Missing at Random** (MAR), **Missing not at Random** (MNAR). In short, these can be described as:

- 1. MCAR: when the probability of data missing is the same across the entire datset.
- 2. MAR: when the probability of data missing is the same across groups within the dataset.
- 3. MNAR: when the probability of data missing depends on an unknown factor.

The current **methodologies** in the literature rely on a **key assumption** choice of either three categories. Hence, certain methods will be more suitable than others depending on the data and whether the assumption is met or not will underpin the validity of statistical inferences.

Another important distinction to make it between **Univariate** and **Multivariate Missing Data**. These are defined as the problem where only one feature is missing and the problem where more than one feature is missing respectively. In our simulation function we will have freedom to choose every time which case to be in.

### In Practice

For this special case, the simulation step is slightly unusual. Normally one would sample from a known distribution, however in this instance the sampling happens as way to identify the location of the values which will be dropped, i.e. become NaN.

### 1. For the **MCAR** Case:

Mathematically, let n be the total number of observations (rows) in the dataset and m be the total number of features (columns) in the dataset. Hence, let  $U_1$  and  $U_2$  be two sets  $U_1 \in {1,...,n}$  and  $U_2 \in {1,...,m}$ . By sampling with replacement k times from these two sets independently, such that any outcome is equally likely, the resulting ordered outcomes indicate the indexing of the element which will be dropped. These indexes can be viewed as the tuples  $(u_{1i}, u_{2i})$  for  $i \in {1,...,k}$ .

## Function to Remove Data (in Python)

```
def remove_random_values(true_df, columns, percentage):
    """
    Funtion the will remove values from the selected columns at random,
    then replace the removed values with np.nan which is NaN.
    PSEUDOCODE:
```

```
1. Random sample row indexes (with replacement)
2. Random sample column labels (with replacement)
3. Extract random entry using 1. and 2. and replace with np.nan
4. Repeat (1-3) until total proportion of data has been extracted
:param df: pandas DataFrame which is the full data
:param columns: list of columns to 'drop' values from. Note: must be string name of columns
:param percentage: float input such as 0.20 (i.e. 20%)
:return: pandas DataFrame of the same shape as the original with NaN entries
# Set up: Focus on subset of data dependent on columns,
          hence extract information about subset and number of values to drop
subset = true_df[columns].copy()
nrow, ncol = true_df[columns].shape
n_samples = int(percentage*nrow)
# 1. Pick out a vector of random samples (row indexes)
row_indexes = np.arange(0, nrow, 1).tolist()
# 2. Sample with replacement from the list of possible indexes
   note, `choices` is used to sample with replacement
sampled_row_indexes = random.choices(row_indexes, k = n_samples)
sampled_columns = random.choices(columns, k = n_samples)
# 3. Extract entry and replace accordingly
for i in range(0, n_samples):
    if sampled_columns[i][0:3] == 'cat':
        subset.loc[sampled_row_indexes[i], sampled_columns[i]] = 'nan'
    else:
        subset.loc[sampled_row_indexes[i], sampled_columns[i]] = -1
        # this is the convention for a number of sklearn packages
# Notes: having it as a loop ensures we are not holding a massive dataframe in memory so its a lot
# Notes: must use 'nan' and numerical as np.nan would not be updated in step 4. otherwise.
# 4. Update data frame with new NaN values
true_df.update(subset)
true_df = true_df.replace('nan', np.nan)
true_df = true_df.replace(-1, np.nan)
return(true_df)
```

# 2. Choosing the Imputation Methods [Methods Step]

### Types of Models

The simplest and most common solutions for missing data problems are

• likewise deletion, which is simply removing the rows with missing elements from the dataset,

- mean or median imputation, which is substituting the missing feature with the sample mean or median over all observed values in that same feature column.
- regression imputation, which is about creating a regression model, hence incorporating the effects of other features, and using the model to predict the missing data.
- stochastic regression imputation, which is similar as above but incorporates variability of the predictions.
- and more...

In general however, these fall into two broad categories: **Univariate** and **Multivariate Imputation Methods**. This simply indicates whether the method takes into account all the data features to determine the missing data (multivariate) or only the column the missing data belongs to (univariate).

There is a key increase in **computational effort when using multivariate methods**, however these often perform better as they take into consideration the relationship between all the data when making a choice. This implies that often **correlation structures** across features are kept intact, compared to univariate methods which completely disregard them.

For the sake of this analysis and simulation study, we will explore both in some level of detail.

Due by multivariate nature of the sampling (i.e. dropping data from more than one feature), there are three main strategies to consider:

- 1. Monotone data imputation. Imputations are created by a sequence of univariate methods;
- 2. Joint modeling. Imputations are drawn from a multivariate model fitted to the data;
- 3. Fully conditional specification, also known as multivariate imputation by chained equations. A multivariate model is implicitly specified by a set of conditional univariate models. Imputations are created by drawing from iterated conditional models.

**Note**: 1. is better suited for monotone missing data patterns and 2. and 3. are suitable for general missing data patterns.

### Univariate Methods to consider:

Univariate imputation methods chosen fall into some of the standard ideas which were mentioned before.

- Median SimpleImputer, this is a standard sklearn imputer which takes the column feature and uses the median value as the imputed value all missing values. *Note*: this only copes with numerical features.
- MostFrequent SimpleImputer, this is another standard sklearn imputer which takes the column feature and uses the most frequent category as the imputed value for all missing values. *Note*: this only copes with categorical features.

### Function to Run Univariate Imputation Methods

```
def univariate_imputation_method(nans_df):
    """
    Function which runs through our chosen three methods: Median Imputation, and Most Frequent Imputati
    Code from https://dzone.com/articles/imputing-missing-data-using-sklearn-simpleimputer helped remov
    when imputing isolated columns alone. Was getting error 'expected 2D array and was provided with 1D
    :param nans_df: pandas DataFrame which has missing data - subset on the columns to impute only
    :return: the imputed dataset
```

```
# Dataset that will be imputed and returned to main
imputed_data = nans_df.copy()

# Looping through each column that has to be imputed.
# Doing this tactic to capture name of column

for column in imputed_data:
    # Checking if the column is categorical data type and using most frequent tactic
    if column[0:3] == 'cat':
        imputer_tactic = SimpleImputer(strategy = "most_frequent")

# Checking if the column is numerical data type and using mean tactic
else:
    imputer_tactic = SimpleImputer(strategy = "mean")

# Imputing the specific column with the appropriate specific tactic
    imputed_data[column] = imputer_tactic.fit_transform(imputed_data[column].values.reshape(-1,1))[
return imputed_data
```

### Multivariate Methods to consider:

For this analysis we focused on two key approaches.

- MICE, Multivariate Imputation by Chained Equations. More info can be found in the docs
- MissForest Imputer, this is a Nonparametric Missing Value Imputation using Random Forest. More info can be found in the docs

Note: both cope with mixed data well.

# MICE Fitting and Function

One of the key assumptions of MICE is that the **response mechanism is MAR**. Hence, that there is value in imputing values using a combination of all other features of the data. Which features are used as the *predictive variables* for the *imputed values* is shown in the output of PredictiveMatrix.

For datasets containing hundreds or thousands of variables, using all predictors may not be feasible (because of multicollinearity and computational problems) to include all these variables. It is also not necessary. [1]

### The recommended steps for large datasets, in short, are:

- 1. Include all features that appear in the model that will be applied to the data after imputation, including the outcome.
- 2. Include any feature which is known to be likely to be a driver for the missing data.
- 3. Include any feature which seems to explain a lot of the variance of the data.

Note: The mice package contains several tools that aid in automatic predictor selection. The quickpred() function is a quick way to define the predictor matrix using the strategy outlined above. [3] ... The mice() function detects multicollinearity, and solves the problem by removing one or more predictors for the model. Each removal is noted in the loggedEvents element of the mids object.

Lastly, there is a wide range of choices in terms of the methods to deploy for this multivarite fitting. More information can be found at [1] - Section 6.3 Model form and predictors.

#### The default methods used are:

- for unordered factor variables with more than two levels : polyreg Bayesian Polytomous Regression
- for continuous variables : ppm Predictive Mean Matching

### Notes;

- Interestingly, when running the MICE imputer on the categorical data only, this error comes up:
  - # Error in edit.setup(data, setup, ...) : mice detected constant and/or collinear variables. No predictors were left after their removal

Which indicates to me that all categorical variables may be drawn from the exact same sampling and that the labelling has no true meaning. Hence, for some meaningful predictions I kept a continuous feature cont0 as the main predictor for both categorical features to impute.

### MissForest Fitting and Function

The machine learning imputation method 'MissForest' follows an iterative lifecycle and is **based on the** 'Random Forest' predictive model. The specific algorithm does not fill missing cells with constants but with predictions. It predicts the missing values using a 'Random Forest' classification or regression model and the final predicted cell value is the average or the majority of predictions obtained.

- Average used when we filling numerical data types.
- Majority is used when we filling categorical data types.

The steps and process of a MissForest lifecycle:

- 1. Fill missing values using the mode ('most frequent') or mean of the column.
- 2. Train a Random Forest model using a complete data set.
- 3. Predict the missing values using the model trained above.
- 4. Re-fit Random Forest with dataset filled with the previous predicts values rather than mean/mode

- 5. Predict missing values again.
- 6. The above process is done in an iterative fashion and stops according to the stopping technique used

The MissForest imputation technique has outperformed other methods multiple times in different researches. Using the Random Forest models allows to take the advantage of out of **bag sample evaluating technique**, which does not need a test set. These techniques help and allow the algorithm to **perform very well when the data follows a complex non linear relation** within it. However when it follows a linear relation, other models are preferred which are created to capture such relations. Another benefit of using the MissForest it that it considers the randomness of data and is computationally efficient, which allows to deal with data of high dimensions. In addition to, the imputation method can deal with categorical and numerical data types. When predicting a categorical data type, classification tree models are used in the forest. On the other hand when predicting a numerical data type, regression tree models are used in the forest.

The main benefit of using a MissForest model is that the algorithm is non-parametric. It does not assume any assumptions on the structural aspects of the data. This has no constraint of when a MissForest is appropriate, as it can deal with structured data and unstructured data. More on the MissForest algorithm can be found on reference [6]

We can apply the MissForest imputation method using R package 'missForest' provided for us.

### Notes;

- By construction of algorithm cannot run on categorical variables with more than 53 factors: drop cat10 and cat8 and cat5. There are different techniques to deal with this phenomeno. We can apply a One v One Strategy but this would make the model more computationl expensive. Since we have many multiple variables, we choose to not include them in the Random Forest and use the rest.
- 3. Evaluating Performance of the Imputation Methods [Evaluation Step]:

Types of Adequate Performance Metrics:

### For Continuous Features:

• Normalized Root Mean Squared Error (NRMSE):

$$\sqrt{mean((X_{true} - X_{imp})^2)/var(X_{true})}$$

where  $X_{true}$  the complete data matrix and  $X_{imp}$  the imputed data matrix For good performance we expect values to be close to zero.

• Difference between true value and imputed value

$$D = x_{true} - x_{imp}$$

where  $x_{true}$  the true value from the data matrix and  $x_{imp}$  the imputed value from the data matrix For good performance we expect values to be concentrated about zero.

• Percentage difference between true value and imputed value

$$PD = 100 * |(x_{true} - x_{imp})/x_{true}|$$

where  $x_{true}$  the true value from the data matrix and  $x_{imp}$  the imputed value from the data matrix For good performance we expect values to be concentrated close to zero.

### For Discrete Features:

• Confusion Matrix: count the number of *True Positives*, False Negatives, False Positives and True Negatives and store them in a matrix to assess performance of predictor.

Good performance is indicated by higher counts in the diagonal of the matrix.

• **Accuracy**: sum the counts in the *diagonal* of the confusion matrix (True Positive and True Negative) and divide by the sum of all entries in the confusion matrix.

Good performance leads to a value close to 1 and bad performance to a value around 0.

# Functions to Evaluate Performance Metrics (Python)

```
def performance_results_py(true_df, imp_df, nans_df, columns):
    Function to extract true values and imputed values and compute performance metrics.
    :param true_df: pandas DataFrame containing true values
    :param imp_df: pandas DataFrame containing imputed values
    :param nans_df: pandas DataFrame containing nans
    :param columns: columns containing imputed values
    :return: list of results such that indexing returns results for each column (in columns) respective
             for each column you then have the output from performance_metrics() which depends on the d
    11 11 11
   results = []
    for column in columns:
        # a. Extract boolean vector (True/False) to identify Imputed True Data Vectors
        # Expect different values in cell, because comparing true with the NaN removed.
        boolean_values = (true_df != nans_df)[column].to_numpy().tolist()
        # c. Extract Imputed Data Vectors
        # Using Boolean Pointer list from above, extract true values
```

```
true_values = true_df[column][boolean_values]
        # Using Boolean Pointer list from above, extract imputed values
        imputed_values = imp_df[column][boolean_values].to_numpy().tolist()
        # d. Evaluate performance metrics
        # Call function the two columns and obtain metrics.
        results.append(performance_metrics(imputed_values, true_values))
   return(results)
def performance_metrics(imputed_values, true_values):
    Function to calculate all performance metrics of interest for each list of imputed vs true values.
    :param imputed_values: list of imputed values
    :param true_values: list of true values
    : return: \ if \ categorical \ - \ measures \ of \ accuracy \ and \ the \ confusion \ matrix
             if numerical - measures of raw difference and percentage difference between
                            imputed value and true value
   Note: each list for now is specific to each column, as it supports univariate imputations.
   Note: depending the on the methods used the columns are independent of each other,
          e.g. the median imputation does not use any other column at all -
          implying that we can assess performance independently of one another.
    # If Categorical Feature
    if isinstance(imputed_values[0], str):
        # Confusion Matrix
        conf_matrix = metrics.confusion_matrix(true_values, imputed_values)
        # Accuracy
        accuracy = metrics.accuracy_score(true_values, imputed_values)
        return(conf_matrix, accuracy)
    # If Numerical Feature
    elif isinstance(imputed_values[0], float):
        # 1. The difference between the imputed value and truth
        D = np.array(imputed_values) - np.array(true_values)
        sample mean = np.mean(D)
        sample_sd = np.sqrt(np.var(D))
        # 2. The percentage difference between the imputed value and truth
        PD = 100 * abs((np.array(imputed_values) - np.array(true_values))/np.array(true_values))
        # 3. Normalised Root Mean Squared Error
        RMSE = sqrt(mean_squared_error(true_values, imputed_values)/np.var(true_values))
        return(D, PD, sample_mean, sample_sd, RMSE)
```

```
else:
return(None)
```

### Function to Evaluate Performance Metrics (R)

```
## FUNCTION
# Function to extract true values and imputed values and compute performance metrics for mice.
## INPUTS
# true_df: data.frame containing true values
# imp_df: output directly from methods `mice()`
# columns: vector input containing strings representing column names which contain imputed values
## OUTPUT
# list containing:
  for categorical - measures of accuracy and the confusion matrix
    for numerical - measures of raw difference and percentage difference between
#
                    imputed value and true value
## NOTE
# number_iterations : Number of iterations mice has done.
# Each iteration computes a different imputation, we will find results from all imputations and get the
performance_results_mice <- function(true_df, imp_df, columns) {</pre>
  # Set up
  number_iterations <- ncol(imp_df$imp[[columns[1]]])</pre>
  all_results <- list()</pre>
  # For each column we imputed values we will found a metric.
  for (column in columns) {
    # 1. Extract True and Imputed
    # a) extract row indexes containing imputed values
    imp_rows <- row.names(imp_df$imp[[column]])</pre>
    # b) use row indexes to find true values
    truth <- data.frame(true_df[[column]])[imp_rows, ]</pre>
    # 2. Check if it is continuous or categorical values to apply appropriate computations.
    # a) If categorical column
    if (startsWith(column, "cat")) {
      accuracies <- c()
      for (i in 1:number_iterations) {
        # extract imputed values for each iteration of mice.
        pred <- imp_df$imp[[column]][[i]]</pre>
        # compute metrics
        analysis table <- table(pred, truth)
        res <- confusionMatrix(analysis_table)</pre>
```

```
accuracies <- c(accuracies, res$overall[1])</pre>
      }
    all_results$conf_matrix[[column]] <- res$table
    all_results$accuracy[[column]] <- mean(accuracies)</pre>
    # b) If numerical column
    } else {
    all_results$D[[column]] <- truth - imp_df$imp[[column]][,]</pre>
    all_results\PD[[column]] <- 100 * abs((truth - imp_df\simp[[column]][,]) / truth)
    all_results$mean[[column]] <- colMeans(all_results$D[[column]])</pre>
    all_results$sd[[column]] <- colVars(as.matrix(all_results$D[[column]]))
    nrmse <- c()
    for (i in 1:number_iterations) {
      # extract imputed values for each iteration of mice.
     pred <- imp_df$imp[[column]][[i]]</pre>
      # compute metrics
      nrmse <- c(nrmse, sqrt((RMSE(pred, truth))^2/var(truth)))</pre>
    all_results$nrmse[[column]] <- mean(nrmse)</pre>
    }
  }
 return(all_results)
}
## FUNCTION
# Function to extract true values and imputed values and compute performance metrics for missForest.
# true_df: data.frame containing true values
# imp_df: output directly from methods `missForest().ximp`
# nans_df: data.frame which we randomly replaced true values with NA's
# columns: vector input containing strings representing column names which contain imputed values
## OUTPUT
# list containing:
     for categorical - measures of accuracy and the confusion matrix
     for numerical - measures of raw difference and percentage difference between
performance_results_missForest <- function(true_df, imp_df, nans_df, columns) {</pre>
  all_results <- list()
  # For each column we imputed values we will find a metric.
  for (column in columns) {
    # 1. Extract True and Imputed
```

```
# a) extract true values to compare with imputed.
    rows_imputed <- nans_df %>%
      mutate(row_names = row.names(nans_df)) # Creating a row_id in data frame.
    # b) Keeps rows that have NA values in column only.
    rows_imputed <- rows_imputed[is.na(rows_imputed[[column]]), ]</pre>
    # c) Extract the cells for true and imputed values to compare between.
    truth <- data.frame(true_df[[column]])[rows_imputed[["row_names"]], ]</pre>
    pred <- data.frame(imp_df[[column]])[rows_imputed[["row_names"]], ]</pre>
    # 2. Check if it is continuous or categorical values to apply appropriate metrics.
    # a) If categorical column
    if (startsWith(column, "cat")) {
      accuracies <- c()
      analysis_table <- table(pred, truth)</pre>
      res <- confusionMatrix(analysis_table)</pre>
      accuracies <- c(accuracies, res$overall[1])
      all_results$conf_matrix[[column]] <- res$table</pre>
      all_results$accuracy[[column]] <- mean(accuracies)</pre>
    # b) If numerical column
    } else {
    all_results$D[[column]] <- truth - pred</pre>
    all_results$PD[[column]] <- 100 * abs((truth - pred) / truth)
    all_results$mean[[column]] <- mean(all_results$D[[column]])</pre>
    all_results$sd[[column]] <- sd(as.matrix(all_results$D[[column]]))</pre>
    all_results$nrmse[[column]] <- sqrt((RMSE(pred, truth))^2/var(truth))</pre>
    }
  }
 return(all_results)
}
```

### Function to Store Performance Metrics (R)

```
## FUNCTION
# Function to extract all results for statistical testing in one dataframe
## INPUTS
# simple_imputer_result: results from fitting
# mice_result: results from fitting
# missforest_result: results from fitting
```

```
# columns: vector input containing strings representing column names which contain imputed values
## OUTPUT
# dataframe
store_performance_results <- function(simple_imputer_result, mice_result, missforest_result, columns) {</pre>
  final_results <- c()</pre>
 i = 0
  # Loop through each column to store respective results of interest
  for (column in columns) {
    i = 1 + i
    if (startsWith(column, "cat")) {
      # - - - - For Categorical Columns
      accuracy <- c(simple_imputer_result[[i]] accuracy, mice_result accuracy[[i]], missforest_result accuracy
      accuracy_results <- data.frame("performance" = accuracy,</pre>
                                      "performance_type" = rep("accuracy", each = 3),
                                      "columns" = rep(column, each = 3),
                                      "model" = c("simple", "mice", "missforest"),
                                      "type" = rep("cat", each = 3))
      final_results <- rbind(final_results, accuracy_results)</pre>
    } else {
      # - - - For Numerical Columns
      if (startsWith(columns[1], "cat") & startsWith(columns[2], "cont")) {
        i = 1 # little fix for when we have one cat and one cont columns to impute as indexing is diffe
        nrmse <- c(simple_imputer_result[[i + 1]]$nrmse, mice_result$nrmse[[i]], missforest_result$nrms</pre>
      } else {nrmse <- c(simple_imputer_result[[i]] nrmse, mice_result nrmse[[i]], missforest_result nrmse
      nrmse_results <- data.frame("performance" = nrmse,</pre>
                                   "performance_type" = rep("nrmse", each = 3),
                                   "columns" = rep(column, each = 3),
                                   "model" = c("simple", "mice", "forest"),
                                   "type" = rep("cont", each = 3))
      final_results <- rbind(final_results, nrmse_results)</pre>
    }
 }
 return(final_results)
}
# Function to extract all mean difference results for statistical testing in one dataframe
```

```
# simple_imputer_result: results from fitting
# mice_result: results from fitting
# missforest_result: results from fitting
# columns: vector input containing strings representing column names which contain imputed values
## OUTPUT
# dataframe
store_mean_diff_results <- function(simple_imputer_result, mice_result, missforest_result, columns) {</pre>
  # 1. IF scenario {both}
  if (startsWith(columns[1], "cat") & startsWith(columns[2], "cont")) {
   mean_diff <- c(simple_imputer_result[[2]]$mean, mice_result$mean[[1]], missforest_result$mean[[1]])</pre>
   mean_diff_results <- data.frame("mean_diff" = mean_diff,</pre>
                                     "model" = c("simple", rep("mice", 3), "forest"))
  # 2. IF scenario {cont_only}
  } else if (startsWith(columns[1], "cont") & startsWith(columns[2], "cont")) {
   mean_diff_col_1 <- c(simple_imputer_result[[1]] mean, mice_result mean[[1]], missforest_result mean
   mean_diff_col_2 <- c(simple_imputer_result[[2]] mean, mice_result mean[[2]], missforest_result mean
   mean_diff_results <- data.frame("mean_diff" = c(mean_diff_col_1, mean_diff_col_2),</pre>
                                     "model" = rep(c("simple", rep("mice", 3), "forest"), 2))
 }
 return(mean_diff_results)
}
```

Note: need to include data wrangling functions to transfer data from python to R

```
## FUNCTION
# Function wrangles the data in appropriate structure for machine learning imputations.
## INPUTS
# nans_df: data.frame containing random NA's values
## OUTPUT
# data frames that are in appropriate state.
wrangle_data_for_na <- function(nans_df) {

# Conversion issue between NaN and NA, hence we can explicitly use
nans_df[nans_df == 'NaN'] = NA

# Converting the data into R data frames
nans_df <- as.data.frame(nans_df)</pre>
```

```
# Unlist all of the columns
columns <- colnames(nans_df)
nans_df <- nans_df %>% mutate(across(all_of(columns), unlist))

# All character types must be changed to factors
nans_df <- nans_df %>% mutate_if(is.character, as.factor)

return(nans_df)
}
```

# 4. Bringing it all together

Need to make a loop over all possible scenario combinations.

First, load in data.

```
original_df <- py$read_true_values('data/train.csv')
```

## Second, set up scenarios.

In order to set up the study, we made a few simplyfing assumptions.

• Choose two columns to be our reference "predictor columns" for all combinations of the simulation. This is due to how computationally expensive the fitting of the study is and by extension requires simplifying assumptions. By choosing a set of consistent predictor columns across the study, we control for any variability that could be introduced if these predictor columns changed at every iteration.

Note, we chose a categorical and a numerical feature for the predictions, this is due to a multicollinearity issue in the prediction of multivariate imputation methods if we had only chosen one type.

```
columns_predictions_if_cat <- c('cont0')
columns_predictions_if_cont <- c('cat0')
columns_predictions_if_both <- c('cat0', 'cont0')</pre>
```

• Block out four categorical variables from the study: namely cat5, cat7, cat8 and cat10. This is because the number of levels of these categorical variables is over 50 and one of the imputation methods cannot handle with this dimensionality.

Note, here we are also removing the predictor columns from the possible random picks of columns to be imputed.

```
cat_vec <- 1:18
cat_vec <- cat_vec[!cat_vec %in% c(5, 7, 8, 10)]
cont_vec <- 1:10</pre>
```

• At this stage we did not focus on identifying predictive power of features and fine-tuning the imputation modelling for a specific set of features. Due to the randomness element of the study, we simply **choose the columns to be imputed at random**. The only requirement of the study is that you have the same number of occurrences of columns to be imputed which are {only continuous, only categorical, both}.

This is controlled by looping through each combination and picking out a number of simulation as defined by num\_simulations.

```
combinations <- c("cont_only", "cat_only", "both")</pre>
num_simulations <- 3</pre>
i = 0
columns_combinations <- list()</pre>
for (combination in combinations) {
  for (iter in 1:num_simulations) {
    i = 1 + i
    if (combination == "cont only") {
      cont_num <- sample(cont_vec, 2)</pre>
      columns_combinations[[i]] <- c(paste0('cont', cont_num[1]), paste0('cont', cont_num[2]))</pre>
    } else if (combination == "cat only") {
      cat_num <- sample(cat_vec, 2)</pre>
      columns_combinations[[i]] <- c(paste0('cat', cat_num[1]), paste0('cat', cat_num[2]))</pre>
    } else if (combination == "both") {
      cat_num <- sample(cat_vec, 1)</pre>
      cont_num <- sample(cont_vec, 1)</pre>
      columns_combinations[[i]] <- c(paste0('cat', cat_num), paste0('cont', cont_num))</pre>
    }
  }
}
```

• Choose a relevant set of percentages of data that should be dropped: this is part of the set up of the simulation study and here we are exploring three cases  $\{0.1, 0.2, 0.3\}$ .

```
percentage_drop <- list("large" = 0.3, "medium" = 0.2, "small" = 0.1)</pre>
```

• Choose a subset of the data: this is due to the computational expense of the runs complessively. In order to be able to run more scenarios in a local machine, we chose to take a subset of the data. This tradeoff was necessary due to the lack of access to a desktop computer and being unable to run this code overnight.

```
original_df <- original_df[0:100000, ]
```

Then, run the full simulation.

### Third, full run.

```
final results <- mean diff results <- c()
# Simulation Loops
i = 0
system.time(for (columns in columns combinations) {
 for (percentage in percentage_drop) {
    i = 1 + i
    cat("Starting run", i)
    start_time <- Sys.time()</pre>
    cat(".")
    ### 1. Drop Values Randomly
    nans_df <- py$remove_random_values(original_df, columns, percentage)</pre>
    cat(".")
    ### 2. Simple Imputer
    simple_imputer <- py$univariate_imputation_method(nans_df[columns])</pre>
    simple_imputer_result <- py$performance_results_py(original_df, simple_imputer, nans_df, columns)
    cat(".", "\n")
    mid_time_1 <- Sys.time()
    cat("Simple Imputer Completed - Time Elapsed :", (mid_time_1 - start_time), '\n')
    ### 3. Data Wrangling (some Python to R conversions)
    # a) Appropriately change features of nans_df to R environment
    nans_df <- wrangle_data_for_na(nans_df)</pre>
    # b) Deal with Single Impute Results
    if (startsWith(columns[1], "cat") & startsWith(columns[2], "cat")) {
      names(simple_imputer_result[[1]]) <- names(simple_imputer_result[[2]]) <- c("conf_matrix", "accur</pre>
    } else if (startsWith(columns[1], "cont") & startsWith(columns[2], "cont")){
      names(simple_imputer_result[[1]]) <- names(simple_imputer_result[[2]]) <- c("D", "PD", "mean", "s</pre>
      names(simple_imputer_result[[1]]) <- c("conf_matrix", "accuracy")</pre>
      names(simple_imputer_result[[2]]) <- c("D", "PD", "mean", "sd", "nrmse")</pre>
    }
    names(simple_imputer_result) <- c(columns[1], columns[2])</pre>
    ### 4. Multivariate Imputers
    # a) Set up prediction columns
    # IF {cat_only} scenario:
    if (startsWith(columns[1], "cat") & startsWith(columns[2], "cat")) {
      pred_columns <- unique(c(columns, columns_predictions_if_cat))</pre>
    # IF {cont_only} scenario:
    } else if (startsWith(columns[1], "cont") & startsWith(columns[2], "cont")) {
      pred_columns <- unique(c(columns, columns_predictions_if_cont))</pre>
```

```
# IF {both} scenario:
    } else { pred_columns <- unique(c(columns, columns_predictions_if_both)) }</pre>
    mice_imputer <- impute_using_mice(nans_df, pred_columns)</pre>
    mice_result <- performance_results_mice(original_df, mice_imputer, columns)</pre>
    mid time 2 <- Sys.time()
    cat("MICE Imputer Completed - Time Elapsed :", (mid_time_2 - mid_time_1), '\n')
    # c) MissForest Imputer
    missforest_imputer <- invisible(impute_using_missForest(nans_df, pred_columns))</pre>
    missforest_result <- performance_results_missForest(original_df, missforest_imputer, nans_df, column
    mid_time_3 <- Sys.time()</pre>
    cat("MissForest Imputer Completed - Time Elapsed :", (mid_time_3 - mid_time_2), '\n')
    ### 5. Store Results
    iter_results <- store_performance_results(simple_imputer_result, mice_result, missforest_result, co
    iter_results <- cbind(iter_results, "% drop" = c(rep(percentage, each = nrow(iter_results))))</pre>
    final_results <- rbind(final_results, iter_results)</pre>
    # IF {cont_only, both} scenarios:
    if (startsWith(columns[1], "cont") & startsWith(columns[2], "cont") | startsWith(columns[1], "cat")
      iter_results <- store_mean_diff_results(simple_imputer_result, mice_result, missforest_result, co</pre>
      iter_results <- cbind(iter_results, "% drop" = c(rep(percentage, each = nrow(iter_results))))
      mean_diff_results <- rbind(mean_diff_results, iter_results)</pre>
    }
    end_time <- Sys.time()</pre>
    cat("Time taken to compute full run", i, "for column combination", columns, "is:", (end_time - sta
 }
})
## Starting run 1...
## Simple Imputer Completed - Time Elapsed : 3.973474
## MICE Imputer Completed - Time Elapsed : 5.787124
    missForest iteration 1 in progress...done!
##
    missForest iteration 2 in progress...done!
    missForest iteration 3 in progress...done!
##
    missForest iteration 4 in progress...done!
## MissForest Imputer Completed - Time Elapsed : 36.32053
## Time taken to compute full run 1 for column combination cont9 cont5 is : 46.10008
## Starting run 2...
\#\# Simple Imputer Completed - Time Elapsed : 2.912039
## MICE Imputer Completed - Time Elapsed : 5.894021
    missForest iteration 1 in progress...done!
##
    missForest iteration 2 in progress...done!
##
##
    missForest iteration 3 in progress...done!
    missForest iteration 4 in progress...done!
    missForest iteration 5 in progress...done!
##
```

```
## MissForest Imputer Completed - Time Elapsed : 44.48171
## Time taken to compute full run 2 for column combination cont9 cont5 is : 53.29961
## Starting run 3...
## Simple Imputer Completed - Time Elapsed : 1.708411
## MICE Imputer Completed - Time Elapsed : 5.362691
    missForest iteration 1 in progress...done!
    missForest iteration 2 in progress...done!
    missForest iteration 3 in progress...done!
##
    missForest iteration 4 in progress...done!
    missForest iteration 5 in progress...done!
## MissForest Imputer Completed - Time Elapsed : 49.16945
## Time taken to compute full run 3 for column combination cont9 cont5 is : 56.24305
## Starting run 4...
## Simple Imputer Completed - Time Elapsed : 3.604003
## MICE Imputer Completed - Time Elapsed : 5.720186
     missForest iteration 1 in progress...done!
##
    missForest iteration 2 in progress...done!
##
    missForest iteration 3 in progress...done!
    missForest iteration 4 in progress...done!
## MissForest Imputer Completed - Time Elapsed : 35.01014
## Time taken to compute full run 4 for column combination cont3 cont7 is : 44.33674
## Starting run 5...
## Simple Imputer Completed - Time Elapsed : 2.702613
## MICE Imputer Completed - Time Elapsed : 4.899619
    missForest iteration 1 in progress...done!
##
    missForest iteration 2 in progress...done!
##
    missForest iteration 3 in progress...done!
    missForest iteration 4 in progress...done!
    missForest iteration 5 in progress...done!
## MissForest Imputer Completed - Time Elapsed : 47.44448
## Time taken to compute full run 5 for column combination cont3 cont7 is : 55.04908
## Starting run 6...
## Simple Imputer Completed - Time Elapsed : 1.723522
## MICE Imputer Completed - Time Elapsed : 5.361894
     missForest iteration 1 in progress...done!
##
    missForest iteration 2 in progress...done!
##
    missForest iteration 3 in progress...done!
##
    missForest iteration 4 in progress...done!
    missForest iteration 5 in progress...done!
## MissForest Imputer Completed - Time Elapsed : 49.0279
## Time taken to compute full run 6 for column combination cont3 cont7 is : 56.1159
## Starting run 7...
## Simple Imputer Completed - Time Elapsed : 3.61621
## MICE Imputer Completed - Time Elapsed : 5.54363
    {\tt missForest\ iteration\ 1\ in\ progress...done!}
##
    missForest iteration 2 in progress...done!
##
    missForest iteration 3 in progress...done!
##
    missForest iteration 4 in progress...done!
    missForest iteration 5 in progress...done!
## MissForest Imputer Completed - Time Elapsed : 43.28939
## Time taken to compute full run 7 for column combination cont2 cont10 is : 52.45152
## Starting run 8...
## Simple Imputer Completed - Time Elapsed : 2.85586
## MICE Imputer Completed - Time Elapsed : 5.105639
```

```
##
     missForest iteration 1 in progress...done!
##
    missForest iteration 2 in progress...done!
##
    missForest iteration 3 in progress...done!
##
    missForest iteration 4 in progress...done!
     missForest iteration 5 in progress...done!
## MissForest Imputer Completed - Time Elapsed : 46.48931
## Time taken to compute full run 8 for column combination cont2 cont10 is : 54.45318
## Starting run 9...
## Simple Imputer Completed - Time Elapsed : 1.74942
## MICE Imputer Completed - Time Elapsed : 5.042664
    missForest iteration 1 in progress...done!
##
    missForest iteration 2 in progress...done!
    missForest iteration 3 in progress...done!
    missForest iteration 4 in progress...done!
## MissForest Imputer Completed - Time Elapsed : 41.52755
## Time taken to compute full run 9 for column combination cont2 cont10 is : 48.32206
## Starting run 10...
## Simple Imputer Completed - Time Elapsed : 3.672622
## MICE Imputer Completed - Time Elapsed : 1.471289
    missForest iteration 1 in progress...done!
##
    missForest iteration 2 in progress...done!
    missForest iteration 3 in progress...done!
##
    missForest iteration 4 in progress...done!
## MissForest Imputer Completed - Time Elapsed : 33.27696
## Time taken to compute full run 10 for column combination cat3 cat14 is : 2.087146
## Starting run 11...
## Simple Imputer Completed - Time Elapsed : 2.77781
## MICE Imputer Completed - Time Elapsed : 1.513019
    missForest iteration 1 in progress...done!
    missForest iteration 2 in progress...done!
    missForest iteration 3 in progress...done!
## MissForest Imputer Completed - Time Elapsed : 26.38234
## Time taken to compute full run 11 for column combination cat3 cat14 is : 1.999052
## Starting run 12...
## Simple Imputer Completed - Time Elapsed : 1.813323
## MICE Imputer Completed - Time Elapsed : 1.526689
##
    missForest iteration 1 in progress...done!
##
    missForest iteration 2 in progress...done!
    missForest iteration 3 in progress...done!
## MissForest Imputer Completed - Time Elapsed : 27.83668
## Time taken to compute full run 12 for column combination cat3 cat14 is : 2.020887
## Starting run 13...
## Simple Imputer Completed - Time Elapsed : 3.581696
## MICE Imputer Completed - Time Elapsed : 2.096893
    missForest iteration 1 in progress...done!
##
    missForest iteration 2 in progress...done!
##
    missForest iteration 3 in progress...done!
##
    missForest iteration 4 in progress...done!
    missForest iteration 5 in progress...done!
\#\# MissForest Imputer Completed - Time Elapsed : 43.56847
## Time taken to compute full run 13 for column combination cat13 cat6 is : 2.882763
## Starting run 14...
## Simple Imputer Completed - Time Elapsed : 2.875359
## MICE Imputer Completed - Time Elapsed : 2.162058
```

```
missForest iteration 1 in progress...done!
##
    missForest iteration 2 in progress...done!
    missForest iteration 3 in progress...done!
##
    missForest iteration 4 in progress...done!
\#\# MissForest Imputer Completed - Time Elapsed : 36.73878
## Time taken to compute full run 14 for column combination cat13 cat6 is : 2.822326
## Starting run 15...
## Simple Imputer Completed - Time Elapsed : 1.803683
## MICE Imputer Completed - Time Elapsed : 2.255289
    missForest iteration 1 in progress...done!
    missForest iteration 2 in progress...done!
    missForest iteration 3 in progress...done!
## MissForest Imputer Completed - Time Elapsed : 28.89967
## Time taken to compute full run 15 for column combination cat13 cat6 is : 2.767044
## Starting run 16...
## Simple Imputer Completed - Time Elapsed : 3.668935
## MICE Imputer Completed - Time Elapsed : 1.840145
    missForest iteration 1 in progress...done!
    missForest iteration 2 in progress...done!
    missForest iteration 3 in progress...done!
## MissForest Imputer Completed - Time Elapsed : 26.10083
## Time taken to compute full run 16 for column combination cat1 cat12 is : 2.336338
## Starting run 17...
## Simple Imputer Completed - Time Elapsed : 2.753283
## MICE Imputer Completed - Time Elapsed : 1.810125
    missForest iteration 1 in progress...done!
##
    missForest iteration 2 in progress...done!
    missForest iteration 3 in progress...done!
    missForest iteration 4 in progress...done!
## MissForest Imputer Completed - Time Elapsed : 35.10009
## Time taken to compute full run 17 for column combination cat1 cat12 is : 2.441047
## Starting run 18...
## Simple Imputer Completed - Time Elapsed : 1.802437
## MICE Imputer Completed - Time Elapsed : 1.991285
     missForest iteration 1 in progress...done!
##
    missForest iteration 2 in progress...done!
##
    missForest iteration 3 in progress...done!
##
    missForest iteration 4 in progress...done!
    missForest iteration 5 in progress...done!
## MissForest Imputer Completed - Time Elapsed : 48.26513
## Time taken to compute full run 18 for column combination cat1 cat12 is : 2.825777
## Starting run 19...
## Simple Imputer Completed - Time Elapsed : 18.08736
## MICE Imputer Completed - Time Elapsed : 1.770916
    missForest iteration 1 in progress...done!
##
    missForest iteration 2 in progress...done!
    missForest iteration 3 in progress...done!
    missForest iteration 4 in progress...done!
## MissForest Imputer Completed - Time Elapsed : 19.0297
## Time taken to compute full run 19 for column combination cat3 cont3 is : 21.10211
## Starting run 20...
## Simple Imputer Completed - Time Elapsed : 12.61352
## MICE Imputer Completed - Time Elapsed : 1.804859
    missForest iteration 1 in progress...done!
```

```
missForest iteration 2 in progress...done!
##
    missForest iteration 3 in progress...done!
##
    missForest iteration 4 in progress...done!
    missForest iteration 5 in progress...done!
## MissForest Imputer Completed - Time Elapsed : 27.26343
## Time taken to compute full run 20 for column combination cat3 cont3 is : 29.27856
## Starting run 21...
## Simple Imputer Completed - Time Elapsed: 8.010869
## MICE Imputer Completed - Time Elapsed : 1.906712
    missForest iteration 1 in progress...done!
    missForest iteration 2 in progress...done!
##
    missForest iteration 3 in progress...done!
    missForest iteration 4 in progress...done!
    missForest iteration 5 in progress...done!
## MissForest Imputer Completed - Time Elapsed : 31.00051
## Time taken to compute full run 21 for column combination cat3 cont3 is : 33.04078
## Starting run 22...
## Simple Imputer Completed - Time Elapsed : 20.05976
## MICE Imputer Completed - Time Elapsed : 2.26607
    missForest iteration 1 in progress...done!
##
    missForest iteration 2 in progress...done!
##
    missForest iteration 3 in progress...done!
##
    missForest iteration 4 in progress...done!
    missForest iteration 5 in progress...done!
## MissForest Imputer Completed - Time Elapsed : 33.86826
## Time taken to compute full run 22 for column combination cat9 cont5 is : 36.46878
## Starting run 23...
## Simple Imputer Completed - Time Elapsed : 14.17356
## MICE Imputer Completed - Time Elapsed : 2.31198
    missForest iteration 1 in progress...done!
##
    missForest iteration 2 in progress...done!
##
    missForest iteration 3 in progress...done!
##
    missForest iteration 4 in progress...done!
    missForest iteration 5 in progress...done!
## MissForest Imputer Completed - Time Elapsed : 37.7613
## Time taken to compute full run 23 for column combination cat9 cont5 is : 40.30955
## Starting run 24...
## Simple Imputer Completed - Time Elapsed : 7.635127
## MICE Imputer Completed - Time Elapsed : 3.163393
##
    missForest iteration 1 in progress...done!
    missForest iteration 2 in progress...done!
##
    missForest iteration 3 in progress...done!
##
    missForest iteration 4 in progress...done!
    missForest iteration 5 in progress...done!
## MissForest Imputer Completed - Time Elapsed : 43.10193
## Time taken to compute full run 24 for column combination cat9 cont5 is : 46.39262
## Starting run 25...
## Simple Imputer Completed - Time Elapsed : 20.57636
## MICE Imputer Completed - Time Elapsed : 32.11855
    missForest iteration 1 in progress...done!
##
    missForest iteration 2 in progress...done!
##
    missForest iteration 3 in progress...done!
##
    missForest iteration 4 in progress...done!
    missForest iteration 5 in progress...done!
```

```
## MissForest Imputer Completed - Time Elapsed : 16.0871
## Time taken to compute full run 25 for column combination cat15 cont1 is : 16.96542
## Starting run 26...
## Simple Imputer Completed - Time Elapsed : 13.06437
## MICE Imputer Completed - Time Elapsed : 29.4412
    missForest iteration 1 in progress...done!
##
    missForest iteration 2 in progress...done!
    missForest iteration 3 in progress...done!
##
##
    missForest iteration 4 in progress...done!
    missForest iteration 5 in progress...done!
## MissForest Imputer Completed - Time Elapsed : 17.12883
## Time taken to compute full run 26 for column combination cat15 cont1 is : 17.8373
## Starting run 27...
## Simple Imputer Completed - Time Elapsed : 6.93474
## MICE Imputer Completed - Time Elapsed : 32.05934
##
     missForest iteration 1 in progress...done!
    missForest iteration 2 in progress...done!
##
##
    missForest iteration 3 in progress...done!
##
    missForest iteration 4 in progress...done!
    missForest iteration 5 in progress...done!
## MissForest Imputer Completed - Time Elapsed : 17.66983
## Time taken to compute full run 27 for column combination cat15 cont1 is : 18.31977
                system
                         elapsed
        user
## 17075.579
               281.797 17380.249
```

A snapshot of the results:

NRMSE: results sorted from best to worst

```
subset <- final_results[final_results$performance_type == 'nrmse', ]
head(subset[order(subset$performance, decreasing = FALSE), c("performance", "performance_type", "model"</pre>
```

```
performance performance_type model type % drop
##
## 36
        0.7255462
                             nrmse forest cont
## 30
        0.7322555
                             nrmse forest cont
                                                   0.2
## 24
        0.7579567
                             nrmse forest cont
                                                   0.3
## 114
        0.7680411
                                                   0.3
                             nrmse forest cont
## 120
        0.7793170
                             nrmse forest cont
                                                   0.2
## 126
        0.7806584
                             nrmse forest cont
                                                   0.1
## 33
        0.7896211
                             nrmse forest cont
                                                   0.1
## 27
        0.7946300
                             nrmse forest cont
                                                   0.2
## 21
        0.8041166
                             nrmse forest cont
                                                   0.3
## 156
        0.8292154
                             nrmse forest cont
                                                   0.2
## 54
        0.8344071
                                                   0.1
                             nrmse forest cont
## 150
        0.8347604
                             nrmse forest cont
                                                   0.3
## 162
        0.8361582
                             nrmse forest cont
                                                   0.1
## 48
        0.8455527
                             nrmse forest cont
                                                   0.2
                              nrmse forest cont
## 42
        0.8540660
                                                   0.3
```

Accuracy: results sorted from best to worst

```
subset <- final_results[final_results$performance_type == 'accuracy', ]</pre>
head(subset[order(subset$performance, decreasing = TRUE), c("performance", "performance_type", "model",
##
      performance performance_type
                                      model type % drop
## 79
        0.9758788
                         accuracy
                                      simple cat
## 73
        0.9758059
                                      simple cat
                                                    0.3
                         accuracy
                                   simple cat
## 85
        0.9755295
                         accuracy
                                                    0.1
## 74
        0.9530363
                         accuracy
                                     mice cat
                                                    0.3
## 80
        0.9519981
                                      mice cat
                                                    0.2
                         accuracy
## 86
        0.9515388
                                       mice cat
                                                    0.1
                          accuracy
## 100
       0.8581267
                         accuracy simple cat
                                                    0.2
                          accuracy simple cat
## 94
        0.8563692
                                                    0.3
## 106
       0.8552795
                         accuracy
                                      simple cat
                                                    0.1
## 101
        0.7914812
                                        mice cat
                                                    0.2
                          accuracy
## 95
        0.7913056
                         accuracy
                                        mice cat
                                                    0.3
## 107
       0.7902692
                                                    0.1
                         accuracy
                                        mice cat
## 96
        0.7375072
                                                    0.3
                         accuracy missforest cat
## 108
       0.7333333
                         accuracy missforest cat
                                                    0.1
## 102
       0.7318288
                         accuracy missforest cat
                                                    0.2
```

Save results!

```
saveRDS(final_results, file = "imputation_results.rds")
saveRDS(mean_diff_results, file = "imputation_results_2.rds")
```