Homework 2

Computer Vision 2023 Spring

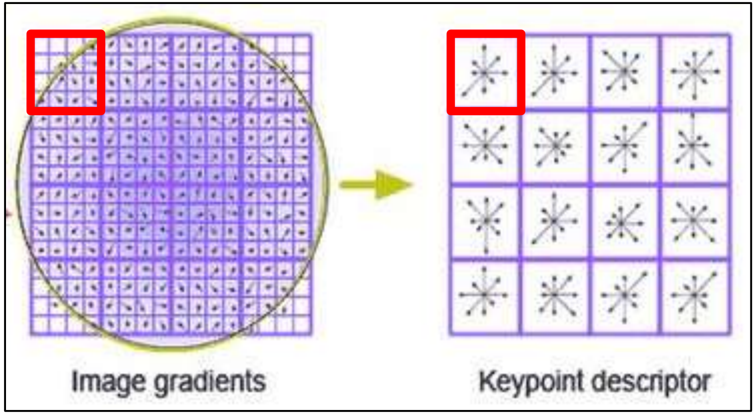
2023.4.20

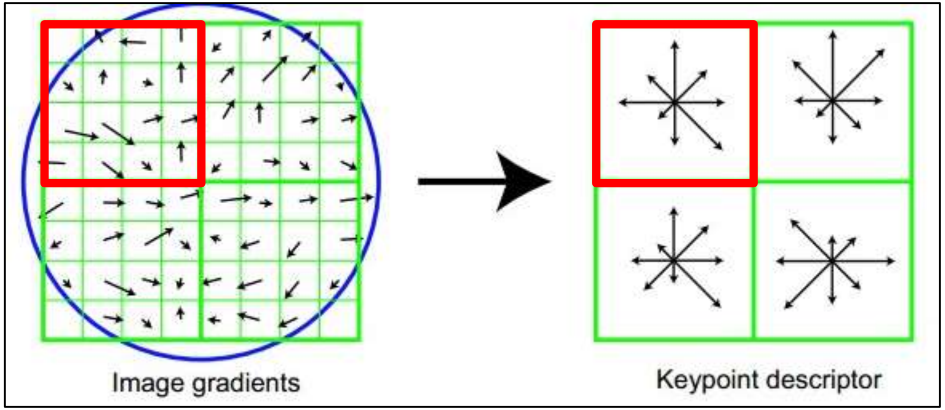
Image stitching

1. Detecting key point(feature) on the images
    - SIFT
2. Finding features correspondences (feature matching)
    - KNN
3. Computing homography matrix.
    - RANSAC
4. Stitching image (warp images into same coordinate system)
    - Homography

Feature Detection

- Finding features correspondences/compute homography matrix.
- SIFT – Scale Invariant Feature Detection
  - detect key points in the image and describe the points as 128-dimensional features (4 * 4 * 8).
- Check Ch.6、7 for more details of SIFT.





1. SIFT in OpenCV

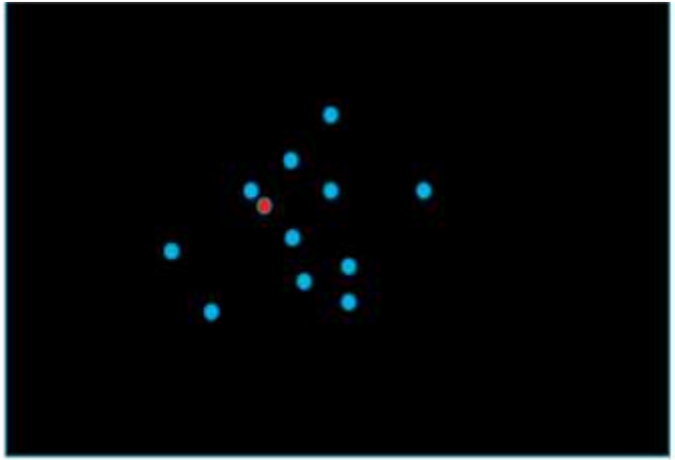
- Using OpenCV to detect SIFT key points of two images
- Input : gray scale image

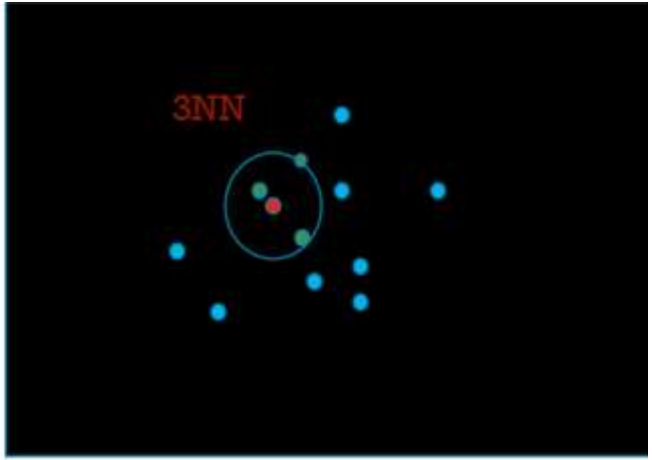
```
SIFT_Detector = cv2.SIFT_create()
kp, des = SIFT_Detector.detectAndCompute(img, None)
```

- output : keypoints (array), Descriptors (array)
- Keypoints store feature points
  - for a single keypoint you can use ".pt" to get the position of this key point on image [Ref]
- Descriptors store the 128-dimensional features
- The function name(detectAndCompute) of SIFT may be different with the version of OpenCV

2. Feature matching - KNN

- K-Nearest Neighbor
  - Finding the K closest neighbors to the target.
  - Brute-force : Comparing with the all 2-norm of SIFT feature (the 2-norm of descriptor)



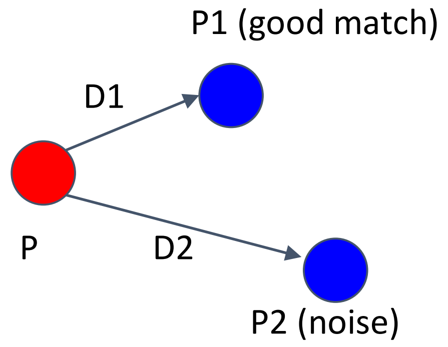


2. Feature matching - Lowe’s Ratio test

- Lowe’s Ratio test for eliminating bad match
  - A good match shold be able to be distinguished from noise

1. For every key point P in image1 using 2NN
   to get 2 matched key points P1 & P2 in image2
2. Computing the 2-norm of P1 & P2 between P named D1 , D2
3. If D1 < threshold * D2 then P1 is a good match

(threshold is a programmer defined ration between 0 to 1 ,
the suggestion of OpenCV tutorial is 0.7~0.8)



3.Homography

- Construct a linear system as: P2=HP1, P2 = (x2,y2,1) , P1 = (x1,y1,1) where P2 and P1 are correspondence points , H is homography matrix.

$$\begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \Leftrightarrow \mathbf{x}_2 = H\mathbf{x}_1$$

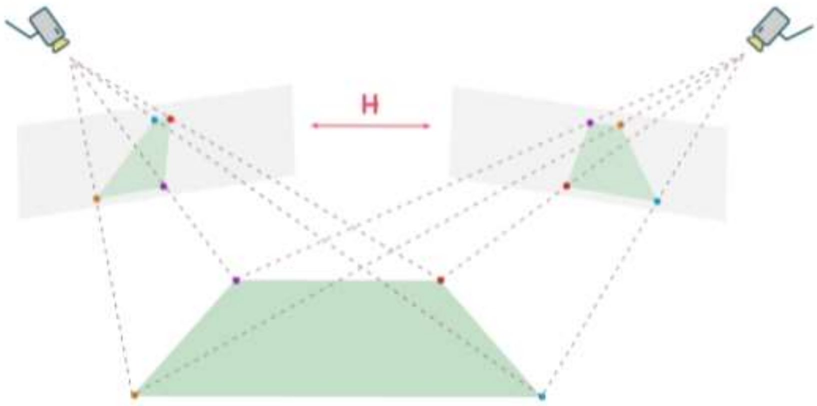
In homogenous coordinates ($x_2' = x_2/z_2$ and $y_2' = y_2/z_2$)

$$x_2' = \frac{H_{11}x_1 + H_{12}y_1 + H_{13}z_1}{H_{31}x_1 + H_{32}y_1 + H_{33}z_1}$$

$$y_2' = \frac{H_{21}x_1 + H_{22}y_1 + H_{23}z_1}{H_{31}x_1 + H_{32}y_1 + H_{33}z_1}$$

$$x_2'(H_{31}x_1 + H_{32}y_1 + H_{33}) = H_{11}x_1 + H_{12}y_1 + H_{13}$$

$$y_2'(H_{31}x_1 + H_{32}y_1 + H_{33}) = H_{21}x_1 + H_{22}y_1 + H_{23}$$



3.Homography

- If we restrict h33 = 1

$$x'_2(H_{31}x_1 + H_{32}y_1 + 1) = H_{11}x_1 + H_{12}y_1 + H_{13}z1$$
$$y'_2(H_{31}x_1 + H_{32}y_1 + 1) = H_{21}x_1 + H_{22}y_1 + H_{23}z1$$

$$x'_2 = H_{11}x_1 + H_{12}y_1 + H_{13}z1 - H_{31}x_1x'_2 - H_{32}y_1x'_2$$
$$y'_2 = H_{21}x_1 + H_{22}y_1 + H_{23}z1 - H_{31}x_1y'_2 - H_{32}y_1y'_2$$

- For perspective transformation , you can use 4 pairs of match result to solve 8 unknown variable in homography matrix

$$\begin{bmatrix} \hat{x}_i z_a \\ \hat{y}_i z_a \\ z_a \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & \boxed{h_{33}} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1\hat{x}_1 & -y_1\hat{x}_1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2\hat{x}_2 & -y_2\hat{x}_2 \\ x_3 & y_3 & 1 & 0 & 0 & 0 & -x_3\hat{x}_3 & -y_3\hat{x}_3 \\ x_4 & y_4 & 1 & 0 & 0 & 0 & -x_4\hat{x}_4 & -y_4\hat{x}_4 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1\hat{y}_1 & -y_1\hat{y}_1 \\ 0 & 0 & 0 & x_2 & y_2 & 1 & -x_2\hat{y}_2 & -y_2\hat{y}_2 \\ 0 & 0 & 0 & x_3 & y_3 & 1 & -x_3\hat{y}_3 & -y_3\hat{y}_3 \\ 0 & 0 & 0 & x_4 & y_4 & 1 & -x_4\hat{y}_4 & -y_4\hat{y}_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix} = h_{33} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \\ \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \\ \hat{y}_4 \end{bmatrix}$$

3.Homography

- Let h33 = 1
- You can solve the equation Ah = b below by pseudo inverse

$$\begin{bmatrix} \hat{x}_i z_a \\ \hat{y}_i z_a \\ z_a \end{bmatrix} = \overset{\textbf{H}}{\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$\overset{\textbf{A}}{\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1\hat{x}_1 & -y_1\hat{x}_1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2\hat{x}_2 & -y_2\hat{x}_2 \\ x_3 & y_3 & 1 & 0 & 0 & 0 & -x_3\hat{x}_3 & -y_3\hat{x}_3 \\ x_4 & y_4 & 1 & 0 & 0 & 0 & -x_4\hat{x}_4 & -y_4\hat{x}_4 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1\hat{y}_1 & -y_1\hat{y}_1 \\ 0 & 0 & 0 & x_2 & y_2 & 1 & -x_2\hat{y}_2 & -y_2\hat{y}_2 \\ 0 & 0 & 0 & x_3 & y_3 & 1 & -x_3\hat{y}_3 & -y_3\hat{y}_3 \\ 0 & 0 & 0 & x_4 & y_4 & 1 & -x_4\hat{y}_4 & -y_4\hat{y}_4 \end{bmatrix}} \overset{\textbf{h}}{\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix}} = h_{33} \overset{\textbf{b}}{\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \\ \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \\ \hat{y}_4 \end{bmatrix}}$$

3.Homography

$$A = U\Sigma V^T$$

- Using SVD decomposition to find Least Squares error solution of Ah = 0
- the solution = eigenvector of A^TA associated with the smallest eigenvalue (V stores the eigenvector of A^TA ,Σ stores the singular value (root of eigen value))
  find the smallest number in Σ and H = corresponding vector in V^T
- Remember to normalize h33 to 1

$$\begin{aligned}
\mathbf{h} &= (H_{11}, H_{12}, H_{13}, H_{21}, H_{22}, H_{23}, H_{31}, H_{32}, H_{33})^T \\
\mathbf{a}_x &= (-x_1, -y_1, -1, 0, 0, 0, x_2'x_1, x_2'y_1, x_2')^T \\
\mathbf{a}_y &= (0, 0, 0, -x_1, -y_1, -1, y_2'x_1, y_2'y_1, y_2')^T .
\end{aligned}$$

$$A = \begin{pmatrix} \mathbf{a}_{x1}^T \\ \mathbf{a}_{y1}^T \\ \vdots \\ \mathbf{a}_{xN}^T \\ \mathbf{a}_{yN}^T \end{pmatrix}.$$

You can multiply a minus to match the form in previous slide

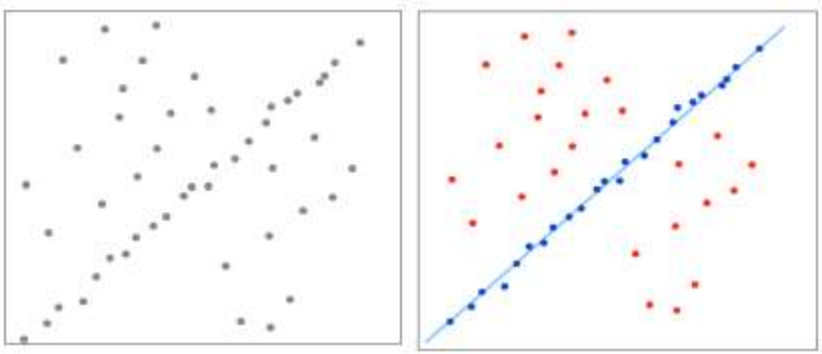
A is a 9 by 9 matrix
(It's similar to A in previous slide)

Reference :

SVD : https://web.mit.edu/be.400/www/SVD/Singular_Value_Decomposition.htm

Homography : https://cseweb.ucsd.edu/classes/wi07/cse252a/homography_estimation/homography_estimation.pdf

3.RANSAC



Random Sample Consensus

Input : M match;

1. Randomly select 4 data points as inliers S.Find a homography matrix H to S.
2. Test all match(p1 , p2) against H, estimate p2' = p1 * H
   - if the distance between p2' and p2 is small , add the match to S, which is called a consensus set.
3. If $|S|$ is larger than ever, mark H as the best estimated H^*.
4. If some stopping criterion is satisfied, end
5. Else go to step 1.

Note that you can re-estimate the models with the consensus sets.

4. Stitching image

1. Using homography matrix H to calculate the position of 4 corners of image1 in the perspective of image2
2. Using image1 after perspective transformation to analyze the size which we need to combine two image together of
3. Using cv2.warpPerspective(src, M, dsize, ...) to warp the whole image1
   - src is source image1 , M is homography matrix H , dsize is output image size

```
warped_l = cv2.warpPerspective(src=img1, M=H, dsize=size)
```

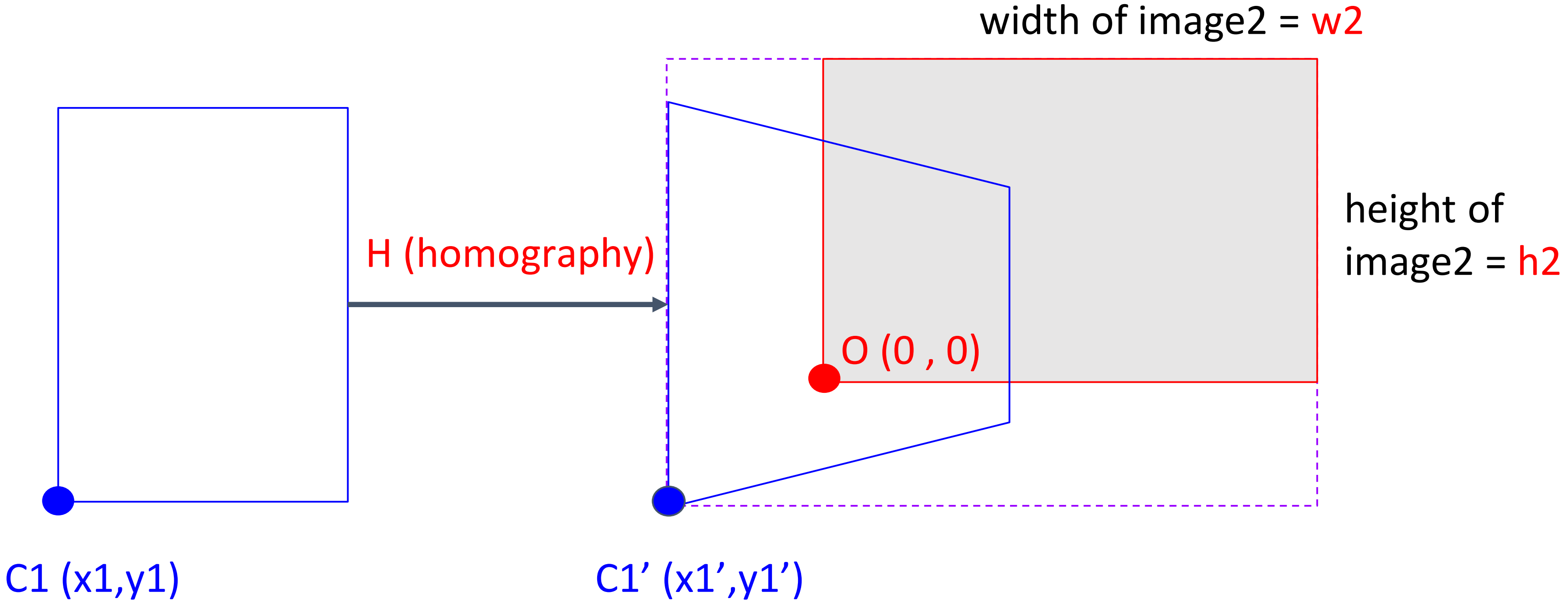
4. Concating two images (for better results you can use blending or some ways to improve the quality of overlap part)

For stitching images you can use any function of OpenCV

4. Stitching image - more detail

corners' = corners * H
x1' = min(min(corners'_x),0)
y1' = min(min(corners'_y),0)

- Assume image1 is on left hand side and image2 is on right hand side
- Size we need = (w2 + abs(x1') , h2 + abs(y1'))

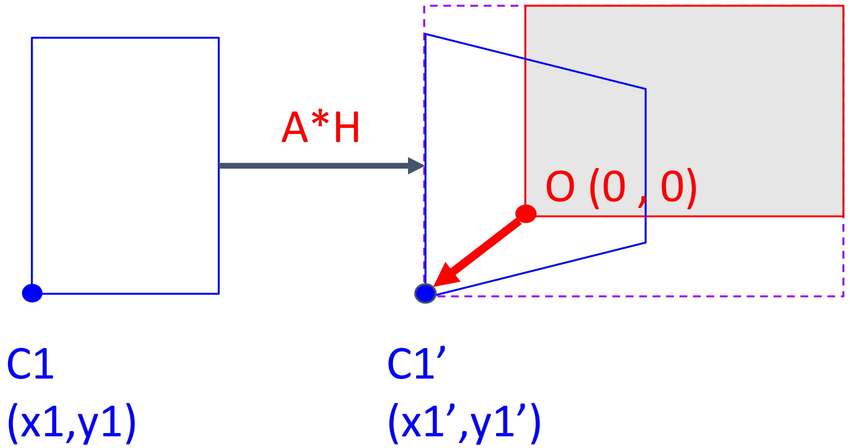


4. Stitching image - more detail

- For both image using affine translation to move the origin O to C1'. This way give some space to image1 and it's easier to combine them in same image size.
- For image1 your homograhpy matrix(H) need multiply translation matrix (A) because we translate the perspective of image2
- For image2 you can directly use warp with affine translation matrix(A)

Affine translation matrix (A) = $\begin{bmatrix} 1 & 0 & \Delta x \\ 0 & 1 & \Delta y \\ 0 & 0 & 1 \end{bmatrix}$

In this case $\Delta x = -x1^{'}, \Delta y = -y1^{'}$
Because origin moves to down left,
image2 needs move to top right relatively



4. Stitching image - more detail

- Example for image1 applys perspective transformation

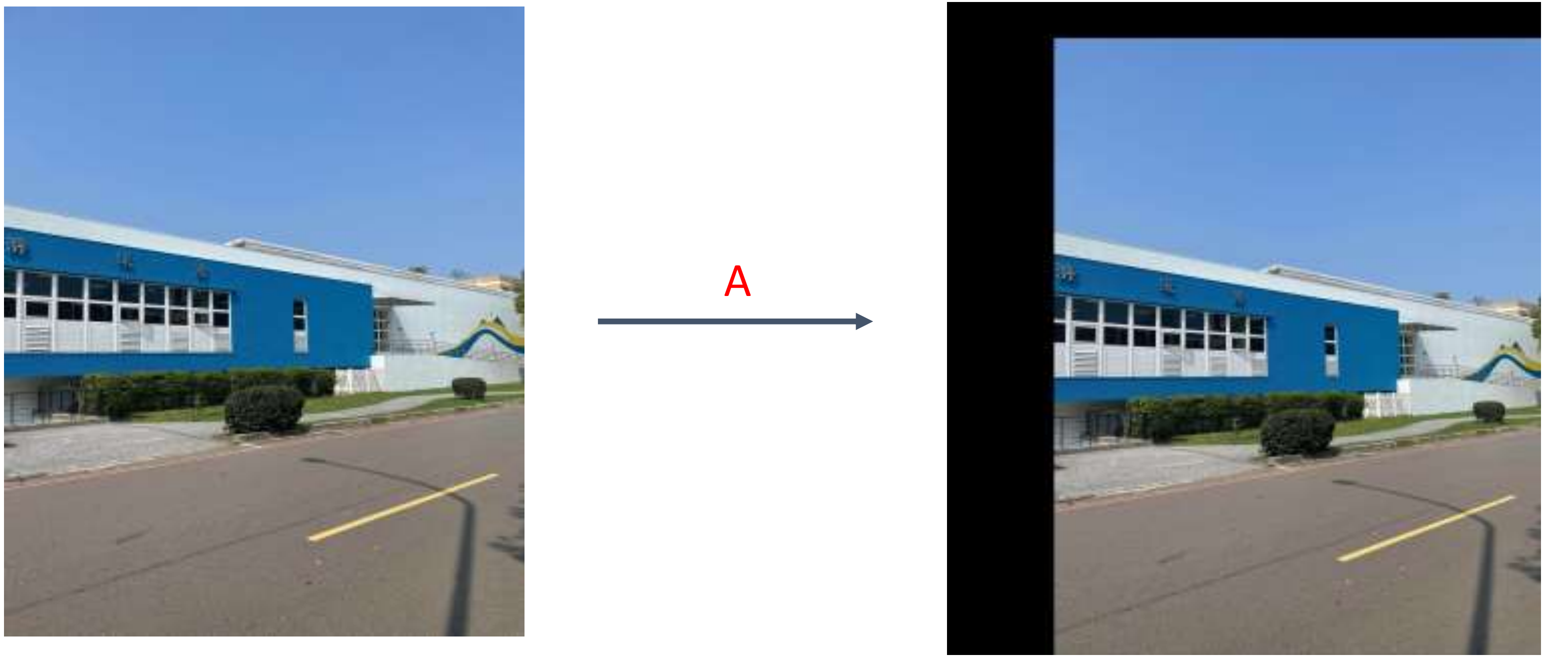
```
warped_l = cv2.warpPerspective(src=img1, M=H, dsize=size)
```

A*H

4. Stitching image - more detail

- Example for using affine translation to move the image2 origin O to C1’

```
warped_r = cv2.warpPerspective(src=img2, M=A, dsize=size)
```



Requirements

- You are only allowed to use the function of OpenCV mentioned in previous slides. Please implement all (key point matching ,RANSAC , Homography ...) by youself
  - For submission you can use :
    - SIFT
  - For debugging only:
    - KNN match : BFMatcher()
    - Homography : findHomography()
- But there is no limitation of "image stitching" only (You can use any function provided by OpenCV)

Other tips

- Using Blending when you concate the two image

Grading

50% Stitching 2 images together

SIFT (10%)

KNN (10%)

RANSAC (15%)

Homography (15%)

30% Report (Don’t just paste the code with comment)

1. explain your implementation
2. show the result of stitching 2 images
3. try to stitch more images as you can and compare with them

10% stitching at least 4 images seamlessly with blending

10% stitching bonus 4 images clearly(bonus)

(Note: the grade you got may vary according to the stitched image quality, methods you used, and so forth.)

Deadline

- Deadline : 2023/05/8 (Mon.) 11:59 pm
- Please zip the all files and name it as {studentID}_HW2.zip :
  ex 311553013_HW2.zip (wrong file format may get -10% panelty)
  - Zip file format:
    - 1. {studentID}_report.pdf
    - 2. your code
- Penalty of 3% of the value of the assignment per late day
  - late a week : your_score * 0.97
  - late two week : your_score * 0.94 …
- Feel free to ask questions on E3 forum, or send e-mails through E3 platform to all TAs for personal questions.

Result

blending

Result

Sample of concatenating 6 images together

Result(bonus)