1 Results

For the following results we consider 3 types of parameterizations for the portfolio problem. The first is a simple case where the assets are identically distributed as seen in (Cai, Judd and Xu 2013), the second is a case where the parameters are chosen to match the parameters in (Schober, Valentin and Pflüger 2022) also seen in (Gaegauf, Scheidegger and Trojani 2023). This is in order to be able to draw correct comparisons between the results. Furthermore this case, displays assets with slight variation in the mean and a small correlation between the assets, and no asset is dominating the others. The last parameterization is a modification of the first case where the correlation between the assets is larger (correlation coefficient of 0.75).

Table 1: Parameters for Examples of Portfolio Problems

	i.i.d Assets	**Schober Parameters**	**High Correlation**
T	6	6	6
k	3	5	3
γ	3.0	3.5	3.0
τ	0.5%	0.5%	0.5%
β	0.97	0.97	0.97
r	3%	4%	3%
$\mu^\top$	(0.07, 0.07)	μ_{Shober}	(0.07, 0.07)
Σ	$\begin{bmatrix} 0.04 & 0.00 \\ 0.00 & 0.04 \end{bmatrix}$	Σ_{Schober}	$\begin{bmatrix} 0.04 & 0.03 \\ 0.03 & 0.04 \end{bmatrix}$

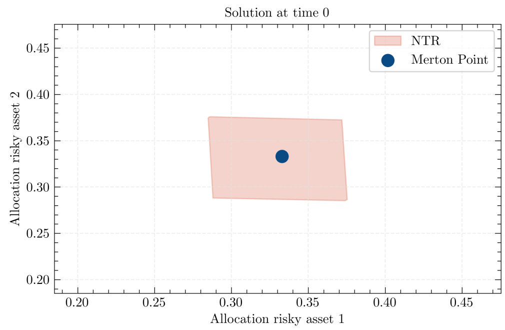
$$\mu_{\text{Schober}}^\top = \begin{bmatrix} 0.0572 & 0.0638 & 0.07 & 0.0764 & 0.0828 \end{bmatrix}$$

$$\Sigma_{\text{Schober}} = \begin{bmatrix} 0.0256 & 0.00576 & 0.00288 & 0.00176 & 0.00096 \\ 0.00576 & 0.0324 & 0.0090432 & 0.010692 & 0.01296 \\ 0.00288 & 0.0090432 & 0.04 & 0.0132 & 0.0168 \\ 0.00176 & 0.010692 & 0.0132 & 0.0484 & 0.02112 \\ 0.00096 & 0.01296 & 0.0168 & 0.02112 & 0.0576 \end{bmatrix}$$

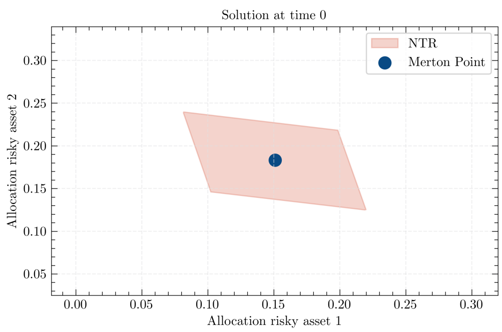
1.1 Dynamic Portfolio Choice without consumption

I first consider the base model with proportional transaction costs and no consumption. In the absence of consumption, the optimal portfolio is the merton points, which we plot in every figure. I plot the No-trade region at time point 0 (initial time point) for each of the parameterizations in Figure 1.1. When using the Schober parameters we select the d first elements of the mean vector, and truncate the covariance matrix to a $d \times d$ matrix, depending on the number of assets d in the model. I note that for each of the

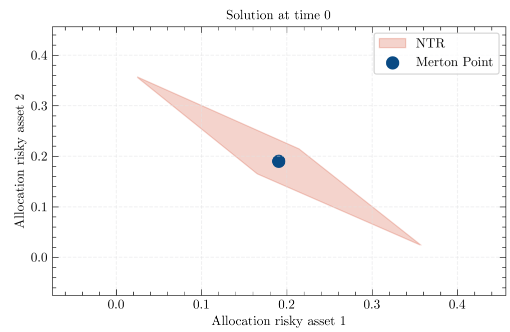
Figure 1.1: Comparison of No Trade Regions.



(a) No Trade Region for Independent Identically Distributed Assets.



(b) No Trade Region for Schober Parameters.



(c) No Trade Region for High Correlation.

parameterizations the No-Trade region is a rectangle or parallelogram. For the case of identical and independent assets, the No-Trade region is a perfect square, whereas for the Schober parameters and the high correlation case, the No-Trade region is a parallelogram. This is due to the correlation between the assets. When some correlation is present, the No-Trade region is skewed, since some allocations which would be optimal in the absence of correlation are no longer optimal.

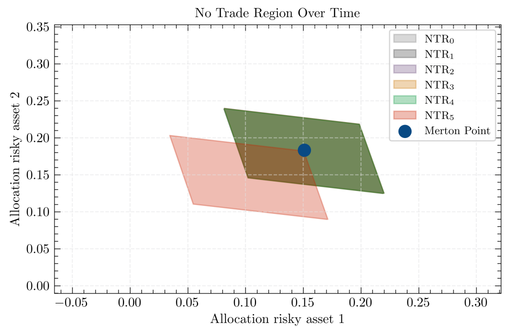
1.1.1 Investigating the No-Trade Region

We now look at the No-Trade region for the base model with proportional transaction costs and no consumption in more detail. Specifically we look at how the region behaves over the entire investment horizon $[0, T]$, and how the region changes with different transaction cost levels. We choose to look at the model with the Schober parameters, as this is a mixture of the other two parameterizations.

I note that at the last time point $t = T$ the NTR moves away from the merton point towards the origin, and the Merton point is is now the upper right corner of the NTR. For all other time periods the NTR is the same, and the Merton point is in the center.

I now investigate how the No-Trade region changes with different transaction cost levels. I do this for the i.i.d. parameters, and plot the NTR for different values of τ in

Figure 1.2: No Trade Region for Schober Parameters over Time.



The No-Trade region is plotted for the Schober parameters over the entire investment horizon $[0, T]$ For time points $t \in [0, T-1]$ the no-trade-region (NTR)s overlap.

Figure 1.3: No Trade Region for the iid Parameters with different values of τ.

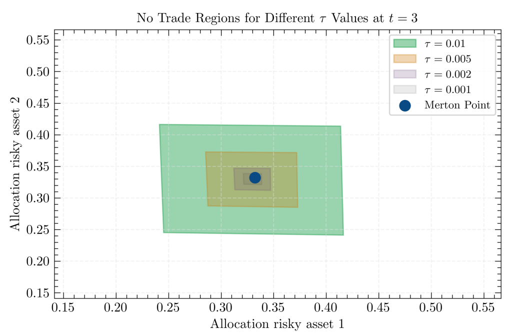


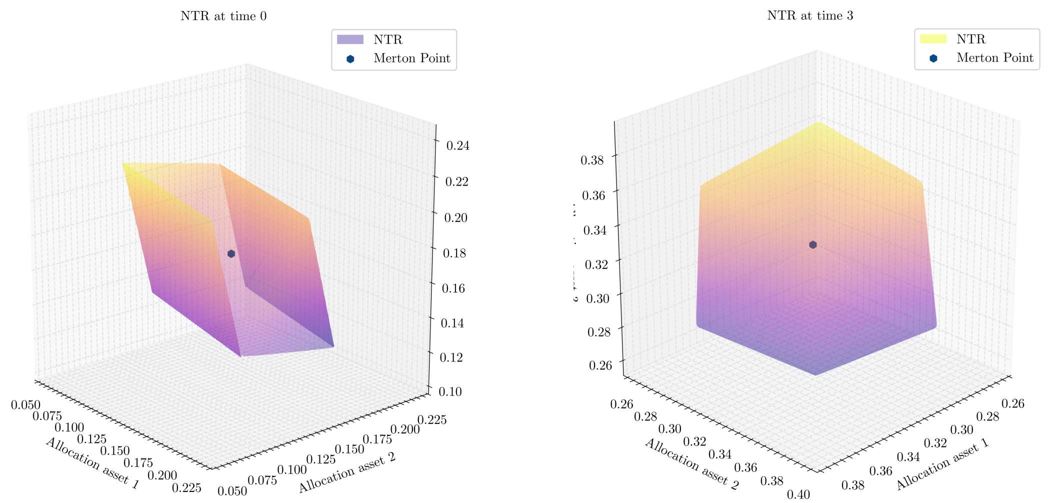
Figure 1.3. When the transaction costs are increased, the NTR increases aswell and vice-versa. I note that for low enough transaction costs, the NTR shrinks towards the Merton point. However when transaction costs are low enough, the Merton point is not in the excact center, which might signify that at low enough values, some numerical instabilities from the minimizer, and function approximation using Gaussian process regression (GPR) might be present.

1.1.2 Increasing the dimensionality of the model

We now increase the dimensionality of the model to $d = 3$ and look at the No-Trade region for the Schober parameters and for the i.i.d parameters.

Note that the i.i.d NTR looks like a skewed cube, whereas this was a perfect square in the 2 dimensional case. Looking that the points forming the convex hull that is the NTR, it is clear that the NTR is restricted by the no-trading and no-borrowing constraints, since one of the border points, which would otherwise form the perfect cube is outside the feasible space. Hence when the risky returns outweigh the risk-free return, to such a

Figure 1.4: Comparison of No Trade Regions.



(a) No Trade Region for Schober Parameters in 3 dimensions.

(b) No Trade Region for i.i.d. Parameters in 3 dimensions

degree that the merton point moves towards the boundary of the feasible space, cube like shapes are no longer possible. This is clear when compared to the Schober parameters, where the merton point is in the center of the NTR, and the NTR is a skewed cube. The merton point in this case suggest lower portfolio allocations to the risky assets, and hence the NTR is not restricted by the no-trading and no-borrowing constraints.