

# 1 Results

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For the following results we consider 3 types of parameterizations for the portfolio problem. The first is a simple case where the assets are identically distributed as seen in (Cai, Judd and Xu 2013), the second is a case where the parameters are chosen to match the parameters in (Schober, Valentin and Pflüger 2022) also seen in (Gaegau, Scheidegger and Trojani 2023). This is in order to be able to draw correct comparisons between the results. Furthermore this case, displays assets with slight variation in the mean and a small correlation between the assets, and no asset is dominating the others. The last parameterization is a modification of the first case where the correlation between the assets is larger (correlation coefficient of 0.75).

**Table 1:** Parameters for Examples of Portfolio Problems

	i.i.d Assets	Schober Parameters	High Correlation
$T$	6	6	6
$k$	3	5	3
$\gamma$	3.0	3.5	3.0
$\tau$	0.5%	0.5%	0.5%
$\beta$	0.97	0.97	0.97
$r$	3%	4%	3%
$\mu^\top$	(0.07, 0.07)	$\mu_{\text{Schober}}$	(0.07, 0.07)
$\Sigma$	$\begin{bmatrix} 0.04 & 0.00 \\ 0.00 & 0.04 \end{bmatrix}$	$\Sigma_{\text{Schober}}$	$\begin{bmatrix} 0.04 & 0.03 \\ 0.03 & 0.04 \end{bmatrix}$

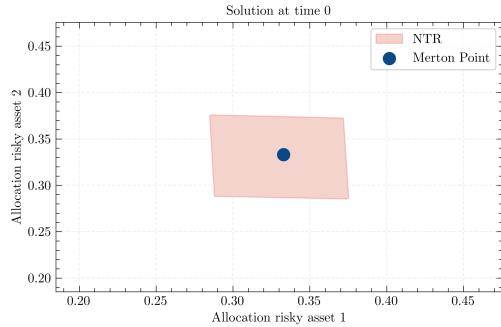
$$\mu_{\text{Schober}}^\top = [0.0572 \ 0.0638 \ 0.07 \ 0.0764 \ 0.0828]$$

$$\Sigma_{\text{Schober}} = \begin{bmatrix} 0.0256 & 0.00576 & 0.00288 & 0.00176 & 0.00096 \\ 0.00576 & 0.0324 & 0.0090432 & 0.010692 & 0.01296 \\ 0.00288 & 0.0090432 & 0.04 & 0.0132 & 0.0168 \\ 0.00176 & 0.010692 & 0.0132 & 0.0484 & 0.02112 \\ 0.00096 & 0.01296 & 0.0168 & 0.02112 & 0.0576 \end{bmatrix}$$

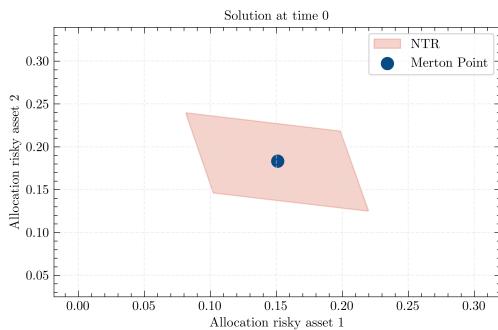
## 1.1 Dynamic Portfolio Choice without consumption

I first consider the base model with proportional transaction costs and no consumption. In the absence of consumption, the optimal portfolio is the merton point, which we plot in every figure. I plot the No-trade region at time point 0 (initial time point) for each of the parameterizations in 1.1. When using the Schober parameters we select the  $d$  first elements of the mean vector, and truncate the covariance matrix to a  $d \times d$  matrix, depending on the number of assets  $d$  in the model. I note that for each of the

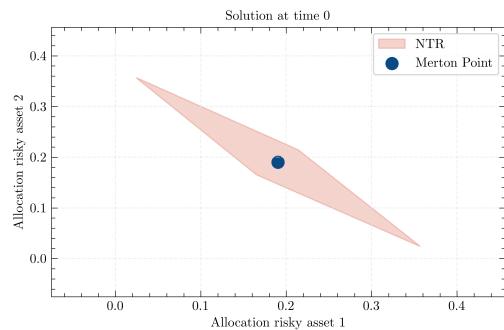
**Figure 1.1:** Comparison of No Trade Regions.



(a) No Trade Region for Independent Identically Distributed Assets.



(b) No Trade Region for Schober Parameters.



(c) No Trade Region for High Correlation.

parameterizations the No-Trade region is a rectangle or parallelogram. For the case of identical and independent assets, the No-Trade region is a perfect square, whereas for the Schober parameters and the high correlation case, the No-Trade region is a parallelogram. This is due to the correlation between the assets. When some correlation is present, the No-Trade region is skewed, since some allocations which would be optimal in the absence of correlation are no longer optimal.

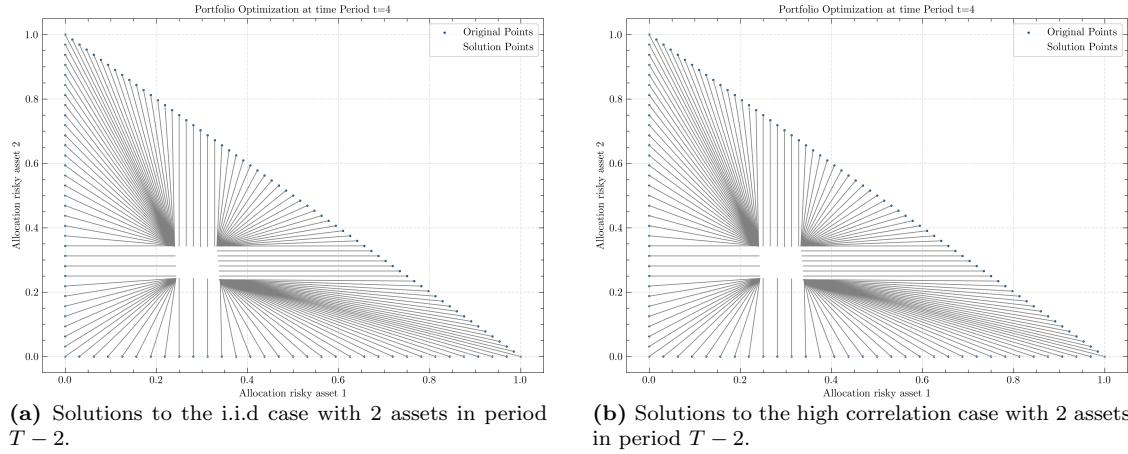
### 1.1.1 Verifying the geometric shape of the No-trade Region

Since much of the procedure for solving this problem, and approximating the NTR, leverages the a-priori assumptions regarding the geometric shape of the NTR, i first want to verify that the NTR indeed has 4 vertices for the 2d case and is a convex hull.

In order to do this, i do a small modification to the solution algorithm proposed earlier. Instead of computing vertices using  $2^d$  predetermined points, i will instead sample a larger set of points, (128) covering the boundaries of the feasible state space. For each of these points i then solve the optimization problem, and plot the solution, from allocations  $\mathbf{x}_t$  and their solutions to the problem  $\hat{\omega}_t$ . I do this by using my original sample scheme, and adding mid-points between points, which either sum to 1.0 or have 0.0 as allocation for one of the assets. I consider the i.i.d case and the high correlation case, with  $tau = 1\%$ . I

have increased the costs slightly in order to increase the size of the NTR. This is to ensure that points also converge towards the faces and not only the vertices. This is akin the the green regions in Figure ???. Otherwise i would need more points. I plot the solutions for next to last period with investment decisions  $T - 2$ . The solutions are plotted below.

**Figure 1.2:** Verifying the assumptions of the NTR in 2 dimensions.



I find that the assumptions regarding the no-trade-region (NTR) are indeed correct in the two dimensional examples i have constructed. Furthermore this verifies that the assumptions also hold for correlated asset, which was only postulated by (Liu 2004). Furthermore these plots also nicely confirm that the optimization process as a whole works as intended. Further verification in higher dimensions are not considered. First of all (Liu 2004) confirms this formally in larger dimensions, for the case of uncorrelated assets, and the intuition regarding the NTR does not change when dimensionality is increased.

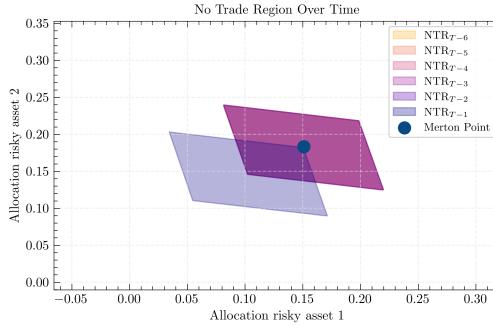
### 1.1.2 Investigating the No-Trade Region

We now look at the No-Trade region for the base model with proportional transaction costs and no consumption in more detail. Specifically we look at how the region behaves over the entire investment horizon  $[0, T]$ , and how the region changes with different transaction cost levels. We choose to look at the model with the Schober parameters, as this is a mixture of the other two parameterizations.

I note that at the last time point  $t = T - 1$  the NTR moves away from the merton point towards the origin, and the Merton point is now the upper right corner of the NTR. For all other time periods the NTR is the same, and the Merton point is in the center.

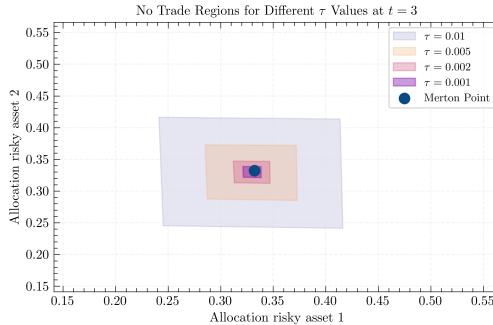
I now investigate how the No-Trade region changes with different transaction cost levels. I do this for the i.i.d. parameters, and plot the NTR for different values of  $\tau$  in

**Figure 1.3:** No Trade Region for Schober Parameters over Time.



The No-Trade region is plotted for the Schober parameters over the entire investment horizon  $[0, T - 1]$ . For time points  $t \in [0, T - 2]$  the NTRs overlap.

**Figure 1.4:** No Trade Region for the iid Parameters with different values of  $\tau$ .



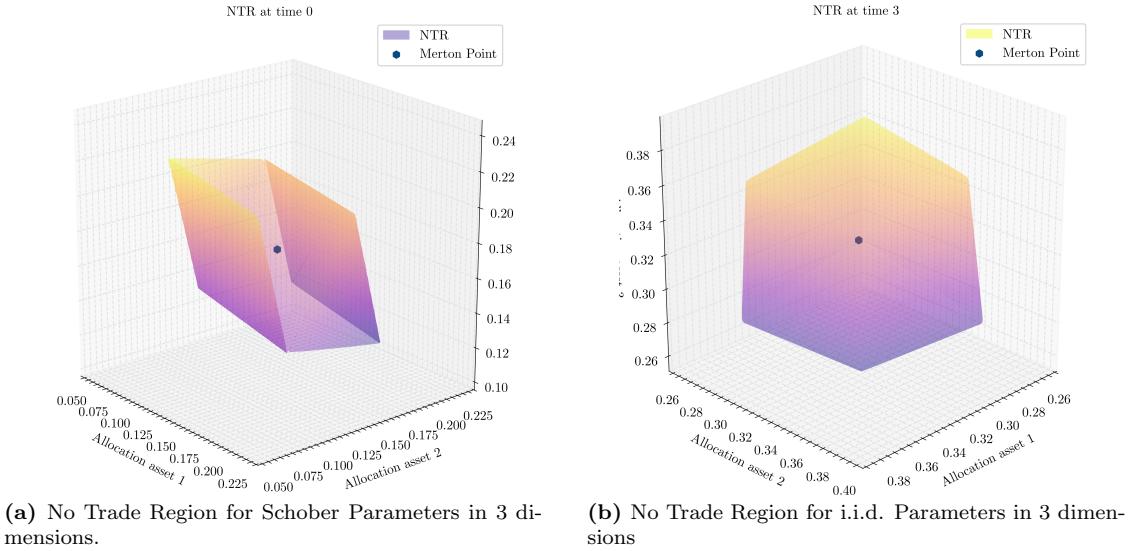
**Figure 1.4.** When the transaction costs are increased, the NTR increases as well and vice-versa. I note that for low enough transaction costs, the NTR shrinks towards the Merton point. However when transaction costs are low enough, the Merton point is not in the exact center, which might signify that at low enough values, some numerical instabilities from the minimizer, and function approximation using Gaussian process regression (GPR) might be present.

### 1.1.3 Increasing the dimensionality of the model

We now increase the dimensionality of the model to  $d = 3$  and look at the No-Trade region for the Schober parameters and for the i.i.d parameters.

Note that the i.i.d NTR looks like a skewed cube, whereas this was a perfect square in the 2 dimensional case. Looking that the points forming the convex hull that is the NTR, it is clear that the NTR is restricted by the no-borrowing constraint, since one of the border points, which would otherwise form the perfect cube, would outside the feasible space if this was possible, and is then projected into the feasible space. Hence when the

**Figure 1.5:** Comparison of No Trade Regions.



risky returns outweigh the risk-free return, to such a degree that the merton point moves towards the boundary of the feasible space, cube like shapes are no longer possible. In the 2 dimension case, this is akin to the NTR being close to the budget line, and the NTR would then form a triangle.

This is clear when compared to the Schober parameters, where the merton point is in the center of the NTR, and the NTR is a skewed cube. The merton point in this case suggest lower portfolio allocations to the risky assets, and hence the NTR is not restricted by the no-trading and no-borrowing constraints.

## 1.2 Dynamic Portfolio Choice with consumption

I now consider the base model with proportional transaction costs which now includes consumption of a non-durable good. This adds an extra decision variable which needs to be solved for, and consumption now adds immediate utility to the investor, in each period.

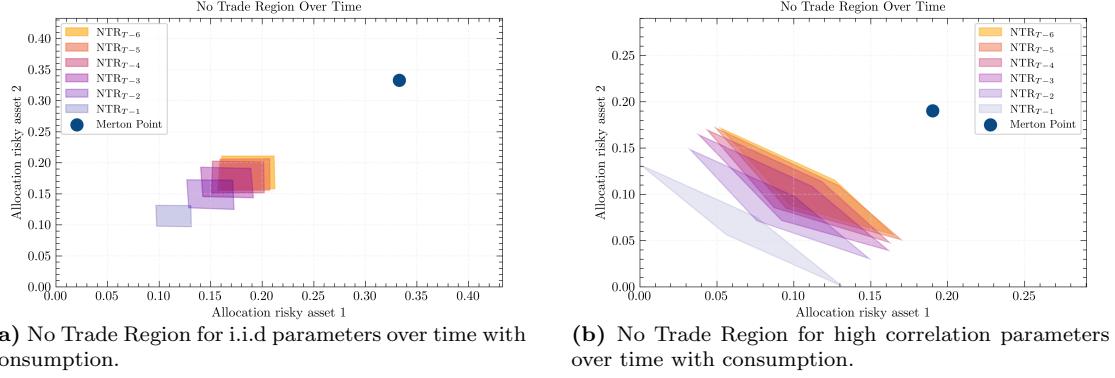
Note that when consumption is included, the NTR no longer encapsulates the Merton point at any time point. Furthermore the NTR now moves over time, towards the origin, as opposed to the case without consumption, where the NTR was static for all time points except the next to last period (last period with trading decisions).

This behaviour is consistent in higher dimensions.

## 1.3 Dynamic Portfolio Choice with fixed costs

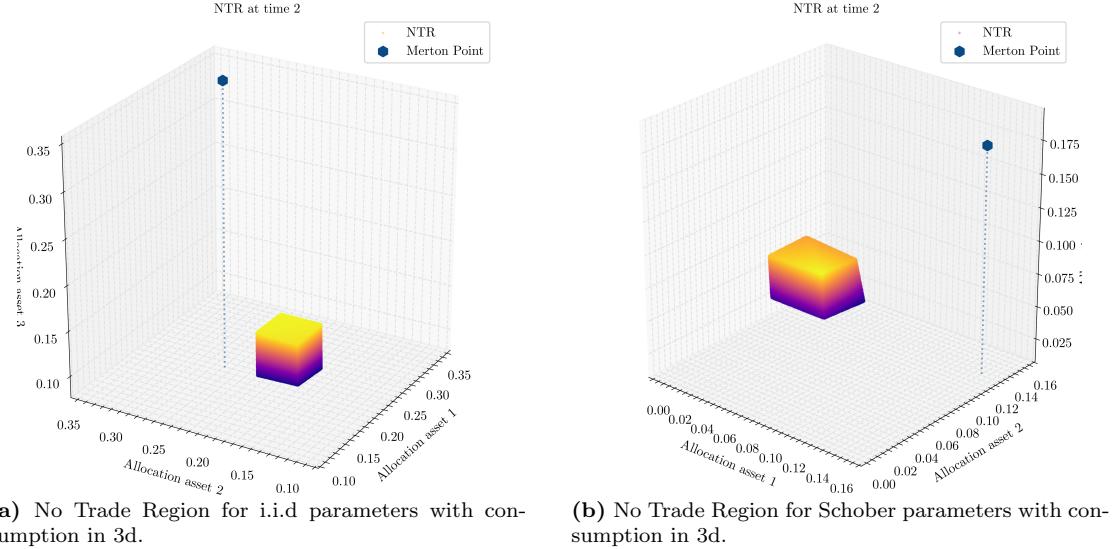
I now consider the base model with fixed transaction costs, and no consumption. From (Dybvig and Pezzo 2020) i know that the NTR is no longer rectangular when we only

**Figure 1.6:** Comparison of No Trade Regions over time with consumption.



The No-Trade regions are plotted over the entire investment horizon  $[0, T - 1]$ .

**Figure 1.7:** No trade regions with consumption in multiple dimensions.



The No-Trade regions are plotted at time  $t = 2$ .

consider fixed costs, but instead circular with the merton point in the middle. This poses a problem for my current sampling scheme, which leverages our predetermined knowledge of the geometric shape of the NTR. As I noted in Section ??, in order to effectively sample points for the NTR approximation, I now need to sample points, such that when they hit the NTR these points are evenly distributed on the sphere, in order to approximate the NTR correctly.

For the case without consumption this is feasible, but for the case with consumption, the NTR moves over time, and locating its center a priori is not possible.