Data structures and algorithms

Tutorial 7 - Introduction to graph theory

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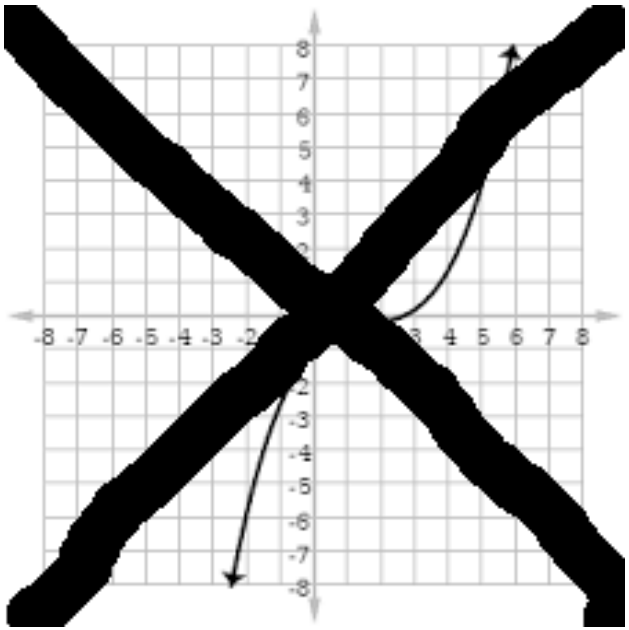
Outline

1 Basic concepts of graph theory
- Terminology
- Types of graphs

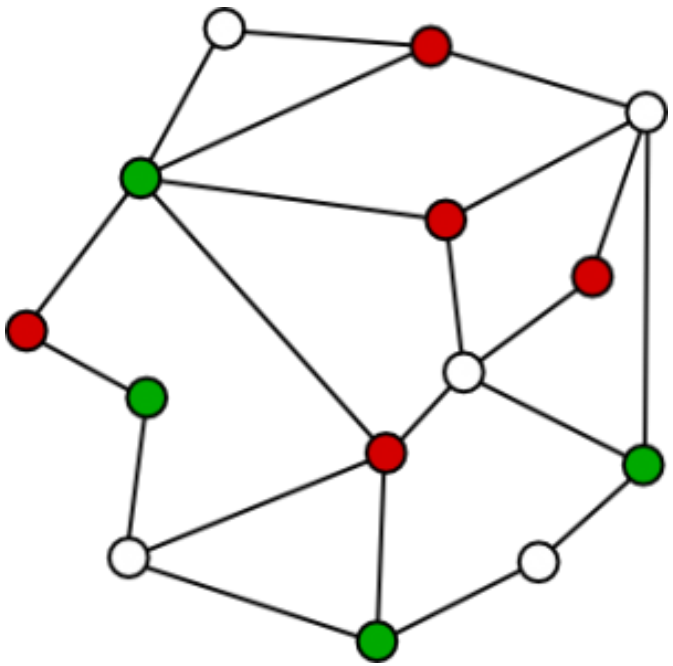
2 Adjacency Matrix

3 Adjacency List

Graph? NO



Graph? YES



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- How to represent a graph using basic data-structures?
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- Node
- Edge: A direct connection between two nodes
- Path: A set of edges connecting source and destination nodes
- Weight: A value give to each edge
- Degree: No. of edges for each node

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Undirected vs Directed

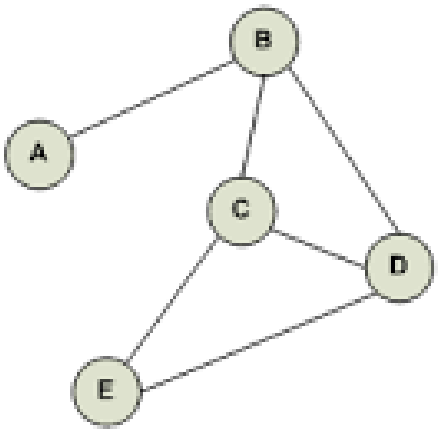


Fig 1. Undirected Graph

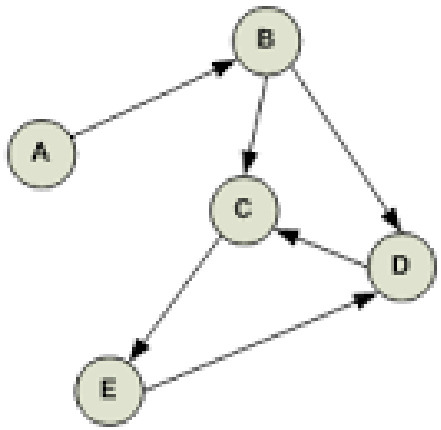
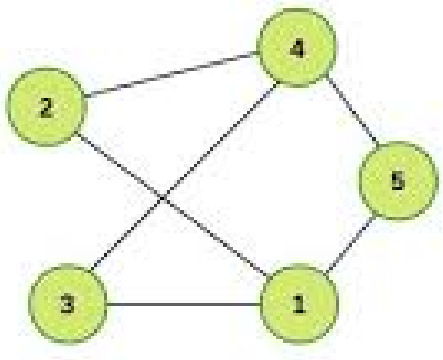
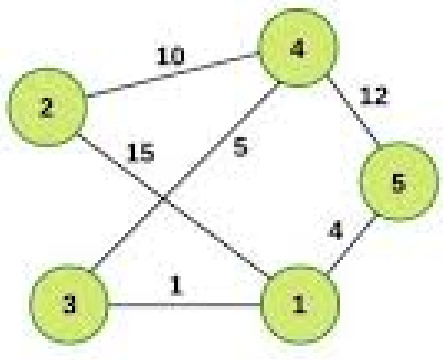


Fig 2. Directed Graph

UnWeighted vs Weighted



unweighted graph



weighted graph

Directed or undirected?

- Followers on Twitter
- Friends on facebook
- Roads of Google Maps

The two properties aren't mutually exclusive.
E.G: A graph can be:

- Directed and Unweighted
- Undirected and Unweighted
- Directed and Weighted
- Undirected and Weighted

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For simplicity and without loss of generality, let's assume that the graph isn't dynamic (We won't need to add or delete nodes after the construction of the graph).

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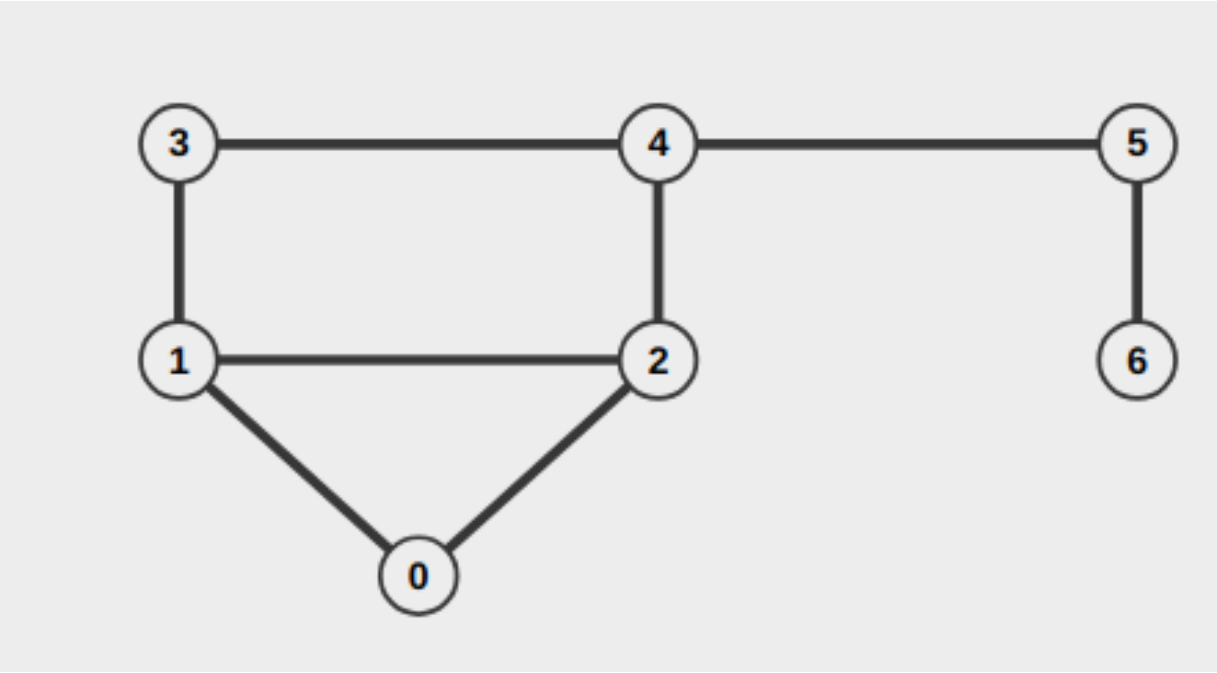
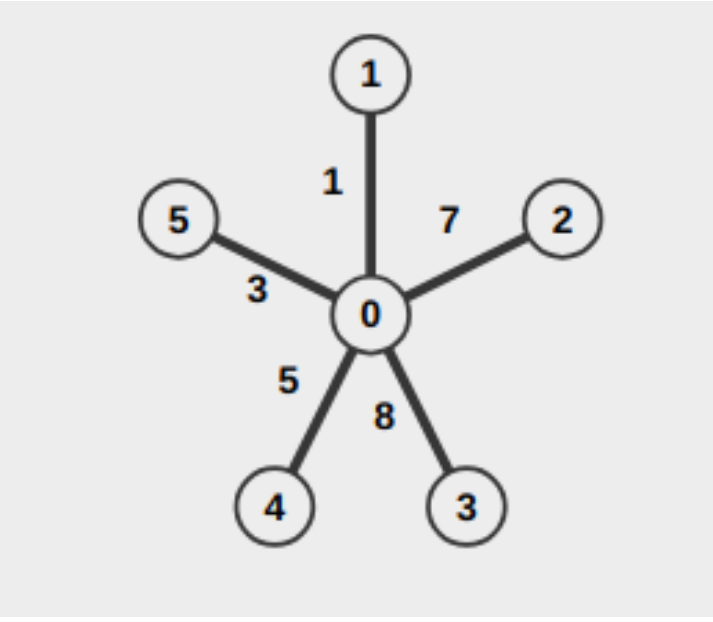
- Wikipedia definition: "In graph theory and computer science, an adjacency matrix is a square matrix used to represent a finite graph. The elements of the matrix indicate whether pairs of vertices are adjacent or not in the graph."
- For a graph of N nodes, build a 2D array (matrix) of size N*N.
- The values stored in the adjacency matrix differs according to the type of the graph.

Generate the adjaceny matrix for the following graph.



Generate the adjaceny matrix for the following graph.



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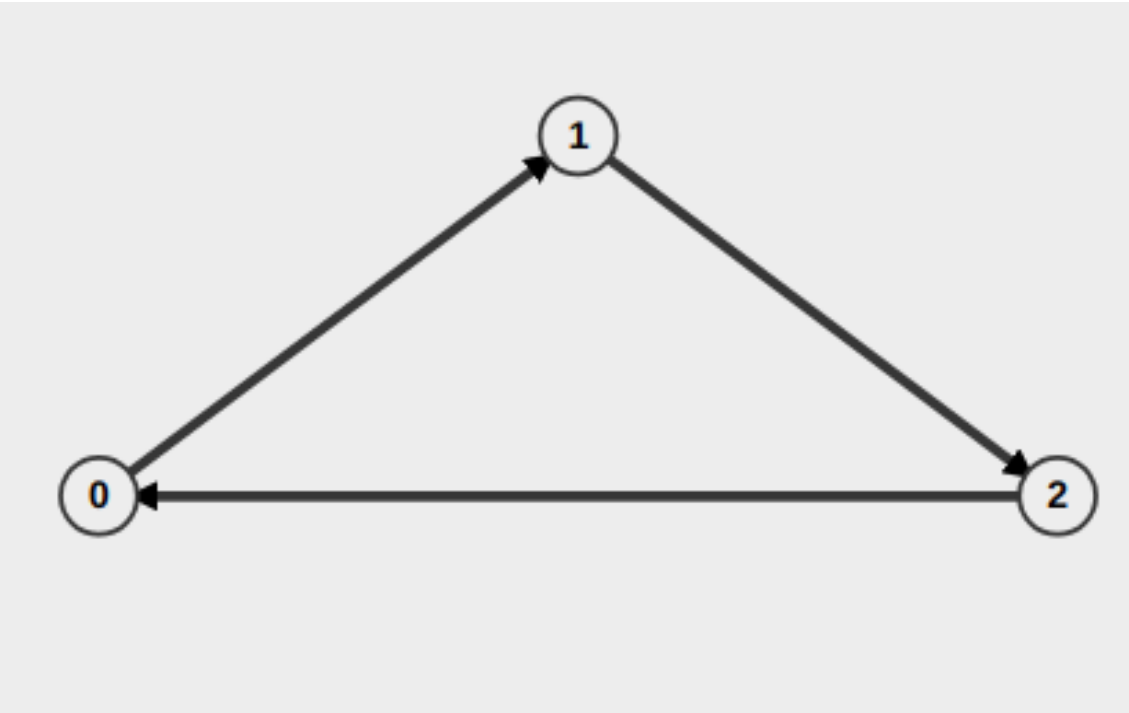
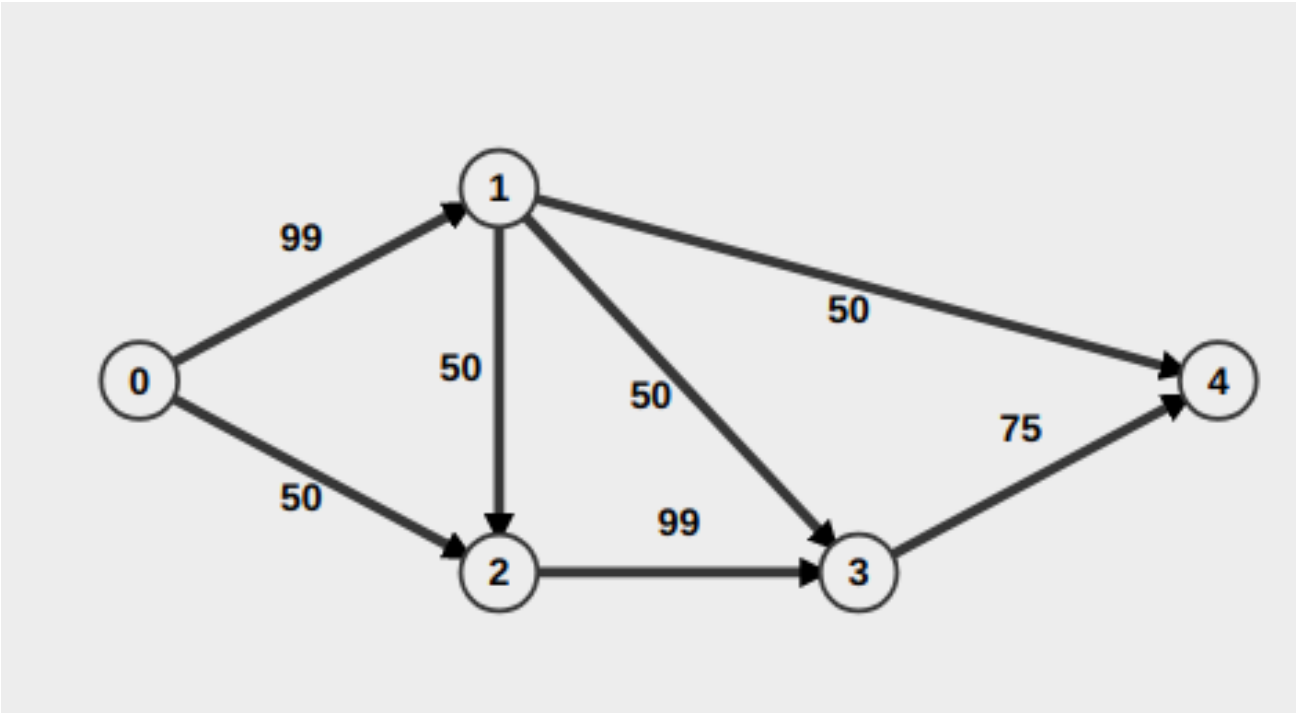


Generate the adjaceny matrix for the following graph.



How to read a description of the graph and load it into an adjacency matrix?

```
// An undirected unweighted graph

3 3
0 1
0 2
1 2
```

How to read a description of the graph and load it into an adjacency matrix?
Note: The lecture uses an array of pointers.

```
// Undirected unweighted
int n, m;
cin>>n>>m;
vector<vector<int> > adj_matrix(n, vector<int> (n, 0))
int a, b;
for (int i=0; i<m ; i++){
   cin>>a>>b;
   adj_matrix[a][b] = 1;
   adj_matrix[b][a] = 1;
}
```

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}
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```
// Undirected Weighted
int n, m;
cin>>n>>m;
vector<vector<int> > adj_matrix(n, vector<int> (n, 0))
int a, b, c;
for (int i=0; i<m ; i++){
   cin>>a>>b>>c;
   adj_matrix[a][b] = c;
   adj_matrix[b][a] = c;
}
```

How to read a description of the graph and load it into an adjacency matrix?

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   cin>>a>>b>>c;
   adj_matrix[a][b] = c;
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One of the questions that DFS can solve is:
"Are nodes src and des connected?"

```
bool dfs(int node, int des,
  const vector<vector<int> > & adj_matrix);
// Return true if there is a path between node and des
```

```
bool dfs(int node, int des,
  const vector<vector<int> > & adj_matrix){

  if (node ==des){
    return true;
  }
  for (int next_node=0;
      next_node<adj_matrix[node].size();
      next_node++){
    if (next_node == node
      || adj_matrix[node][next_node] == 0)
      continue;
    if (dfs(next_node, des, adj_matrix))
      return true;
  }
  return false;
}
```

```
bool dfs(int node, int des,
  const vector<vector<int> > & adj_matrix,
  vector<bool> & visited){

  if (node ==des){
    return true;
  }
  visited[node] = true;
  for (int next_node=0;
      next_node<adj_matrix[node].size();
      next_node++){
    if (next_node == node
      || adj_matrix[node][next_node] == 0
      || visited[next_node])
      continue;
    if (dfs(next_node, des, adj_matrix, visited))
      return true;
  }
  return false;
}
```

```
bool dfs(int node, int des,
  const vector<vector<int> > & adj_matrix,
  vector<bool> & visited);

int main(){
  int n, m;
  cin>>n>>m;
  vector<vector<int> > adj_matrix(n, vector<int> (n, 0))
  ... // Build the graph
  int src, des;
  cin>>src>>des;
  vector<bool>visited(n, false);
  if(dfs(src, des, adj_matrix, visited))
    cout<<"Connected";
  else
    cout<<"Not connected";
}
```

```
bool dfs(int node, int des,
  const vector<vector<int> > & adj_matrix,
  vector<bool> & visited){

  stack<int> nodes_stack;  nodes_stack.push(node);
  while(!nodes_stack.empty()){
    int cur_node = nodes_stack.top();
    nodes_stack.pop(); visited[cur_node] = 1;
    if (cur_node==des) return true;
    for (int next_node=0;
     next_node<adj_matrix[node].size();
     next_node++){
      if (next_node == node
        || adj_matrix[node][next_node] == 0
        || visited[next_node])
        continue;
      nodes_stack.push(next_node);
    }
  }
  return false;
```

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- Adjacency matrix isn't suitable for sparse graphs.
- For each node, store only information about its adjacent nodes.

```
// For an Undirected Unweighted graph
int n, m;
cin>>n>>m;
vector<vector<int> > adj_list(n, vector<int> ());
int a, b;
for (int i=0; i<m ;i++){
   cin>>a>>b;
   adj_list[a].push_back(b);
   adj_list[b].push_back(a);
}
```

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```
bool dfs(int node, int des,
  const vector<vector<int> > & adj_list,
  vector<bool> & visited){

  if (node ==des) return true;

  visited[node] = true;
  for (int next_node_index=0;
      next_node_index<adj_list[node].size();
      next_node_index++){

    int next_node = adj_list[next_node_index];
    if (visited[next_node])
      continue;
    if (dfs(next_node, des, adj_list, visited))
      return true;
  }
  return false;
}
```

Feedback form: `https://forms.gle/JG9JY7a6ZrsipWDB9`