Logic Design

Tutorial 4

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Outline

1 Logic gates
- Motivation
- Axioms of Boolean Algebra
- Sheet 3 - Question 4
- Sheet 3 - Question 7
- Sheet 3 - Question 9
- Sheet 3 - Question 15

2 Gray code

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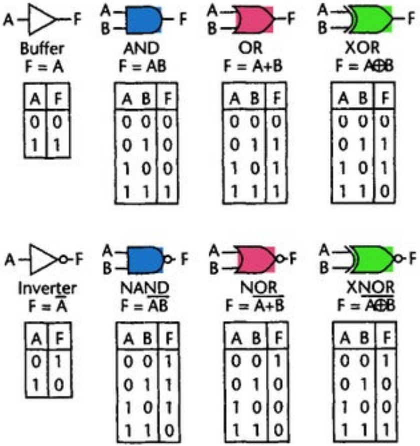
2 Gray code
- Sheet 2 - Question 2



A	F
0	0
1	1

A	B	F (AND)
0	0	0
0	1	0
1	0	0
1	1	1

A	B	F (OR)
0	0	0
0	1	1
1	0	1
1	1	1

A	B	F (XOR)
0	0	0
0	1	1
1	0	1
1	1	0

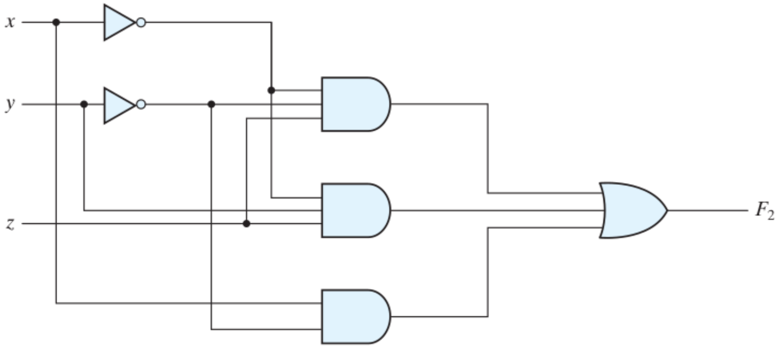
A	F
0	1
1	0

A	B	F (NAND)
0	0	1
0	1	1
1	0	1
1	1	0

A	B	F (NOR)
0	0	1
0	1	0
1	0	0
1	1	0

A	B	F (XNOR)
0	0	1
0	1	0
1	0	0
1	1	1

What is the output of this circuit for the following input X=0, Y=0, Z=1?



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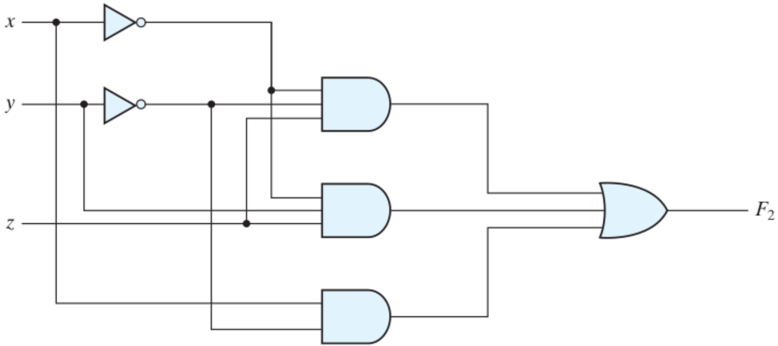

$$F2 = \overline{X}.\overline{Y}.Z + \overline{X}.Y.Z + X.\overline{Y}$$

Using Boolean Algebra, we can simplify the function to
$F2 = X.\overline{Y} + \overline{X}.Z$

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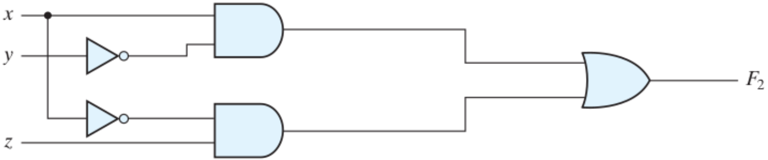

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Table 2.1
Postulates and Theorems of Boolean Algebra

Postulate 2	(a)	$x + 0 = x$	(b)	$x \cdot 1 = x$
Postulate 5	(a)	$x + x' = 1$	(b)	$x \cdot x' = 0$
Theorem 1	(a)	$x + x = x$	(b)	$x \cdot x = x$
Theorem 2	(a)	$x + 1 = 1$	(b)	$x \cdot 0 = 0$
Theorem 3, involution		$(x')' = x$		
Postulate 3, commutative	(a)	$x + y = y + x$	(b)	$xy = yx$
Theorem 4, associative	(a)	$x + (y + z) = (x + y) + z$	(b)	$x(yz) = (xy)z$
Postulate 4, distributive	(a)	$x(y + z) = xy + xz$	(b)	$x + yz = (x + y)(x + z)$
Theorem 5, DeMorgan	(a)	$(x + y)' = x'y'$	(b)	$(xy)' = x' + y'$
Theorem 6, absorption	(a)	$x + xy = x$	(b)	$x(x + y) = x$

How to prove something like $X + (Y + Z) = (X + Y) + Z$?

How to prove something like $X + (Y + Z) = (X + Y) + Z$?
Using Truth Table:

X	Y	Z	(Y+Z)	**X + (Y+Z)**	(X+Y)	**(X+Y) + Z**
0	0	0	0	**0**	0	**0**
0	0	1	1	**1**	0	**1**
0	1	0	1	**1**	1	**1**
0	1	1	1	**1**	1	**1**
1	0	0	0	**1**	1	**1**
1	0	1	1	**1**	1	**1**
1	1	0	1	**1**	1	**1**
1	1	1	1	**1**	1	**1**

I will wear a jacket ($Jacket$) if:

- It rains ($RAIN$)
- It doesn't rain ($\overline{RAIN}$) and it's cold ($COLD$)

$Jacket =$

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$Jacket =$
$RAIN + \overline{RAIN}.COLD=$

I will wear a jacket ($Jacket$) if:

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$Jacket =$
$RAIN + \overline{RAIN}.COLD =$
$RAIN + COLD$

I will wear a jacket ($Jacket$) if:

- It rains ($RAIN$)
- It doesn't rain ($\overline{RAIN}$) and it's cold ($COLD$)

$Jacket =$
$RAIN + \overline{RAIN}.COLD =$
$RAIN + COLD$

Generally: $X + \overline{X}.Y = X + Y$

I will wear a jacket (*Jacket*) if:

- It rains (*RAIN*)
- It doesn't rain ($\overline{RAIN}$) and it's cold (*COLD*)

$Jacket =$
$RAIN + \overline{RAIN}.COLD =$
$RAIN + COLD$

Generally: $X + \overline{X}.Y = X + Y$
AND $\overline{X} + X.Y = \overline{X} + Y$

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Simplify the following equations:

- c) $\overline{A}B(\overline{D} + \overline{C}D) + B(A + \overline{A}CD)$ (to one literal)

Simplify the following equations:

- c) $\overline{A}B(\overline{D}+\overline{C}D)+B(A+\overline{A}CD)$ (to one literal)
  $=\overline{A}B\overline{D}+\overline{A}B\overline{C}D+AB+\overline{A}BCD$

Simplify the following equations:

- c) $\overline{A}B(\overline{D} + \overline{C}D) + B(A + \overline{A}CD)$ (to one literal)
  $= \overline{A}B\overline{D} + \overline{A}B\overline{C}D + AB + \overline{A}BCD$
  $= \overline{A}B(\overline{D} + \overline{C}D + CD) + AB$

Simplify the following equations:

- c) $\overline{A}B(\overline{D} + \overline{C}D) + B(A + \overline{A}CD)$ (to one literal)
  $= \overline{A}B\overline{D} + \overline{A}B\overline{C}D + AB + \overline{A}BCD$
  $= \overline{A}B(\overline{D} + \overline{C}D + CD) + AB$
  $= \overline{A}B(\overline{D} + D.(\overline{C} + C)) + AB$

Simplify the following equations:

- c) $\overline{A}B(\overline{D} + \overline{C}D) + B(A + \overline{A}CD)$ (to one literal)
  $= \overline{A}B\overline{D} + \overline{A}B\overline{C}D + AB + \overline{A}BCD$
  $= \overline{A}B(\overline{D} + \overline{C}D + CD) + AB$
  $= \overline{A}B(\overline{D} + D.(\overline{C} + C)) + AB$
  $= \overline{A}B(\overline{D} + D) + AB$

Simplify the following equations:

- c) $\overline{A}B(\overline{D}+\overline{C}D)+B(A+\overline{A}CD)$ (to one literal)
  $=\overline{A}B\overline{D}+\overline{A}B\overline{C}D+AB+\overline{A}BCD$
  $=\overline{A}B(\overline{D}+\overline{C}D+CD)+AB$
  $=\overline{A}B(\overline{D}+D.(\overline{C}+C))+AB$
  $=\overline{A}B(\overline{D}+D)+AB$
  $=\overline{A}B+AB$

Simplify the following equations:

- c) $\overline{A}B(\overline{D}+\overline{C}D)+B(A+\overline{A}CD)$ (to one literal)
  $=\overline{A}B\overline{D}+\overline{A}B\overline{C}D+AB+\overline{A}BCD$
  $=\overline{A}B(\overline{D}+\overline{C}D+CD)+AB$
  $=\overline{A}B(\overline{D}+D.(\overline{C}+C))+AB$
  $=\overline{A}B(\overline{D}+D)+AB$
  $=\overline{A}B+AB$
  $=B(\overline{A}+A)$

Simplify the following equations:

- c) $\overline{A}B(\overline{D}+\overline{C}D)+B(A+\overline{A}CD)$ (to one literal)
  $=\overline{A}B\overline{D}+\overline{A}B\overline{C}D+AB+\overline{A}BCD$
  $=\overline{A}B(\overline{D}+\overline{C}D+CD)+AB$
  $=\overline{A}B(\overline{D}+D.(\overline{C}+C))+AB$
  $=\overline{A}B(\overline{D}+D)+AB$
  $=\overline{A}B+AB$
  $=B(\overline{A}+A)$
  $=B$

Simplify the following equations:

- a) $\overline{A}\,\overline{C} + ABC + A\overline{C}$ (to three literals) $=$

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- a) $\overline{A}\,\overline{C} + ABC + A\overline{C}$ (to three literals) $=$
  $\overline{C}.(\overline{A} + A) + ABC =$

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- a) $\overline{A}\,\overline{C} + ABC + A\overline{C}$ (to three literals) $=$
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Simplify the following equations:

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  $\overline{C}.(\overline{A} + A) + ABC =$
  $\overline{C}.(1) + ABC =$
  $\overline{C} + ABC =$

Simplify the following equations:

- a) $\overline{A}\,\overline{C} + ABC + A\overline{C}$ (to three literals) $=$
  $\overline{C}.(\overline{A} + A) + ABC =$
  $\overline{C}.(1) + ABC =$
  $\overline{C} + ABC =$
  $\overline{C} + AB$

Simplify the following equations:

- e) $AB\overline{C}D + \overline{A}BD + ABCD$ (to two literals)

Simplify the following equations:

- e) $AB\overline{C}D + \overline{A}BD + ABCD$ (to two literals)
  $= BD(A\overline{C} + \overline{A} + AC)$

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- e) $AB\overline{C}D + \overline{A}BD + ABCD$ (to two literals)
  $= BD(A\overline{C} + \overline{A} + AC)$
  $= BD(A(\overline{C} + C) + \overline{A})$

Simplify the following equations:

- e) $AB\overline{C}D + \overline{A}BD + ABCD$ (to two literals)
  $= BD(A\overline{C} + \overline{A} + AC)$
  $= BD(A(\overline{C} + C) + \overline{A})$
  $= BD(A + \overline{A})$

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- e) $AB\overline{C}D + \overline{A}BD + ABCD$ (to two literals)
  $= BD(A\overline{C} + \overline{A} + AC)$
  $= BD(A(\overline{C} + C) + \overline{A})$
  $= BD(A + \overline{A})$
  $= BD$

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Draw logic diagrams of the circuits that implement the original and simplified expressions in Problem 2.4
c) $\overline{A}B(\overline{D} + \overline{C}D) + B(A + \overline{A}CD)$ and B

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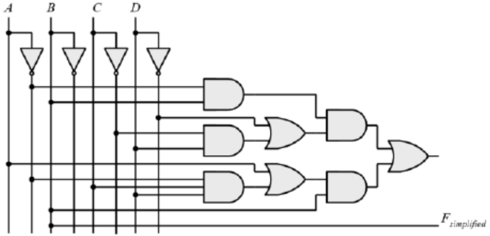

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Find the complement of the following expressions:

- a) $X\overline{Y} + \overline{X}Y$

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  Complement $= \overline{(X\overline{Y}) + (\overline{X}Y)}$
  $=\overline{(X\overline{Y})}\,.\,\overline{(\overline{X}Y)}$
  $=(\overline{X} + Y)\,.\,(X + \overline{Y})$

Find the complement of the following expressions:

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  Complement $= \overline{(X\overline{Y}) + (\overline{X}Y)}$
  $= \overline{(X\overline{Y})} \,.\, \overline{(\overline{X}Y)}$
  $= (\overline{X} + Y) \,.\, (X + \overline{Y})$
  $= (\overline{X}X + \overline{X}\,\overline{Y} + YX + Y\overline{Y})$

Find the complement of the following expressions:

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  Complement $= \overline{(X\overline{Y}) + (\overline{X}Y)}$
  $=\overline{(X\overline{Y})}\,.\,\overline{(\overline{X}Y)}$
  $=(\overline{X} + Y)\,.\,(X + \overline{Y})$
  $=(\overline{X}X + \overline{X}\,\overline{Y} + YX + Y\overline{Y})$
  $=(0 + \overline{X}\,\overline{Y} + XY + 0)$

Find the complement of the following expressions:

- a) $X\overline{Y} + \overline{X}Y$

  Complement $= \overline{(X\overline{Y}) + (\overline{X}Y)}$
  $= \overline{(X\overline{Y})} \, . \, \overline{(\overline{X}Y)}$
  $= (\overline{X} + Y) \, . \, (X + \overline{Y})$
  $= (\overline{X}X + \overline{X}\,\overline{Y} + YX + Y\overline{Y})$
  $= (0 + \overline{X}\,\overline{Y} + XY + 0)$
  $= (\overline{X}\,\overline{Y} + XY)$

Find the complement of the following expressions:

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- b) $(a+c)(a+\overline{b})(\overline{a}+b+\overline{c})$
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  $= \overline{((a+c).(a+\overline{b}))}\,.\,\overline{(\overline{a}+b+\overline{c})}$

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  Complement $= \overline{(a+c)(a+\overline{b})(\overline{a}+b+\overline{c})}$

  $= \overline{(\,(a+c).(a+\overline{b})).(\overline{a}+b+\overline{c})}$

  $= \overline{(\,(a+c).(a+\overline{b}))} + \overline{(\overline{a}+b+\overline{c})}$

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- b) $(a+c)(a+\overline{b})(\overline{a}+b+\overline{c})$

  Complement $=\overline{(a+c)(a+\overline{b})(\overline{a}+b+\overline{c})}$

  $=\overline{((a+c).(a+\overline{b})).(\overline{a}+b+\overline{c})}$

  $=\overline{((a+c).(a+\overline{b}))}+\overline{(\overline{a}+b+\overline{c})}$

  $=\overline{(a+c)}+\overline{(a+\overline{b})}+\overline{(\overline{a}+b+\overline{c})}$

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  $= \overline{(\,(a+c).(a+\overline{b}))\,.\,(\overline{a}+b+\overline{c})}$

  $= \overline{(\,(a+c).(a+\overline{b}))} + \overline{(\overline{a}+b+\overline{c})}$

  $= \overline{(a+c)} + \overline{(a+\overline{b})} + \overline{(\overline{a}+b+\overline{c})}$

  $= \overline{a}\,\overline{c} + \overline{a}b + a\overline{b}c$

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15* Simplify the following Boolean functions T_1 and T_2 to a minimum number of literals:

A	*B*	*C*	T_1	T_2
0	0	0	1	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
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1	0	1	0	1
1	1	0	0	1
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T1 = ?

15* Simplify the following Boolean functions T_1 and T_2 to a minimum number of literals:

A	B	C	T_1	T_2
0	0	0	1	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
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0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	0	1

$$\mathrm{T1} = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}\,B\overline{C}$$

$$\mathrm{T1} = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}\,B\overline{C}$$

$$\mathrm{T1} = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}\,B\overline{C}$$
$$= \overline{A}\,\overline{C}(\overline{B} + B) + \overline{A}\,\overline{B}C$$

$$
\begin{aligned}
\mathrm{T1} &= \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}\,B\overline{C} \\
&= \overline{A}\,\overline{C}(\overline{B} + B) + \overline{A}\,\overline{B}C \\
&= \overline{A}\,\overline{C} + \overline{A}\,\overline{B}C
\end{aligned}
$$

$$\begin{aligned}
\mathrm{T1} &= \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}\,B\overline{C} \\
&= \overline{A}\,\overline{C}(\overline{B} + B) + \overline{A}\,\overline{B}C \\
&= \overline{A}\,\overline{C} + \overline{A}\,\overline{B}C \\
&= \overline{A}(\overline{C} + \overline{B}C)
\end{aligned}$$

$$\mathrm{T1} = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}\,B\overline{C}$$
$$= \overline{A}\,\overline{C}(\overline{B} + B) + \overline{A}\,\overline{B}C$$
$$= \overline{A}\,\overline{C} + \overline{A}\,\overline{B}C$$
$$= \overline{A}(\overline{C} + \overline{B}C)$$
$$= \overline{A}(\overline{C} + \overline{B})$$

$$
\begin{aligned}
\mathrm{T1} &= \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}\,B\overline{C} \\
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&= \overline{A}\,\overline{C} + \overline{A}\,\overline{B}C \\
&= \overline{A}(\overline{C} + \overline{B}C) \\
&= \overline{A}(\overline{C} + \overline{B}) \\
&= \overline{A}(\overline{B} + \overline{C})
\end{aligned}
$$

15* Simplify the following Boolean functions T_1 and T_2 to a minimum number of literals:

A	B	C	T_1	T_2
0	0	0	1	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	0	1

T2 = ?

$$\mathrm{T2} = \overline{T1} = \overline{\overline{A}(\overline{B} + \overline{C})}$$

$$\text{T2} = \overline{T1} = \overline{\overline{A}(\overline{B} + \overline{C})} = A + BC$$

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Q2) Write an algorithm for converting from binary to gray code. You can draw a flow chart.

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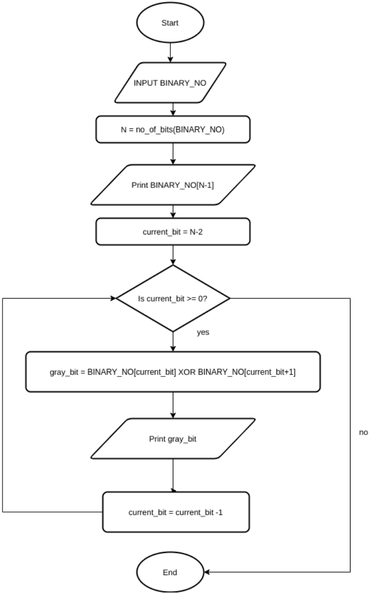
Index	3	2	1	0
Binary	1	0	1	1
Gray	1	1	1	0

Q2) Write an algorithm for converting from binary to gray code. You can draw a flow chart.

Index	3	2	1	0
Binary	1	0	1	1
Gray	1	1	1	0

Gray[3] = Binary[3]
Gray[2] = Binary[2] XOR Binary[3]
Gray[1] = Binary[1] XOR Binary[2]
Gray[0] = Binary[0] XOR Binary[1]


Start
INPUT BINARY_NO
N = no_of_bits(BINARY_NO)
Print BINARY_NO[N-1]
current_bit = N-2
Is current_bit >= 0?
yes
gray_bit = BINARY_NO[current_bit] XOR BINARY_NO[current_bit+1]
Print gray_bit
current_bit = current_bit -1
no
End