Logic Design

Tutorial 1

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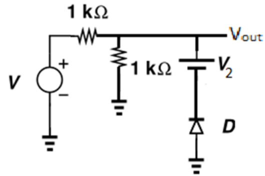
Outline

Outline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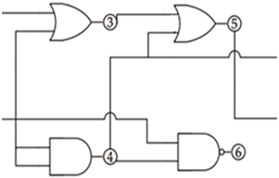
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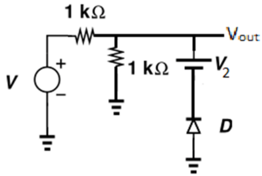

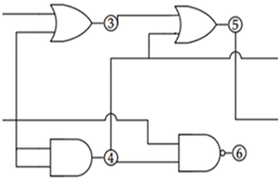

Computers are large digital circuits.

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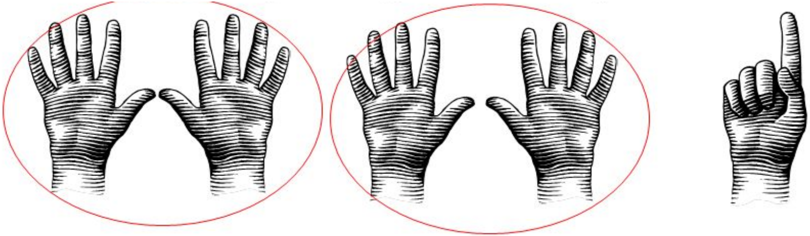
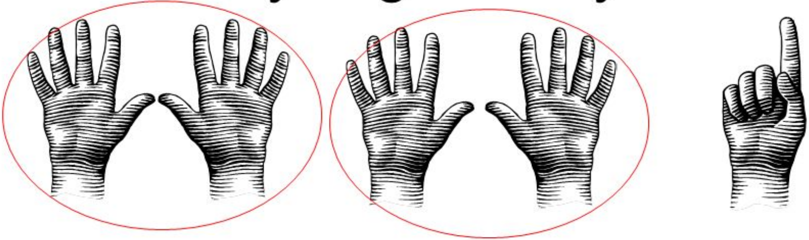
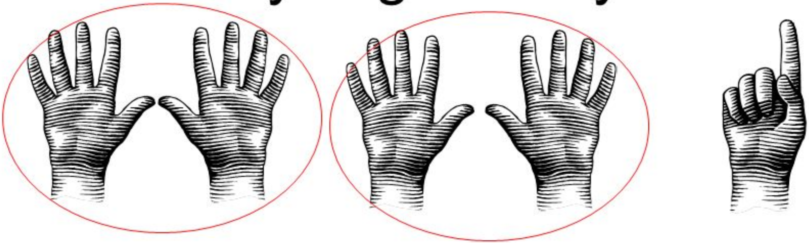


How many fingers do you see?



How many fingers do you see?



- Did you have to count them all? (I hope not)
- How did you know there were 21?
- We count how many groups of 10, and then add the single fingers. 2 groups of 10 and 1 extra finger.

How to define a numbering system?

- Specify the no. of symbols for a single digit (We call it the **Base**).
- Select unique symbols

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Example: The decimal system has 10 symbols for each digit (0-9).

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- No of symbol per digit **Base** (2): 2
- The symbols are: 0, 1

Example of binary numbers are 10, 10010, ...

- No of symbol per digit **Base** (8): 8
- The symbols are: 0, 1, 2, 3, 4, 5, 6, 7

Example of octal numbers are 253, 7716, ...

- No of symbol per digit **Base** (16): 16
- The symbols are: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, ??????

- No of symbol per digit **Base** (16): 16
- The symbols are: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, ??????
- The symbols are: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Example of hexadecimal numbers are 19, 106, ABC, D08, FF, ...

Outline

Q1) List the octal and hexadecimal numbers from 16 to 32.

Decimal(10)	Octal(8)	Hexadecimal(16)
0	0	0
1	1	1
2	2	2
3	3	3
4	4	4
5	5	5
6	6	6
7	7	7

Decimal(10)	Octal(8)	Hexadecimal(16)
7	7	7
8	??	8
9	??	9
10	??	??

Decimal(10)	Octal(8)	Hexadecimal(16)
7	7	7
8	??	8
9	??	9
10	??	??

Decimal(10)	Octal(8)	Hexadecimal(16)
7	7	7
8	10	8
9	11	9
10	12	A

Decimal(10)	Octal(8)	Hexadecimal(16)
11	13	B
12	14	C
13	15	D
14	16	E
15	17	F
16	20	10
17	21	11
18	22	12
19	23	13

What is the value of the following operations: $17 + 2$ in Base (8) AND $FE + 2$ in Base (16)
Hint: For these examples, you can do this operations as two successive $+1$ operations (i.e: $17 + 2 = (17 + 1) + 1$).

Outline

Apples	Binary	Decimal
	0	0
	1	1
	10	2
	11	3

Apples	Binary	Decimal
	0	0
	1	1
	10	2
	11	3

Therefore:

- 10_2 is equivalent to 2_{10}
- 11_2 is equivalent to 3_{10}

- We know that the value 278_{10} actually means:
  $2 * 10^2 + 7 * 10^1 + 8 * 10^0$

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- Similarly the value 1011_2 actually means:
  $1 * 2^3 + 0 * 2^2 + 1 * 2^1 + 1 * 2^0$

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Convert the following numbers with the indicated bases to decimal:

- f) $1011001_2 =$

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- a) $4310_5 =$

Convert the following numbers with the indicated bases to decimal:

- f) $1011001_2 = 1 * 2^6 + 1 * 2^4 + 1 * 2^3 + 1 * 2^0 = 89_{10}$
- a) $4310_5 = 4 * 5^3 + 3 * 5^2 + 1 * 5^1 =$

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- f) $1011001_2 = 1 * 2^6 + 1 * 2^4 + 1 * 2^3 + 1 * 2^0 = 89_{10}$
- a) $4310_5 = 4 * 5^3 + 3 * 5^2 + 1 * 5^1 = 580_{10}$
- e) $AC5_{16} =$

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- e) $AC5_{16} = 10 * 16^2 + 12 * 16^1 + 5 * 16^0 =$

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- e) $AC5_{16} = 10 * 16^2 + 12 * 16^1 + 5 * 16^0 = 2757_{10}$

Outline

Q6) Determine the base of the numbers in each case for the following operations to be correct:

- a) $14_{??}/2_{??} = 5_{??}$

Q6) Determine the base of the numbers in each case for the following operations to be correct:

- a) $14_{??}/2_{??} = 5_{??}$ (We know that the Base is >= 6 (why?))
  Answer: Let the unknown base to be "B".
  $14_B = (1 * B^1 + 4 * B^0)_{10} = (B+4)_{10}$
  $2_B = 2_{10}$
  $5_B = 5_{10}$
  Then we have the decimal equation: $(B+4)/2 = 5$
  $(B+4) = 10$
  $B = 6$

Outline

Q7) The solutions of the quadratic equation $x^2 - 11 * x + 22 = 0$ are $x = 3$ and $x = 6$. What is the base of the numbers?
Note: 11 means 11_B and not 11_{10}.

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(We know that the Base "B" is >= 7).
$11_B = (B+1)_{10}$
$22_B = (2B+2)_{10}$
$5_B = 5_{10}$
$6_B = 6_{10}$

Q7) The solutions of the quadratic equation x^2 - 11 * x + 22 = 0 are x = 3 and x = 6. What is the base of the numbers?
Note: 11 means 11_B and not 11_{10}.
(We know that the Base "B" is >= 7).
$11_B = (B+1)_{10}$
$22_B = (2B+2)_{10}$
$5_B = 5_{10}$
$6_B = 6_{10}$

Decimal Equation: x^2 - (B+1) * x + (2B + 2) = 0
for $x = 3_{10}$ and $x = 6_{10}$

1 Equation in 1 unknown, Substitute by any of the roots in the equation:

$3^2 - (B+1) * 3 + (2B + 2) = 0$
B = 8

Outline

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Q9) Convert the decimal number 431 to binary.

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Let the equivalent binary number is $B_9B_8B_7B_6B_5B_4B_3B_2B_1B_0$
Therefore $431_{10} =$
$B_9 * 2^9 + B_8 * 2^8 + B_7 * 2^7 + B_6 * 2^6 + B_5 * 2^5 + B_4 * 2^4 + B_3 * 2^3 + B_2 * 2^2 + B_1 * 2^1 + B_0$

What can we know given that 431_{10} is an odd number?

$B_0 = 1$

431_{10} - $B_0 =$

$B_9 * 2^{9} + B_8 * 2^{8} + B_7 * 2^{7} + B_6 * 2^{6} + B_5 * 2^{5} + B_4 * 2^{4} + B_3 * 2^{3} + B_2 * 2^{2} + B_1 * 2^{1}$

$430_{10} =$

$B_9 * 2^{9} + B_8 * 2^{8} + B_7 * 2^{7} + B_6 * 2^{6} + B_5 * 2^{5} + B_4 * 2^{4} + B_3 * 2^{3} + B_2 * 2^{2} + B_1 * 2^{1}$

Divide both sides by 2:

$B_0 = 1$
431_{10} - $B_0 =$
$B_9 * 2^{9} + B_8 * 2^{8} + B_7 * 2^{7} + B_6 * 2^{6} + B_5 * 2^{5} + B_4 * 2^{4} + B_3 * 2^{3} + B_2 * 2^{2} + B_1 * 2^{1}$

$430_{10} =$
$B_9 * 2^{9} + B_8 * 2^{8} + B_7 * 2^{7} + B_6 * 2^{6} + B_5 * 2^{5} + B_4 * 2^{4} + B_3 * 2^{3} + B_2 * 2^{2} + B_1 * 2^{1}$

Divide both sides by 2:
$215_{10} =$
$B_9 * 2^{8} + B_8 * 2^{7} + B_7 * 2^{5} + B_6 * 2^{5} + B_5 * 2^{3} + B_4 * 2^{4} + B_3 * 2^{2} + B_2 * 2^{1} + B_1 * 2^{0}$

What can we know given that 215_{10} is an odd number?

Value	Value/ base	Coefficient = (decimal value * base)
431	215.5	$(0.5 * 2) = 1$

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431	215.5	$(0.5 * 2) = 1$
215	107.5	$(0.5 * 2) = 1$

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215	107.5	$(0.5 * 2) = 1$
107	53.5	$(0.5 * 2) = 1$

Value	Value/ base	Coefficient = (decimal value * base)
431	215.5	$(0.5 * 2) = 1$
215	107.5	$(0.5 * 2) = 1$
107	53.5	$(0.5 * 2) = 1$
53	26.5	$(0.5 * 2) = 1$

Value	Value/ base	Coefficient = (decimal value * base)
431	215.5	$(0.5 * 2) = 1$
215	107.5	$(0.5 * 2) = 1$
107	53.5	$(0.5 * 2) = 1$
53	26.5	$(0.5 * 2) = 1$
26	13	$(0 * 2) = 0$

Value	Value/ base	Coefficient = (decimal value * base)
431	215.5	(0.5 * 2) = 1
215	107.5	(0.5 * 2) = 1
107	53.5	(0.5 * 2) = 1
53	26.5	(0.5 * 2) = 1
26	13	(0 * 2) = 0
13	6.5	(0.5 * 2) = 1

Value	Value/ base	Coefficient = (decimal value * base)
431	215.5	(0.5 * 2) = 1
215	107.5	(0.5 * 2) = 1
107	53.5	(0.5 * 2) = 1
53	26.5	(0.5 * 2) = 1
26	13	(0 * 2) = 0
13	6.5	(0.5 * 2) = 1
6	3	(0 * 2) = 0

Value	Value/ base	Coefficient = (decimal value * base)
431	215.5	$(0.5 * 2) = 1$
215	107.5	$(0.5 * 2) = 1$
107	53.5	$(0.5 * 2) = 1$
53	26.5	$(0.5 * 2) = 1$
26	13	$(0 * 2) = 0$
13	6.5	$(0.5 * 2) = 1$
6	3	$(0 * 2) = 0$
3	1.5	$(0.5 * 2) = 1$

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431	215.5	$(0.5 * 2) = 1$
215	107.5	$(0.5 * 2) = 1$
107	53.5	$(0.5 * 2) = 1$
53	26.5	$(0.5 * 2) = 1$
26	13	$(0 * 2) = 0$
13	6.5	$(0.5 * 2) = 1$
6	3	$(0 * 2) = 0$
3	1.5	$(0.5 * 2) = 1$
1	0.5	$(0.5 * 2) = 1$

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53	26.5	$(0.5 * 2) = 1$
26	13	$(0 * 2) = 0$
13	6.5	$(0.5 * 2) = 1$
6	3	$(0 * 2) = 0$
3	1.5	$(0.5 * 2) = 1$
1	0.5	$(0.5 * 2) = 1$
0	X	X

The binary number is (bottom to top): 110101111_2

Value	Value/ base	Coefficient = (decimal value * base)
431	26.9375	(0.9375 * 16) = 15 "F"

Value	Value/ base	Coefficient = (decimal value * base)
431	26.9375	(0.9375 * 16) = 15 "F"
26	1.625	(0.625 * 16) = 10 "A"

Value	Value/ base	Coefficient = (decimal value * base)
431	26.9375	(0.9375 * 16) = 15 "F"
26	1.625	(0.625 * 16) = 10 "A"
1	0.0625	(0.0625 * 16) = 1

Value	Value/ base	Coefficient = (decimal value * base)
431	26.9375	(0.9375 * 16) = 15 "F"
26	1.625	(0.625 * 16) = 10 "A"
1	0.0625	(0.0625 * 16) = 1
0	X	X

The hexadecimal number is (bottom to top): $1AF_{16}$
Each hexadecimal digit represents 4 binary digits (Try listing the binary and hexadecimal values in range 0-48 and check the values):

1AF is equivalent to (????) (????) (????)

Value	Value/ base	Coefficient = (decimal value * base)
431	26.9375	(0.9375 * 16) = 15 "F"
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0	X	X

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Each hexadecimal digit represents 4 binary digits (Try listing the binary and hexadecimal values in range 0-48 and check the values):

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1AF is equivalent to (0001) (1010) (1111)

Outline

Q8) Convert the hexadecimal number 64CD to binary, and then convert it from binary to octal.

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Similary each 3 binary digit represent an octal digit:
$(0)(110)(010)(011)(001)(101) = (000)(110)(010)(011)(001)(101)$

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Similary each 3 binary digit represent an octal digit:
$(0)(110)(010)(011)(001)(101) = (000)(110)(010)(011)(001)(101)$

$(000)(110)(010)(011)(001)(101) = (062315)_8$

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- We know that the value 278_{10} actually means:
  $2 * 10^2 + 7 * 10^1 + 8 * 10^0$

- We know that the value 278_{10} actually means:
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- Similarly the value 1011_2 actually means:
  $1 * 2^3 + 0 * 2^2 + 1 * 2^1 + 1 * 2^0$

Table: How is 278_{10} represented in decimal?

Coefficients	2	7	8
Weights	10^2	10^1	10^0
Values	$2 * 10^2$	$7 * 10^1$	$8 * 10^0$

Table: How is 278_{10} represented in decimal?

Coefficients	2	7	8
Weights	10^2	10^1	10^0
Values	$2 * 10^2$	$7 * 10^1$	$8 * 10^0$

Table: How is 278.56_{10} represented in decimal?

Coefficients	2	7	8	5	6
Weights	10^2	10^1	10^0	10^{-1}	10^{-2}
Values	$2 * 10^2$	$7 * 10^1$	$8 * 10^0$	$5 * 10^{-1}$	$6 * 10^{-2}$

Similarly for all the other number systems:

Table: How is 132_4 represented in Base 4?

Coefficients	1	3	2
Weights	4^2	4^1	4^0
Values	$1 * 4^2$	$3 * 4^1$	$2 * 4^0$

Similarly for all the other number systems:

Table: How is 132_4 represented in Base 4?

Coefficients	1	3	2
Weights	4^2	4^1	4^0
Values	$1 * 4^2$	$3 * 4^1$	$2 * 4^0$

Table: How is 132.02_4 represented in Base 4?

Coefficients	1	3	2	0	2
Weights	4^2	4^1	4^0	4^{-1}	4^{-2}
Values	$1 * 4^2$	$3 * 4^1$	$2 * 4^0$	$0 * 4^{-1}$	$2 * 4^{-2}$

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Q10) Express the following numbers in decimal:

- a) $(10110.0101)_2$

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Coefficients	1	0	1	1	0.	0	1	0	1
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Q10) Express the following numbers in decimal:

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Weights	2^{4}	2^{3}	2^{2}	2^{1}	2^{0}	2^{-1}	2^{-2}	2^{-3}	2^{-4}

$$= 2^{4} + 2^{2} + 2^{1} + 2^{-2} + 2^{-4} = 22.3125_{10}$$

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$= 2^{4} + 2^{2} + 2^{1} + 2^{-2} + 2^{-4} = 22.3125_{10}$

- d) $(DADA.B)_{16}$

Q10) Express the following numbers in decimal:

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Weights	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}

$= 2^4 + 2^2 + 2^1 + 2^{-2} + 2^{-4} = 22.3125_{10}$

- d) $(DADA.B)_{16}$
  Remember: A_{16} is 10_{10}, B_{16} is 11_{10}, D_{16} is 13_{10}

Q10) Express the following numbers in decimal:

- a) $(10110.0101)_2$

Coefficients	1	0	1	1	0.	0	1	0	1
Weights	2^{4}	2^{3}	2^{2}	2^{1}	2^{0}	2^{-1}	2^{-2}	2^{-3}	2^{-4}

$= 2^{4} + 2^{2} + 2^{1} + 2^{-2} + 2^{-4} = 22.3125_{10}$

- d) $(DADA.B)_{16}$
  Remember: A_{16} is 10_{10}, B_{16} is 11_{10}, D_{16} is 13_{10}
  $= 13 * 16^3 + 10 * 16^2 + 13 * 16^1 + 10 * 16^0 + 11 * 16^{-1} =$
  56026.6875_{10}

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Q11) Convert the following binary numbers to hexadecimal and to decimal: (a) 1.10010, (b) 110.010

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Remember that we can convert a binary number to hexadecimal by grouping each four consecutive binary bits.
(e.g.: 11001001_2 is $(1100)(1001)_2$ which is $\mathbf{C9}_{16}$)

Q11) Convert the following binary numbers to hexadecimal and to decimal: (a) 1.10010, (b) 110.010

Remember that we can convert a binary number to hexadecimal by grouping each four consecutive binary bits.
(e.g.: 11001001_2 is $(1100)(1001)_2$ which is $\mathbf{C9}_{16}$)

Similarly for floating point values
1100.1001_2 is $(1100).(1001)_2$ which is $\mathbf{C.9_{16}}$

(a) 1.10010

(a) 1.10010

$$\mathbf{000}1.10010\mathbf{000} = (\mathbf{000}1).(1001)(0\mathbf{000})_2 = 1.90_{16} = 1.9_{16}$$

$$1.9_{16} = 1 + 9 * 16^{-1} = 1.5625_{10}$$

(a) 1.10010

$\mathbf{000}1.1001\mathbf{000} = (\mathbf{000}1).(1001)(0\mathbf{000})_2 = 1.90_{16} = 1.9_{16}$

$1.9_{16} = 1 + 9 * 16^{-1} = 1.5625_{10}$

(b) 110.010

$\mathbf{0}110.010\mathbf{0} = (\mathbf{0}110).(010\mathbf{0})_2 = 6.4_{16}$

$6.4_{16} = 6 + 4 * 16^{-1} = 6.25_{10}$

Explain why the decimal answer in (b)$110.010_2 = 6.25_{10}$ is 4 times that in (a)$1.10010_2 = 1.5625_{10}$

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Let's think of this case first:

$11_{10} =$

Explain why the decimal answer in (b)$110.010_2 = 6.25_{10}$ is 4 times that in (a)$1.10010_2 = 1.5625_{10}$

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$11_{10} = 1011_2$

Explain why the decimal answer in (b)$110.010_2 = 6.25_{10}$ is 4 times that in (a)$1.10010_2 = 1.5625_{10}$

Let's think of this case first:
$11_{10} = 1011_2$ AND $22_{10} =$

Explain why the decimal answer in (b)$110.010_2 = 6.25_{10}$ is 4 times that in (a)$1.10010_2 = 1.5625_{10}$

Let's think of this case first:
$11_{10} = 1011_2$ AND $22_{10} = 10110_2$
Hmmm, why is this happening??

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Let's think of this case first:
$11_{10} = 1011_2$ AND $22_{10} = 10110_2$
Hmmm, why is this happening??

Table: Binary representation of 1011_2

Coefficients	0	1	0	1	1
Weights	2^4	2^3	2^2	2^1	2^0

Explain why the decimal answer in (b)$110.010_2 = 6.25_{10}$ is 4 times that in (a)$1.10010_2 = 1.5625_{10}$

Let's think of this case first:
$11_{10} = 1011_2$ AND $22_{10} = 10110_2$
Hmmm, why is this happening??

Table: Binary representation of 1011_2

Coefficients	0	1	0	1	1
Weights	2^4	2^3	2^2	2^1	2^0

Table: Binary representation of $1011_2 * 2_{10}$

Coefficients	0	1	0	1	1	??
Weights	2^5	2^4	2^3	2^2	2^1	2^0