Logic Design

Tutorial 3

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Outline

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Assume we have 4 bits to represent numbers on our system.

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Table 1.3
Signed Binary Numbers

Decimal	Signed Magnitude
+7	0111
+6	0110
+5	0101
+4	0100
+3	0011
+2	0010
+1	0001
+0	0000
−0	1000
−1	1001
−2	1010
−3	1011
−4	1100
−5	1101
−6	1110
−7	1111
−8	—

Table 1.3
Signed Binary Numbers

Decimal	Signed-2's Complement	Signed Magnitude
+7	0111	0111
+6	0110	0110
+5	0101	0101
+4	0100	0100
+3	0011	0011
+2	0010	0010
+1	0001	0001
+0	0000	0000
−0	—	1000
−1	1111	1001
−2	1110	1010
−3	1101	1011
−4	1100	1100
−5	1011	1101
−6	1010	1110
−7	1001	1111
−8	1000	—

How to add two numbers in signed two's complement representation?

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- The result is positive since the sign bit is equal to zero
- The value is $8 + 4 + 2 = 14$ (Makes sense now)

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- The result is negative since the sign bit is equal to one
- Its two's complement is 01110
- The value is - (8 + 4 + 2) = -14 (Makes sense now)

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Q17) Convert decimal +49 and +29 to binary, using the signed 2's complement representation and enough digits to accommodate the numbers. Then perform the binary equivalent of (+29) + (-49), (-29) + (+49), and (-29) + (-49). Convert the answers back to decimal and verify that they are correct.

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- c) 2421 code[1]: Instead of considering the weights of the 4 bits to be 8, 4, 2, 1 make them 2, 4, 2, 1
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- $(398)_{10} = (0011, 1001, 1000)_{BCD}$
- $(198)_{10} = (0001, 1001, 1000)_{BCD}$

$$\begin{array}{rrr} 0011 & 1001 & 1000 \\ & & + \\ 0001 & 1001 & 1000 \\ \hline \end{array}$$

$$
\begin{array}{r r r}
0011 & 1001 & 1000 \\
 & & + \\
0001 & 1001 & 1000 \\
\hline
 & & 10000
\end{array}
$$

(result is more than 9, Add 6 to the value)

$$
\begin{array}{rrr}
0011 & 1001 & 1000 \\
 & & + \\
0001 & 1001 & 1000 \\
\hline
 & & 10000
\end{array}
$$

(result is more than 9, Add 6 to the value)

$$
\begin{array}{r}
+ \\
0110 \\
\hline
\end{array}
$$

0011	1001	1000
		+
0001	1001	1000
		10000

(result is more than 9, Add 6 to the value)

	+
	0110
1	0110

$$
\begin{array}{rrr}
 & 1 & \\
0011 & 1001 & 1000 \\
 & & + \\
0001 & 1001 & 1000 \\
\hline
\end{array}
$$

$$
\begin{array}{rrr}
 & 1 & \\
0011 & 1001 & 1000 \\
 & & + \\
0001 & 1001 & 1000 \\
\hline
 & 10011 & 0110
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 & & + \\
 & 0110 & \\
\hline
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	1	
0011	1001	1000
		+
0001	1001	1000
	10011	0110

(result is more than 9, Add 6 to the value)

		+
	0110	
1	1001	0110

$$
\begin{array}{rrr}
1 & & \\
0011 & 1001 & 1000 \\
 & & + \\
0001 & 1001 & 1000 \\
\hline
\end{array}
$$

1		
0011	1001	1000
		+
0001	1001	1000
0101	1001	0110

(result is less than 9, Do thing)

1		
0011	1001	1000
		+
0001	1001	1000
0101	1001	0110

(result is less than 9, Do thing)

0101	1001	0110

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- Note: Convert the values to decimal and check that the result is correct.

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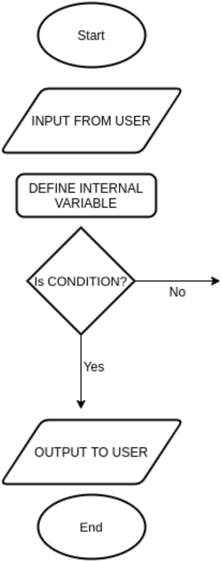
- Note: Convert the values to decimal and check that the result is correct.

Thus, binary multiplication depends on two operations:
Shift-left (multiplication by 2) and Binary-addition.

Outline

The main blocks are:



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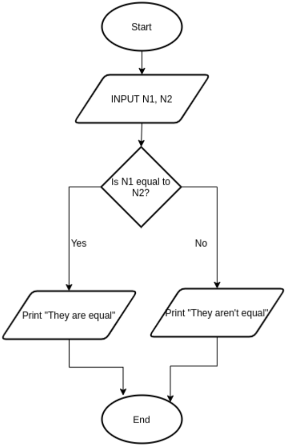

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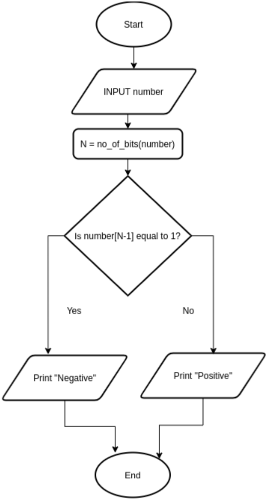

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Note: For binary non-floating numbers:

- A shift left operation adds a zero to the right of the number (e.g: 101 becomes 1010).
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Note: Assume that we want to compute (A * B) where A, B are two binary numbers of the equal length.

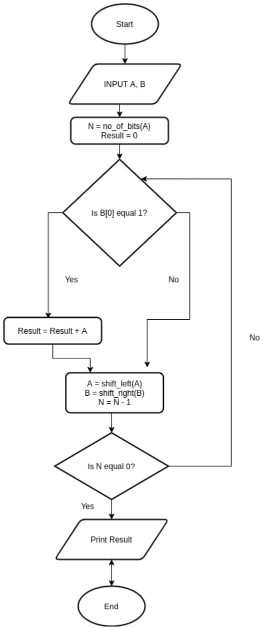
Start
INPUT A, B
N = no_of_bits(A)
Result = 0
Is B[0] equal 1?
Yes
No
Result = Result + A
No
A = shift_left(A)
B = shift_right(B)
N = N - 1
Is N equal 0?
Yes
Print Result
End

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Table 1.6
Gray Code

Gray Code	Decimal Equivalent
0000	0
0001	1
0011	2
0010	3
0110	4
0111	5
0101	6
0100	7
1100	8
1101	9
1111	10
1110	11
1010	12
1011	13
1001	14
1000	15

https://en.wikipedia.org/wiki/Gray_code