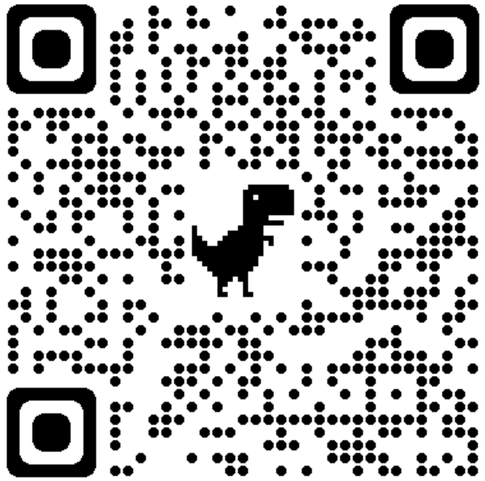
Eikonal Solver Implementation

Advanced Methods for Scientific Computing (AMSC) Hands-on Project



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Description

Eikonal Equation

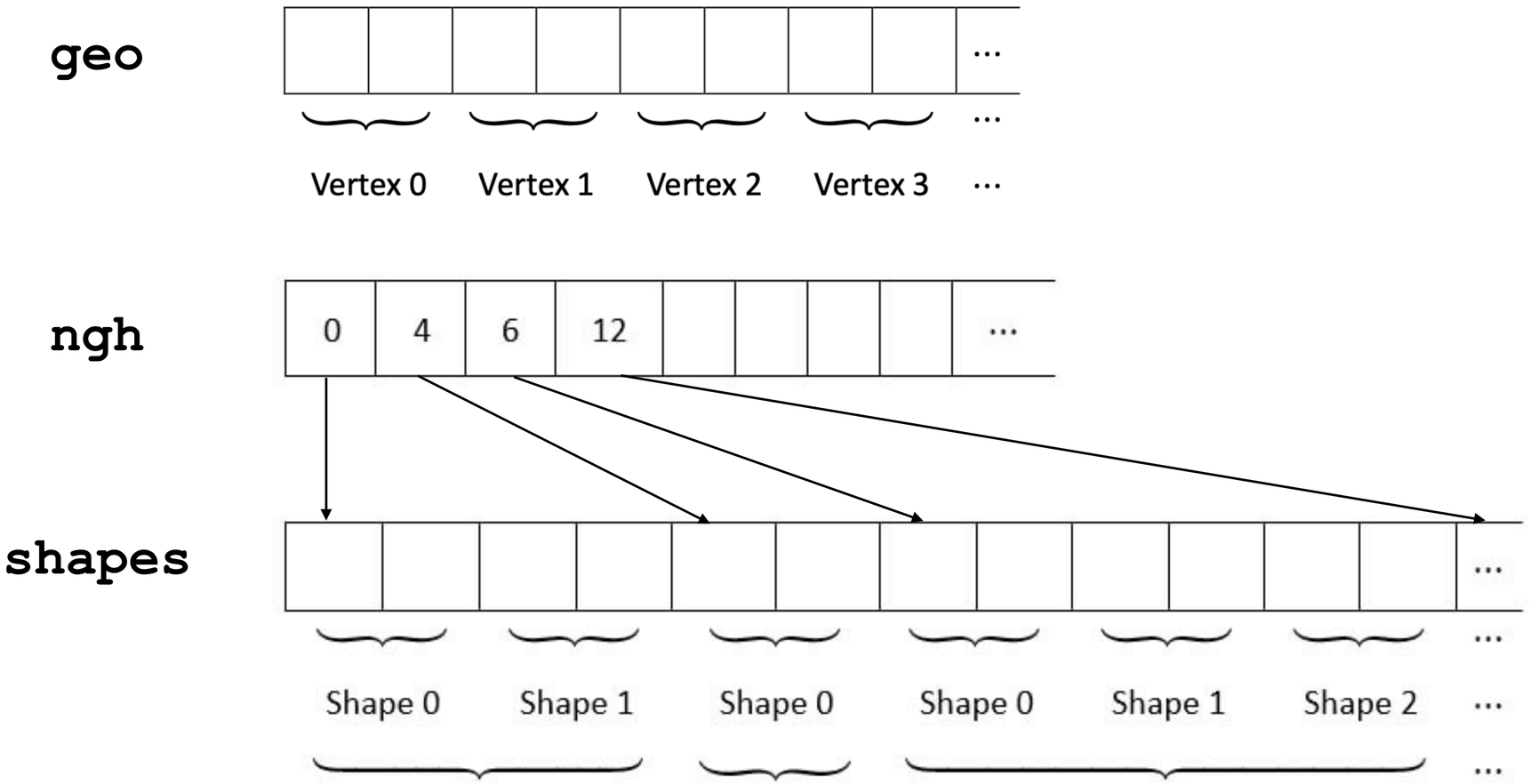
$$\begin{cases} H(x, \nabla u(x)) = 1 & x \in \Omega \subset \mathbb{R}^d \\ u(x) = g(x) & x \in \Gamma \subset \partial\Omega \end{cases}$$

where

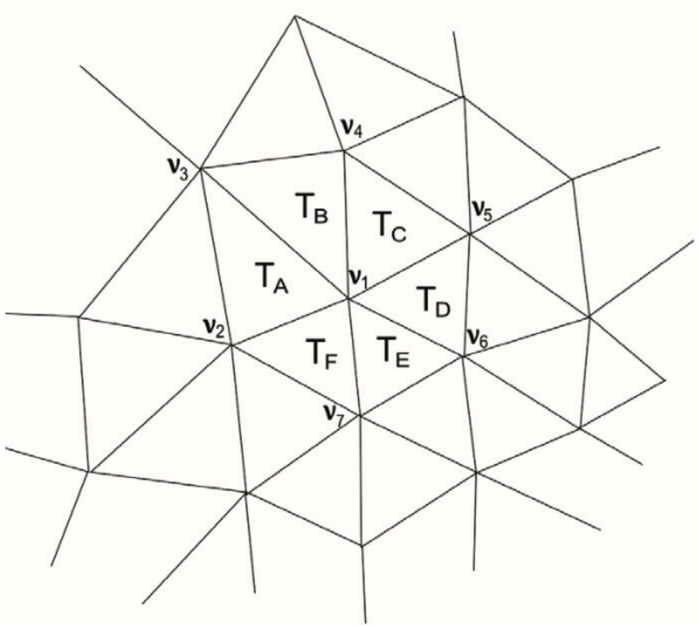
- d is the dimension of the problem, either 2 or 3;
- u is the eikonal function, representing the travel time of a wave;
- $\nabla u(x)$ is the gradient of u, a vector that points in the direction of the wavefront;
- H is the Hamiltonian, which is a function of the spatial coordinates x and the gradient ∇u;
- Γ is a set smooth boundary conditions.

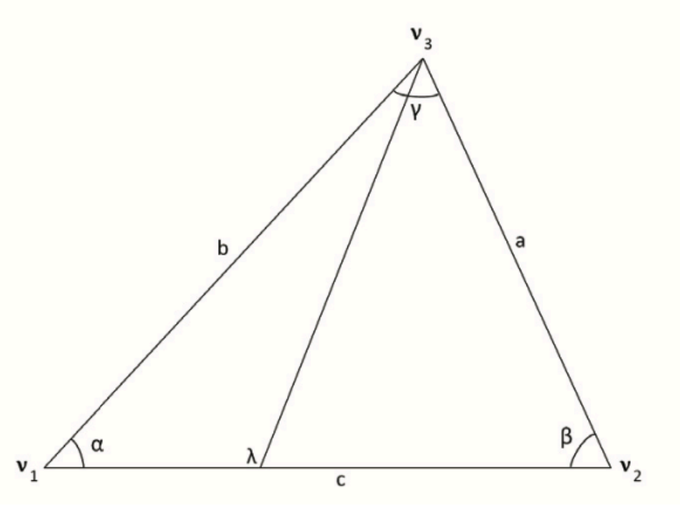
Mesh

Data Structures



Local Solver





For each vertex, it is essential to solve a local problem, specifically determining the updated value u, by considering the upwind neighbors.

Serial Implementation

Algorithm 2.1. MESHFIM(V, B, L)

comment: 1. Initialization (V : all vertices, L : active list, B: seed vertices)

for each $v \in V$
do **if** $v \in B$
then $\Phi_v \leftarrow 0$
else $\Phi_v \leftarrow \infty$

for each $v \in V$
do **if** any 1-ring vertex of $v \in B$
then add v to L

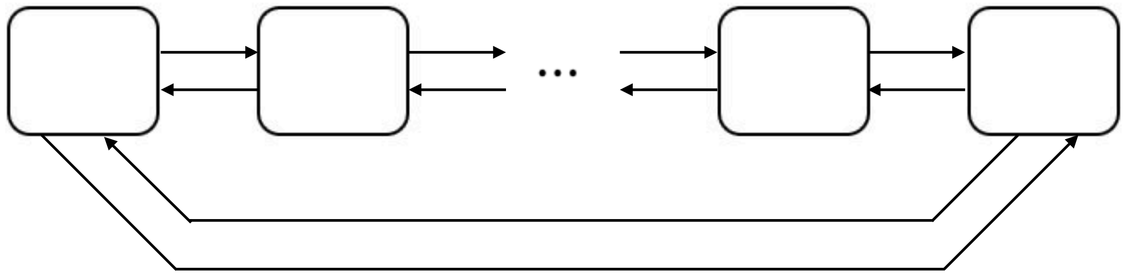
comment: 2. Update vertices in L

while L is not empty
do **for each** $v \in V$
do $p \leftarrow \Phi_v$
$q \leftarrow \text{Update}(v)$
if $|p - q| < \varepsilon$
then **for each** adjacent neighbor v_{nb} of v
do **if** v_{nb} is not in L
then $p \leftarrow \Phi_{v_{nb}}$
$q \leftarrow \text{Update}(v_{nb})$
if $p > q$
then $\Phi_{v_{nb}} \leftarrow q$
add v_{nb} to L
remove v from L

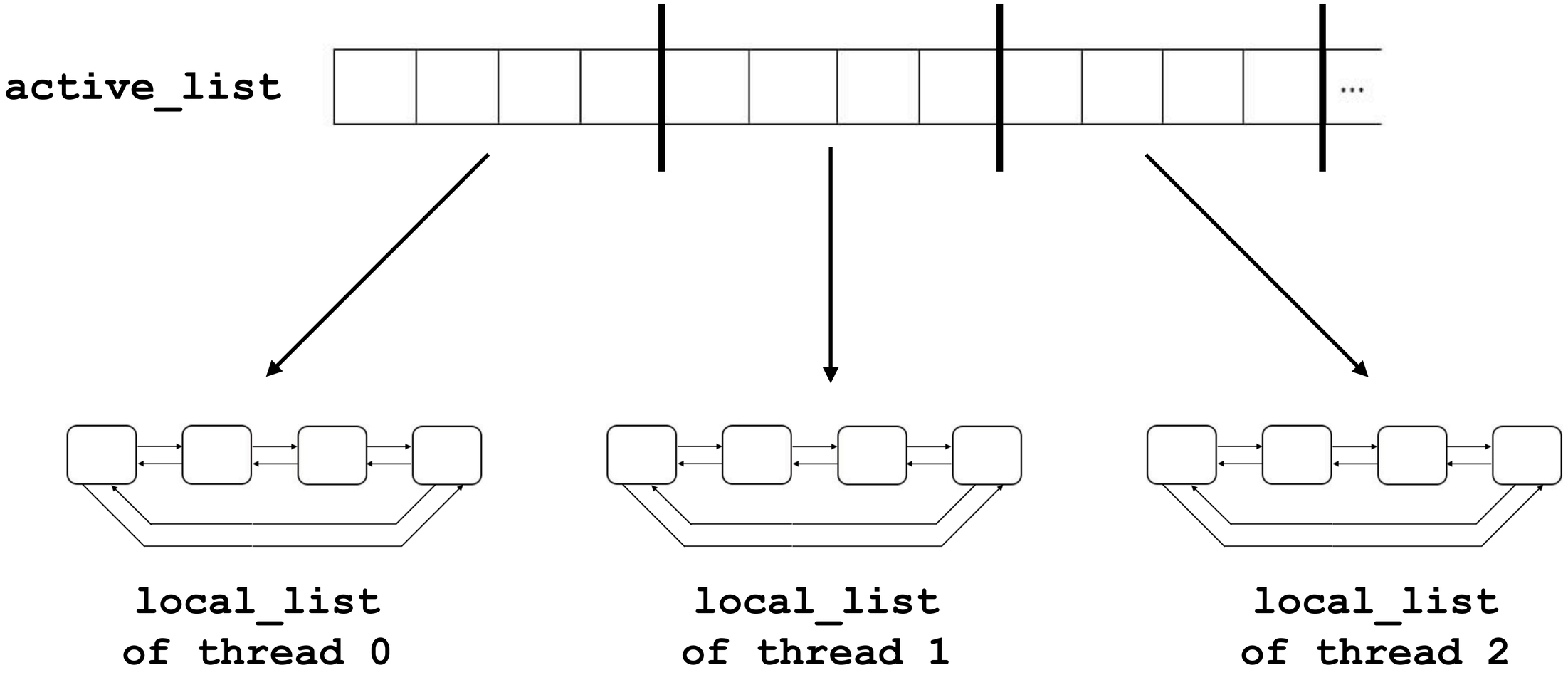
Data Structures

`boundary_vertices`

`active_list`



Parallel Implementation

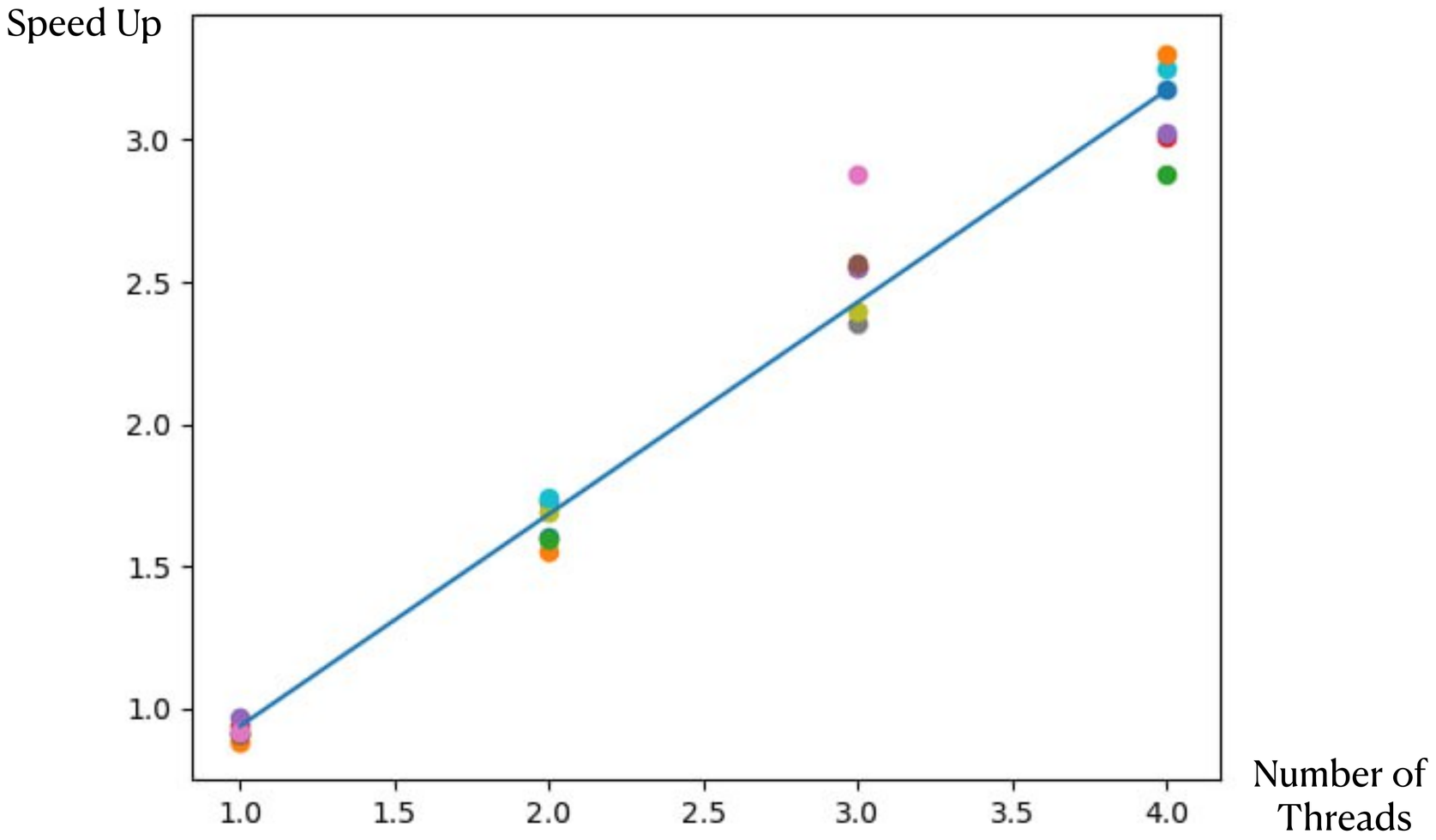


Concurrent Read and Write

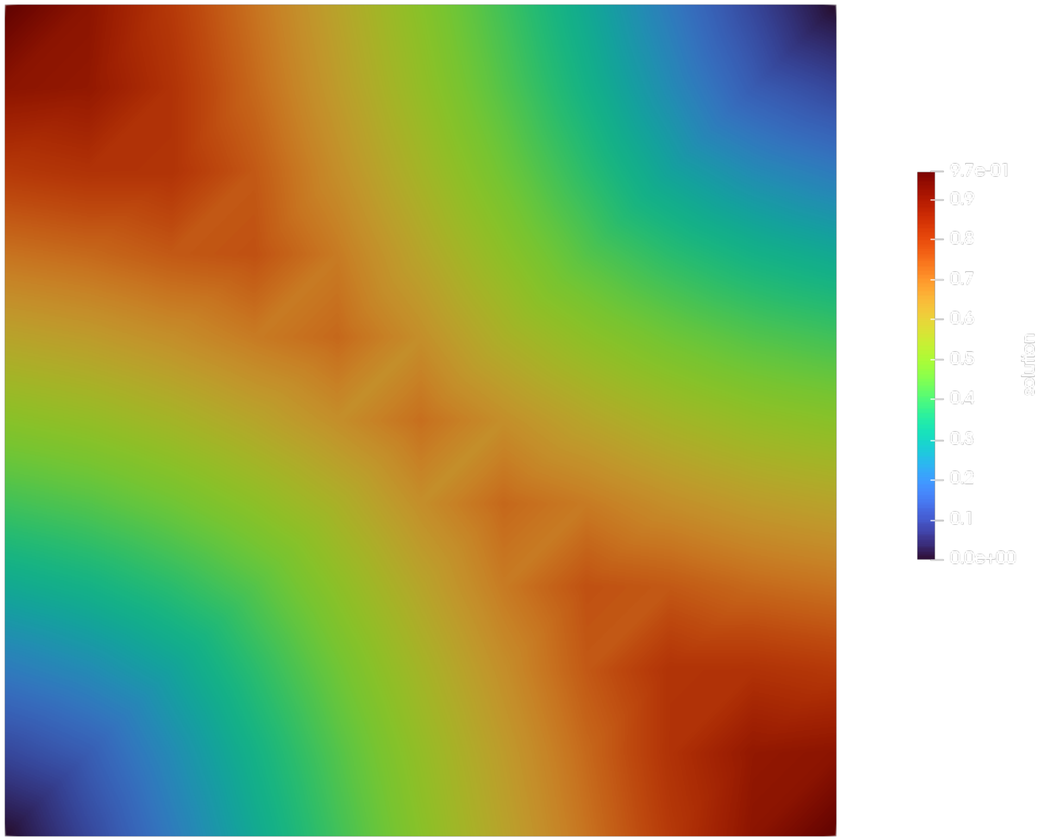
- Atomic operations are used to avoid data races when accessing the global solutions vector.
- To avoid too much of them, a map for each thread is used to cache the intermediate local solutions.

Results

Speed Up Table



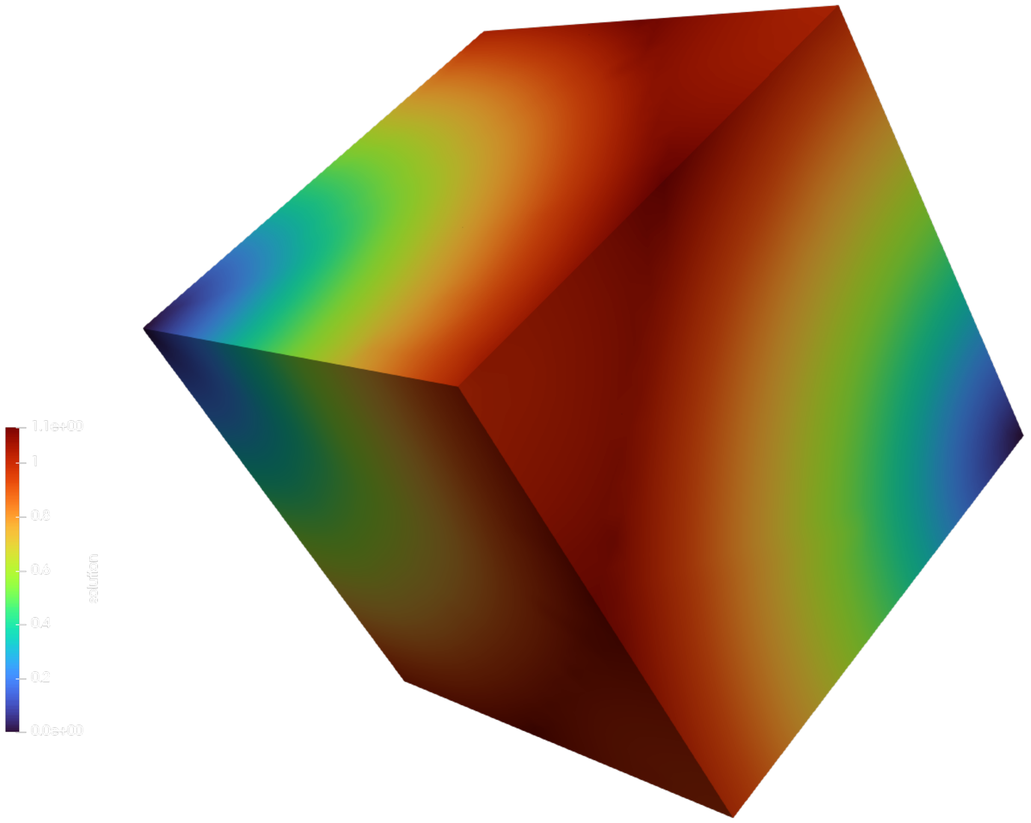
Some Examples



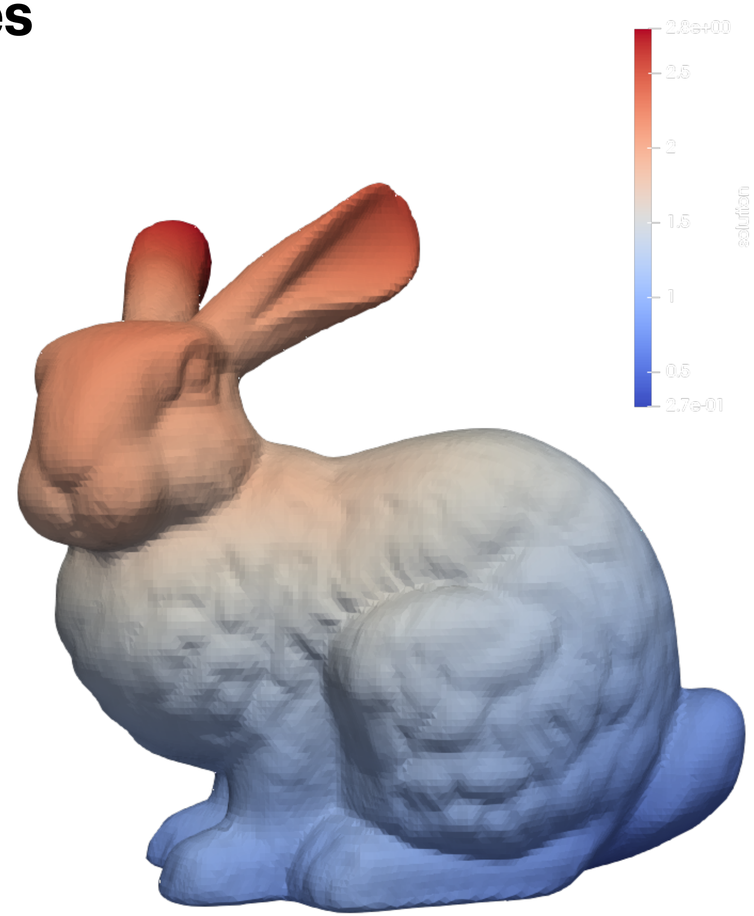
2D square model with two wave sources

Lucy model with one wave source

Some more examples



3D cube with two wave sources



Stanford Bunny model with one wave source

Thank you for your attention!