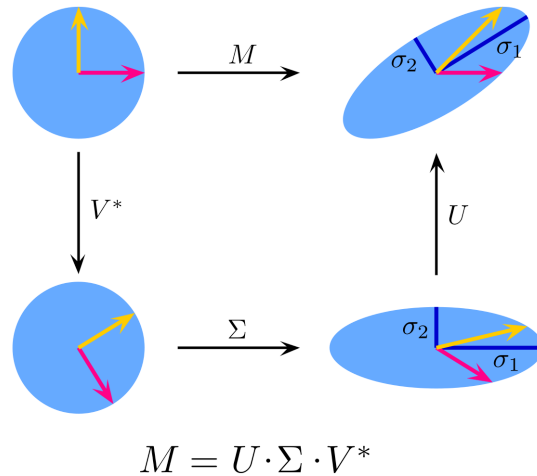


rSVD

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1 Introduction

Low-rank matrix approximations, such as the truncated singular value decomposition and the rank-revealing QR decomposition, play a central role in data analysis and scientific computing. Randomization offers a powerful tool for performing low-rank matrix approximation. These techniques exploit modern computational architectures more fully than classical methods and open the possibility of dealing with truly massive data sets.

2 Objectives

Write the code to perform the random SVD (rSVD) [1]. Start by writing a code that performs the SVD exploiting the power method [2], check that your implementation matches the one of reliable linear algebra libraries (like Matlab or numpy). Apply your SVD to compute rSVD using the QR decomposition implemented through Givens rotations (also this in parallel) [3].

3 Ideas for an exam project

Apply the rSVD to a massive dataset and evaluate its performance. Among other things, rSVD is can be applied to

- principal components analysis (PCA)
- dynamic mode decomposition (DMD)
- image compression
- proper orthogonal decomposition (POD)

References

- [1] Finding structure with randomness: Probabilistic algorithms for constructing approximate matrix decompositions Nathan Halko, Per-Gunnar Martinsson, Joel A. Tropp. (<https://arxiv.org/abs/0909.4061>)
- [2] https://www.cs.yale.edu/homes/el327/datamining2013aFiles/07_singular_value_decomposition.pdf
- [3] https://courses.engr.illinois.edu/cs554/fa2015/notes/11_qr.pdf