IBNR PROJECT

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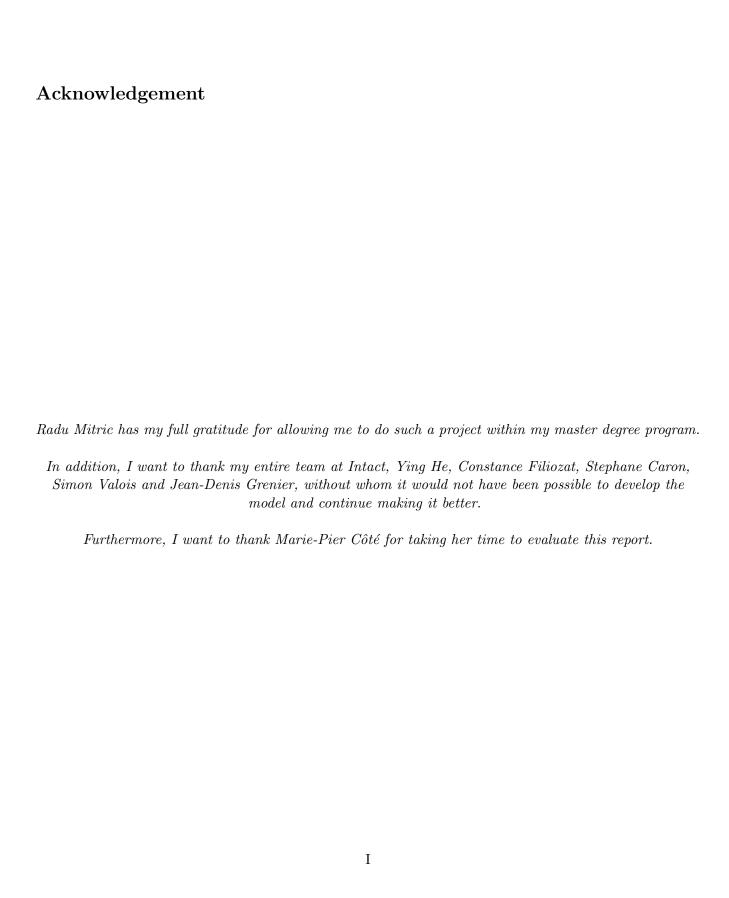
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1 Introduction

This winter I was working for the Data Lab of Intact Insurance in the claims teams. The team develops programs and models which are used internally for process optimization and cost reduction. The claims team was mandated to develop an IBNR prediction model to validate the results of the existing model from the actuarial department. Based on the analysis of the data, we use a hierarchical approach, where we cluster the data into more homogenous groups. In addition, we do not use the typical chain-ladder method. Instead, we utilize the predictive power of key information such as the garage estimate or the market value of the vehicle.

The report is organized into four main sections. First, in section 2 we provide the context of the project and its objectives. Then, we deep dive into the dataset in section 3. Next, in section 4, we discuss the methodology for the model. Finally, we show the results in section 5.

2 Project objectives

In this section, we briefly discuss the project and its objectives.

	2018	2017	2016	2015	2014		average Canada % NEP¹
PYD P&C Canada P&C U.S.	(185) (181) (4)	(238) (253) 15	(389) (389) -	(477) (477) -	(364) (364) -	(333) n/a	(4.3)% n/a
By line of business Personal auto Personal property Commercial lines – Canada Commercial lines – U.S.	49	10	(115)	(212)	(141)	(82)	(1.0)%
	(78)	(62)	(88)	(70)	(71)	(74)	(1.0)%
	(152)	(201)	(186)	(195)	(152)	(177)	(2.3)%
	(4)	15	n/a	n/a	n/a	n/a	n/a
By quarter Q1 Q2 Q3 Q4	(75)	(82)	(163)	(189)	(141)	(130)	(1.7)%
	(32)	(41)	(93)	(106)	(65)	(67)	(0.8)%
	(28)	(53)	(71)	(107)	(80)	(68)	(0.9)%
	(50)	(62)	(62)	(75)	(78)	(70)	(0.9)%

Figure 1: Unfavourable (favourable) prior year development, [Intact, 2018]

The IBNR project arises from the results of [Intact, 2018] annual report, see figure 1. The prior year development (PYD) of the Personal auto line is at 49 million of which 20 million are auto physical damage. PYD represents the change in total prior year claims liabilities during a specific period, in this case 2018. An increase in claims liabilities is referred to as an unfavourable prior year development. This means that the actuarial department underestimated the claim losses by 49 million. Even if percentage-wise this is not very significant, it still is a large amount for a line of business, which should not be fluctuating as much. Such unfavourable development is not desirable and therefore Intact's higher management launched an investigation regarding the origin of this issue. They decided that the claims team should investigate the issue and develop a new model which should exist in parallel with the model of the actuarial department.

This project started in summer 2019, while I got involved by autumn 2019.

The idea is to have a second model which allows the actuarial department to assess if their model works correctly. If both models converge, they can have more confidence in their booked numbers. If the discrepancies are too large, it will trigger further investigation. It is important to note that the booked PYD is shown in the Intact annual reports and is often used by investors to determine Intact's performance. In addition, the actuarial department wants a model which is interpretable and comprehensive. At this stage, a black box model is not a solution, since it does not allow an exact understanding of the results. The model uses historical claims data in order to predict the incurred but not reported (IBNR) personal auto claims for a specific month. The actuarial department uses an advanced chain-ladder approach. We were asked to find a different method which we discuss in more detail in section 4. Consequently, the main objective of this project is:

develop a model which outperforms the current model used for booking the PYD. This model has be interpretable and dynamic enought to be able of capturing recent data changes.

Before diving into the model itself, we have to fully understand the data used for the predictions. Thus, in the next section, we analyze the data we use for the model.

3 Data Analysis

In this section, we analyse the data in more detail by viewing a sample and visualizing aggregation results.

The dataset consistes of over one million lines of Canada-wide monthly claims data, starting in January 2016 until today. There is no data earlier than 2016, since prior 2016 claims were registered in an older system. This significantly changes the underlying claim distribution and makes prior 2016 data non-representative of future data.

The data is divided into databases for each region and line of business. The regions we cover are Quebec ("QC"), Ontario ("ON") and Alberta ("AB"). The two lines of business we cover are physical damage ("PHYSDAM") and liability ("LIPD"). The former consists of collisions and comprehensive coverage (theft, vandalism, etc.), while the latter includes all damage caused by the insured to a third party. Note that, in Quebec, due to regulatory differences there is no separation between the two lines of businesses. In Quebec, the insurance company covers the loss only for its own insured independent of the responsibility and accountability. For this reason, the Quebec region is treated differently with a single line of business called "PDPD". Thus, the adjustment of the model hyper-parameters is carried out according to each of the regions and business lines.

3.1 Data sample

The dataset has over 120 columns, so we have to first determine what variables are relevant. The figures 2 and 3 show an extract for a fictive claim number. The claim number is unique for each claim. Each line represents a month of observation (obs_month) and is the snapshot of that claim in that specific

month. With exception of the variables last_closed_month, FINAL_NET_PAID_AMT and FINAL_ALAE_AMT, all the information shown is the information we would have for that month of observation, while the three mentioned variables is information we know today (after the observation month). sf_status is the variable that indicated if the claim is open or closed at the observation month. On figure 2, we also have the month of loss (MOL), the reported date (CLM_REPORTED_DT) and the month of closure (closed_month). last_closed_month is the month at which the claim closed for the last time, since claims can reopen, this is important information we don't have when we predict the IBNR. reported_dev is the number of months since the claim has been reported, i.e. the age of the claim. CATASTROPHE_IND, TOTAL_LOSS_IND, GLASS_IND, flag_43 and luxury_ind are variables we used to classify our data into leaf. We discuss this in further detail in section 4.

CLM_NBR	sf_status (obs_month	MOL	CLM_REPORTE D_DT	closed_month		reported_mon th	reported_dev	dev_group	CATASTROPHE _IND	TOTAL_LOSS_I	GLASS_IND flag43	luxury_ind leaf
123456789	OP	201711	201711	2017-11-10	PAR	201712	201711	0	0	N	Т	0 N	0 ti_n43
123456789	a	201712	201711	2017-11-10	201712	201712	201711	1	1	N	T	0 N	0 ti_n43
123456789	a	201801	201711	2017-11-10	201712	201712	201711	2	2	N	T	0 N	0 ti_n43
123456789	a	201802	201711	2017-11-10	201712	201712	201711	3	3	N	T	0 N	0 ti_n43
123456789	a	201803	201711	2017-11-10	201712	201712	201711	4	4	N	T	0 N	0 ti_n43
123456789	a	201804	201711	2017-11-10	201712	201712	201711	5	5	N	T	0 N	0 ti_n43
123456789	a	201805	201711	2017-11-10	201712	201712	201711	6	6	N	T	0 N	0 tl_n43
123456789	a	201806	201711	2017-11-10	201712	201712	201711	7	7+	N	T	0 N	0 tl_n43
123456789	a	201807	201711	2017-11-10	201712	201712	201711	8	7+	N	T	0 N	0 tl_n43
123456789	a	201808	201711	2017-11-10	201712	201712	201711	9	7+	N	T	0 N	0 tl_n43
123456789	a	201809	201711	2017-11-10	201712	201712	201711	10	7+	N	T	0 N	0 tl_n43
123456789	a	201810	201711	2017-11-10	201712	201712	201711	11	7+	N	T	0 N	0 tl_n43
123456789	a	201811	201711	2017-11-10	201712	201712	201711	12	7+	N	T	0 N	0 tl_n43
123456789	a	201812	201711	2017-11-10	201712	201712	201711	13	7+	N	T	0 N	0 tl_n43
123456789	a	201901	201711	2017-11-10	201712	201712	201711	14	7+	N	T	0 N	0 ti_n43
123456789	a	201902	201711	2017-11-10	201712	201712	201711	15	7+	N	T	0 N	0 tl_n43
123456789	a	201903	201711	2017-11-10	201712	201712	201711	16	7+	N	T	0 N	0 ti_n43
123456789	a	201904	201711	2017-11-10	201712	201712	201711	17	7+	N	T	0 N	0 ti_n43
123456789	a	201905	201711	2017-11-10	201712	201712	201711	18	7+	N	T	0 N	0 ti_n43
123456789	a	201906	201711	2017-11-10	201712	201712	201711	19	7+	N	T	0 N	0 ti_n43
123456789	a	201907	201711	2017-11-10	201712	201712	201711	20	7+	N	T	0 N	0 ti_n43
123456789	a	201908	201711	2017-11-10	201712	201712	201711	21	7+	N	T	0 N	0 tl_n43
123456789	a	201909	201711	2017-11-10	201712	201712	201711	22	7+	N	T	0 N	0 tl_n43
123456789	a	201910	201711	2017-11-10	201712	201712	201711	23	7+	N	T	0 N	0 tl_n43
123456789	a	201911	201711	2017-11-10	201712	201712	201711	24	7+	N	T	0 N	0 tl_n43
123456789	a	201912	201711	2017-11-10	201712	201712	201711	25	7+	N	T	0 N	0 tl_n43
123456789	a	202001	201711	2017-11-10	201712	201712	201711	26	7+	N	T	0 N	0 tl_n43
123456789	a	202002	201711	2017-11-10	201712	201712	201711	27	7+	N	T	0 N	0 tl_n43
123456789	a	202003	201711	2017-11-10	201712	201712	201711	28	7+	N	T	0 N	0 tl_n43

Figure 2: Sample from database

In figure 3, we have, for each month of observation, the amounts paid for the loss and ALAE.

AUTO_LTD_NET_LOSS_PAID_AMT is the paid amount known at the observation month. Any type of recovery can decrease the paid amount. AUTO_LTD_LOSS_RES_CHG_AMT and AUTO_LTD_LOSS_RES_CHG_AMT are the case reserve amounts at a given observation month. AUTO_LTD_LOSS_INCURRED_AMT and AUTO_LTD_ALAE_INCURRED_AMT, represent the incurred at the given observation month. The variables FINAL_NET_PAID_AMT and FINAL_ALAE_AMT are the final amounts we know today, they are also called the ultimate amount for that claim. AvgTypicalCarValue

(ACV) is an estimate of the market value of the accident vehicle. TotalGAV (GAV) is the gross appraisal value, which is the garage cost estimate to repair the vehicle. IBC_PRICE is the initial purchasing price of the vehicle.

CIM NRR -	-6 -b-b			AUTO_LTD_LO				FINAL_NET_PAI	FINAL_ALAE_A	AvgTypical <u>Car</u>	T-1-15AV	IDC DDICE
CLM_NBR	sf_status	obs_month	T_LOSS_PAID_ AMT	SS_INCURRED _AMT	AE_INCURRED _AMT	SS_RES_CHG_ AMT	AE_RES_CHG_ AMT	D_AMT	MT	Value	TotalGAV	IBC_PRICE
123456789	ОР	201711	11414.35	17964.35	0	6550	0	11213.87	0	8007	11481.67	33729
123456789	a	201712	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	201801	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	201802	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	201803	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	201804	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	201805	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	201806	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	201807	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	201808	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	201809	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	201810	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	201811	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	201812	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	201901	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	201902	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	201903	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	201904	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	201905	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	201906	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	201907	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	201908	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	201909	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	201910	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	α	201911	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	α	201912	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	202001	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	202002	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729
123456789	a	202003	11213.87	11213.87	0	0	0	11213.87	0	8007	11481.67	33729

Figure 3: Severity sample from database

Already with this single extract we can get a small understanding of our data. Specifically, we notice that the final paid amount, i.e. the ultimate, is close to ACV and GAV amount. This might indicate that the ACV and GAV are good predictors for our model. Consequently, we want to identify the dependence structure between the ultimate amount and the ACV or GAV.

3.2 Dependence structure

[Embrechts et al., 2001] show how copulas are used for modeling dependence between random variables. Even though, we do not plan to model the dependence structure itself, we use quantile plotting to visualize

the dependence. Since we are interested in more than only linear dependence, we use Kendall's tau as dependence measure. The definition of Kendall's tau for a random vector pair (X, Y) is given as

$$\tau(X,Y) = \Pr((X - \widetilde{X})(Y - \widetilde{Y}) > 0) - \Pr((X - \widetilde{X})(Y - \widetilde{Y}) < 0),$$

where $(\widetilde{X}, \widetilde{Y})$ is an independent copy of (X, Y).

It is the probability of concordance minus the probability of discordance. Concordance measures how X and Y move in the same direction relative to their independent copy. Discordance measures how X and Y move in opposite direction relative to their independent copy. It can also be interpreted as the correlation coefficient between the quantiles of X and Y, which have a relationship defined by a copula. Kendall's tau has a value between -1 and 1. -1 indicates perfect negative dependence, also called countermonotonic, while 1 indicates perfect positive dependence, comonotonic. If Kendall's tau is close to 0, the pairs are likely independent.

A copula is a cumulative distribution function of a multivariate uniformed distribution. The copula of two independent uniform distribution $U_1 \sim U(0,1)$ and $U_2 \sim U(0,1)$ is defined as

$$C(u_1, u_2) = u_1 \times u_2.$$

A copula can be visualized by plotting pairs of quantiles of the uniform distributions. For the bivariate independent copula, the pairs are evenly distributed on the graph. Kendall's tau should be close to 0 since it measures the correlation coefficient of these pairs.

Before we can plot the ultimate and the ACV/GAV, we need to find their empirical quantile values between 0 and 1. The values are ranked according to their relative size and divided by the total number of observations. In addition, the figure only shows a sample of 10,000 pairs, one pair for each claim number, because there would be too many data points on each grpaph. Furthermore, we group the data into age since reported date categories.

Starting with Quebec, figures 4 to 5 shows the relationship of the quantile pairs for each age grouping. The x-axis consists of the quantiles of the ultimate amount (paid loss + ALAE) and the y-axis plots the quantiles of the GAV or ACV. We use the GAV for claims with repairable vehicles and the ACV for claims with vehicles that are total loss. On figure 4 we can observe the relationship between the ultimate and GAV. The relationship seems strong especially for younger claims. Note that, as claims get older, Kendall's tau decreases, indicating that older claims become more complex and incurred additional payments or recoveries which do not depend on the damage estimation. Figure 5 demonstrates a weaker dependence between the ACV and the ultimate. However, the dependence is still positive and not negligible. Further, it shows that the ACV might not always be a good estimation of the actual market value. The weaker dependence is therefore only caused by additional fees but also by intrinsic estimation error of the actual market value of the vehicle.

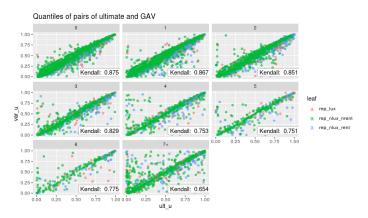


Figure 4: Quantiles pairs for Quebec repairable claims. The x-axis stands for the ultimate quantiles and the y-axis is associated to the GAV quantiles

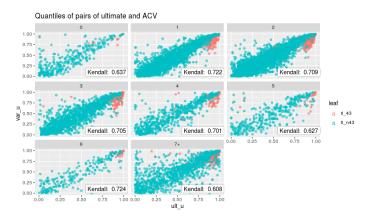


Figure 5: Quantiles pairs for Quebec total loss claims. The x-axis stands for the ultimate quantiles and the y-axis is associated to the ACV quantiles

In figure 6 to 7, one can see that the patterns observed for the Quebec region are similar to those of Ontario. However, since we have two different lines of business PHYSDAM (physical damage) and LIPD (liabilities), it is interesting to observe the difference in pattern. LIPD tends to be more on the lower half of the diagonal. Third party liabilities seem to incur higher losses than the GAV would suggest and that it incurs more additional fees.

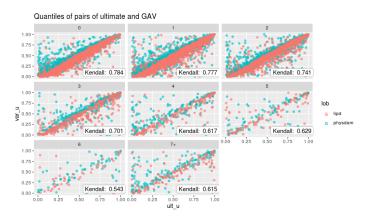


Figure 6: Quantiles pairs for Ontario repairable claims. The x-axis stands for the ultimate quantiles and the y-axis is associated to the GAV quantiles

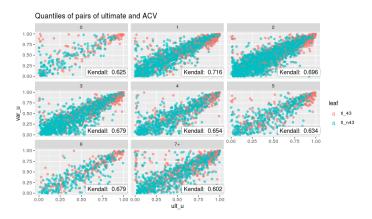


Figure 7: Quantiles pairs for Ontario total loss claims. The x-axis stands for the ultimate quantiles and the y-axis is associated to the ACV quantiles

As shown in figure 8 to 9, Alberta has an interesting pattern. Again, we see weaker dependence for older claims. Albeit, there is a descriptive force which seems to strongly impact the dependence structure and leads to more claims with very low ultimate compared to the GAV or ACV. We identified this disruptive force as subrogation and recoveries. Subrogation is a slow process at which the insurance company can recover paid losses if the insured was not responsible for the accident. This means that if Intact paid the entire loss, they might be able to recover a part or the entire loss with a lawsuit.

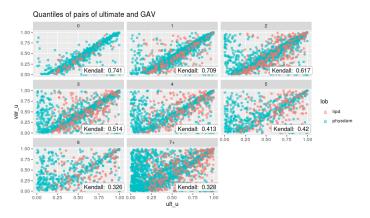


Figure 8: Quantiles pairs for Alberta repairable claims. The x-axis stands for the ultimate quantiles and the y-axis is associated to the GAV quantiles

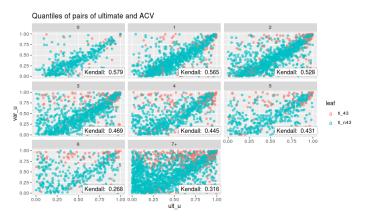


Figure 9: Quantiles pairs for Alberta total loss claims. The x-axis stands for the ultimate quantiles and the y-axis is associated to the ACV quantiles

The data indicates that it can take more than a year until the subrogation process is finished. Consequently, a proportion of claims in Alberta need much longer to fully develop to ultimate which might even fall to 0 or negative. This can be problematic, since it proves difficult to predict, if the claim falls into the subrogation category or not. These claims cannot be filtered out, because, for a given observation month, we do not know which claims are affected. We can partly mitigate this issue by aggregating the data using averages. Consequently, we can expect higher volatility in the model for Alberta. All of these figures also indicate a slightly stronger dependence for large values.

3.3 Trend analysis

The model is not designed to estimate the ultimate on a claim by claim basis. Therefore, the data is aggregated. When aggregating data, it is important to verify potential trends. If we use the GAV and the ACV as a predictor for the ultimate, we should validate that the mean growth rates are similar. For each observation month, we calculate the mean ultimate and the mean GAV/ACV of open claims. Then, we compare their monthly growth rates. We could also do the same per month of loss; however, we want to analyze the underlying distribution of what we are trying to predict. The model predicts the ultimate based on observation month data. Each observation month will contain a proportion of claims with different months of loss. While using aggregation per observation month, we have to be aware of possible fluctuations related to different number of claims and different mixtures of months of loss. When looking at the figure 10, we can observe that in Quebec the average ultimate grows faster than the average GAV/ACV. Consequently, using the GAV and ACV as predictors might tend to underestimate the ultimate if using past averages.

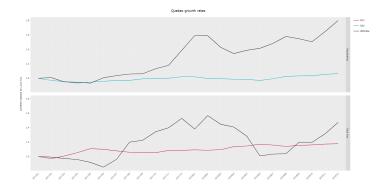


Figure 10: Quebec ultimate (black) and GAV/ACV in (blue/red respectively) growth relative to January 2017

Figure 11, shows that for Ontario the opposite seems to happen for total loss claims. Thus, we might tend to overestimate total loss claims ultimate.

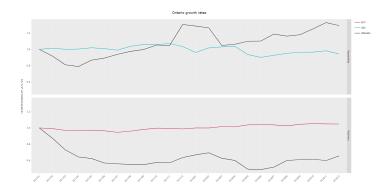


Figure 11: Ontario ultimate (black) and GAV/ACV in (blue/red respectively) growth relative to January 2017

On figure 12, Alberta has a similar but reversed pattern. Total loss claims ultimate growth fluctuates around the 1 value. While for repairables, growth rates for the ultimate are lower than for the GAV.

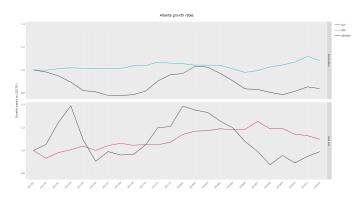


Figure 12: Alberta ultimate (black) and GAV/ACV in (blue/red respectively) growth relative to January 2017

Furthermore, the GAV and ACV might not capture the additional costs related to the claim. The ultimate allocated loss adjustment expense (ALAE) is often unrelated to the GAV or ACV. A larger proportion of ALAE can cause greater estimation error. The figures are shown in the appendix. Figure 31 shows the ALAE to loss ratio for Quebec. The average is around 0.0125. In Ontario, seen in figure 32, the ALAE to loss ratio is similar to the Quebec, although after December 2017 there is clearly a spike which might cause prediction errors. Alberta, in figure 33, shows again a different pattern than the other 2 regions. While most ratios are lower than in Quebec and Ontario, the non-luxury non-rental repairable vehicles show proportionally larger ratios.

In order to better understand the impact of recovery on the data, figures 34 to 36 shows the recovery

to ultimate ratio for all 3 regions. Quebec and Ontario both have a ratio below 0.17, while Alberta has ratios between 1 and 0.3. The discrepancy is very significant and has to be considered in the model.

Now that we have a better understanding of the data, we discuss the model structure and methodology.

4 Model methodologie

4.1 Incurred but not reported

The task of the actuarial model is to predict the IBNR, the incurred but not reported claims. The IBNR can be divided into 3 distinct elements, which we defined as pure IBNR, IBNER and IBNR on reopen. Pure IBNR are claims which are not reported at the observation date, meaning the insurer has no information on them. The insurer only knows that a claim happened when it is reported. IBNER, incurred but not enough reported, are claims which have been reported and the insurer has information on the claims in their database. IBNR on reopen consist of closed claims which might reopen at any given time. This mean that a claim which closed in 2017 might reopen in 2018 or 2019. IBNR on reopen is a small proportion of the total IBNR, but still should be considered in the model. Figure 13 gives a visual representation of these categories.

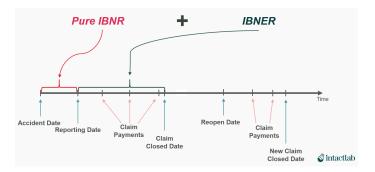


Figure 13: Timeline of a claim

4.2 Hierarchical approach

The actuarial department uses a modified chain-ladder method for their model. This model uses a more hierarchical approach, where the data is clustered into more homogeneous groups. First, we develop a model for each of the three IBNR types. The claims team focuses on the IBNER part, while the pure and IBNR on reopen models are still chain-ladder based and were developed by the actuarial department. For the IBNER model, we grouped the data according to the following claims characteristics, called leaf: total loss (43), total loss without replacement cost endorsement (n43), luxury repairable vehicles (rep_lux), non-luxury non-rental repairable vehicles (rep_nlux_nrent) and non-luxury rental repairable vehicles (rep_nlux_rent). We suppose that the frequency and severity distributions are very similar within these groups. Figure 14 gives an overview of the hierarchical approach, including small statistics.

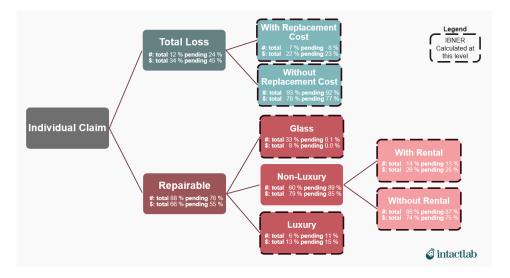


Figure 14: Hierarchical model structure. The statistics give the percentage proportion of claims in number of claims (#) and in dollars amount (\$). The total percentage is based on the previous subgroup total. The pending percentage is based on the pending of the previous subgroup.

4.3 Key formulas

As mentioned in section 3, the ACV and GAV have strong predictive strength. We use a basic formula to link the GAV/ACV with the ultimate and use historical information of claims.

Definition 1. We define \tilde{L}_i as ultimate loss prediction for claims pending/open in period i and X_i as the predictor, in our case GAV or ACV used to predict period i. Their relationship is defined as

$$\hat{L}_i = \hat{\Theta}_i \times X_i,$$

where $\hat{\Theta}_i$ is the factor for the time period i.

We need to calculate the factor $\hat{\Theta}_i$ with the available historical data.

Definition 2. We define $\widetilde{L}_{j,i}$ as total incurred for claims in the time period j as of i. X_j is the predictor in the time period j. Thus, the factor is defined as

$$\hat{\theta}_i = \frac{\widetilde{L}_{j,i}}{X_j}.$$

Note the difference between \hat{L}_i and $\tilde{L}_{j,i}$. The former is the ultimate we want to predict, so we do not know its value in observation month i. The latter is the total incurred for claims in period j we know as

of i. For illustration in the sample data of figures 2 and 3, we want to calculate the factor $\hat{\theta}_{201804}$. We suppose we want to use open claim with CLM_NBR = 123456789 to calculate this factor, then

```
\begin{split} \widetilde{L}_{201711,201804} &= \texttt{AUTO\_LTD\_NET\_LOSS\_PAID\_AMT} + \texttt{AUTO\_LTD\_ALAE\_INCURRED\_AMT} \\ &+ \texttt{AUTO\_LTD\_LOSS\_RES\_CHG\_AMT} \\ &= 11213.87 \end{split}
```

as of 201804 (obs_month = 201804) and $X_j = \text{AvgTypicalCarValue} = 8007$. As the example illustrated, the ultimate and the predictor are historical values which should be fully developed. It is necessary to have a least 5 to 12 months of development, so that the factors are stable enough. The difference in time between the moment we want to predict the pending and the historical data, i-j is defined as the lag. How many historical observation month j we use for the calculation is defined as period length. Figure 15 shows a 5-month lag and 3-month period length. We want to predict the ultimate of the December 2018, i = 201812 pending (open) claims. We go back 5 months and use the historical claim data from open claims in May, June and July 2018, j = 201805, 201806, 201807. This means that their sf_status needs to be either "OP" for open or "RO" for reopen in the observation month j = 201805, 201806, 201807.

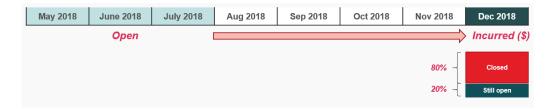


Figure 15: Lag and period length visualization

This approach is similar to a lagged moving average model. We use the incurred as of December 2018 of claims that are open at least once in May, June and July. Thus, the incurred had a minimum of 5 months to develop. Then, we divide this incurred by the aggregated GAV or ACV in May, June and July. We only keep the most recent data line for each claim, in other words, we don't have any duplicates per claim number. Figure 16 gives a numerical example.

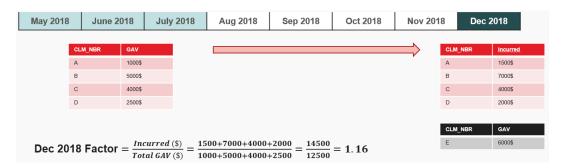


Figure 16: Factor calculation example

Note that in figure 15 the incurred is subdivided into open and closed claims, since it is possible that claims remain open even after 5 months. Therefore, we calculate a claim number weighted average of closed and open factors. As final factor we get definition 3.

Definition 3. $\hat{\Theta}_i$ is the final factor used for the prediction. $\hat{\theta}_{i,open}$ is the factor for claims that are still open during i and $\hat{\theta}_{i,closed}$ is the factor for claims that are closed during i. Furthermore, n_{open} is the number of open claims in period i and n_{closed} is the number of closed claims in period i. Thus, we have

$$\hat{\Theta}_{i} = \frac{n_{open}}{n_{open} + n_{closed}} \hat{\theta}_{i,open} + \frac{n_{closed}}{n_{open} + n_{closed}} \hat{\theta}_{i,closed}.$$

We multiply this factor by the aggregated GAV or ACV of pending claims (December 2018 in the previous example) to get the ultimate amount.

Definition 4. We define the IBNER in period i as

$$IBNER_i = \hat{L}_i - I_i$$

where I_i is the incurred payments and reserve for claims pending in period i.

Note that once claims are fully developed $L_i = I_i$, where L_i is the real observed ultimate loss for claims open in period i. The prediction results for the model are discussed in section 5.

4.4 Imputation methodology

It is important to note that no data is perfect. The dataset contains a non-negligible amount of missing GAV and ACV values. About 17% of the claims have missing GAV or ACV, some regions and line of business are worse than others. Figure 17 demonstrates how missing values are imputed. If we have missing values, we calculate the median of the existing values in the time window. We replace the missing values by the median and execute the factor calculations without any further readjustment. This method is not perfect and should be revised. For instance, in the previous example, we would overestimate the factor since we impute with a lower value.

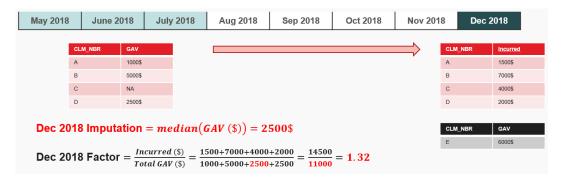


Figure 17: Factor calculation example with missing values

4.5 Adjusting the lag and period length

As we have noticed in the analysis of the data, Alberta clearly has different patterns than the other regions. Claims in Quebec and Ontario settle considerably faster (after 4-5 months) and recovery is less impactful. Note that in section 5, figure 24, we have an December 2019 and an March 2020 model. The main different between the two are the Alberta hyper-parameters. In December 2019, all regions had the same lag of 5 months. A more detailed analysis shows an advantage to increase the lag for Alberta to 10 months. In Alberta having a lag of 5 is clearly insufficient, thus using a lag of 10 is necessary. This gives the claims at least 10 months to fully develop. In addition, a lag and period length that includes the 12th month, is beneficial if we have seasonal effects.

4.6 Second model iteration: Historical pending but now closed claims

After having used the first iteration of the model for December 2020 and March 2020. We noticed systematic error, i.e. constant over- or underestimation. This might be related to the open claims (after the lag) which are used to calculate the factors. Since open claims still have a case reserve, the incurred used for the factor might be inflated, thus explaining overestimation. Indeed, the factor for open claims is always larger than the factor for closed. If we overestimate constantly, we attribute too much weight to still open claims. In the second iteration of the model, we discard the weighted average and we only use the factor of closed claims. In short, based on definition 3, we have

$$\hat{\Theta}_i = \hat{\theta}_{i,closed}.$$

This also has the benefit that closed claims usually should not develop further and thus have less volatility in the calculation of the factors. The results are discussed in section 5.

4.7 Third model iteration: Historical closed claims

The final iteration of the model is based on the idea of the second iteration. The idea is to only use closed claims. However, in the second iteration we use the factor of closed claims of the lagged 3-month

period, while in this iteration we want to increase the number of claims used in the calculations and add more recent claims. Consequently, instead of using the pending claims in a historical time window, we calculate factors for all claims that closed in the time window. This simply implies changing the filter on sf_status. If we have used the previous example, we want to predict the pending claims in December 2018, we use all claims that closed between May and July for the factor calculations. Since the claims are closed, it is unlikely that we have still open claims in December.

5 Results

In this section, we analyze the results of all three model iterations. For the first two versions, we show the prediction error on the ultimate for each leaf and province, while for the third version we only give the average monthly error. It should be noted that we only show the ultimate predictions for claims pending in 2017 and 2018, although we use the incurred as of March 2020. Pending claims in December 2018 had more than 14 months to develop to the ultimate.

5.1 First model version: Historical pending claims

Quebec: For the first model, the average monthly prediction error is 957, 758\$, based on a total volume of about 100 million \$. Figure 18 shows that the error has a light seasonal pattern. In winter, we tend to slightly underestimate, while in most months we overestimate the ultimate loss. The seasonality is difficult to confirm, since we only have 2 years of fully credible data. The error does not seem to be all random and thus should be further investigated. We do notice less cyclic pattern when using a lag and period length which includes the same month of the previous year. However, it does not neutralize it and actually increases the overall error.

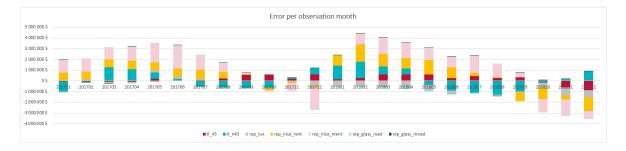


Figure 18: Quebec "Historical pending claims" model, prediction error by observation month

Figure 19 indicates that the largest proportion of error comes from tl_n43 and rep_nlux_nrent. These two leaf are also the largest in terms of volume.

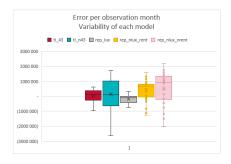


Figure 19: Quebec "Historical pending claims" model, monthly prediction error boxplot

Ontario: In Ontario, the model has the tendency to underestimate the ultimate amount. The average error per observation month shown in figure 20 is at -4,632,282\$.from the total volume of 100 million \$. Specifically, the winter proves difficult for the model. We are currently investigating this issue, but it might be related to the lag we use and/or the trends observed in the data.

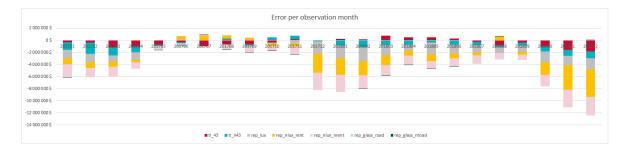


Figure 20: Ontario "Historical pending claims" model, prediction error by observation month

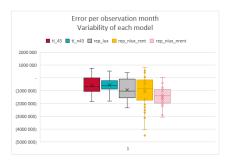


Figure 21: Ontario "Historical pending claims" model, monthly prediction error boxplot

Alberta: The average monthly prediction error amounts to 3,547,626\$ on a total volume of about 80 million \$. There is no clear seasonal pattern. However the model constantly overestimates. The cause of the overestimation is the subrogation, which even after 12 months is not completed. We found about 10% of the claims in Alberta still receive recovery payments after 12 months. Figure 23 shows that rep_nlux_rent is the biggest source of error for this model.

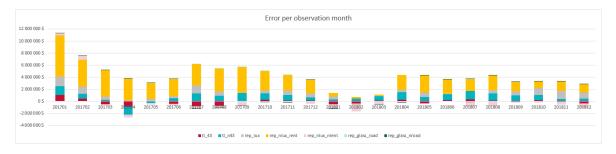


Figure 22: Alberta "Historical pending claims" model, prediction error by observation month

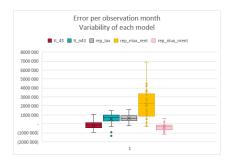


Figure 23: Alberta "Historical pending claims" model, monthly prediction error boxplot

In section 3, it was shown that the ALAE might have an impact on the model. However, the model uses a factor proportionnal to the ultimate including ALAE. If the ALAE to loss ratio is stable in time, the factor should account for the ALAE. Thus, the instability of the ALAE to loss ratio observed in the data could cause between 0.2 to 1% of estimation error.

Overall, when combining all results we can observe that Onatrio and Alberta almost equalize. Figure 24 shows the results for the December 2019 model (upper graph) and the current model (lower graph). The current model is a significant improvement over the previous model. The December 2019 model uses a different lag than the March 2020 version, as mentioned in section 4.5. The error is rarely greater than 2 million \$. However, the red square indicates a zone of bad predictions amounting to error as high as 18 million \$. One should note that the real IBNER after February 2019 is not reliable, since it is not yet fully developed in some cases.



Figure 24: IBNER prediction vs real IBNER ($IBNER_i$ from definition 4, for $i = 201701, \dots, 201812$)

5.2 Second model version: Historical pending but now closed claims

Excluding open claims in the factor calculations has a strong impact on the model. The average monthly errors are -1,379,997\$, -5,134,386\$ and -431,989\$, for Quebec, Ontario and Alberta respectively. This model has a strong tendency to underestimate. Albeit, for Alberta, when looking at the figure 29, the results are considerably better than the first model version. Adding open claims seems to multiply the error by a factor of 10. This indicated that once a claim is closed, the subrogation process seems to also be finished or partly finished. For the other two provinces, it shows that adding open claims captures something we are not yet able to fully explain. The issue might be related to the imputation method and observed trends.

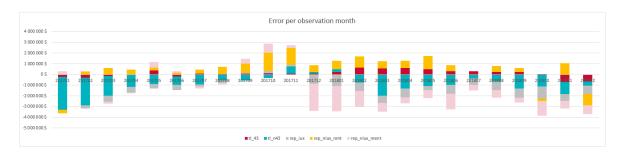


Figure 25: Quebec "Historical pending but now closed claims" model, prediction error by observation month

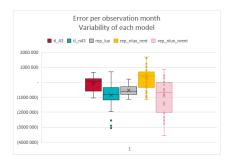


Figure 26: Quebec "Historical pending but now closed claims" model, monthly prediction error boxplot



Figure 27: Ontario "Historical pending but now closed claims" model, prediction error by observation month

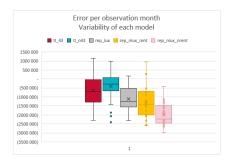


Figure 28: Ontario "Historical pending but now closed claims" model, monthly prediction error boxplot

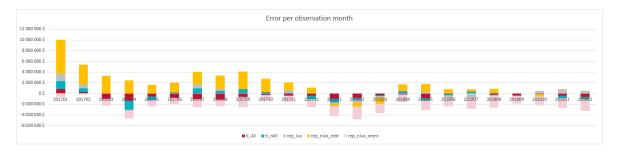


Figure 29: Alberta "Historical pending but now closed claims" model, prediction error by observation month

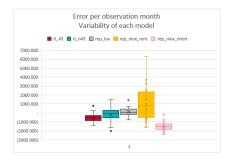


Figure 30: Alberta "Historical pending but now closed claims" model, monthly prediction error boxplot

5.3 Third model version: Historical closed claims

Lastly, the third model version seems not to function as well as expected. Quebec has an average monthly error of -7,196,717, Ontario of -13,153,106 and Alberta of 4,148,104. One issue with this model

is related to the claims in the window. While the two previous model uses a small window of pending claims, this model uses all claims that closed in the time window. Thus, we are using claims that can be very old relative to the observation month or claims that are not representative of current pending claims.

6 Conclusion

To summarize, we were able to develop a functional and practical model for December 2019 and thus delivered 2019 year end predictions to compare with the corporative actuarial department booked numbers. The results were positive and the model is mathematically simple and interpretable. However, we were not fully satisfied with the model and wanted to increase accuracy and consistency. After a few months, we manage to even further reduced the error, although we still have some potential areas of improvement. The imputation method can be potentially improved by taking into account the month of loss, the cause of missing values (accident type, etc.) and other variables. Maybe simply removing the missing values for the factor calculation might be feasible. Systematic overestimation and underestimation depending on the region is the biggest concern and we still have to further investigate the cause. However, it might be related to the observed data trends. In addition, we might rapidly reach a limit for this simplistic model. We consider developing a more advanced machine learning model for a claim by claim prediction approach.

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[Intact, 2018] Intact (2018). 2018 annual report.

A Appendix ALAE to loss and recovery to loss ratios

Reminder for the leaf grouping: total loss (43), total loss without replacement cost endorsement (n43), luxury repairable vehicles (rep_lux), non-luxury non-rental repairable vehicles (rep_nlux_rent) and non-luxury rental repairable vehicles (rep_nlux_rent).



Figure 31: Quebec ALAE to loss ratio per leaf



Figure 32: Ontario ALAE to loss ratio per leaf

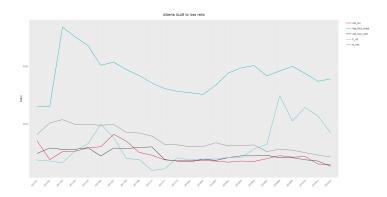


Figure 33: Alberta ALAE to loss ratio per leaf

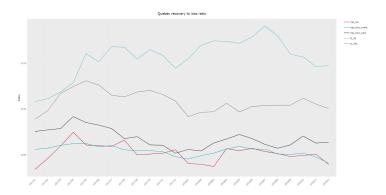


Figure 34: Quebec recovery to loss ratio per leaf

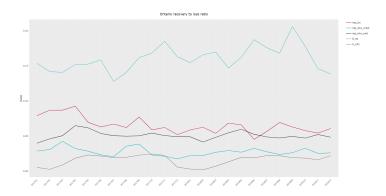


Figure 35: Ontario recovery to loss ratio per leaf

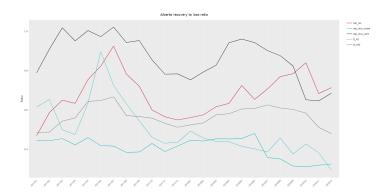


Figure 36: Alberta recovery to loss ratio per leaf