MF4052/MF6012 - Computational Finance

Assignment 1 – Binomial tree methods (10 marks)

In Theorem 2.1 of Notes Set 1 we showed that by selecting (u, d, p) to be

$$\begin{aligned} u &= e^{\sigma\sqrt{\delta t}+(r-\sigma^2/2)\delta t}; \\ d &= e^{-\sigma\sqrt{\delta t}+(r-\sigma^2/2)\delta t}; \\ p &= 1/2, \end{aligned}$$

we could ensure that the final asset price values S_T generated by the binomial tree method converge in distribution to the final asset price values of the Black Scholes model. This convergence takes place as the step size $\delta t \to 0$ and hence as the number of steps $M \to \infty$.

In this assignment, you will use the code you developed for Labs 1 & 2 to investigate the error in the option values produced by the binomial method for increasing values of M, where that error is defined to be the absolute difference between the option value produced by the Black-Scholes formula, and that produced by the binomial tree method.

1. For a European put, produce a plot of error against M for values of M between 100 and 400. Use the values of (u, d, p) given above, with

$$T = 1, E = 20, S_0 = 19, r = 0.03, \sigma = 0.2.$$

(3 marks)

2. Repeat using the values of (u, d, p) given by

$$\begin{aligned} A &= (e^{(r+\sigma^2)\delta t} + e^{-r\delta t})/2; \\ u &= A + \sqrt{A^2 - 1}; \\ d &= A - \sqrt{A^2 - 1}; \\ p &= \frac{e^{r\delta t} - d}{u - d}, \end{aligned}$$

and comment. (3 marks)

3. Investigate further. For example, does varying any of the parameters have any effect on the nature of the convergence? Does the inclusion of a dividend yield make a difference? Can you demonstrate the phenomenon for an American put option?

(4 marks)

Submit your assignment individually by email (put *MF4052/MF6012 Assignment 1* in the subject line (delete whichever course code does not apply) to

`conall.kelly@ucc.ie`

before 4pm on Monday, November 1, 2020. Late submissions without acceptable documentation will be awarded a mark of zero.

Your submission should consist of Jupyter notebook containing a short written report providing answers to Q1-3 above, and including any code that you used to generate plots or inform your investigation in a clear, readable and executable form.