MF4052/MF6012 - Computational Finance

Assignment 2 – Finite difference methods for options (10 marks)

In Lab 5, we solve the transformed Black-Scholes PDE for European call and put options using a BTCS finite difference scheme applied to the resulting diffusion equation, and reversed the transformation to produce a plot of the Black-Scholes surface corresponding to options prices for asset values over the interval $S \in [0, 100]$.

In this assignment we will do two things. First, we will implement the Crank-Nicolson method for the pricing of European options. Second, we will apply the finite difference approach to the pricing of a binary option.

Q1: Modify the code from the file `BTCSBlackScholesEquation.py` to use a **Crank-Nicolson** finite difference scheme, rather than BTCS, to produce surface plots of prices for both a European call and a European put option with parameter values

$$E = 5, T = 1.5, r = 0.01, \sigma = 0.5,$$

over the approximate range $S \in [0, 10]$.

Q2: Produce a surface plot of a European put with parameters as above, but instead of approximating V over a large set of values for S and zooming in to $S \in [0, 10]$, approximate V **only over the smaller range**. Explain the resulting output.

Q3: Consider a **cash-or-nothing call** option with payoff

$$\Lambda(S) = \begin{cases} B, & S(T) > E; \\ B/2, & S(T) = E; \\ 0, & S(T) < E. \end{cases}$$

where $B = 20$, $E = 5$, $T = 1.5$, $r = 0.01$, $\sigma = 0.5$.

(i) By reference to the class notes for Section 6 in Notes Set 3 of MF3052/MF6011, identify the initial function $u_0(x)$ associated with $\Lambda(S)$ and modify the calculation of `U[:,0]` in Step 4 of the sample code accordingly.

(ii) By reference to Point 2 of the 'Implementation Notes' section of Lab 5 of MF4052/MF6012, identify suitable boundary conditions for $u(x, \tau)$ and modify the calculation of `a`, `b` in Step 4 of the sample code accordingly.

(iii) Use either BTCS or Crank-Nicolson to produce a surface plot of prices for the cash-or-nothing call over the approximate range $S \in [0, 10]$. Comment on your output.

Instructions:

1. Submit your assignment in groups of two by email (put *MF4052/MF6012 Assignment 2* in the subject line and include the names and student IDs of both group members in the email body) to

   `conall.kelly@ucc.ie`

   before 4pm on Monday, November 30, 2020. Late submissions without acceptable documentation will be awarded a mark of zero.

2. Your submission should be clearly documented and be entirely contained within a single Jupyter notebook.

3. All code contained in the notebook is expected to work when run in sequence from a reset kernel.