HOMEWORK 5

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INNOPOLIS UNIVERSITY Dynamic of Nonlinear Robotic Systems

GitHub: https://github.com/Errors-Everywhere/Dynamics-of-RP-Robot.git

Question #1: Solve the direct dynamics of the robot

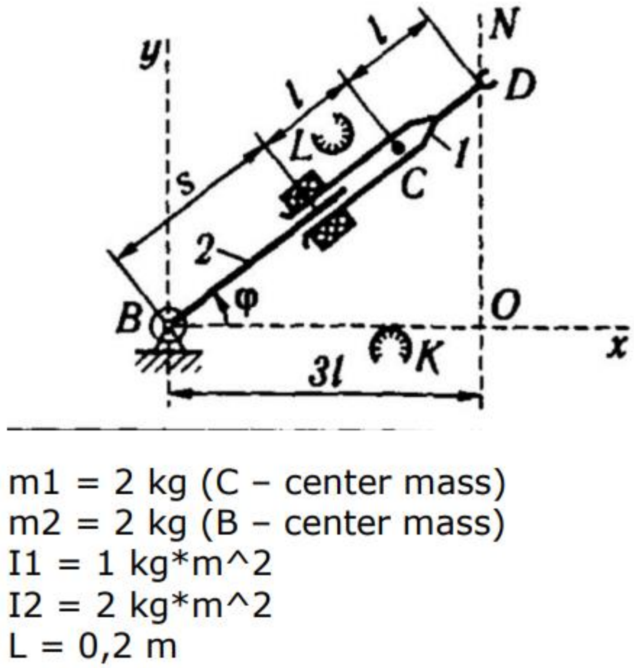

Figure 1 RP robot.

The robot consists of two joints, one is revolute and located at the origin of the world frame, and the other is a prismatic joint and also can be considered to be in the origin of the world frame.

In order to solve the dynamics of this robot using Lagrange-Euler method, we need the following steps:

1- Find forward kinematics to the center of mass of each link
2- Calculate the jacobians
3- Find the inertia matrix D
4- Find coriolos matrix C
5- Find gradient of potential energy matrix G
6- Use the Lagrange-Euler equation to represent the torque and forces as a function of the joints.

Step 1: Find forward kinematics to the center of mass of each link

For this is step it is possible to find the origin of each center of mass manually or by using the transformation matrices and extracting the translation's column out of that matrix as follow.

$$T_{c2} = R_z(\varphi) \cdot T_x(s) \cdot T_x(l) \tag{1}$$

The above transformation matrix will locate the rotation and translation elements of center of mass of the second link with respect to the world frame, thus the last column can be used as the position of this center of mass with respect to the world frame.

Step 2: Calculate the jacobians:

For this step, it is required to find the jacobians of each joint. It will be the same as if we were to find the jacobian of a whole robot that has its end-effector located at the center of mass of the link. So for J1, we will cut the robot from the center of mass and remove any parts after that and then find the jacobian of this new robot. The same will be applied to the second center of mass. And the jacobian itself will be calculated using the following formula:

$$J_v^{(i)} = \begin{cases} z_{i-1}^0 & \text{for prismatic joint} \\ z_{i-1}^0 \times \left[o_c^0 - o_{i-1}^0\right] & \text{for revolute joint} \end{cases}$$
$$J_\omega^{(i)} = \begin{cases} 0 & \text{for prismatic joint} \\ z_{i-1}^0 & \text{for revolute joint} \end{cases}$$

Step 3: Find the inertia matrix D:

The D matrix will be calculated using the following formula (which is based on the kinetic energy's formula):

$$\left[\sum_{i=1}^{n} m_i J_{v_i}(q)^T J_{v_i}(q) + J_{\omega_i}(q)^T R_i(q) I R_i(q)^T J_{\omega_i}(q)\right]$$

Where Jv and Jw are calculated from the previous step. And m, R, I are the mass, rotaiona and inertial matrices of the link. So the D matrix will be calculated for each link, and then the summation of these Ds will be the final D matrix of the robot.

Step 4: Find Coriolos matrix C:

The Coriolos matrix C depends on the matrix D. And it is calculated using the following rule:

$$\sum_{j=1}^{n} d_{kj}(q)\ddot{q}_j + \sum_{j=1}^{n}\sum_{i=1}^{n} c_{ijk}(q)\dot{q}_i\dot{q}_j + g_k(q) = \tau_k\,, \quad k = 1, \ldots, n$$

with Christoffel symbols $c_{ijk} = c_{jik}$ and gradient of the potential energy

$$c_{ijk}(q) = \frac{1}{2}\left(\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right), \quad g_k(q) = \frac{\partial}{\partial q_k}\mathcal{P}$$

Step 5: Find gradient of potential energy matrix G:

The potential energy's equation is very basic. It depends on the mass the height of the body from ground. And it is calculated using the following equation:

$$P = m \cdot g \cdot h \tag{2}$$

Where m,g and h are the mass, acceleration due to gravity and height of body from ground respectively.

The height can be found by extracting the element that corresponds to the axis of gravity out of the transformation matrix of the robot. After finding the potential energy, now it is time to get the G matrix using the following rule:

$$G = \frac{\partial}{\partial q} P \tag{3}$$

Step 6: Use the Lagrange-Euler equation to represent the torque and forces as a function of the joints:

The D, C and G matrices will be used directly in the following formula to get a relation between the torques and forces of the joints with the joint's position, velocity and acceleration. This relation can be depicted from the following equation:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{4}$$

Question 2: Resulted Equation of Motion:

After substituting with the inertias and lengths of the robot links, we will get the following equation of motion:

$$\begin{bmatrix} 2q_2^2 + 0.8q_2 + 3.08 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} \dot{q}_2(2q_2 + 0.4) & \dot{q}_1(2q_2 + 0.4) \\ -\dot{q}_1(2q_2 + 0.4) & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} 19.62 \cdot \cos(q_1)(q_2 + 0.2) \\ 19.62 \cdot \sin(q_1) \end{bmatrix} = \tau(t) = \begin{bmatrix} \tau(t) \\ f(t) \end{bmatrix}$$

Question 3: Results:

For this task, a sin function was applied to the torque that controls the first revolute joint and another cos function was applied to the force function as follow:

$$\tau = \sin(t)$$

$$f = \cos(t) + 10$$

And the following plots shows these functions along with the response of the robot as a result of these forces:

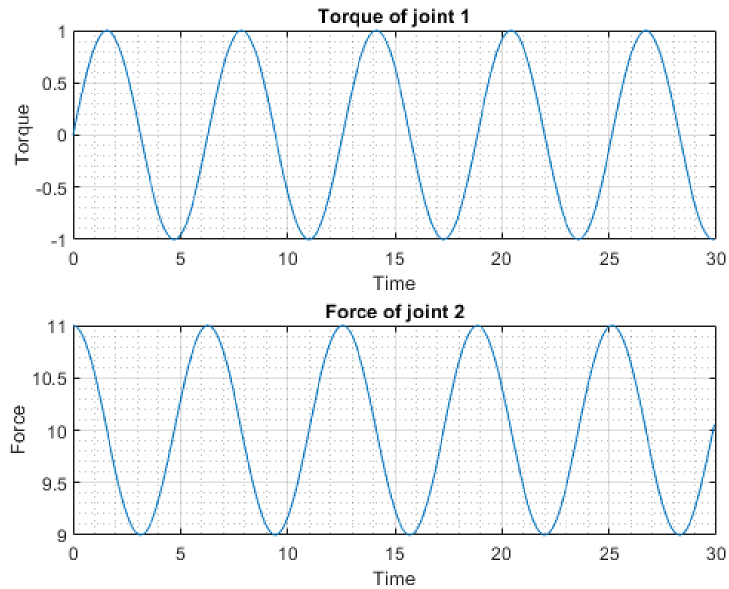

Figure 2 Plots of the force and torque applied

And this was the response of the robot:

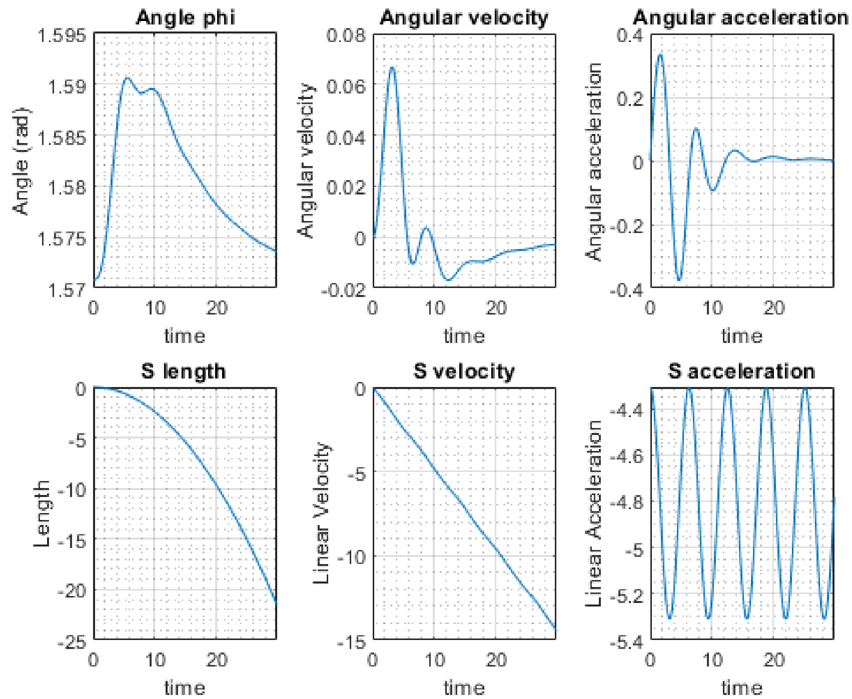

Figure 3 Robot's response

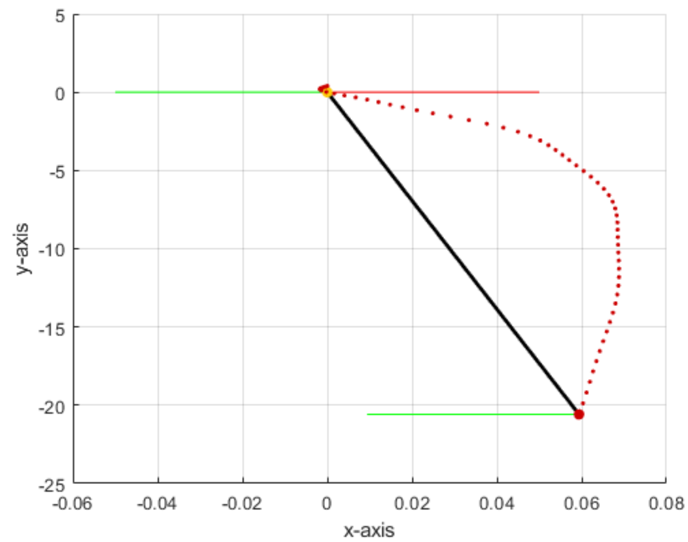

Figure 4 Trajectory in response to the applied force and torque functions

The following are additional tests applied to test the robot's behavior:

Test #1: Stabilization

In this test, I have applied no forces nor torques on the joints but initiated the robot from the stable vertical position. This showed that the angle phi will not change with time as expected and that the second prismatic joint will keep extending as a result of gravity as shown below:

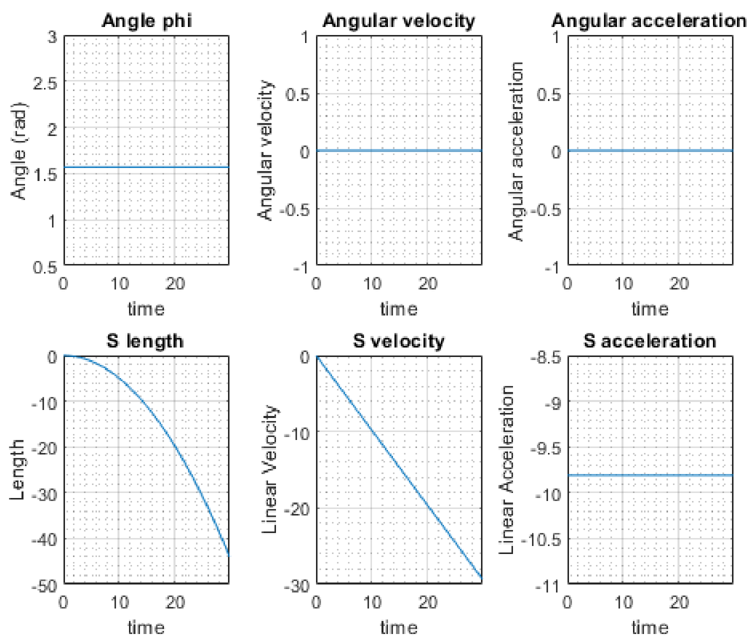

Figure 5 Test 1 plots

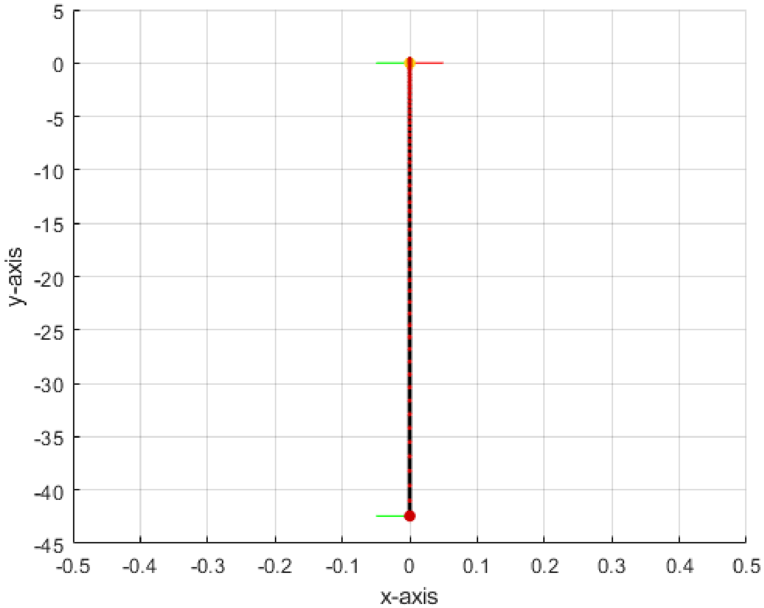

Figure 6 Test 1 response

Test #2: Pendulum first test:

In this test, I forced the acceleration of the second joint to 0, which in turn affected the velocity to be never get updated and the position as well, since both of them were initiated with zero values. This test was performed to test if the robot will behave as a pendulum in the case of having a fixed link's length or not, which in normal case should happen. And the following figures shows the response to that test.

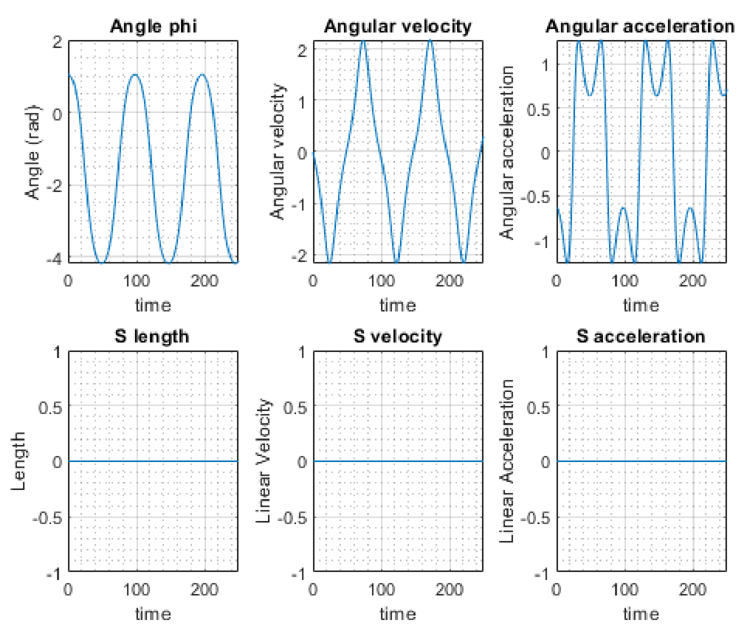

Figure 7 Pendulum 1 test plots

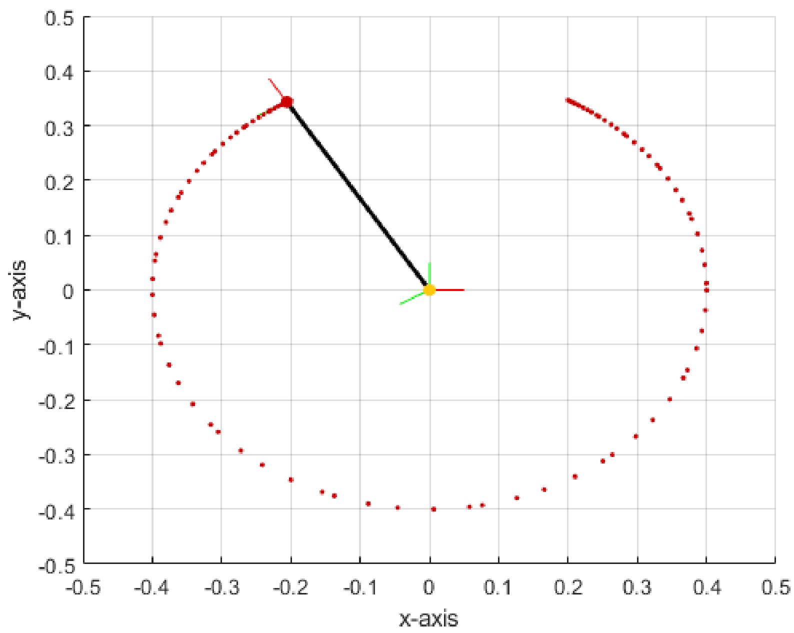

Figure 8 Robot's response to the pendulum test

Test #3: Pendulum second test:

This is the same as the previous test, but with additional increasing torque function. So the torque of the first joint is increasing with time, which resulted in the following response:

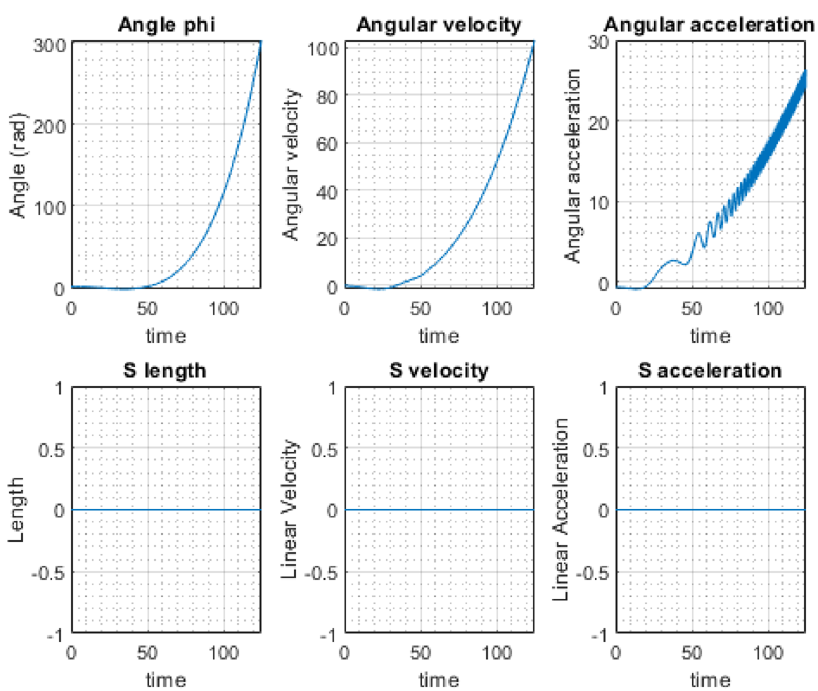

Figure 9 Pendulum 2 test plots

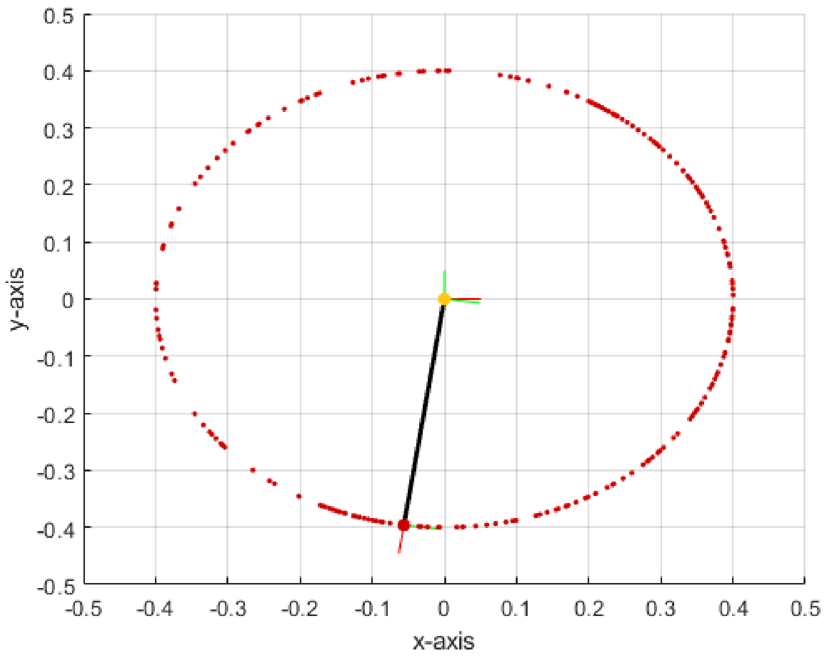

Figure 10 Robot's response to the second pendulum test