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Partitioning large signed two-mode networks: Problems and prospects

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ABSTRACT

While a substantial amount of attention within social network analysis (SNA) has been given to the study of one-mode networks, there is an increasing consideration of two-mode networks. Recent research on signed networks resulted in the relaxed structural balance (RSB) approach and its subsequent extension to signed two-mode networks involving social actors and social objects. We extend this approach to large signed two-mode networks, and address the methodological issues that arise. We develop tools to partition these types of networks and compare them with other approaches using a recently collected dataset of United Nations General Assembly roll call votes. Although our primary purpose is methodological, we take the first step towards bridging Heider's structural balance theory with recent theorizing in international relations on soft balancing of power processes.

1. Introduction

While a substantial amount of attention within social network analysis (SNA) has been given to the study of one-mode networks, there is an increasing consideration of two-mode networks. It is now recognized that such data are particularly important (see Borgatti and Everett, 1992, 1997; Doreian et al., 2004; Latapy et al., 2008). Here we focus our attention on signed two-mode networks. While our primary focus here is methodological, we stress that the technical issues are driven by substantive concerns.

The substantive problem described in Section 2.5 implies a need for tools designed to partition large[1] two-mode signed networks. One source for such a partitioning tool is found in the work of Heider (1946, 1958). He focused on two types of triples involving signed relations: those involving three individuals and those of two individuals and a social object such as a belief. His work was foundational for structural balance theory. A central assumption of this theory is that there is a tendency towards balance in these triples of relations. The outcomes of these tendencies for triples involving three actors are expressed in four folk aphorisms: "a friend of a friend is a friend"; "a friend of an enemy is an enemy"; "an enemy of a friend is an enemy"; and "an enemy of an enemy is a friend". Such outcomes and the dynamics leading to network structures consistent with balance are for one-mode signed relations. Noting that many, if not most, signed networks do not have this exact form, Doreian and Mrvar (1996) proposed an algorithm for partitioning signed one-mode networks to obtain partition structures that were as close as possible to those predicted by structural balance theory. Recognizing that multiple processes can operate to generate signed structures, Doreian and Mrvar (2009) generalized structural balance to relaxed structural balance for one-mode networks to accommodate more complex signed block structures.

Similar dynamics hold for networks involving social actors and social objects like beliefs or statements. These involve two-mode relations and, arguably, were more important in Heider's formulation using unit formation relations (between social actors and social objects). This led to an expansion of the notion of relaxed structural balance (Mrvar and Doreian, 2009) through the development of a method for delineating the partition structure of two-mode signed networks. This paper extends that approach to address problems that may be encountered when partitioning signed two-mode data. We illustrate our approach with a recently collected dataset of United Nations General Assembly (UNGA) roll call votes, as they provide a natural example of this signed data type with *states* (as social actors) voting for or against *resolutions* (as social objects). In addition, the diversity among states and resolutions means countries are likely to have overlapping and even conflicting loyalties

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[1] Large is an inherently ambiguous term because it is a function of the size of the network, the algorithm used, the speed of a machine, and its memory. Intuitively, we think a network is 'large' when, for running a program, it ushers in significant computational burdens. Direct blockmodeling is a computationally burdensome approach and we restrict the term 'large' for networks that greatly lengthen the running time of the relocation algorithm we use. Operationally, this can be specified as sizes above $n_1 > 100$, $n_2 > 100$ (using the notation introduced below) for two-mode networks. The islands technique (Zaveršnik and Batagelj, 2004) that we use in Section 6.3 can easily handle networks with millions of vertices because it is a linear-in-time algorithm.

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that lead to more complex processes and outcomes—items that tend not to be considered within structural balance approaches. Typically, UNGA voting data have been analyzed using methods that locate primary divisions among states in an attempt to identify potential fault lines for conflict. Our approach to these data is unique in that we exploit their structural characteristics to further explore structural balance theory and the tools needed to partition large signed two-mode networks.

The rest of this paper is organized as follows: Section 2 elaborates relaxed structural balance theory, its natural application to international relations data, and explores its complementarity with balance of power ideas. We describe the data in Section 3, elaborate our primary methodological approach in Section 4, and we provide the results from this approach in Section 5. We then explore the data using other partitioning approaches and compare the results with our own in Section 6. In Section 7 we discuss some of the methodological problems with using a blockmodeling approach to partition large signed two-mode data, as well as the potential for its use in exploring soft balancing of power processes. We conclude with recommendations for further methodological development of blockmodeling approaches to signed data.

2. Applying structural balance theory to international relations data

We couple relaxed structural balance to balance of power ideas to provide a substantive foundation for the partitioning methods we consider here.

2.1. Relaxed structural balance

Heider (1946, 1958) provided the foundational statements for a major approach to signed networks known as structural balance theory. He focused on signed relations between people, signed relations between people and social objects, and the implications of social structures involving these signed relations. These structures of relations are in the form of particular triples. Triples involving three social actors (p, o, and q) are *poq*-triples and triples involving two social actors, p and o, and a social object, x, are *pox*-triples. Beliefs and ideas as examples of social objects are particularly salient for our consideration—and extension—of structural balance. Relations between actors are social relations and Heider labeled the ties between actors and social objects as *unit formation* relations. Examples of signed social relations between social actors are like/dislike, love/hate and respect/disrespect. For the unit formation relations, examples include accepting/rejecting beliefs and supporting/opposing ideas.

There are 8 types of triples for a set of three relations between three actors (each tie can be positive or negative). Heider divided these triples into two types: balanced and unbalanced. Denoting positive ties by 1 and negative ties by -1, the sign of a triple is the product of the signs of the ties in the triple. A triple is defined as *balanced* if the product of the signs in the triple is 1 and *imbalanced* if this product is -1. According to Heider, imbalanced triples are a source of strain for the individuals in them and the individuals will attempt to move from having imbalanced triples to having balanced triples through changes in the sign(s) of relations. For p in a *poq*-triple, if p has positive ties to o and q but knows that there is a negative tie between o and q, the sign of the triple is -1 and is imbalanced. According to Heider, p will attempt to balance the triple by changing the sign of a tie. However, if there are imbalanced triples in sets of either *poq*-triples or *pox*-triples then there will be many attempts, by different actors, to achieve balance. This makes achieving balance over a whole network of social actors a difficult and error prone process (Hummon and Doreian, 2003).

Cartwright and Harary (1956) formalized Heider's theory and focused on signed ties between social actors and, in effect, discarded unit formation relations. A *binary signed network* is an ordered pair $(\mathcal{G}, \sigma)$, where:

1. $\mathcal{G} = (\mathcal{U}, \mathcal{A})$ is a digraph, without loops, having a set of vertices, $\mathcal{U}$, and a set of arcs, $\mathcal{A}$, where $\mathcal{A}$ is a subset of $\mathcal{U} \times \mathcal{U}$; and
2. $\sigma : \mathcal{A} \to \{+1, -1\}$ is a sign function where positive arcs have the sign +1 and negative arcs have the sign -1.

If a network has multiple weak components, these components can be considered separately as distinct networks. Here, we assume that the digraph is weakly connected.[2] Cartwright and Harary (1956) proved the following:

Theorem 1. *For a balanced signed network, $(\mathcal{G}, \sigma)$, the vertices in $\mathcal{U}$ can be partitioned into two subsets[3] such that each positive arc joins vertices in the same subset and each negative arc joins vertices in different subsets.*

Harary et al. (1965, pp. 342–3) prove that for all pairs of vertices in a balanced network, all semi-paths joining them have the same sign. Davis (1967) observed that, despite the appeal of Theorem 1, there are social groups where there are more than two subsets of mutually hostile groups of social actors. He proposed that the all negative triple (with the sign of -1) not be classified as imbalanced. Consistent with this, he defined a signed network as *clusterable* if it contains no semi-cycle with exactly one negative tie. He then proved:

Theorem 2. *For a clusterable signed network,[4] the set of vertices, $\mathcal{U}$, can be partitioned into two or more subsets such that every positive arc joins vertices in the same subset and every negative arc joins vertices in different subsets.*

The network structure implied by the above (structure) theorems can be described also in terms of blockmodeling (see Doreian et al., 2005: Chapter 10). We use the term *position* for a cluster of vertices (representing actors) and, given a set of positions, a *block* is a set of ties between positions. The diagonal blocks are positive (with only positive ties between vertices within subsets) and the off-diagonal blocks will be negative (with the negative ties between vertices in different subsets). A *positive block* is one that has only positive or null ties and a *negative block* has only negative or null ties. Rather than use both balanced and clusterable as terms, we use *k-balance* to cover both of them.[5]

However, most empirical signed networks do not have a structure consistent with either of these two theorems. Put differently, most empirical signed networks are not k-balanced.[6] Yet, if there are balance processes that are operative, they ought to leave

[2] The presence of multiple weak components requires some mild restatement of the results but do not alter their intrinsic content.

[3] If every tie in the network is positive then one of the two subsets is empty.

[4] By definition, a signed network must contain at least one negative tie: if a network contains no negative ties it is an unsigned network. None of the balance theoretic processes involving *both* positive and negative ties have relevance for unsigned networks. By the same token, attempts to assess structural balance theory using unsigned data cannot provide a useful assessment. The only occasion when an unsigned network is relevant with regard to structural balance is when it is the outcome of balancing processes.

[5] If there are only two subsets then we are dealing with balance (k-balance, $k = 2$) and if there are more than two subsets then we are dealing with clusterability (k-balance, $k > 2$).

[6] This calls into question the validity of a general claim of a tendency towards balance. Doreian and Krackhardt (2001) examined the trajectories of the eight types of triples over time in the Newcomb data (Newcomb, 1961, Nordlie, 1958). Two of the balanced triples increased in number of time, consistent with Heider's theory, but the other two balanced triples decreased in frequency over time. Two types of imbalanced triples decreased in frequency over time, also consistent with Heider,

some structural traces. Instead of asking if signed networks are k-balanced or not, it is more fruitful, empirically, to seek partitions of signed networks that are as *close to being k-balanced as possible*. If an empirical network is k-balanced then there will be no inconsistent ties within either type of block. But if the network is not k-balanced then there will be some positive ties in negative blocks, or some negative ties in positive blocks, or both. We denote the number of negative inconsistencies by $\mathcal{N}$ and the number of positive inconsistencies by $\mathcal{P}$. To locate partitions as close as possible to a perfect balanced partition, it is necessary to measure the extent to which a partition departs from zero inconsistencies. A simple and direct measure of this is $(\alpha\mathcal{N} + (1-\alpha)\mathcal{P})$ with $0 < \alpha < 1$ (varying α permits a differential weighting of $\mathcal{N}$ and $\mathcal{P}$). All that is needed is an algorithm to locate a partition(s) that minimizes this measure. Doreian and Mrvar (1996) proposed a partitioning algorithm—described in Section 4.1—that does exactly this.

Doreian and Mrvar (2009) reconsidered the problem that most empirical signed networks are not exactly balanced. While balance theoretic mechanisms may be operative, they need not be the only mechanisms in play. Some members of a signed network may be universally regarded in positive terms even though they may belong to different positions with negative ties between the positions. If present, such actors imply positive blocks off the main diagonal. Expressed differently, differential popularity (Feld and Elmore, 1982) implies positive blocks off the main diagonal. Actors who play a mediating role between mutually hostile subsets also imply off-diagonal positive blocks and sets of mutually hostile actors (not blocks) imply negative blocks on the diagonal. In response to these considerations, Doreian and Mrvar (2009) proposed the notion of *relaxed structural balance* (RSB). This idea has three components: (i) the idea of positive and negative blocks is retained; (ii) these blocks can appear *anywhere* in the blockmodel; and (iii) the measure of inconsistency is retained. For such a blockmodel, then if $(\alpha\mathcal{N} + (1-\alpha)\mathcal{P}) = 0$ for a signed network then the network is a *relaxed structurally balanced* network. They show that RSB is a formal generalization of structural balance where the block structures anticipated by Theorems 1 and 2 are special cases. On fitting the RSB model to some of the classical signed networks in the literature, they obtained better fitting models that permitted more nuanced interpretations of the prior partitions of these signed networks. More importantly, thinking of positive and negative block types being located anywhere in a blockmodel opens the way to partitioning signed two-mode networks.

2.2. Relaxed structural balance for signed two-mode networks

For two-mode networks (also called affiliation networks), there are two sets of social actors. Typical examples include people attending events, individuals sitting on organizational boards of directors, and people belonging to multiple social or recreational organizations. Mrvar and Doreian (2009) extended relaxed structural balance to signed two-mode networks to consider US Supreme Court Justices (as social actors) supporting or dissenting from Supreme Court decisions (as social objects). In doing so, they returned to Heider's unit formation relations and used signed two-mode networks to formalize that aspect of Heider's theory. The general formalism extends straightforwardly.

Let $\mathcal{U} = (u_1, u_2, \ldots, u_{n_1})$ denotes the set of social actors (represented as vertices) and $\mathcal{V} = (v_1, v_2, \ldots, v_{n_2})$ denotes the set of social objects (represented as vertices). The number of vertices in $\mathcal{U}$, is denoted by n_1 and the number of vertices in $\mathcal{V}$, is denoted by n_2. An *undirected binary signed two-mode network* is an ordered pair, $(\mathcal{G}, \sigma)$, where:

1. $\mathcal{G} = (\mathcal{U}, \mathcal{V}, \mathcal{E})$ is a bipartite network having two sets of vertices, $\mathcal{U}$ and $\mathcal{V}$, and a set of edges,[7] $\mathcal{E}$, where $\mathcal{E} \subseteq \mathcal{U} \times \mathcal{V}$ (where $\mathcal{U} \cap \mathcal{V}$ is empty); and
2. $\sigma : \mathcal{E} \to \{+1, -1\}$ is a sign function where positive edges have the sign +1 and negative edges have the sign -1.

Clearly, with two-mode networks, the idea of diagonal and off-diagonal blocks does not apply and having positive and negative blocks appearing anywhere follows naturally. All of the formal development for one-mode networks extends straightforwardly to two-mode networks. Instead of partitioning a one-mode network into k clusters, a two-mode network is partitioned into k_1 clusters of rows (social actors) and k_2 clusters of columns (social objects). We define a (k_1, k_2) *partition* of $(\mathcal{G}, \sigma)$ as one where there are k_1 clusters in the partition of $\mathcal{U}$ and k_2 clusters in the partitions of $\mathcal{V}$. Let $\mathcal{C} = (\mathcal{C}_1, \mathcal{C}_2)$ denote a (k_1, k_2) partition of $(\mathcal{G}, \sigma)$ where $\mathcal{C}_1$ is a partition of $\mathcal{V}$ and $\mathcal{C}_2$ is a partition of $\mathcal{V}$. The measure of inconsistency for the (k_1, k_2) partition of $(\mathcal{G}, \sigma)$ remains $(\alpha\mathcal{N} + (1-\alpha)\mathcal{P})$. Mrvar and Doreian (2009) adapted their one-mode algorithm to identify partitions of two-mode signed networks that minimize this measure of inconsistency.

2.3. Coupling relaxed structural balance with balance of power processes

Heider's (1946) approach to structural balance theory is social psychological which, at face value, is quite different from the ideas in social network analysis. Yet, they can be coupled in fruitful ways (Robins and Kashima, 2008). There are micro-level processes operating at the social actor (vertex) level—what an actor does in specific situations—and macro-level structural processes affecting the structure as a whole. The two processes act to constrain each other: while actors are free to do whatever they want to do (given their interests), they need to be mindful of what others are doing and the nature of the social relations within which they are located.[8] Robins and Kashima point out that accepting this approach is to accept also a dynamic view of these structural processes. We note that Heider's theory about empirical triples embraces a very dynamic approach featuring change as an essential part of generating structural outcomes.

We explore the macro-level implications of Heider's structural balance theory in two ways. First, we extend Heider's assumption of a tendency towards balance among multiple actors to balance among state actors in the international system of states. Balance of power theories in International Relations research share assumptions of balancing processes among states, and alliance formation implied by balancing processes evokes the same four folk aphorisms (e.g., "a friend of an enemy is an enemy"...). In the post-Cold War period of unipolar military power, international relations theorists have proposed the idea of "soft power" balancing through

but the other two types of imbalanced triples increased and became more frequent over time.

[7] Our choice of using edges rather than arcs is driven primarily by the empirical example. With resolutions and states, there is no obvious way of determining the arcs linking them. If (the representatives of) states cast their votes about the resolution the arcs could go from states to resolutions. But if resolutions prompt the states to vote in certain ways the arcs go from resolutions to states. It is simpler to couple states and resolutions by edges where the signs indicating how they voted. There are no edges between states and none between resolutions. The formalization goes through with little change if arcs are used instead of edges.

[8] As individuals, Romeo and Juliet were free to fall in love with each other despite belonging to two different mutually hostile families. This macro-feature was a powerful constraint on their choices through the actions of others with whom they were linked. The imbalance implied by their love drives the drama of Shakespeare's *Romeo and Juliet*.

norms. We explore this idea using UN General Assembly (UNGA) military resolutions as they contain evidence of both hard (decisions to go to war) and soft (norms and laws to constrain military power) balancing of power processes.

Second, we show how the relaxed structural balance approach can be used to explore balancing of power processes. We do this in the course of developing the methodological contribution of this paper, namely, identification of and solutions for problems with applying the relaxed structural balance approach to large signed two-mode data.

2.4. Balance of power and international relations

Balance of power is an International Relations (IR) theory that addresses the process of building coalitions among states to prevent one state from becoming too powerful. There are two variants of this theory—'hard' and 'soft'. Hard balance of power theory is rooted in military alliances and arms build-ups associated with them. Soft balancing behavior, in contrast, is considered a state strategy, or foreign policy conduct, to prevent a rising power from assuming hegemony but without a primary emphasis on force. Some balance of power is the outcome at the systemic or sub-systemic levels that result in power equilibria among key states. Indeed, recent debates in the IR literature question whether in an age of a unipolar (U.S. hegemonic) power, states are currently engaging in traditional hard balancing behavior. Instead, some have argued that diplomatic coalitions and norms building (as soft balancing) define much of state behavior.[9] Soft balancing strategies can include informal alliances, collaboration in regional or international institutions, or voting and veto power in international organizations such as the United Nations (UN) (Art, 2005/2006; Brooks and Wohlforth, 2005/2006; Paul, 2004). Hard balancing for social relations in one-mode networks leads to states within an alliance having positive ties to other states in the alliance and negative ties to states in other alliances. The core idea is *consistency* in the relations formed. Soft balancing also involves consistency in the sense of states in blocs voting in the same way on resolutions of central concern to bloc members and voting against resolutions supported by (some) other blocs of states.

It is generally accepted that the UN is both the pre-eminent international organization (IO), and that it plays a central role in the maintenance of international security. Although the UN Security Council (UNSC) is the UN organ most associated with the use of, or constraints on, the exercise of military power, the UNGA has also played an important, if less publicly visible, role. The First Committee, one of the six permanent committees of the UNGA, has worked to develop international norms and laws aimed at restricting the exercise of military power. Although its resolutions are non-binding, many have crystallized into customary law while others have become the basis for treaties that limit military power either by prohibiting the use of some weapons (e.g., anti-personnel mines, chemical or biological weapons) to efforts to create zones of peace that prohibit nuclear weapons altogether.

Voting analyses on military resolutions typically reveal voting similarity across key issues by states who share alliances or key attributes and affiliations. Consequently, these data lend themselves to an exploration of the relaxed structural balance approach, and provide an opportunity to explore the relationship of Heider's structural balance theory with primarily soft balancing of power processes theorized in IR theory. We compare two time periods in order to delineate the structure of the voting array at each time point and to assess change between them. The first is in the late Cold War period characterized by ideologically and militarily opposed groups of states. The second time period examines the 5–10 years after the end of the Cold War. This period includes the dissolution of the Warsaw Pact and the re-alignment of many former Soviet bloc countries with NATO and European Union member states. We expect empirically to find similar clustering of states as prior studies as far as the large clusters of states (often seen as voting blocs) are concerned,[10] but also evidence of additional sub-clustering within the major blocs that can be exploited for our theoretical purposes. Part of this further exploration concerns the clusters of resolutions in relation to the voting blocs. To the extent that hard balancing involves military issues with a real possibility of warfare, states 'choosing' sides face huge consequences for their choices. Voting on military resolutions before UNGA, in terms of committed resources, is less consequential but far more subtle in its implications. The notion of being consistent for states within voting blocs remains in place but permits greater flexibility when there are cross-cutting issues.

2.5. Empirical implications

Balance of power arguments are made in terms of blocs (clusters as positions) of states allied with each other and mobilized against blocs of other states. With alliance-based sets of interests, members of an alliance are expected to act together in opposition to members of other alliances. Members of the other alliances also act in concert. If the interests of alliance members are activated by resolutions before the UNGA, then these members are expected to vote similarly to other members on these resolutions. This can take the form of all voting in support of some resolutions and voting against other resolutions. During the era of the Cold War this idealized voting pattern could be illustrated by NATO states voting together and members of the Warsaw Pact voting together and in opposition to NATO.

However, members of one alliance can have interests beyond those implied by membership in a particular military or political alliance—or in an international organization. Some specific issues could divide alliance members so that members of an alliance do not vote in the same fashion across all issues. Generally, we expect patterns in voting behavior to reveal long-standing coalitions of states that are similar on attributes or affiliations, but we also expect a certain amount of self-interest, logrolling, and bargaining to result in other temporary coalitions. Our aim is to show how the RSB approach can be used to locate balancing of power processes. Specifically, we expect that simultaneously partitioning of *both* states and resolutions will highlight broad patterns of joint voting and also lesser patterns of temporary coalition formation. We illustrate our approach in Fig. 1.

Fig. 1 shows 12 states and 13 resolutions in a hypothetical voting array. The top panel shows a two-mode network composed of states, s_1 through s_{12}, that are represented by circles, and resolutions, r_1 through r_{13}, represented by squares. Solid lines between states and resolutions represent votes in favor of resolutions. In contrast, the dashed lines represent votes against resolutions. The states have been placed into three subsets that can be labeled as S_1, S_2 and S_3. The states in S_1 are s_1 through s_7; the states in S_2 are s_6 though s_9; and S_3 is composed of s_{10} through s_{12}. The resolutions have also been placed in three subsets: R_1 contains r_1 through r_7; R_2 contains r_8 through r_{10}; and R_3 contains r_{11} through r_{13}. The structure of this network can be described simply. States in S_1 tend

[9] See for example, World Politics 61 (1), January 2009, and International Security 30 (1), Summer 2005; both are special issues devoted to examining the continual utility of the balance of power concept in international relations given the characteristics of current world politics.

[10] See Kim and Russett (1996) and Voeten (2000) for example.

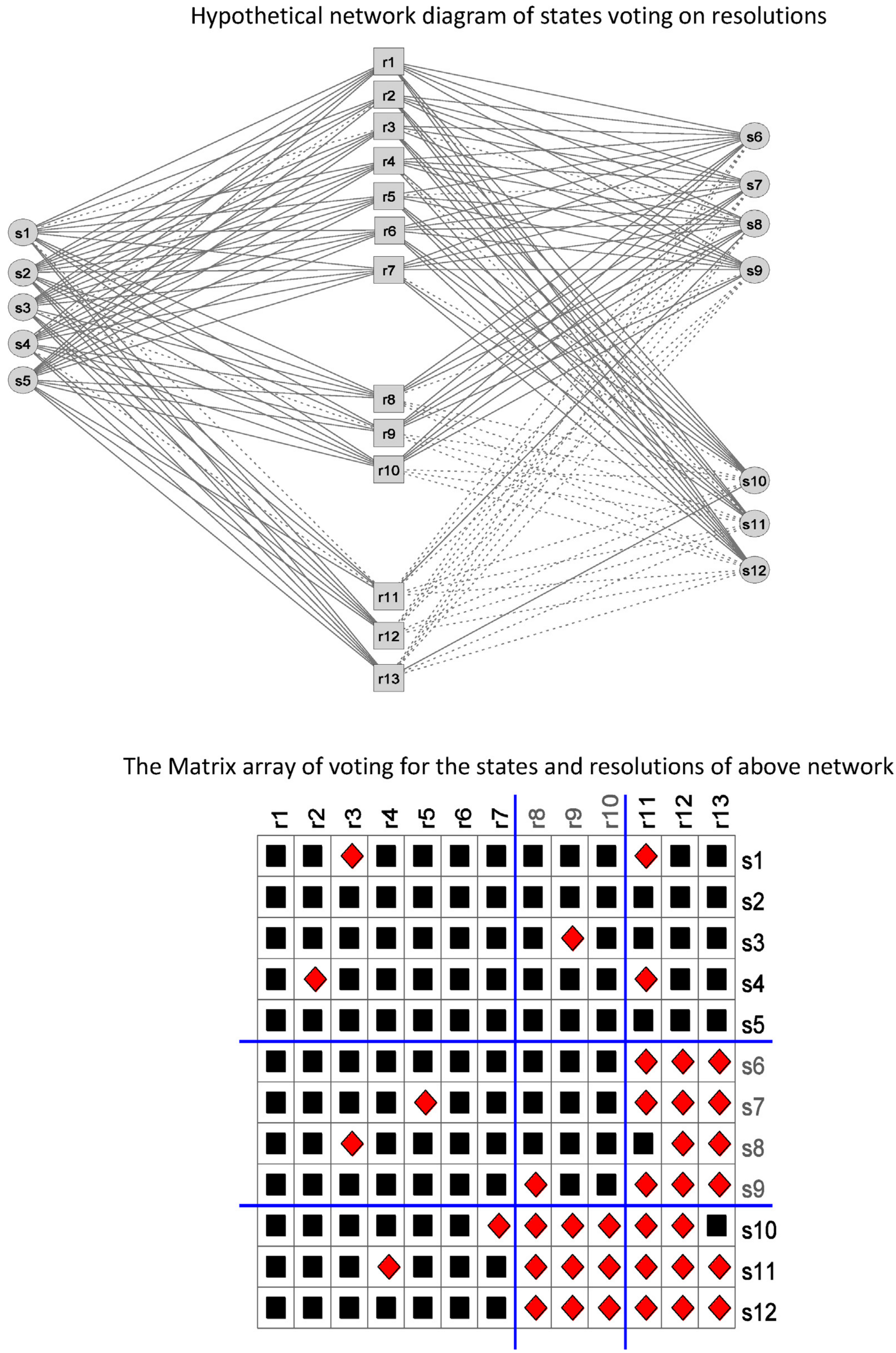


Fig. 1. Illustrative network and formatted array for states and voting on resolutions.

to vote in support of all resolutions; states in S_2 vote in support of resolutions in R_1 and R_2 but tend to vote against resolutions in R_3; and states in S_3 tend support only resolutions in R_1 and tend to oppose resolutions in both R_2 and R_3. There are also some votes not fitting with this general description.

The lower panel of Fig. 1 contains an alternative representation of exactly the same information but in a formatted array. Black squares represent votes in support of resolutions and red diamonds represent votes against resolutions. The array has been formatted to that members of the above clusters of states and resolutions are placed together. Solid lines mark the boundaries between the clusters. The names of the resolutions are written across the top and the names of states are on the right side of the array. The signed block structure for this hypothetical example, using the labeling of

P for positive blocks and N for negative blocks, is as follows: in the top row, the blocks are PPP; the blocks in the second row are PPN; and the blocks in the bottom row are PNN. When the two-mode networks are large, the network diagrams get to be unwieldy and difficult to read. For large datasets, the formatted arrays are simpler to read and form a more compact representation.

The real data for the UNGA voting are more complex in two ways: (i) trivially, they are much larger and (ii) exceptions are far more likely to occur than revealed in the hypothetical motivating example. In short, the real data are messy but, we expect blockmodeling to reveal distinct patterns in the form of positive and negative blocks, despite expected exceptions. A disadvantage is that the exceptions will make the network diagrams challenging to read in the standard blockmodeling format; therefore we will report the majority of our graphical results using the formatted (blockmodeling) arrays.

3. Data

We use a dataset of roll call votes for the United Nations General Assembly (UNGA) covering the decades before and after the end of the Cold War and the dissolution of the Warsaw Pact.[11] The data were divided into four time periods: 1981–1985, late Cold War; 1986–1990, a first transition period; 1991–1995, a second transition period; and 1996–2001, a "settled" period, albeit at the juncture of 9/11. As noted above, we select the first and last periods which we label as Time 1 and Time 2, respectively.

We focus on military resolutions because this subset of resolutions is the most substantively appropriate for testing our coupling of balance of power ideas with relaxed structural balance theory.[12] Given the type of data—international voting data—where states have overlapping and even conflicting loyalties (Turkey with NATO and the Organization of Islamic Conference (OIC) for example)—voting blocs are likely to shift by issue areas. The potential advantage of blockmodeling is that it can identify and define blocs by the resolutions favored by the states in those blocs. Other methods can also accomplish this but blockmodeling has a direct way of grouping these together for further exploration by representing the underlying network structure in image matrices using rows of block types (PPP, PPN, etc.).

The Time 1 data set has 141 states and 276 resolutions. While there were 159 UN member states during this time period, we deleted 18 states because they were absent for 25% or more of the votes during this time period. Our second time period (Time 2) had 153 states and 150 resolutions.[13] There were 189 UN member states as of 2001; we removed 34 states that were absent for 25% or more of the votes during this time period. A full list of the names is provided in Appendix A and a summary of the resolutions is provided in Appendix B. The continual process of decolonization and increase in membership of newly independent states, including those resulting from the dissolution of the Soviet Union resulted in an increase of roughly 30 new member states for our second time period. Because our focus is on balancing processes and changing alliances, we think it is important to include all of the new states rather than focus on the same subset of states for both time periods. This is particularly important given the enlargement of NATO and the EU with former Soviet bloc states. The number of resolutions *decreased* because of a deliberate effort by the UNGA to reach consensus on resolutions once the Cold War impediment to international policy-making had ended. This results in a significant reduction in roll call votes. There were a total of 725 roll call votes during our first time period and 406 in our second. Of these, military resolutions were the most numerous (38% and 37% respectively), highlighting the importance of these issues to the UN mandate to promote international peace and security.

Coding the votes cast on military (and other types of) resolutions is not straightforward. Ideally, there are only votes for a resolution and votes against a resolution. However, states can abstain from voting on certain resolutions and their representatives may choose to be absent when a vote is taken on other resolutions. These absences could be coded as 0, but this decision would obscure deliberate absences. Interviews with UN delegates and permanent UNGA personnel[14] indicate that frequently absences can be grouped with abstentions as a "weak no" vote. Therefore we made the choice of first removing all countries that were absent 25% or more of the time for a given subset of roll call votes. We then recoded the few remaining absences as a "−1". We coded votes in support of a resolution as "+1". We believe this coding accurately captures state voting patterns.

4. Methods

4.1. Direct signed blockmodeling

Signed blockmodeling is a label for the partitioning of signed networks and *direct signed blockmodeling* is the approach based on the ideas described in Sections 2.1–2.4 and used here. The Doreian and Mrvar algorithm for doing this has the following features:

1. The measure of inconsistency described earlier can be used as a *criterion function* (Doreian et al., 2005) measuring departures from exact balance. Departures from exact balance take only two forms: positive ties in negative blocks and negative ties in positive blocks. As before, $\mathcal{N}$ denotes the number of negative ties in positive blocks and $\mathcal{P}$ denote the number of positive ties in negative blocks. Let $\mathcal{C}$ denote a *clustering* of the vertices into mutually exclusive subsets and let $P(\mathcal{C})$ denote the value of the *criterion function* for that clustering, $\mathcal{C}$, then, $P(\mathcal{C}) = \alpha\mathcal{N} + (1-\alpha)\mathcal{P}$ where $0 < \alpha < 1$.
2. Two ranges and one value for α can be distinguished: (i) $0 < \alpha < 0.5$ (where positive inconsistencies are weighted more

[11] The data were collected by the second author from 2000 to 2002. Roll call votes from 1983 to 2001 had been digitized. A summary of all resolutions on roll call votes is found at http://www.un.org/documents/resga.htm and specifics on roll call votes are found at: http://unbisnet.un.org/. Roll call votes from 1981 to 1983 were found in hard copy at Yale University Social Science Library, 140 Prospect Street, Government Documents collection, United Nations Collection; the Government Librarian was very helpful. Assistance was also provided by the United Nations Dag Hammarskjold Library. UN Librarians and Personnel in the Department of Public Information (UN Headquarters, 1st Avenue between 45th and 46th Street, New York City) were very helpful in obtaining and verifying information.

[12] See http://www.un.org/en/ga/maincommittees/index.shtml (accessed 7/4/11) for information on the main committees and http://www.un.org/documents/resga.htm to access the resolutions. Most years contain a summary of the resolutions deliberated upon that year, including information on where the resolution was debated, e.g., one of the permanent committees or the Plenary; information on the voting; draft documents that includes sponsorship information and the general topic of the resolution.

[13] There were 129 military resolutions for the period 1996–2000 and 150 if we included the 21 resolutions for 2001. We experimented with both in case there were 9/11 effects but found the output to be substantially similar. We report the 129 resolutions for the blockmodeling results because we matched 5 year periods for both time points.

[14] The second author interviewed a number of UN delegates and permanent personnel when she collected her voting data during the 2001 UNGA session. Without exception, interviewees noted that both an "abstain" and frequently an absence was a "weak no" vote. Similarly, Voeten (2000, p. 193) argues that there is little practical difference between a "no" vote and an abstention. What matters is the willingness of a state to go on record for supporting a resolution. To separate out a situation of high absenteeism from a voting choice, we omitted all states absent for 25% of the time. This took care of the vast majority of absences that would be coded as a "no" vote.

heavily); (ii) $\alpha = 0.5$ (where positive and negative inconsistencies are weighted equally); and iii) $0.5 < \alpha < 1$ (where negative inconsistencies are weighted more heavily).

3. There is a relocation algorithm that is based on a neighborhood of a clustering, $\mathcal{C}$, defined by two transformations: (i) the movement of one vertex from one plus-set to another plus-set in $\mathcal{C}$; and (ii) the interchange of two vertices between two plus sets in $\mathcal{C}$. Given a partition, $\mathcal{C}$, this leads to a local optimization procedure where these two types of transformations are the only ones considered.
4. Given a partition, $\mathcal{C}$, the effect of these local transformations is examined in terms of their impact on $P(\mathcal{C})$. If a transformation leads to another clustering, $\mathcal{C}'$, such that $P(\mathcal{C}) > P(\mathcal{C}')$ for $\mathcal{C}'$ in the neighborhood of $\mathcal{C}$, then $\mathcal{C}'$ replaces $\mathcal{C}$ as the active clustering. This is continued until no further drop in the value of criterion function is possible.
5. The whole procedure is repeated *many* times[15] (because it is a local optimization procedure) until no further improvement is possible. The final partition(s) is (are) the *optimal*[16] *partition*(*s*) where optimal means only that the partition the minimal value of the criterion function.

For structural balance, the existence of a globally minimized partition is guaranteed, regardless of the size of the network. We use k to denote the number of subsets in a partition (clustering) $\mathcal{C}$ where $1 \leq k \leq n$ and n denotes the number of vertices in the network. A partition with $(k+1)$ clusters is an *adjacent* partition. Doreian et al. (2005, p. 305) provide:

Theorem 3. *For any signed network* $(\mathcal{G}, \sigma)$*, there will be a unique lowest value of the criterion function, denoted by* $P(\mathcal{C}^{\min})$*, that occurs for partitions with k subsets or adjacent partitions.*[17]

Note that a unique minimum value, $P(\mathcal{C}^{\min})$, does not imply that there is one unique partition with this optimal value. Theorems 1 and 2 formed a major development for the formulation of structural balance and Theorem 3 provides a clear statement about the behavior of the criterion function. Brusco et al. (2011) showed that, for networks where $n \leq 40$, the local optimization procedure locates the optimal partition(s) of signed one-mode networks. However, for relaxed structural balance, while the criterion function is defined in the same fashion, its behavior as the number of clusters, k, is increased is different. Denoting the value of the optimal value of the criterion function for partitions with k clusters by $P(\mathcal{C}_k)$, Doreian and Mrvar (2009) prove:

Theorem 4. *For establishing optimal partitions using the relaxed structural balance blockmodel, the values of* $P(\mathcal{C}_k)$ *decline monotonically as k increases.*

The nice behavior of the criterion function for structural balance (Theorem 3) is lost when relaxed structural balance is considered (unless an empirical network conforms to, or closely approximates, structural balance).[18] The declining monotonic property of the criterion function for relaxed structural balance for signed one-mode networks holds also for signed two-mode networks. Mrvar and Doreian (2009) prove:

Theorem 5. *Given a signed two-mode network* $\mathcal{G} = (\mathcal{U}, \mathcal{V}, \mathcal{E})$ *and a set of optimal partitions for the* (k_1, k_2)*-partitions of* $\mathcal{G}$*, with* $1 \leq k_1 \leq n_1$ *and with* $1 \leq k_2 \leq n_2$*, the optimal values of* $P(\mathcal{C})$ *decline monotonically with* k_1 *for each value of* k_2 *and monotonically with* k_2 *for each value of* k_1.

While relaxed structural balance, as a generalization of structural balance, has useful properties that include being the foundation of a method for partitioning signed two-mode networks, the behavior of the criterion function in relation to k_1 and k_2, as described in Theorem 5, ushers in some serious problems. These problems are the methodological focus of this paper. To state the first problem, we define the *grain* of a partition in terms of the number of clusters in a partition (for both one-mode and two-mode networks). Loosely, partitions having many clusters are *fine-grained* and partitions with few clusters are *coarse-grained*. The minimum value of the criterion function for relaxed structural balance is 0 and this value must occur when every vertex is a singleton in a cluster. This is the most fine-grained partition possible but is useless as a blockmodel. Empirically, it is possible that this value is reached for extremely fine grained partitions that also have little utility for blockmodeling. This implies that to establish a reasonable coarse-grained partition, some judgment is required.[19] When the implicit 3D plot (of the criterion function against k_1 and k_2) becomes flatter it suggests that the coarsest-grained partition for this flattened curve is an appropriate partition. Second, for signed networks that have the size of the UNGA voting arrays considered here, the guarantee offered by the results of Brusco et al. (2011) no longer holds.[20] It follows that the resulting partitions be subject to additional scrutiny in an effort to ensure a useful partition. In general, this involves some consideration of alternative partitions near a candidate partition. Finally, using the local optimization algorithm includes some searching over different values of k_1 and k_2 is computationally demanding when k_1 and k_2 are large.[21] It follows that partitioning over large ranges of k_1 and k_2 would be extremely time consuming. Indirect blockmodeling may be a way of seeking guidance as to where to focus attention in terms of k_1 and k_2.

4.2. Indirect signed blockmodeling

Doreian et al. (2005) distinguish direct blockmodeling, as described above, from indirect blockmodeling where summaries of the data, in the form of (dis)similarity measures constructed from a network, are used to partition the network data with some standard clustering method (Doreian et al., 2005, pp. 177–84).

[15] At a minimum, we recommend using many thousands of repetitions.

[16] There are two meanings for this term. One is the best that was found by using the local optimization method (thus far). A more general meaning for *optimal partition* is for the globally optimal partition(s)—which might not be obtained in a specific analysis. All theorems regarding optimal partitions apply for the latter meaning of this term.

[17] The result is intuitively reasonable. If $k = 1$, then every negative tie is an inconsistency and, at the other extreme, if $k = n$, then every positive tie is an inconsistency. As k increases from 1, in general, $P(\mathcal{C})$ decreases monotonically until $(P(\mathcal{C}^{\min}))$ is reached and as k is increased further then the value of $P(\mathcal{C})$ increases. However, it is possible that for adjacent partitions the value of $P(\mathcal{C})$ remains at the optimal value and the plot of $P(\mathcal{C})$ against k is flat for these values before increasing as k is increased.

[18] Given a partition whose criterion function is optimal value is 0 under structural balance; increases in the value of k, under relaxed structural balance will produce more fine grained partitions with the same value of the criterion function. If the optimized value of the criterion function under structural balance is not 0 then, under relaxed structural balance, the value of $P(\mathcal{C})$ will decline monotonically, consistent with Theorem 4.

[19] The same property holds for structural equivalence although this is seldom recognized.

[20] For the smaller networks that Brusco et al. (2011) considered, the results of using the local optimization algorithm were compared with those obtained from an exact (branch-and-bound) algorithm guaranteed to identify the optimal partitions. In all of the networks they considered, the returned partitions from both algorithms were identical. Alas, the exact algorithm is completely impractical for large networks and such comparisons cannot be made. While this suggests another definition of 'large'—a signed network is large ($n > 40$ for one-mode networks) when this guarantee is no longer available—we think that this is overly conservative with 'large' being used for rather small networks.

[21] For the P4 data with $k_1 = 7$ and $k_2 = 7$ 100,000 repetitions, using an HP HDX18 Notebook PC with an Intel Core 2 CPU Q9000 running at 2.00 GHz and with installed memory of 4.96 GB, a single run took about 4 h and 40 min. Seeking more efficient methods seems merited provided that the resulting partitions are the best possible partitions.

Doreian et al. (1994) compared the direct and indirect approaches for some well known small networks. They report that, most of the time, the direct approach produced better fitting partitions, in terms of the values of the criterion function reported for each method, than those obtained by the indirect approach. Further, the indirect approach never outperformed the direct approach for these small networks. However, the direct blockmodeling approach can be extremely burdensome computationally and this problem becomes acute as network sizes increases. Given that the signed UNGA voting data set that we use is much larger than the signed networks considered hitherto, it seems prudent to consider *indirect signed blockmodeling*—blockmodeling signed data using indirect methods—as well.

The data described in Section 3 are very close to being complete (having zero or near-zero null ties). One faster indirect way of partitioning the rows and columns is to partition them separately and then fuse the two partitions to form a joint partition of the two-mode signed network. For the rows, we computed the Euclidean distance[22] for the vectors of each row. Letting g_{ik} denote the (i, k) element of G, the Euclidean distance between u_i and u_j, $d(u_i, u_j)$, is obtained from $d^2(u_i, u_j) = \sum(g_{ik} - g_{jk})^2$ and the Euclidean distance between v_i and v_j, $d(v_i, v_j)$, is obtained from $d^2(v_i, u_j) = \sum(g_{ki} - g_{kj})^2$ in separate computations. These were used as input for a standard hierarchical clustering program where we used the Ward clustering method (Ward, 1963). The two hierarchical clustering algorithms each produce a dendrogram that can be visually inspected to determine the number of clusters (k_1 for rows and k_2 for columns). Of course, there is flexibility here and this introduces another element of judgment.

4.3. Comparing partitions

Our primary goal is to establish blockmodels of signed two-mode data that are meaningful and well defined. This implies that comparisons of partitions have to be made. These comparisons arise in three ways. First, given the establishment of a blockmodeling partition, it is necessary to establish that it is better than a random partition with the same numbers of clusters. Second, given the result of Theorem 5, and the implication that an element of judgment is needed to choose from among fitted blockmodels (with different values of k_1 and k_2). Given a move from a coarse-grained partition to a more fine-grained partition—where Theorem 5 points to a smaller value of the criterion function for the finer-grained partition—it is necessary to check that the difference is large enough to justify the move. Third, as we do use other partitioning methods, we need a way of comparing partitions obtained from using different methods.

Direct blockmodeling has a trio of criteria for evaluating partitions: (i) the form of the blockmodel in terms of its block structure; (ii) the agreement (or not) of the composition of the clusters; and (iii) the value of the criterion function implied by each partition.

The blockmodeling approach is designed to use all of the signed two-mode data directly and does not use a low(er) dimensional representation. The expectation is that blockmodeling will produce better partitions in the sense of: (i) having a lower value of the criterion function, $P(\mathcal{C})$; (ii) will have a cleaner block structure (more blocks in their correct location); and (iii) will have more coherent—internally, more consistent—blocks in the returned blockmodel. However, given the problems outlined in Section 4.1, there is no guarantee that this will be the case. The size of the data sets and the computational burden that they imply for the blockmodeling approach can mean that the optimal partition(s) are not reached. The value of $P(\mathcal{C})$ for all of the partitions that we report is one criterion for evaluating the reported partitions. The smaller the value of $P(\mathcal{C})$, the better is the partition with that value. A second criterion measures the consistency of the composition of the clusters in a partition. The gold standard for this is the Adjusted Rand Index (ARI): see Hubert and Arabie, 1985; Saporta and Youness, 2002; Warrens, 2008; and Santos and Embrechts, 2009. Here we use the ARI to measure the correspondence of two partitions of the rows and columns of a partition of a two-mode structure.

Steinley (2004), based on an extended simulation study, has provided a set of guidelines for assessing the correspondence (or not) of two partitions of a set of vertices. For ARI ≥ 0.9, the correspondence between partitions is deemed *excellent*. When 0.9 > ARI > 0.8, the correspondence is said to be *acceptable*. For this range of the values of ARI, the memberships of the two partitions are deemed to be close enough to be considered the same. For lower values of ARI (ARI ≤ 0.8) the correspondence of the two partitions is *unacceptable*.

The blockmodeling approach starts with a random partition of the vertices in a two-mode network and proceeds from there. An obvious question for any established partition is simple to state: does it differ from a random partition of the vertices? In Pajek, a random partition of the vertices into k_1 clusters of rows and k_2 clusters of columns produces k_1 clusters of about the same size (n_1/k_1) of the rows and k_2 clusters of columns of about the same size (n_2/k_2) of the columns. The ARI can be used to measure the difference in the composition of the (starting) clusters and the ones established by blockmodeling. In general, the sizes of row clusters for the optimal partitions differ from the sizes of the random row clusters, as is the case for columns. Another measure of departures from randomness is to use randomly generated clusters having the same sizes as were established for the optimal partition.[23]

In the blockmodeling procedure, a value of the criterion function for the initial random partition is computed. We denote this value by $P(\mathcal{C}_r)$ and the value of the optimal criterion function by $P(\mathcal{C}_o)$. A simple measure of the difference between these two values of the criterion function is a proportional reduction of error measure: $PRE = (P(\mathcal{C}_r) - P(\mathcal{C}_o))/P(\mathcal{C}_r)$. Following experiments with random networks, values of $PRE \geq 0.2$ indicate departures from randomness and are therefore noteworthy. When viable partitions were obtained using other methods either ARI or $P(\mathcal{C}_r)$ are used as evaluative criteria.

5. Results

Here, we present our blockmodeling results and stress two features of this approach. One is that blockmodeling is a *full information* approach in the sense that all of the data are analyzed all of the time. No data reduction occurs when partitioning because the method uses no summaries of the data. Second, as used here, the focus is placed on the data being two-mode data. Both clusters of states and clusters of resolutions are important: the two partitions (of states and resolutions) get their full coherence when they are considered together.

5.1. Time 1: Late cold war period

5.1.1. Partitioning the full array of states and resolutions

Fitting blockmodels to a network array can be done within two distinct strategies. One is purely inductive and the other uses pre-specification. An inductive specification for signed two-mode

[22] Because the data are so close to being complete, Euclidean distance can be used even though for most incomplete signed data sets it cannot be used fruitfully.

[23] For example, if there are 100 vertices and a random partition is created, the sizes of the clusters are 34, 33 and 33. But if the blockmodeling partition returned clusters of sizes 60, 30 and 10, this alternative value of ARI is computed with a randomly created partition into clusters with sizes of 60, 30 and 10.

networks states only that there will be positive or negative blocks. In contrast, pre-specification can be used when analysts possess substantive or empirical knowledge (Doreian et al., 2005, pp. 233–5) regarding the network to which blockmodels can be fitted.

Using pre-specification requires, ahead of analysis, the statement of a partial or complete blockmodel. A complete blockmodel gives the distribution of all block types in their locations within the blockmodel. Although the term *model based* is often used to describe only statistical modeling where one or more equations or some probability structure are used, blockmodeling is model based when a complete blockmodel is pre-specified. The nature of UNGA voting dictates that some form of pre-specification be used. The reason is simple. For Time 1 there are 30,468 positive votes and 8,448 negative votes implying that the proportion of positive votes is 0.783. The inductive use of blockmodeling (with only P and N blocks specified) leads to a blockmodel with only P blocks.

Of course, the form of the actual blockmodel becomes important and formulating one ahead of time is difficult. We know that some states tend to vote in support of the majority of resolutions while no states vote against all resolutions. Other states vote in ways that span a range between these extremes. This leads to a simple specification that takes the following form when expressed in block types:

	R_1	R_2	R_3	R_4	R_5	R_6	R_7
S_1	P	P	P	P	P	P	P
S_2	P	P	P	P	P	P	N
S_3	P	P	P	P	P	N	N
S_4	P	P	P	P	N	N	N
S_5	P	P	P	N	N	N	N
S_6	P	P	N	N	N	N	N
S_7	P	N	N	N	N	N	N

We call this a *generic* blockmodel form regardless of the number of clusters. To simplify notation, we write $k = k_1 = k_2$. In the above illustration, $k = 7$. The first row of positive (P) blocks is for states in cluster S_1 that tended to support all resolutions. In contrast, the row corresponding to cluster S_7 is for states voting against all resolutions except those in cluster R_1. The remaining rows have systematic differences regarding the number of resolutions that are supported and opposed by the states in each of the row clusters. Similarly, resolutions differ systematically in the ways that states support or oppose them.

The results from using the generic pre-specified blockmodel are shown in Table 1A. Six variants of the generic pre-specification are shown. The number of partitions, the values of the criterion function, and the values of the fit indices (PRE and ARI) are shown. The decline in the values of the criterion function is consistent with Theorem 5 and the declines show diminishing returns from increasing the number of clusters.[24] Also, the behavior of PRE and ARI make it clear that the obtained partitions are far from random partitions. All looks good with this information until we compare the values of the criterion function for the indirect approach using Euclidean distance whose implied criterion function values are shown in the last column. In the main, the performance using Euclidean distance is poor. Not only are the implied values of the criterion function higher, the blockmodel structures have duplicate rows and columns of block types which imply that there are many other partitions with equally well fitting blockmodels. The surprise comes with the (7, 7) partition where the criterion function is lower than for using the generic blockmodel.

Table 1B shows the implied blockmodel for the Euclidean (7, 7) partition where, in contrast to the generic blockmodel, there are some P blocks among the N blocks in the lower right of the array of blocks. The generic blockmodel was quite good but not good enough for the Time 1 voting array. As a result, we used it as an empirically based pre-specified blockmodel and fitted that to the data. The results are shown in Table 1C. The partition is unique and the criterion function has dropped even further to 1881.5 and, again, the PRE and ARI indices conform that the partition is far from random. Further, the value of the ARI when this partition is compared with the partition from the generic blockmodel is 0.552 indicating that the two are different partitions.

We show the partition of the full set of states and military resolutions for Time 1 in Appendix C (Fig. C1) to demonstrate how the *empirically informed pre-specification* in Table 1 is used. It shows the nature of the partition even though the labels for states and resolutions cannot be read. The black squares represent positive votes and the red diamonds represent negative votes. There is a clear patterning where there are concentrations of ties of a type within blocks. Table 2 gives the partition of the states[25] and Table C1 gives the partition of resolutions. In short, this gives the 'big picture' of key voting clusters. This overall partition provides the departure point for focusing on parts within the big picture.

We note that the clusters of states labeled S_5 and S_6 feature primarily the Industrialized (and mainly Western) states. Most members of NATO are in S_6 including the US. The cluster labeled S_7 has the (former) Soviet Union, members of the Warsaw Pact and other leftist/communist states. The remaining clusters have the rest of the world's states that are in the dataset. The clusters of resolutions reported in Table 3 are labeled R_1 through R_7. The states in S_6 solidly oppose the resolutions in R_3, R_4, R_6, and R_7. The four states in S_5 have the same pattern except for tending to support the resolutions in R_3. The states in S_7 and S_6 tend to vote in support of resolutions in R_1—as do all states—and against those in R_6 but for resolutions on all of the other five clusters of resolutions their voting is diametrically opposed. The states in S_4 tend to support all resolutions except those in R_5 (although the resolutions in R_2 draw a mixed response from this set of states). States in S_2 tend to oppose resolutions in R_7 and support those in the remaining clusters while states in S_1 tend to support all resolutions.

In addition to obtaining clusters of states as potential blocs (not blocks), and the clusters of the resolutions, this two-mode partition opens the way to examining *simultaneously* the states and resolutions to facilitate the exploration of the exact nature of the resolutions that distinguished the states that oppose or support them.[26]

We noted earlier that 78% of the votes cast at Time 1 were positive. Blocks of a signed blockmodel are distinguished as positive or null blocks. Well fitting blockmodels ought to have blocks that are filled with ties having the same sign: positive blocks have primarily positive ties and negative blocks primarily have negative ties. Table 3 presents the block densities of the ties consistent with the block type for each block together with a label for its sign. The higher the density of the correctly signed ties in a block, the fewer inconsistencies this block contributes to the criterion function. For a well fitting blockmodel, positive blocks must have densities well above 0.78 and negative blocks must have densities well above 0.22. There are 28 (bolded) blocks having densities above 0.9. Of these, 20 of them are positive blocks and 8 are negative blocks. They provide evidence that the blockmodel partition in Fig. C1 has created many blocks with ties largely of the same sign. The 20 P blocks have

[24] The values of the criterion function for the (8, 8), (9, 9) and (10, 10) partitions are 2148, 2137.5 and 2124 respectively.

[25] The partitioning returned Papua New Guinea as a singleton in a cluster. We do not take this seriously as a real cluster and treat it as not belonging to any potential voting bloc. However, we take some comfort in noting that the methods we use did distinguish it from all of the identified voting blocs.

[26] This is done elsewhere because the focus here is on the methods that facilitate such comparisons.

Table 1
Summary of the blockmodeling partitioning for Time 1.

Partition	Number of partitions	Value of criterion function	Fit indices		Euclidean criterion function values
			PRE	ARI	
(A) Summary output and fit indices for Time 1					
(2, 2)	1	2743.5	0.611	0.344	2934.0
(3, 3)	1	2452.0	0.686	0.204	2648.0
(4, 4)	1	2353.0	0.717	0.734	2353.0
(5, 5)	4	2262.5	0.734	0.111	2284.0
(6, 6)	4	2197.5	0.762	0.096	2284.0
(7, 7)	2	2164.0	0.753	0.080	2117.5

	R_1	R_2	R_3	R_4	R_5	R_6	R_7
(B) Inductively established blockmodel							
S_1	P	P	P	P	P	P	P
S_2	P	P	P	P	P	P	N
S_3	P	P	P	P	P	N	N
S_4	P	P	P	P	N	P	P
S_5	P	P	P	N	P	N	N
S_6	P	P	N	N	P	N	N
S_7	P	N	P	P	N	N	P

Partition	Number of partitions	Value of criterion function	Fit indices	
			PRE	ARI
(C) The final fitted blockmodel for Time 1				
(7, 7)	1	1881.5	0.748	0.456

densities well above the overall density of 0.78 for positive ties and the 8 N blocks are well above the overall density of 0.22 for negative ties. The concentration of negative ties primarily into only 8 N blocks is the most interesting because they show *joint opposition to resolutions supported by most other states*.

While Fig. 2 presents the big picture of alliances, blockmodeling also permits "drilling down" to examine what issues cause particular states to depart from the voting patterns of other bloc members. This examination could be driven by an interest in particular blocs, interest in specific states, or voting cohesion of regional, international, or cross-regional organizations such as the EU, NATO, or the OIC.

5.1.2. Some detailed further partitions

One notable feature of Table 3 is that six of the blocks for the Leftist or Communist states of the era (in the cluster labeled S_7 in Table 3) have block densities above 0.9 (and the seventh has a density of 0.869) showing highly consistent bloc voting. Indeed, these densities point to this bloc has having the most consistently coherent voting pattern illustrative of Cold War power

Table 2
Seven clusters of states from Fig. 2.

S_1		S_2	S_4	S_5	S_6	S_7
Bahrain	Malta	Bahamas	Algeria	Austria	Australia	Afghanistan
Bangladesh	Mauritania	BurmaMyanmar	Angola	Finland	Belgium	Bulgaria
Barbados	Mauritius	Chile	Argentina	Greece	Canada	ByeloBelarus
Bolivia	Morocco	China	Benin	Ireland	Denmark	Cuba
Botswana	Niger	Colombia	Bhutan	Sweden	France	Czech
Burundi	Nigeria	CostaRica	Brazil		GerFedRep	DemYemen
Cameroon	Oman	CotedIvoire	CapeVerde		Iceland	GerDemRep
CenAfrRep	Pakistan	DKCambodia	Congo		Israel	Hungary
Chad	Panama	DominicanRep	Cyprus		Italy	India
Djibouti	Peru	ElSalvador	Ethiopia		Japan	Laos
Ecuador	Qatar	Fiji	GuineaBissau		Luxembourg	Mongolia
Egypt	Romania	Guatemala	Indonesia		Netherlands	Mozambique
Gabon	Rwanda	Haiti	LibyanAJ		NewZealand	Poland
Ghana	SaudiArabia	Honduras	Madagascar		Norway	USSRussianFe
Guinea	Senegal	Jamaica	Mexico		Portugal	Ukraine
Guyana	SierraLeone	Liberia	Nicaragua		Spain	Vietnam
Iran	Somalia	Malawi	SaoTomePrinc		Turkey	
Iraq	SriLanka	Nepal	SyrianArabRe		UK	
Jordan	Sudan	Paraguay	UVBurkinoFas		US	
Kenya	Thailand	Philippines	Uganda			
Kuwait	Togo	Singapore	Yemen			
Lebanon	TrinidadTobago	StLucia	Yugoslavia			
Lesotho	Tunisia	Suriname				
Malaysia	UAE	Uruguay				
Maldives	URTanzania	ZaireDRC				
Mali	Venezuela					
	Zambia					

Note: A singleton cluster S_3 (with Papua New Guinea) was identified but excluded from this and subsequent tables.

Table 3
Block densities for the 42 blocks in Fig. 2.

	R_1	R_2	R_3	R_4	R_5	R_6	R_7
S_1	**P 0.958**	**P 0.929**	**P 0.961**	**P 0.932**	P 0.838	**P 0.947**	P 0.769
S_2	P 0.898	**P 0.918**	P 0.845	P 0.789	P 0.863	P 0.836	N 0.761
S_4	**P 0.931**	P 0.778	**P 0.966**	**P 0.954**	N 0.770	**P 0.914**	P 0.868
S_5	**P 0.983**	**P 0.992**	**P 0.904**	N 0.833	P 0.789	N 0.680	N 0.790
S_6	P 0.848	**P 0.905**	N 0.856	**N 0.952**	P 0.889	**N 0.952**	**N 0.955**
S_7	**P 0.951**	**N 0.913**	**P 0.986**	**P 0.992**	**N 0.977**	N 0.869	**P 0.982**

The singleton cluster (S_3) was removed but the labels for Clusters S_4, S_5, S_6 and S_7 were retained. Bolded densities are all above 0.9.

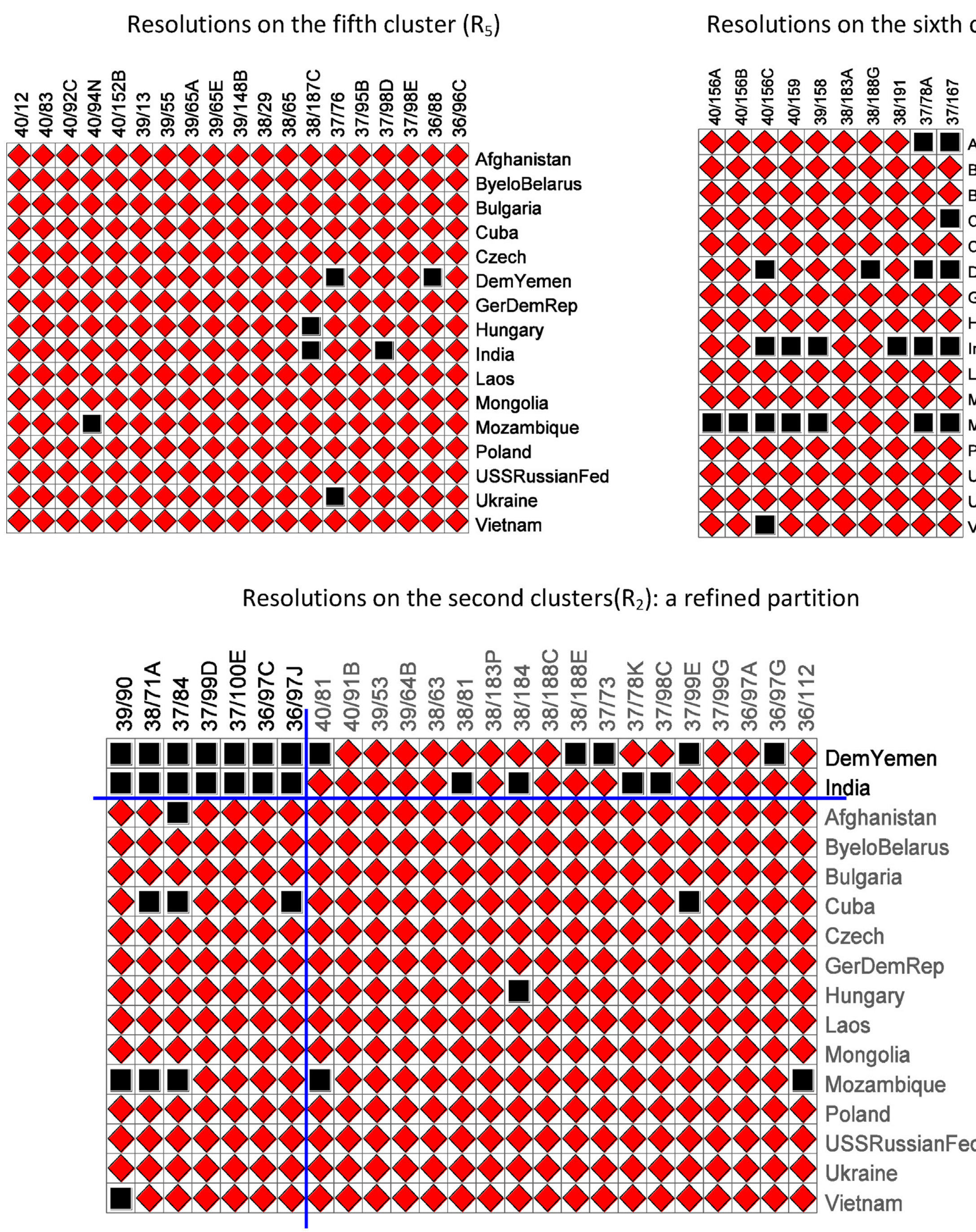


Fig. 2. Negative blocks for communist and leftist states voting against resolutions for Time 1.

Fig. 3. Details Fig. 4 negative voting blocks for the western and industrial states for Time 1.

struggles. Although the Western and/or Industrial states generally vote against military resolutions, particularly those that impact their military power, there are resolutions that the Communist bloc also votes against in consistent fashion. This is shown in Fig. 3 for the three negative blocks for these states. The first two blocks in Fig. 2 are reproduced as is from Fig. C1, but the third has been refined slightly (via blockmodeling) to show that both India and the Democratic Republic of the Yemen are slightly different in their detailed voting patterns. The resolutions identifying this block define the issues where these two states depart from the other members of the larger bloc. These two states tended to support issues related to disarmament and development and complete disarmament. We include this to show that, if needed and relevant, more detailed selective partitioning is possible to identify both the clusters of states and resolutions for further study, including any implications for balancing processes involving military resolutions as the soft balancing counterpart to hard balancing.

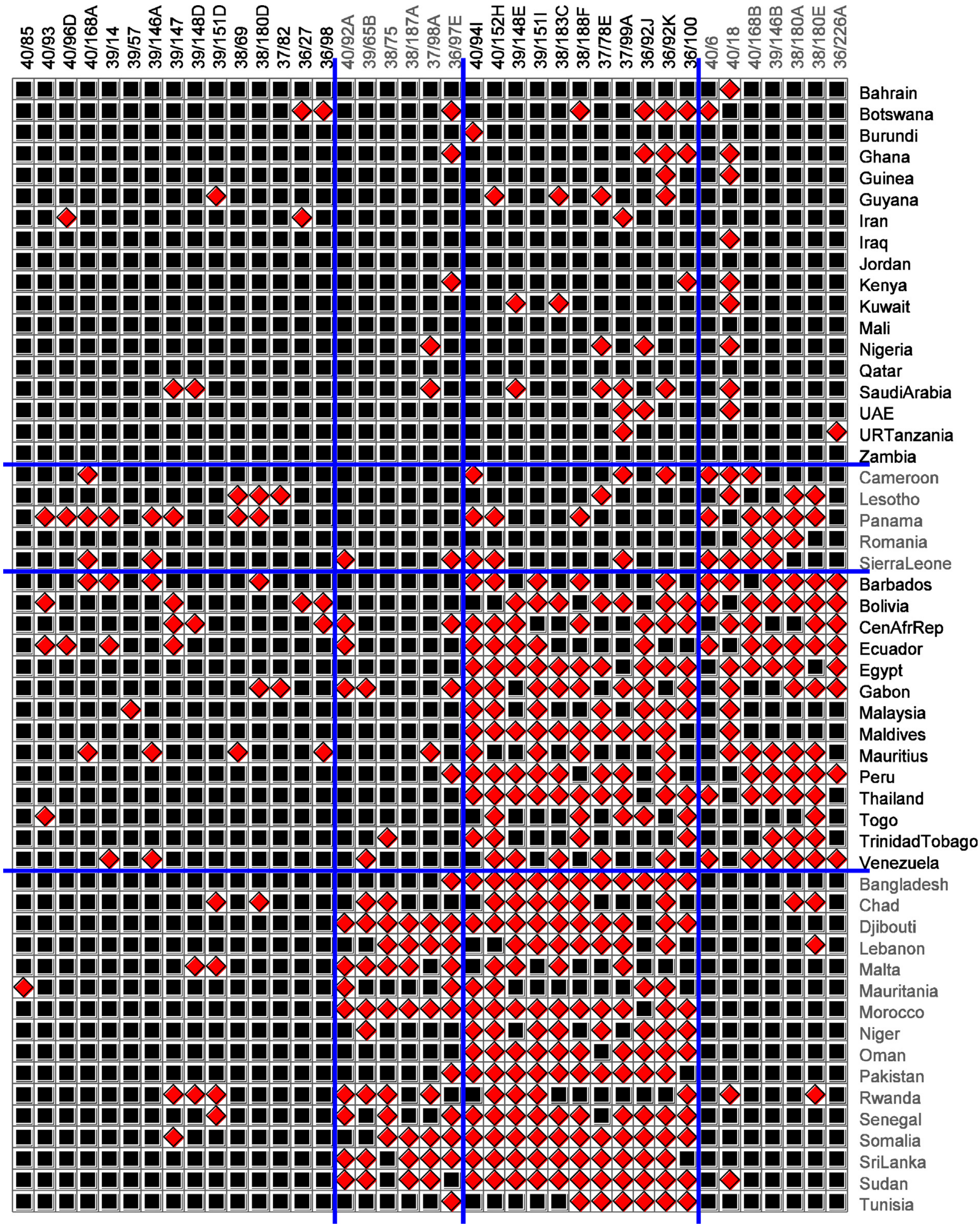


Fig. 4. Partition of one bloc of states from the non-western states and resolutions in R7.

Fig. 3 presents some corresponding formatted arrays for the Western, NATO and/or Industrial states for three of the clusters of resolutions that they vote against as a voting bloc. In the big picture of Fig. C1, these are simply negative blocks of votes illustrating voting patterns typical of Cold War voting where alliances oppose one another on military policy issues. As noted above, the majority of Western states oppose the majority of military resolutions (particularly those on disarmament—a direct threat to their military power). Yet the Western bloc—including NATO member-states—are divided on several resolutions, illustrating that states voted for other reasons than standing solidly with alliance members. For example, the block of resolutions defined by the cluster R_3 shows a subset of 23 resolutions for which 5 states (Australia, Denmark, Iceland, Norway and Spain—all but Australia are NATO member states) tend to support. In the second panel of Fig. 3, there are 7 resolutions supported by Portugal, Spain and Turkey that other members of the EU and NATO vote against. And in the final panel of Fig. 4, there are 17 resolutions where Turkey votes in opposition to the majority of Western states. The resolutions involving these positive votes by Turkey reflect *a conflict between NATO and OIC interests.* In each of these three examples, the joint partition of states and resolutions provide important information on voting (dis)similarity. We expected on the whole during this period to find voting defined largely by the two Cold War military alliances—and this was confirmed. We also found that some issues divided alliance members and the full information provided by our approach allows the interested researcher to explore the factors that lead to this divide.

Examining the top row of Table 3, for a cluster of non-aligned member states suggests that the block defined by S_1 and R_7 (with a positive tie density of 0.769) has enough negative ties (467 of them) that merit further attention. The blockmodel in Fig. 4 shows that there is some pattern to these ties (PRE = 0.626 and ARI = 0.162) in the sense of there being clear N blocks in an otherwise large P block for a coarser-grained partition. Again, the clusters of resolutions and states can be examined further to determine what links these states to oppose resolutions together that are supported by most other non-aligned member states.

5.1.3. Balancing of power revealed

As we noted earlier, the blockmodeling provides full information (on states and resolutions) which provides a basis for further exploration, including locating *clear balancing processes among rival alliances*. Fig. 6 provides an example of doing this. We look more closely at the following clusters: S_6 (the primary Western bloc containing most of the NATO members) and S_7 (contains Warsaw Pact Soviet bloc members and allies) from Table 2 and R_7 (the resolutions that the Western bloc opposes and the Soviet bloc supports) from Table 3. These are highlighted in the lower right part of the overall partitioned array in Fig. C1. Denoting this two-mode subnetwork by G, we multiply it by its transpose to obtain GG^T which we depict as a valued and signed one-mode network[27] for states. This is drawn in the top panel of Fig. 5 with the S_6 (Western, largely NATO member) states depicted by circles on the right and S_7 (Soviet bloc and allied) states depicted as squares in the left—a clear reflection of the Cold War political divide that was present at Time 1. All but one of the positive ties are within these two clusters of states. The only exception is a positive tie between India and Turkey. The lower panel of Fig. 6 shows how this positive tie was generated (e.g., the pattern of agreement on some resolutions).

Turkey votes with the 'NATO bloc' on most issues (25 of 39 resolutions); but diverges on a number of others for this subset of resolutions.[28] Similarly, India votes largely in alignment with Soviet bloc and allied states (29 of 39 resolutions).[29] India and Turkey vote identically on the 24 (14 for Turkey and 10 for India) resolutions for which they vote differently than their respective bloc members. An advantage of blockmodeling is the ability to identify specific resolutions which both identify bloc voting and the issues that divide bloc members. Once identified, the content of the resolutions and the temporary coalitions supporting them can be explored to pinpoint the reasons for the divergence and to identify issues that can potentially fracture alliances.

5.2. Time 2: Post cold war period

Fig. 6 contains the detailed unique (7, 7) partition for Time 2 that was obtained using the generic pre-specified blockmodel, and Table 4 lists the seven clusters of states. The pattern is similar to that of the Cold War period in identifying clear blocks of states that voted highly similar (as well as the divergences from bloc voting). It is also similar in that the Western or developed states are more likely to vote against military resolutions in the UNGA and to vote as a block. There are a number of important differences, however. First, S_6 is now composed of both Western European states and newly independent Eastern and Central European states that formed after the dissolution of the Soviet Union and Yugoslavia. Many of these states have now been incorporated into both the EU and NATO. Interestingly, we also see a distinct voting pattern of the US, which votes more independently from the rest of the developed and Western states and quite similarly with Israel (in S_7). In some ways, the blocks reflect Huntington's "clash of civilizations" thesis in that there is a clearly expanded group of largely Western states that vote quite distinctly from "the rest" on these military issues.

Blockmodeling allows further drilling down to identify the distinct cluster of resolutions that divide these Western/developed states from each other and from the rest of UN member-states. This feature is useful for identifying the issues for which NATO members are united and those which may suggest some balancing between the US (as the sole military hegemon) and EU-members states. A more in-depth exploration of this balancing process is beyond the scope of this paper but we have provided the tools by which this can be pursued.

Table 5 contains the summary of fitting this pre-specified blockmodel. As was the case for Time 1, the criterion function drops in a fashion consistent with Theorem 5. The fit statistics show that each obtained partition for the grains shown in the table is far from being random: these blocks of voting ties represent real differences in the patterns of voting by states within clusters. Again, there are diminishing returns for increasing the number of clusters and further partitioning of selected blocks is a better strategy than increasing the number of clusters for the full partitioning. In contrast to Time 1, the indirect blockmodels perform poorly for all grains ($2 \le k \le 7$). The values of the criterion function are higher and there are identical rows and columns of block types.[30] The computed value for the second ARI is 0.118 for the two (7, 7) partitions, an extremely low value indicating that the indirect and direct (7, 7) partitions are very different.

5.3. Partial summary

A theoretical goal of our paper was to demonstrate the utility of the relaxed structural balance approach to the analysis of UNGA roll call votes, in potentially coupling balance of power theory in international relations with Heider's structural balance theory. Both theories share an assumption of a tendency towards balance among multiple actors resulting in an alliance formation. International relations scholars have proposed the idea of "soft power" balancing through norms, and adopted resolutions represent a consensus on norms underlying particular policy recommendations regarding issues of interest to the international community. Voting on these military resolutions provide information on the coalitions that support or oppose particular norms about the use of military power by UN member-states. Examining the roll call votes on military issues at two time points spanning the Cold War political

[27] We do not use the values in drawing Fig. 6 but they are useful for subsequent analyses.

[28] Turkey votes differently from its NATO and Western allies on the following 14 resolutions: 40/6, 40/93, 40/96D, 40/168A, 40/168B, 39/14, 38/69, 38/180A, 38/180D, 38/180E, 37/82, 36/27, 36/98 and 36/226A.

[29] India votes differently from Soviet and allied states on the following 10 resolutions: 40/18, 40/85, 40/92A, 40/94I, 39/57, 39/65B, 39/151I, 38/187A, 38/188F and 36/92J.

[30] This helps account for the criterion function being unchanged for the (3, 3), (4, 4), and (5, 5) partitions because splitting a block into blocks with the same sign will not change the value of the criterion function.

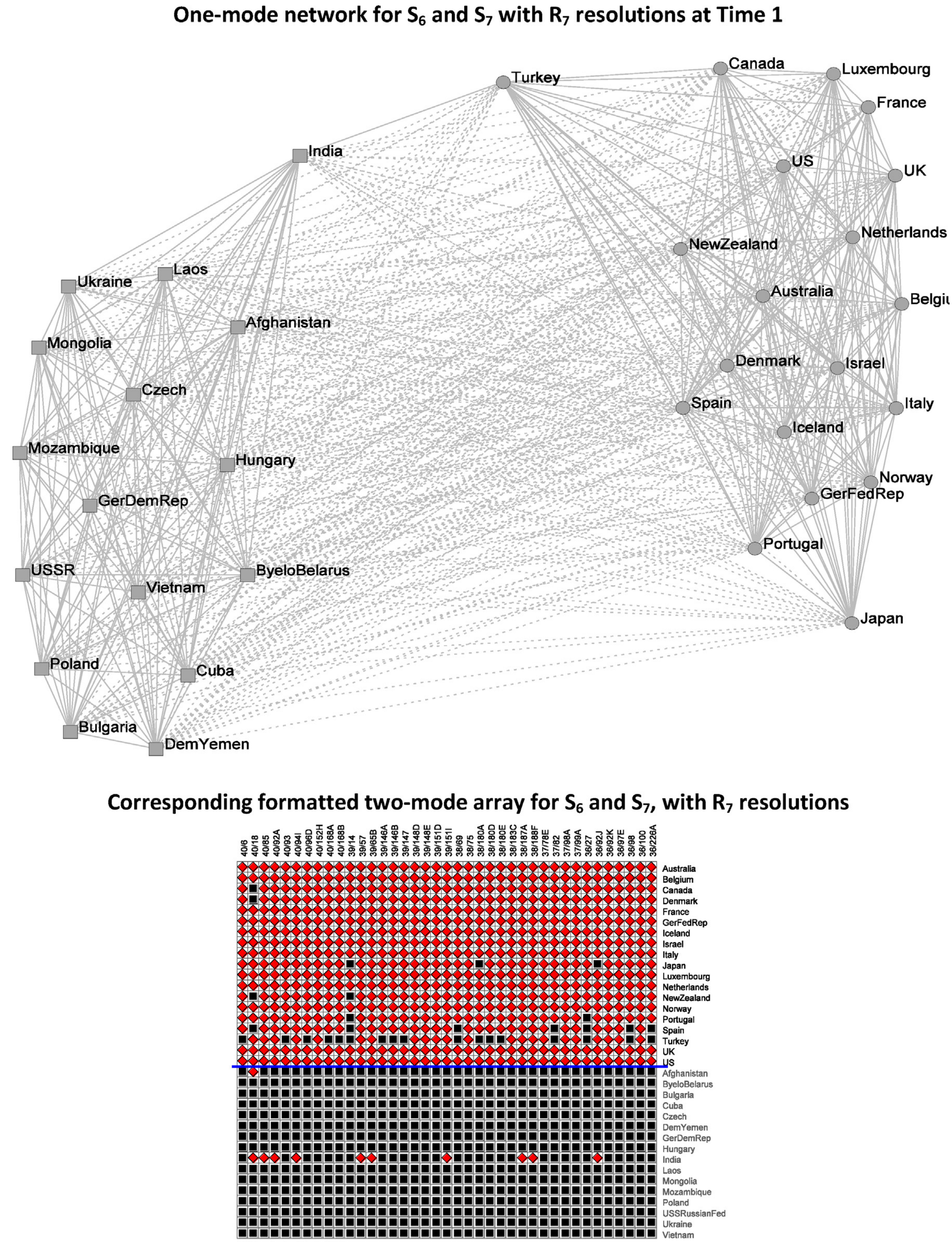


Fig. 5. The one-mode network for S_6 and S_7 based on R_7 and the two-mode network for S_6, S_7 and R_7.

ideological divide illustrated changing coalitions after a major shock to the international system. The overall structure of state relations impacted the relations among coalition members and the balance of power among them. As might be expected, the US and EU member states vote more similarly than other UN-member states overall, but also less similarly than they did during the Cold War when they shared a common enemy. We argue that our technique and results provide an important first step in exploring the coupling of these two important theories, and encourage other scholars to continue in this direction.

We also progressed in our methodological goals, namely, identification of and solutions for problems with applying the relaxed structural balance approach to large signed two-mode data. We demonstrated that direct signed blockmodeling is very useful for

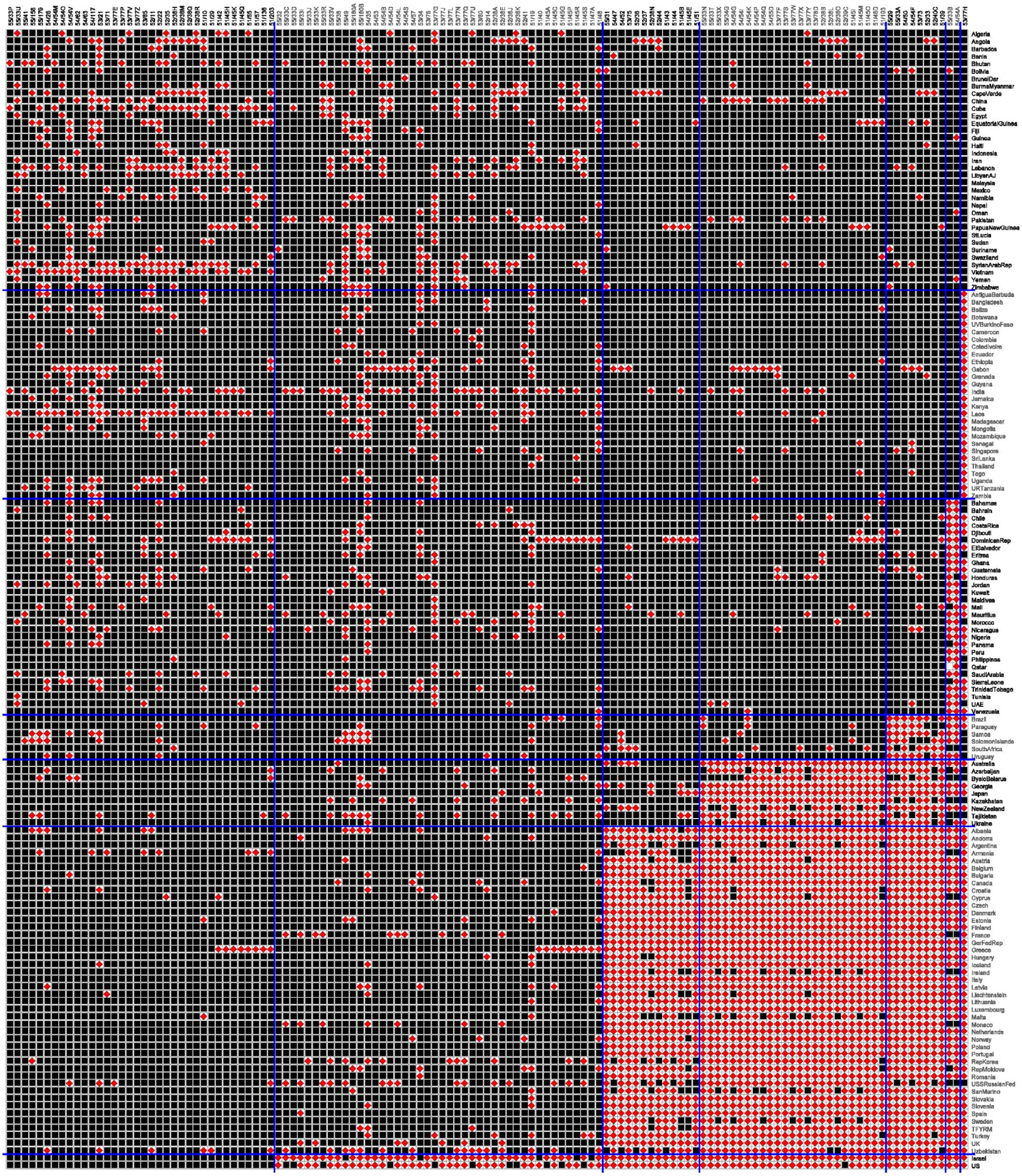


Fig. 6. A partition of the full set of military resolutions for Time 2.

partitioning larger signed two-mode networks, but we also identified several methodological issues that must be resolved. We suggested several solutions, and found that despite the problems highlighted, the results produced are coherent, and reflect divisions located in prior UNGA voting analyses for both time periods examined. However, despite having provided a substantively driven method that produces coherent and useful partitions of signed two-mode data, and illustrating how blockmodeling can identify smaller clusters that may be fruitful for examining the implications of temporary or more permanent coalitions, several problems remain. First, when the data arrays are large, the partitions obtained by using direct blockmodeling methods required long computational times, which is problematic in two ways: the length of time involved, and the difficulty with obtaining optimal partitions if either the time for the partitioning is reduced or the size of the two-mode array is raised too much. Second, we were surprised to find that the result for the (indirect) Euclidean distance (7, 7) partition was superior to the direct partition for the Time 1. This was a salutary reminder that the direct blockmodeling method need not always locate the best partition. In this instance, using indirect blockmodeling had great value in not only providing a better fitting partition but also in providing a pre-specified

Table 4
Seven Clusters of States corresponding to Fig. 6.

S_1 ($N=35$)		S_2 ($N=28$)		S_3 ($N=29$)	
Algeria	Lebanon	AntiguaBarbuda	Mongolia	Bahamas	Nigeria
Angola	LibyanAJ	Bangladesh	Mozambique	Bahrain	Panama
Barbados	Malaysia	Belize	Senegal	Chile	Peru
Benin	Mexico	Botswana	Singapore	CostaRica	Philippines
Bhutan	Namibia	UVBurkinoFaso	SriLanka	Djibouti	Qatar
Bolivia	Nepal	Cameroon	Thailand	DominicanRep	SaudiArabia
BruneiDar	Oman	Colombia	Togo	ElSalvador	SierraLeone
BurmaMyanmar	Pakistan	CotedIvoire	Uganda	Eritrea	TrinidadTobago
CapeVerde	PapuaNewGuinea	Ecuador	URTanzania	Ghana	Tunisia
China	StLucia	Ethiopia	Zambia	Guatemala	UAE
Cuba	Sudan	Gabon		Honduras	Venezuela
Egypt	Suriname	Grenada		Jordan	
EquatorialGuinea	Swaziland	Guyana		Kuwait	
Fiji	SyrianArabRep	India		Maldives	
Guinea	Vietnam	Jamaica		Mali	
Haiti	Yemen	Kenya		Mauritius	
Indonesia	Zimbabwe	Laos		Morocco	
Iran		Madagascar		Nicaragua	

S_4 ($N=6$)	S_5 ($N=9$)	S_6 ($N=44$)				S_7 ($N=2$)
Brazil	Australia	Albania	France	Norway	UK	Israel
Paraguay	Azerbaijan	Andorra	GerFedRep	Poland	Uzbekistan	US
Samoa	ByeloBelarus	Argentina	Greece	Portugal		
SolomonIslands	Georgia	Armenia	Hungary	RepKorea		
SouthAfrica	Japan	Austria	Iceland	RepMoldova		
Uruguay	Kazakhstan	Belgium	Ireland	Romania		
	NewZealand	Bulgaria	Italy	USSRussianFed		
	Tajikistan	Canada	Latvia	SanMarino		
	Ukraine	Croatia	Liechtenstein	Slovakia		
		Cyprus	Lithuania	Slovenia		
		Czech	Luxembourg	Spain		
		Denmark	Malta	Sweden		
		Estonia	Monaco	TFYRM		
		Finland	Netherlands	Turkey		

blockmodel as the start point for getting an even better fitting partitions.

It is possible that using methods having a very different rationale could be useful. Alternative methods also may provide different but complementary information to help understand bloc voting in the UNGA. For these reasons, we make a preliminary exploration of several alternative approaches. We do *not* claim to take a complete survey of alternative methods because they are too numerous and a single paper could not do justice to what they have to offer. Nevertheless, we think it instructional to compare our results with that of some alternative popular approaches. Not all of these will advance directly our theoretical interest but they may have the potential to do so.

6. Alternative clustering approaches

We consider briefly alternative tools for analyzing these data that have been used in UNGA voting studies or were developed as different ways of partitioning or representing two-mode data. However, this not intended as a complete survey.

6.1. *Geometric data analysis*

Spatial models of multivariate data produce graphical geometric displays that facilitate the exploration of multivariate structure. For example, principal component analysis (PCA) produces a low-dimensional summary of a high-dimensional Euclidean space, multidimensional scaling (MDS) constructs a low-dimensional Euclidean representation of dissimilarity data, and multiple correspondence analysis (MCA) constructs a low-dimensional Euclidean representation of categorical data. Gower and Hand (1995) and Gower et al. (2011) provide a unified treatment of these techniques within the rubric of geometric data analysis.

Spatial models of voting data were introduced and popularized by Downs (1957) who applied Hotelling's (1929) model of spatial competition. Such spatial models represent both voters and candidates as ideal points in a low-dimensional Euclidean space and posit that voters tend to prefer candidates who are closer over candidates who are further away. Mapping the ideal points exemplifies an approach called "unfolding" in the MDS literature. See Groenen (2005: Part III) for an introduction to the notion of

Table 5
Summary of full partitioning of all military resolutions for Time 2.

Partition	Number of partitions	Value of criterion function	Fit indices		Euclidean criterion function values
			PRE	ARI	
(2, 2)	1	972.0	0.732	0.362	972.0
(3, 3)	1	932.0	0.758	0.245	974.5
(4, 4)	1	902.0	0.778	0.164	974.5
(5, 5)	1	873.5	0.785	0.128	974.5
(6, 6)	1	859.5	0.807	0.102	947.0
(7, 7)	1	848.0	0.804	0.081	947.5

Value of the second ARI for the (7, 7)-partition is 0.118.

unfolding. Cahoon (1975) developed an early example of an unfolding method for voting data.

For analysis of legislative voting, candidates are replaced by policy positions—for example, Yea or Nay on a resolution. An unfolding analysis of legislative voting data represents each legislative representative (voter) and each policy position as an ideal point in a low-dimensional Euclidean space. The premise is that a voter will tend to vote Yea if s/he is closer to the ideal point for Yea than s/he is to the ideal point for Nay. We adopt this approach here.

Not all ideal-point spatial models of legislative voting assign an ideal point to each policy position. The popular NOMINATE procedure of Poole and Rosenthal, described in detail by Poole (2005), assigns an ideal point to Yea and posits that voters will to tend to vote Yea if they are sufficiently close to the ideal point and Nay if they are sufficiently far from it. Erik Voeten (2000) introduces this method in his analysis of UNGA voting.

For this paper, we used the R package HOMALS (De Leeuw and Mair, 2007, 2009) to perform MCA. In the context of HOMALS, MCA is called a simple homogeneity analysis. See Gifi (1990) and Michailidis and de Leeuw (1998) for detailed descriptions of homogeneity analysis. Previous applications of MCA/homogeneity analysis to legislative voting data include Desposato's (1997) study of party switching in Brazil and de Leeuw's (2005) analysis of Senate voting patterns.[31]

These methods permit the simultaneous spatial locations for, in this case, states and resolutions. Fig. 8 shows the results for our first (late Cold War) period, using a two-dimensional space and plotting the coordinates estimated using the Homals method.[32] For visual reasons we only display the locations of states. The main clusters are highly similar to the blockmodeling results for this time period. Although the resolutions are suppressed, Homals locates countries near the resolutions for which they vote yes. Cluster 1 contains members of the NATO alliance and their allies; cluster 4 (bottom right hand side of graph) contains the Soviet bloc (Warsaw Pact member-) states and their allies. A large group of leftist states is located just above this group (cluster 3). The large group of largely non-aligned member states (with China nearby) is in and around cluster 2. The classic horseshoe pattern positions countries most dissimilar from one another at opposite ends, with the Western bloc opposite the Soviet bloc countries on both dimensions. The non-aligned member states are between these two ideologically opposed blocs. Interestingly, the three Western states P_5 (UNSC Permanent 5 members) are clustered more closely than the rest of the Western states (China and the Soviet Union are the other two P_5 states) (Fig. 7).

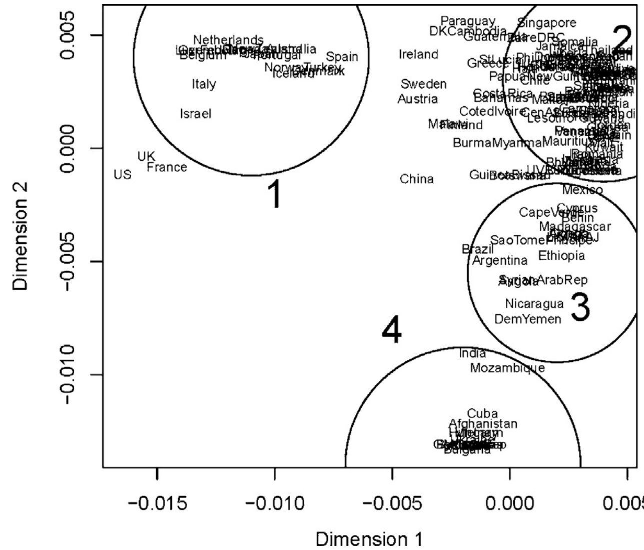


Fig. 7. Homals voting map, Time 1: 1981–1985.

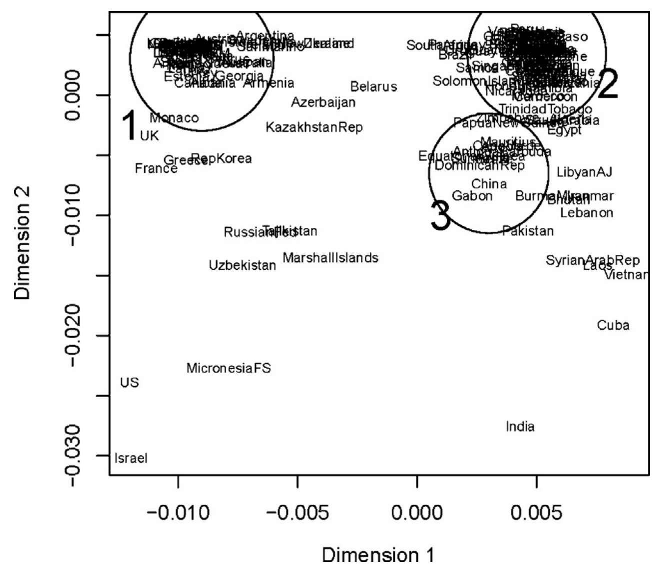


Fig. 8. Homals voting map, Time 2: 1996–2001.

The two-dimensional Homals plot for Time 2 (Fig. 8) reveals a change in the voting structures beyond the addition of about 30 new UN member-states. The graph indicates that most of the difference in voting is along the first dimension where an expanded "West" (Cluster 1) containing many of the former Soviet Union bloc states—and representing an expanded NATO and EU—is distinct from "the rest" of the states. As in the blockmodeling analysis, the US and Israel are distinct from the other Western states on the second dimension. The five remaining communist states—Cuba, China, Laos, North Korea (omitted due to high absenteeism), and Vietnam—are located within or just below cluster 3 and distinct from the large group of developing states. Not surprisingly, the distinct pattern of Western states (including differences within this bloc), and the separation of states described as "counterhegemonic" (remaining communist states and those with which the US and other Western states have been in conflict)[33] from the rest of the non-Western states was consistent across analyses.

There is no exact way to fully compare the results for using different methods. However, we can compare some of the results from using other methods with our own in order to independently evaluate the blocs produced by blockmodeling. We found that the results across methods were highly similar providing some validation for the clusters we identified. One advantage of the geographic methods was the production of coordinates for both resolutions and states that can be used as variables in statistical models. However, voting bloc memberships as identified by blockmodeling also can be used as dummy variables in the same fashion as done in Snyder and Kick (1979).

[31] We also used the R package WNOMINATE (Poole et al., 2011) to perform the NOMINATE procedure and compare the results from using it with results with our Homals output but do not provide the results here because of space constraints. The results are available upon request.

[32] The NOMINATE results produced similar clusters to that of Homals; the plots look different because of how the ideal points are plotted but the correlation between clusters are high.

[33] See Voeten (2000) for similar findings using NOMINATE.

6.2. Negative eigenvector centrality

An approach that can be used to both identify cliques of states voting similarly, and to examine balancing behavior, is the negative eigenvector centrality approach created by Bonacich and Lloyd (2004).[34] This approach illustrates competing alliances through the use of positive and negative ties. The clustering is highly similar to that found in the blockmodel results. The two approaches can be complementary in two ways. First, blockmodeling provides full information results which can be used to explore the key issues that define the divide among opposing groups of states. Second, the eigenvector centrality approach can be used to explore subsets of states. We provide an example using a subset of "European" (or Western) states.[35]

Fig. 9 illustrates voting similarity at Time 1 among European and other Western states and clearly reflects what would be expected by balancing processes described in Theorems 1 and 2 above. Note that all of the positive ties (solid lines) are among countries within each cluster, and all of the negative ties are between the group of Western (and NATO member) states on the right and Warsaw Pact members and their allies on the left.

Fig. 10 illustrates voting similarity in Time 2, the period after the Cold War has ended and states have re-aligned. The graph reveals first, a much denser graph as there are a number of newly independent states; and second, a largely cohesive Europe (and "West"). It is not surprising that the incorporation of many former Soviet bloc Eastern European states into NATO and EU would result in the peripheral status of Russia, Belarus and former Soviet bloc Central Asian states. Blockmodeling can be explored to locate the key issues that keep these states distinct from the rest.

6.3. Islands

Similar to the Eigenvector Centrality approach, the islands technique (Zaveršnik and Batagelj, 2004) begins with the transformation of the data from a two-mode to a one-mode network. As noted in that section, we begin with a matrix G that denotes the array of ties for a signed two-mode network $G = (U, V, E)$. The matrix GG^T is a valued network whose vertices are the elements in U and the values on the ties are integers (that can be positive or negative). More formally, the network created by forming GG^T is $M = (U, E_m, w)$ where E_m is the set of edges and w maps these edges to the set of integers. A visual imagery for the concept of islands is to think of a network where the lines differ in height according to the values of the edges in E_m. Imagine the whole network is covered by water in a tank. As the level of water drops, the highest valued edges appear first (along with the vertices directly linked by them) above the level of the water. If the level drops further, more valued edges appear and they form one of more islands visible above the threshold.

Let t be a threshold (the level of the water in the visual imagery). Let e denote an element of E_m ($e \,\varepsilon\, E_m$) and let $w(e)$ denote the value of an edge, e. A line cut of $M = (U, E_m)$ is a subnetwork, $M(t) = (U(t), E_m(t))$ where the elements of $E_m(t)$ are such that $e \,\varepsilon\, E_m(t)$ if and only if $w(e) \geq t$ and $U(t)$ is the set of elements of U having some edges, e, incident at them where $w(e) \geq t$. The vertices $U(t)$ that are linked by valued edges in $E_m(t)$ form islands. Given a threshold t, there can be one or many islands that are determined solely by being connected by the edges of $E_m(t)$. Note that this criterion for island membership is stringent. The set of vertices is *edge island* if the corresponding induced subgraph is connected and there exists a spanning tree, such that the values of ties with exactly one endpoint in island are less than or equal to the values of ties of the tree in the island (see Zaveršnik and Batagelj, 2004 and De Nooy et al., 2011, pp. 124–32). Island detection is implemented in Pajek (Batagelj and Mrvar, 1998).

In terms of the UNGA voting arrays, the values of the ties in GG^T capture the net number of times that two states vote the same way over the set of resolutions in the array. The higher the values in GG^T, the more often the states vote in the same fashion overall. An analysis using islands seeks to locate those islands composed of states that vote together the most often. In practice in Pajek, the threshold t is left implicit by specifying the minimum and maximum sizes of islands. Determining island sizes is done by inspecting the distribution of their sizes under different pairs of minimum and maximum sizes. The algorithm is extremely fast and islands analyses done on the UNGA voting matrices are completed within seconds rather than hours for many of the blockmodeling analyses.

Because of space constraints, we summarize our results here. The analysis in terms of islands produced partitions that differ in two ways from those obtained by using blockmodeling. First, the islands procedure allows for vertices to be unclassified. For Time 1, there were 5 identified islands and one residual cluster of states that were not classified. And for resolutions there were four islands of classified resolutions and one residual cluster. The implied value of the criterion function for this partition is 3130, a value well above all values of the criterion function reported in Table 1. The values of ARI when the islands partition is compared with the (5, 5), (6, 6) and (7, 7) partitions are 0.148, 0.262, and 0.235 respectively. The partition provided by the island detection method differs from all of the blockmodeling partitions. While it does cluster all of the Western and/or Industrial states together, it is less nuanced in the clustering of the remaining states.

An analysis using Islands for Time 2 led to a partition with 8 clusters of states and a partition with 7 clusters of resolutions.[36] The implied value of the criterion function was 1016.5, a value well above the values reported in Table 5. When this partition was compared with the (7, 7) and (8, 8) partitions, the values of the ARI measure were 0.455 and 0.416, respectively. The clusters detected by the islands method were very different to those obtained from blockmodeling and inferior in terms of the criterion function used to evaluate joint partitions.

6.4. Community detection

At face value, there is a potential connection to be made with the community detection literature developed primarily by researchers coming from physics. See, for example, Fortunado (2009), Girvan and Newman (2002) and Newman (2006). The essential idea of community detection for one-mode networks is to locate one or more subsets of vertices such that the ties between vertices inside the subsets are larger (if valued) and/or more dense than the ties from these vertices to the rest of the network. The potential connection is that within voting blocs there will be dense ties of a particular type. We explore the popular Girvan–Newman (2002) algorithm[37] implemented in UCInet (Borgatti et al., 2002).

[34] Lloyd (2007) also developed the application of this approach to international relations data.

[35] The UN has two regional classifications: one is defined by the ECOSOC regional commissions and includes all of Europe plus the US, Canada, and Israel; the other is the regional caucusing group which separates Eastern and Western Europe and places countries in the latter category (WEOG—Western European and Other Group) along with the U.S., Canada, Israel, Australia, New Zealand, Denmark, Finland, and Norway. We chose to use the ECOSOC European category that included both Eastern and Western Europe.

[36] As noted, we do not report the full results because of space constraints but they are available upon request from the authors.

[37] We thank Steve Borgatti for implementing this algorithm into Ucinet (Borgatti et al., 2002).

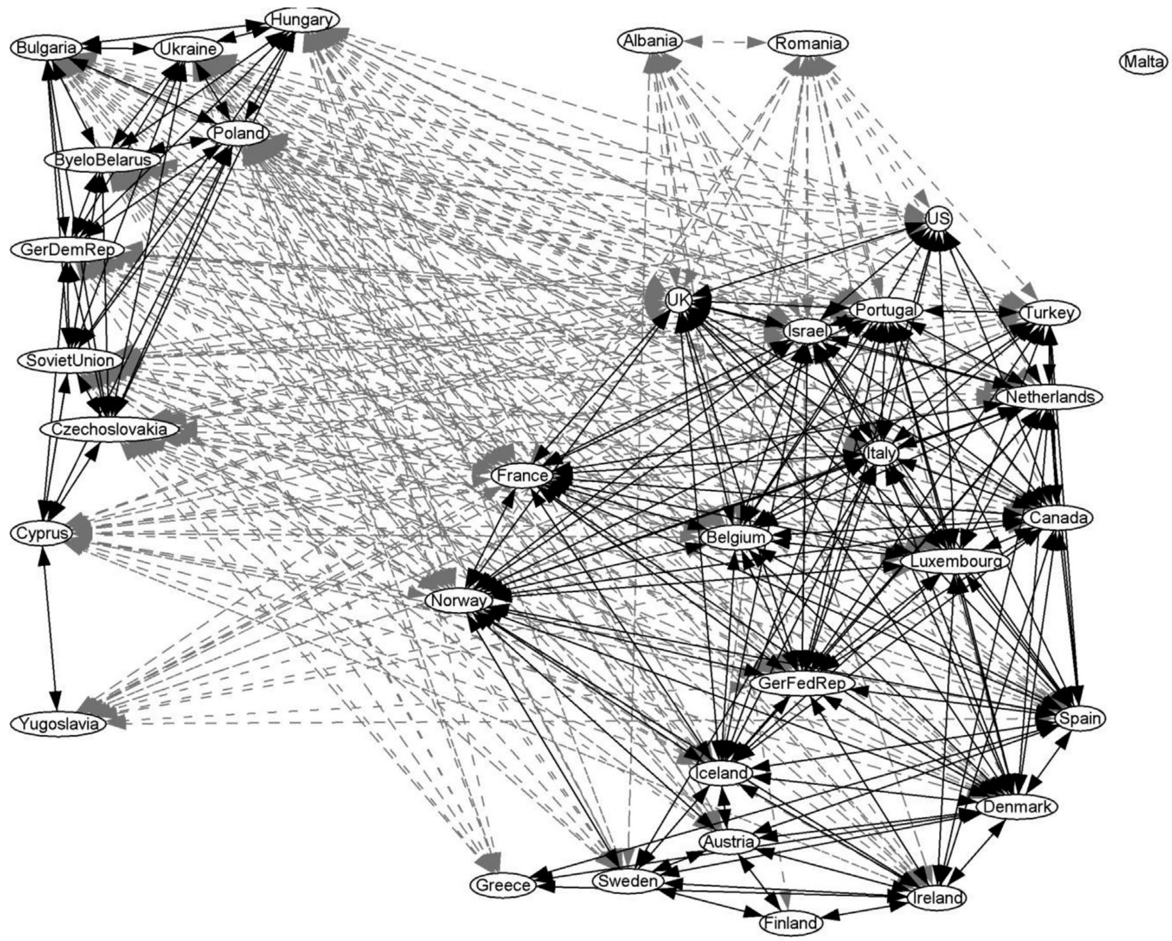


Fig. 9. Eigenvector centrality voting map for "Europe", Time 1 (1981–1985).

This also required conversion of our two-mode data into two one-mode networks.

When the Girvan–Newman CD algorithm is used for the one-mode network GG^{T} constructed from the two-mode network G, there is one large cluster with a few (Western) states not in the large cluster. The reason for this is straightforward: the counts of states voting together on resolutions ranges from 276 (for states that supported all resolutions) to −222 (for 4 pairs of states whose net joint voting together was against military resolutions considered in the UNGA). The majority of values in this one-mode matrix are large and there is no real surprise with the detection of a single 'community'. For Time 2, this algorithm detected one large community containing all states except a few non-Western states.

We note two things. First, the Girvan–Newman algorithm is formulated for binary networks and it is straightforward to convert the valued network into a binary one. Some threshold is selected and values above this threshold are coded "1" while values below it are coded "0." The problem lies in selecting the threshold and different thresholds lead to detecting different communities. This is an option that could be explored. Second, and perhaps more importantly, this algorithm was designed for sparse networks and the UNGA voting arrays are far from sparse. Indeed, they are close to being complete. Indeed, the near complete nature of the data used here are not appropriate for using this algorithm. A better option might be in Newman (2006). Many community detection methods are covered by Fortunado (2009) and some could be useful here. Also, Tragg and Bruggeman (2009) propose an approach for signed networks. These are among many potential algorithms that could prove useful. Certainly, the literature on community detection could be very useful. We return to this in our final section.

7. Summary

We have presented results stemming from the partitioning of signed two-mode data in the form of states voting on military resolutions that came before the UNGA for two time periods (1981–85 and 1996–2001). One period was well before the dissolution of the Soviet Union and the other period was defined for a time period that was as far after this dissolution as the first period was before it. Unsurprisingly, the voting structure of states differed considerably for the two time periods. But a focus on voting analysis was not our primary intent. Rather, our purpose was to consider seriously some of the methodological problems of a blockmodeling approach to partitioning large signed two-mode data. While the UNGA voting data for military resolutions could be seen as no more than providing a 'convenient' data set for doing this, these data are far more than a demonstration database. In addition to being important in their own right, these data exemplify the inherent problems stemming from moving from small data sets to data that stretch the bounds of what can be done with blockmodeling. Hence, the preliminary effort to evaluate our results using alternative approaches. A second motivation was to evaluate what can be gained in the trade-off of using an approach with high computational times for larger datasets. Our desire was to 'bring back in' the less studied triple of two actors and a social object that blockmodeling allows in order to explore balancing of power processes using IR data.

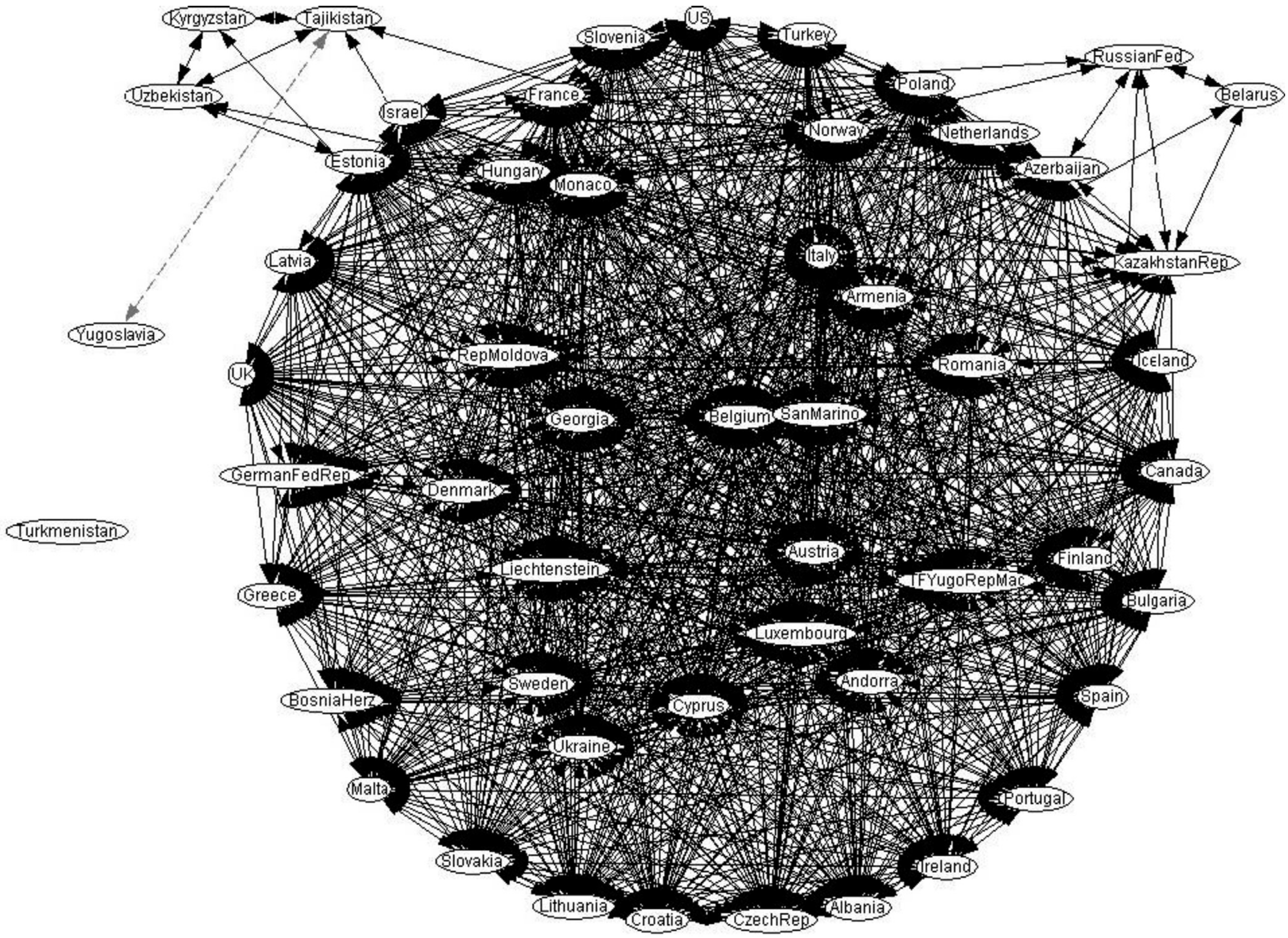


Fig. 10. Eigenvector centrality voting map for "Europe", Time 2 (1996–2001).

We also considered some other approaches to these data, and presented comparisons of the different partitions that are identified using these alternative methods. Direct comparisons, however, were difficult because of one fundamental difference in the methodological approaches used in the analysis of signed two-mode data. Blockmodeling is a full information approach that deals directly with *all of the data* and does not use any summarization of them for obtaining partitions or any general conclusions about these data. The cost of doing this comes in the form of increased computational times for completing the analyses of data. Even so, for the UNGA voting data for military resolutions, the more extensive analyses can be done. Of course, this flies in the face of a drive to analyze very large data sets in a very fast fashion and one of the large questions is whether or not the extra time is worth it.

In terms of characterizing the big picture, the blockmodeling approach considered here appears to provide the same general characterization of the structure of UNGA voting as some of the other approaches. The potential contribution of blockmodeling is to go beyond this type of characterization to probe for further details within these broad characterizations. While we think that the more detailed partitions that we have provided as examples do lay the foundations for doing this, a decision about the value of examining these fine-grained details is up to individual researchers.

We think that the joint partition of states and resolutions has intrinsic value because when considering the blocs of states with regard to voting, a deeper understanding comes from knowing where the exact differences are located. Blockmodeling provides a coherent way of doing this. For example, it is one thing to note that the states belonging to different broad alliances face competing pressures to vote according to the multiple memberships that they have. It is another thing to identify responses to these kinds of pressures. We think that the refined partition in the bottom panel of Fig. 4 takes a useful step in this direction. This sets up a more demanding task to see exactly what it is in the content of the resolutions that leads states to vote the way they do, especially when subject to conflicting pressures, in the UNGA. But as illustrated in Fig. 5, it provides the means to explore soft balancing processes, e.g., efforts by states to exert their policy preferences. The drilling down feature of blockmodeling allows for the identification of the complete set of resolutions that distinguish the voting among states and to identify the issues that distinguish typical voting blocs and that may contribute to balancing processes among competing states or alliances. For example, the complete information analysis provides the information needed to determine whether voting differences are due to a particular conflict (differences in how to resolve the ongoing Middle East conflict for example); or reflect more general challenges to a state or an alliances' military power by constraining its use, e.g., through promotion of norms that prohibit the possession or use of particular weaponry.

8. Discussion

During the analyses that led to the results shown here, a number of difficult issues emerged. Key to fitting blockmodels is the distinction between inductive and deductive blockmodeling. The former forms a strategy that can be construed as based on ignorance because all that is expressed is that certain blocks could be present. Of course, there is a role for inductive blockmodeling, especially when it is applied in new empirical contexts. Yet, we often know more about empirical phenomena than the relevance of particular block types. This knowledge can be expressed in a pre-specified blockmodel. Our experience with the generic blockmodel described in Section 4 is a sobering reminder that a prespecified model, while immensely plausible, can never be the end of a structural story. The use of what we thought would be an inferior indirect approach using Euclidean distance, while much inferior most of the time, did surprise us once in Time 1. It suggested an alternative, and better, pre-specified blockmodel that we were compelled to use. This suggests that if there are multiple approaches to obtaining a partition of two-mode signed (or any) structural data, there is a benefit to moving between the different methods rather than sticking to one preferred approach.[38]

One little explored feature of the blockmodeling of signed data, regardless of whether they are one-mode or two-mode, is found in the criterion function, $\alpha\mathcal{N}+(1-\alpha)\mathcal{P}$ that we have used. Nearly all empirical applications have featured partitioning with $\alpha=0.5$ because it is safe and there is no obvious reason for weighting $\mathcal{N}$ and $\mathcal{P}$ differently. Yet, when there is a large imbalance between the number of positive and negative ties in a signed data set, it seems reasonable to explore different values for α. Indeed, some of the refined partitions that we reported for partitioning within previously established blocks were done with varying α. There are at least two compelling reasons for doing this. One is pragmatic in that values of α that depart from 0.5 are much more likely to produce unique partitions. More importantly, when seeking 'minority' patterns within large blocks, it is important to make sure that the sub-blocks are really homogeneous. So, when the minority different blocks are expected to be N blocks then positive inconsistencies were weighted more heavily and, when the minority blocks were expected to be P blocks, the negative inconsistencies were weighted more heavily. The detailed impact of differing values of α is an issue that merits exploration.

As data sets considered by network analysts expand in size, and blockmodeling two-mode data structures is seen as having value, there is a need for faster algorithms. In terms of speed, the approaches considered here range from the blazingly fast Islands algorithm to the heavy computational demands of direct blockmodeling. Choosing a preferred method(s) will be the decision of individual researchers according to their overall objectives. Blockmodeling locates the same general clustering and also provides the means for drilling down into fine-grained details with greater ease or clarity than some other approaches. On the other hand, blockmodeling may lead to extremely fine-grained partitions with the potential to overwhelm the researcher with this detail. Still, this detail may be of value for those interested in the position of particular states or groups of states on particular policies.

We emphasize that our comparisons with other methods are far from complete. Space constraints prevented us from fully comparing alternative approaches. The fact that these methods did not work as well in the data considered here, in terms of poor ARI scores or not providing partitions has no implications for their general utility. In the RSB approach to partitioning 2-mode networks, the primary evaluative emphasis is on the value of the criterion functions for established partitions, something that is absent from the other approaches that we have considered. We acknowledge that given the presence of many other algorithms, such as used in the community detection approach, there may be more efficient methods to identify clusters in large datasets. But our motivation for writing this paper was twofold: to explore solutions for problems that occur when applying the relaxed structural balance approach to large signed two-mode data, and to contribute to recent applications of social network analysis to IR research by highlighting its potential to uncover soft balancing of power processes. Soft balancing implies alliances with consistency with regard to joint voting behavior and, to capture this, the signed blockmodeling of two-mode data has been operationalized in a specific and substantive driven way.

[38] It raises also the daunting problem of specifying the form of two-mode data structures where an indirect approach, such as the one using Euclidean distance, performs very well rather than very poorly.

Appendix A. Countries included in the analyses

T1 countries, $N=141$			
Afghanistan	DKCambodia	Kenya	Portugal
Algeria	DemYemen	Kuwait	Qatar
Angola	Denmark	Laos	Romania
Argentina	Djibouti	Lebanon	USSR
Australia	DominicanRep	Lesotho	Rwanda
Austria	Ecuador	Liberia	StLucia
Bahamas	Egypt	Libya	SaoTomePrincipe
Bahrain	ElSalvador	Luxembourg	SaudiArabia
Bangladesh	Ethiopia	Madagascar	Senegal
Barbados	Fiji	Malawi	SierraLeone
Byelorussia	Finland	Malaysia	Singapore
Belgium	France	Maldives	Somalia
Benin	Gabon	Mali	Spain
Bhutan	GerDemRep	Malta	SriLanka
Bolivia	GerFedRep	Mauritania	Sudan
Botswana	Ghana	Mauritius	Suriname
Brazil	Greece	Mexico	Sweden
Bulgaria	Guatemala	Mongolia	SyrianArabRep
UVBurkinoFaso	Guinea	Morocco	Thailand
BurmaMyanmar	GuineaBissau	Mozambique	Togo
Burundi	Guyana	Nepal	TrinidadTobago
Cameroon	Haiti	Netherlands	Tunisia
Canada	Honduras	NewZealand	Turkey
CapeVerde	Hungary	Nicaragua	Uganda
CenAfrRep	Iceland	Niger	Ukraine
Chad	India	Nigeria	UAE
Chile	Indonesia	Norway	UK
China	Iran	Oman	URTanzania
Colombia	Iraq	Pakistan	US
Congo	Ireland	Panama	Uruguay
CostaRica	Israel	PapuaNewGuinea	Venezuela
CotedIvoire	Italy	Paraguay	Vietnam
Cuba	Jamaica	Peru	Yemen
Cyprus	Japan	Philippines	Yugoslavia
Czechoslovakia	Jordan	Poland	ZaireDRC
			Zambia
T2 Countries, $N=153$			
Albania	Djibouti	Lebanon	StLucia
Algeria	DominicanRep	LibyanAJ	Samoa
Andorra	Ecuador	Liechtenstein	SanMarino
Angola	Egypt	Lithuania	SaudiArabia
AntiguaBarbuda	ElSalvador	Luxembourg	Senegal
Argentina	EquatorialGuinea	Madagascar	SierraLeone
Armenia	Eritrea	Malaysia	Singapore
Australia	Estonia	Maldives	Slovakia
Austria	Ethiopia	Mali	Slovenia
Azerbaijan	Fiji	Malta	SolomonIslands
Bahamas	Finland	Mauritius	SouthAfrica
Bahrain	France	Mexico	Spain
Bangladesh	Gabon	Monaco	SriLanka
Barbados	Georgia	Mongolia	Sudan
Belarus	GerFedRep	Morocco	Suriname
Belgium	Ghana	Mozambique	Swaziland
Belize	Greece	Namibia	Sweden
Benin	Grenada	Nepal	SyrianArabRep
Bhutan	Guatemala	Netherlands	Tajikistan
Bolivia	Guinea	NewZealand	Thailand
Botswana	Guyana	Nicaragua	TFYRM
Brazil	Haiti	Nigeria	Togo
BruneiDar	Honduras	Norway	TrinidadTobago
Bulgaria	Hungary	Oman	Tunisia
UVBurkinoFaso	Iceland	Pakistan	Turkey

Appendix A (*Continued*)

BurmaMyanmar	India	Panama	Uganda
Cameroon	Indonesia	PapuaNewGuinea	Ukraine
Canada	Iran	Paraguay	UAE
CapeVerde	Ireland	Peru	UK
Chile	Israel	Philippines	URTanzania
China	Italy	Poland	US
Colombia	Jamaica	Portugal	Uruguay
CostaRica	Japan	Qatar	Uzbekistan
CotedIvoire	Jordan	RepKorea	Venezuela
Croatia	Kazakhstan	RepMoldova	Vietnam
Cuba	Kenya	Romania	Yemen
Cyprus	Kuwait	RussianFed	Zambia
CzechRep	Laos	Monaco	Zimbabwe
Denmark	Latvia	Mongolia	

Appendix B. UNGA military resolutions for each time period

Time 1: 1981–1985, $N = 276$ resolutions

(1) *National and regional concerns*: a number of resolutions addressed apartheid. South Africa was under apartheid from 1948 to early 1994 and it was sanctioned from voting in the UN from September 1974 until June 1994. The issue of apartheid was raised in the first UNGA session in 1946 but it was seen as an internal issue until the 1960 Sharpeville massacre brought greater international attention to the issue. It was not until 1974, that the UNGA voted to expel South Africa from the UN. The UNSC 418 in 1977 created a mandatory arms embargo. The following resolutions focused on national and regional concerns: South Africa apartheid regime against Angola and other independent African states (36/172C, 1981); arms embargo against South Africa (36/172E; 40/6, 1985); condemnation of military and nuclear collaboration with South Africa (37/69D); armed conflict between Iran and Iraq (37/3); Israeli aggression against Iraqi nuclear installations (36/27, 1981; 38/9, 1983; 39/14, 1984); Israeli nuclear armament (36/98, 1981; 37/82, 1982; 38/69, 1983; 39/147, 1984; 40/93, 1985); using Antarctica for peaceful purposes (3 in 1985); Falkland islands (37/9, 1982; 38/12, 1983; 39/6, 1984; 40/21, 1985); 13 addressing Palestine; 13 addressing the Middle East situation; 3 on Afghanistan (very divisive (38/29, 1983; 39/13, 1984; 40/12, 1985); 2 to establish nuclear-weapon-free-zone in the Middle East; 4 to establish a nuclear-weapon-free-zone in South Asia.

(2) *Resolutions related to conventions*: chemical and biological weapons (the following resolutions revealed a strong division in voting: 36/96 A, B, C; 37/98 A, C, D, E; 38/187 A, C; 36/65 A, B, E; 40/92 A, C). The Convention on Biological Weapons opened for signature April 1972; entered into force March 1975; 171 signatories and 155 ratifications; Israel is one of the states that have not ratified or signed; 4 resolutions address the "urgent need" for a comprehensive nuclear-test-ban treaty, 1982–1985 (a Convention was opened for signature in 1996 but is not yet in force; the US signed it but did has not ratified it as of yet; see http://www.ctbto.org/faqs/?uid=3&cHash=11241d850); 10 resolutions call for the creation of an international convention to assure non-nuclear weapon states against the use or threat of use of nuclear weapons; 1 resolutions calls for a treaty to prohibit stationing weapons of any kind in outer space.

(3) *Resolutions addressing particular security issues and policies*: 36 resolutions focus on general and complete disarmament with multiple related issues; 4 on removing landmines as remnants of war; 4 on reducing military budgets; 2 reviewing multilateral treaty-making process; 6 prohibiting the development and manufacture of new types of weapons of mass destruction; 4 on preventing an arms race in outer space; 1 on nuclear weapon freeze; 3 calling for an immediate end to testing nuclear weapons; 4 calling for implementation of resolutions concerning ratification of Additional Protocol I of the Treaty for the Prohibition of nuclear weapons in Latin America; 1 on declaration to prevent nuclear catastrophe; 1 on comprehensive review of all peacekeeping operations.

(4) *Resolutions from the 10th and 12th special session on disarmament*: 55 and 23 resolutions, respectively; these address multiple issues from general disarmament to chemical, biological and small arm weapons, and the need for conventions.

(5) *Special reports on disarmament and security*: 5 (one a year) on enhancing the effectiveness of the principle of non-use of force in international relations.

(6) *UN conferences*: 1 on promotion of international cooperation in peaceful uses of nuclear energy.

(7) *Normative and rights issues*: 2 relating disarmament and development; 2 on right of people to peace.

Time 2: 1996–2001, $N = 150$ resolutions

(1) *National and regional concerns*: 2 resolutions focus on financing of a UN interim force in Lebanon; 1 on the ME peace process; 6 on the risk of nuclear proliferation in the ME; 1 on establishing nuclear-weapon-free-zone in the ME region; 1 on maintenance of international security in the development of South-Eastern Europe; 1 on Bosnia and Herzegovina; 6 to establish a nuclear-weapon-free-zone in South Atlantic; 1 to establish a nuclear-weapon-free-zone in South Asia; 4 on implementing declaration of Indian Ocean as a zone of peace.

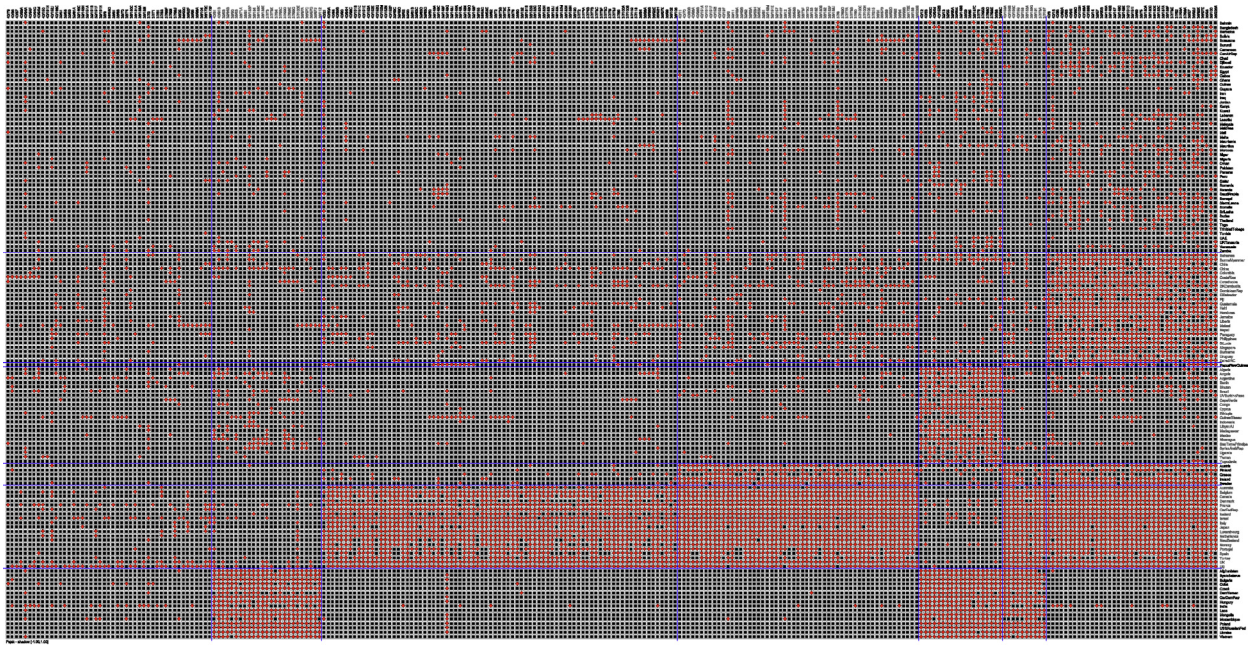
(2) *Resolutions related to conventions*: these are all addressed under the general and complete disarmament resolutions detailed in #4 below. The Convention on Chemical Weapons opened for signature January 1993; entered into force April 1997; as of May 2009, 188 signatures and 186 ratifications; Israel and Myanmar have not signed; Angola, N Korea, Egypt, Somalia, Syria have not signed or ratified it.

(3) *Resolutions addressing particular security issues and policies*: 82 resolutions focused on general and complete disarmament with multiple issues. Notably, this is a 4-fold increase; several resolutions addressed the ICJ advisory opinion on the Legality of the threat or use of nuclear weapons and reveal very divisive voting; other issues covered a wide range of disarmament issues including the Nuclear non-Proliferation Treaty; conventional arms control at the regional level; transparency in armaments; environmental norms in disarmaments; small arms; nuclear testing; 1 on arms race prevention in outer space; 1 on comprehensive review of peacekeeping operations; 1 on maintenance of international security; 7 on scientific and technological developments and their impact on international security; 5 on preventing an arms race in outer space; prevent violent disintegration of states; 1 on measures to eliminate international terrorism.

(4) *Resolutions from the 10th and 12th special session on disarmament*: 2 and 6 resolutions respectively (continual reduction); 5 from the 12th session is on a nuclear weapons convention; other issues discuss disarmament generally.

(5) *Special reports on disarmament and security*: 4 on the International Atomic Energy Agency (IAEA); 1 on the report of the Security Council.

(6) *Cooperation with other agencies*: 6 on cooperation between the UN and the Organization for Security and Cooperation in Europe (OSCE); 1 on cooperation between the UN and the Preparatory Commission for the Comprehensive nuclear test ban treaty organization.



PPPPPPP
PPPPPPN
PPPPPNN
PPPPNPP
PPPNPNN
PPNNPNN
PNPPNNP

Fig. C1. A partition of the full set of states and military resolutions for Time 1.

Appendix C. Ancillary figures and tables

Fig. C1 and Table C1.

Table C1
Seven clusters of resolutions from Fig. C1.

R1	R2	R3		R4		R5	R6	R7
36/104	36/112	36/102	38/73E	36/172E	40/11	36/88	37/167	36/100
36/172C	36/97A	36/106	38/73H	36/188	40/151A	36/96C	37/78A	36/226A
36/172F	36/97C	36/84	38/76	36/226B	40/151D	37/76	38/183A	36/27
36/25	36/97G	36/86A	38/80	36/31	40/151F	37/95B	38/188G	36/92J
36/92C	36/97J	36/86B	39/148A	36/87B	40/152I	37/98D	38/191	36/92K
36/92F	37/100E	36/92E	39/148C	36/89	40/158	37/98E	39/158	36/97E
36/95	37/73	36/92H	39/148F	36/92D	40/197	38/187C	40/156A	36/98
36/96A	37/78K	36/97K	39/148G	36/92I	40/70	38/29	40/156B	37/78E
37/3	37/84	36/99	39/148J	36/94	40/80B	38/65	40/156C	37/82
37/71	37/98C	37/100A	39/148K	36/96B	40/96A	39/13	40/159	37/98A
37/81	37/99D	37/100B	39/148L	37/100C		39/148B		37/99A
37/83	37/99E	37/102	39/148N	37/100H		39/55		38/180A
37/99F	37/99G	37/118	39/148O	37/105		39/65A		38/180D
37/99I	38/183P	37/72	39/148P	37/215		39/65E		38/180E
37/99J	38/184	37/74A	39/155	37/69D		40/12		38/183C
38/180C	38/188C	37/74B	39/49C	37/77A		40/152B		38/187A
38/181A	38/188E	37/77B	39/49D	37/78B		40/83		38/188F
38/188A	38/63	37/78C	39/52	37/80		40/92C		38/69
38/188I	38/71A	37/78F	39/60	37/9		40/94N		38/75
38/58D	38/81	37/78G	39/61B	38/12				39/
38/61	39/53	37/78I	39/63C	38/133				39/6A
38/68	39/64B	37/78J	39/63D	38/162				39/6B
38/70	39/90	37/85	39/63G	38/180B				39/7
38/74	40/81	38/132	39/80	38/182				39/8D
38/9	40/91B	38/181B	40/151B	38/183F				39/8E
39/146C		38/183B	40/151C	38/188J				39/151D
39/148H		38/183D	40/151E	38/58A				39/151I
39/151A		38/183G	40/152A	38/58B				39/57

Table C1 (*Continued*)

R1	R2	R3		R4	R5	R6	R7
39/151B		38/183H	40/152E	38/67			39/65B
39/151F		38/183I	40/152G	38/73F			40/152H
39/151H		38/183J	40/152M	38/73G			40/168A
39/51		38/183L	40/152N	39/11			40/168B
39/58		38/183M	40/152P	39/148M			40/18
39/59		38/183N	40/21	39/151E			40/6
39/61A		38/188H	40/80A	39/157			40/85
40/150		38/190	40/88	39/167			40/92A
40/151H		38/58C	40/89B	39/49A			40/93
40/152J		38/58E	40/90	39/49B			40/94I
40/168C		38/62	40/94H	39/6			40/96D
40/79		38/72	40/96C	39/62			
40/86		38/73B		39/63A			
40/87				39/63H			
40/89A				39/63K			
40/94A				39/81			
40/94F							
40/94G							
40/94M							

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