Discrete Structures

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Lecture 18 Counting

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An example

- A child is allowed to choose one jellybean out of two jelly beans, one red and one black, and one gummy bear out of three gummy bears, yellow, green, and white. How many different sets of candy can the child have?

Multiplication Principle

- If there are n_1 possible outcomes for a first event and n_2 possible outcomes for a second event, then there are $n_1 * n_2$ possible outcomes for the sequence of the two events.

- Example

  How many 4 digit numbers are there? (Include those which begin with 0)

  Solution:

  There are 10 ways to select the first digit, 10 ways to select the second digit, 10 ways to select the third digit and 10 ways to select the fourth digit. As per the multiplication principle, there are 10 **x** 10 **x** 10 **x** 10 = 10000 ways to construct a 4 digit number.

Examples of the Multiplication Principle

- How many 4 digits are there, if no digit can be repeated?

  Solution: $10 \times 9 \times 8 \times 7 = 5040$.

- Let A and B denote two sets. How many elements does A $\times$ B have?

  Solution: Using the multiplication principle, $|A| \times |B|$.

Addition Principle

- If A and B are disjoint events with n_1 and n_2 possible outcomes respectively, then the total number of possible outcomes for the event "A **or** B" is $n_1 + n_2$.

- Example
  - A customer wishes to purchase a vehicle from a dealer. The dealer has 10 trucks and 5 cars. How many choices does the customer have?
    Solution: $10 + 5 = 15$.
  - Let A and B be two disjoint sets. What is $|A \cup B|$?
    Solution: By the addition principle, $|A| + |B|$.
  - Let A and B denote two finite sets; show that
  - $|A - B| = |A| - |A \cap B|$
    Solution: The key observation is that $(A - B)$ and $A \cap B$ are disjoint. Further, the union of $(A - B)$ and $A \cap B$ is A! Therefore, by the addition principle, $|A| = |A - B| - |A \cap B|$.

Combining addition and multiplication principles

- How many 4 digit numbers begin with a 4 or a 5?

  Solution: Using the multiplication principle, the number of 4 digit numbers which begin with 4 is $1 \times 10 \times 10 \times 10 = 1000$. Likewise, the number of 4 digit numbers which begin with 5 is 1000. Thus the number of 4 digit numbers which begin with a 4 or a 5 is $1000 + 1000 = 2000$, using the addition principle.

- How many 3 digit numbers between 100 and 999 (inclusive) are even?

  Solution: Every even number ends in 0, 2, 4, 6 or 8. Use multiplication principle to compute the number of even numbers that end in 0, that end in 2 and so on ($9 \times 10 \times 1$). Use the addition principle to get the total number of even numbers (450).

- How many 4 digits are there in which at least one digit is repeated?

  Solution: Find the total number of 4 digit numbers and subtract the 4 digit numbers with no repetitions!

Decision Trees

- The multiplication principle cannot be used if the number of choices at a given stage depends upon the exact choice made in the previous stage.

- Examples:
  - In the 4 digit problems, it did not matter what digit was chosen in the current stage.
  - In how many ways can you toss a coin 5 times, so that the head side does not show up in consecutive tosses?