Len Lye's *Grass* Scaling Proposal

Angus McGregor, Jan. 2022

Summary

Len Lye's kinetic artwork *Grass* features a series of highly flexible cantilevered beams. A enlarged version of the artwork, featuring enlarged cantilever elements between three and four metres is desired. This report presents draft design requirements for a proposed version of the work, and investigates the feasibility of scaling the artwork using static similarity to ensure a similar artistic performance between the original and scaled sizes.

From this report, artistic design requirements should be verified by the Len Lye Foundation, and it is recommended that a wand of enlarged specification be tested.

Description of Lye's original Artwork

Grass features approximately 120 cantilevers arranged in two parallel rows, in a 'SS' pattern. These cantilevers were often referred to as *Wands* by Lye.

The cantilevers – or *Wands* – are mounted into an aesthetically designed wooden mounting plate. Each wand is approximately 900mm long, and of a $1/32"$ diameter. The mounting plate is tilted back and forth about a longitudinal axis, upto an angle of approximately 3^o.

The rocking action may be achieved by a number of suitable methods, and is not critical to the performance. Lye chose the speed of the rocking motion to coincide with the resonant speed of the cantilever beams. Due to the eccentricity of each wand from the neutral rocking axis, and occasional contact with nearly wands, the dynamic motion of each wand varies.

Description of the intended installation location

The Len Lye foundation has proposed this work as a candidate for a site on a coastal walkway near Ferrymead in Christchurch, New Zealand. If fabricated, this would be the first permanent outdoor exhibit in Lye's birthplace.

This location features occasional strong winds, with continuous exposure to corrosive salt breeze and abrasion from sand.

As an outdoor sculpture, it is prone to interference from wildlife of gaian – and urban – origin.

Design Requirements

To successfully produce and enlarged version of *Grass*, enlarged wands must

- Be able to produce a *similar* performance to the original artwork.
- Withstand loading generated by a performance
- Withstand environmental loading that may be produced at the intended installation site.

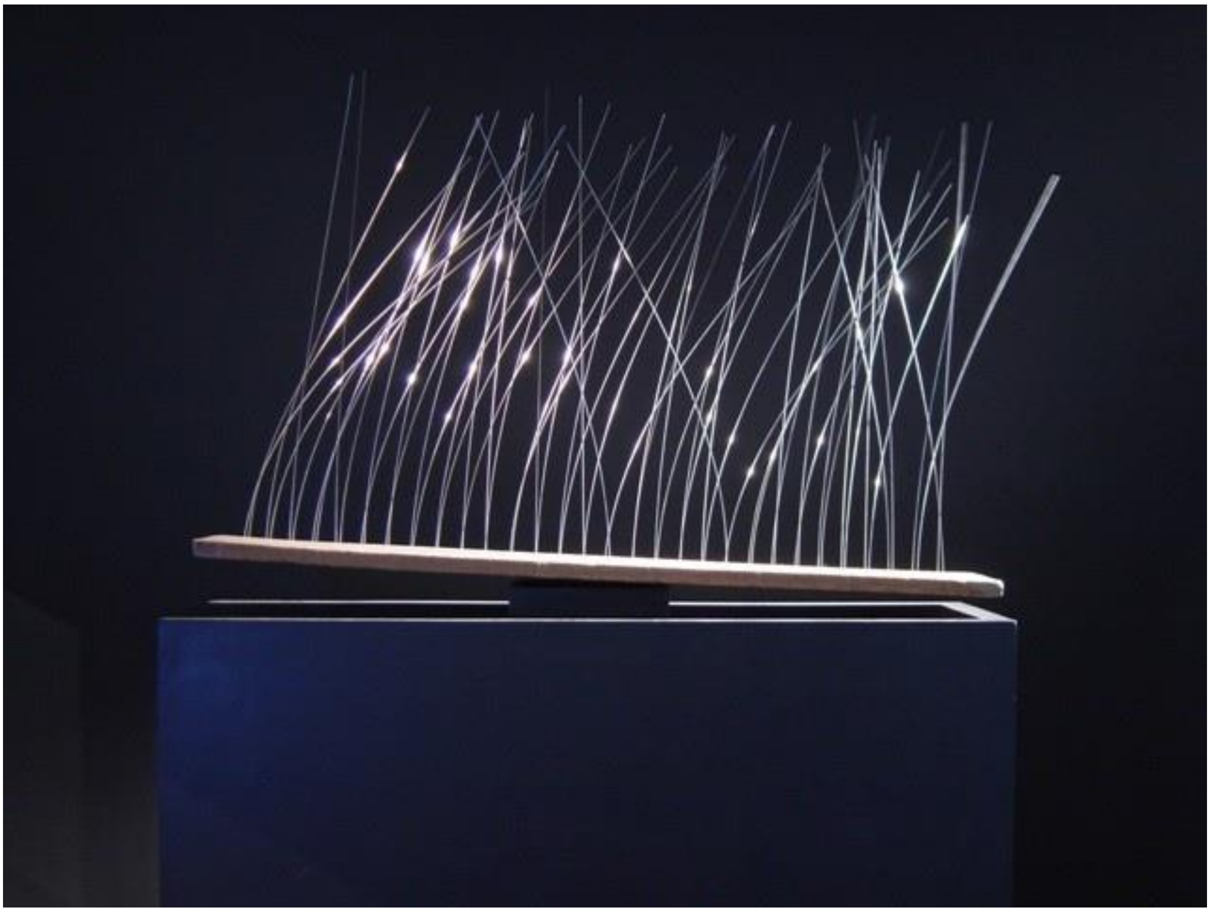


Figure 1 – Grass, Len Lye. (Govett Brewster)

Design Requirements for the Wand

D(emand) / W(ish)	Criteria	Source
D	Cantilevers to be between 3-4m	**EW**
D	Cantilevers able to support 120km/hr wind loading	**EW**
D	Cantilevers fabricated from a bright, shiny material of similar lustre to Stainless Steel	**AM**
D	Cantilevers capable of resisting visible scratching from salt / sand breeze / surface marring from contact with other cantilevers	
D	Material must have low damping coefficient $\zeta \leq 5\%$	
W(H)	Cantilevers to have an expected life of 5 years prior to replacement	**AM**
W(H)	Be resistant to casual contact with people.	**AM**
W(M)	**Cantilevers to have infinite fatigue lifespan**	**AM**
W(H)	**Cantilevers to be fabricated from 316 stainless steel**	**EW**
W(M)	Cantilevers to produce a geometrically similar performance to the original *Grass*	**AM**

Scaling Approach

Typically in engineering design, when an object is produced at a different scale, the intention is to hold a particular engineering characteristic constant. Often, this relates to the spatial proportions of an object, or the timescale of motions produced.

Therefore, the two primary means of scaling *Grass* might involve either holding the resonance frequency constant, or the amplitude and vibratory form constant between scales.

Lye gives evidence that scaling to hold the vibratory form constant between scales is an artistically acceptable method for scaling, as reported by Gooch [1] in the design of *Big Blade; O'Keefe [2]* in the design of *Sun, Land, and Sea* and McGregor [3] In the design of *Flaming Harmonic.*

Lye experimented with thin rods of spring steel in a local workshop near his residence in New York, USA [4]. By testing the stability of various rods by holding them vertically and shaking them to form mode shapes, Lye would increase the length of the vertical cantilever rod until it buckled under its own weight. Then, he would reduce the length of the rod to obtain the correct stability. Lye's intuitive experimental method can be replicated mathematically by quantifying the stability of a vertical cantilever at the design height and comparing this with the stability at the onset of buckling. Timoshenko and Gere [5] define the dimensionless gravity parameter for a vertical cantilever as:

$$\gamma = \frac{mgl^3}{EI} \tag{1}$$

m represents mass per unit length, g is acceleration due to gravity, l is the cantilever length, and EI is the flexural rigidity of the beam. If the value of the gravity parameter exceeds $\gamma = 7.827$, the beam will be unstable. In order to scale wands for grass, the intention is to maintain the gravity parameter γ of the original wands in the enlarged versions.

If the gravity parameter is held between scales, then geometric similarity between Lye's original *Grass* and the new work is possible.

To calculate the gravity parameter of the original *Grass*, Table 2 presents relevant specifications for the original work

Table 2 - Grass Wand Specifications for the original and desired sculptures

	Original	Desired (316)	Units
Length	900	3000-4000	mm
Diameter	$.79$	$-$	mm
Density	7850	7850	kg/m^3
Modulus	190	190	GPa

Using the data from Table 2, we calculate the flexural rigidity, *EI* of the original work as

$$EI = [\ 200e9\ ] * [\frac{\pi}{64}(.00079^4)]\ \ \mathrm{Nm}^2$$

$$EI = .0038\ \mathrm{Nm}^2$$

Further, the mass per unit length of the original cantilever is found by

$$m = [7850] * [\frac{\pi}{4}(.00079^2]\ \mathrm{kg/m}$$

$$m = .0038\ \mathrm{kg/m}$$

Given the wand specification in Table 2, the gravity parameter for Lye's original Grass may be established.

$$\gamma = \frac{.0038 * 9.81 * \ 0.9^3}{.0036}$$

$$\gamma = 6.7$$

From this result, it is clear that the wands for grass are chosen to be near the limit of stability.

1) How will scaling affect the operating loading
2) Can the sculpture support environmental loading?
3) Do Lye's notes allow for *some* flexibility in the flexibility intended for larger wands

1. Influence of Scale on Environmental and Operating Loads

During an artistic performance, grass makes a mode shape consistent with the first bending mode of a cantilever beam. For the case of a vertical cantilever, Naguleswaran [6] presented a method to produce the resonant frequencies for a wand of given γ required to produce the correct mode shape for an artistic performance.

McGregor [7] developed equations to assess structural loads generated by a cantilever with a circular cross section, when vibrated at a resonant frequency to produce a mode shape. This work describes the operating loads of Len Lye's Grass at any scale, given γ, and the maximum amplitude of a wand during a performance.

1.1 Desired operating Amplitude

During a performance of the original grass, each wand oscillates through some amplitude. Video footage of the original grass yields a maximum amplitude of about 0.5m.

1.2 Optimising Material

In the design of Grass, the objective criterion for an optimal wand is to maximise lifespan, by maximising the ratio of endurance strength to maximum stress developed during a performance.

$$\Psi = \frac{\sigma_{endurance}}{\sigma_b} \tag{2}$$

Applying the definition for bending strength in a flexible cantilever, and allowing some stress concentration factor K at some length along the beam, the optimisation criterion becomes

$$\Psi = \frac{1}{\sqrt{2}} \underbrace{\left(\frac{1}{Kl^3}\frac{1}{\frac{d^2v}{dx^2}}\right)}_{\text{Geometry}} \underbrace{\sqrt{\frac{\gamma}{g}}}_{\text{Stability}} \underbrace{\left(\frac{\sigma_{endurance}}{\sqrt{E\rho}}\right)}_{\text{Material}} \tag{3}$$

- The material parameter here is related to maximising the ratios of strength to density, and strength to stiffness, allowing us to perform a search for candidate materials which achieve high values for both indices.
- The stability parameter indicates that artworks closer to the limit of stability are inherently stronger Usually in scaling Lye sculptures, this is fixed.
- The geometry parameter indicates that shorter works at low amplitudes are stronger, as we might expect.

High performance materials include:

Composites such as fibreglass, carbon fibre

Titanium

Medium Performance materials include

Tools steels (716/420), 5160 steel

Aluminium alloys

Lower performance materials include

Stainless steel,

Mild steel

These material performance indications do not consider environmental loading, corrosion resistance, or manufacturability.

In the following analysis, we will compare performance characteristics for three candidate materials.

- 6Al4V Titanium
- Fibreglass
- 316 Stainless steel

1.3 Influence of scale on the operating loads of *Grass*

The scale factor ($S.F.$) of Grass is defined as the length of a proposed artwork compared to that of Lye's original, such that

$$S.F. = \frac{l_{s(caled)}}{l_{o(riginal)}} \quad (4)$$

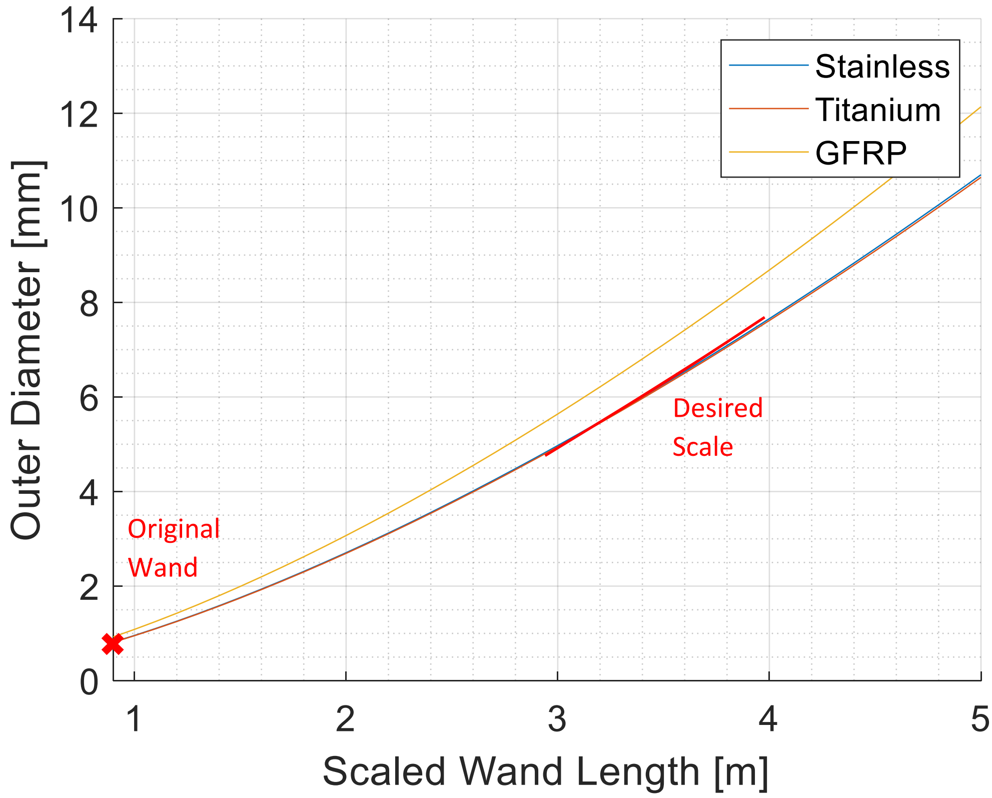
As the scale of the artwork is increased, the length of a rod increases proportionally. In order to hold $\gamma_s = \gamma_o$ to satisfy our desired scaling method, this requires particular choice of the wand diameter.

Following McGregor [8], an equation for the diameter of scaled wands with respect to the original is given by rearranging Equation (1) for the gravity parameter.

$$d_o = \sqrt{\frac{16\rho g l^3}{\gamma E}} \quad (5)$$

Solving Equation (X) and selecting a real, positive root returns a value of d_o for a given d_i. Different materials have different properties, which alters the desired diameter.

For a 3m wand,



- Titanium or steel rod approximately 5mm in diameter
- A fibreglass rod of 5.8mm in diameter.

For a 4m wand,

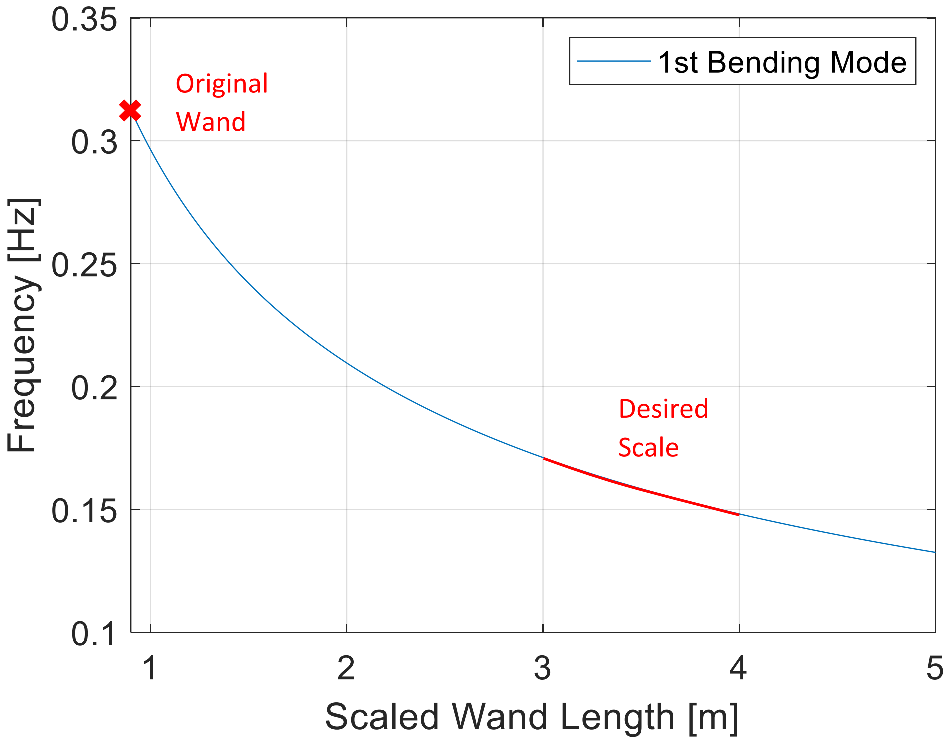
- Titanium or steel wand of 7.6mm in diameter
- A fibreglass wand of 8.6mm in diameter

1.4 Changing Frequency

As the scale of a work changes, the apparent displaced form is held constant between scales. However, this changes the resonant frequencies ω of each wand by a ratio of the length scales for the original and scaled wands.

$$\frac{\omega_s}{\omega_o} = \sqrt{\frac{l_o}{l_s}} \tag{6}$$

If the wands are held rigidly in a fixed support, then the maximum amplitude is limited by the allowable stress. In the case of a 0.5m tip amplitude, which seems to be about the maximum exhibited in Lye's original



For a 3m work, expect a frequency reduction to 0.17 Hz (55% original speed)

For a 4m work, expect a frequency reduction to .15 Hz (48% original speed)

1.5 Limitations on operating amplitude

Stresses are induced in the wand by the elastic deformation caused during a performance. Because the wands are long and slender. If stresses exceed the elastic limit for the material, the wand will deform permanently, and fail. Stress caused during a performance are defined as

$$\sigma_b = \frac{Md}{2I} \tag{7}$$

McGregor [7] gives the change in stress as the artwork increases in scale.

$$\frac{\sigma_{b,s}}{\sigma_{b,o}} = \frac{\rho_s l_s^2 d_s (d_o^2)}{\rho_o l_o^2 d_o (d_s^2)}$$

If the stress for the scaled work is below the allowable stress limit for the material, then the artwork may operate at a geometrically similar amplitude to the original work.

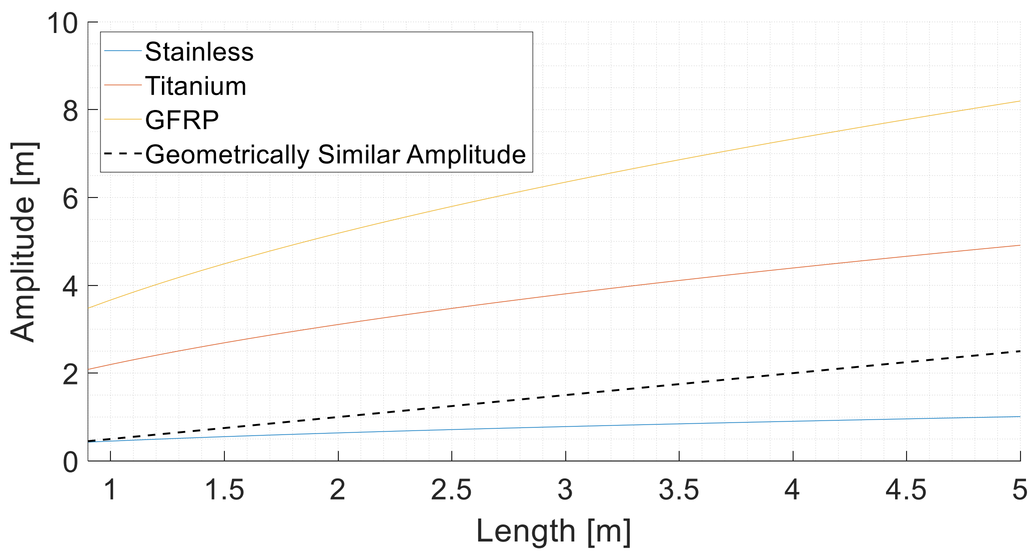
If the stress for the scaled work exceeds the allowable stress, then the amplitude for the work may have to be reduced to offer a suitable life for the artwork.

Noting the dimensionless amplitude is constant between scales, as

$$\frac{A_s}{l_s} = \frac{A_o}{l_o}$$

Also noting that stress increases linearly with amplitude by the definition of geometric similarity [9], the maximum amplitude a wand may support may be defined

$$A_{s,lim} = A_o \frac{l_s}{l_o} \frac{\sigma_{b,s\ lim}}{\sigma_{b,s}}$$



The amplitude for titanium and GFRP is lower than may be desired. There are several recourse options, connected to a material optimisation criterion for the artwork.

Design considerations this optimisation criterion supports, useful for the design of *Grass* are:

- Flexibility in the clamping geometry, to reduce peak stresses
  - This may be achieved by spacing a pair of supports to allow flexure in the clamp.

    - This may result in a 20-30% increase in amplitude.
- A change in material
    - Pultruded, uniaxial fibreglass or Titanium have capacity to exceed the desired amplitude.
    - Risks around paint chipping exist with fibreglass.
    - Titanium 6Al4V would have the requisite strength to produce large amplitude, while avoiding the issues of chipping and rust.
- A change in wand geometry near the fixed end.
    - A tapered sleeve or wand may have some effect.
    - This will require a length of ~10% wand length to have effect, and have a limited effect.

2 Wind loading

Wind places a distributed load on a wand. If wind is present while the sculpture operates, then loading on the artwork is a combination of stress induced during operation, and the wind loading.

Initially, we consider only the influence of wind, to determine whether the sculpture is resilient to environmental loading.

The maximum moment a wand may support is given by rearranging the formula for bending stress, such that

$$M_{failure} = \frac{2\sigma_{failure}I}{d} \tag{8}$$

A uniformly distributed load sufficient to generate M_{fail} can be calculated as

$$w_{failure} = \frac{2M_{failure}}{l^2} \tag{9}$$

From the drag equation,

$$F_d = \frac{1}{2}\rho C_d A v^2 \tag{10}$$

Defining wind loading per unit length as $w = F_D/l$ and frontal area of the cantilever as $A = ld$ allows us to relate the critical load per unit length , $w_{failure}$ to a wind speed.

$$w = \frac{1}{2}\rho C_d d v^2 \tag{11}$$

Rearranging, we can find the wind speed to cause material failure of a cantilever

$$v_{failure} = \sqrt{\frac{2w_{failure}}{\rho_{air}C_d d}} \tag{12}$$

For stainless steel, the strength of the artwork limits both its ability to support the desired amplitude, and the ability to support suitably large wind loading. This suggests the material is unacceptable.

For Fibreglass or Titanium, the wand is limited in its ability to support wind loading, but is able to support substantial deflections, much larger than desired.

This data has some caveats

- It is assumed that the frontal area is constant. As wind loading increases, the sculpture deforms, which reduces its effective length. The reduction in length reduces the moment on the base. This effect may double the allowable wind speed from the predicted value, assuming the sculpture halves in effective length.
- Physical model testing is required to confirm the magnitude of the effect. For materials able to support larger deflections, the effective length reduction will be larger, which will increase supported wind loading further.

In the case of a wand capable of supporting a maximum deflection of $1/\beta^{th}$ the length of the wand, then at a minimum, an effective length of $l^* = l/\beta$ may be exposed to wind. However, the frontal area per unit length will increase by the factor β such that $A^* = \beta A$.

In general,

$$v_f^* = \sqrt{\frac{\frac{4M_f}{(l/\beta)^2}}{\rho_{air} C_d (\beta d)}}$$

$$v_f^* = \sqrt{\frac{4M_f \beta}{\rho_{air} C_d d l^2}}$$

$$v_f^* = \sqrt{\beta} v_f \tag{13}$$

In the case of a 4m stainless wand, a theoretical maximum 2m deflection may be supported. A minimum wand length of less than 2m ($\beta = 2$) may be expected, which would offer a supported wind speed of $\sqrt{2} v_f$, or 88km/h .

In the case of a 4m titanium wand, a theoretical maximum 8.8m deflection may be supported. A minimum wand length of less than 1m ($\beta = 4$) may be expected, which would offer a supported wind speed of $\sqrt{4} v_f$, or 202km/h.

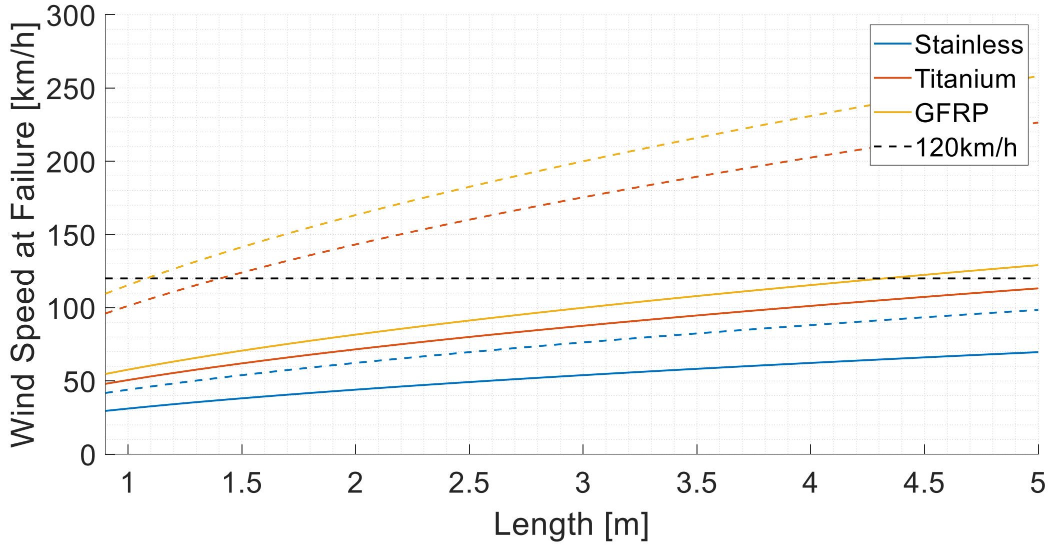


Figure 2 - Maximum Supported wind speeds. (-) Lower bound (--) Upper bound

Use of a mechanical fuse to isolate the sculpture from elevated wind loading

Some type of mechanical fuse to isolate the artwork from high wind loading has been suggested as a means to allow the sculpture to operate during normal loading conditions, and fold under high winds.

Due to the large spacing between relatively low operating loads and relatively high wind loads for fibreglass and titanium rods, this requires a relatively low precision device to achieve.

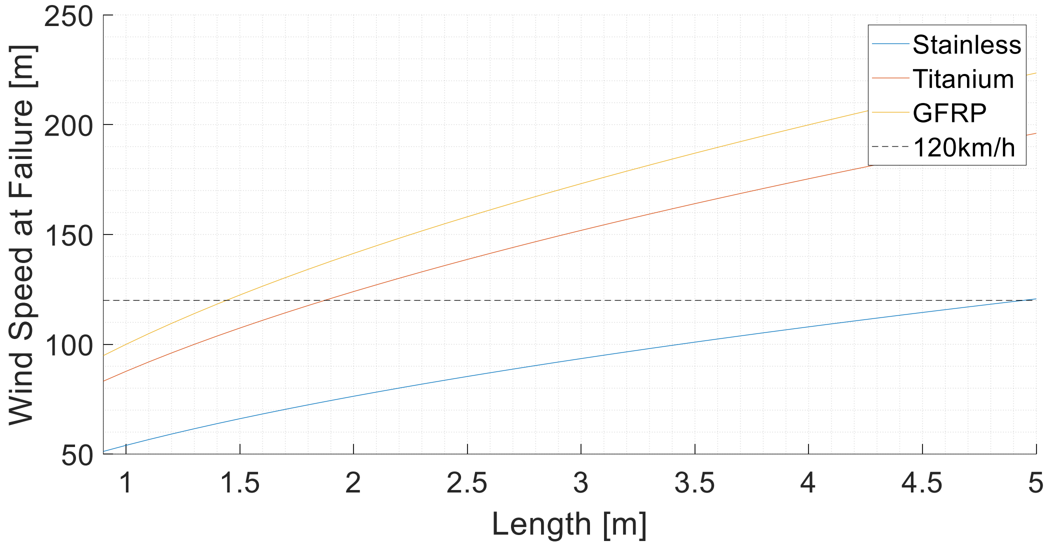
For the case of stainless wands, where excess wind loading is 1.5-2x larger than the operating loads, a higher precision device may be required.

In both cases, this is a plausible option.

Use of tapering to reduce wind loading

If the wands presented in the previous section were tapered, then the effective allowable wind load w is tripled.

Cantilever, tapered (cone)



Tapering a beam provides substantial improvement to strength in windy conditions.

In standard operating conditions, tapering decreases stability (requiring a larger diameter for the same length) which mitigates these benefits to an undetermined degree.

Further, while viable – the production costs of producing wands tapered sufficiently to be beneficial, and to the aesthetic standard required are likely to exceed the cost of using, titanium rods.

If it is decided that scaling using the original *Grass* stability parameter is too delicate for a large outdoor work, or that the resonant frequency is too low – then another consideration is to scale holding the resonant frequency constant.

In order to hold frequencies constant between sizes of *Grass,* we hold a *frequency parameter* constant between scales. The frequency parameter Ω is given by

$$\Omega = \frac{\mu\omega^2 l^4}{EI}$$

Holding the frequency parameter constant between scales gives

Both the dimensionless frequencies and natural frequencies of works scaled by this method should be equal therefore $\omega_s = \omega_o$

$$\frac{\mu_s l_s^4}{(EI)_s} = \frac{\mu_o l_o^4}{(EI)_o}$$

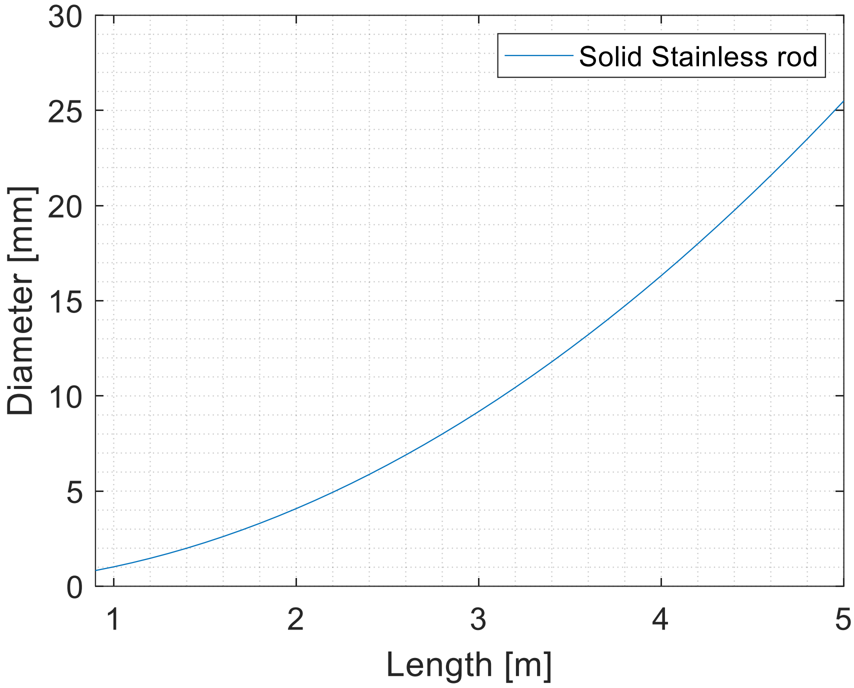
In the case of a cantilever beam,

$$\frac{64\rho_s \pi d_s^2 l_s^4}{4E_s \pi d_s^4} = \frac{64\rho_o \pi d_o^2 l_o^4}{4E_o \pi d_o^4}$$

$$\frac{16\rho_s l_s^4}{E_s d_s^2} = \frac{16\rho_o l_o^4}{E_o d_o^2}$$

Therefore the ratio of diameters to hold resonant frequency constant between scales is

$$\frac{d_s}{d_o} = \sqrt{\frac{\rho_s l_s^4 E_o}{\rho_o l_o^4 E_s}}$$



The corresponding stress is given by Gooch [9], if the original and scaled variants are made from the same material, is a linear increase by the scale factor, as

$$\frac{\sigma_s}{\sigma_o} = \frac{l_s}{l_o}\sqrt{\frac{E_s \rho_s}{E_o \rho_o}}$$

[1] S. D. Gooch and J. K. Raine, "The Dynamics and Limits on the Scaling of a Flexible Kinetic Sculpture," in *Proc. Instn. Mech Engrs.*, 2000.

[2] A. O'Keefe, *Determining the Feasibility of Building Len Lye's Kinetic Artwork Sun, Land, and Sea*. Christchurch: Department of Mechanical Engineering, University of Canterbury, 2015.

[3] A. McGregor and C. Balmer, "Len Lye's Rotating Harmonic," Department of Mechanical Engineering, University of Canterbury, Christchurch, 2015.

[4] R. Horrocks, *Len Lye: A Biography*. Auckland: Auckland University Press, 2002.

[5] S. Timoshenko and J. Gere, *Theory of Elastic Stability*. New York: McGraw-Hill, 1961.

[6] S. Naguleswaran, "Vibration of a Vertical Cantilever with and without Axial Freedom at Clamped End," *Sound and Vibration,* vol. 146, 2, pp. 191-198, 1991.

[7] A. McGregor, "On the Design of Len Lye's Harmonic Sculptures at the Maximum Feasible Size," Doctor of Philosophy, Dept. of Mechanical Engineering, University of Canterbury, Christchurch, New Zealand, 2021.

[8] A. McGregor, S. D. Gooch, and E. Webb, "On the Design of Len Lye's Flaming Harmonic," in *Proceedings of the 21st International Conference on Engineering Design (ICED 17)*, Vancouver, 2017.

[9] S. D. Gooch, *Design and Mathematical Modelling of the Kinetic Sculpture Blade*. Christchurch: Department of Mechanical Engineering, University of Canterbury, 2001.