和差化积 & 积化和差：

$\alpha \quad \beta \quad \alpha+\beta \quad \alpha-\beta$

$S \times C = \frac{1}{2}(S + S)$

$C \times S = \frac{1}{2}(S - S)$

$C \times C = \frac{1}{2}(C + C)$

$S \times S = -\frac{1}{2}(C - C)$

$\alpha \quad \beta \quad \frac{\alpha+\beta}{2} \quad \frac{\alpha-\beta}{2}$

$S + S = 2\,S\,C$

$S - S = 2\,C\,S$

$C + C = 2\,C\,C$

$C - C = -2\,S\,S$

万能公式：

$$\sin\alpha = \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$

$$\cos\alpha = \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$

$$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}$$

三倍角公式：

$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$.

$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$.

$$\tan 3\alpha = \frac{3\tan\alpha - \tan^3\alpha}{1-3\tan^2\alpha}$$

三倍角公式：

$\sin 3\alpha = 4\sin\alpha\sin(\frac{\pi}{3}-\alpha)\sin(\frac{\pi}{3}+\alpha)$

$\cos 3\alpha = 4\cos\alpha\cos(\frac{\pi}{3}-\alpha)\cos(\frac{\pi}{3}+\alpha)$

$\tan 3\alpha = \tan\alpha\tan(\frac{\pi}{3}-\alpha)\tan(\frac{\pi}{3}+\alpha)$

$$\tan(\alpha+\beta+\gamma) = \frac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\tan\alpha\tan\beta-\tan\beta\tan\gamma-\tan\gamma\tan\alpha}$$

$$\sin\alpha+\sin(\alpha+2d)+\cdots+\sin(\alpha+2nd) = \frac{\sin(n+1)d\cdot\sin(\alpha+nd)}{\sin d}$$

$$\cos\alpha+\cos(\alpha+2d)+\cdots+\cos(\alpha+2nd) = \frac{\sin(n+1)d\cdot\cos(\alpha+nd)}{\sin d}$$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$

$1+\cos\alpha = 2\cos^2\frac{\alpha}{2}$. $1-\cos\alpha = 2\sin^2\frac{\alpha}{2}$

$\cos^2\alpha = \frac{\cos 2\alpha+1}{2}$. $\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$

$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$, $\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha}.$$

三角形要點:

在 $\triangle ABC$ 中,

$\tan A+\tan B+\tan C=\tan A\tan B\tan C$

$\cot A\cot B+\cot B\cot C+\cot C\cot A=1$

$\dfrac{1}{\tan A\tan B}+\dfrac{1}{\tan B\tan C}+\dfrac{1}{\tan C\tan A}=1$

$\sin^2 A+\sin^2 B+\sin^2 C=2(1+\cos A\cos B\cos C)$

$\cos^2 A+\cos^2 B+\cos^2 C=1-2\cos A\cos B\cos C$

$2\sin\frac{A+B}{2}\cos\frac{A-B}{2}=\tan A\cos A+\tan B\cos B$

$\dfrac{\cot A+\cot B}{\tan A+\tan B}+\dfrac{\cot B+\cot C}{\tan B+\tan C}+\dfrac{\cot C+\cot A}{\tan C+\tan A}=1$

不等式:

若 $x\in(0,\frac{\pi}{2})$, 则 $\sin x<x<\tan x$.

若 $\alpha,\beta>0$, $\alpha+\beta<\pi$, 则 $\sin\alpha<\sin\beta \Leftrightarrow \alpha<\beta$.

若 $A+B+C=\pi$, 则 $\tan^2\frac{A}{2}+\tan^2\frac{B}{2}+\tan^2\frac{C}{2}\geqslant 1$

$x^2+y^2=r^2 \Rightarrow \begin{cases}x=r\cos\alpha\\ y=r\sin\alpha\end{cases}$ | $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \Rightarrow \begin{cases}x=a\cos\alpha\\ y=b\sin\alpha\end{cases}$ | $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \Rightarrow \begin{cases}x=a\sec\alpha\\ y=b\tan\alpha\end{cases}$

若 $|x|\geqslant a \Rightarrow x=a\sec\alpha$ 或 $x=a\csc\alpha$.

若 $b=\frac{a+c}{1-ac} \Rightarrow a=\tan\alpha, b=\tan\beta, c=\tan\gamma$.

若 $x,y,z\geqslant 0$, $x+y+z=1 \Rightarrow x=\sin^2\alpha\cos^2\beta, y=\cos^2\alpha\cos^2\beta, z=\sin^2\beta$

$y=\frac{a\sin x+b}{c\cos x+d} \Rightarrow$ 斜率公式, $a=c$ 时, 单位圆与点的斜率.

$\sin x\pm\cos x$, $\sin x\cos x$ 的函数, 令 $\sin x\pm\cos x=t \Rightarrow (\sin x\pm\cos x)^2=1\pm2\sin x\cos x$