Elements of Macroeconomics Spring 2024

Week 5

10 Long Term Growth

10.1 Simple Interest vs Compounding

When working with growth rates, we need to multiply growth rates due to **compounding**! We receive interest from interest. In finance, we do not care that much about it. Why? Because adding growth rates is a good *approximation* **IF GROWTH RATES ARE SMALL AND THE TIME HORIZON SHORT**!

Why? The approximation

$$(1+a)*(1+b) \approx 1+a+b \tag{1}$$

holds very well for very small numbers for a and b.

To see this take a \$100 bond with a 2% yield. If we want to know what we get after 5 years, we could either do it correctly with compounding or adding them up

$$\$100*(1.02)^5 = 110.41 \tag{2}$$
$$\$100*(1+0.02*5) = 110.00 \qquad \rightarrow \text{error is } 0.4\% \tag{3}$$

But, if interest rates or timer horizon increase, the approximation gets much worse!

$$\$100*(1.02)^{50} = 269.16 \tag{4}$$
$$\$100*(1+0.02*50) = 200.00 \qquad \rightarrow \text{error is } 34.6\% \tag{5}$$

$$\$100*(1.20)^5 = 248.83 \tag{6}$$
$$\$100*(1+0.2*50) = 200.00 \qquad \rightarrow \text{error is } 24.4\% \tag{7}$$

10.2 Long Term Growth

The most important formula is:

$$\%\Delta Y = \%\Delta L + \%\Delta LP \tag{8}$$

In words, change in long term output growth can only be achieved through either a change in labor growth or a change in labor productivity growth.