Assignment #1

Student #: [redacted]

Name: Zaid Albirawi

UWO email: zalbiraw@uwo.ca

1. Translate each of the following sentences into the language of propositional calculus using the indicated letters for atomic formulas. Indicate explicitly, for each letter, the statement it stands for.

    1. Not only is the dolphin air-breathing, it is viviparous as well. (A, V)
       A = air-breathing.
       V = viviparous as well.

       $A \wedge V$

    2. John attends the ballet whenever he has the money for the ticket. (A, H)
       A = attends the ballet.
       H = has the money for the ticket.

       $H \rightarrow A$

    3. If logic is difficult, Thea and Lori will pass only if they study. (D, T, L, S)
       D = difficult.
       T = Thea will pass.
       L = Lori will pass.
       S = if they study.

       $D \rightarrow ((T \wedge L) \rightarrow S)$

    4. Passing the logic course is a necessary condition for a student to obtain a Computer Science degree. (L, P)
       L = Passing logic.
       P = obtain a Computer Science Degree.

       $P \rightarrow L$

    5. If Q is a quadrilateral, then Q is a parallelogram if and only if its opposite sides are both equal and parallel. (Q, P, E, L)
       Q = quadrilateral.
       P = parallelogram.
       E = opposite sides are equal.
       L = opposite sides are parallel.
       $Q \rightarrow \left(P \leftrightarrow (E \wedge L)\right)$

6. A necessary condition for two lines to be parallel is that they neither intersect nor coincide. (P, I, C)
   P = parallel.
   I = intersect.
   C = coincide.

   $P \rightarrow \neg(I \vee C)$

7. If the derivative of f is defined on the interval (a, b), a necessary condition for f to be increasing is that its derivative is positive on (a, b). (D, I, P)
   D = derivative of f is defined on the interval (a, b).
   I = for f to be increasing.
   P = its derivative is positive on (a, b).

   $D \rightarrow (I \rightarrow P)$

8. A sufficient condition for the function f to have a maximum on [a, b] is that f is continuous on (a, b) and f is continuous at both a and b. (M, C, A, B)
   M = to have a maximum on [a, b]
   C = continuous.
   A = continuous at a.
   B = continuous at b.

   $(C \wedge A \wedge B) \rightarrow M$

2. Determine the validity (soundness) of the following arguments. If not already indicated, state explicitly for each letter the proposition it stands for. Use:
   - the tautology method for **(a);**
   - the contradiction method for **(b);**
   - An argument based on value assignments (but not using truth tables) to show that the conclusion is a tautological consequence of the premises for **(c)**.

   Justify your answers.

   Argument (a)

   Premise 1. If knowing is a state of mind (like feeling a pain), then I could always tell by introspection whether I know.

   Premise 2. If I could always tell by introspection whether I know, then I'd never mistakenly think that I know.

   Premise 3. I sometimes mistakenly think that I know.

   Conclusion. Knowing is not a state of mind.

   [Use S, I and M]

   $P_1: S \rightarrow I$

   $P_2: I \rightarrow \neg M$

   $P_3: M$

   $Con: \neg S$

S	I	M	P_1	P_2	P_3	Con	$(P_1 \wedge P_2 \wedge P_3) \rightarrow C$
0	0	0	1	1	0	1	1
0	0	1	**1**	**1**	**1**	**1**	**1**
0	1	0	1	1	0	1	1
0	1	1	1	0	1	1	1
1	0	0	0	1	0	0	1
1	0	1	0	1	1	0	1
1	1	0	1	1	0	0	1
1	1	1	1	0	1	0	1

Therefore, this argument is a tautology because row 2s premises and conclusion are consistent. Hence, the argument does hold.

Argument (b)

Premise 1. If you hold a moral belief and don't act on it, then you're inconsistent.

Premise 2. If you're inconsistent, then you're doing wrong.

Conclusion. Therefore, if you hold a moral belief and act on it, then you aren't doing wrong.

[Use M, A, I and W]

$P_1: (M \wedge \neg A) \rightarrow I$

$P_2: I \rightarrow W$

$Con: (M \wedge A) \rightarrow \neg W$

M	A	I	W	P_1	P_2	Con	$\neg Con$	$P_1 \wedge P_2 \wedge \neg Con$
0	0	0	0	1	1	1	0	0
0	0	0	1	1	1	1	0	0
0	0	1	0	1	0	1	0	0
0	0	1	1	1	1	1	0	0
0	1	0	0	1	1	1	0	0
0	1	0	1	1	1	1	0	0
0	1	1	0	1	0	1	0	0
0	1	1	1	1	1	1	0	0
1	0	0	0	0	1	1	0	0
1	0	0	1	0	1	1	0	0
1	0	1	0	1	0	1	0	0
1	0	1	1	1	1	1	0	0
1	**1**	**0**	**0**	**1**	**1**	**0**	**1**	**1**
1	1	0	1	1	1	1	0	0
1	1	1	0	1	0	0	1	0
1	1	1	1	1	1	1	0	0

Therefore, the argument is invalid, because there exists a set of values that make premise 1, 2 and the negation of conclusion true.

Argument (c)

Premise 1. If Socrates didn't approve of the laws of Athens, then he'd either have left Athens or else have tried to change the laws.

Premise 2. If Socrates didn't leave Athens and didn't try to change the laws, then he agreed to obey the laws.

Premise 3. Socrates didn't leave Athens.

Conclusion. If Socrates didn't try to change the laws, then he approved of the laws and agreed to obey them.

[Use A, L, C and O as propositional variables. From Plato's Crito.]

$P_1: \neg A \rightarrow (L \oplus C)$

$P_2: \neg(L \vee C) \rightarrow O$

$P_3: \neg L$

$Con: \neg C \rightarrow (A \wedge O)$

Proof by contradiction: Set premises 1, 2 and 3 to 1 and the conclusion to 0.

$P_1: \neg A \rightarrow (L \oplus C) = 1$

$P_2: \neg(L \vee C) \rightarrow O = 1$

$P_3: \neg L = 1$

$Con: \neg C \rightarrow (A \wedge O) = 0$
$\neg\big(\neg C \rightarrow (A \wedge O)\big) = 1$
$\neg\big(C \vee (A \wedge O)\big) = 1$
$\neg C \wedge (\neg A \vee \neg O) = 1$
Therefore, $Con: \neg C \wedge (\neg A \vee \neg O) = 1$

Firstly, since $\neg L = 1$, then $\neg(L \vee C) = 1$, which forces O to equal 1 in the second premise. Furthermore, for $\neg C \wedge (\neg A \vee \neg O)$ to equal to 1, both $\neg C$ and $\neg A \vee \neg O$ have to equal 1. In this case, $C = 0$ and since $\neg O$ is already equal to 0 because $O = 1$, we can conclude that $\neg A$ must equal 1. Finally, with $\neg A = 1$, $L \oplus C$ must equal 1 to satisfy premise 1, but since $L = 0$ and $C = 0$, the premise is not satisfiable, hence contradiction reached.

The argument is valid, proven by contradiction.