UNIVERSITY OF WESTERN ONTARIO

Computer Science 2214a, Fall 2013 - 2014
Discrete Structures for Computing

ASSIGNMENT 4
Given: Wed. Nov. 13, Due: Wed. Nov. 20, 6:00pm

1. Use mathematical induction to show that

$$\Sigma_{j=n}^{2n-1}(2j+1) = 3n^2,$$

for all positive integers n. Provide detailed justifications for your answer.

2. Use structural induction to prove that $l(T)$, the number of leaves of a full binary tree T, is 1 more than $i(T)$, the number of internal vertices of T.

The set of leaves and the set of internal vertices of a full binary tree are defined recursively as follows:

Basis step: The root r is a leaf of the full binary tree with exactly one vertex r. This tree has no internal vertices.
Recursive step: The set of leaves of the tree $T = T_1 \cdot T_2$ is the union of the sets of leaves of T_1 and T_2. The internal vertices of T are the root r of T and the union of the set of internal vertices of T_1 and the set of internal vertices f T_2.

Provide detailed justifications for your answer.

3. Consider all bit strings of length 12.

1. How many begin with 110?
2. How many begin with 11 and end with 10?
3. How many begin with 11 or end with 10?
4. How many have exactly five 1's?
5. How many have exactly four 1's such as none of these 1's are adjacent to each other?

Provide detailed justifications for your answers.

4. Solve the following counting problems:

1. How many permutations of the seven letters A, B, C, D, E, F, G have E in the first position?

2. How many permutations of the seven letters A, B, C, D, E, F, G have E in one of the first two positions?

3. How many permutations of the seven letters A, B, C, D, E, F, G have the two vowels before the five consonants?

4. How many permutations of the seven letters A, B, C, D, E, F, G neither begin nor end with A?

5. How many permutations of the seven letters A, B, C, D, E, F, G do not have the vowels next to each other?

Provide detailed justifications for your answers.

5. A student council consists of 15 students.

(a) In how may ways can a committee of six be selected from the membership of the council?

(b) Two council members have the same major and are not permitted to serve together on a committee. How many ways can a committee of six be selected from the membership of the council?

(c) Two council members always insist on serving on committees together. If they cannot serve together, they won't serve at all. How many ways can a committee of six be selected from the council membership?

(d) Supose the council contains eight men and seven women. How many committes of six contain three men and three women? How many committees of six contain at least one woman?

(e) Suppose the council consists of three freshmen, four sophomores, three juniors and five seniors. How many committees of eight contain two representatives from each class?

Provide detailed justifications for your answers.