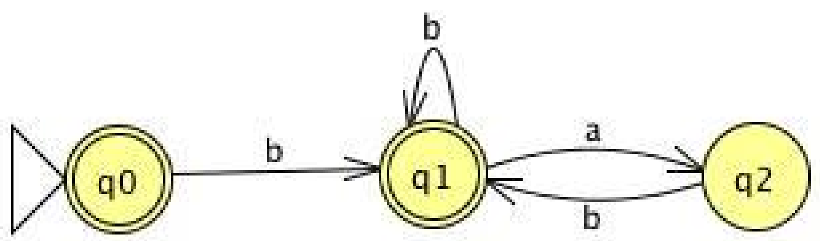
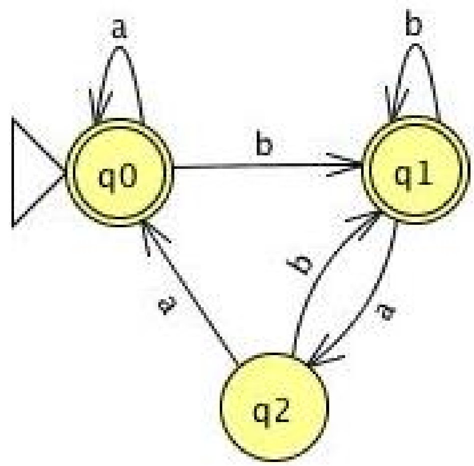
CS3331 – Assignment 1
due Oct. 14, 2014 (latest to submit: Oct. 17)

1. (20pt) Build DFSMs for the following languages. Explain why your construction is correct.

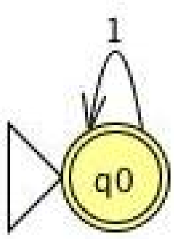
   (a) $\{w \in \{a,b\}^* \mid$ every a in w is immediately preceded and followed by $b\}$.



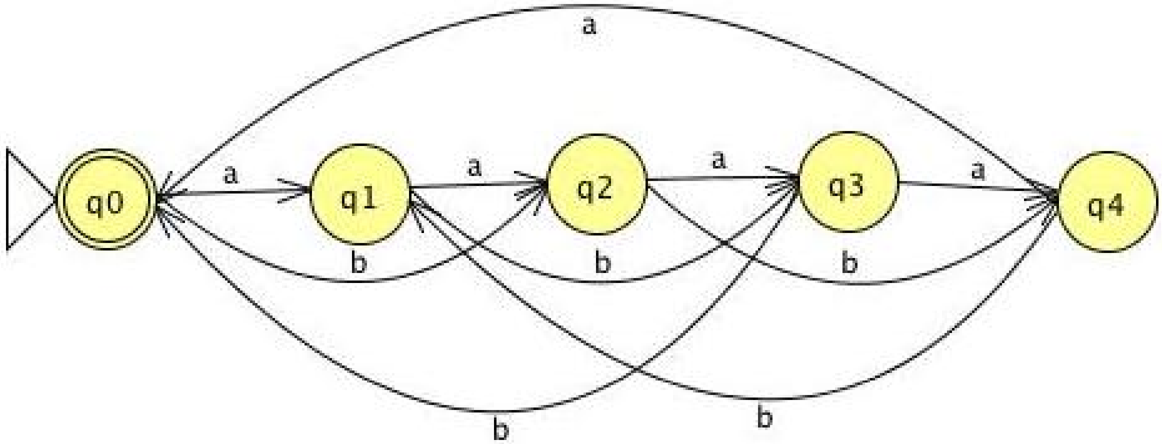
   (b) $\{w \in \{a,b\}^* \mid$ w does not end in $ba\}$.



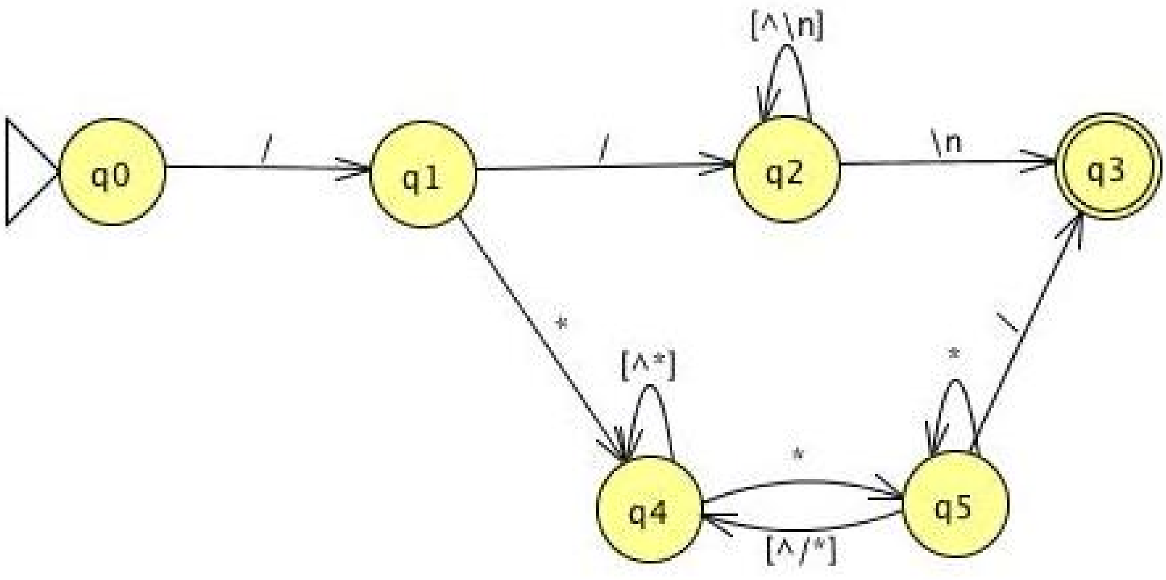
   (c) $\{w \in \{0,1\}^* \mid$ none of the prefixes of w ends in $0\}$.



   (d) $\{w \in \{a,b\}^* \mid (\#_a(w) + 2\#_b(w)) \equiv 0 \pmod 5\}$. ($\#_a(w)$ is the number of a's in w).



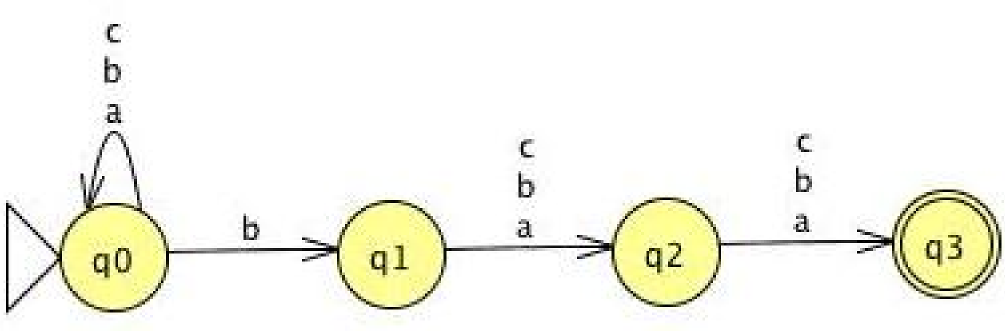
(e) C++ comments: `/* ... comment ... */` or `// ... comment ... \n`.



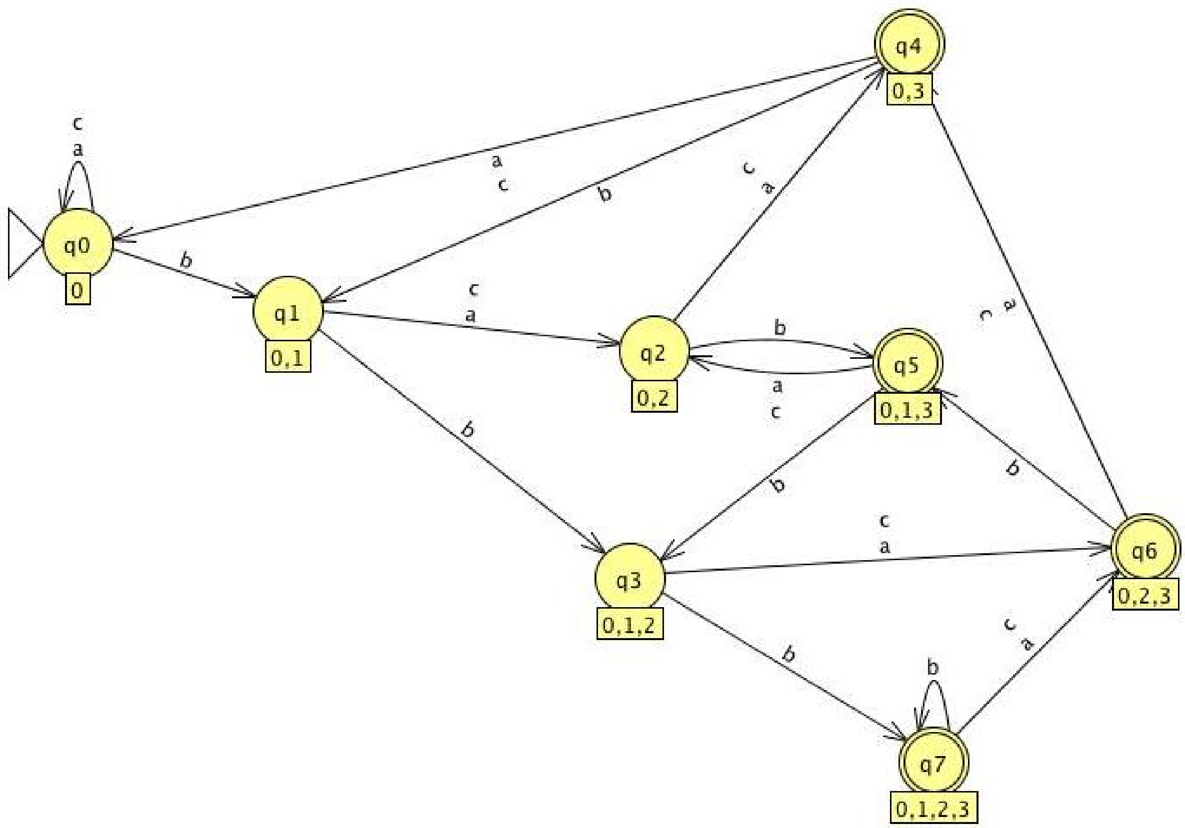
2. (20pt) Consider the language:

$$L = \{w \in \{a, b, c\}^* \mid \text{ the third from the last character is } b\}$$

(a) Build a NDFSM for L.



(b) Transform it into a DFSM.



(c) Build an equivalent regular expression from one of the two FSM above. (*Hint:* It makes a big difference which FSM you choose.)

Solution: Choose the NDFSM. The regular expression is $(a+b+c)^*b(a+b+c)(a+b+c)$.

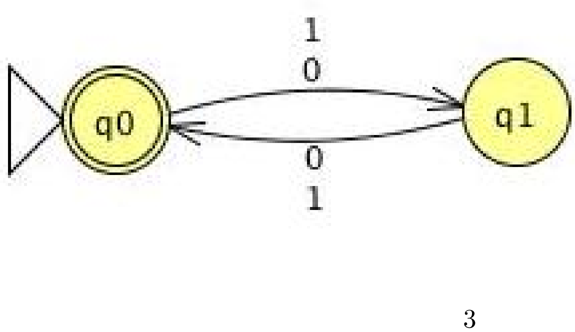
3. (15pt) For the following languages L, describe the equivalence classes of $\approx_L$. If there are finitely many classes, then build a minimal DFSM that accepts L.

(a) $L = \{ww^R \mid w \in \{a,b\}^*\}$

Solution: Any $x \neq y$ are not in the same class since adding x^R keeps one in L and not the other one. So, the language is not regular.

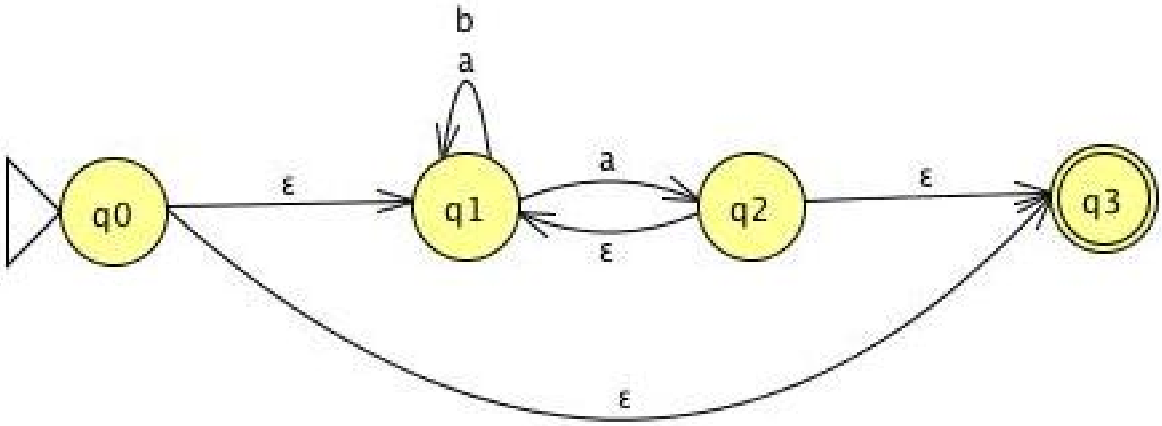
(b) $L = \{w \in \{0,1\}^* \mid \#_0(w) \text{ and } \#_1(w) \text{ are both even or both odd}\}$

Solution: It appears there are four classes, corresponding to all four combinations. However, there are only two: one with even length strings, and the other with odd. That's because $\#_0(w)$ and $\#_1(w)$ are both even or both odd iff $|w|$ is even.

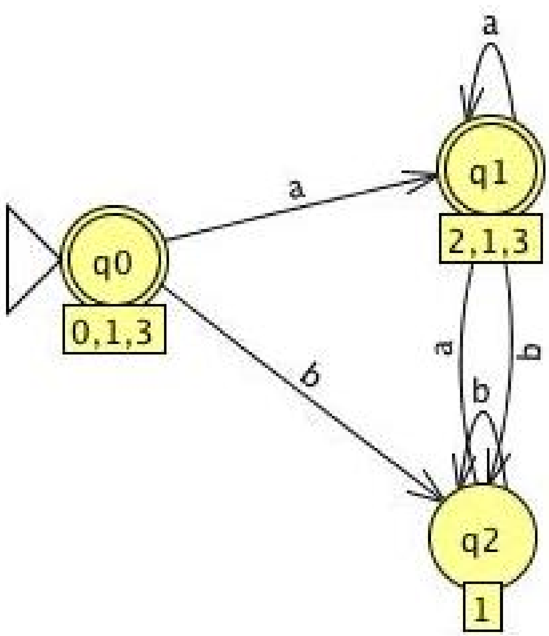


4. (20pt) Consider the regular expression $\alpha = ((a \cup b)^* a)^*$.

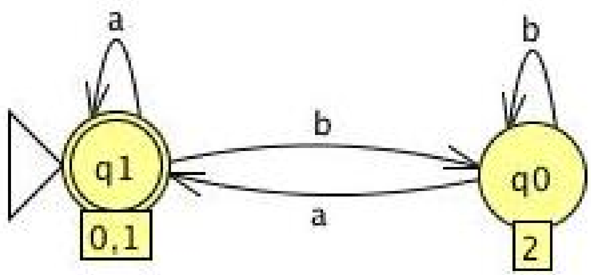
   (a) Construct a NDFSM that accepts $L(\alpha)$. (You can use Thompson's construction but you don't have to.)



   (b) Transform it into a DFSM.



   (c) Minimize it.



5. (15pt) For each of the following languages L, prove whether L is regular or not:

   (a) $\{a^i b^j \mid i, j \geq 0 \text{ and } i - j = 5\}$.
   Solution: The language is not regular. We use pumping theorem to prove it. So, we assume it is regular and consider $w = a^k b^{k-5}$, where k is the constant in the theorem. We have $w = xyz$, with $|xy| < k$, $y \neq \varepsilon$, and $xy^q z$ in the language for any $q \geq 0$. Since $|xy| < k$, we must have $y = a^p$, for some $p > 0$, and so $a^{k-p+qp} b^{k-5}$ has to be in the language for any $q > 0$. But for $q = 0$, we have $a^{k-p+qp} b^{k-5} = a^{k-p} b^{k-5}$ which is not in the language. The contradiction obtained shows that the language is not regular.

   (b) $\{w = xyzy^R x \mid x, y, z \in \{a, b\}^*\}$.
   Solution: The language is actually $\{a, b\}^*$ because you can have $x = y = \varepsilon$. So, it is regular.

6. (10pt) Show that the following problem is decidable: Given $\Sigma = \{a, b\}$ and α a regular expression, does the language defined by α contain all the even length strings in Σ^*?

   Solution: Algorithm to decide:

   1. Construct a DFSM M_α that accepts $L(\alpha)$.
   2. Construct a DFSM M_{even} that accepts $L_{\text{even}} = \{w \in \Sigma^* \mid |w| \text{ is even}\}$.
   3. Construct a DFMS M for $L_{\text{even}} - L(\alpha)$.
   4. Return $L(M) \stackrel{?}{=} \emptyset$.

   The answer is correct because $L(\alpha)$ contains all strings in L_{even} iff $L(M) = \emptyset$.

Note Submit your solution as a pdf file on `owl.uwo.ca`.