

UNIVERSITY OF WESTERN ONTARIO

Computer Science 2214a, Fall 2013 - 2014
Discrete Structures for Computing

ASSIGNMENT 5

Given: Wednesday Nov.27, Due: Wednesday Dec.4, 6:00pm

1. You pick a bit string from the set of all bit strings of length 10.
 - a) What is the probability that the bit string has exactly two 1s, given that the string begins with a 1?
 - b) What is the probability that the bit string begins and ends with 0?
 - c) What is the probability that the bit string has more 0s than 1s?
 - d) What is the probability that the bit string has the sum of its digits equal to seven?
 - e) What is the probability that the bit string begins with 111?

Provide detailed justifications for your answers.

2. What is the probability of these events when we randomly select a permutation of the 26 lowercase letters of the English alphabet?
 - (a) The first 13 letters in the permutation are in alphabetical order.
 - (b) a is the first letter of the permutation and z is the last letter.
 - (c) a and z are next to each other in the permutation.
 - (d) a and z are separated by at least 23 letters in the permutation.
 - (e) z precedes both a and b in the permutation.

Provide detailed justifications for your answers.

3. Suppose that 4% of the patients tested in a clinic are infected with avian influenza. Furthermore, suppose that when a blood test for avian influenza is given, 97% of the patients infected with avian influenza test positive and that 2% of the patients not infected with avian influenza test positive. What is the probability that:

- (a) a patient testing positive for avian influenza with this test is infected with it?

- (b) a patient testing positive for avian influenza with this test is not infected with it?
- (c) a patient testing negative for avian influenza with this test is infected with it?
- (d) a patient testing negative for avian influenza with this test is not infected with it?

Provide detailed justifications for your answers.

4. Suppose that a Bayesian spam filter is trained on a set of 500 spam messages and 200 messages that are not spam. The word “exciting” appears in 40 spam messages and in 25 messages that are not spam. Would an incoming message be rejected as spam if it contains the word “exciting” and the threshold for rejecting spam is 0.9? (Assume, for simplicity, that the message is equally likely to be spam as it is not to be spam.) Provide detailed justifications for your answers.

5. Let A be the set of all ordered pairs of integers for which the second element of the pair is nonzero. Symbolically,

$$A = \mathbf{Z} \times (\mathbf{Z} \setminus \{0\}).$$

Define a binary relation R on A as follows: For all $(a, b), (c, d) \in A$,

$$(a, b) R (c, d) \Leftrightarrow ad = bc.$$

- a) Is R reflexive?
- b) Is R symmetric?
- c) Is R antisymmetric?
- d) Is R transitive?
- e) Is R an equivalence relation, a partial order, neither, or both?
- f) List four distinct elements in the equivalence class $[(1, 3)]$.
- g) List four distinct elements in the equivalence class $[(2, 5)]$.
- h) Describe the equivalence classes of R .

Provide detailed justifications for your answers.