UNIVERSITY OF WESTERN ONTARIO

Computer Science 2209b, Winter 2013-2014

Applied Logic for Computer Science

ASSIGNMENT 4

Given: Monday, March 17, Due: Monday. March 24, 5pm

(1). Translate each of the following sentences into the language of the predicate calculus, using the indicated letters for predicate symbols.

$M(x)$: "x is mysterious"
$S(x)$: "x is a student"
$B(x)$: "x is beautiful"
The universe of discourse is the set of all people.

a) All who aren't students are mysterious.
b) Some students are mysterious and beautiful.
c) No student who is beautiful is mysterious.
d) Not all who are mysterious or beautiful are students.
e) Not all students are mysterious.

(2). Translate the following sentences in the languages of predicate calculus. Use the given predicate and function symbols. The universe of discourse is the set of positive integers.

$L(x, y)$: x is less than y
$E(x)$: x is even
$P(x)$: x is prime
$O(x)$: x is odd
$Pro(x, y) = x \times y$

i) Every positive integer is less than some positive integer.
ii) There is a positive integer with no positive integer less than it.
iii) 2 is even and prime and it is the only positive integer that is both even and prime.
iv) The product of any pair of odd positive integers is itself odd.
v) If either of a pair of positive integers is even, their product is even.

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(3). For <u>each</u> of the following five sets of formulas, find a domain and interpretation in which the last formula is false, but the other formulas are true.

a). $\exists x P(f(x), x)$, $\forall x \forall y (Q(x) \wedge Q(y) \rightarrow P(x, y))$, $\exists x Q(f(x))$, $\exists x Q(x)$

b). $\exists x Q(x) \rightarrow \exists x P(x)$, $P(a)$, $\forall x (Q(x) \rightarrow P(x))$

c). $\forall x (R(x) \rightarrow \neg P(x))$, $\exists x (R(x) \wedge \neg P(x))$

d). $\exists x \forall y Q(x, y)$, $\forall x Q(x, f(x))$

e). $\forall x (R(x) \rightarrow \neg P(x))$, $\forall x (P(x) \rightarrow Q(x))$, $\exists x (R(x) \wedge \neg Q(x))$,
$\forall x (R(x) \rightarrow \neg Q(x))$

(4). Translate the following argument in the language of predicate calculus choosing appropriate symbols for constants, variables, predicates, or functions. The universe of discourse is the set of all teenagers. Prove that the conclusion is a logical consequence ($\models$) of the premises. Justify your answers.

Premise 1: Teenagers who play tennis are in good health.
Premise 2: Teenagers who play football have team spirit.
Premise 3: Anyone who has team spirit is in good health.
Premise 4: All teenagers who go to college either play tennis or football.
Conclusion: If some teenager goes to college, then some teenager is in good health.

(5). Consider the following argument:

Premise 1: $\forall y \forall x (P(x, y) \rightarrow P(y, x))$
Premise 2: $\forall z \forall y \forall x (P(x, y) \wedge P(y, z) \rightarrow P(x, z))$
Conclusion: $\forall y \forall x (P(x, y) \rightarrow P(x, x))$

Find out whether or not the argument is valid, i.e. whether or not the conclusion is a logical consequence ($\models$) of the premises. Justify your answer.