

Assignment #3

Student #: [REDACTED]

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1. $F(x) = x \bmod 7$

Hashtable:

n	F(x)	
0	>	Null
1	>	15
2	>	Null
3	>	Null
4	>	Null
5	>	47
6	>	27

>	12	>	19
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2. $F(x) = x \bmod 7$

Hashtable:

n	F(x)
0	12
1	47
2	15
3	Null
4	Null
5	19
6	27

3. $F(x) = x \bmod 7, F'(x) = 5 - (x \bmod 5)$

Hashtable:

n	F(x)
0	Null
1	12
2	15
3	Null
4	47
5	19
6	27

4. $F(1) = 1$

$$F(n) = F(n-1) + 2(n-1)$$

$$F(1) = 1$$

$$F(2) = F(2-1) + 2(2-1)$$

$$= F(1) + 2(1)$$

$$= 1 + 2 = 3$$

$$F(3) = F(3-1) + 2(3-1)$$

$$= F(2) + 2(2)$$

$$= 3 + 4 = 7$$

$$F(4) = F(4-1) + 2(4-1)$$

$$= F(3) + 2(3)$$

$$= 7 + 6 = 13$$

Therefore, since $F(4) = F(3) + 2(3)$

Then, $= F(2) + 2(2) + 2(3)$

$$= F(1) + 2(1) + 2(2) + 2(3)$$

$$= 1 + 2(1) + 2(2) + 2(3)$$

Therefore, $F(4) = 2(1+2+3) + 1$

Therefore, $F(n) = 2(1+2+3...+n-1) + 1$

$$= 2 \sum + 1$$

* $\sum = (1+2+3...+n-1)$, the sum of all values from 1 to n

Since $\sum = n(n-1)/2$

Therefore, $F(n) = 2[n(n-1)/2] + 1$

$$= n(n-1) + 1$$

$$= n^2 - n + 1$$

5. Algorithm count (root r, int k)

Input: root r of a proper Tree T, int k an integer that represents the leaves distance from the tree.

Output: the number for leaves.

int leaves=0

```

If (k==0)
    If (r.isLeaf ())
        leaves++
Else
    If (r.hasChild ())
        leaves+=count (r.getLeft (), k-1)
        leaves+=count (r.getRight (), k-1)
return leaves

```

Time Complexity:

count (root r, int k)

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int leaves=0 } 1
If (k==0)
    If (r.isLeaf ())
        leaves++ } 1 } c } c'
Else
    If (!r.isLeaf())
        leaves+=count (r.getLeft (), k-1) } 1 } c'' } c'''
        leaves+=count (r.getRight (), k-1) } 1 }
return leaves } 1

```

We divide the problem into two parts, the algorithm and the recursive calls

$$\begin{aligned}
 F(n) \text{ for the algorithm, } F(n) &= 1 + c' * c + c''' * (2 * c'') + 1 \\
 &= (2c''' * c'') + (c' * c) + 2
 \end{aligned}$$

Since this algorithm recursively visits every node once, then the number of recursive calls should be equal to the number of nodes of this tree. Furthermore, the algorithm is a traversal algorithm which implies that every node has to be visited once. Therefore, $F(n) = n(2c''' * c'' + c' * c + 2)$, $O(n) = n$.