Date Structures and Analysis of Algorithms

Midterm Examination (Two Hours)

1. (10 marks)
a). (5 marks) Fill in the relationship between each pair of the functions below.
(Use Ω, O, Θ, or none of the above)

$$
\begin{align}
n &= \text{---} & n + n/\log_2(n) & \qquad (1) \\
n + \sqrt{n} &= \text{---} & n & \qquad (2) \\
n * \log_2(n^2) &= \text{---} & n^{\log_2(n)} & \qquad (3) \\
(2/3)^n &= \text{---} & n & \qquad (4) \\
2^n &= \text{---} & (3/2)^n & \qquad (5)
\end{align}
$$

b). (5 marks) Answer following questions.
  What is the worst case time complexity of constructing Huffman coding tree?
  What is the average case time complexity of quicksort?
  What is the worst case time complexity of heap construction (heapification) ?
  What is the worst case time complexity of sequence comparison algorithm?
  What is the lower bound for sorting algorithm based on comparison?

2. (10 marks)
a). (4 marks) Prove, by using the definitions of O and Ω, the following:
    $\sqrt{n} = O\left(\sqrt{n} - 5\right)$
    $\log_2(\max\{f(n), g(n)\}) = \Omega(\log_2(f(n) * g(n)))$
b). (6 marks) Find a closed form for the following recursive equation.
    $T(n) = T(n/2) + 1$; if $n > 1$ and n is of power of 2.
    $T(1) = 1$;

3. (10 marks)
a) (5 marks) For a given string $S = p_1 p_2 ... p_n$, the definition for $next(i)$, $2 \le i \le n$, is as follows:
  $next(i) = $ the maximum j $(0 < j < i - 1)$ such that $p_1 p_2 ... p_j = p_{i-j} p_{i-j+1} ... p_{i-1}$,

        0 if no such j exists.
        ($next(1)$ is defined as -1.)
Compute the $next$ function for string "$abababc$".

b) (5 marks) Given two strings, $A = a_1, a_2, ...a_m$ and $B = b_1, b_2, ...b_n$, we define a matrix $C[0..m, 0..n]$ as follows: $(1 \le i \le m, 1 \le j \le n)$
$C[0, 0] = 0$, $C[0, j] = j$, $C[i, 0] = i$.

$C[i, j] = \min\{C[i - 1, j] + 1, C[i, j - 1] + 1, C[i - 1, j - 1] + r_{ij}\}$ where $r_{ij} = 0$, if $a_i = b_j$, or $=1$, if $a_i \ne b_j$.
  Compute the C matrix for strings $A = ababab$, and $B = babaaa$. What is the meaning of
the value in $C[m, n]$?

4. (10 marks)
Given a set of characters $A = \{a_1, a_2, ...a_n\}$, where $n > 2$. Let f_i and f_j be the frequencies of a_i and a_j, and d_i and d_j be the depths of a_i and a_j in a Huffman encoding tree for A. Prove or disprove (by counter example) the following statement.

    a. (5 marks) if $d_i = d_j$ then $f_i = f_j$.

    b. (5 marks) if $f_i > f_j$ then $d_i \leq d_j$.

5. (10 marks) Suppose that you are given a sorted list of n elements followed by $f(n)$ randomly ordered elements. How would you sort the entire list if

    a. (3 marks) $f(n) = 2$?

    b. (4 marks) $f(n) = \sqrt{n}$?

    c. (3 marks) How large can $f(n)$ be for the entire list to be sorted in $O(n)$ time?