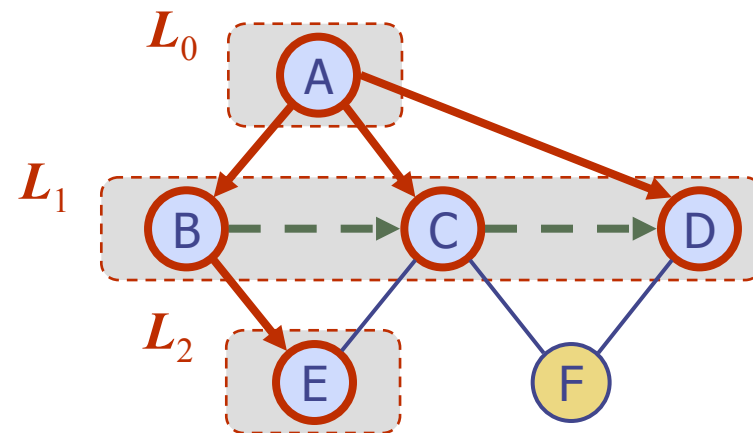


# Breadth-First Search



# Breadth-First Search (§ 12.3.3)

- ◆ Breadth-first search (BFS) is a general technique for traversing a graph
- ◆ A BFS traversal of a graph  $G$ 
  - Visits all the vertices and edges of  $G$
  - Determines whether  $G$  is connected
  - Computes the connected components of  $G$
  - Computes a spanning forest of  $G$
- ◆ BFS on a graph with  $n$  vertices and  $m$  edges takes  $O(n + m)$  time
- ◆ BFS can be further extended to solve other graph problems
  - Find and report a path with the minimum number of edges between two given vertices
  - Find a simple cycle, if there is one

# BFS Algorithm

- ◆ The algorithm uses a mechanism for setting and getting “labels” of vertices and edges

## Algorithm **BFS**( $G$ )

**Input** graph  $G$

**Output** labeling of the edges and partition of the vertices of  $G$

```
for all  $u \in G.vertices()$ 
     $setLabel(u, UNEXPLORED)$ 
for all  $e \in G.edges()$ 
     $setLabel(e, UNEXPLORED)$ 
for all  $v \in G.vertices()$ 
    if  $getLabel(v) = UNEXPLORED$ 
         $BFS(G, v)$ 
```

## Algorithm **BFS**( $G, s$ )

```
 $L_0 \leftarrow$  new empty sequence
 $L_0.insertLast(s)$ 
 $setLabel(s, VISITED)$ 
 $i \leftarrow 0$ 
while  $\neg L_i.isEmpty()$ 
     $L_{i+1} \leftarrow$  new empty sequence
    for all  $v \in L_i.elements()$ 
        for all  $e \in G.incidentEdges(v)$ 
            if  $getLabel(e) = UNEXPLORED$ 
                 $w \leftarrow opposite(v, e)$ 
                if  $getLabel(w) = UNEXPLORED$ 
                     $setLabel(e, DISCOVERY)$ 
                     $setLabel(w, VISITED)$ 
                     $L_{i+1}.insertLast(w)$ 
                else
                     $setLabel(e, CROSS)$ 
     $i \leftarrow i + 1$ 
```

# Example



unexplored vertex



visited vertex



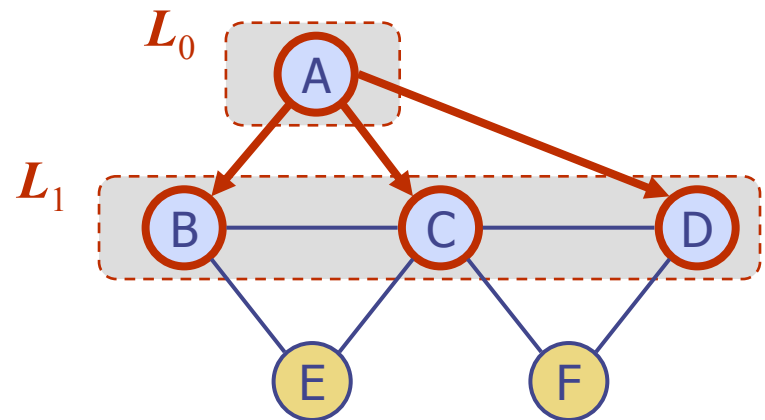
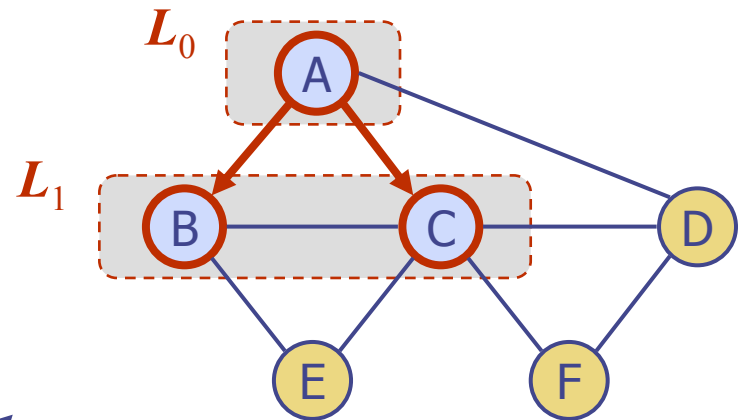
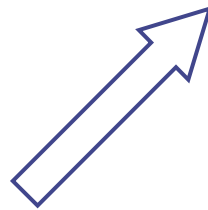
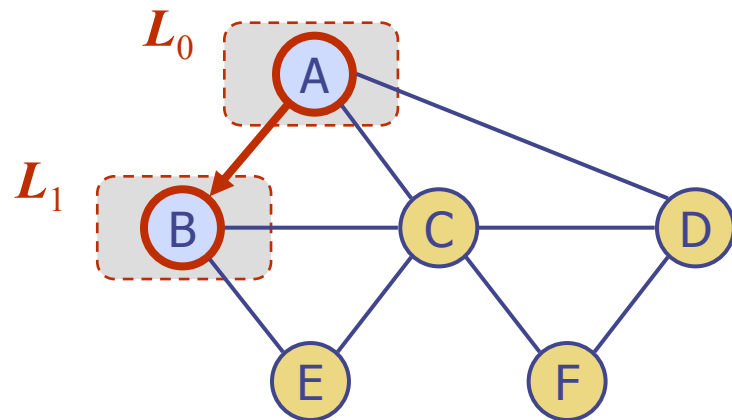
unexplored edge



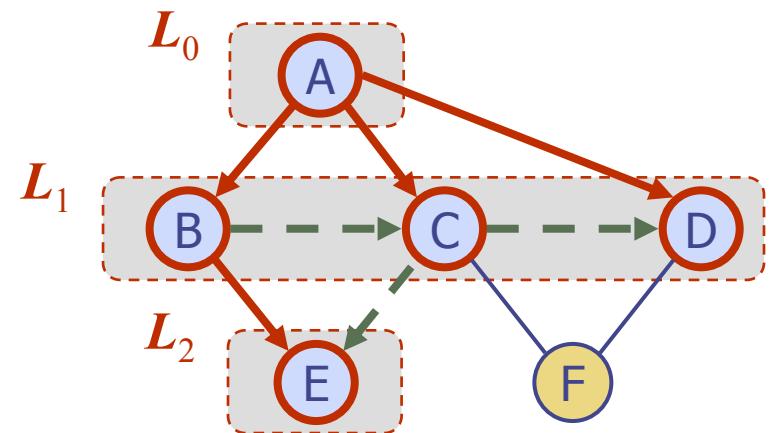
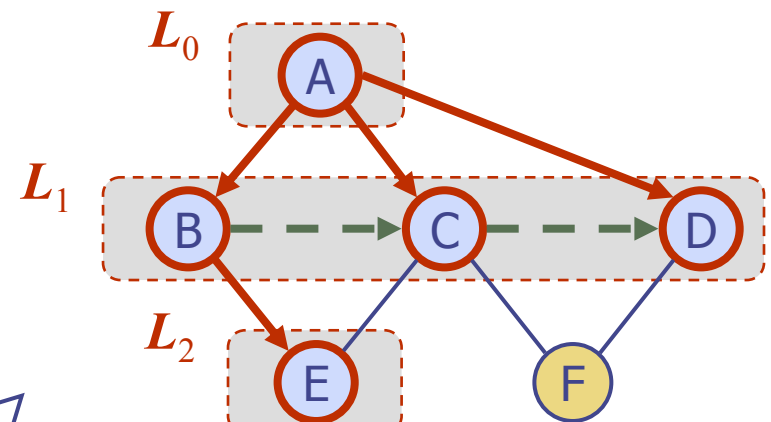
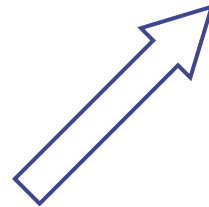
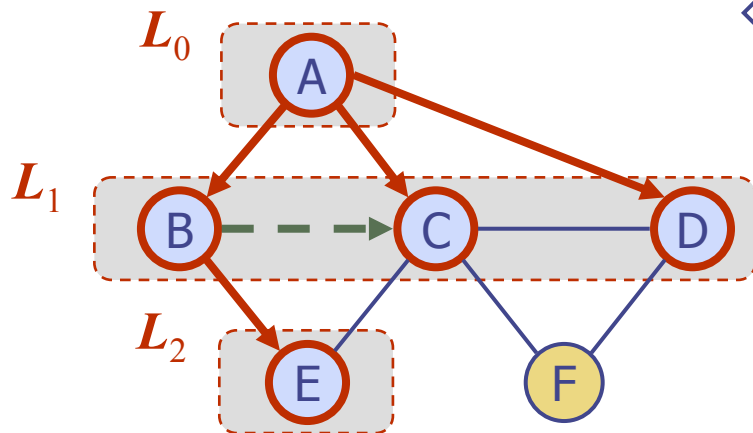
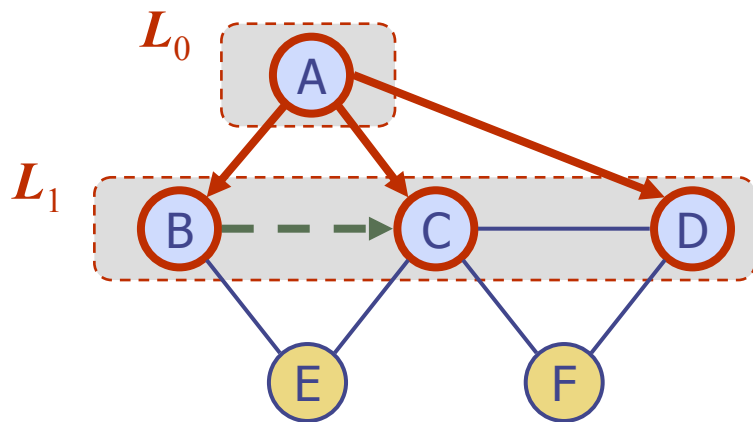
discovery edge



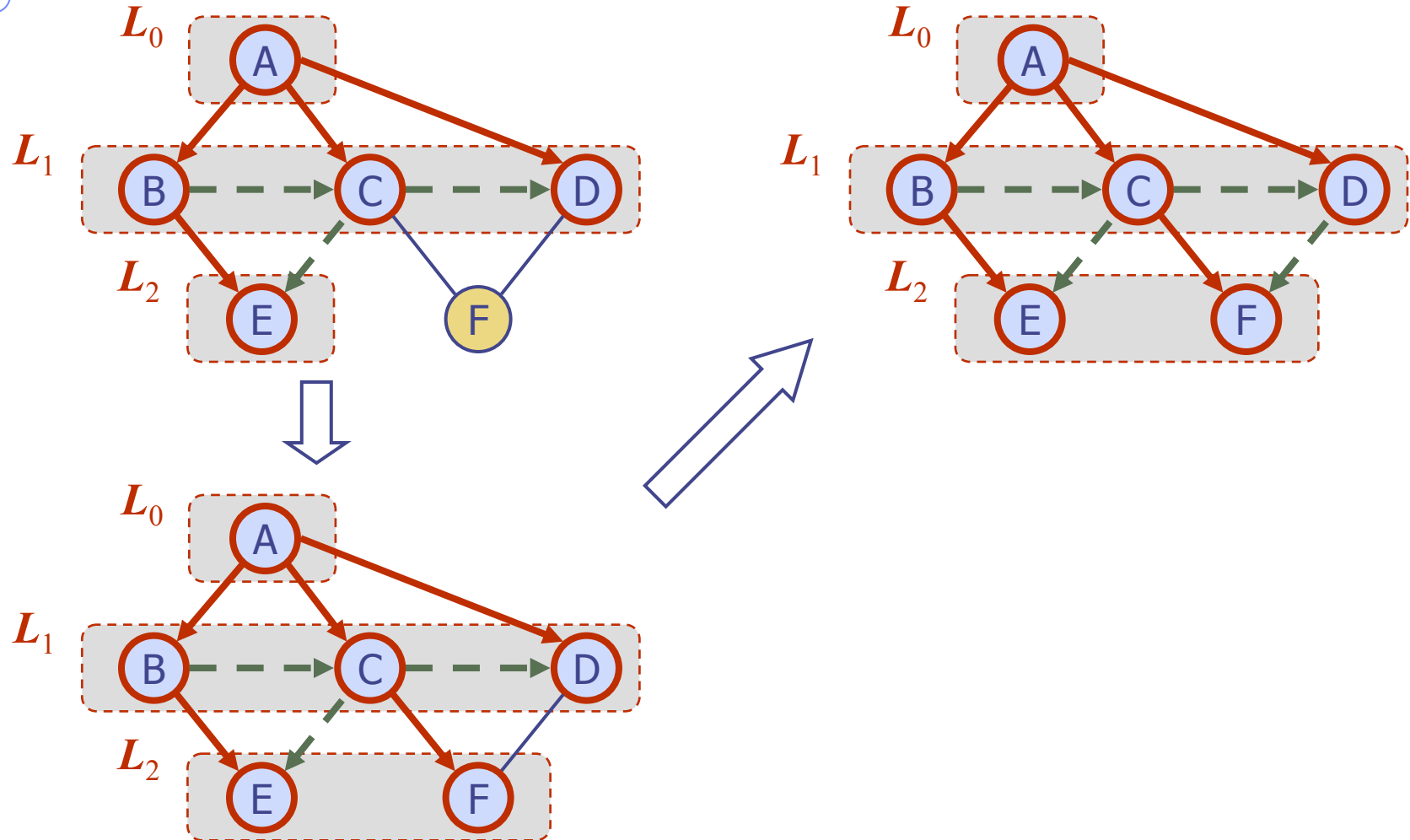
cross edge



# Example (cont.)



# Example (cont.)



# Properties

## Notation

$G_s$ : connected component of  $s$

## Property 1

$BFS(G, s)$  visits all the vertices and edges of  $G_s$

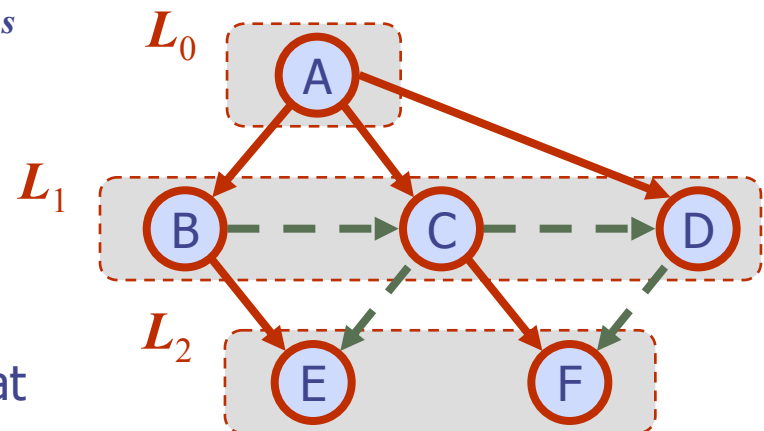
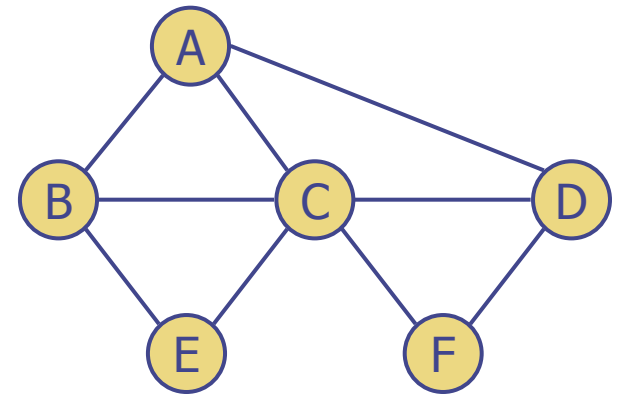
## Property 2

The discovery edges labeled by  $BFS(G, s)$  form a spanning tree  $T_s$  of  $G_s$

## Property 3

For each vertex  $v$  in  $L_i$

- The path of  $T_s$  from  $s$  to  $v$  has  $i$  edges
- Every path from  $s$  to  $v$  in  $G_s$  has at least  $i$  edges



# Analysis

- ◆ Setting/getting a vertex/edge label takes  $O(1)$  time
- ◆ Each vertex is labeled twice
  - once as UNEXPLORED
  - once as VISITED
- ◆ Each edge is labeled twice
  - once as UNEXPLORED
  - once as DISCOVERY or CROSS
- ◆ Each vertex is inserted once into a sequence  $L_i$
- ◆ Method incidentEdges is called once for each vertex
- ◆ BFS runs in  $O(n + m)$  time provided the graph is represented by the adjacency list structure
  - Recall that  $\sum_v \deg(v) = 2m$

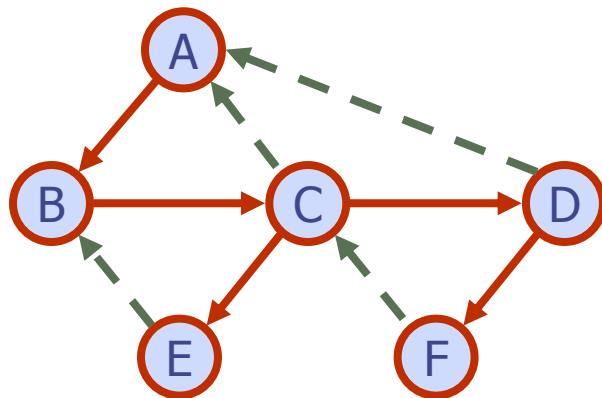


# Applications

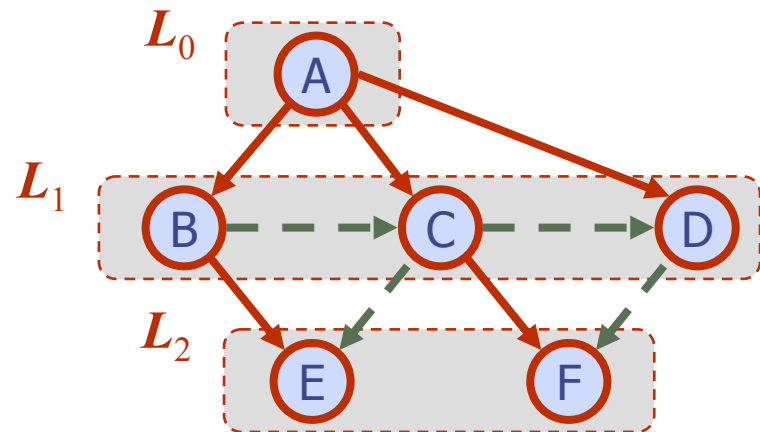
- ◆ Using the template method pattern, we can specialize the BFS traversal of a graph  $G$  to solve the following problems in  $O(n + m)$  time
  - Compute the connected components of  $G$
  - Compute a spanning forest of  $G$
  - Find a simple cycle in  $G$ , or report that  $G$  is a forest
  - Given two vertices of  $G$ , find a path in  $G$  between them with the minimum number of edges, or report that no such path exists

# DFS vs. BFS

Applications	DFS	BFS
Spanning forest, connected components, paths, cycles	✓	✓
Shortest paths		✓
Biconnected components	✓	



DFS

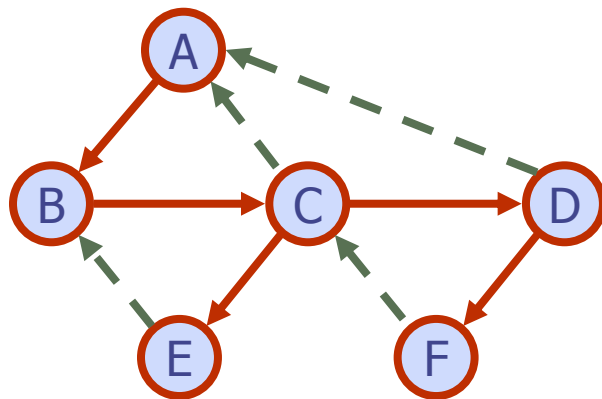


BFS

# DFS vs. BFS (cont.)

## Back edge $(v, w)$

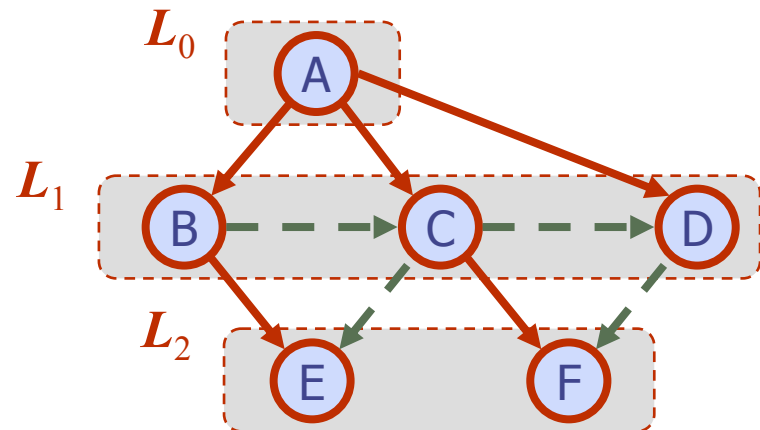
- $w$  is an ancestor of  $v$  in the tree of discovery edges



DFS

## Cross edge $(v, w)$

- $w$  is in the same level as  $v$  or in the next level in the tree of discovery edges



BFS