CS3331 – Assignment 2
due Oct. 21, 2014 (latest to submit: Oct. 24)

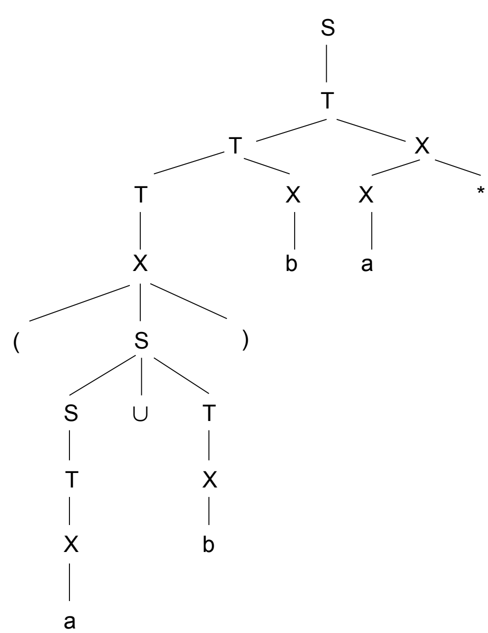
1. (30pt) Let $L = \{w \in \{a, b, \cup, \varepsilon, (,), ^*, ^+\}^* \mid w$ is a syntactically legal regular expression over $\Sigma = \{a, b\}\}$.

   (a) Give an unambiguous context-free grammar that generates L. The grammar should
   - use the following precedence levels, from highest to lowest: (1) $*$ and $+$ (equal precedence), (2) concatenation, and (3) $\cup$ and
   - left-associate operators of equal precedence.

   Solution:

$$
\begin{array}{ccl}
S & \longrightarrow & S \cup T \\
S & \longrightarrow & T \\
T & \longrightarrow & TX \\
T & \longrightarrow & X \\
X & \longrightarrow & X^* \\
X & \longrightarrow & X^+ \\
X & \longrightarrow & a \\
X & \longrightarrow & b \\
X & \longrightarrow & \varepsilon \\
X & \longrightarrow & (S)
\end{array}
$$

   (b) Show the parse tree that your grammar produces for the string $(a \cup b)ba^*$.
   Solution:



2. (40pt) For each of the following languages L, prove whether L is regular, context-free but not regular, or not context-free:

   (a) $\{xy \mid x, y \in \{a, b\}^*$ and $|x| = |y|\}$.
   Solution: Regular; $L = \{w \in \{a, b\}^* \mid |w|$ even$\} = (aa \cup ab \cup ba \cup bb)^*$.

(b) $\{a^i b^n \mid i, n > 0 \text{ and } i = n \text{ or } i = 2n\}$.

Solution: Context-free non-regular. Here is a context-free grammar for L:

$$
\begin{array}{lcl}
S & \longrightarrow & A \\
S & \longrightarrow & B \\
A & \longrightarrow & aAb \\
A & \longrightarrow & ab \\
B & \longrightarrow & aaBb \\
B & \longrightarrow & aab
\end{array}
$$

To show it is not regular, use pumping theorem for $w = a^k b^k$.

(c) $\{(ab)^n a^n b^n \mid n > 0\}$.

Solution: Non context-free. Use pumping theorem for $w = (ab)^k a^k b^k$. Consider the three regions: $(ab)^k$, a^k, and b^k. Any pumping in-between these regions will make the letters out of order. Therefore, any pumping can be done only within one region. But then the string is not in the language because the powers are not equal.

(d) $\{ww^R w \mid w \in \{a, b\}^*\}$.

Solution: Non context-free. Use pumping theorem for $w_0 = a^k bba^k a^k b$. Since any string in L must have a multiple of 3 b's, the b's in w_0 cannot be affected by pumping. That is because not all b's can be affected by pumping (they are too far) and therefore we can pump one or two b's only, thus producing a non-multiple of 3. Therefore the b's must act as delimiters for the three parts of w_0. The pumping affects a's only, easily producing strings not in the language.

3. (30pt) Show that the following problem is decidable: Given a context-free grammar G, does G generate any even length strings?

Solution: Consider $E = \{w \in \Sigma^* \mid |w| \text{ is even}\}$. E is a regular language. Therefore $L(G) \cap E$ is context-free and G generate some even strings iff $L(G) \cap E \neq \emptyset$. This gives the algorithm:

(a) Construct a FSM M for E.

(b) Construct a PDA P for $L(G)$.

(c) Construct a PDA P' for $L(G) \cap L(M)$.

(d) Return $L(P') \stackrel{?}{\neq} \emptyset$.

Note Submit your solution as a pdf file on `owl.uwo.ca`.