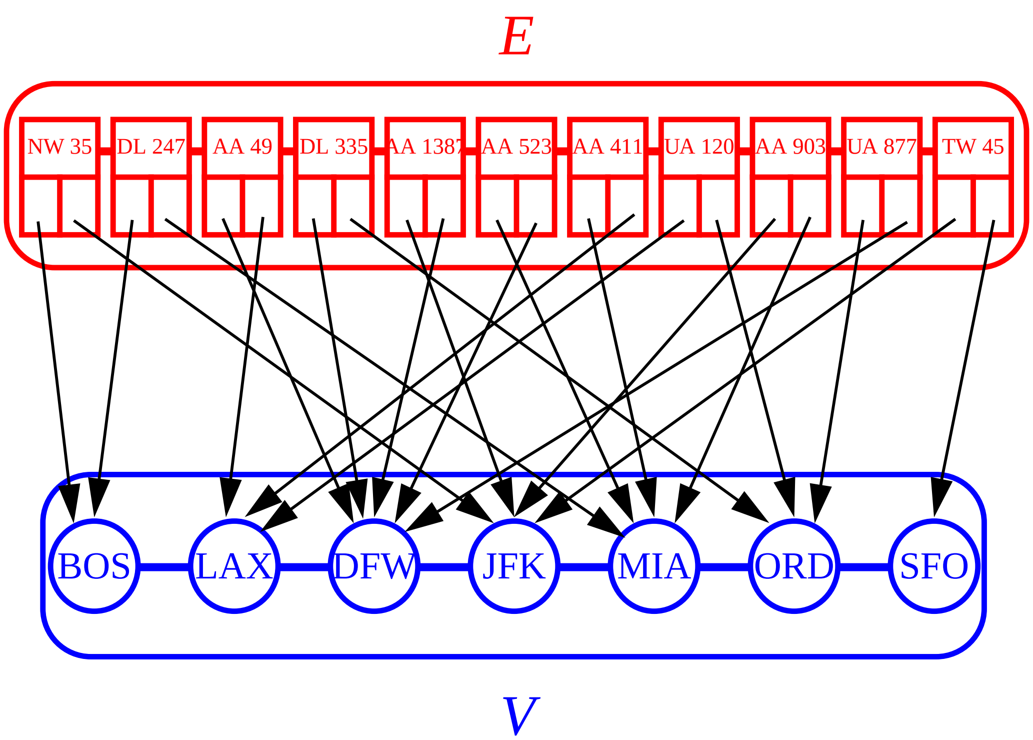
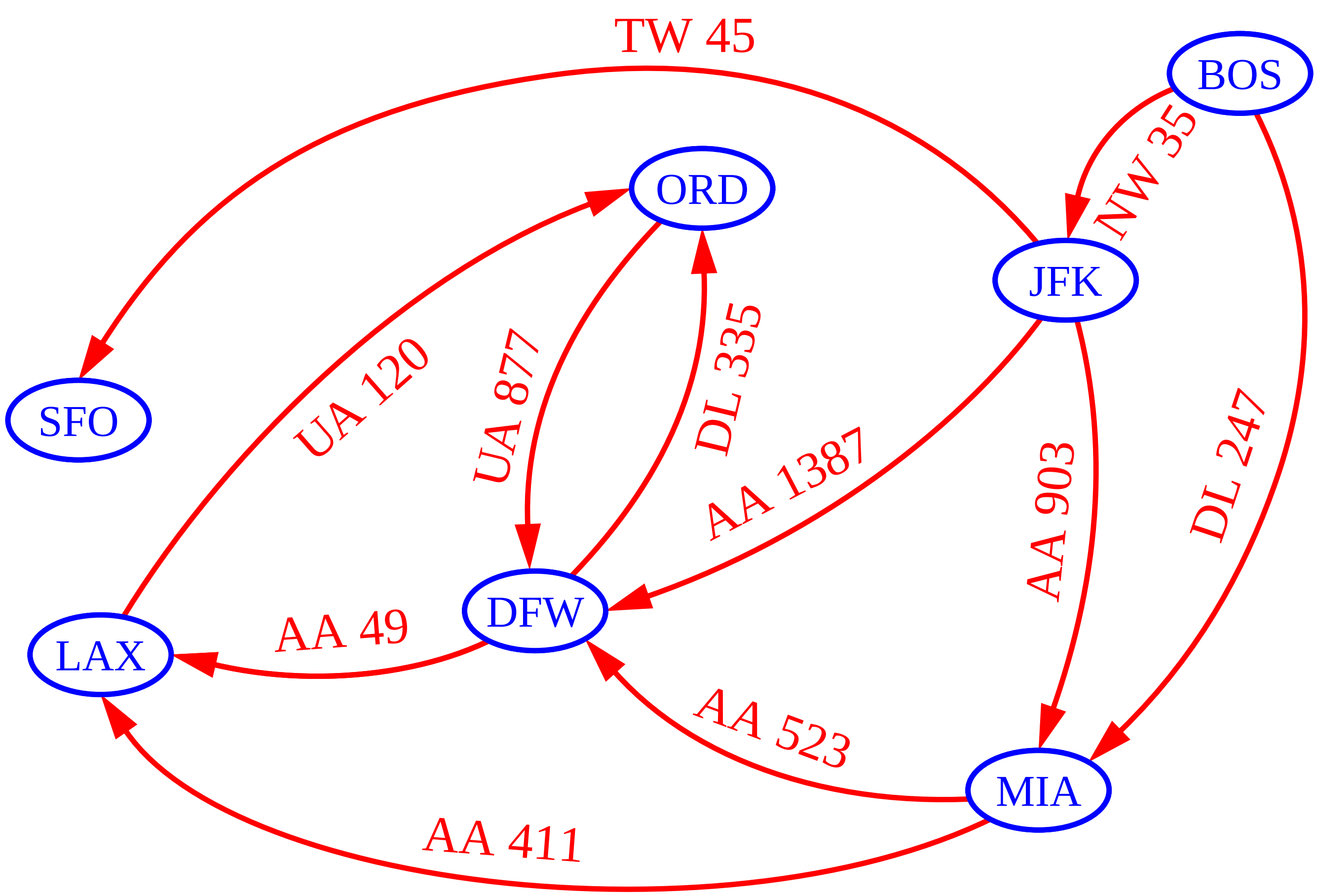
DATA STRUCTURES FOR GRAPHS

- Edge list
- Adjacency lists
- Adjacency matrix



Data Structures for Graphs

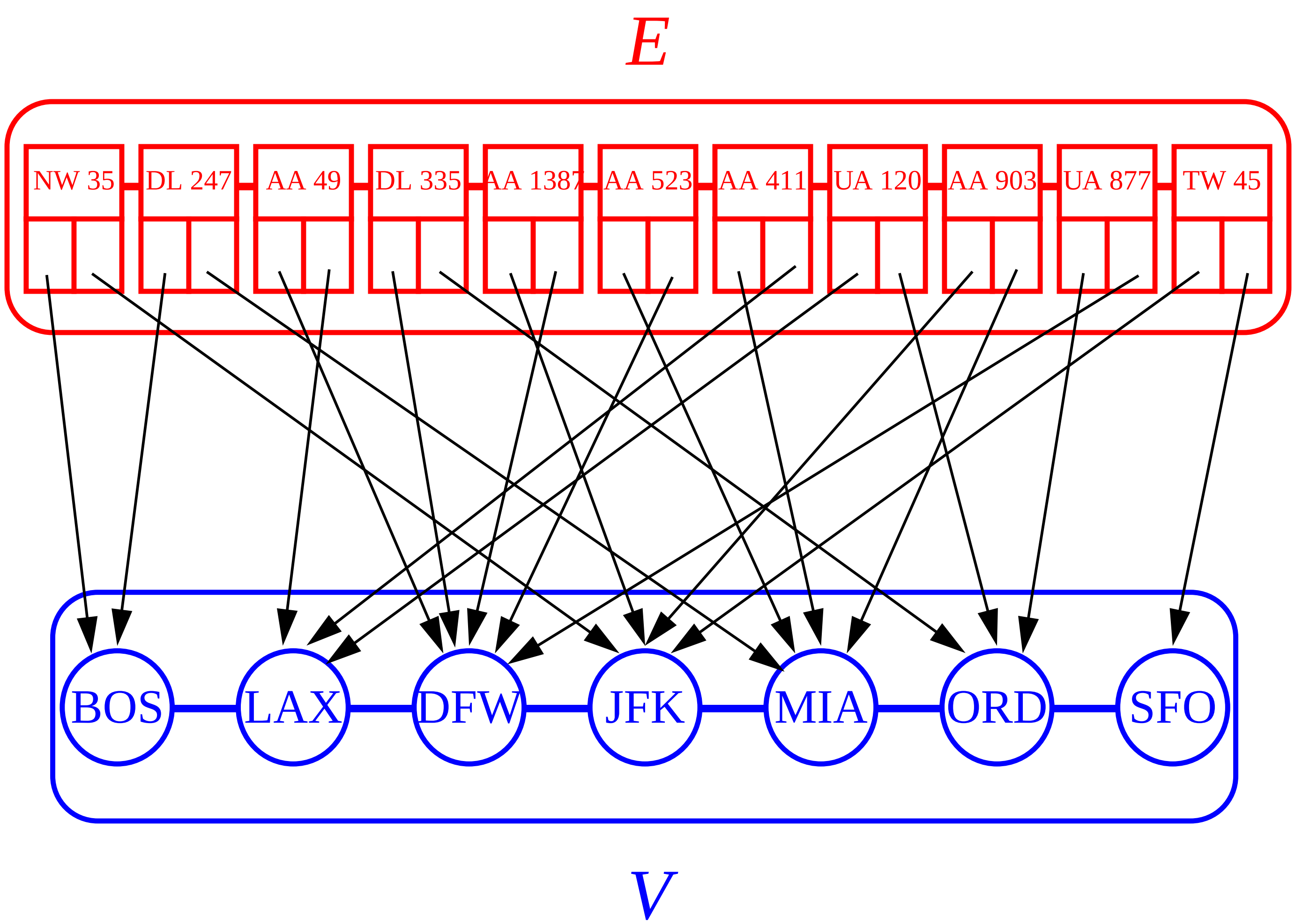
- A Graph! How can we represent it?
- To start with, we store the vertices and the edges into two containers, and each edge object has references to the vertices it connects.



- Additional structures can be used to perform efficiently the methods of the Graph ADT

Edge List

- The **edge list** structure simply stores the vertices and the edges into unsorted sequences.
- Easy to implement.
- Finding the edges incident on a given vertex is inefficient since it requires examining the entire edge sequence

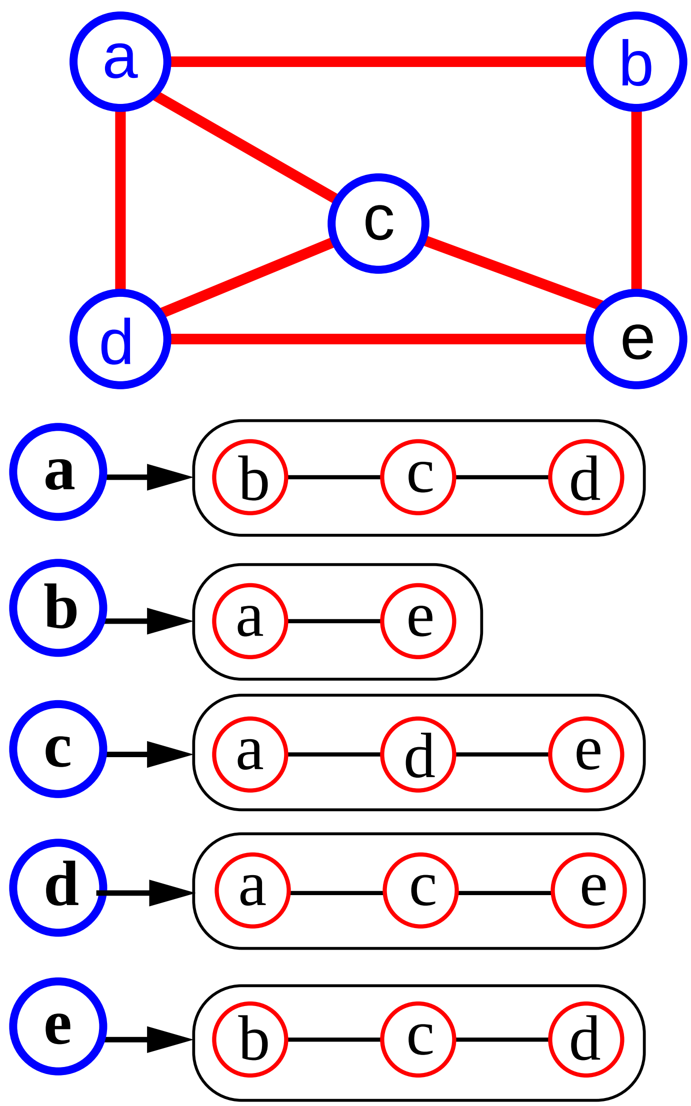


Performance of the Edge List Structure

Operation	Time
size, isEmpty, replaceElement, swap	O(1)
numVertices, numEdges	O(1)
vertices	O(n)
edges, directedEdges, undirectedEdges	O(m)
elements, positions	O(n+m)
endVertices, opposite, origin, destination, isDirected	O(1)
incidentEdges, inIncidentEdges, outIncidentEdges, adjacentVertices, inAdjacentVertices, outAdjacentVertices, areAdjacent, degree, inDegree, outDegree	O(m)
insertVertex, insertEdge, insertDirectedEdge, removeEdge, makeUndirected, reverseDirection, setDirectionFrom, setDirectionTo	O(1)
removeVertex	O(m)

Adjacency List (traditional)

- adjacency list of a vertex v:
  sequence of vertices adjacent to v
- represent the graph by the adjacency lists of all the vertices

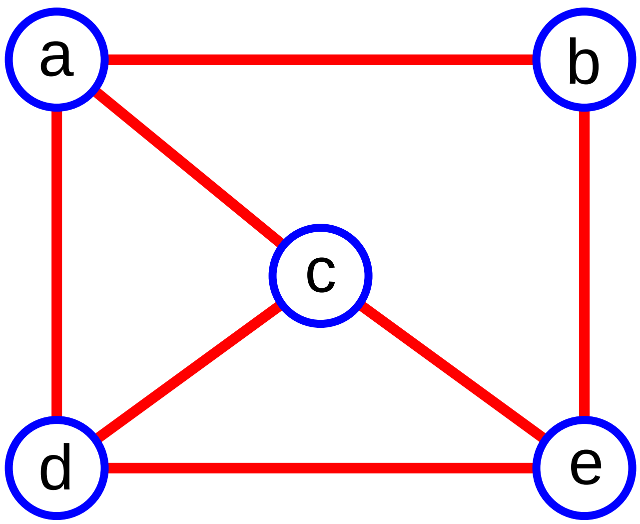


- Space = $\Theta(\mathbf{N} + \sum \deg(\mathbf{v})) = \Theta(\mathbf{N} + \mathbf{M})$

Performance of the Adjacency List Structure

Operation	Time
size, isEmpty, replaceElement, swap	O(1)
numVertices, numEdges	O(1)
vertices	O(n)
edges, directedEdges, undirectedEdges	O(m)
elements, positions	O(n+m)
endVertices, opposite, origin, destination, isDirected, degree, inDegree, outDegree	O(1)
incidentEdges(v), inIncidentEdges(v), outIncidentEdges(v), adjacentVertices(v), inAdjacentVertices(v), outAdjacentVertices(v)	O(deg(v))
areAdjacent(u, v)	O(min(deg(u), deg(v)))
insertVertex, insertEdge, insertDirectedEdge, removeEdge, makeUndirected, reverseDirection,	O(1)
removeVertex(v)	O(deg(v))

Adjacency Matrix (traditional)



	a	b	c	d	e
a	F	T	T	T	F
b	T	F	F	F	T
c	T	F	F	T	T
d	T	F	T	F	T
e	F	T	T	T	F

- matrix M with entries for all pairs of vertices
- M[i,j] = true means that there is an edge (i,j) in the graph.
- M[i,j] = false means that there is no edge (i,j) in the graph.
- There is an entry for every possible edge, therefore:

  Space = $\Theta(\mathbf{N^2})$

Adjacency Matrix
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- The adjacency matrix structures augments the edge list structure with a matrix where each row and column corresponds to a vertex.

	0	1	2	3	4	5	6
0	Ø	Ø	NW 35	Ø	DL 247	Ø	Ø
1	Ø	Ø	Ø	AA 49	Ø	DL 335	Ø
2	Ø	AA 1387	Ø	Ø	AA 903	Ø	TW 45
3	Ø	Ø	Ø	Ø	Ø	UA 120	Ø
4	Ø	AA 523	Ø	AA 411	Ø	Ø	Ø
5	Ø	UA 877	Ø	Ø	Ø	Ø	Ø
6	Ø	Ø	Ø	Ø	Ø	Ø	Ø

BOS	DFW	JFK	LAX	MIA	ORD	SFO
0	1	2	3	4	5	6

- The space requirement is $O(n^2 + m)$

Performance of the Adjacency Matrix Structure

Operation	Time
size, isEmpty, replaceElement, swap	O(1)
numVertices, numEdges	O(1)
vertices	O(n)
edges, directedEdges, undirectedEdges	O(m)
elements, positions	O(n+m)
endVertices, opposite, origin, destination, isDirected, degree, inDegree, outDegree	O(1)
incidentEdges, inIncidentEdges, outIncidentEdges, adjacentVertices, inAdjacentVertices, outAdjacentVertices,	O(n)
areAdjacent	O(1)
insertEdge, insertDirectedEdge, removeEdge, makeUndirected, reverseDirection, setDirectionFrom, setDirectionTo	O(1)
insertVertex, removeVertex	O(n^2)