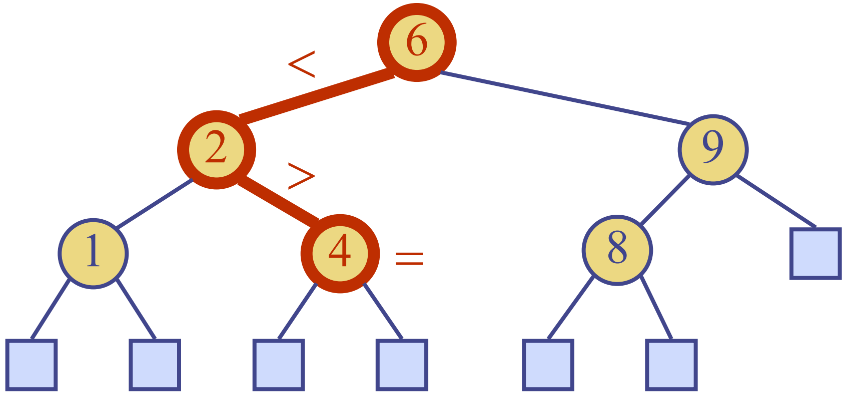
# Binary Search Trees

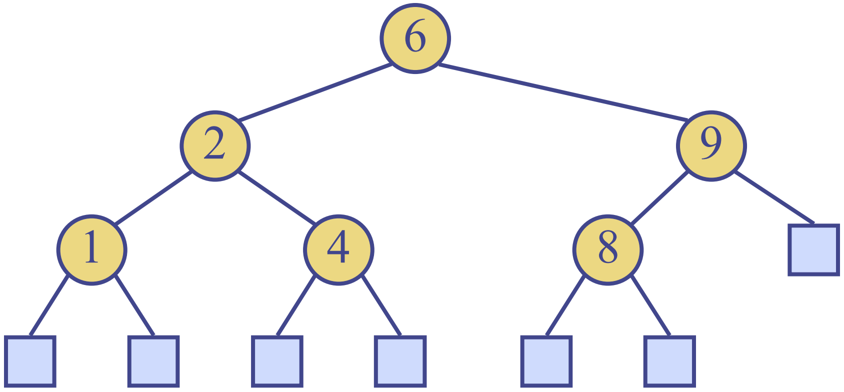


# Ordered Dictionaries

- Keys are assumed to come from a total order.
- New operations:
  - first(): first entry in the dictionary ordering
  - last(): last entry in the dictionary ordering
  - successor(k): first entry with key greater than or equal to k
  - predecessor(k): last entry with key less than or equal to k

# Binary Search Trees

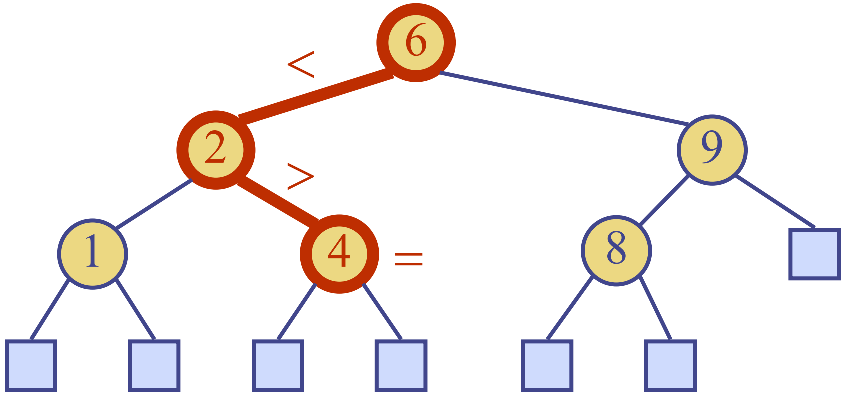
- A binary search tree is a binary tree storing keys (or key-value entries) at its internal nodes and satisfying the following property:
  - Let $u$, $v$, and $w$ be three nodes such that $u$ is in the left subtree of $v$ and $w$ is in the right subtree of $v$. We have $key(u) \leq key(v) < key(w)$
- External nodes do not store items
- An inorder traversal of a binary search trees visits the keys in increasing order



# Search

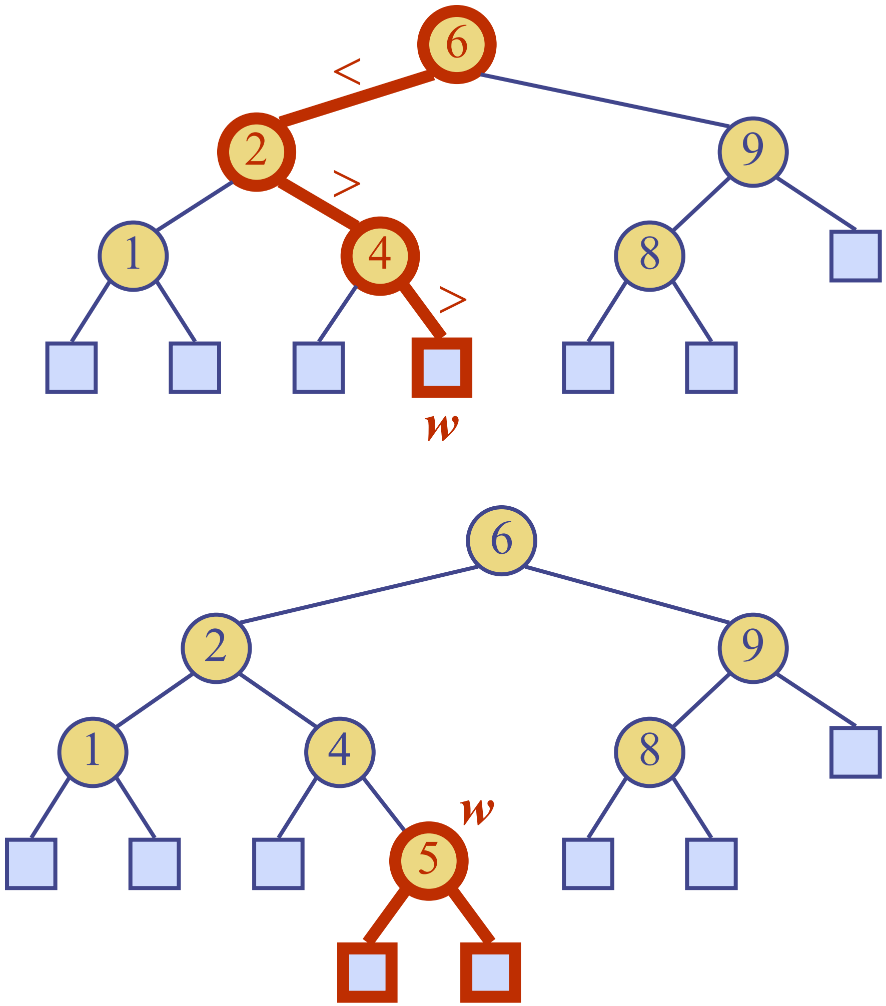
- To search for a key ***k***, we trace a downward path starting at the root
- The next node visited depends on the outcome of the comparison of ***k*** with the key of the current node
- If we reach a leaf, the key is not found and we return null
- Example: find(4):
  - Call TreeSearch(4,root)

**Algorithm** *TreeSearch*($k$, $v$)
  **if** *T.isExternal* ($v$)
    **return** $v$
  **if** $k$ < *key*($v$)
    **return** *TreeSearch*($k$, *T.left*($v$))
  **else if** $k$ = *key*($v$)
    **return** $v$
  **else** { $k$ > *key*($v$) }
    **return** *TreeSearch*($k$, *T.right*($v$))



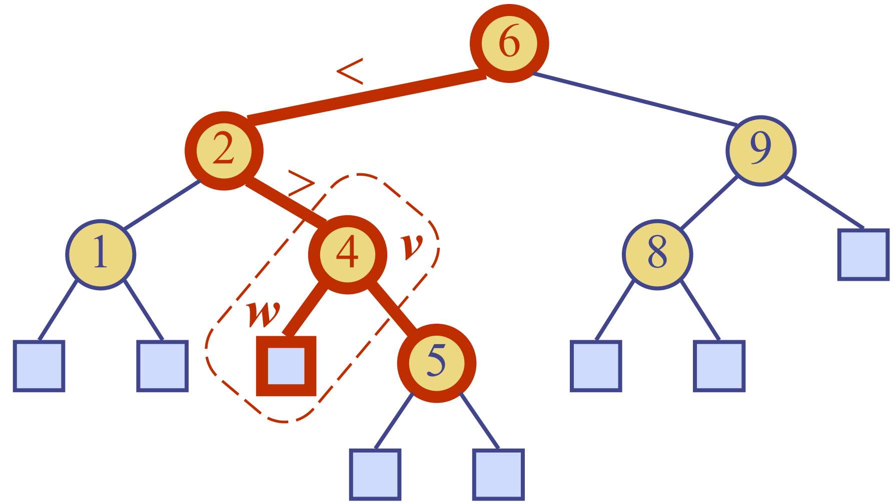
# Insertion

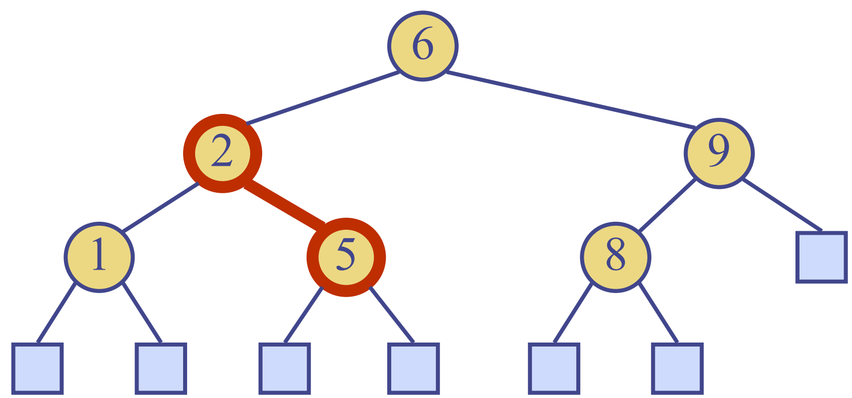
- To perform operation insert(k, o), we search for key k (using TreeSearch)
- Assume k is not already in the tree, and let let w be the leaf reached by the search
- We insert k at node w and expand w into an internal node
- Example: insert 5



# Deletion

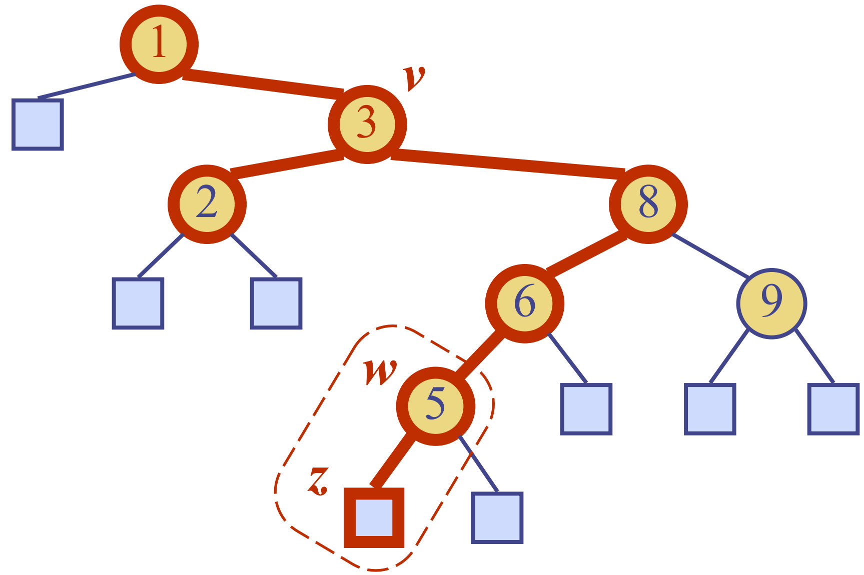
- To perform operation remove($k$), we search for key $k$
- Assume key $k$ is in the tree, and let let $v$ be the node storing $k$
- If node $v$ has a leaf child $w$, we remove $v$ and $w$ from the tree with operation removeExternal($w$), which removes $w$ and its parent
- Example: remove 4

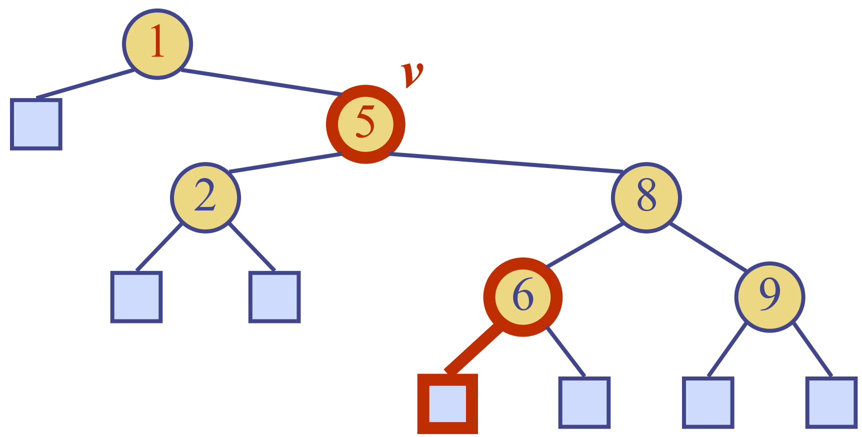


# Deletion (cont.)

- We consider the case where the key $k$ to be removed is stored at a node $v$ whose children are both internal
  - we find the internal node $w$ that follows $v$ in an inorder traversal
  - we copy $key(w)$ into node $v$
  - we remove node $w$ and its left child $z$ (which must be a leaf) by means of operation removeExternal($z$)
- Example: remove 3



# Performance

- Consider a dictionary with $n$ items implemented by means of a binary search tree of height $h$
  - the space used is $O(n)$
  - methods find, insert and remove take $O(h)$ time
- The height $h$ is $O(n)$ in the worst case and $O(\log n)$ in the best case