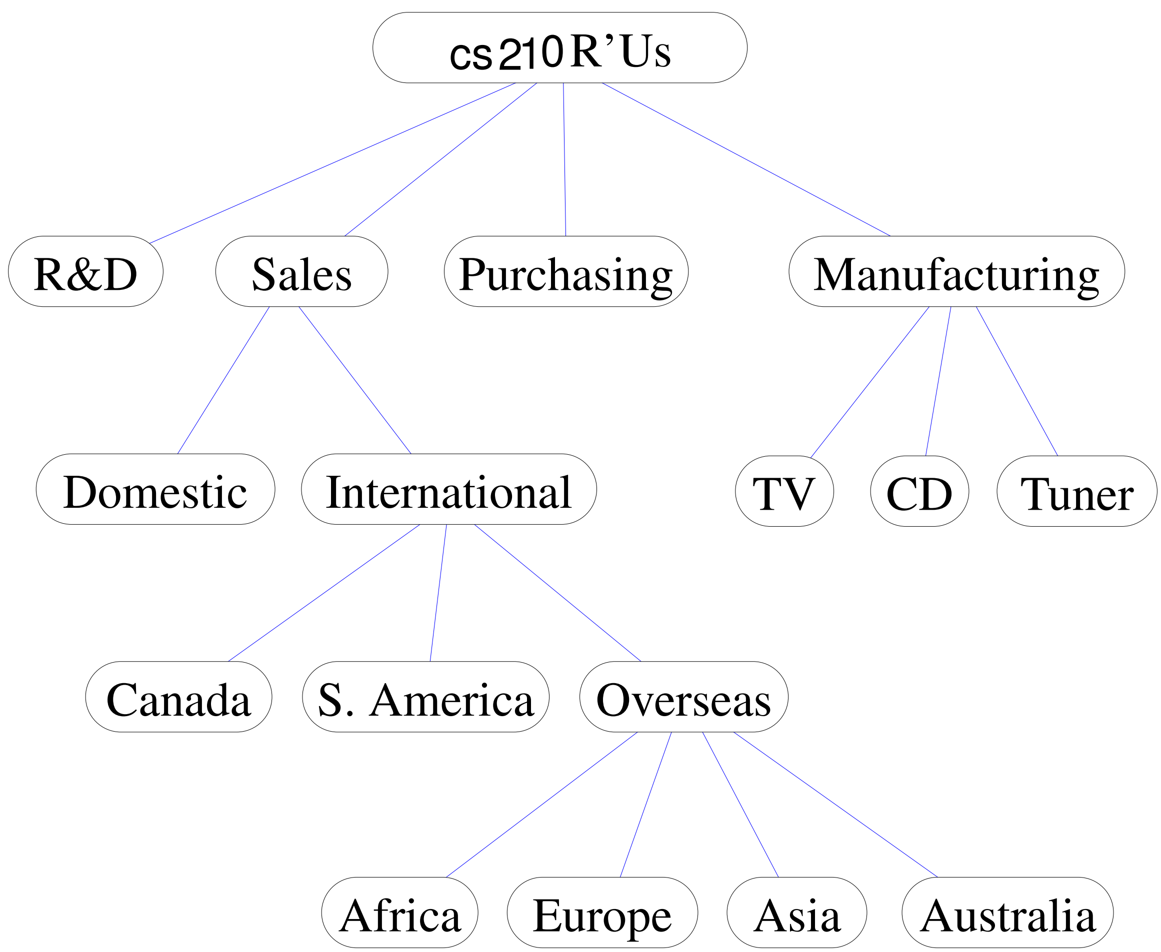
TREES

- trees
- binary trees
- traversals of trees
- template method pattern
- data structures for trees

Trees

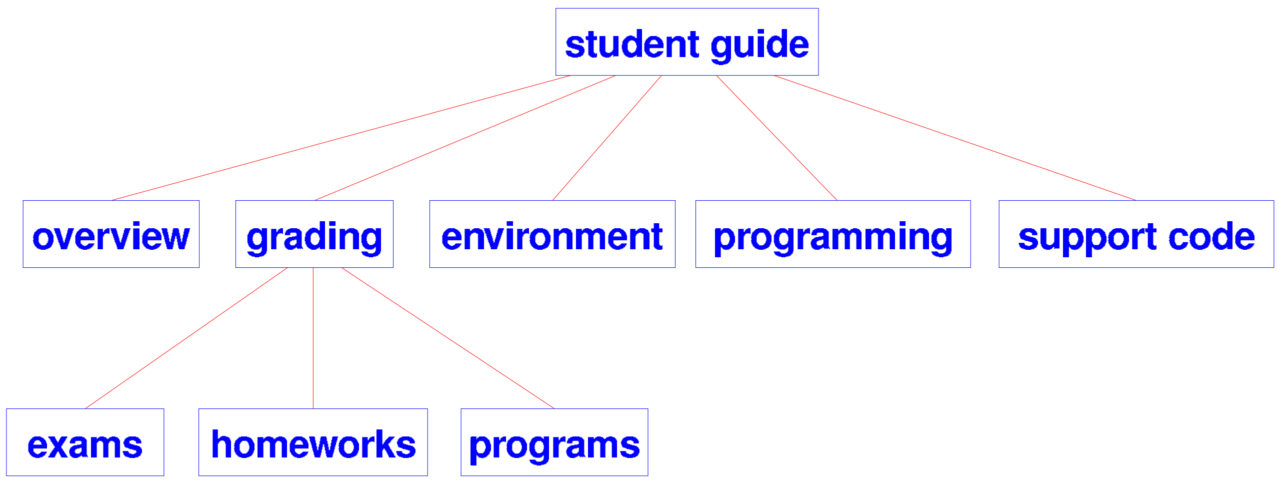
- In computer science, a tree is an abstract model of a hierarchical structure
- A tree consists of nodes with a parent-child relation
- Applications:

Organization of a company



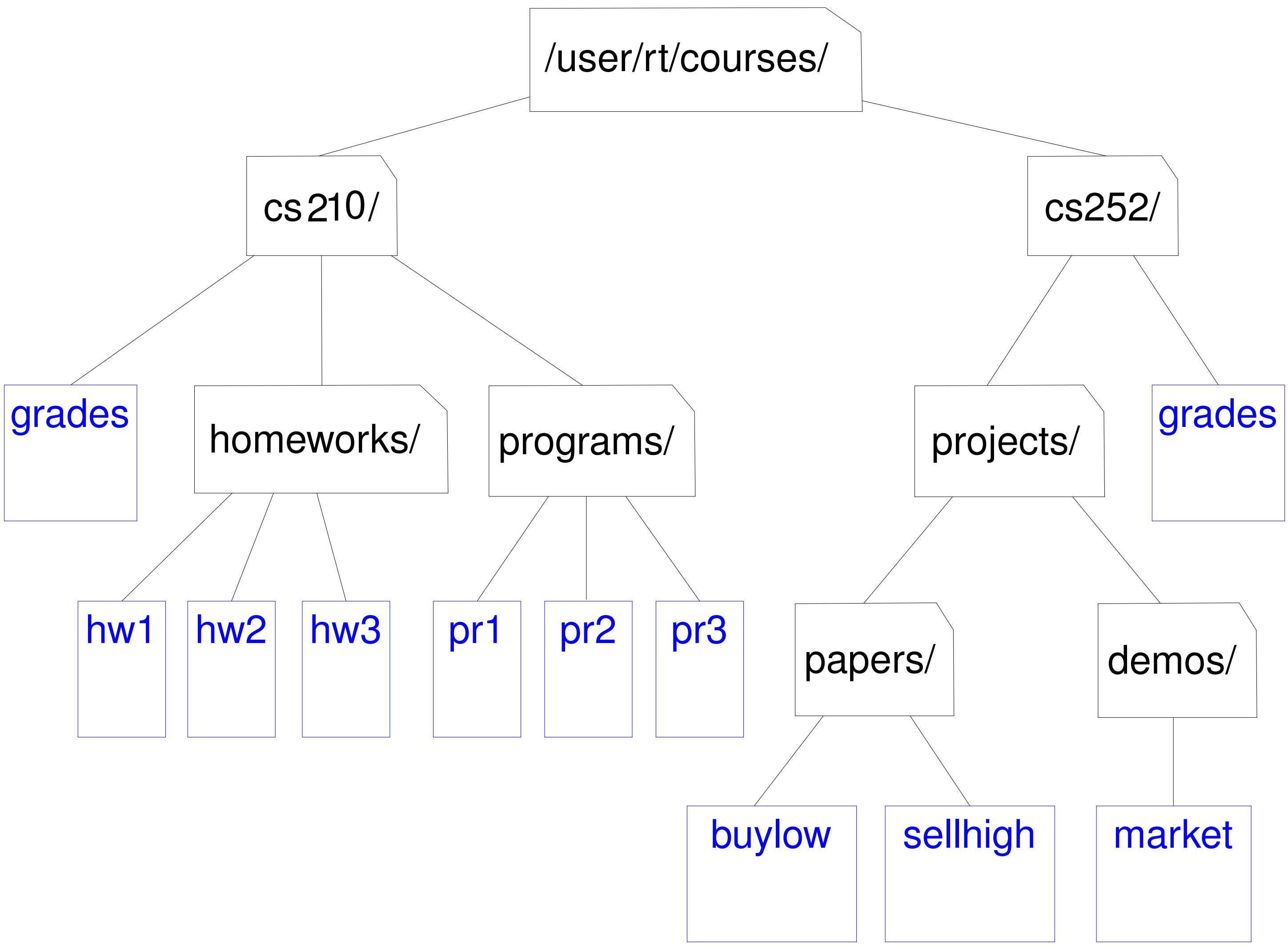
Another Example

- table of contents of a book



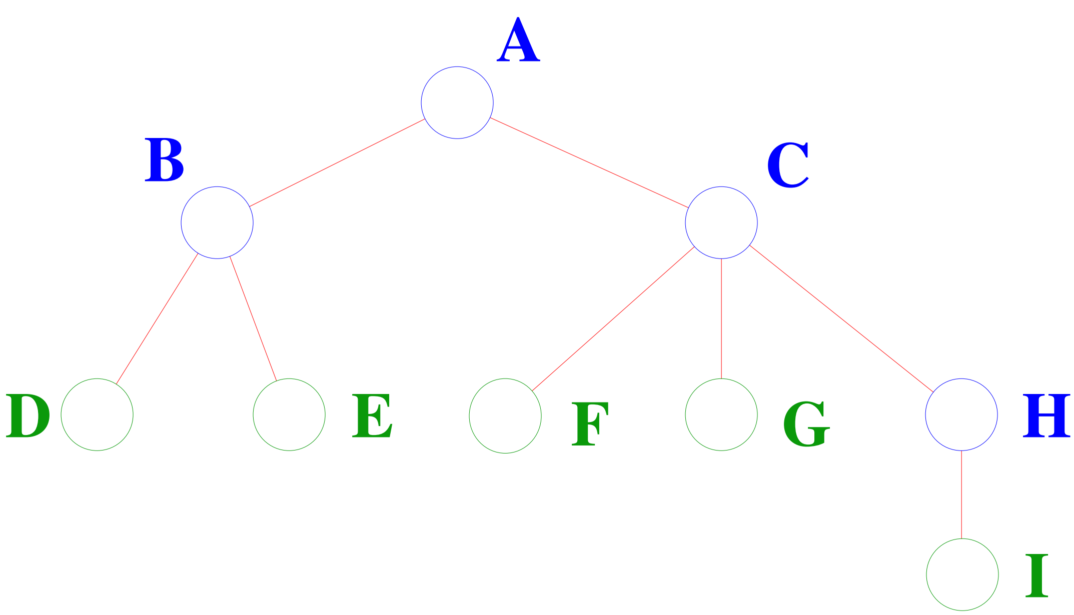
Another Example

- Unix or DOS/Windows file system



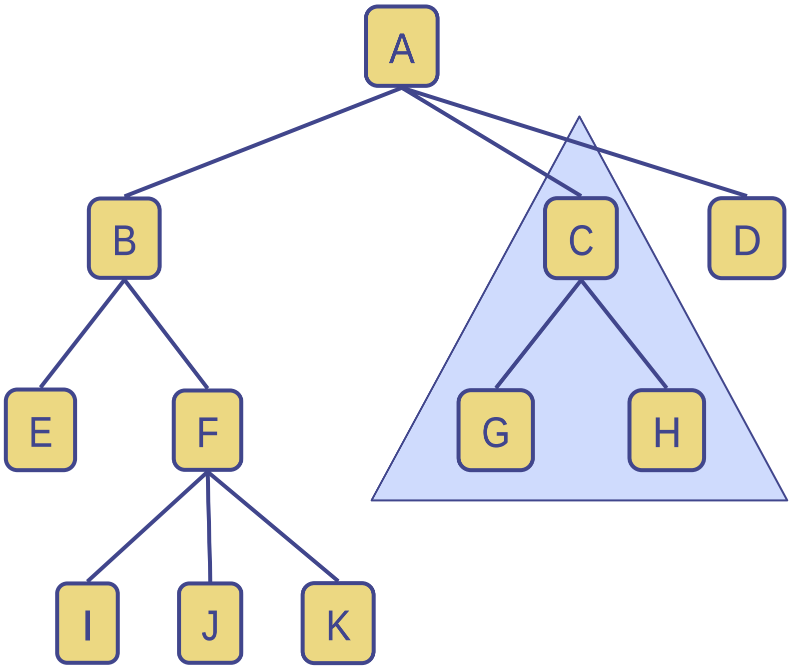
Terminology

- ***A*** is the ***root*** node.
- ***B*** is the ***parent*** of D and E.
- ***C*** is the ***sibling*** of B
- ***D*** and ***E*** are the ***children*** of B
- ***D, E, F, G***, ***I*** are ***external nodes***, **or** ***leaves***
- ***A, B, C, H*** are ***internal nodes***
- The ***depth*** (***level***) of ***E*** is ***2***
- The ***height*** of the tree is ***3***
- The ***degree*** of node ***B*** is ***2***

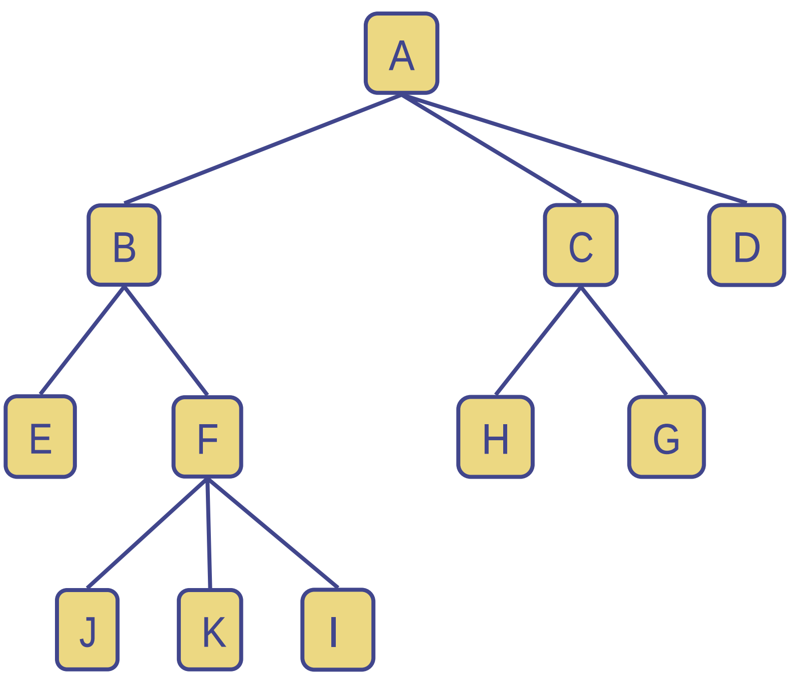


Property: (*# edges*) = (*#nodes*) − 1

- Subtree: tree consisting of a node and its descendants



- Ordered Tree: the children of a node are ordered



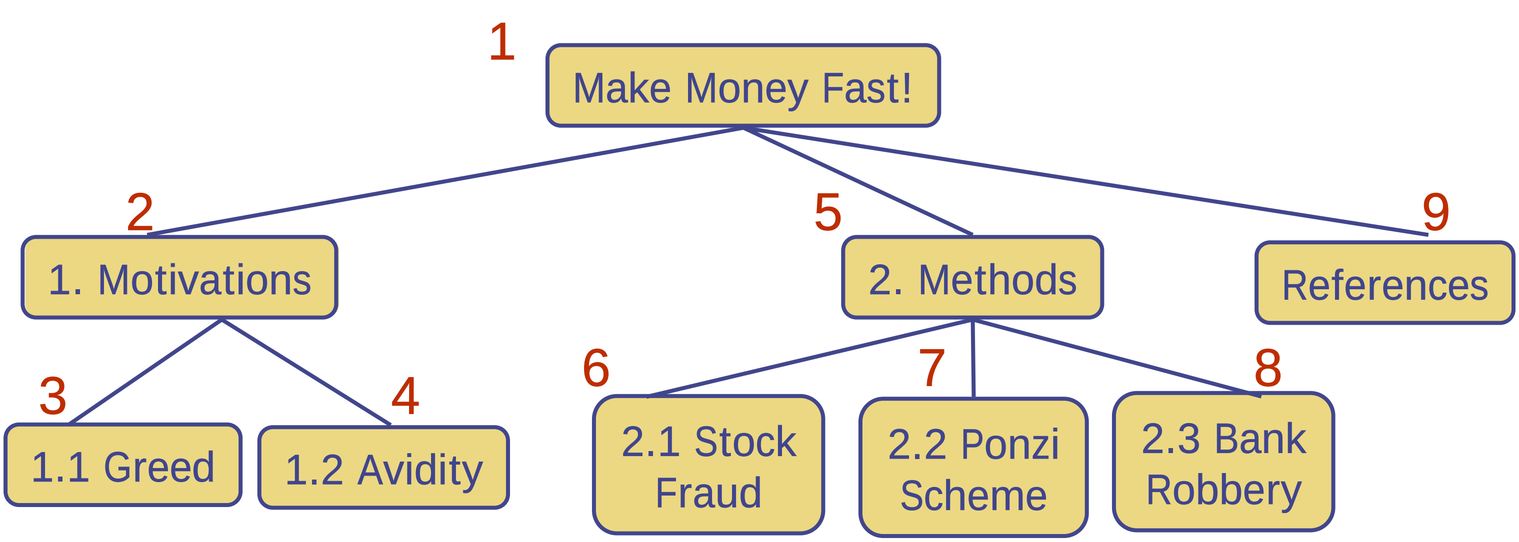
Tree ADT

- We use positions to abstract nodes
- Generic methods:
  - integer size()
  - boolean isEmpty()
  - Iterator elements()
  - Iterator positions()
- Accessor methods:
  - position root()
  - position parent(p)
  - positionIterator children(p)
- Query methods:
  - boolean isInternal(p)
  - boolean isExternal(p)
  - boolean isRoot(p)
- Update method:
  - object replace (p, o)
- Additional update methods may be defined by data structures implementing the Tree ADT

Preorder Traversal

- A traversal visits the nodes of a tree in a systematic manner
- In a preorder traversal, a node is visited before its descendants
- Application: print a structured document

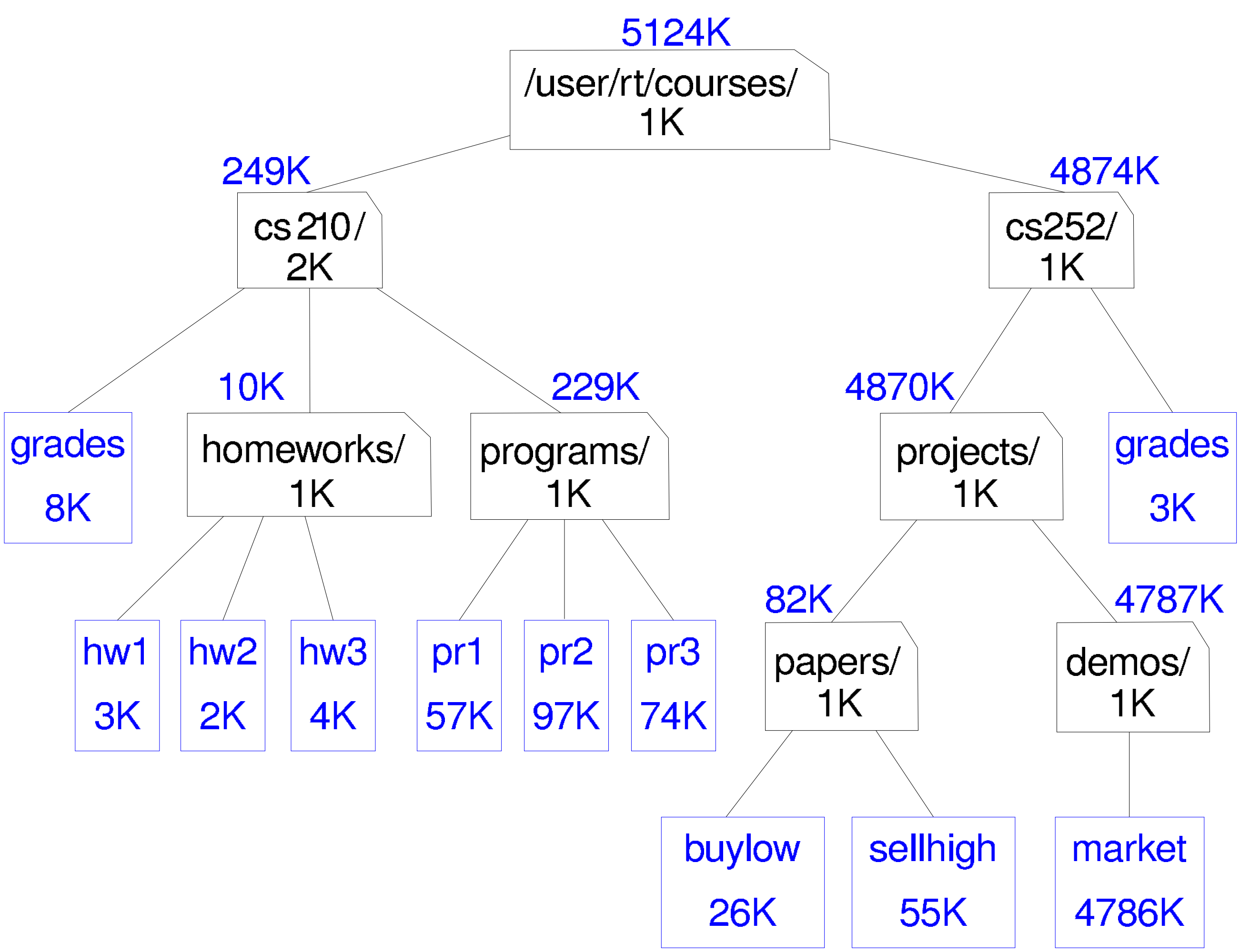
```
Algorithm preOrder(v)
    visit(v)
    for each child w of v
        preOrder (w)
```



Postorder Traversal

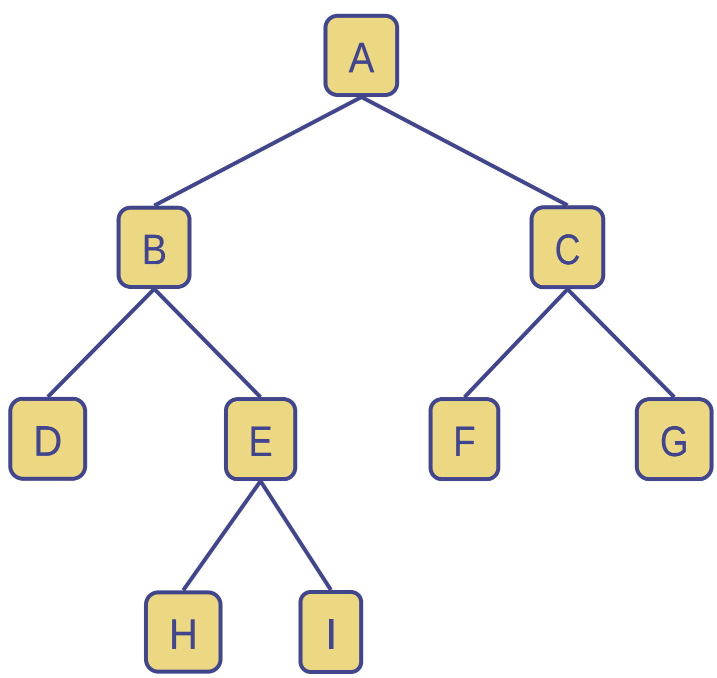
- In a postorder traversal, a node is visited after its descendants
- Application: compute space used by files in a directory and its subdirectories

Algorithm *postOrder*(*v*)
    for each child *w* of *v*
        postOrder (*w*)
    visit(*v*)



Binary Trees

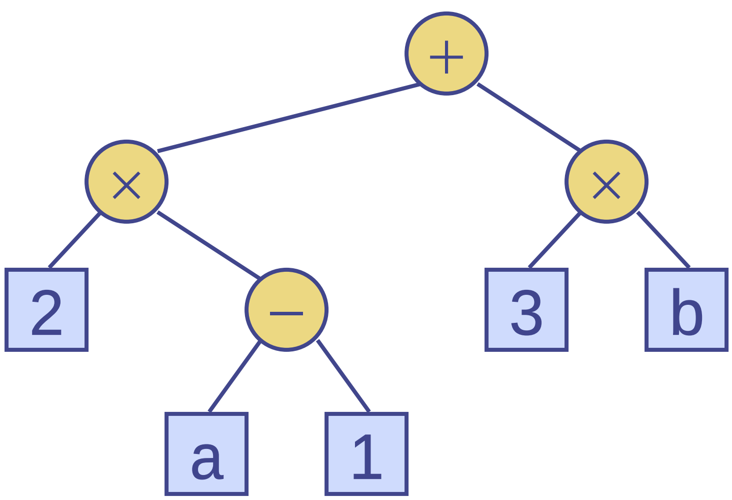
- A binary tree is a tree with the following properties:
  - Each internal node has at most two children (exactly two for **proper** binary trees)
  - The children of a node are an ordered pair
- We call the children of an internal node left child and right child
- Alternative recursive definition: a binary tree is either
  - a tree consisting of a single node, or
  - a tree whose root has an ordered pair of children, each of which is a binary tree.



Examples of Binary Trees

Arithmetic Expression Tree

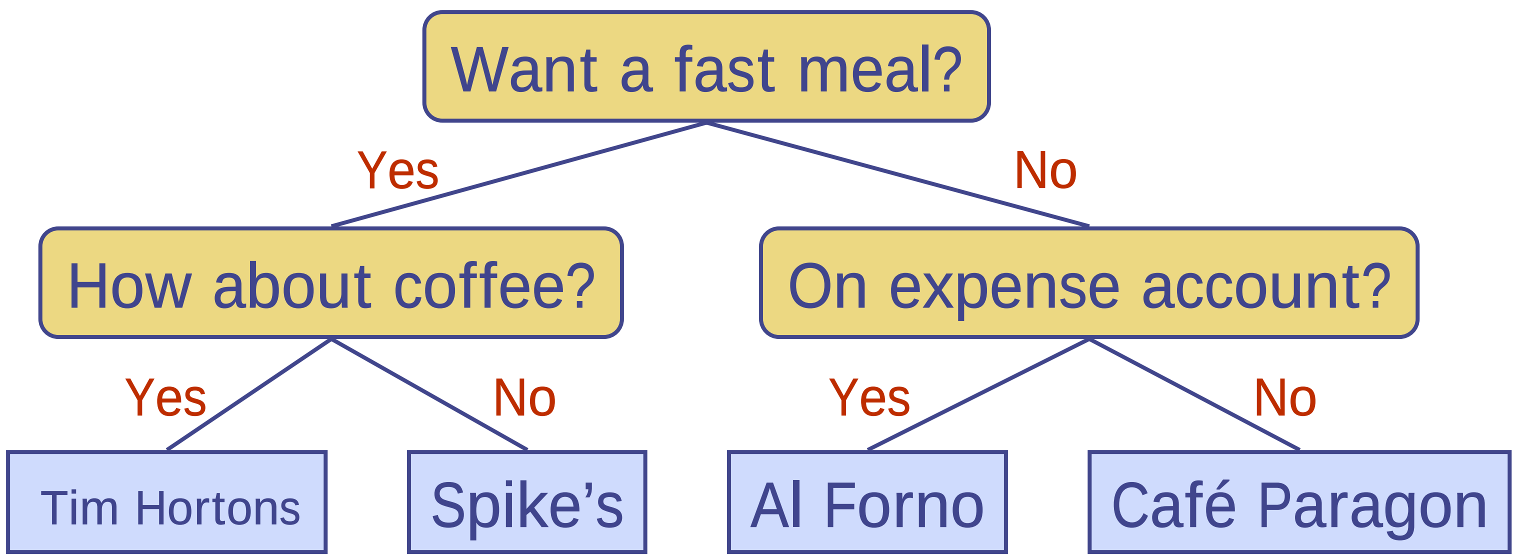
- Binary tree for an arithmetic expression
  - internal nodes: operators
  - external nodes: operands
- Example: arithmetic expression tree for the expression $(2 \times (a - 1) + (3 \times b))$



Examples of Binary Trees

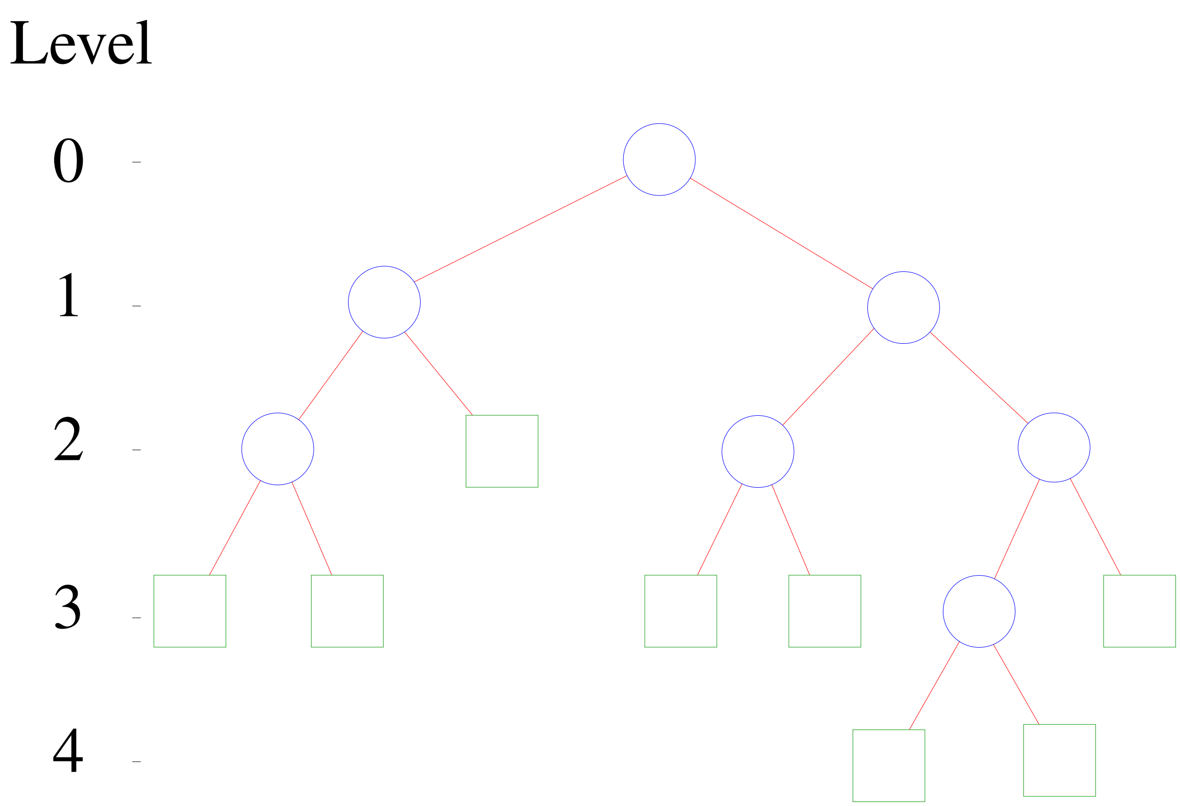
Decision Tree

- Binary tree associated with a decision process
  - internal nodes: questions with yes/no answer
  - external nodes: decisions
- Example: dining decision



Properties of Binary Trees

- (# external nodes) = (# internal nodes) + 1
- (# nodes at level $\boldsymbol{i}$) $\leq 2^{\boldsymbol{i}}$
- (# external nodes) $\leq 2^{(\text{height})}$
- (height) $\geq \log_2$ (# external nodes)
- (height) $\geq \log_2$ (# nodes) – 1
- (height) ≤ (# internal nodes) = ((# nodes) – 1)/2



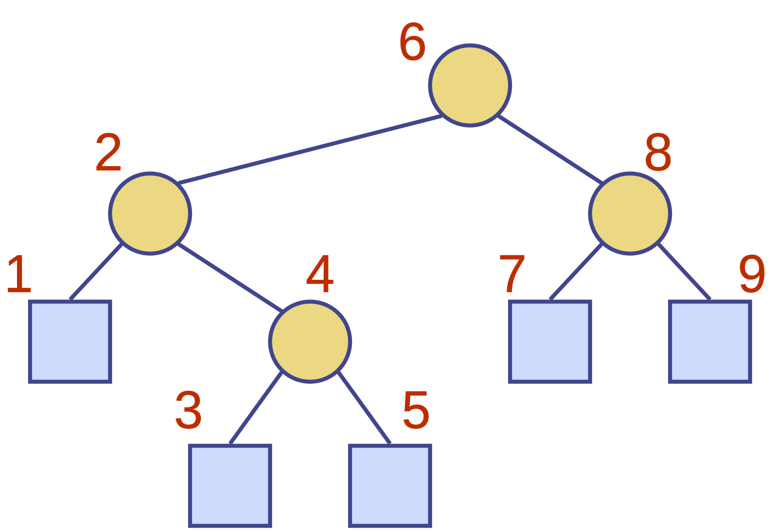
BinaryTree ADT

- The BinaryTree ADT extends the Tree ADT, i.e., it inherits all the methods of the Tree ADT
- Additional methods:
  - position left(p)
  - position right(p)
  - boolean hasLeft(p)
  - boolean hasRight(p)
- Update methods may be defined by data structures implementing the BinaryTree ADT

Inorder Traversal

- In an inorder traversal a node is visited after its left subtree and before its right subtree

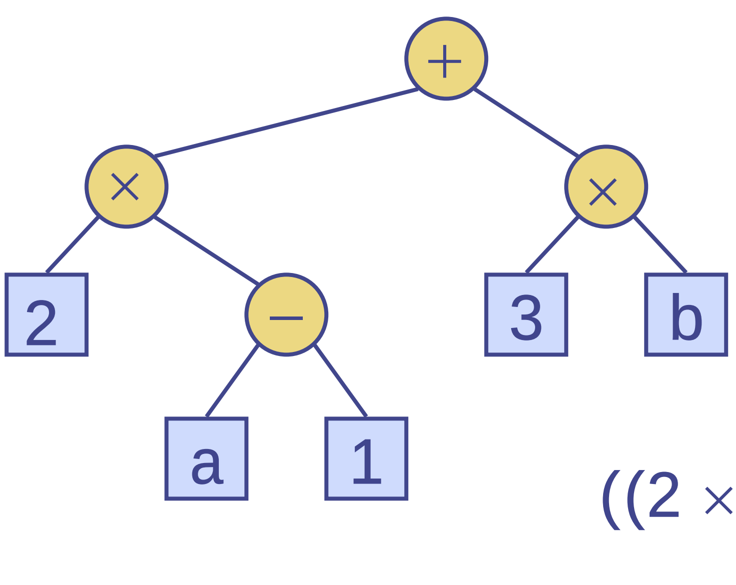
```
Algorithm inOrder(v)
    if hasLeft (v)
        inOrder (left (v))
    visit(v)
    if hasRight (v)
        inOrder (right (v))
```



Print Arithmetic Expressions

- Specialization of an inorder traversal
  - print operand or operator when visiting node
  - print “(“ before traversing left subtree
  - print “)“ after traversing right subtree

Algorithm *printExpression*(v)
  if *hasLeft* (v) **then** {
    print(“(”)
    printExpression(*left*(v)) }
  print(v.*element* ())
  if *hasRight* (v) **then** {
    printExpression (*right*(v))
    print (“)”) }

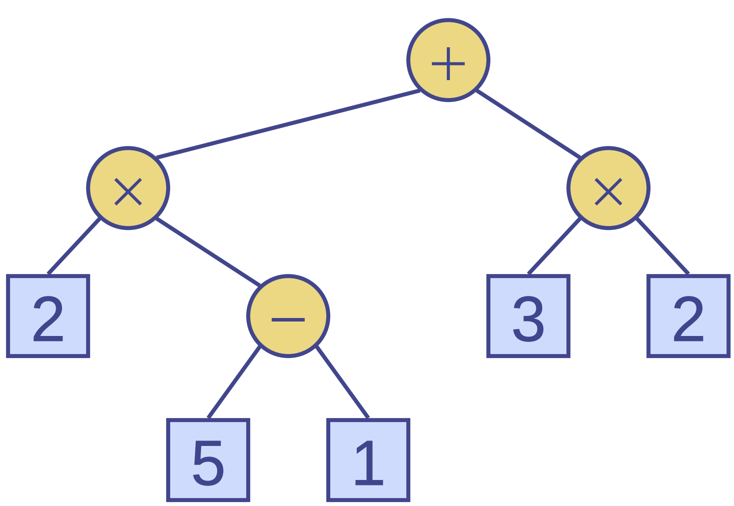


$((2 \times (a - 1)) + (3 \times b))$

Evaluate Arithmetic Expressions

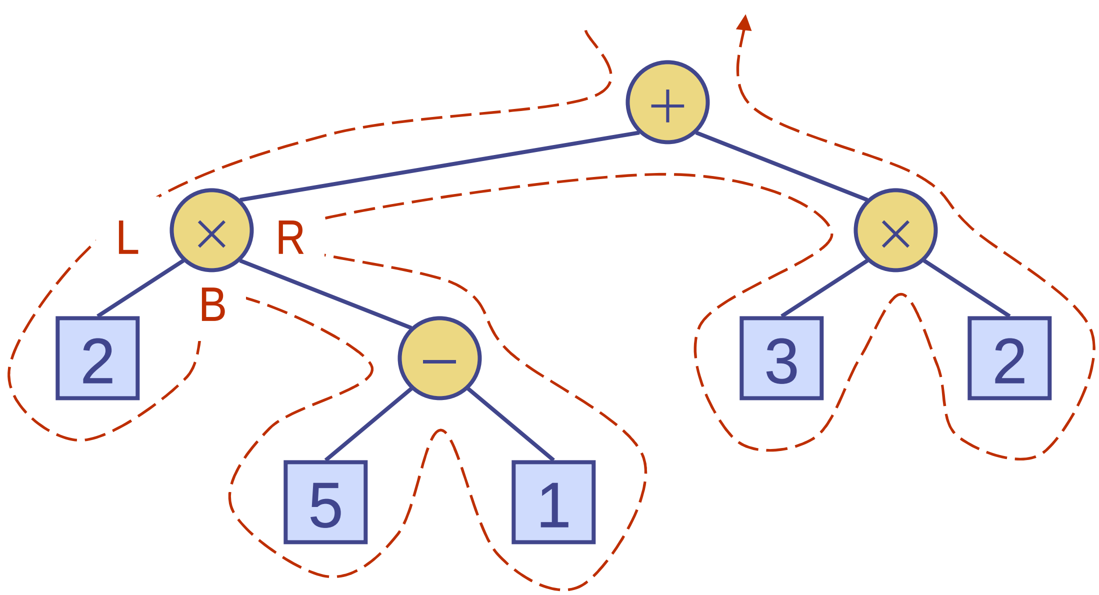
- Specialization of a postorder traversal
  - recursive method returning the value of a subtree
  - when visiting an internal node, combine the values of the subtrees

Algorithm *evalExpr*(v)
    if *isExternal* (v)
        return *v.element* ()
    else {
        $x \leftarrow$ *evalExpr*(*leftChild* (v))
        $y \leftarrow$ *evalExpr*(*rightChild* (v))
        $\Diamond \leftarrow$ operator stored at v
        return $x \Diamond y$
    }



Euler Tour Traversal

- Generic traversal of a binary tree
- Includes a special cases the preorder, postorder and inorder traversals
- Walk around the tree and visit each node three times:
  - on the left (preorder)
  - from below (inorder)
  - on the right (postorder)



Template Method Pattern

- Generic algorithm that can be specialized by redefining certain steps
- Implemented by means of an abstract Java class
- Visit methods that can be redefined by subclasses
- Template method eulerTour
  - Recursively called on the left and right children
  - A Result object with fields leftResult, rightResult and finalResult keeps track of the output of the recursive calls to eulerTour

```java
public abstract class EulerTour {
  protected BinaryTree tree;
  protected void visitExternal(Position p, Result r) { }
  protected void visitLeft(Position p, Result r) { }
  protected void visitBelow(Position p, Result r) { }
  protected void visitRight(Position p, Result r) { }

  protected Object eulerTour(Position p) {
    Result r = new Result();
    if tree.isExternal(p) { visitExternal(p, r); }
      else {
        visitLeft(p, r);
        r.leftResult = eulerTour(tree.left(p));
        visitBelow(p, r);
        r.rightResult = eulerTour(tree.right(p));
        visitRight(p, r);
        return r.finalResult;
      } …
```

Specializations of EulerTour

- We show how to specialize class EulerTour to evaluate an arithmetic expression
- Assumptions
  - External nodes store Integer objects
  - Internal nodes store Operator objects supporting method operation (Integer, Integer)

```
public class EvaluateExpression
                        extends EulerTour {
    protected void visitExternal(Position p, Result r) {
        r.finalResult = (Integer) p.element();
     }
    protected void visitRight(Position p, Result r) {
        Operator op = (Operator) p.element();
        r.finalResult = op.operation(
                            (Integer) r.leftResult,
                            (Integer) r.rightResult
                            );
    }
    ...
}
```

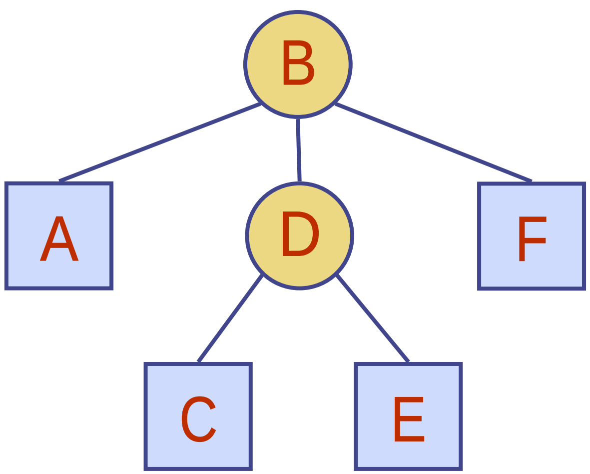
Specializations of EulerTour

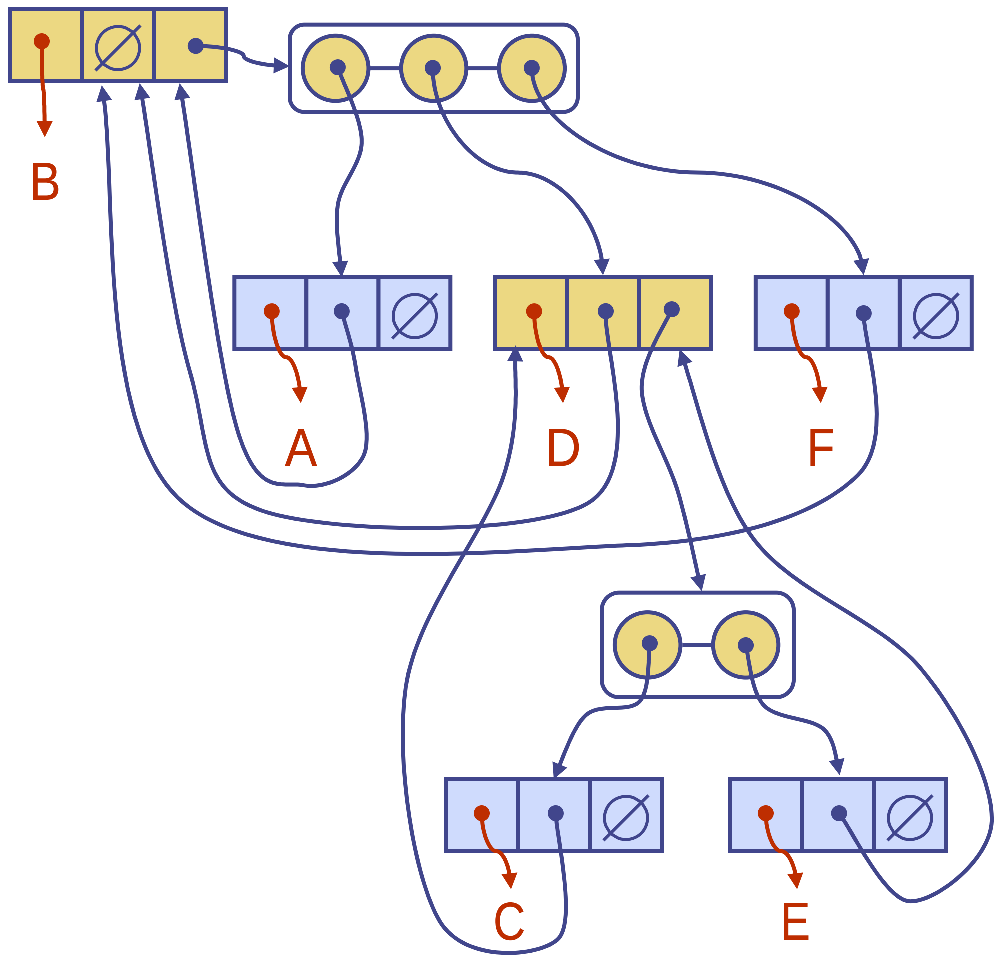
- We show how to specialize class EulerTour to print an arithmetic expression
- Assumptions
  - Method print prints any Object.

```
public class PrintExpression
                    extends EulerTour {

  protected void visitExternal(Position p, Result r) {
      print(p.element());
  }

  protected void vistLeft(Position p, Result r) {
      print("(");
  }

  protected void visitRight(Position p, Result r) {
    print(")");
  }

  protected void visitBelow(Position p, Result r) {
      print(p.element());
  }
}
```

Linked Structure for Trees

- A node is represented by an object storing
  - Element
  - Parent node
  - Sequence of children nodes
- Node objects implement the Position ADT





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Linked Structure for Binary Trees

- A node is represented by an object storing
  - Element
  - Parent node
  - Left child node
  - Right child node
- Node objects implement the Position ADT