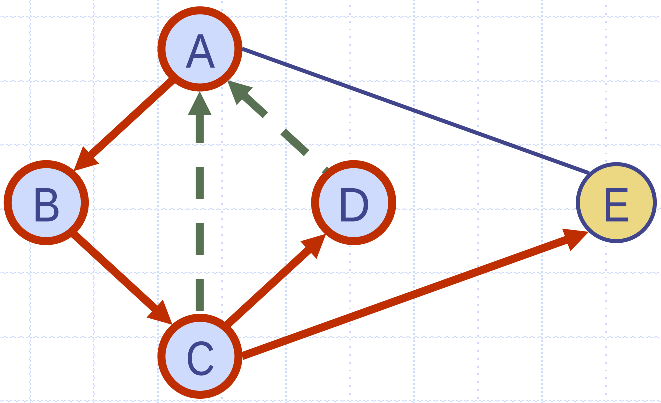
Depth-First Search



Depth-First Search (§ 13.3.1)

- Depth-first search (DFS) is a general technique for traversing a graph
- A DFS traversal of a graph G
  - Visits all the vertices and edges of G
  - Determines whether G is connected
  - Computes the connected components of G
  - Computes a spanning forest of G
- DFS on a graph with n vertices and m edges takes $O(n + m)$ time
- DFS can be further extended to solve other graph problems
  - Find and report a path between two given vertices
  - Find a cycle in the graph
- Depth-first search is to graphs what Euler tour is to binary trees

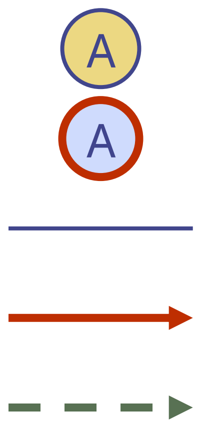
DFS Algorithm

- The algorithm uses a mechanism for setting and getting "labels" of vertices and edges

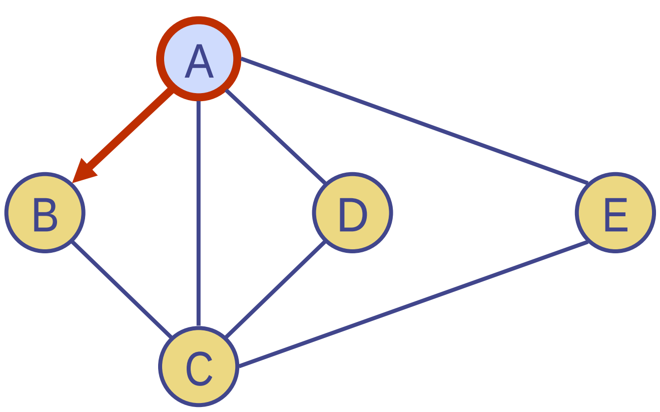
Algorithm *DFS*(*G*)
    Input graph *G*
    Output labeling of the edges of *G* as discovery edges and back edges
  for all $u \in$ *G.vertices*()
    setLabel(*u, UNEXPLORED*)
  for all $e \in$ *G.edges*()
    setLabel(*e, UNEXPLORED*)
  for all $v \in$ *G.vertices*()
    if *getLabel*(*v*) = *UNEXPLORED*
      DFS(*G, v*)

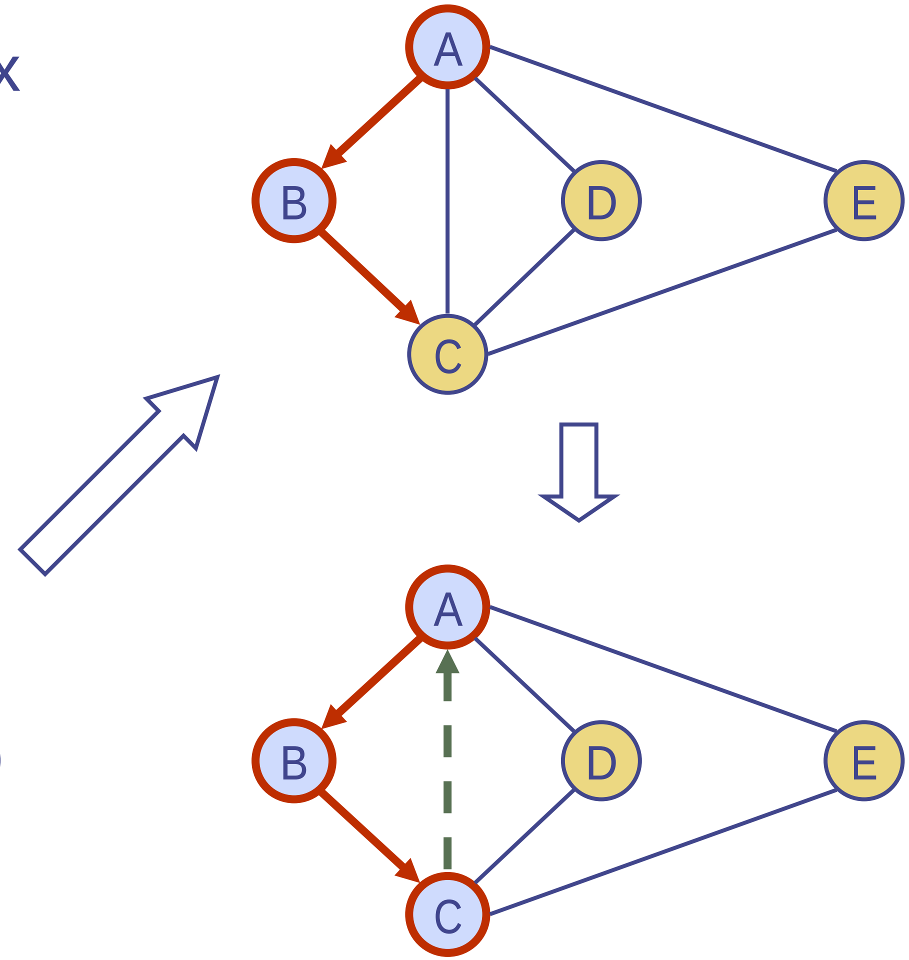
Algorithm *DFS*(*G, v*)
  Input graph *G* and a start vertex *v* of *G*
  Output labeling of the edges of *G* in the connected component of *v* as discovery edges and back edges
  setLabel(*v, VISITED*)
  for all $e \in$ *G.incidentEdges*(*v*)
    if *getLabel*(*e*) = *UNEXPLORED*
      $w \leftarrow$ *opposite*(*v,e*)
      if *getLabel*(*w*) = *UNEXPLORED*
        setLabel(*e, DISCOVERY*)
        DFS(*G, w*)
      else
        setLabel(*e, BACK*)

Example

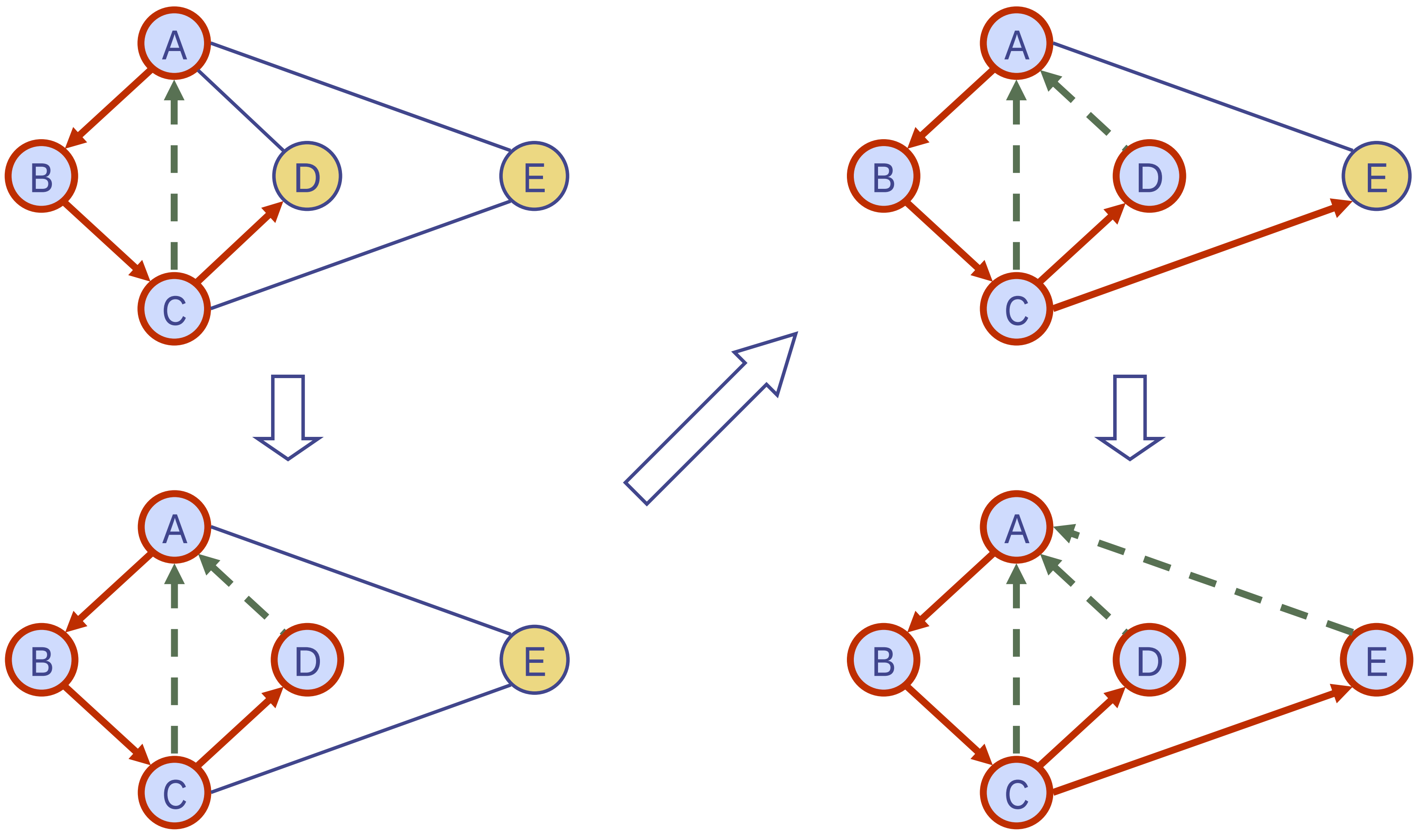


unexplored vertex
visited vertex
unexplored edge
discovery edge
back edge

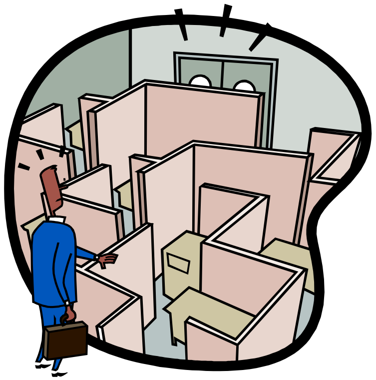




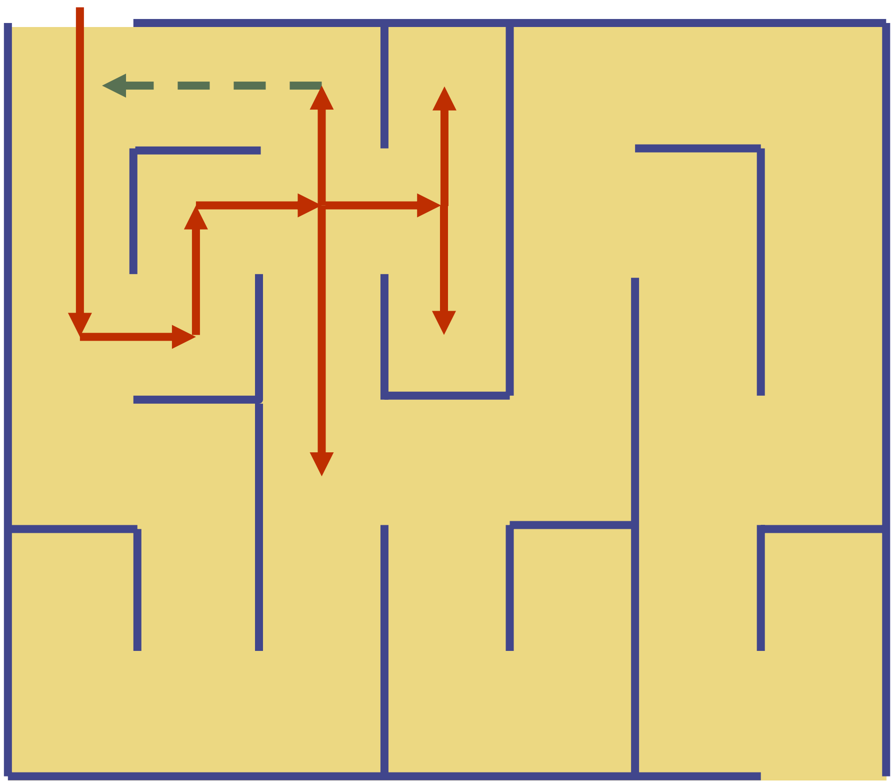
Example (cont.)



DFS and Maze Traversal



- The DFS algorithm is similar to a classic strategy for exploring a maze
  - We mark each intersection, corner and dead end (vertex) visited
  - We mark each corridor (edge) traversed
  - We keep track of the path back to the entrance (start vertex) by means of a rope (recursion stack)



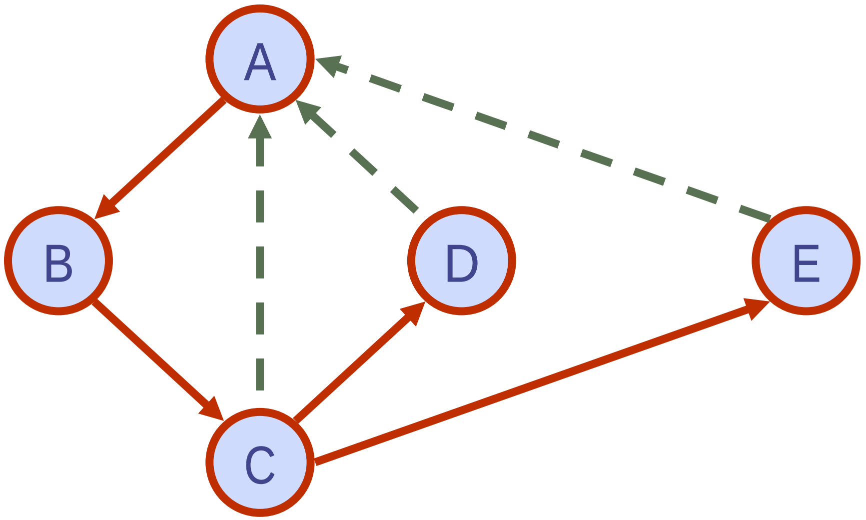
Properties of DFS

Property 1

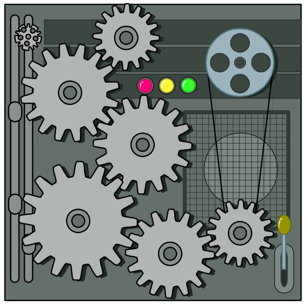
DFS(*G*, *v*) visits all the vertices and edges in the connected component of *v*

Property 2

The discovery edges labeled by *DFS*(*G*, *v*) form a spanning tree of the connected component of *v*



Analysis of DFS



- Setting/getting a vertex/edge label takes $O(1)$ time
- Each vertex is labeled twice
  - once as UNEXPLORED
  - once as VISITED
- Each edge is labeled twice
  - once as UNEXPLORED
  - once as DISCOVERY or BACK
- Method incidentEdges is called once for each vertex
- DFS runs in $O(n + m)$ time provided the graph is represented by the adjacency list structure
  - Recall that $\Sigma_v \deg(v) = 2m$

Path Finding

- We can specialize the DFS algorithm to find a path between two given vertices ***u*** and ***z*** using the template method pattern
- We use a stack ***S*** to keep track of the path between the start vertex and the current vertex
- As soon as destination vertex ***z*** is encountered, we return the path as the contents of the stack

```
Algorithm pathDFS(G, v, z)
    setLabel(v, VISITED)
    S.push(v)
    if  v = z
        return S.elements()
    for all  e ∈ G.incidentEdges(v)
        if  getLabel(e) = UNEXPLORED {
            w ← opposite(v,e)
            if getLabel(w) = UNEXPLORED {
                 setLabel(e, DISCOVERY)
                S.push(e)
                p = pathDFS(G, w, z)
                if p != NULL return p
                else S.pop(e)
            } else setLabel(e, BACK)
    }
    S.pop(v)
    return NULL
```

Cycle Finding

- We can specialize the DFS algorithm to find a simple cycle using the template method pattern
- We use a stack S to keep track of the path between the start vertex and the current vertex
- As soon as a back edge (v, w) is encountered, we return the cycle as the portion of the stack from the top to vertex w

```
Algorithm cycleDFS(G, v)
  setLabel(v, VISITED)
  S.push(v)
  for all  e ∈ G.incidentEdges(v)
    if  getLabel(e) = UNEXPLORED
      w ← opposite(v,e)
      S.push(e)
      if getLabel(w) = UNEXPLORED {
         setLabel(e, DISCOVERY)
        p = cycleDFS(G, w)
         if p != NULL return p
        else S.pop(e)
      } else {
        T ← new empty stack
        repeat
          o ← S.pop()
          T.push(o)
        until o = w
        return T.elements()
      }
  S.pop(v)
  return NULL
```