1. Translate the following sentences into the language of propositional calculus using the indicated notations for atomic formulas.
    a. Harry is short, Bill is tall, and Marry is not blond. (S, T, B)
    $S \wedge T \wedge \neg B$
    b. Mary is sick (S) and it is raining (R) implies that Bob stayed (T) up late last night. (S, R, T)
    $S \wedge R \rightarrow T$
    c. It is not the case that it is raining, and John will not take an umbrella. (R, T)
    $\neg R \wedge \neg T$
    d. I will explore (E) canyons during spring break only if I get Grand Canyon reservations (R) and get a group (G) together. (E, R, G)
    $E \rightarrow R \wedge G$
    e. You get mashed potatoes or French fries but not both. (M, F)
    $M \vee F \wedge \neg M \vee \neg F$
    f. Oscar will not pass the logic course unless he does his homework and studies. (P, H, S)
    $H \wedge S \rightarrow P$
    g. If logic is difficult, Oscar and Virginia will pass only if they study. (D, O, V, S)
    $D \rightarrow (O \wedge V \rightarrow S)$
    h. If I bring (B) my digital camera, then if my batteries don't die (D) then I'll take (T) pictures of my backpack trip and put (P) the pictures on my Website. (B, D, T, P)
    $B \rightarrow (D \rightarrow T \wedge P)$
    i. If Q is quadrilateral (Q), then Q is a parallelogram (P) if and only if its opposite sides are both equal (E) and parallel (L). (Q, P, E, L)
    $Q \rightarrow (P \leftrightarrow E \wedge L)$

2. Use the laws of propositional calculus to decide whether the following sentences are logically equivalent. Use the atomic propositions C, B, and W.
    a. If you are not crustacean then you have bones or you would wiggle along the ground.
    $(\neg C \rightarrow B) \vee W$
    b. If you are not crustacean and you do not have bones then you wriggle along the ground.
    $\neg C \wedge \neg B \rightarrow W$

    $(\neg C \rightarrow B) \vee W \vdash \neg C \wedge \neg B \rightarrow W$
    $(C \vee B) \vee W \vdash \neg(C \vee B) \rightarrow W$
    $C \vee B \vee W \vdash (C \vee B) \vee W$
    $C \vee B \vee W \vdash C \vee B \vee W$

3. Use the truth table method to decide whether the conclusion D is a tautological consequence of the premises $A \leftrightarrow B, B \rightarrow C, \neg C \rightarrow \neg D, \neg A \rightarrow D$. You can only consider the value assignments that make all the premises true.

$\boldsymbol{A}$	$\boldsymbol{B}$	$\boldsymbol{C}$	$\boldsymbol{D}$	$\boldsymbol{A \leftrightarrow B}$			$\boldsymbol{B \rightarrow C}$			$\boldsymbol{\neg C \rightarrow \neg D}$			$\boldsymbol{\neg A \rightarrow D}$			$\boldsymbol{Con}$	$\boldsymbol{P_1 \wedge P_2 \wedge P_3 \wedge P_4 \wedge Con}$
				A	B		B	C		$\neg C$	$\neg D$		$\neg A$	D			
0	0	0	0	0	0	1	0	0	1	1	1	1	1	0	0		
0	0	0	1	0	0	1	0	0	1	1	0	0					
0	0	1	0	0	0	1	0	1	1	0	1	1	1	0	0		
0	0	1	1	0	0	1	0	1	1	0	0	1	1	1	1	1	1
0	1	0	0	0	1	0											
0	1	0	1	0	1	0											
0	1	1	0	0	1	0											
0	1	1	1	0	1	0											
1	0	0	0	1	0	0											
1	0	0	1	1	0	0											
1	0	1	0	1	0	0											
1	0	1	1	1	0	0											
1	1	0	0	1	1	1	1	0	0								
1	1	0	1	1	1	1	1	0	0								
1	1	1	0	1	1	1	1	1	1	0	1	1	0	0	1	0	**0**
1	1	1	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1

Therefore, the argument is invalid because there exists a set of values that make the premises true but not the conclusion, row 15.

4. Give the truth table for each of the following formulas:
   a. $A \rightarrow (B \rightarrow \neg A)$

$\boldsymbol{A}$	$\boldsymbol{B}$	$\boldsymbol{C}$	$\boldsymbol{B \rightarrow \neg A}$			$\boldsymbol{A \rightarrow (B \rightarrow \neg A)}$			$\boldsymbol{\neg F}$
			B	$\neg A$		A	$B \rightarrow \neg A$		
0	0	0	0	1	1	0	1	1	0
0	0	1	0	1	1	0	1	1	0
0	1	0	1	1	1	0	1	1	0
0	1	1	1	1	1	0	1	1	0
1	0	0	0	0	1	1	1	1	0
1	0	1	0	0	1	1	1	1	0
1	1	0	1	0	0	1	0	0	1
1	1	1	1	0	0	1	0	0	1

b. $A \vee \neg B \leftrightarrow C \rightarrow \neg A$

$\boldsymbol{A}$	$\boldsymbol{B}$	$\boldsymbol{C}$	$\boldsymbol{A \vee \neg B}$			$\boldsymbol{C \rightarrow \neg A}$			$\boldsymbol{A \vee \neg B \leftrightarrow C \rightarrow \neg A}$			$\boldsymbol{\neg F}$
			A	$\neg B$		C	$\neg A$		$A \vee \neg B$	$C \rightarrow \neg A$		
0	0	0	0	1	1	0	1	1	1	1	1	0
0	0	1	0	1	1	1	1	1	1	1	1	0
0	1	0	0	0	0	0	1	1	0	1	0	1
0	1	1	0	0	0	1	1	1	0	1	0	1
1	0	0	1	1	1	0	0	1	1	1	1	0
1	0	1	1	1	1	1	0	0	1	0	0	1
1	1	0	1	0	1	0	0	1	1	1	1	0
1	1	1	1	0	1	1	0	0	1	0	0	1

5. Using the truth tables, find the DNF and CNF for each of the formulas in Question 4.
   a. DNF: $(\neg A \wedge \neg B \wedge \neg C) \vee (\neg A \wedge \neg B \wedge C) \vee (\neg A \wedge B \wedge \neg C) \vee (\neg A \wedge B \wedge C) \vee (A \wedge \neg B \wedge \neg C) \vee (A \wedge \neg B \wedge C)$

      CNF: $\neg\big((A \wedge B \wedge \neg C) \vee (A \wedge B \wedge C)\big) \vdash$
      $(\neg A \vee \neg B \vee C) \wedge (\neg A \vee \neg B \vee \neg C)$

   b. DNF: $(\neg A \wedge \neg B \wedge \neg C) \vee (\neg A \wedge \neg B \wedge C) \vee (A \wedge \neg B \wedge \neg C) \vee (A \wedge B \wedge \neg C)$

      CNF: $\neg\big((\neg A \wedge B \wedge \neg C) \vee (\neg A \wedge B \wedge C) \vee (A \wedge \neg B \wedge C) \vee (A \wedge B \wedge C)\big) \vdash$
      $(A \vee \neg B \vee C) \wedge (A \vee \neg B \vee \neg C) \wedge (\neg A \vee B \vee \neg C) \wedge (\neg A \vee \neg B \vee \neg C)$

6. Using the laws of propositional calculus for the formula, $F_1 \rightarrow (F_2 \vee \neg F_3) \rightarrow F_4$, finds its
   a. Disjunctive normal form.

      $\neg F_1 \vee \neg(F_2 \vee \neg F_3) \vee F_4 \vdash$
      $\neg F_1 \vee \neg F_2 \wedge F_3 \vee F_4 \vdash$
      $\neg F_1 \vee (\neg F_2 \wedge F_3) \vee F_4$

   b. Conjunctive normal form.

      $\neg F_1 \vee \neg(F_2 \vee \neg F_3) \vee F_4 \vdash$
      $\neg F_1 \vee \neg F_2 \wedge F_3 \vee F_4 \vdash$
      $(\neg F_1 \vee F_4) \vee (\neg F_2 \wedge F_3) \vdash$
      $(\neg F_1 \vee F_4 \vee \neg F_2) \wedge (\neg F_1 \vee F_4 \vee F_3)$

7. For each of the following formulas, find an equivalent formula that only contains connectives $\{\neg, \rightarrow\}$.
   a. $a \vee b \vdash \neg a \rightarrow b$
   b. $a \wedge b \vdash \neg(\neg a \vee \neg b) \vdash \neg(a \rightarrow \neg b)$
   c. $a \leftrightarrow b \vdash (a \rightarrow b) \wedge (b \rightarrow a) \vdash \neg\neg\big((a \rightarrow b) \wedge (b \rightarrow a)\big) \vdash$
   $\neg\big(\neg(a \rightarrow b) \vee \neg(b \rightarrow a)\big) \vdash \neg\big((a \rightarrow b) \rightarrow \neg(b \rightarrow a)\big)$

8. Use formal deduction to prove the following statements. Identify the laws that you are using.
   a. $\{A \rightarrow B \vee C, A \wedge \neg B\} \vdash C$
   Let $\{A \rightarrow B \vee C, A \wedge \neg B\} = \sum$

1. $\sum \vdash A \wedge \neg B$	$(\in)$
2. $\sum \vdash A$	$(1, -\wedge)$
3. $\sum \vdash \neg B$	$(1, -\wedge)$
4. $\sum \vdash A \rightarrow B \vee C$	$(\in)$
5. $\sum \vdash B \vee C$	$(4, 2, -\rightarrow)$
6. $\sum \vdash C$	$(5, diss.solly.)$

   b. $\{A \rightarrow B, C \rightarrow A, \neg C \rightarrow D, \neg D\} \vdash B$
   Let $\{A \rightarrow B, C \rightarrow A, \neg C \rightarrow D, \neg D\} = \sum$

1. $\sum \vdash \neg D$	$(\in)$
2. $\sum \vdash \neg C \rightarrow D$	$(\in)$
3. $\sum \vdash C$	$(1, 2, modus\ tollens)$
4. $\sum \vdash C \rightarrow A$	$(\in)$
5. $\sum \vdash A$	$(4, 3, -\rightarrow)$
6. $\sum \vdash A \rightarrow B$	$(\in)$
7. $\sum \vdash B$	$(5, 6, -\rightarrow)$

   c. $\{(A \rightarrow B) \wedge (C \rightarrow D), D \wedge B \rightarrow E, \neg E\} \vdash \neg C \vee \neg A$
   Let $\{(A \rightarrow B) \wedge (C \rightarrow D), \neg E\} = \sum$

1. $\sum \vdash (A \rightarrow B) \wedge (C \rightarrow D)$	$(\in)$
2. $\sum, \neg B \vdash A \rightarrow B$	$(1, -\wedge)$
3. $\sum, \neg B \vdash \neg A$	$(2, modus\ tollens)$
4. $\sum, \neg D \vdash C \rightarrow D$	$(1, -\wedge)$
5. $\sum, \neg D \vdash \neg C$	$(4, modus\ tollens)$
6. $\sum, \neg D \vee \neg B \vdash \neg C \vee \neg A$	$(3, 5\ -\vee, +\vee)$
7. $\sum, \neg D \vee \neg B \vdash \neg D \vee \neg B$	$(\in)$
8. $\sum \vdash \neg(D \wedge B)$	$(7, De\ Morgan)$
9. $\sum \vdash \neg E$	$(\in)$
10. $\sum \vdash \neg E \rightarrow \neg(D \wedge B)$	$(8, 9, +\rightarrow)$
11. $\sum \vdash (D \wedge B) \rightarrow E$	$(10, con.pos.)$
12. $\sum, (D \wedge B) \rightarrow E \vdash \neg C \vee \neg A$	$(+, \in)$

9. Provide the formula for each of the logic circuits
   a. $A \vee (B \wedge C) \ \vee \neg D$
   b. $(\neg X \wedge \neg Y \wedge Z) \vee (\neg X \wedge Y \wedge \neg Z) \vee (X \wedge \neg Y \wedge \neg Z) \vee (X \wedge Y \wedge Z)$

10. Simplify each of the following formulas using Karnaugh maps
    a. $A\bar{B}CD \vee A\bar{B}C\bar{D} \vee A\bar{B}\bar{C}\bar{D} \vee A\overline{BC}D$
       $A\bar{B}CD \vee A\bar{B}C\bar{D} \vee A\bar{B}\bar{C}\bar{D} \vee A(\bar{B} \vee \bar{C})D$
       $A\bar{B}CD \vee A\bar{B}C\bar{D} \vee A\bar{B}\bar{C}\bar{D} \vee AD(\bar{B} \vee \bar{C})$
       $A\bar{B}CD \vee A\bar{B}C\bar{D} \vee A\bar{B}\bar{C}\bar{D} \vee AD\big(\bar{B}(C \vee \bar{C}) \vee \bar{C}(B \vee \bar{B})\big)$
       $A\bar{B}CD \vee A\bar{B}C\bar{D} \vee A\bar{B}\bar{C}\bar{D} \vee A\bar{B}\bar{C}D \vee AB\bar{C}D$

	CD	$\boldsymbol{C\bar{D}}$	$\boldsymbol{\bar{C}\bar{D}}$	$\boldsymbol{\bar{C}D}$
$\boldsymbol{AB}$				1
$\boldsymbol{A\bar{B}}$	1	1	1	1
$\boldsymbol{\bar{A}\bar{B}}$				
$\boldsymbol{\bar{A}B}$				

$A\bar{B} \vee A\bar{C}D$

    b. $AD \vee ABCD \vee AB\bar{C}D \vee \bar{A}BCD \vee \bar{A}B\bar{C}D$
       $A(BC \vee B\bar{C} \vee \bar{B}\bar{C} \vee \bar{B}C)D \vee ABCD \vee AB\bar{C}D \vee \bar{A}BCD \vee \bar{A}B\bar{C}D$
       $ABCD \vee AB\bar{C}D \vee A\bar{B}\bar{C}D \vee A\bar{B}CD \vee \bar{A}BCD \vee \bar{A}B\bar{C}D$

	CD	$\boldsymbol{C\bar{D}}$	$\boldsymbol{\bar{C}\bar{D}}$	$\boldsymbol{\bar{C}D}$
$\boldsymbol{AB}$	1			1
$\boldsymbol{A\bar{B}}$	1			1
$\boldsymbol{\bar{A}\bar{B}}$				
$\boldsymbol{\bar{A}B}$	1			1

$ACD \vee A\bar{C}D \vee \bar{A}BD$

11. Translate the following sentences into the language of predicate calculus, using the indicated letters for constants, functions symbols, and predicate symbols.
    a. Some cats do not have tails. $\big(C(x), T(x)\big)$
       $\exists x\big(C(x) \wedge \neg T(x)\big)$
    b. No student in the class likes physics. $(S(x), C(x), P(x))$
       $\neg\exists x(S(x) \wedge C(x) \wedge P(x))$
    c. Oscar and Miriam are students who like mathematics. $\big(o, m, S(x), M(x)\big)$
       $\exists x\big(\ S(x) \wedge M(x) \wedge (x = o \vee x = m)\big)$
    d. Any student who likes every professor also likes himself or herself. $(S(x), P(x), L(x,y))$
       $\exists x\big(S(x) \wedge \forall y(P(y) \rightarrow L(x,y)) \wedge L(x,x)\big)$

e. In the following question use these predicate symbols:
$F(x)$: x is a faculty member
$S(x)$: x is a student
$M(x)$: x attended the meeting
$A(x, y)$: x accompanied by y
$P(x, y)$: x is a parent of y

i. All students and faculty members attended the meeting.
$\forall x\big(F(x) \vee S(x) \rightarrow M(x)\big)$

ii. No student who attended the meeting is accompanied by a faculty member.
$\neg\exists x\left(S(x) \wedge M(x) \wedge \forall y\big(F(y) \rightarrow A(y, x)\big)\right)$

iii. Some parent of a student did not attend the meeting.
$\exists x\left(S(x) \wedge \forall y\big(P(y, x) \rightarrow \neg M(x)\big)\right)$

iv. If all faculty members attended the meeting, then some student is accompanied by a faculty member.
$\forall x\left(\big(F(x) \rightarrow M(x)\big) \rightarrow \exists y(S(y) \wedge A(y, x))\right)$

12. Use the set of support strategy resolution to prove the validity of the argument:
Premise: $\{(A \rightarrow B) \wedge (C \rightarrow D), (D \wedge B \rightarrow M), \neg M\}$
Conclusion: $(\neg A \vee \neg C)$

$(A \rightarrow B) \wedge (C \rightarrow D) \equiv (\neg A \vee B) \wedge (\neg C \vee D)$
$(D \wedge B \rightarrow M) \equiv (\neg(D \wedge B) \vee M) \equiv (\neg D \vee \neg B \vee M)$
$\neg M$
$\neg(\neg A \vee \neg C) \equiv A \wedge C$

1. $\neg A \vee B$	
2. $\neg C \vee D$	
3. $\neg D \vee \neg B \vee M$	
4. $\neg M$	
5. A	
6. C	
7. $\neg D \vee \neg B$	Resolve: M $(3, 4)$
8. $\neg C \vee \neg B$	Resolve: D $(7, 2)$
9. $\neg B$	Resolve: C $(8, 6)$
10. $\neg A$	Resolve: B $(9, 1)$
11. $\emptyset$	Resolve: A $(10, 5)$

Contradiction reached, argument is valid.