UNIVERSITY OF WESTERN ONTARIO

Computer Science 2209b, Winter 2013-2014

Applied Logic for Computer Science

ASSIGNMENT 5

Given: Monday, March 31st, Due: Monday, April 7, 5:30pm

(1). Use **formal deduction** to prove the validity of the following argument:

Premise 1: $\forall x(P(x) \to Q(x))$
Premise 2: $\exists x(R(x) \wedge \neg Q(x))$
Premise 3: $\forall x(R(x) \to P(x) \vee S(x))$
Conclusion: $\exists x(R(x) \wedge S(x))$

(2). Use **resolution for propositional calculus with the set of support strategy** to show that the following argument is valid:

Premise 1: $\neg R \to S$
Premise 2: $R \to P$
Premise 3: $P \vee S \to Q$
Conclusion: Q

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(3). Consider that by translating an argument into the language of propositional calculus, and by adding the negation of the conclusion to the set of premises we obtained the set $\mathcal{S}$ of clauses:

$$\begin{array}{llll} \{Q,\neg S\} & \{\neg Q,T\} & \{\neg Q,P\} & \{Q,R,T\} \\ \{\neg Q,\neg S\} & \{\neg R,\neg P\} & \{\neg R,S,P\} & \{Q,S,\neg T\} \\ \{S,\neg T,\neg P\} & & & \end{array}$$

Apply the **Davis-Putnam procedure** to find out whether or not the original argument was valid, i.e. whether or not the set $\mathcal{S}$ is satisfiable. Show in detail all the intermediary steps. In particular, for each elimination of a variable, show which are the sets S_i, S_i', T_i and U_i. For each resolvent indicate what the parent clauses are. Eliminate the variables in the order Q, R, S, T, P.

(4). Prove the validity of the following argument by using **resolution for predicate calculus**. Use the method described in class.

Premise 1: $\exists x[P(x) \land \forall y(Q(y) \to R(x,y))]$
Premise 2: $\forall x \forall y(P(x) \land R(x,y) \to S(y))$
Premise 3: $\forall x(S(x) \to Q(x))$
Conclusion: $\forall x(Q(x) \leftrightarrow S(x))$