Western University
Computer Science Department

CS 2209A: Applied Logic for Computer Science
Final 2012
Midterm
2:00pm - 5:00pm, Saturday, December 15, 2012

1. Translate each of the following sentences into the language of propositional calculus using the indicated notations for atomic formulas.

   (a) Harry is short, Bill is tall and Mary is not blond. (S, T, B)

   (b) Mary is sick (S) and it is raining (R) implies that Bob stayed (T) up late last night. (S, R, T)

   (c) It is not the case that it is raining, and John will not take an umbrella. (R, T)

   (d) I will explore (E) canyons during spring break only if I get Grand Canyon reservations (R) and get a group (G) together. (E, R, G)

   (e) You get mashed potatoes or french fries but not both. (M, F)

   (f) Oscar will not pass the logic course unless he does his homework and studies. (P, H, S)

   (g) If logic is difficult, Oscar and Virginia will pass only if they study. (D, O, V, S)

   (h) If I bring (B) my digital camera, then if my batteries dont die (D) then Ill take (T) pictures of my backpack trip and put (P) the pictures on my Web site. (B, D, T, P)

   (i) If Q is quadrilateral (Q), then Q is a parallelogram (P) if and only if its opposite sides are both equal (E) and parallel (L). (Q, P, E, L)

2. Use the laws of propositional calculus to decide whether the following sentences are logically equivalent. Use atomic propositions C, B and W.

   (a) *If you are not a crustacean then you have bones or you would wriggle along the ground.*

   (b) *If you are not a crustacean and you do not have bones then you would wriggle along the ground.*

3. Use the truth table method to decide whether the conclusion D is a tautological consequence of the premises $A \leftrightarrow B$, $B \rightarrow C$, $\neg C \rightarrow \neg D$, $\neg A \rightarrow D$. You can only consider the value assignments that make all the premises true.

4. Give a truth table for each of the following formulas:

   (a) $A \rightarrow (B \rightarrow \neg A)$

   (b) $A \vee \neg B \leftrightarrow C \rightarrow \neg A$

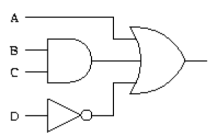
5. Using the truth tables, find the DNF and CNF for each of the formulas in Question 4.

6. Using the laws of propositional calculus for the formula $F_1 \to (F_2 \vee \neg F_3) \to F_4$, find its
   (a) Disjunctive normal form.
   (b) Conjunctive normal form.

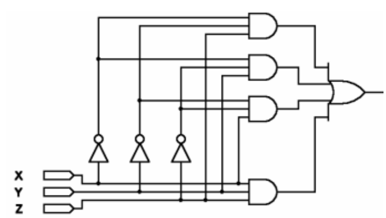
7. For each of the following formulas, find an equivalent formula that only contains connectives $\{\neg, \to\}$
   (a) $a \vee b$
   (b) $a \wedge b$
   (c) $a \leftrightarrow b$

8. Use formal deduction to prove the following statements. Identify the laws that you are using.
   (a) $\{A \to (B \vee C),\ A \wedge \neg B\} \vdash C$
   (b) $\{A \to B, C \to A, \neg C \to D, \neg D\} \vdash B$
   (c) $\{(A \to B) \wedge (C \to D), (D \wedge B \to E), \neg E\} \vdash (\neg C \vee \neg A)$

9. Provide the formula for each of the logic circuits
   (a)



   (b)



10. Simplify each of the following formulas using Karnaugh maps
    (a) $A\bar{B}CD \vee A\bar{B}C\bar{D} \vee A\bar{B}\bar{C}\bar{D} \vee A\overline{BC}D$
    (b) $AD \vee ABCD \vee AB\bar{C}D \vee \bar{A}BCD \vee \bar{A}B\bar{C}D$

11. Translate the following sentences into the language of predicate calculus, using the indicated letters for constants, function symbols, and predicate symbols.

(a) Some cats do not have tails. (C(x), T(x))
(b) No student in the class likes physics. (S(x), C(x), P(x))
(c) Oscar and Miriam are students who like mathematics. (o, m, S(x), M(x))
(d) Any student who likes every professor also likes himself or herself. (S(x), P(x), L(x,y))
(e) In the following questions use these predicate symbols:
$F(x)$: x is a faculty member
$S(x)$: x is a student
$M(x)$: x attended the meeting
$A(x, y)$: x accompanied by y
$P(x, y)$: x is a parent of y
   i. All students and faculty members attended the meeting.
   ii. No student who attended the meeting is accompanied by a faculty member.
   iii. Some parent of a student did not attend the meeting.
   iv. If all faculty members attended the meeting, then some student is accompanied by a faculty member.

12. Use the set of support strategy resolution to prove the validity of the argument:
Premises: $\{(A \to B) \wedge (C \to D), (D \wedge B \to M), \neg M\}$
Conclusion: $(\neg A \vee \neg C)$

13. Consider that by translating an argument into the language of propositional calculus, and by adding the negation of the conclusion to the set of premises we obtained the set S of clauses

$$\{P, \neg R\}, \{\neg P, S\}, \{\neg P, T\}, \{P, Q, S\}, \{\neg P, \neg R\}, \{\neg Q, \neg T\}, \{\neg Q, R, T\},$$
$$\{P, R, \neg S\}, \{R, \neg S, \neg T\}$$

Apply the Davis-Putnam procedure to find out whether or not the original argument was valid. For each elimination of a variable, show the sets S_i, S_i', T_i and U_i. Eliminate the variables in the order P, Q, R, S, T.

14. Find the prenex normal form of

$$\forall x(\exists y R(x, y) \wedge \forall y \neg S(x, y) \to \neg(\exists y R(x, y) \wedge P))$$

15. Use formal deduction for predicate calculus to prove the following. Identify the laws that you are using.

(a) $\forall x(C(x) \to A(x)), \exists x(C(x) \wedge T(x)) \vdash \exists x(A(x) \wedge T(x))$
(b) $\exists y \forall x R(x, y) \vdash \forall x \exists y R(x, y)$
(c) $\exists x(P(x) \wedge \forall y(P(y) \wedge R(x, y) \to Q(y, a))), \exists x(P(x) \wedge \neg Q(x, a)) \vdash \exists x \exists y(P(x) \wedge P(y) \wedge R(x, y))$

16. Use the set of support strategy resolution to prove the validity of the arguments. Make sure to show the $\exists$-free PNFs and clauses.

(a) Premises: $\{\forall x(C(x) \to A(x)), \exists x(C(x) \wedge T(x)\}$
Conclusion: $\exists x(A(x) \wedge T(x))$
(b) Premises: $\{\exists y \forall x R(x, y)\}$
Conclusion: $\forall x \exists y R(x, y)$