Date structures and analysis of algorithms (CS340a)
Final Examination (Three Hour)

Problem 1. (10 marks) In big O notation for worst case, what is the time complexity of:
a) mergsort.
[redacted]
c) Maximum-Cost Spanning Tree.
d) Single-Source Longest Paths in a weighted directed graph with non-positive weights.
e) All-pair shortest paths.

Problem 2. (5 marks) a). (4 marks) Prove, by using the definitions of O and Ω, the following:

$$n = O\left(n - \log_2(n)\right)$$
$$2^n - n = \Omega(2^n)$$

Problem 3. (10 marks) (Short answer. Your answer should be yes or no. If the answer is no, explain why)
a) Dijkstra algorithm for Single-Source Shortest Paths works even if there are negative weights.
b) Dijkstra method for finding Single-Source Shortest Paths can be used to find the Single-Source *Longest* Paths assuming that all the weights are non-negative.
c) Minimum-Cost Spanning Tree for a graph will not change if *all* the weights are multiplied by a constant c.
d) Graphs G_1 and G_2 have the same transitive closure iff $G_1 = G_2$.
e) If 3SAT is not in P then Graph 3-coloring is in P.

Problem 4. (15 marks) A directed graph $G = (V, E)$ is given in the figure.

a) Show an adjacency list presentation of the graph.
b) Find a DFS tree starting from vertex v using the adjacency list representation in a).
c) Show the shortest-path tree from v to all the other vertices.
d) Now assume that the same graph is undirected, show the minimum cost spanning tree of G.

Problem 5. (15 marks) Given a connected, undirected, weighted graph $G = (V, E)$, define the cost of a spanning tree to be the maximum weight among the weights associated with the edges of the spanning tree.
Design an efficient algorithm to find the spanning tree of G which maximize above defined cost. What is the complexity of your algorithm.

Problem 6. (15 marks) Let $G = (V, E)$ be a connected, undirected, weighted graph with non-negative weights. Let v be a vertex of G. Given a spanning tree T of G, denote by $c(v, w)$ the summation of the weights along the simple path from v to w on T ($c(v, v)$=0)). We then define the cost of spanning tree T by

$$cost(T) = \max_{w \in V} c(v, w).$$

Design an efficient algorithm to find the spanning tree of G which minimize this cost.

[redacted]

Problem 8. (15 marks)

a) Let $G = (V, E)$ be an undirected graph such that, for any vertex v, degree $d(v)$ (number of edges incident to v) is bounded by 3. Given such a graph and an integer k we want to determine if G contains a clique of size $\geq k$. Show that this problem is in P. Dose this contradict to the fact that Clique Problem is $NP-complete$.

b) The composite numbers problem is define as follows:

Given an positive integer K, determine if there are integers $m, n > 1$ such that $K = m \cdot n$.

There is an $O(\sqrt{K})$ time algorithm solving this problem. Is this problem in P? Why?