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Selected Problems

Problem: Demonstrate that the 2D Gaussian function

$$f(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(\frac{-x^2 + y^2}{2\sigma^2}\right)$$

is a separable function.

Problem: Given a 1D Gaussian function

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-x^2}{2\sigma^2}\right)$$

compute a Gaussian kernel of size 5 for 1D convolution with $\sigma^2 = 1$.

Problem: Create a 2D Gaussian convolution kernel using the 1D kernel from the previous problem.

Problem: Convolve a normalized 1D Gaussian kernel of size 5 with $\sigma^2=1$ with the following row of pixels:

0	0	0	0	0	10	20	30	40	50

and put the result in this table:

NULL	NULL							NULL	NULL

Problem: Let's say we have 1000 image points that we think fit a 2D line in the resolution of a given problem. Also suppose we know about 30 percent of that data is corrupt. No matter what, we decide we are going to use RanSaC and iterate ten times, no more. Hence, ten times we are going to select 2 image points and fit a line, and evaluate how good that fit is with the rest of the points. What is the probability that one of these ten random choices of two points yields the right solution (i.e. the two chosen points are inliers)?

Problem: How many iterations would be needed to be sure at 90% that the best solution is correct?

Problem: Given a 5-point central difference formula:

$$f'(x)=\frac{-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h)}{12h}$$

Find the corresponding derivative kernel.

Problem: Derive the following signal at $i=6$ using the kernel obtained in the previous question:

1	2	3	4	5	6	7	8	9	10	11	12	13

Problem: Suppose we have these two 3D points $P_1=(X_{1,}Y_{1,}Z_1)=(0,0,50)$ and $P_2=(X_{2,}Y_{2,}Z_2)=(1,1,50)$ in a camera coordinate system with focal length $f=10$. What is the perspective projection of these two points onto the imaging plane of the camera?

Problem: What is the image disparity between these two points?

Problem: Suppose a 3D point $P_1=(10,10,100)$ in camera coordinates with $f=10$ as before. Let this point translate by $\vec{T}=(1.0,0.0,0.0)^T$ and rotate by $\vec{\omega}=(0.0,0.0,0.01)^T$. Compute the 3D velocity of this point.

Problem: Prove that:

$$f\frac{P}{Z}\frac{d}{dt}=\begin{bmatrix}\frac{T_z x-f T_x}{Z} & - & f\omega_y & + & \omega_z y & + & \frac{\omega_x x y}{f} & - & \frac{\omega_y x^2}{f} \\ \frac{T_z y-f T_y}{Z} & + & f\omega_x & - & \omega_z x & - & \frac{\omega_y x y}{f} & + & \frac{\omega_x y^2}{f}\end{bmatrix}=\begin{bmatrix}v_x \\ v_y\end{bmatrix}$$

Problem: At an image point, and given the following values for image derivatives $I_x = 10,\ I_y = 5,\ I_t = 1$, compute the normal image velocity.

Problem: Give the vector for normal velocity.

Problem: Suppose we have spatiotemporal derivatives I_{x1}, I_{y1}, I_{t1} and I_{x2}, I_{y2}, I_{t2} for two adjacent image points, all non-zero. Form a system of equations that yields the full velocity of this image neighborhood.

Problem: In the development of the simple Fourier transform of 1D signal $f(x)=A\cos(k_0 x)$, indicate which properties are applied (in the order that they are) to obtain the Fourier spectrum.

Problem: What happens to the 2D power spectrum of a 2D signal as it starts translating?

Problem: Suppose we have a stereo system that is calibrated and rectified with both focal length $f=1$ and baseline $T=10$. A stereo disparity is found as $(x_l, y_l) = (0.2, 0.2)$ and $(x_r, y_r) = (0.0, 0.2)$. What are the coordinates of the 3D point giving rise to this detected image disparity?

Problem: Given two points $p_1=(x_1, y_1)=(1,3)$ and $p_2=(x_2, y_2)=(3,5)$ that are on a 2D line, use the Hough transform to obtain the equation of this line.