Monte_Carlo

January 18, 2019

1 Mini Project: Monte Carlo Methods

In this notebook, you will write your own implementations of many Monte Carlo (MC) algorithms.

While we have provided some starter code, you are welcome to erase these hints and write your code from scratch.

1.0.1 Part 0: Explore BlackjackEnv

Use the code cell below to create an instance of the Blackjack environment.

```
In [1]: import gym
        env = gym.make('Blackjack-v0')
```

Each state is a 3-tuple of: - the player's current sum $\in \{0, 1, \ldots, 31\}$, - the dealer's face up card $\in \{1, \ldots, 10\}$, and - whether or not the player has a usable ace (no $= 0$, yes $= 1$).

The agent has two potential actions:

```
STICK = 0
HIT = 1
```

Verify this by running the code cell below.

```
In [2]: print(env.observation_space)
        print(env.action_space)
```

```
Tuple(Discrete(32), Discrete(11), Discrete(2))
Discrete(2)
```

Execute the code cell below to play Blackjack with a random policy.

(*The code currently plays Blackjack three times - feel free to change this number, or to run the cell multiple times. The cell is designed for you to get some experience with the output that is returned as the agent interacts with the environment.*)

```
In [3]: for i_episode in range(3):
            state = env.reset()
            print('initial state =', state)
            while True:
```

```
                action = env.action_space.sample()
                print('action =',action)
                state, reward, done, info = env.step(action)
                print('state =', state)
                if done:
                    print('End game! Reward: ', reward)
                    print('You won :)\n') if reward > 0 else print('You lost :(\n')
                    break
```

```
initial state = (16, 10, False)
action = 0
state = (16, 10, False)
End game! Reward:  -1.0
You lost :(

initial state = (14, 5, True)
action = 1
state = (12, 5, False)
action = 1
state = (20, 5, False)
action = 0
state = (20, 5, False)
End game! Reward:  1.0
You won :)

initial state = (20, 10, True)
action = 1
state = (15, 10, False)
action = 1
state = (18, 10, False)
action = 1
state = (20, 10, False)
action = 1
state = (28, 10, False)
End game! Reward:  -1
You lost :(
```

1.0.2 Part 1: MC Prediction: State Values

In this section, you will write your own implementation of MC prediction (for estimating the state-value function).

We will begin by investigating a policy where the player always sticks if the sum of her cards exceeds 18. The function `generate_episode_from_limit` samples an episode using this policy.

The function accepts as **input**: - `bj_env`: This is an instance of OpenAI Gym's Blackjack environment.

It returns as **output**: - `episode`: This is a list of (state, action, reward) tuples (of tuples) and corresponds to $(S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T)$, where T is the final time step. In particular, `episode[i]` returns (S_i, A_i, R_{i+1}), and `episode[i][0]`, `episode[i][1]`, and `episode[i][2]` return S_i, A_i, and R_{i+1}, respectively.

```
In [4]: def generate_episode_from_limit(bj_env):
            episode = []
            state = bj_env.reset()
            while True:
                action = 0 if state[0] > 18 else 1
                next_state, reward, done, info = bj_env.step(action)
                episode.append((state, action, reward))
                state = next_state
                if done:
                    break
            return episode
```

Execute the code cell below to play Blackjack with the policy.

(*The code currently plays Blackjack three times - feel free to change this number, or to run the cell multiple times. The cell is designed for you to gain some familiarity with the output of the* `generate_episode_from_limit` *function.*)

```
In [5]: for i in range(3):
            print(generate_episode_from_limit(env))

[((5, 8, False), 1, 0), ((8, 8, False), 1, 0), ((16, 8, False), 1, -1)]
[((20, 1, False), 0, 1.0)]
[((20, 7, False), 0, 1.0)]
```

Now, you are ready to write your own implementation of MC prediction. Feel free to implement either first-visit or every-visit MC prediction; in the case of the Blackjack environment, the techniques are equivalent.

Your algorithm has three arguments: - `env`: This is an instance of an OpenAI Gym environment. - `num_episodes`: This is the number of episodes that are generated through agent-environment interaction. - `generate_episode`: This is a function that returns an episode of interaction. - `gamma`: This is the discount rate. It must be a value between 0 and 1, inclusive (default value: `1`).

The algorithm returns as output: - `V`: This is a dictionary where `V[s]` is the estimated value of state `s`. For example, if your code returns the following output:

```
{(4, 7, False): -0.38775510204081631, (18, 6, False): -0.58434296365330851, (13, 2, False): -0.4
```

then the value of state `(4, 7, False)` was estimated to be `-0.38775510204081631`.

If you are unfamiliar with how to use `defaultdict` in Python, you are encouraged to check out this source.

```
In [6]: from collections import defaultdict
        import numpy as np
        import sys
```

```
def mc_prediction_v(env, num_episodes, generate_episode, gamma=1.0):
    # initialize empty dictionary of lists
    returns = defaultdict(list)
    # loop over episodes
    for i_episode in range(1, num_episodes+1):
        # monitor progress
        if i_episode % 1000 == 0:
            print("\rEpisode {}/{}.".format(i_episode, num_episodes), end="")
            sys.stdout.flush()

        ## TODO: complete the function
        episode = generate_episode(env)
        states = []
        #actions = []
        rewards = []

        n = len(episode)
        for t in range(n):
            states.append(episode[t][0])
            #actions.append(episode[t][1])
            rewards.append(episode[t][2])

        # prepare for discounting
        discounts = np.array([gamma**i for i in range(len(rewards)+1)])
        # calculate and store the return for each visit in the episode
        for i, state in enumerate(states):
            returns[state].append(sum(rewards[i:]*discounts[0:n-i]))
    # calculate the state-value function estimate
    V = {key: np.mean(value) for key, value in returns.items()}

    return V
```

Use the cell below to calculate and plot the state-value function estimate. (*The code for plotting the value function has been borrowed from this source and slightly adapted.*)

To check the accuracy of your implementation, compare the plot below to the corresponding plot in the solutions notebook **Monte_Carlo_Solution.ipynb**.

```
In [7]: from plot_utils import plot_blackjack_values

        # obtain the value function
        V = mc_prediction_v(env, 500000, generate_episode_from_limit)

        # plot the value function
        plot_blackjack_values(V);
```

```
Episode 500000/500000.
```

```
<matplotlib.figure.Figure at 0x7f02270643c8>
```

1.0.3 Part 2: MC Prediction: Action Values

In this section, you will write your own implementation of MC prediction (for estimating the action-value function).

We will begin by investigating a policy where the player *almost* always sticks if the sum of her cards exceeds 18. In particular, she selects action `STICK` with 80% probability if the sum is greater than 18; and, if the sum is 18 or below, she selects action `HIT` with 80% probability. The function `generate_episode_from_limit_stochastic` samples an episode using this policy.

The function accepts as **input**: - `bj_env`: This is an instance of OpenAI Gym's Blackjack environment.

It returns as **output**: - `episode`: This is a list of (state, action, reward) tuples (of tuples) and corresponds to $(S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T)$, where T is the final time step. In particular, `episode[i]` returns (S_i, A_i, R_{i+1}), and `episode[i][0]`, `episode[i][1]`, and `episode[i][2]` return S_i, A_i, and R_{i+1}, respectively.

```
In [8]: def generate_episode_from_limit_stochastic(bj_env):
            episode = []
            state = bj_env.reset()
            while True:
                probs = [0.8, 0.2] if state[0] > 18 else [0.2, 0.8]
                action = np.random.choice(np.arange(2), p=probs)
                next_state, reward, done, info = bj_env.step(action)
                episode.append((state, action, reward))
                state = next_state
                if done:
                    break
            return episode
```

Now, you are ready to write your own implementation of MC prediction. Feel free to implement either first-visit or every-visit MC prediction; in the case of the Blackjack environment, the techniques are equivalent.

Your algorithm has three arguments: - `env`: This is an instance of an OpenAI Gym environment. - `num_episodes`: This is the number of episodes that are generated through agent-environment interaction. - `generate_episode`: This is a function that returns an episode of interaction. - `gamma`: This is the discount rate. It must be a value between 0 and 1, inclusive (default value: `1`).

The algorithm returns as output: - `Q`: This is a dictionary (of one-dimensional arrays) where `Q[s][a]` is the estimated action value corresponding to state `s` and action `a`.

```
In [9]: def mc_prediction_q(env, num_episodes, generate_episode, gamma=1.0):
            # initialize empty dictionaries of arrays
            returns_sum = defaultdict(lambda: np.zeros(env.action_space.n))
            N = defaultdict(lambda: np.zeros(env.action_space.n))
            Q = defaultdict(lambda: np.zeros(env.action_space.n))
            # loop over episodes
            for i_episode in range(1, num_episodes+1):
                # monitor progress
                if i_episode % 1000 == 0:
                    print("\rEpisode {}/{}.".format(i_episode, num_episodes), end="")
```

```
                sys.stdout.flush()

            ## TODO: complete the function
            episode = generate_episode(env)
            states = []
            actions = []
            rewards = []

            n = len(episode)
            for t in range(n):
                states.append(episode[t][0])
                actions.append(episode[t][1])
                rewards.append(episode[t][2])

            # prepare for discounting
            discounts = np.array([gamma**i for i in range(len(rewards)+1)])
            # calculate the action-value function estimate
            for i, state in enumerate(states):
                action = actions[i] # corresponding action taken at this state
                returns_sum[state][action] += sum(rewards[i:]*discounts[0:n-i])
                N[state][action] += 1
                Q[state][action] = returns_sum[state][action] / N[state][action]

        return Q
```

Use the cell below to obtain the action-value function estimate Q. We have also plotted the corresponding state-value function.

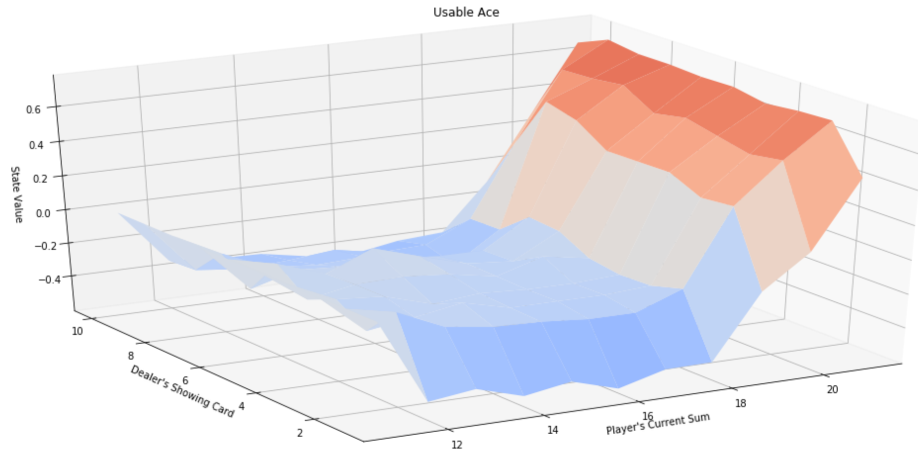
To check the accuracy of your implementation, compare the plot below to the corresponding plot in the solutions notebook **Monte_Carlo_Solution.ipynb**.

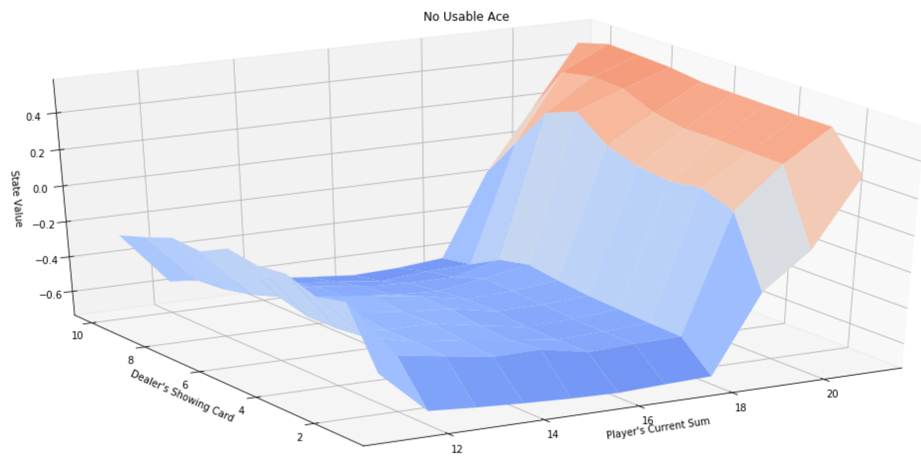
```
In [10]: # obtain the action-value function
         Q = mc_prediction_q(env, 500000, generate_episode_from_limit_stochastic)

         # obtain the state-value function
         V_to_plot = dict((k,(k[0]>18)*(np.dot([0.8, 0.2],v)) + (k[0]<=18)*(np.dot([0.2, 0.8],v)
                  for k, v in Q.items())

         # plot the state-value function
         plot_blackjack_values(V_to_plot)
```

```
Episode 500000/500000.
```





1.0.4 Part 3: MC Control: GLIE

In this section, you will write your own implementation of constant-α MC control.

Your algorithm has three arguments: - `env`: This is an instance of an OpenAI Gym environment. - `num_episodes`: This is the number of episodes that are generated through agent-environment interaction. - `gamma`: This is the discount rate. It must be a value between 0 and 1, inclusive (default value: `1`).

The algorithm returns as output: - `Q`: This is a dictionary (of one-dimensional arrays) where `Q[s][a]` is the estimated action value corresponding to state `s` and action `a`. - `policy`: This is a dictionary where `policy[s]` returns the action that the agent chooses after observing state `s`.

(*Feel free to define additional functions to help you to organize your code.*)

```
In [27]: def generate_episode_eps_greedy(bj_env, Q, eps, n_actions):
             episode = []
```

```
        state = bj_env.reset()
        while True:
            probs = np.ones(n_actions)*eps/n_actions
            p1 = 1 - eps + eps/n_actions
            action = np.random.choice(np.arange(n_actions), p=get_probs(Q[state], eps, n_ac
                                    if state in Q else env.action_space.sample()
            next_state, reward, done, info = bj_env.step(action)
            episode.append((state, action, reward))
            state = next_state
            if done:
                break
        return episode

    def get_probs(Q_s, eps, n_actions):
        """ obtains the action probabilities corresponding to epsilon-greedy policy """
        policy_s = np.ones(n_actions) * eps / n_actions
        best_a = np.argmax(Q_s)
        policy_s[best_a] = 1 - eps + (eps / n_actions)
        return policy_s
```

In [32]:
```
def mc_control_GLIE(env, num_episodes, gamma=1.0):
    nA = env.action_space.n
    # initialize empty dictionaries of arrays
    Q = defaultdict(lambda: np.zeros(nA))
    N = defaultdict(lambda: np.zeros(nA))
    # loop over episodes
    for i_episode in range(1, num_episodes+1):
        # monitor progress
        if i_episode % 1000 == 0:
            print("\rEpisode {}/{}.".format(i_episode, num_episodes), end="")
            sys.stdout.flush()

        ## TODO: complete the function
        #eps = 1./i_episode
        eps = 1.0/((i_episode/8000)+1)
        n_actions = 2

        # generate episode with epsilon-greedy policy
        episode = generate_episode_eps_greedy(env, Q, eps, n_actions)

        states = []
        actions = []
        rewards = []
        n = len(episode)
        for t in range(n):
            states.append(episode[t][0])
            actions.append(episode[t][1])
            rewards.append(episode[t][2])
```

```
            # prepare for discounting
            discounts = np.array([gamma**i for i in range(len(rewards)+1)])
            # calculate the action-value function estimate
            for i, state in enumerate(states):
                action = actions[i] # corresponding action taken at this state
                old_Q = Q[state][action]
                old_N = N[state][action]
                N[state][action] += 1
                Q[state][action] = old_Q + 1/N[state][action]*(sum(rewards[i:]*discounts[0:

        # determine the policy corresponding to the final action-value function estimate
        policy = dict((key, np.argmax(value)) for key, value in Q.items())

        return policy, Q
```

Use the cell below to obtain the estimated optimal policy and action-value function.

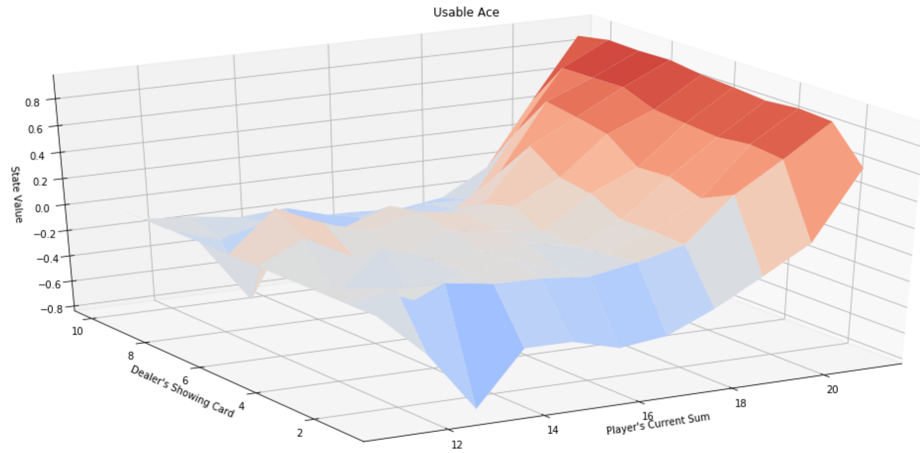
```
In [33]: # obtain the estimated optimal policy and action-value function
         policy_glie, Q_glie = mc_control_GLIE(env, 500000)
```

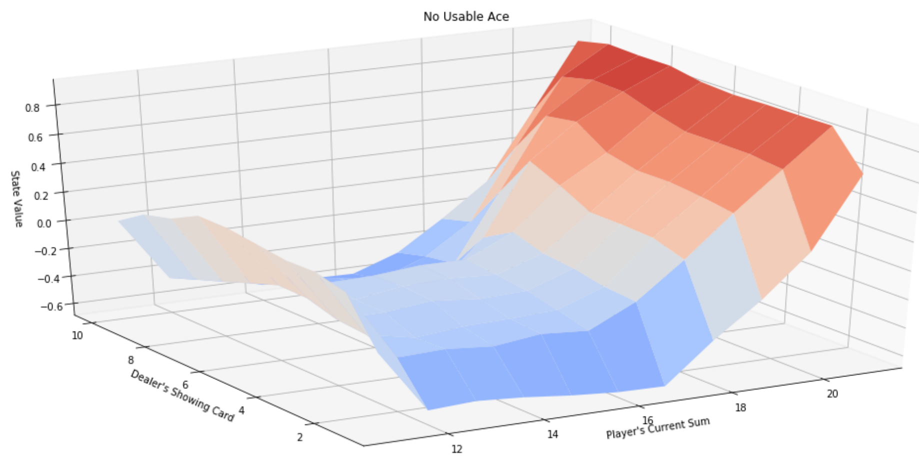
```
Episode 500000/500000.
```

Next, we plot the corresponding state-value function.

```
In [34]: # obtain the state-value function
         V_glie = dict((k,np.max(v)) for k, v in Q_glie.items())

         # plot the state-value function
         plot_blackjack_values(V_glie)
```

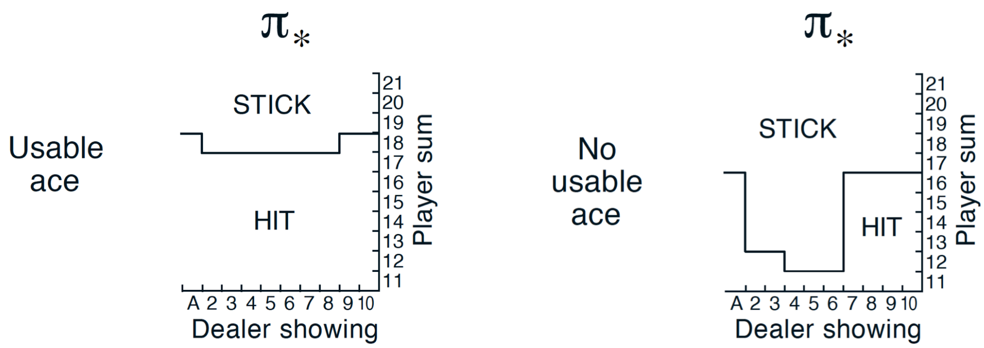




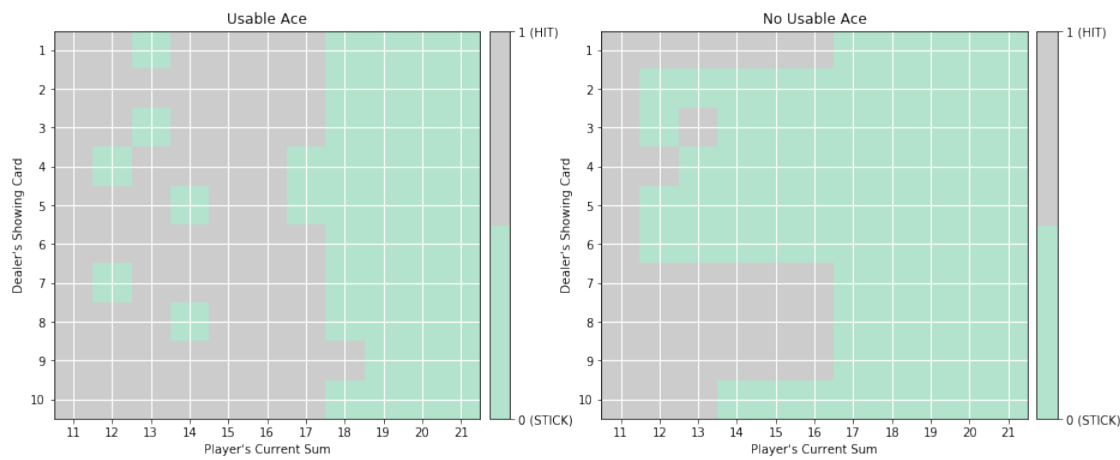
Finally, we visualize the policy that is estimated to be optimal.

In [35]:
```
from plot_utils import plot_policy

# plot the policy
plot_policy(policy_glie)
```



True Optimal Policy



The **true** optimal policy π_* can be found on page 82 of the textbook (and appears below). Compare your final estimate to the optimal policy - how close are you able to get? If you are not happy with the performance of your algorithm, take the time to tweak the decay rate of ϵ and/or run the algorithm for more episodes to attain better results.

1.0.5 Part 4: MC Control: Constant-α

In this section, you will write your own implementation of constant-α MC control.

Your algorithm has four arguments: - `env`: This is an instance of an OpenAI Gym environment. - `num_episodes`: This is the number of episodes that are generated through agent-environment interaction. - `alpha`: This is the step-size parameter for the update step. - `gamma`: This is the discount rate. It must be a value between 0 and 1, inclusive (default value: `1`).

The algorithm returns as output: - `Q`: This is a dictionary (of one-dimensional arrays) where `Q[s][a]` is the estimated action value corresponding to state `s` and action `a`. - `policy`: This is a dictionary where `policy[s]` returns the action that the agent chooses after observing state `s`.

(*Feel free to define additional functions to help you to organize your code.*)

```
In [36]: def mc_control_alpha(env, num_episodes, alpha, gamma=1.0):
             nA = env.action_space.n
```

```
    # initialize empty dictionary of arrays
    Q = defaultdict(lambda: np.zeros(nA))
    # loop over episodes
    for i_episode in range(1, num_episodes+1):
        # monitor progress
        if i_episode % 1000 == 0:
            print("\rEpisode {}/{}.".format(i_episode, num_episodes), end="")
            sys.stdout.flush()

        ## TODO: complete the function
        #eps = 1./i_episode
        eps = 1.0/((i_episode/8000)+1)
        n_actions = 2

        # generate episode with epsilon-greedy policy
        episode = generate_episode_eps_greedy(env, Q, eps, n_actions)

        states = []
        actions = []
        rewards = []
        n = len(episode)
        for t in range(n):
            states.append(episode[t][0])
            actions.append(episode[t][1])
            rewards.append(episode[t][2])

        # prepare for discounting
        discounts = np.array([gamma**i for i in range(len(rewards)+1)])
        # calculate the action-value function estimate
        for i, state in enumerate(states):
            action = actions[i] # corresponding action taken at this state
            old_Q = Q[state][action]
            Q[state][action] = old_Q + alpha*(sum(rewards[i:]*discounts[0:n-i]) - old_Q

    # determine the policy corresponding to the final action-value function estimate
    policy = dict((key, np.argmax(value)) for key, value in Q.items())

    return policy, Q
```

Use the cell below to obtain the estimated optimal policy and action-value function.

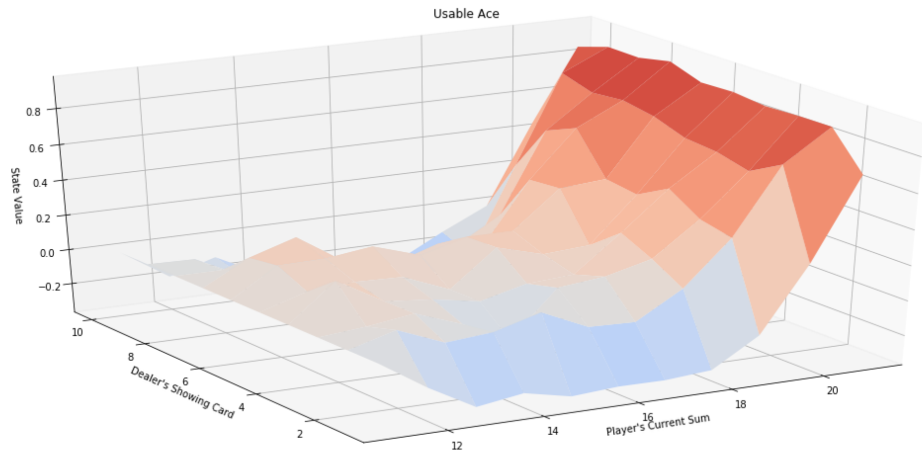
```
In [37]: # obtain the estimated optimal policy and action-value function
         policy_alpha, Q_alpha = mc_control_alpha(env, 500000, 0.008)
```

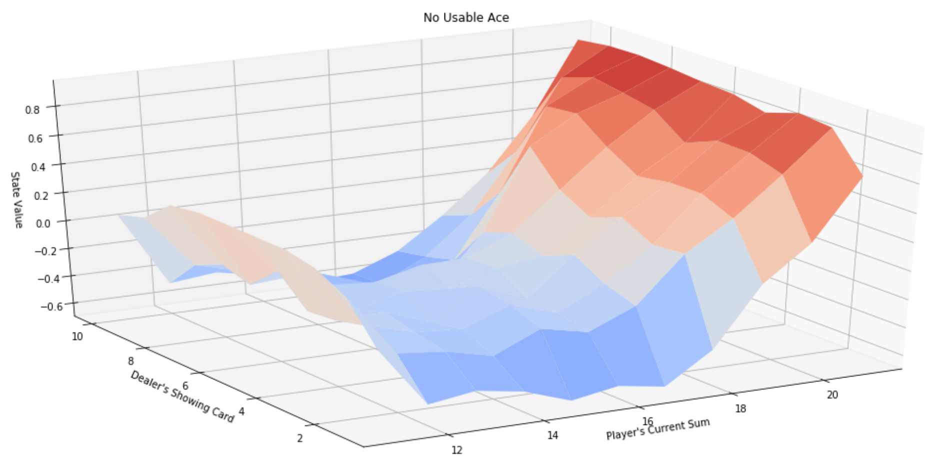
```
Episode 500000/500000.
```

Next, we plot the corresponding state-value function.

```
In [38]: # obtain the state-value function
         V_alpha = dict((k,np.max(v)) for k, v in Q_alpha.items())
```

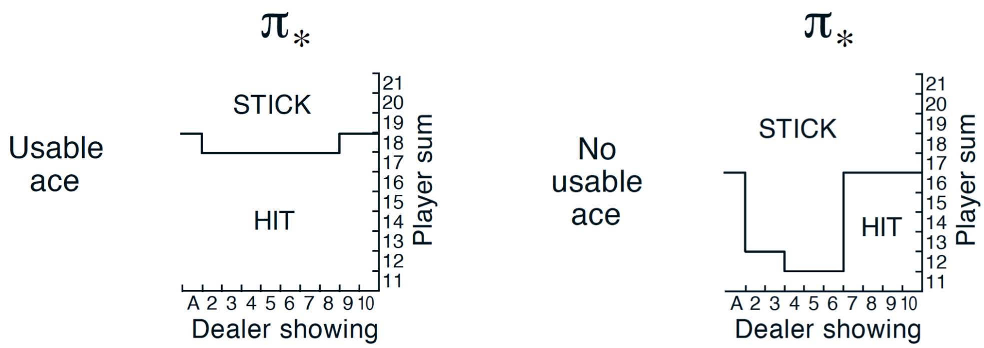
```
# plot the state-value function
plot_blackjack_values(V_alpha)
```



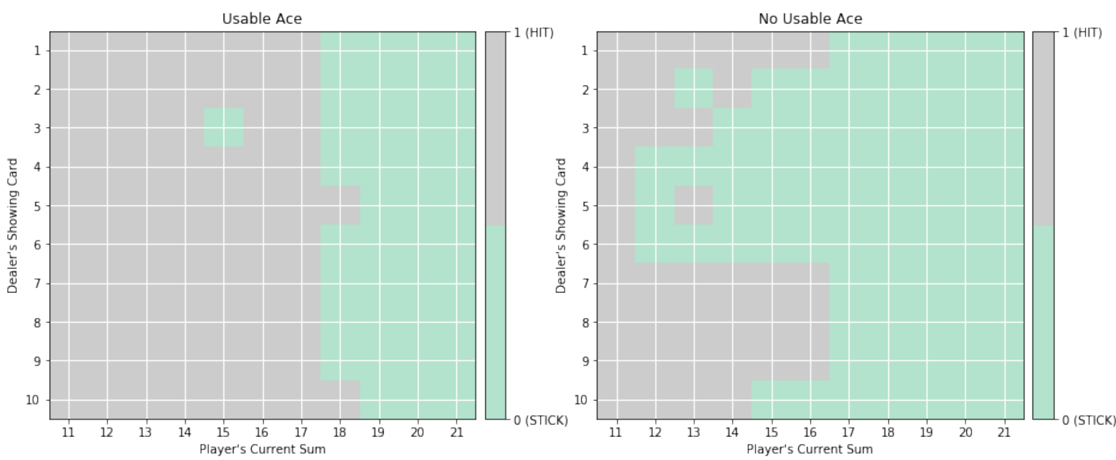


Finally, we visualize the policy that is estimated to be optimal.

```
In [39]: # plot the policy
         plot_policy(policy_alpha)
```



True Optimal Policy



The **true** optimal policy π_* can be found on page 82 of the textbook (and appears below). Compare your final estimate to the optimal policy - how close are you able to get? If you are not happy with the performance of your algorithm, take the time to tweak the decay rate of ϵ, change the value of α, and/or run the algorithm for more episodes to attain better results.