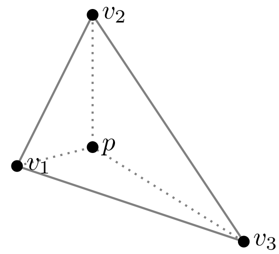
1 Barycentric Coordinates



$$U = (u_1, u_2, u_3) \tag{1}$$
$$v_1 = (x_1, y_1) \tag{2}$$
$$v_2 = (x_2, y_2) \tag{3}$$
$$v_3 = (x_3, y_3) \tag{4}$$
$$p = (x_p, y_p) \tag{5}$$

The Barycentric coordinates can be defined in terms of the following relationships:

$$\begin{cases} u_1 + u_2 + u_3 = 1 \\ u_1x_1 + u_2x_2 + u_3x_3 = x_p \\ u_1y_1 + u_2y_2 + u_3y_3 = y_p \end{cases} \tag{6}$$

Let's reduce the amount of varibles in these equations:

$$u_3 = 1 - u_1 - u_2 \tag{7}$$
$$\begin{cases} u_1(x_1 - x_3) + u_2(x_2 - x_3) & = x_p - x_3 \\ u_1(y_1 - y_3) + u_2(y_2 - y_3) & = y_p - y_3 \end{cases} \tag{8}$$

Now we can turn the system of equations into matrix form:

$$T = \begin{bmatrix} x_1 - x_3 & x_2 - x_3 \\ y_1 - y_3 & y_2 - y_3 \end{bmatrix} \tag{9}$$
$$U = \begin{bmatrix} u1 \\ u2 \end{bmatrix} \tag{10}$$
$$R = \begin{bmatrix} x_p - x_3 \\ y_p - y_3 \end{bmatrix} \tag{11}$$
$$D = 1 \tag{12}$$
$$T \cdot U = R \cdot D \tag{13}$$

So the solution is

$$U = T^{-1} \cdot R \tag{14}$$

So the main effort goes towards finding T^{-1}

$$T^{-1} = \frac{adj(T)}{det(T)} \tag{15}$$

$$det(T) = (x_1 - x_3)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_3) \tag{16}$$

$$adj(T) = \begin{bmatrix} y_2 - y_3 & x_3 - x_2 \\ y_3 - y_1 & x_1 - x_3 \end{bmatrix} \tag{17}$$

$$T^{-1} = \frac{D}{det(T)} \cdot \begin{bmatrix} y_2 - y_3 & x_3 - x_2 \\ y_3 - y_1 & x_1 - x_3 \end{bmatrix} \tag{18}$$

$$T^{-1} \cdot R = \frac{D}{det(T)} \cdot \begin{bmatrix} (y_2 - y_3)(x_p - x_3) + (x_3 - x_2)(y_p - y_3) \\ (y_3 - y_1)(x_p - x_3) + (x_1 - x_3)(y_p - y_3) \end{bmatrix} \tag{19}$$

So, the final formula you need to find (u_1, u_2, u_3) given points v_1, v_2, v_3, p is

$$u_1 = \frac{(y_2 - y_3)(x_p - x_3) + (x_3 - x_2)(y_p - y_3)}{(x_1 - x_3)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_3)} \tag{20}$$

$$u_2 = \frac{(y_3 - y_1)(x_p - x_3) + (x_1 - x_3)(y_p - y_3)}{(x_1 - x_3)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_3)} \tag{21}$$

$$u_3 = 1 - u_2 - u_1 \tag{22}$$