Credit Risk Modelling and Simulation

1. Introduction:

The risk of monetary loss due to the default, or a change in the perceived likelihood of default, of a counterparty to a contract is called credit risk. Unlike shares, bonds accompany a risk of default. Hence modelling this follows different methods. It is classified in Non-Business Risk (Regulatory Capital).

The objective of this assignment is to model a credit risky portfolio of corporate bonds. The structural model is used to infer a counterparty's (CP) one year in future credit state from a continuous random variable known as creditworthiness index (W). The mathematical form of the creditworthiness index of counterparty j is given as:

- Creditworthiness index of counterparty j :

sensitivity of CP j to the credit driver

$$\sigma_j = \sqrt{1-\beta_j^2}$$

$$w_j = \beta_j y_{j(k)} + \sigma_j z_j$$

systemic risk credit driver factor correlated credit drivers

$$\mathcal{N}(0,1)$$

idiosyncratic component independent across CPs j

2. Model Formulation:

Since the true distribution of losses is not known, we simulate standard normal random variables for systemic and idiosyncratic components many times and assume it as the true distribution. In this assignment we run the simulation 100,000 times for generating true distribution. Following that, to check if a smaller sample could represent the true distribution, we consider generating samples of 5000 in two methods as follows:

1. 1000 systemic scenarios for 5 idiosyncratic scenarios (1000x5=5000) termed as in sample MC1.
2. 5000 systemic scenarios for 1 idiosyncratic scenarios (5000x1=5000) termed as in sample MC2.

Model error and sampling errors are calculated to evaluate the simulations generated and the assumptions made on distribution as such for two portfolios as follows

1. Initial portfolio whose weights can be calculated by individual counterparty values.
2. Equally weighted portfolio, calculated by distributing the weights equally among available parties.

3. Model Implementation:

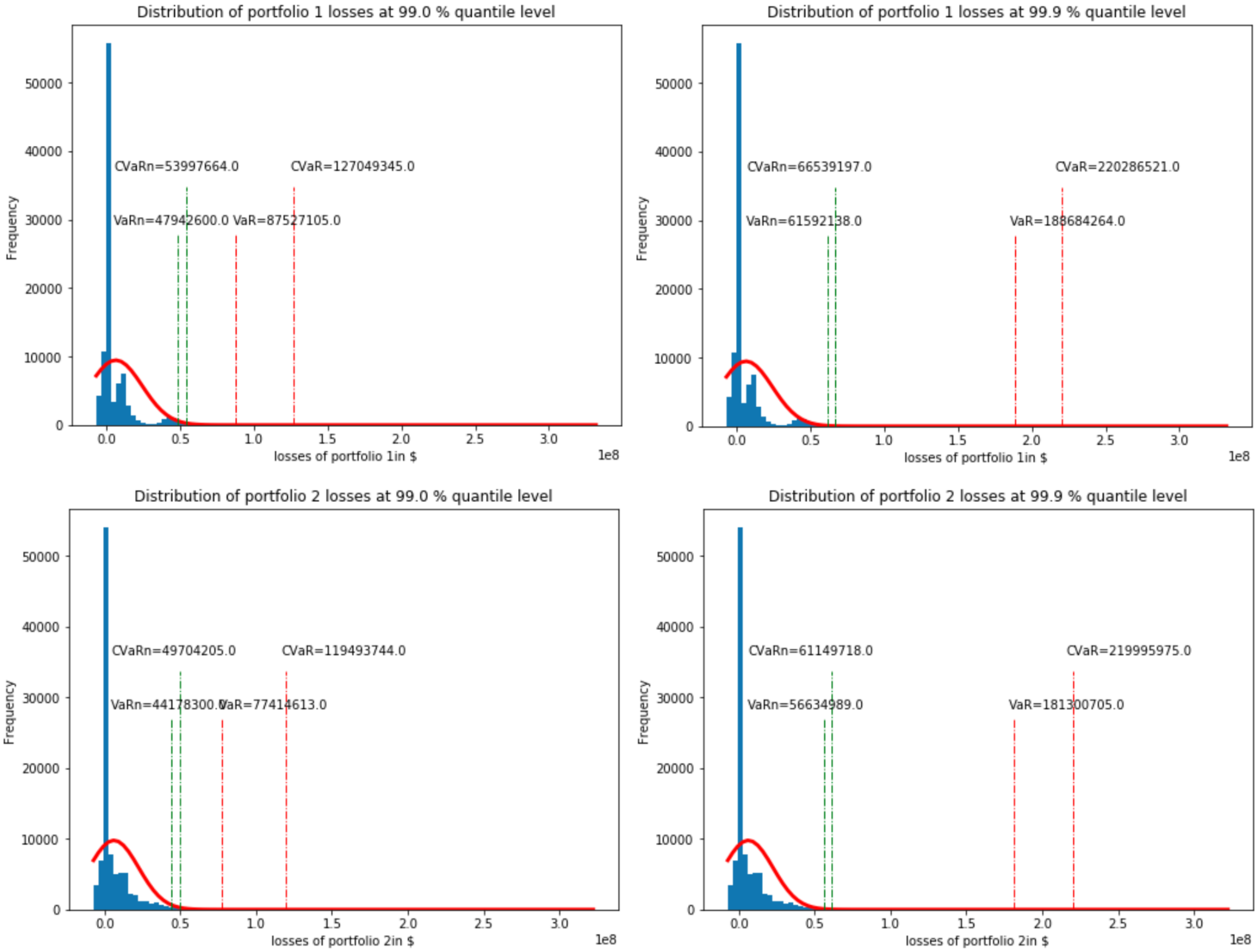
1. Generate random variables from assumed distribution for both systemic and idiosyncratic scenarios.
2. Derive correlated random variable for systemic part from the Cholesky decomposition provided
3. Calculate credit worthiness index.
4. Based on the arrived index, losses are computed from the exposure values given.
   *For this, **np.searchsorted** function is used*
5. Calculate portfolio losses using counter party losses and positions.

6. Calculate VAR, CVAR from sorted list of losses.
7. Calculate VAR and CVAR assuming the distribution of losses to be normal.

The difference between out of sample and in sample simulations are only step 1,2 and 3 where the random variables are generated as per the model formulation and the rest follows.

In in-sample simulations, we perform 100 trails to partially average out the error due to different samples in each trial.

The ***out of sample losses distribution*** thus obtained are as follows:

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Portfolio 1:

Out-of-sample: VaR 99.0% = $87527105.24, CVaR 99.0% = $127049344.94

Portfolio 2:

Out-of-sample: VaR 99.0% = $77414612.81, CVaR 99.0% = $119493744.43
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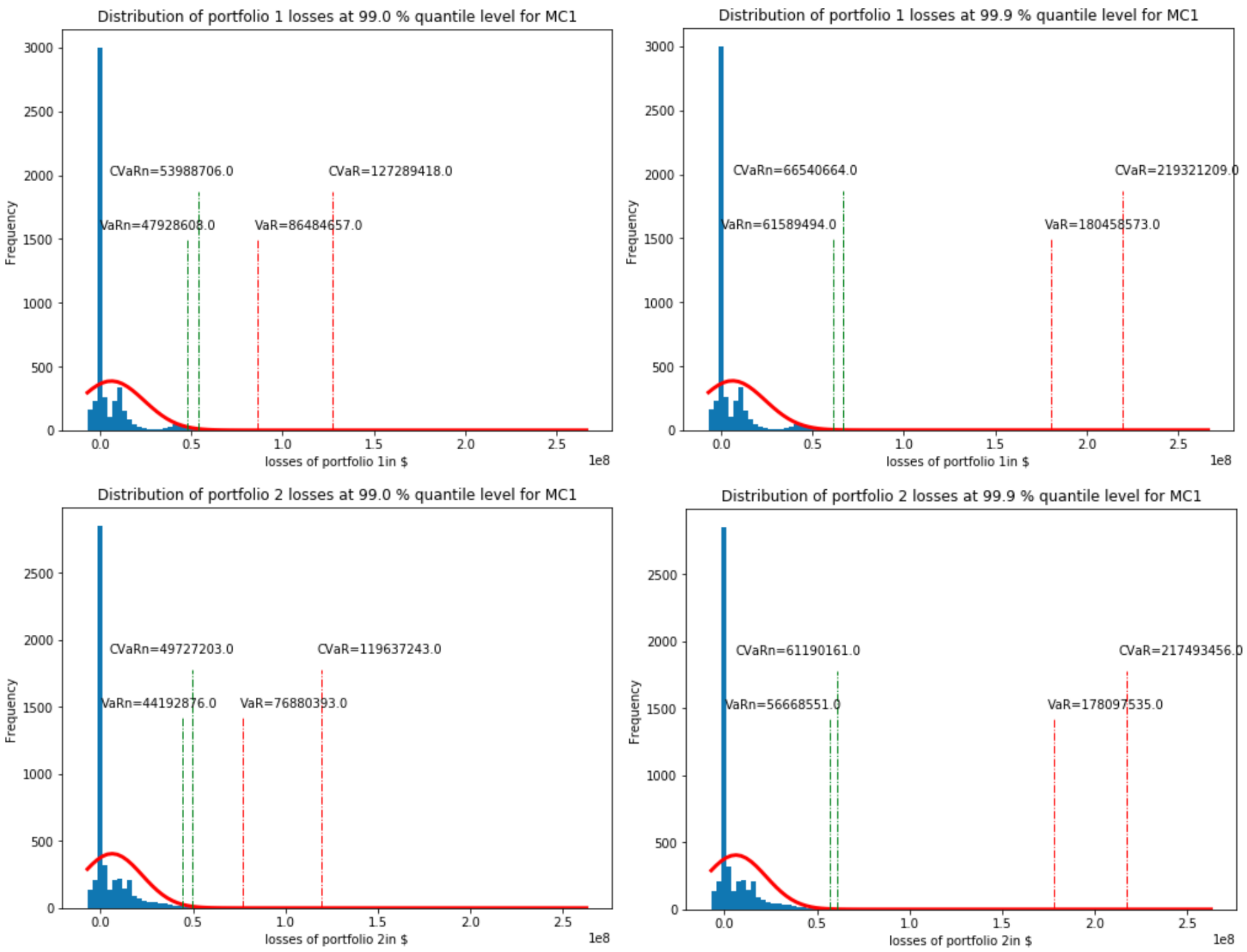
The ***In sample losses distribution for Monte Carlo simulation 1*** thus obtained are as follows:

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Portfolio 1:

Out-of-sample: VaR 99.0% = $87527105.24, CVaR 99.0% = $127049344.94
In-sample MC1: VaR 99.0% = $86484657.27, CVaR 99.0% = $127289418.36
In-sample MC2: VaR 99.0% = $88081642.62, CVaR 99.0% = $129689038.92
Out-of-sample N: VaR 99.0% = $47942600.01, CVaR 99.0% = $53997664.12
In-sample N1: VaR 99.0% = $47928608.30, CVaR 99.0% = $53988706.05
In-sample N2: VaR 99.0% = $48660137.73, CVaR 99.0% = $54813816.30

Out-of-sample: VaR 99.9% = $188684264.10, CVaR 99.9% = $220286520.58
In-sample MC1: VaR 99.9% = $180458572.54, CVaR 99.9% = $219321208.65
In-sample MC2: VaR 99.9% = $184711213.71, CVaR 99.9% = $223406836.48
Out-of-sample N: VaR 99.9% = $61592138.41, CVaR 99.9% = $66539196.65
In-sample N1: VaR 99.9% = $61589493.70, CVaR 99.9% = $66540664.49
In-sample N2: VaR 99.9% = $62531976.32, CVaR 99.9% = $67559603.72
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Portfolio 2:

Out-of-sample: VaR 99.0% = $77414612.81, CVaR 99.0% = $119493744.43
In-sample MC1: VaR 99.0% = $76880393.47, CVaR 99.0% = $119637243.34
In-sample MC2: VaR 99.0% = $78875998.77, CVaR 99.0% = $122230909.72
Out-of-sample N: VaR 99.0% = $44178300.17, CVaR 99.0% = $49704205.00
In-sample N1: VaR 99.0% = $44192876.37, CVaR 99.0% = $49727203.32
In-sample N2: VaR 99.0% = $44937598.06, CVaR 99.0% = $50567669.80

Out-of-sample: VaR 99.9% = $181300705.46, CVaR 99.9% = $219995975.33
In-sample MC1: VaR 99.9% = $178097535.03, CVaR 99.9% = $217493455.65
In-sample MC2: VaR 99.9% = $181046923.81, CVaR 99.9% = $222250774.60
Out-of-sample N: VaR 99.9% = $56634989.12, CVaR 99.9% = $61149718.04
In-sample N1: VaR 99.9% = $56668550.76, CVaR 99.9% = $61190160.65
In-sample N2: VaR 99.9% = $57629103.71, CVaR 99.9% = $62228938.21
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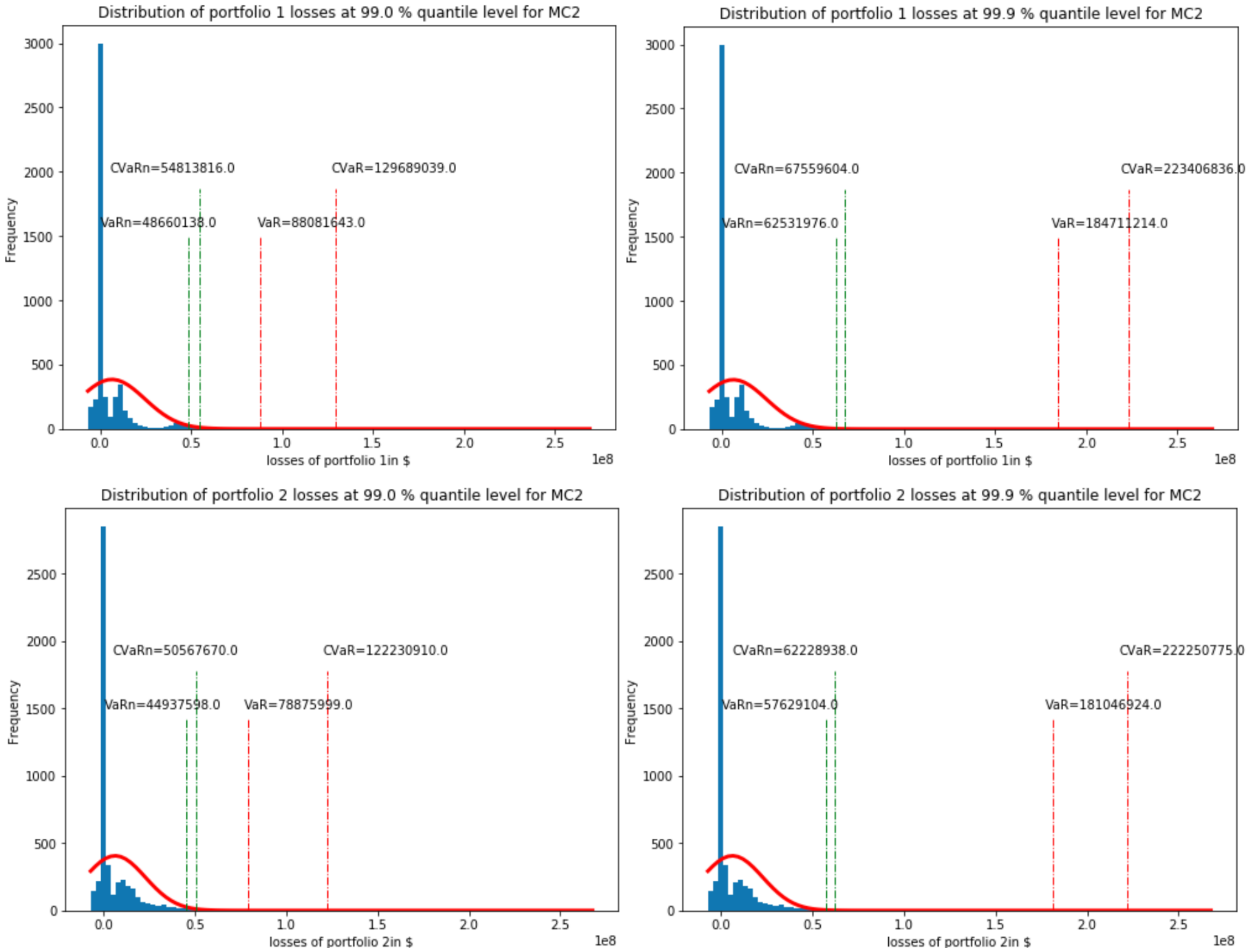
The ***In sample losses distribution for Monte Carlo simulation 2*** thus obtained are as follows:



4. Results Analysis:

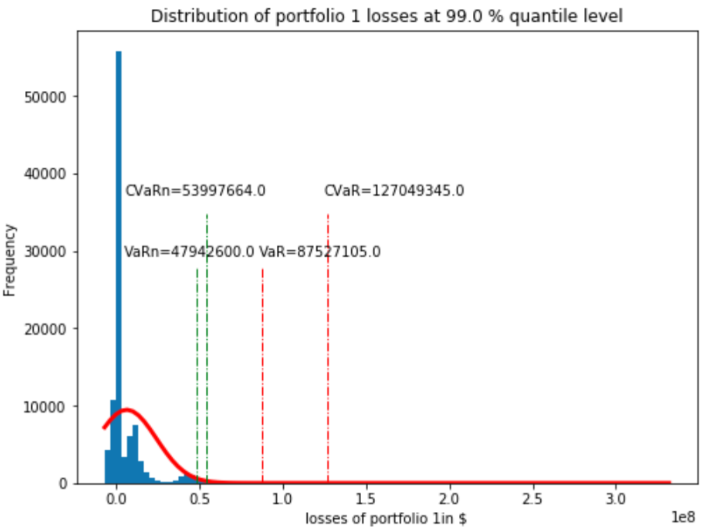
4.1. Sampling Error:

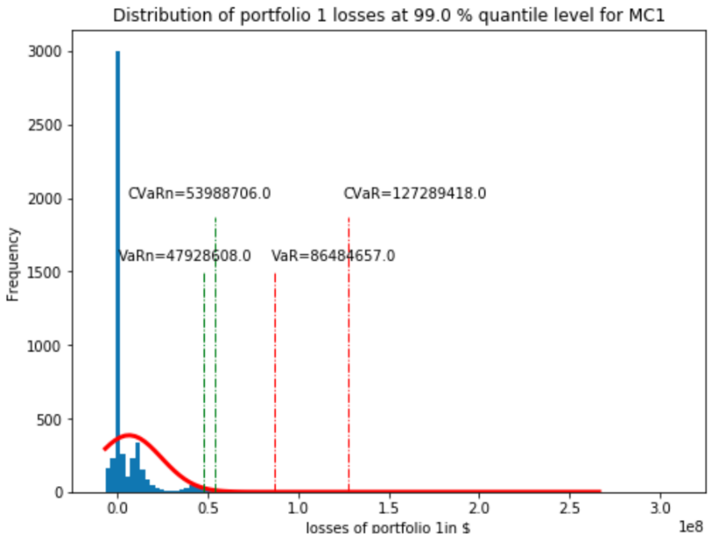
Error from different samplings compared to the true distribution is considered as sampling error. Here, the Out of sample scenarios are true distribution and MC1, MC2 are two samples. Errors are calculated based on the VAR and CVAR values obtained in both methods.

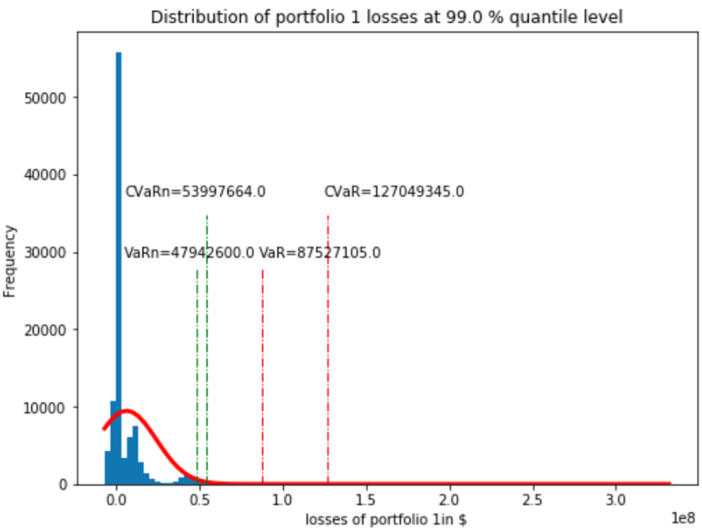
$$Sampling\ error_{(VAR)} = \frac{VAR_{true} - VAR_{sample}}{VAR_{true}}\ x\ 100\ \%$$

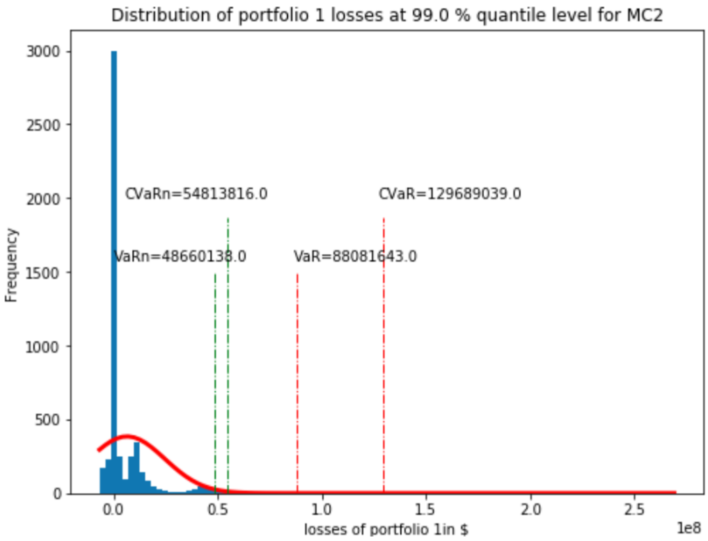
$$Sampling\ error_{(CVAR)} = \frac{CVAR_{true} - CVAR_{sample}}{CVAR_{true}}\ x\ 100\ \%$$

The ***Out of sample vs in sample losses 1 &2 distribution comparison*** thus obtained are as follows:









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Portfolio 1:

Alpha 99.0:

==================================================================
Out-of-sample: VaR 99.0% = $87527105.24, CVaR 99.0% = $127049344.94
In-sample MC1: VaR 99.0% = $86484657.27, CVaR 99.0% = $127289418.36
In-sample MC2: VaR 99.0% = $88081642.62, CVaR 99.0% = $129689038.92
======================================================================
Sampling percentage error for MC1: VAR 99.0% =  1.19, CVaR 99.0% =  0.19
Sampling percentage error for MC2: VAR 99.0% =  0.63, CVaR 99.0% =  2.08
======================================================================

Alpha 99.9:

==================================================================
Out-of-sample: VaR 99.9% = $188684264.10, CVaR 99.9% = $220286520.58
In-sample MC1: VaR 99.9% = $180458572.54, CVaR 99.9% = $219321208.65
In-sample MC2: VaR 99.9% = $184711213.71, CVaR 99.9% = $223406836.48
======================================================================
Sampling percentage error for MC1: VAR 99.9% =  4.36, CVaR 99.9% =  0.44
Sampling percentage error for MC2: VAR 99.9% =  2.11, CVaR 99.9% =  1.42
======================================================================

Portfolio 2:

Alpha 99.0:

==================================================================
Out-of-sample: VaR 99.0% = $77414612.81, CVaR 99.0% = $119493744.43
In-sample MC1: VaR 99.0% = $76880393.47, CVaR 99.0% = $119637243.34
In-sample MC2: VaR 99.0% = $78875998.77, CVaR 99.0% = $122230909.72
======================================================================
Sampling percentage error for MC1: VAR 99.0% =  0.69, CVaR 99.0% =  0.12
Sampling percentage error for MC2: VAR 99.0% =  1.89, CVaR 99.0% =  2.29
======================================================================

Alpha 99.9:

==================================================================
Out-of-sample: VaR 99.9% = $181300705.46, CVaR 99.9% = $219995975.33
In-sample MC1: VaR 99.9% = $178097535.03, CVaR 99.9% = $217493455.65
In-sample MC2: VaR 99.9% = $181046923.81, CVaR 99.9% = $222250774.60
======================================================================
Sampling percentage error for MC1: VAR 99.9% =  1.77, CVaR 99.9% =  1.14
Sampling percentage error for MC2: VAR 99.9% =  0.14, CVaR 99.9% =  1.02
======================================================================
```

It can be observed that the sampling error from both VAR and CVAR for 99.9 percentile is greater than that of errors for 99.0 percentile. **Hence sampling error increases with increase in alpha.** This is due to the position of VAR and CVAR for higher alphas will be at extreme tail ends which can be estimated better if the number of samples is large.

Also**, errors from MC2 are less than that of MC1 sampling** in most of the cases since in MC2, we are generating idiosyncratic variable for each systemic random number which produces better approximation of the true distribution whereas in MC1, only 1 idiosyncratic variable is generated for 1000 systemic random numbers. Whereas, MC1 sampling is computationally beneficial.

4.2. Model Error:

Error from different models compared to the true distribution is considered as model error. Here, the Out of sample scenarios are true distribution and normal distribution assumption is another model. Errors are calculated based on the VAR and CVAR values obtained as follows:

$$Model\ error_{(VAR)} = \frac{VAR_{true} - VAR_{normal}}{VAR_{true}}\ x\ 100\ \%$$

$$Model\ error_{(CVAR)} = \frac{CVAR_{true} - CVAR_{normal}}{CVAR_{true}}\ x\ 100\ \%$$

```
Portfolio 1:


Alpha 99.0:

===================================================================
Out-of-sample: VaR 99.0% = $87527105.24, CVaR 99.0% = $127049344.94
Out-of-sample N: VaR 99.0% = $47942600.01, CVaR 99.0% = $53997664.12
====================================================================================
Model percentage error for Normal distribution: VAR 99.0% = 45.23, CVaR 99.0% = 57.50
====================================================================================

Alpha 99.9:

===================================================================
Out-of-sample: VaR 99.9% = $188684264.10, CVaR 99.9% = $220286520.58
Out-of-sample N: VaR 99.9% = $61592138.41, CVaR 99.9% = $66539196.65
====================================================================================
Model percentage error for Normal distribution: VAR 99.9% = 67.36, CVaR 99.9% = 69.79
====================================================================================

Portfolio 2:


Alpha 99.0:

===================================================================
Out-of-sample: VaR 99.0% = $77414612.81, CVaR 99.0% = $119493744.43
Out-of-sample N: VaR 99.0% = $44178300.17, CVaR 99.0% = $49704205.00
====================================================================================
Model percentage error for Normal distribution: VAR 99.0% = 42.93, CVaR 99.0% = 58.40
====================================================================================

Alpha 99.9:

===================================================================
Out-of-sample: VaR 99.9% = $181300705.46, CVaR 99.9% = $219995975.33
Out-of-sample N: VaR 99.9% = $56634989.12, CVaR 99.9% = $61149718.04
====================================================================================
Model percentage error for Normal distribution: VAR 99.9% = 68.76, CVaR 99.9% = 72.20
====================================================================================
```

As observed in the case of sampling error, **model error is also high for higher values of alpha**. It can be also seen that normal distribution highly underestimates the tail risk in all the cases. Model error is very high for this assumption, hence assuming **the losses are normal may not be very useful**.

5. Discussion:

5.1. Consequences of reporting In-sample VAR and CVAR:

Capital Requirement or regulatory capital is the amount of capital a bank or other financial institution must hold as per the regulations to ensure that they do not take excess leverage and become insolvent. As seen from our example, in-sample VAR and CVAR are low compared to the true estimates of VAR and CVAR.

Hence if we report in-sample quantities to the decision makers of a bank, they could underestimate the risk and become more probable to default. At a higher alpha, the risk is even more as the sampling error is more. Hence, using in-sample estimates to take decisions is not recommended.

5.2. Minimizing sampling and model errors :

1. Increasing number of samples gives better estimates of the true distribution. But, computation time as well as cost will also be increased.
2. Since normal distribution highly underestimates tail risk it is recommended not use normal assumptions.
3. Avoiding use of VAR and using CVAR as the risk estimate.
4. Apart from the credit risk in the model, we need to take in account the market accounting.
5. Our model assumes that the change in the monetary value of the contract is only based on the credit rating. But other factors can also be included in the model.