Materials Today Communications 16 (2018) 142–151

Contents lists available at ScienceDirect

Materials Today Communications

journal homepage: www.elsevier.com/locate/mtcomm

Thermal buckling behaviour of variable stiffness laminated composite plates

Ganapathi Manickam*, Anirudh Bharath[1], Aditya Narayan Das[1], Anant Chandra[1], Pradyumna Barua[1]

School of Mechanical Engineering, Vellore Institute of Technology, Deemed University, Vellore, 632014, India

A R T I C L E I N F O

Keywords:
Thermal buckling
Variable stiffness composites
Finite element
Curvilinear fibre
Shear deformation theory
Multi-layers
Fibre angle

A B S T R A C T

Here, the thermal buckling behaviour of variable stiffness laminated composite plates subjected to thermal loads is numerically studied employing finite element approach based on first-order shear deformation theory.

In the composite laminate considered here, the fibre orientation varies continuously within the layer leading to spatial variation of stiffness in the laminate. Different types of thermal loadings such as uniform and non-uniform temperature distributions are assumed in the analysis. The governing equations developed, applying the principle of minimization of total potential energy, are solved through an eigenvalue approach. The displacement field of pre-buckling of the laminate is evaluated before proceeding for the thermal buckling analysis depending on the type of temperature distributions. The formulation is tested against problems for which the solutions are available. A detailed study considering various design parameters such as curvilinear fibre angles at the centre and edge of the lamina, lay-up, thickness ratio, coefficients of thermal expansion, and modular ratio is made on the critical buckling temperature. The present analysis shows the significant change in the critical thermal buckling parameter while varying the curvilinear fibre angles and lay-up of the laminate.

1. Introduction

The use of advanced composite materials in aerospace, automotive and marine technologies, etc. has been growing rapidly due to its excellent directional stiffness properties. Furthermore, the efficient structural designs using such materials are obtained by introducing simulation based on classic lamination or shear deformation theory. Various shear deformation theories are considered depending on the accurate prediction of either global or local behaviour of laminated composite structures. In general, composite laminates having many layers with different ply orientations make use of spatially straight fibres. Various applications and studies based on spatially straight fibres are reviewed and well documented in the literature [1–4].

Laminated composite structures with varying stiffness have received considerable attention among researchers as they may lead to better and efficient design. The stiffness of the composite laminate can be altered by varying the volume fraction of fibre, fibre spacing, dropping or adding plies to the laminate, and attaching discrete stiffeners to the laminate. More recent advancements in manufacturing processes have demonstrated the possibility of realising the variable fibre angle composites, wherein the fibre angle varies spatially in the layers, thus leading to variable stiffness composite laminates (VSCL). The potential advantage of using such VSCLs is the possibility of altering the load paths. These types of composite laminates were further shown to have greater resilience to buckling, better thermal properties, and were found to be flexible, in the sense that their deformation under different loadings can be controlled to produce favourable geometric configurations. For instance, Murugan and Friswell [5] have investigated the application of these composites in morphing wing structures. In this study, composites with curvilinear fibre paths have been examined to enhance the conflicting structural requirements, which require a low in-plane stiffness and high out of plane bending stiffness.

The available materials in the literature on the subject of variable stiffness composite structures with curvilinear fibres are mainly dealt with the mechanical and vibration characteristics, optimization of stacking, buckling and postbuckling analysis. A study on the use of curvilinear fibres to improve mechanical buckling resistance of composite plates was first conducted by Hyer and Lee [6]. Their investigation was pertaining to the strengthening of plates with central circular holes using sensitivity analysis. Hao et al. [7] have investigated the buckling analysis of VSCL employing first-order shear deformation and isogeometric analysis. The static analysis prior to the buckling analysis is carried out to improve the accuracy of the solutions. A variable-kinematic model for buckling and vibration analyses of variable stiffness plates was introduced in the work of Vescovini and Dozio [8]. The formulation developed in the context of a variational

* Corresponding author.
E-mail address: Ganapathi.M@vit.ac.in (G. Manickam).
[1] Graduate.

https://doi.org/10.1016/j.mtcomm.2018.05.003
Received 2 April 2018; Accepted 18 May 2018
Available online 19 May 2018

framework together with the method of Ritz is adopted to solve the problem. Marouene et al. [9] have carried out experimental study on the buckling behaviour of VSCL including panels containing gaps or overlaps and the specific responses of the out-of-plane deflections were tracked using laser sensors.

The postbuckling analysis of variable stiffness composite laminates with curved fibres is attempted by Madeo et al. [10] using Koiter's approach in conjunction with a mixed finite element method. White et al. [11] have analysed the postbuckling of curved panels with curvilinear fibres whereas the compressive buckling and first-ply failure of VSCL were carried out based on numerical simulations in Ref. [12]. Setoodeh et al. [13] have investigated the optimization of the curvilinear fibre laminates under buckling load. The fibre angles for the discretized domains were updated using the post-deformation for realistic modelling. Ghiasi et al. [14] have further developed the optimization of the sequence of stacking for a variable stiffness plate. Falcó et al. [15] have given an in-depth analytical method to predict the failure characteristics of a curvilinear fibre ply based on the fibre path and curvature. The study of the effects of defects on the material properties was performed by Arian Nik et al. [16] while dealing with an optimization of the laminates. Similar study was extended to the optimization of laminatesunder mechanical buckling loads with induced defects by Venkatachari at al. [17]. Recently, Wu et al. [18] have dealt with the optimization of postbuckling behaviour of variable thickness composite panels with variable angle tows and obtained the first-level optimal postbuckling solutions in terms of lamination parameters. The influence of various structural models in predicting the mechanical and thermal behaviour of curvilinear fibres reinforced laminates is assessed in the work of Venlatachari et al. [19]. Ribeiro et al. [20] have reviewed and documented the available work pertaining to the mechanical behaviour of curvilinear fibre composite laminated panels.

Research importance is also given to the investigation of vibration and dynamic response of variable stiffness composite laminates. Linear and non-linear vibration analyses of curvilinear fibre composite panels were performed by Abdalla et al. [21], and Ribeiro and Akhavan [22] considering different geometric configurations. Houmat [23] has studied the effects of forced vibrations on a curvilinear fibre reinforced rectangular plate. Ribeiro and Stoykov [24] have presented the results on the behaviour of a cylindrical structure with curvilinear fibres under free and forced vibration studies. The effect of environmental conditions on the free vibrations of curvilinear composite plates with cut-out was investigated by Venkatachari et al. [25]. The variation of natural frequency is highlighted by changing the curved fibre tow angles and the cut-out ratio. Tan and Nie [26] have carried out the free and forced vibrations of variable thickness composite annular thin plates with elastically restrained edges employing the method of weighted residuals. Loja et al. [27] investigated the dynamic Instability of VSCL using Rayleigh-Ritz method to perform buckling, free vibrations and dynamic instability analyses. Venkatachari et al. [28] have recently performed the free vibration characteristics variable stiffness laminated composite shells based on a higher-order structural theory and brought out the influence of environmental factors such as the temperature and the presence of moisture on the vibration behaviour.

The structural elements such as plates and panels may get exposed to the thermal environment. For instance, when these structures exposed to supersonic or hypersonic airflow, the aerodynamic heating is generated and the temperature can be high and the pattern may be of non-uniform distributions across the panel. Therefore, it is important to maximize the stiffness of the structure while minimizing the weight of the composite structures under such environment. The scope of such studies has been extended to laminates made use of straight fibres composites and received significant importance in the literature [29,30]. Some of the notable contributions are presented here.

Chen et al. [31] have examined the influence of thermal buckling of various configurations of composite plates whereas Ganapathi and Touratier [32] have carried out the post buckling of composite laminates assuming different temperature loads. Kant and Babu [33] solved the thermal buckling of skew fibre-reinforced composites and sandwich plates using high-order shear deformation theories. Vosoughi et al. [34] extended the investigation of the effect of thermal loading on laminated composite skew plates by evaluating the post-buckling phenomenon. Makhecha et al. [35] have dealt with the effects of dynamic thermal and mechanical loadings on the laminated composite plates using a higher-order shear deformation theory. The influence of material properties and lay-up sequences on the thermo-elastic stability behaviour of composite was examined in the work of Shiau et al. [36]. Recently using the approach of sublaminates in conjunction with variable-kinematic formulation, the thermal buckling analysis of composite plates and sandwich panels was carried out in Ref. [37]. Based on a layer-wise displacement model, this class of problem was studied by Cetkovic [38]. Singha et al. [39] considered the post-buckling characteristics of laminated composite plates with different boundary conditions. Patel et al. [40] have worked on the thermo-flexural analysis of thick laminates of bi-modulus composite materials whereas the study of hygrothermalelastic stability of thick laminated composite is shown in Ref. [41]. The research work on the thermal buckling and post buckling of functionally graded rectangular composite laminated plates is also available in Refs. [42–47] using analytical/numerical approaches. Thermal buckling analysis of FG plates with internal defects was performed by Yu et al. [48] using an extended iso-geometric analysis. Recently, Haldar et al. [49] have attempted to study the thermally induced multistable configurations of variable stiffness composite plates using semi-analytical and finite element approaches. It is clearly seen from the work carried out in the literature while solving the thermally stressed laminates, a comprehensive insight into the thermo-structural behaviour of variable stiffness plates appears to be limited in the literature.

This paper aims at studying the thermal buckling of variable stiffness composite flat panels introducing curvilinear fibres under uniform and non-uniform temperature fields. The mathematical model derived assuming first-order Reisser-Mindlin plate theory, is solved by employing an eight-noded C° quadrilateral plate element [50,51]. Different types of nonuniform temperature distributions like linearly varying and sinusoidal temperature distributions are considered in the present analysis. Since pre-buckling deformation may not be applicable for buckling of laminates under non-uniform temperature loading cases, thermal stress analysis is carried out before evaluating the buckling values. The influence of the important parameters such as lay-up sequence, number of layers, thickness ratio, material and geometric properties on the thermal buckling behaviour of simply supported curvilinear fibre composite laminates are brought out and certain conclusion are made.

This paper is structured with Section 2 presenting the theoretical formulation accounting for shear deformation, variation of curvilinear fibre and the governing equation. Section 3 deals with results and discussion based on a systematic parametric study on the buckling behaviour; lastly, Section 4 concludes the paper based on the assessment of various design parameters.

2. Theoretical formulation

Considering a composite laminate having length a, width b, thickness h, and n layers with x_1, x_2 as coordinates along the in-plane directions and x_3 through the thickness direction (see Fig.1a), the displacement functions (u, v, w) at any position from the mid-surface are presumed as function of in-planedisplacements (u_0, v_0, w_0), and normal rotations (φ_{x_1} and φ_{x_2} about the transverse planes (x_1x_3 & x_2x_3) respectively as

$$\begin{Bmatrix} u(x_1, x_2, x_3) \\ v(x_1, x_2, x_3) \\ w(x_1, x_2, x_3) \end{Bmatrix} = \begin{Bmatrix} u_0(x_1, x_2) \\ v_0(x_1, x_2) \\ w_0(x_1, x_2) \end{Bmatrix} + x_3 \begin{Bmatrix} \varphi_{x_1}(x_1, x_2) \\ \varphi_{x_2}(x_1, x_2) \\ 0 \end{Bmatrix} \tag{1}$$

The different strain vectors such as the membrane, bending and

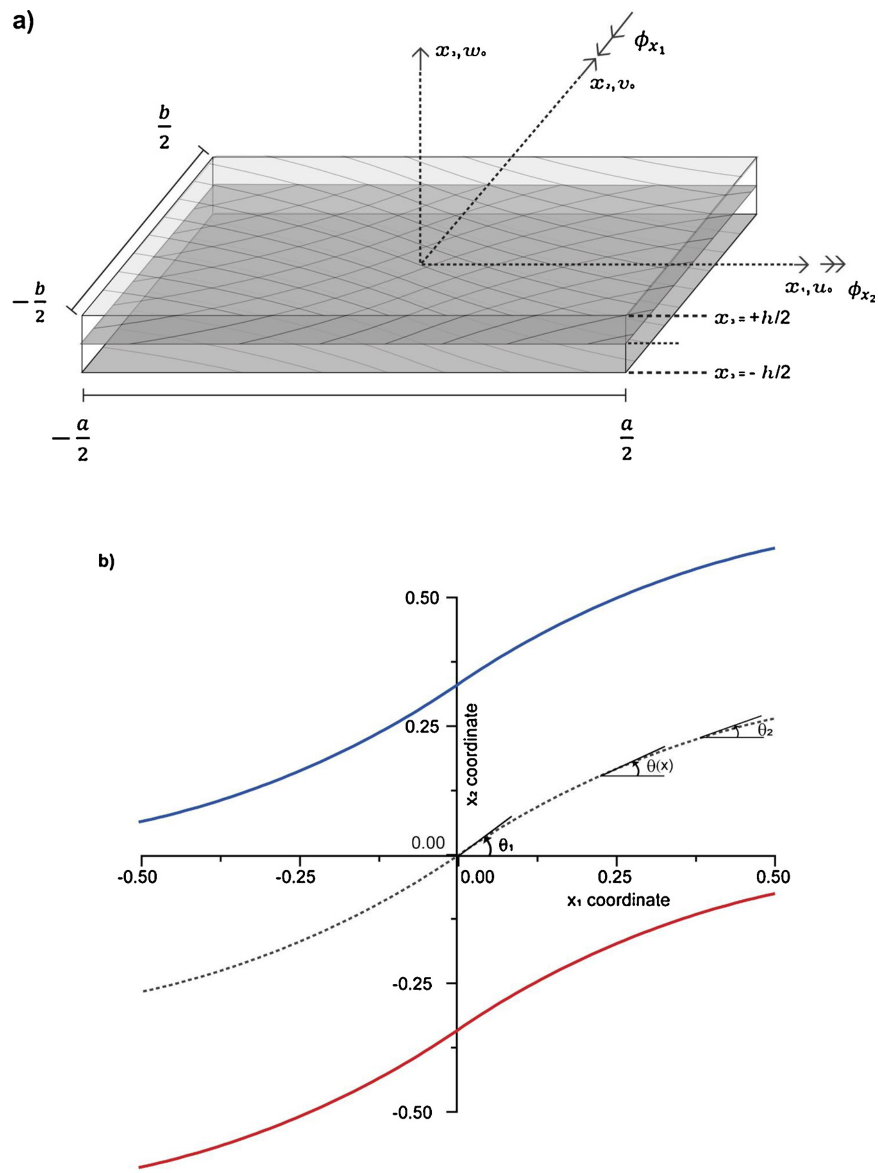

Fig. 1. a) Plate coordinates and b) VSCL-curved fibres: Reference and shifted fibres.

transverse shear strains, and thermal strains ($\{\varepsilon_m\}$, $\{\varepsilon_b\}$, $\{\varepsilon_s\}$ & $\{\varepsilon_t\}$) are respectively, presented in the composite laminate bending analysis based on the assumed displacement kinematics as

$$\{\varepsilon\} = \{\varepsilon_{x_1x_1} \quad \varepsilon_{x_2x_2} \quad \gamma_{x_1x_2} \quad \gamma_{x_1x_3} \quad \gamma_{x_2x_3}\}^T = \begin{Bmatrix} \varepsilon_m \\ 0 \end{Bmatrix} + x_3 \begin{Bmatrix} \varepsilon_b \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ \varepsilon_s \end{Bmatrix} - \begin{Bmatrix} \varepsilon_t \\ 0 \end{Bmatrix} \tag{2}$$

where

$$\{\varepsilon_m\} = \begin{Bmatrix} u_{0,x_1} \\ v_{0,x_2} \\ v_{0,x_2} + u_{0,x_1} \end{Bmatrix}; \quad \{\varepsilon_b\} = \begin{Bmatrix} \kappa_{x_1} \\ \kappa_{x_2} \\ \kappa_{x_1x_2} \end{Bmatrix} = \begin{Bmatrix} \varphi_{x_1,x_1} \\ \varphi_{x_2,x_2} \\ \varphi_{x_1,x_2} + \varphi_{x_2,x_1} \end{Bmatrix}; \quad \{\varepsilon_t\} = \begin{Bmatrix} \varepsilon_{x_1}^t \\ \varepsilon_{x_2}^t \\ \varepsilon_{x_1x_2}^t \end{Bmatrix} = \begin{Bmatrix} \alpha_{11} \\ \alpha_{22} \\ 2\alpha_{12} \end{Bmatrix} \Delta T \tag{3a}$$

$$\{\varepsilon_s\} = \begin{Bmatrix} \gamma_{x_1x_3} \\ \gamma_{x_2x_3} \end{Bmatrix} = \begin{Bmatrix} w_{0,x_1} + \varphi_{x_1} \\ w_{0,x_2} + \varphi_{x_2} \end{Bmatrix} \tag{3b}$$

Here, $(\,)_{,x_1}$ and $(\,)_{,x_2}$ represent the partial differentiation with respect to x_1 and x_2; respectively. $\Delta T\,(=T - T_a)$ is the temperature rise from the reference temperature T_a at which there are no thermal strains; α_{11}, α_{22} are the coefficients of thermal expansion in the principle material directions (Fig. 2).

The constitutive equation for any layer k in the laminate can be represented as

$$\begin{aligned} \{\sigma\} &= \{\sigma_{x_1x_1} \quad \sigma_{x_2x_2} \quad \tau_{x_1x_2} \quad \tau_{x_1x_3} \quad \tau_{x_2x_3}\}^T \\ &= [\overline{Q}_k]\ \{\varepsilon_{x_1x_1} \quad \varepsilon_{x_2x_2} \quad \gamma_{x_1x_2} \quad \gamma_{x_1x_3} \quad \gamma_{x_2x_3}\}^T \end{aligned} \tag{4}$$

Here, T,the superscript represents the transpose of a vector or matrix; {σ} is the stress vector; the stiffness $[\overline{Q}_k]$ matrix of the kth layer are obtained with respect to laminate axes and can be derived from the $[\overline{Q}_k]$ provided with respect to fibre directions [52]. Since the variable stiffness composite panel is based on curvilinear fibres, the fibre angle changes in the plane of the lamina (Fig. 1b). The fibre angle is further assumed as a linear variation along the x-axis that permits for a wide range of properties and is defined in the kth lamina as [17,19,20]

$$\theta(x) = \begin{cases} -\frac{2}{a}(\theta_2 - \theta_1)\,x + \theta_1 \ldots\ldots\ldots\ldots -\frac{a}{2} \le x \le 0 \\ \frac{2}{a}(\theta_2 - \theta_1)\,x + \theta_1 \ldots\ldots\ldots\ldots\ 0 \le x \le \frac{a}{2} \end{cases} \tag{5}$$

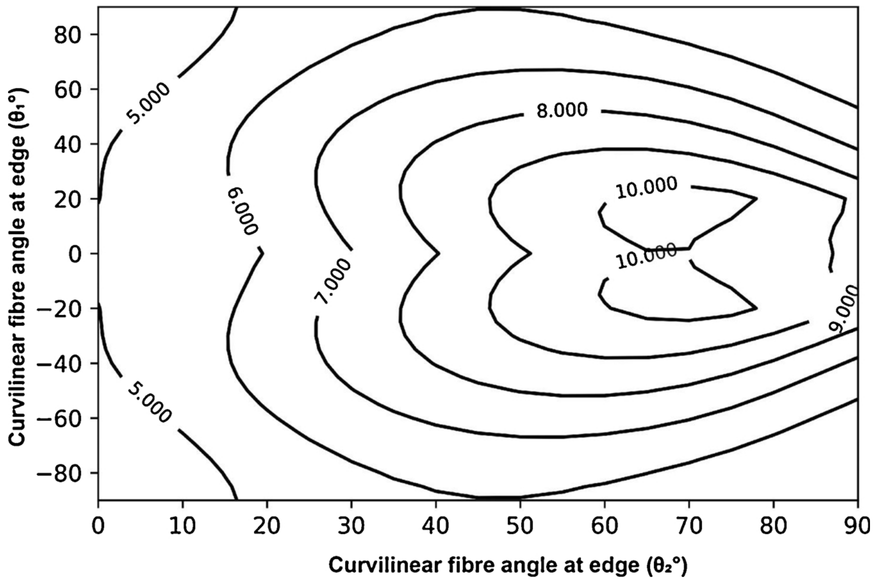

Fig. 2. Contour plot of non-dimensional thermal buckling parameter ($\lambda_{cr}\,\alpha_{11}\,10^3$) of variable stiffness composite plate (a/h = 10, [$\mp$ (θ_1/θ_2)]$_2$) varying curvilinear fibre angles at the centre & edge of the layers under non-uniform thermal distribution (case-2).

where θ_1&θ_2 are the fibre angles defined at the lamina centre $x_1 = 0$ and at the laminate ends $x_1 = \pm a/2$, respectively and is referred by <θ_1 θ_2>. It can be noted that $[\overline{Q}_k]$ varies along the surface of the plane x_1x_2, within a lamina, as θ is a function of spatial coordinates.

The strain energy function U pertaining to the variable stiffness composite laminate is expressed by,

$$U(\delta) = \frac{1}{2}\iint\left[\sum_{k=1}^{L}\int_{h_k}^{h_{k+1}}\{\sigma\}^T\{\varepsilon\}\,dx_3\right]dx_1\,dx_2 \tag{6}$$

where h_k, h_{k+1} are the $x_3$coordinates of laminate corresponding to the bottom and top surfaces of the kth layer. δ is the vector of the degrees of freedom associated to the displacement field in a finite element discretisation.

Substituting Eq. (3) into the constitutive relation Eq. (4), one can rewrite strain energy functional Uas

$$U(\delta) = \frac{1}{2}\{\delta\}^T[K]\{\delta\} - \{\delta\}^T[\{N^t\} + \{M^t\}] \tag{7}$$

where $[K]$ presents the stiffness matrix; $\{N^t\}$ & $\{M^t\}$ are the thermal load vectors due to thermal stress resultants and moment resultants, respectively.

The composite panel is subjected to temperature field and this, in turn, results in thermal stress and moment resultants $\{N^t\} = \{N^{th}_{x_1x_1}\ \ N^{th}_{x_2x_2}\ \ N^{th}_{x_1x_2}\}$ and $\{M^t\} = \{M^{th}_{x_1x_1}\ \ M^{th}_{x_2x_2}\ \ M^{th}_{x_1x_2}\}$., respectively. Thus, the potential energy (V) due to the thermal pre-buckling stresses developed under assumed thermal field can be written as

$$\begin{aligned}V(\delta) &= \tfrac{1}{2}\iint\left[N^{th}_{x_1x_1}\left(\frac{\partial w}{\partial x_1}\right)^2 + N^{th}_{x_2x_2}\left(\frac{\partial w}{\partial x_2}\right)^2 + 2N^{th}_{x_1x_2}\left(\frac{\partial w}{\partial x_1}\right)\left(\frac{\partial w}{\partial x_2}\right)\right]dx_1\,dx_2\\
&+\tfrac{h^3}{24}\iint\left[N^{th}_{x_1x_1}\left\{\left(\frac{\partial\phi_{x_1}}{\partial x_1}\right)^2 + \left(\frac{\partial\phi_{x_2}}{\partial x_1}\right)^2\right\} + N^{th}_{x_2x_2}\left\{\left(\frac{\partial\phi_{x_1}}{\partial x_2}\right)^2 + \left(\frac{\partial\phi_{x_2}}{\partial x_2}\right)^2\right\}\right]dx_1\,dx_2\\
&+\tfrac{h^3}{24}\iint\left[2N^{th}_{x_1x_2}\left\{\left(\frac{\partial\phi_{x_1}}{\partial x_1}\right)\left(\frac{\partial\phi_{x_1}}{\partial x_2}\right) + \left(\frac{\partial\phi_{x_2}}{\partial x_1}\right)\left(\frac{\partial\phi_{x_2}}{\partial x_2}\right)\right\}\right]dx_1\,dx_2\\
&= \tfrac{1}{2}\{\delta\}^T[K_{GT}]\{\delta\}\end{aligned} \tag{8}$$

Where $[\boldsymbol{K_{GT}}]$ is the geometric stiffness matrix due to unit temperature rise

For the thermo-flexural case, the minimization of the energy functional given in Eq. (7) leads to the governing equations as outlined in Zienkiewicz and Taylor [53]

$$[[K]]\{\delta\} = \{N^t\} + \{M^t\} \tag{9}$$

The displacement field of pre-buckling of the laminate can be evaluated bysolving Eq. (9). For thermal buckling case, the equilibrium equation is obtained by minimizing the total potential energy associated with Eqs. (7) and (8), $U_T = (U\text{-}V)$ as follows:

$$[[K] + \lambda\,[K_{GT}]]\{\delta\} = \{0\} \tag{10}$$

where λ is the critical temperature rise.

The governing equilibrium equations given in Eq. (10) are treated as an eigenvalue problem. It is numerically solved using standard eigenvalue approach and finite element procedure with a C° continuous, eight-noded serendipity quadrilateral shear flexible plate element developed in [45,50,51]. There are five nodal degrees of freedom per node (u_0, v_0, w_0,φ_{x_1}, φ_{x_2}) in the element as observed from the displacement kinematic function given in Eq. (1). Since the element employed here is formulated using field-consistency approach, the element is free from shear locking syndrome and does not require reduced numerical integration scheme to alleviate the shear locking phenomenon and exact integration is numerically carried out. The element has very good convergence properties and no spurious rigid body modes.

3. Results and discussion

Here, the analysis is focused on the thermo-elastic stability of variable stiffness composite laminates made up of curvilinear fibres. Since the formulation accounts for shear deformation effects, the numerical experimentation is conducted for fairly thick to thin VSCL with exact integration scheme. The shear correction factor is taken as $k_1^2 = k_2^2 = 5/6$. The influences of different parameters like the thickness and aspect ratios, material anisotropy, lay-up, temperature distribution and curvilinear fibre angles at centre as well as at the edge of the laminate on the thermal buckling loads of VSCL are discussed. Unless specified otherwise, the following material properties, typical of high modulus graphite-epoxy, are considered in the present study [31]:

$E_{11}/E_{22} = 40$; $G_{12}/E_{22} = G_{13}/E_{22} = 0.6$, $G_{23}/E_{22} = 0.5$; $\nu_{12} = \nu_{13} = \nu_{23} = 0.25$; $E_{22} = 10^{09}$GPa

$\alpha_{22}/\alpha_{11} = 2$; $\alpha_{11} = \alpha_{33}$; $\alpha_{11} = 1 \times 10^{-06}(1/°K)$,where E, G and ν are Young's modulus, shear modulus and Poisson's ratio. Subscripts 1 and 2 denote the longitudinal and transverse directions, respectively, with respect to the fibre directions at any point in the plane x_1x_2. The layers of the laminate are numbered from the bottom-most layer and the fibre-angle is calculated with respect to $x_1$axis in an anti-clockwise direction. Layers of equal thickness are considered. The boundary conditions considered here is simply supported one and it is defined as follows:

$$u_0 = w_0 = \varphi_{x_2} = 0 \text{at } x_1 = \pm a/2; v_0 = w_0 = \varphi_{x_1} = 0 \text{at } x_2 = \pm b/2$$

The thermal loads assumed to be applied over the laminate for the present analysis are

Uniform distribution: $T(x_1, x_2) = T_1$

Non-uniform distribution:

case 1: (linear form) $T(x_1, x_2) = \begin{cases} T_0 + (1 + 2T_1 x_2/b)\cdots \cdots -b/2 < x_2 < 0 \\ T_0 + (1 - 2T_1 x_2/b)\cdots\cdots\cdots 0 < x_2 < b/2 \end{cases}$

case 2: (cosine function form) $T(x_1, x_2) = T_0 + T_1 \cos(\pi x_1/a) \cos(\pi x_2/b) \cdots\cdots -a/2 < x_1 < a/2$ & $-b/2 < x_2 < b/2$

Firstly, the finite element mesh-size required for the present investigation is determined based on the mesh convergence study as depicted in Table 1 for thermal buckling of curvilinear fibres composite subjected to uniform thermal load and presuming 8-layered anti-symmetric square laminates with various thickness ratio, and fibre path angle $<\theta_1\ \theta_2>$. It is observed that 8 × 8 mesh provides an acceptable converged result and the same mesh is considered for detailed investigation. The formulation developed herein is further examined considering the buckling behaviour of an isotropic plate with different aspect ratios and the non-dimensional critical buckling temperatures ($\lambda_{cr}\ \alpha_{11}\ 10^3$) evaluated are compared in Table 2 along with those of analytical and numerical solutions provided in Refs. [54] and [31], respectively. They compared very well and the small discrepancy in the results for low aspect ratio may be attributed to the classical plate theory employed in [54], and the mesh size used in Ref. [31]. Also, for multi-layered laminate cases with straight fibres, the performance of the present model is highlighted against the numerical solutions [31] in Fig. 3 by varying the ply-angle and they match well.

Next, the buckling characteristics of curvilinear fibre composite laminate under uniform and non-uniform temperature distributions are analysed varying lay-up and ply-angle. In curvilinear composite case, the lay-up sequence for symmetric $[\mp(\theta_1/\theta_2)]_s$ is expanded as $\{+(\theta_1/\theta_2), -(\theta_1/\theta_2), -(\theta_1/\theta_2), +(\theta_1/\theta_2)\}$ whereas for anti-symmetric case $[\mp(\theta_1/\theta_2)]_2$ represents $\{+(\theta_1/\theta_2), -(\theta_1/\theta_2), +(\theta_1/\theta_2), -(\theta_1/\theta_2)\}$, respectively. To start with, the thermal elastic stability study is conducted selecting different values for the fibre angles at the centre and edge of the lamina in the laminate. The buckling resisting strength contours are shown in Fig. 4 considering 4-layered anti-symmetric case with non-uniform (case 2) thermal distribution. It is viewed from this Figure that the non-dimensional critical values ($\lambda_{cr}\ \alpha_{11}\ 10^3$) with respect to the fibre angle θ_1 about 15° and $\theta_2 > 0$ yields a higher range of values. Hence, these values are taken for the further studies related to detailed numerical experiments.

Table 1
Convergence study of non-dimensional thermal buckling parameter ($\lambda_{cr}\ \alpha_{11}\ 10^3$) for a four-layered anti-symmetric $[\pm(\theta_1/\theta_2)]_4$ square laminates (a/h = 10, 20, 40).

$[\pm(\theta_1/\theta_2)]_4$	Mesh	Thickness Ratio (a/h)		
		10	20	40
$[\pm(15°/15°)]_4$	2 × 2	2.5908	1.0735	0.3811
	4 × 4	6.2373	1.9749	0.6027
	6 × 6	6.2221	2.1623	0.6001
	8 × 8	6.2192	2.1609	0.5997
	10 × 10	6.2184	2.1605	0.6010
$[\pm(15°/45°)]_4$	2 × 2	3.8435	1.4139	0.4322
	4 × 4	9.4094	3.5621	1.0115
	6 × 6	9.5679	3.5547	1.0119
	8 × 8	9.5645	3.5527	1.0112
	10 × 10	9.5633	3.5521	1.0131
$[\pm(15°/75°)]_4$	2 × 2	4.3741	2.1184	0.5030
	4 × 4	10.3460	3.6146	1.1471
	6 × 6	10.6060	4.0929	1.1427
	8 × 8	10.6410	4.0868	1.1408
	10 × 10	10.6480	4.0844	1.1425

Table 2
Comparison of non-dimensional thermal buckling load with analytical and numerical results [54,31] for an isotropic ($E = 10^9$ Pa; $\nu = 0.3$, $a/h = 100$) plate with different aspect ratios.

a/b	Present	Ref [31]	Ref [54]
0.25	0.673	0.691	0.686
0.5	0.792	0.814	0.808
1.0	1.266	1.319	1.283
1.5	2.057	2.101	2.073
2.0	3.163	3.191	3.179
2.5	4.583	4.601	4.599
3.0	6.316	6.330	6.332

Table 3 presents the non-dimensional critical buckling thermal loads ($\lambda_{cr}\ \alpha_{11}\ 10^3$) of VSCLs subjected to non-uniform temperature distribution (case 1) with $T_0 = 0$considering four- and eight-layered symmetric/anti-symmetric lay-ups. The fibre angles at the centre and edge of the laminate $<\theta_1\ \theta_2>$ are also varied keeping $\theta_1 = 15°$, 45°, 75°& 90°and assuming different values for θ_2, respectively. It can be noticed from this Table that, for the chosen fibre angle at the centre of the lamina θ_1 and fairly thick case ($a/h = 10$), the non-dimensional critical buckling temperature increases with the increase in fibre angle at the edge of the layer, θ_2 up to certain value and then it decreases. The angle θ_2 at which the maximum value occurs significantly decreases with the increase in the value of angle θ_1, and, to some extent, it also depends on the thickness ratio. This is because of the change in the stiffness in the in-plane of laminate due to the variation of the fibre angle. It is further observed that the resisting load against the buckling is, in general, more for the anti-symmetric case. It is also inferred that, with the increase in layers, the critical temperature increases as expected.

The effect of VSCL exposed to uniform, and non-uniform (case 2) temperature distributions with $T_0 = 0$ is presented for $a/h = 10$ in Fig. 5 by changing the lay-up sequence and ply-angle, respectively. The influence of increasing the layers enhances the bending stability of VSCL against thermal load and it is similar to the behaviour of composite plate with straight fibres. The two-layered case yields the lowest buckling value due to the presence of severe bending-stretching coupling as generally viewed. The fibre edge angle θ_2 of the laminate corresponding to the maximum non-dimensional buckling temperature depends on the type of temperature distribution applied to the curvilinear fibres structure and it will be normally higher for the non-uniform distribution case compared to those of uniformly thermally stressed plate. Furthermore, it can be opined that the stability characteristic of the VSCL is strong against the applied non-uniform temperature distribution because of the continuous changing of stiffness in the pane of laminate and the maximum temperature region is focussed around the centre of the plate wherein the laminate is rather stiff compared to those of region towards the edge of the laminate. In general, it can be concluded that the thermo-elastic stability behaviour of curvilinear fibre composite is qualitatively similar to that of straight fibre, irrespective of temperature distributions laminate as exhibited in Table 3 and Fig. 4. However, the magnitude of critical load of VSCL strongly depends on the curvilinear fibre angles in the plane of lamina and its overall effect rests on the lay-up and the number of layers in the laminate.

A similar study is done to check the influence of thickness ratio of

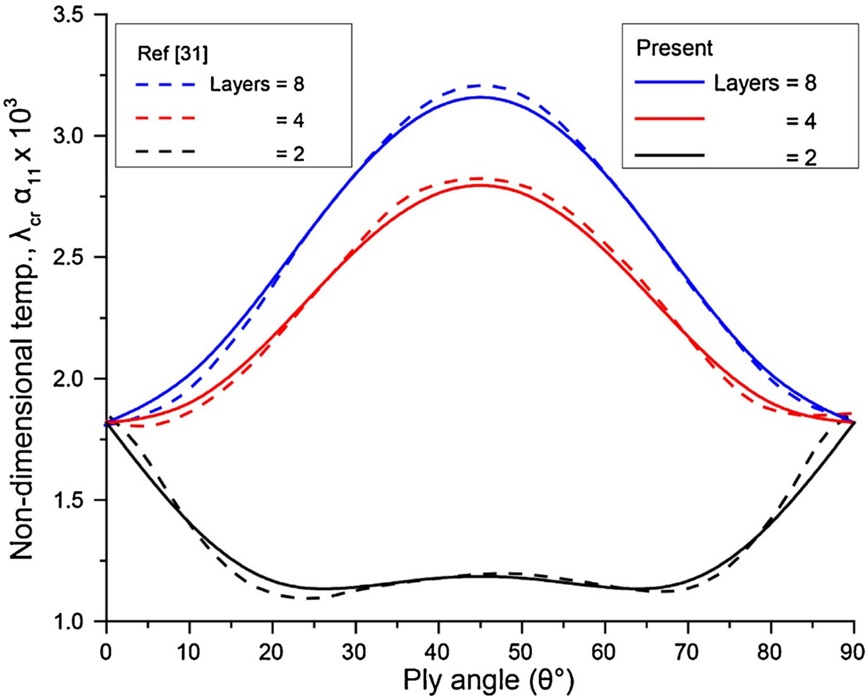

Fig. 3. Comparison of non-dimensional thermal buckling parameter ($\lambda_{cr}\ \alpha_{11}\ 10^3$) for the multi-layered square laminates (a/h = 10) with straight fibres under uniform thermal load [31].

VSCL by choosing a range of values for a/h = 10, 20,& 40 and selecting non-uniform (case 2) temperature distribution with $T_0 = 0$ on the thermal buckling load. The results are plotted in Fig. 5 for different lay-up and for the selected curvilinear fibre angles $<\theta_1\ \theta_2>$ of VSCL. It is however revealed that the thermo-elastic stability of VSCL gets weakened with the increase in thickness ratio as expected and the variation in the thermal buckling behaviour does not change with respect to the curvilinear fibre angle θ_2 at the edge of the laminate. Figs. 6 and 7 illustrate the importance of the coefficients of thermal expansion ratio α_{22}/α_{11} and material anisotropicity on the buckling parameter for fairly thick laminates. It is indicated from this Fig. 6 that the increase in the coefficients of thermal expansion ratio affects the critical value like in the case of increasing in thickness ratio (Fig. 5). Modular ratio E_1/E_2 can change the elastic stability of VSCL under thermal environment as brought out in Fig. 7. It is seen from Fig. 7 that the higher the modular ratio produces lesser the non-dimensional critical value because of, in general, increases in the thermal stress for the chosen thermal field. However, due to the combination of continuous change in the laminate stiffness in VSCL and the temperature profile change, the qualitative buckling characteristic may get altered with the increase in curvilinear fibre edge angle θ_2, in particular, while keeping the fibre angle at the centre low.

4. Conclusion

The thermal buckling characteristics of VSCL with spatially distributed curvilinear fibres are examined using first-order shear deformation theory and considering different types of thermal fields. The formulation accounts numerically the spatial variation of fibre angles through updating the laminate stiffness coefficients. The solutions are obtained using standard eigenvalue procedure. The influence of curvilinear fibre centre and edge angles, side-to-thickness and aspect ratios, lay-up and coefficient of thermal expansion, and material anisotropy on the thermo-elastic stability behaviour is assessed by conducting a detailed numerical experimentation. It is hoped that the results presented here can form benchmark solutions in assessing such studies either based on higher-order formulations or other numerical approaches. Some of the important conclusion from this study are as follows:

i) Lower the fibre angle at the centre, a range of higher angle at the edge of the lamina in the laminate leads to higher non-dimensional critical buckling temperature.
ii) The fibre edge angle θ_2 at which the maximum buckling value occurs significantly decreases with the increase in the value of fibre centre angle θ_1 in the lamina.
iii) Thermal buckling load is, in general, more for anti-symmetric VSCL case compared to those of the symmetric laminate.
iv) VSCL with the non-uniform temperature distribution considered here yields higher critical load carrying capacity compared to those of uniformly thermally stressed laminate.
v) Thermo-elastic stability of VSCL gets reduced with the increase in thickness ratio.
vi) Increase in the number of layers in the laminate significantly enhances the buckling stability behaviour.
vii) Higher the modulus ratio and the thermal expansion coefficient ratio lead to drastically reducing the non-dimensional critical thermal buckling parameter.
viii) Decrease in the aspect ratio results in increasing of the degree of hardening nonlinear behaviour.
ix) The qualitative thermal buckling characteristics may change with the combination of continuous change in the laminate stiffness in VSCL and the temperature profile variations.

Data availability

The material data used for the present study is taken from published literature which are referenced in the manuscript.

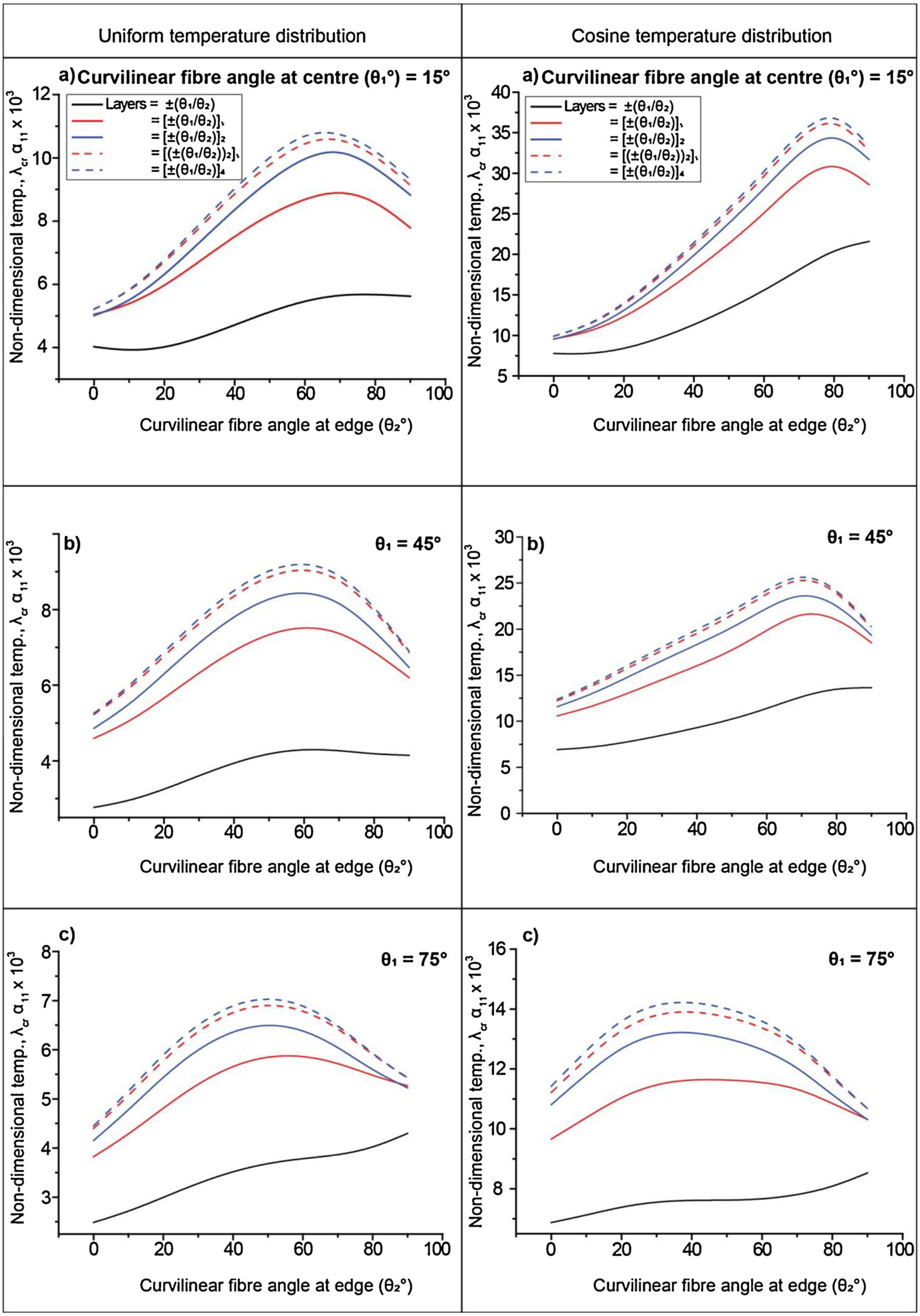

Fig. 4. Non-dimensional thermal buckling parameter ($\lambda_{cr}\ \alpha_{11}\ 10^3$) of multi-layered variable stiffness laminates (a/h = 10) varying lay-up & curvilinear fibre angles <$\theta_1\ \theta_2$>: Left side column: uniform thermal case; Right side column: non-uniform thermal (case-2).

Table 3
Non-dimensional thermal buckling parameters ($\lambda_{cr}\,\alpha_{11}\,10^3$) for the 4- & 8-layered variable stiffness laminates (a/h = 10 & 20) subjected to non-uniform thermal load (case 1-linear) and by varying curvilinear fibre angles<θ_1 θ_2>.

θ_1°	θ_2°	4 Layer ($\lambda_{cr}\alpha_{11}$ x 10^3)				8 Layer ($\lambda_{cr}\alpha_{11}$ x 10^3)			
		$[\pm(\theta_1/\theta_2)]_s$		$[\pm(\theta_1/\theta_2)]_2$		$[(\pm(\theta_1/\theta_2))_2]_s$		$[\pm(\theta_1/\theta_2)]_4$	
		a/h = 10	a/h = 20	a/h = 10	a/h = 20	a/h = 10	a/h = 20	a/h = 10	a/h = 20
15	0	7.4730	2.6009	7.4746	2.5418	7.7434	2.6959	7.7397	2.6806
	15	8.7746	3.0332	9.1562	3.1042	9.6698	3.3778	9.7439	3.3880
	40	13.7870	4.8933	15.2120	5.3597	16.1280	5.8954	16.3980	5.9860
	45	14.8300	5.2894	16.4500	5.8252	17.4180	6.4094	17.7260	6.5152
	60	18.0480	6.4194	20.0720	7.0883	21.2290	7.8186	21.6090	7.9487
	75	21.2980	7.3483	23.2880	7.9181	24.7040	8.7431	25.0560	8.8382
	80	21.3220	7.2332	23.3440	7.7344	24.6390	8.4773	25.0110	7.8926
	90	19.1080	6.2658	20.9640	6.5719	21.5660	7.0109	21.946	7.0643
45	0	9.3109	3.1141	10.1570	3.3307	10.7280	3.6723	10.8680	3.6984
	15	10.7720	3.6169	12.0920	4.0420	12.8970	4.4734	13.1260	4.5426
	40	13.5940	4.7168	15.3800	5.3716	16.4560	5.9509	16.7590	6.0704
	45	14.0580	4.8904	15.8730	5.5584	16.9880	6.1592	17.2950	6.2815
	60	15.4830	5.3436	17.1720	5.9457	18.4360	6.6087	18.7080	6.7125
	75	16.3130	5.5281	17.4470	5.8269	18.6890	6.4657	18.8590	6.5027
	80	15.8210	4.9885	16.7730	5.4976	17.8260	6.0526	17.9760	6.0700
	90	14.1570	4.6888	14.6990	4.6804	15.2440	5.0245	15.3380	5.0116
75	0	9.8394	3.3968	10.9510	3.8550	11.3780	4.1753	11.5780	4.2577
	15	11.0140	3.7917	12.5720	4.3857	13.1780	4.7651	13.4810	4.8820
	40	11.7550	4.0761	13.3380	4.6637	14.0670	5.0775	14.3790	5.1991
	45	11.6630	4.0410	13.1350	4.5902	13.8950	5.0002	14.1900	5.1136
	60	11.3070	3.9000	12.3880	4.2634	13.1470	4.6573	13.3600	4.7296
	75	11.0190	3.8167	11.4370	3.8850	12.1400	4.2380	12.2160	4.2456
	80	10.8300	3.7619	11.0260	3.7379	11.6420	4.0514	11.6730	4.0395
	90	10.4480	3.6470	10.3470	3.5331	10.7510	3.7351	10.7290	3.7053
90	0	10.9110	3.8069	12.1340	4.3550	12.4200	4.5667	12.6620	4.6824
	15	11.8440	4.0845	13.4980	4.7115	13.8790	4.9484	14.2300	5.0870
	40	11.8840	4.1040	13.2760	4.5771	13.6570	4.7961	13.9600	4.9035
	45	11.6790	4.0367	12.9240	4.4522	13.2960	4.6596	13.5690	4.7541
	60	11.0150	3.8258	11.7350	4.0496	12.0590	4.2112	12.2200	4.2622
	75	11.4710	3.6780	10.6790	3.7299	10.8660	3.8167	10.9150	3.8288
	80	10.3020	3.6286	10.3950	3.6481	10.5040	3.6983	10.5260	3.7030
	90	10.1210	3.5692	10.1210	3.5692	10.1210	3.5692	10.1210	3.5692

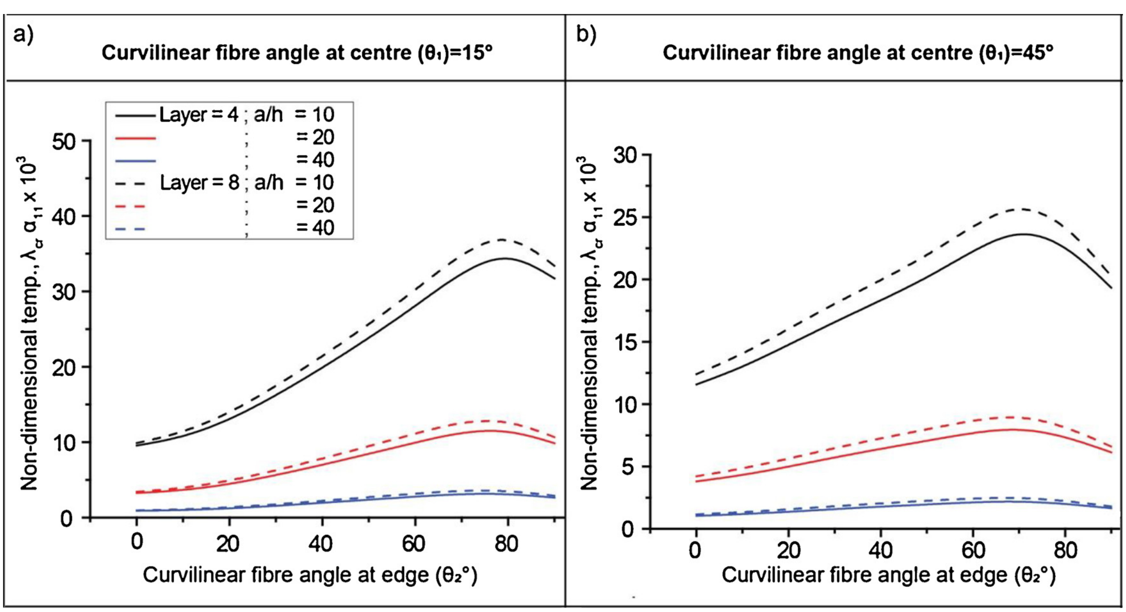

Fig. 5. Effect of thickness ratio on non-dimensional critical thermal buckling parameter ($\lambda_{cr}\,\alpha_{11}\,10^3$) of variable stiffness laminate (a/h = 10; $[\mp(\theta_1/\theta_2)]_4$) under non-uniform thermal distribution (case-2) and for two different values for curvilinear fibre angle at the centre of laminate θ_1 (15°& 45°).

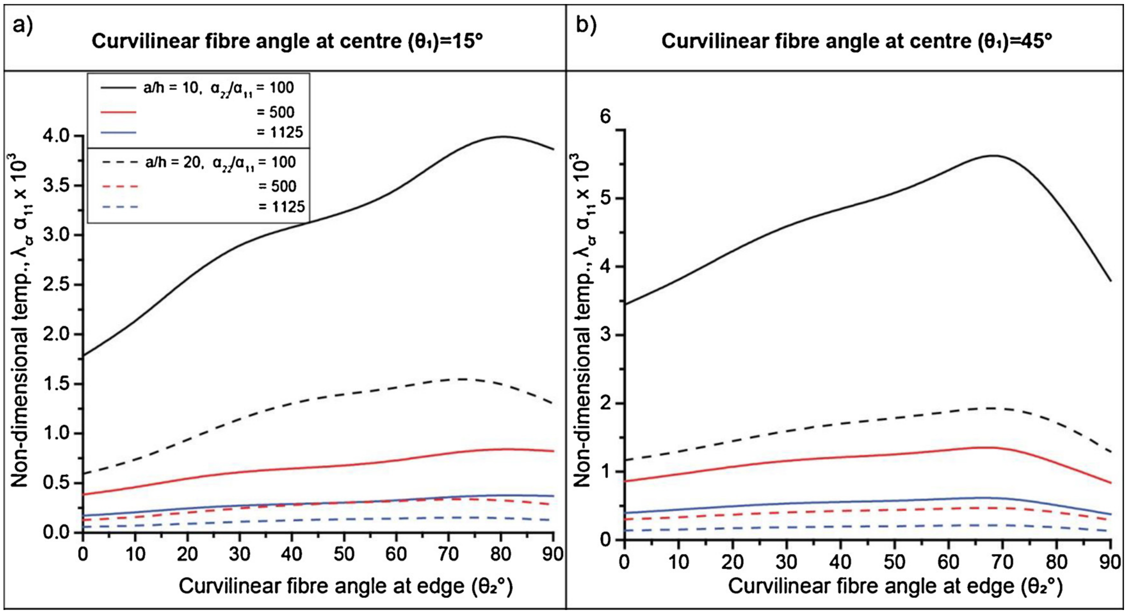

Fig. 6. The influence of thermal coefficient ratios (α_{22}/α_{11}) on the non-dimensional critical thermal buckling parameter ($\lambda_{cr}\ \alpha_{11}\ 10^3$) of variable stiffness laminate ($a/h = 10$; $[\ \mp\ (\theta_1/\theta_2)\ ]_4$) under non-uniform thermal distribution (case-2) and for two different values for curvilinear fibre angle at the centre of laminate θ_1 (15°& 45°).

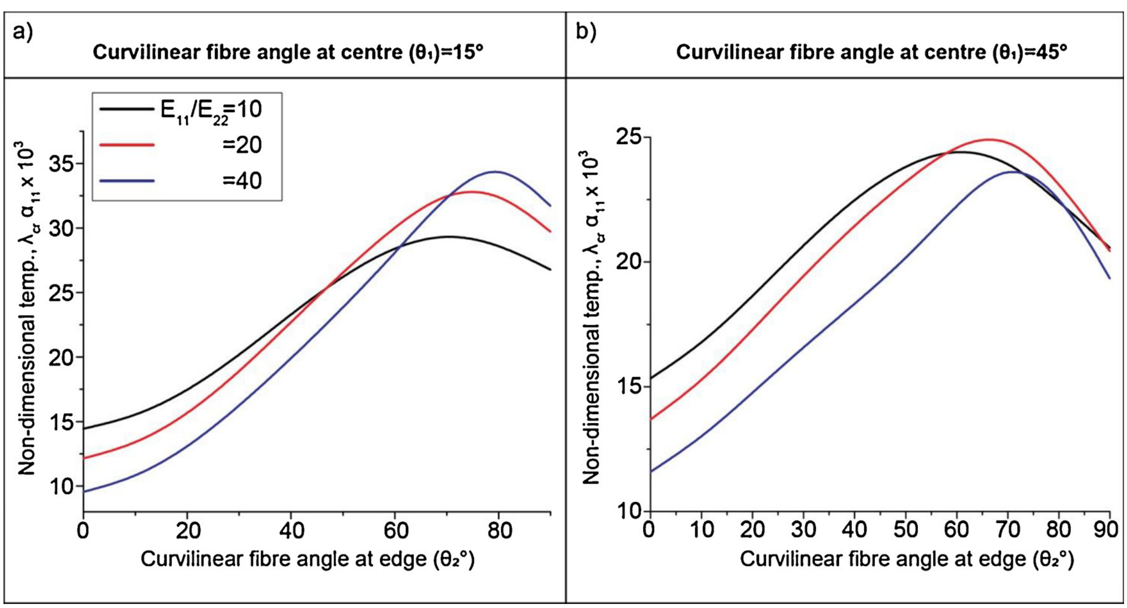

Fig. 7. Effect of modular ratios (E_{11}/E_{22}) on the non-dimensional critical thermal buckling parameter ($\lambda_{cr}\ \alpha_{11}\ 10^3$) of variable stiffness laminate ($a/h = 10$; $[\ \mp\ (\theta_1/\theta_2)\ ]_4$) under non-uniform thermal distribution (case-2) and two different values for curvilinear fibre angle at the centre of laminate θ_1 (15°& 45°).

Acknowledgement

This research work did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors. The authors also declare that they have no conflict of interest and the declaration of interest is none.

References

[1] A.K. Noor, W.S. Burton, Assessment of shear deformation theories for multilayered composite plates, ASME Appl. Mech. Rev. 42 (1989) 1–13.
[2] A.K. Noor, W.S. Burton, Assessment of computational models for multilayered anisotropic plates, Compos. Struct. 14 (1990) 233–265.
[3] R.K. Kapania, S. Raciti, Recent advances in analysis of laminated beams and plates, part I: shear effects and buckling, AIAA J. 27 (1989) 923–935.
[4] R.K. Kapania, A review on the analysis of laminated shells, J. Press. Vessel Technol. 111 (1989) 88–96.
[5] S. Murugan, M.I. Friswell, Morphing wing flexible skins with curvilinear fiber composites, Compos. Struct. 99 (2013) 69–77.
[6] M.W. Hyer, H.H. Lee, The use of curvilinear fiber format to improve buckling resistance of composite plates with central circular holes, Compos. Struct. 18 (1991) 239–261.
[7] P. Hao, X. Yuan, H. Liu, B. Wang, C. Liu, D. Yang, S. Zhan, Isogeometric buckling analysis of composite variable-stiffness panels, Compos. Struct. 165 (2017) 192–208.
[8] R. Vescovini, L. Dozio, A variable-kinematic model for variable stiffness plates: vibration and buckling analysis, Compos. Struct. 142 (2016) 15–26.
[9] A. Marouene, R. Boukhili, J. Chen, A. Yousefpour, Buckling behavior of variable-stiffness composite laminates manufactured by the tow-drop method, Compos. Struct. 139 (2016) 243–253.
[10] A. Madeo, R.M.J. Groh, G. Zucco, P.M. Weaver, G. Zagari, R. Zinno, Post-buckling analysis of variable-angle tow composite plates using Koiter's approach and the finite element method, Thin-Walled Struct. 110 (2017) 1–13.
[11] S.C. White, G. Raju, P.M. Weaver, Initial post-buckling of variable-stiffness curved panels, J. Mech. Phys. Solids 71 (2014) 132–155.
[12] C.S. Lopes, P.P. Camanho, Z. Gürdal, B.F. Tatting, Progressive failure analysis of tow-placed, variable-stiffness composite panels, Int. J. Solids Struct. 44 (2007)

8493–8516.
[13] S. Setoodeh, M.M. Abdalla, S.T. Ijsselmuiden, Z. Gürdal, Design of variable-stiffness composite panels for maximum buckling load, Compos. Struct. 87 (2009) 109–117.
[14] H. Ghiasi, K. Fayazbakhsh, D. Pasini, L. Lessard, Optimum stacking sequence design of composite materials part II: variable stiffness design, Compos. Struct. 93 (2010) 1–13.
[15] O. Falcó, J.A. Mayugo, C.S. Lopes, N. Gascons, A. Turon, J. Costa, Variable-stiffness composite panels: as-manufactured modeling and its influence on the failure behaviour, Compos. Part B 56 (2014) 660–669.
[16] M. Arian Nik, K. Fayazbakhsh, D. Pasini, L. Lessard, Optimization of variable stiffness composites with embedded defects induced by automated fiber placement, Compos. Struct. 107 (2014) 160–166.
[17] A. Venkatachari, S. Natarajan, M. Ganapathi, M. Haboussi, Mechanical buckling of curvilinear fibre composite laminate with material discontinuities and environmental effects, Compos. Struct. 131 (2015) 790–798.
[18] A. Venkatachari, S. Natarajan, K. Ramajeyathilagam, M. Ganapathi, Assessment of certain higher-order structural models based on global approach for bending analysis of curvilinear composite laminates, Compos. Struct. 118 (2014) 548–559.
[19] P. Ribeiro, H. Akhavan, A. Teter, J. Warmiński, A review on the mechanical behaviour of curvilinear fibre composite laminated panels, J. Compos. Mater. 48 (2014) 2761–2777.
[20] Z. Wu, G. Raju, P.M. Weaver, Optimization of postbuckling behaviour of variable thickness composite panels with variable angle tows: towards “Buckle-Free” design concept, Int. J. Sol. Struct. 132–133 (2018) 66–79.
[21] M.M. Abdalla, S. Setoodeh, Z. Gürdal, Design of variable stiffness composite panels for maximum fundamental frequency using lamination parameters, Compos. Struct. 81 (2007) 283–291.
[22] P. Ribeiro, H. Akhavan, Non-linear vibrations of variable stiffness composite laminated plates, Compos. Struct. 94 (2012) 2424–2432.
[23] A. Houmat, Nonlinear free vibration of laminated composite rectangular plates with curvilinear fibers, Compos. Struct. 106 (2013) 211–224.
[24] P. Ribeiro, S. Stoykov, Forced periodic vibrations of cylindrical shells in laminated composites with curvilinear fibres, Compos. Struct. 131 (2015) 462–478.
[25] A. Venkatachari, S. Natarajan, M. Haboussi, M. Ganapathi, Environmental effects on the free vibration of curvilinear fibre composite laminates with cutouts, Compos. Part B 88 (2016) 131–138.
[26] P. Tan, G.J. Nie, Free and forced vibrations of variable thickness composite annular thin plates with elastically restrained edges, Compos. Struct. 149 (2016) 398–407.
[27] M.A.R. Loja, J.I. Barbosa, C.M. Mota, Dynamic instability of variable stiffness composite plates, Compos. Struct. 182 (2017) 402–411.
[28] A. Venkatachari, N. Sundararajan, M. Ganapathi, Variable stiffness laminated composite shells – Free vibration characteristics based on higher-order structural theory, Compos. Struct. 188 (2018) 407–414.
[29] T.R. Tauchert, Thermally induced induced flexure, buckling and vibration of plates, ASME Appl. Mech. Rev. 44 (1991) 347–360.
[30] A.K. Noor, W.S. Burton, Computational models for high temperature multi-layered plates and shells, ASME Appl. Mech. Rev. 45 (1992) 419–446.
[31] W.J. Chen, P.D. Lin, L.W. Chen, Thermal buckling behavior of thick composite laminated plates under nonuniform temperature distribution, Compos. Struct. 41 (1991) 637–645.
[32] M. Ganapathi, M. Touratier, A study on thermalpostbuckling behaviour of laminated composite plates using a shear-flexible finite element, Finite Elem. Anal. Des. 28 (1997) 115–135.
[33] T. Kant, C.S. Babu, Thermal buckling analysis of skew fibre-reinforced composite and sandwich plates using shear deformable finite element models, Compos. Struct. 49 (2000) 77–85.
[34] A.R. Vosough, P. Malekzadeh, M.R. Banan, Thermal postbuckling of laminated composite skew plates with temperature-dependent properties, Thin-Walled Struct. 49 (2011) 913–922.
[35] D.P. Makhecha, M. Ganapathi, B.P. Patel, Dynamic analysis of laminated composite plates subjected to thermal/mechanical loads using an accurate theory, Compos. Struct. 51 (2001) 221–236.
[36] L.C. Shiau, S.Y. Kuo, C.Y. Chen, Thermal buckling behaviour of composite laminated plates, Compos. Struct. 92 (2010) 508–514.
[37] R. Vescovini, M. D’Ottavio, L. Dozio, O. Polit, Thermal buckling response of laminated and sandwich plates using refined 2-D models, Compos. Struct. 176 (2017) 313–328.
[38] M. Cetkovic, Thermal buckling of laminated composite plates using layerwise displacement model, Compos. Struct. 142 (2016) 238–253.
[39] M.K. Singha, L.S. Ramachandra, J.N. Bandyopadhyay, Thermal postbuckling analysis of laminated composite plates, Compos. Struct. 54 (2001) (2001) 453–458.
[40] B.P. Patel, A.V. Lele, M. Ganapathi, S.S. Gupta, C.T. Sambandam, Thermo-flexural analysis of thick laminates of bimodulus composite materials, Compos. Struct. 63 (2004) 11–20.
[41] B.P. Patel, M. Ganapathi, D.P. Makhecha, Hygrothermal effects on the structural behaviour of thick composite laminates using higher-order theory, Compos. Struct. 56 (2002) 25–34.
[42] H.-S. Shen, Thermal buckling and postbuckling of functionally graded fiber-reinforced composite laminated plates, J. Compos. Mater. 47 (2013) 2783–2795.
[43] K.M. Liew, J. Yang, S. Kitipornchai, Thermal postbuckling of laminated plates comprising functionally graded materials with temperature dependent properties, ASME J. Appl. Mech. 71 (2004) 839–850.
[44] R. Javaheri, M.R. Eslami, Thermal buckling of functionally graded plates, AIAA J. 40 (2002) 162–169.
[45] M. Ganapathi, T. Prakash, Thermal buckling of simply supported functionally graded skew plates, Compos. Struct. 74 (2006) 247–250.
[46] T. Prakash, M.K. Singha, M. Ganapathi, Thermal postbuckling analysis of FGM skew plates, Eng. Struct. 30 (2008) 22–32.
[47] T. Prakash, N. Sundararajan, M. Ganapathi, On the nonlinear axisymmetric dynamic buckling behaviour of clamped functionally graded spherical caps, J. Sound Vib. 299 (2007) 36–43.
[48] T. Yu, T.Q. Bui, S. Yin, D.H. Doan, C.T. Wu, T. Van Do, S. Tanaka, On the thermal buckling analysis of functionally graded plates with internal defects using extended isogeometric analysis, Compos. Struct. 136 (2016) 684–695.
[49] A. Haldar, J. Reinoso, E. Jansen, R. Rolfes, Thermally induced multistable configurations of variable stiffness composite plates: semi-analytical and finite element investigation, Compos. Struct. 183 (2018) 161–175.
[50] G. Prathap, B.P. Naganarayana, B.R. Somashekar, A field consistency analysis of the isoparametric eight-noded plate bending elements, Comput. Struct. 29 (1988) 857–874.
[51] M. Ganapathi, P. Boisse, D. Solaut, Non-linear dynamic stability analysis of composite laminates under periodic in-plane compressive loads, Int. J. Numer. Method Eng. 46 (1999) 943–956.
[52] R.M. Jones, Mechanics of Composite Materials, McGraw-Hill, New York, 1975.
[53] O.C. Zienkiewicz, R.L. Taylor, The Finite Element Method, McGraw-Hill, Singapore, 1989.
[54] B.A. Boley, J.J. Weiner, Theory of Thermal Stress, Wiley, New York, 1960.