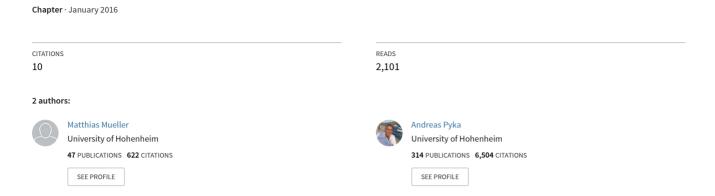
# Economic Behaviour and Agent-Based Modelling



# Chapter: Economic Behaviour and Agent-Based Modelling

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Published in: Routledge Handbook of Behavioral Economics

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#### **Abstract:**

Agent-based modelling (ABM) has been acknowledged by many scholars and various scientific disciplines as a promising tool to investigate the dynamics of complex systems, such as economic systems. In particular in economics, ABM increasingly gains momentum and is considered as an approach with the potential to overcome the shortcomings of the mainstream approach with its optimization- and equilibrium-oriented framework which requires substantial simplifications of the phenomena under investigation. This chapter describes the main characteristics of the ABM approach. We focusing on the role of ABM as a scientific tool, addressing also common caveats and pitfalls and emphasizing the complementary relationship of ABM and behavioural economics.

#### 1. Introduction

Since the 1980s, continuous improvements in computer technologies have remarkably changed scientists' possibilities to develop and apply computer simulations. Simulations as a scientific tool offer new ways to explore the dynamics of complex models in various disciplines. They enable us to process scientists' thought models into artificial test laboratories in which we can systematically analyse the models' outcomes.

Within the broad field of simulation techniques, the so-called agent-based modelling (ABM) approach has gained increasing momentum, not only for economics but also in many other scientific disciplines. The ABM approach takes the perspective of the system building elements and focuses on the actions and interactions of these entities as the relevant actors within the system. This perspective of ABM, is accompanied by the attempt to represent actors of economic systems in a more realistic fashion, thereby overcoming the shortcomings of approaches limited to representative agents which by definition ignore heterogeneity and the related implications of interacting heterogeneous agents. In this vein, the research objectives of ABM show a strong similarity with behavioural economics, where deviations from the assumed theoretical behaviour play an outstanding role.

The value of such the computational modelling approach becomes clear when we look at the experience gained during the 2010 financial crisis. As Jean-Claude Trichet phrased it:

"When the crisis came, the serious limitations of existing economic and financial models immediately became apparent. [...] Macro models failed to predict the crisis and seemed incapable of explaining what was happening to the economy in a convincing manner. [...] We need to deal better with heterogeneity across agents and the interaction among those heterogeneous agents. [...] Agent-based modelling dispenses with the optimization assumption and allows for a more complex interactions between agents. Such approaches are worthy of our attention." (Trichet 2010)

One reason for the dissatisfaction with the currently dominant paradigm in economics is substantiated in the compulsory assumption of "rational expectations" (Farmer and Foley 2009). Empirical evidence shows that this assumption clearly is at odds with real human behaviour (see for example the experiments conducted by Güth et al. 1982, Berg et al. 1995). To overcome this constrictive paradigm, we clearly need new insight into human behaviour

and the development of institutions, which are prominently addressed by the behavioural economics approach. Given the enormous degree of complexity, however, we also need a methodological framework, such as ABM, to incorporate the individual and heterogeneous behaviour of economic actors into models which are capable to portray the emerging dynamics.

In an early attempt Bob Axelrod characterizes ABM by "the existence of many agents who interact with each other with little or no central direction" (Axelrod 1997). A final consensus, of what agent-based modelling is and how scientists can use it to increase our understanding of the complex economic system so far is not existent. The aim of this chapter is to shed some light on the agent-based modelling approach as a promising and possibly necessary tool in the scientist's toolbox. We focus, therefore, on the most central methodological issues, reviewing the current state of literature and highlighting the possibilities of this still somewhat disputed approach and emphasizing the complementary relationship of ABM and behavioural economics.

To do so, we start in section two with an introduction to the three pillars of agent-based modelling: *modelling*, *agents and simulation*, which are characteristic for all agent-based models. In section three we then introduce important methodological aspects, every agent-based modeller must be aware of. Finally, we conclude with a number of remarks and a brief outline of the complementary relationship between ABM and behavioural economics.

### 2. The three pillars of ABM

Despite the awareness ABM has gained for many years, a common protocol for this method is still missing. Driven by the increasing computer resources, different scientific disciplines discovered the versatile possibilities offered by this modelling approach, which lead to different notions and understandings of agent-based modelling used in economic science.

To illustrate the diversity of notions and emphasis, agent-based modelling also is also labelled as: **agent-based modeling** (Epstein and Axtell 1996), **agent-based simulation modeling** (Polhill et al. 2001), **multi-agent simulation** (Ferber 1999, Gilbert and Troitzsch 2005), **multi-agent-based simulation** (Edmonds 2001), **agent-based social simulation** (Doran 2001, Downing et al. (2001), **individual-based configuration modeling** (Judson

1994), multi-agent systems (Bousquet and Le Page 2004), and agent-based computational economics (Tesfatsion 2002).

Despite missing consent concerning the appropriate notion for this scientific method, the understanding of ABM is very similar (see also Hare and Deadman (2004) for a discussion on this issue). In the following section we will describe three pillars of agent-based modelling: *modelling*, *agents and simulation*, which will frame our understanding of ABM.

#### 2.1 Modelling from an agent-based perspective

The most important element of agent-based modelling is its *bottom-up perspective* - describing a system from the perspective of its constituent units, i.e. the agents (Bonabeau 2002). In short, building the model from the bottom-up means, letting complex macroscopic systems emerge from the interactions of microscopic entities (Epstein and Axtell 1996, Axelrod 1997). A good example illustrates this underlying principle is the artificial life program BOIDS by Reynolds (1987), which reproduces the complex behaviour of a flock of birds. Instead of treating a flock as a self-contained unit, Reynolds was able to recreate the behaviour of a flock of birds by disaggregating the flock into birds and therefore building the model from the bottom-up using only three simple behavioural rules for the birds:

- 1. Separation avoid crowding neighbours (short range repulsion),
- 2. Alignment steer toward average heading of neighbours and
- Cohesion steer toward average position of neighbours (long range attraction)
  (Reynolds 1987)

<!Fig. 1 near here!>

Figure 1: Flock of birds created by the BOIDS algorithm.

Although this example could be considered trivial, it shows that the behaviour of complex systems can only be reproduced adequately by taking into account the individual behaviour, i.e. the actions and interactions of the system building units. This also illustrates how ABM

differs from another numerical method which enjoys a certain popularity in economics, namely system dynamics. On the system level, the rich patterns of possible dynamics remains hidden, whereas it is in the explicit focus of ABM.

In economics, the ABM approach follows the same idea as Reynolds proposed, modelling (macroscopic) e.g. economic systems through the actions and interactions of (micro-) entities e.g. firms, universities etc. ABM hereby depart from the top-down perspective of mainstream economic models. Instead of representative individuals and constrained by strong consistency requirements associated with equilibrium and an Olympic rationality, ABM describes *heterogeneous* entities living in complex systems that evolve through time (Windrum 2007).

## 2.2 A definition of agents

Although the bottom-up perspective of ABM is without doubt a key characteristic, there are other approaches following the same logic of reasoning. In contrast for example to the related family of Cellular Automates (Wolfram 1986) or Microsimulations (Orcutt et al. 1986), the ABM approach places central a representation of agents in a more realistic way. Focusing on the individual behaviour of economic actors, agent-based models can display important concepts such as for example *heterogeneity* and *bounded rational behaviour* of agents which leads to all sorts of variation in their modelled behaviour.

An agent represents a dynamic entity, which can be assigned by the modeller with an individual role exhibiting a variety of characteristics. Although a final consensus has not been reached, in the literature it is often claimed that agents may possess the following properties (Woolridge & Jennings 1995):

**Autonomy:** Agents are autonomous entities with little or no central direction and have control of their actions.

**Social ability:** Agents can interact with other agents (e.g. receiving or sending information about locations or other internal states of others).

**Reactivity:** Agents have a perception of their environment (e.g. the landscape they are in).

**Pro-activeness:** Agents exhibit goal-directed behaviour, taking the initiative.

The list of possible characteristics of agents can be extended in several ways. Already in their original paper, Woolridge and Jennings (1995) named more human characteristics, for example knowledge, belief, intention, obligation and emotions etc. as possible additional features of agents (Woolridge & Jennings 1995, Jennings et al. 1998).

In ABM approaches economic actors are what they are, i.e. autonomous and heterogeneous entities embedded in an environment which is created by the actions and interactions of these agents (Gilbert and Troitzsch 2005). In contrast to traditional modelling approaches, the variety of agents and their behaviour are not restricted to fit into an analytical framework. Depending on the problem under investigation that is investigated and the scope of the corresponding model, agents can flexibly represent any kind of economic actor. On an aggregated level, this can be firms, universities, governmental bodies etc.; on an individual level, agents can be employees, scientists, consumers, households etc. From an abstract point of view, however, basically any independent component of a system can be considered as an agent (Bonabeau 2002, Macal and North 2010).

It is important to emphasize that with ABM we can specifically relax unrealistic assumptions about the agents and their behaviour. While in most mainstream models strong assumptions are required in order to guarantee in principle an analytical solvability (Farmer and Foley 2009), in ABM every agent is endowed with an individual set of initial states, which allows for the representation of characteristic features and a representation of individual behaviour. As a consequence, the ABM approach is able to incorporate the manifold insights from behavioural economics about human and institutional behaviour into fruitful models.

In particular, agents within an agent-based model can be assumed to have only limited information about the environment and the behaviour of other agents, limited foresight about the scope of decisions or other resource limitations, such as memory, etc. (Edmonds 1999). Building on that, Agent-based models are capable of displaying true heterogeneity of agents (Macal and North 2010). Heterogeneity in this sense means that agents are modelled as individual entities with individual states, and also with individual behaviours. Heterogeneity in the model can be assigned by the modeller according to the requirements of the problem under investigation, i.e. agents may be endowed with different levels of resources, initial knowledge stocks, strategies, reference systems etc. Additionally, heterogeneity is

endogenously created within the model through the actions and interactions of agents themselves.

Second, as the behaviour of agent-based models is not restricted to obtain an analytical solution, agents may be assumed to behave as rationally-bound entities (Pyka and Fagiolo 2007, see also Das 2006 for a detailed discussion). As the ABM approach focuses directly on the individual, it allows for an intentional non-rational design of economic decisions which e.g. allows for experimental adaption and learning. This enables us to model the effects of psychological principles as for example *reference dependence*, *loss aversion* and *non-linear probability weighting* postulated by the famous prospect theory by Kahneman and Tversky (1979).

The complexity of a model with individual and heterogeneous agents, however, quickly reaches a level where for example the traditional analytical framework fails to offer any solution. A way-out for this problem are computational simulation environments where even complex models can be studied in detail.

#### 2.3 Simulation as *in-silicio* laboratories

As a third pillar in our understanding of the ABM approach, we have to consider that agent-based models often are implemented within a simulation environment. Seeing simulation as a form of quasi-experiments, in principle, simple agent-based models can be carried out without the help of simulation tools. A prominent example for this is the famous *Segregation model* by Schelling (1969), which was originally conducted on a chessboard using coins of different colours.

The complexity of any model, however, grows exponentially with the magnitude of the model's assumptions, quickly reaching a level where computational support is necessary, such as in the processing of experimental results. Especially through the steady improvements in computer performance, but also in the progress made on the software side (e.g. object-oriented languages and simulation environments especially dedicated to ABM as for example NetLogo, LSD - Laboratory for Simulation Development, Repast - Recursive Porous Agent Simulation Toolkit etc.), today's simulations act as laboratories where agent-based models can be created and studied *in silicio* (Pyka and Fagiolo 2007).

For ABM simulation, tools facilitate a detailed look into the complex interplay between the model's assumptions and the resulting outcome. By building an agent-based model within a computer simulation environment, we have a tool at hand, which helps us to systematically observe and analyse the complex dynamics created by the actions and interactions of agents both on a macro as well as on a micro level. With a computational simulation, we are able to observe all relevant information of the simulation as it progresses. In contrast to real world experiments, simulations offer the possibility of recreating and repeating experiments with the same initial conditions. This gives us the opportunity to systematically alter model parameters and assumptions, and hence leads us to a comprehensive understanding of the model's outcome.

The complexity involved, however, still limits the possible scope of ABM. Although today's computer performance allows for models with an unforeseen range, the complexity of models will always be limited by the capabilities to process the data obtained by the simulation, especially on the researchers' side.

# 3. Using ABM as a scientific tool

Despite the new and promising perspective ABM offers, it is also necessary to deal with the question of how ABM can contribute to the scientific endeavour. If used properly, ABM offers researchers a new perspective on complex interplay within economic systems.

#### 3.1 Managing the complexity

Although ABM at its core makes a huge step towards a more realistic model of economic systems, one cannot expect a fully detailed picture. As with any model, an agent-based model is designed as a purposeful representation of a system rather than an exact and precise attempt to display real systems (Starfield et al. 1990). Purposeful in the broadest possible sense can be understood that a model helps the scientist to answer questions that are of interest (Minsky 1965).

Managing the complexity within a model, i.e. finding the right level of complexity, is one of the key challenges for any agent-based modeller. As previously stated, the possibility to implement agent-based models within a simulation environment, the increasing performance of computer systems, and constantly improving methods for data analysis and visualisation enable researchers to create models of unforeseen complexity and detail.

To start, almost by definition, we will be hardly able to define something as the optimum level of complexity that should be strived for by ABM in general. Yet there is an extreme we need to be aware of: if the complexity of the model is reaching a level where we ae no longer able to understand the processes involved, the experiments conducted are of little interest and we cannot understand these artificial complex systems any better than we understand the real ones (Gilbert and Terna 1999, Axtell and Epstein 1994).

The level of complexity of an ABM is determined by the actors that the modeller aims to include and the number of assumptions we consider to be relevant for the model. In particular, for models of economic systems there is a broad range of possible actors and assumptions which can be relevant. The modeller must decide carefully to what extend elements of the model are necessary or negligible, facing a common trade-off: while models aiming at prediction need descriptive accuracy, models designed for explanatory power should be rather simple (Axelrod 1997).

Considering the right strategy for building the complexity within ABM, the debate has triggered a rich methodological discussion in which we find three distinct modelling strategies. First, following the KISS (Keep It Simple, Stupid) strategy one should start with a simple model, which may be extended if necessary. A special case of the KISS strategy is the so called TAPAS approach ("Take A Previous model and Add Something"). Here one starts with an existing model and successively complicate it with incremental additions (Pyka & Fagiolo 2005).

In contrast, the *KIDS* (*Keep-it-descriptive, stupid*) strategy follows the idea of starting with a descriptive model first, which is then, if possible, simplified (Edmonds and Moos 2005). An illustrative example for the *KISS* modelling strategy is Schelling's model of segregation in North-American cities (Schelling 1969). In this model, Schelling uses a simple grid for the representation of a city in order to model neighbourhood relationships. He succeeds in identifying the mechanisms, which lead to strong clustering patterns of ethnic groups, even if this was only mildly intended in the individual behaviour of agents (Schelling 1969).

A good example for the *KIDS* strategy is the model of water demand by Edmonds and Moss (2004), which includes an extremely rich set of varying behaviour rules, preference systems, water consuming devices (power showers, water-saving washing machines etc.), pricing systems, and policy options. With the help of this rich set of elements, backed by empirical observations, the authors intend to model water demand in a region close to the real water demand.

It is important to understand that, in general, the KISS and the KIDS strategies do not differ concerning the degree of complexity. In principle, it is rather a question of how to get there, although in reality it is probably inevitable that the choice for a strategy concerning building the relevant assumptions will end in models with considerably different levels of complexity.

Without going into too much detail, the debate has many facets and a set of additional strategies must be included for a full picture (Pyka and Fagiolo 2007). It is, however, important to note, that agent-based research shouldn't be oppressed by the fear of complexity. Any claim that ABM is limited to investigate only simple dynamics in small systems is neglecting the possibilities of ABM. Applying ABM for the study of economic systems, however, requires a detailed methodological understanding of this research method and additional inputs from other disciplines, i.e. through a profound understanding of realistic behavioural heuristics.

#### 3.2 Two ways of using agent-based models

As stated before, the bottom-up perspective of ABM builds on the actions and interactions between heterogeneous agents to analyse the multilevel effects on the overall system, the environment and the agents themselves. Through this, ABM gives a unique understanding of the processes of the interplay between micro and macro levels of a complex system.

The particular role of ABM for the scientific endeavour still is under debate. Facing critiques questioning the scientific value of ABM the modeller needs to be aware of how ABM can be used to deepen our understanding. Based on the literature we can distinguish between two, as will be argued later, complementary perspectives facing different questions.

On the one hand, following the idea introduced by Epstein and Axtell (1999), ABM can be used in a *generative* perspective. ABM in this sense focuses on the possibilities to display

the emergence of complex macroscopic system behaviour by the actions and interactions of agents on a micro level. In other words, agent-based models can be designed to find microspecifications that can explain (from a generative perspective *grow*) a macro-level phenomenon of interest (Epstein 1999). In the literature this often is put on a level of reproducing stylised facts through ABM to validate models.

To name just a few examples for interesting models reproducing stylised facts, Thomas Schelling (1969) showed, as already mentioned, with his famous Segregation model, that the macroscopic pattern of segregation in cities can be explained by even a minor preferences of inhabitants for neighbourhoods with the same colour. In 1996 Epstein and Axtell (1996) reproduced amongst other things right-skewed wealth distributions based on their seminal Sugarscape model. The scope of the model, has been extended during the past years continuously, implementing new features carrying forward new aspects to the model (see Epstein 1999 for a list of other interesting models reproducing stylised facts). The logic of reasoning behind the generative perspective of modelling stylized facts constitutes also a severe methodological caveat of the ABM approach which is stressed also by Epstein (1999) or Gilbert and Terna (2000): In principle it is possible that similar macroscopic system behaviour might be generated by models which refer to different assumptions with respect to the behavioural rules of agents on the micro-level. The agent-based modeller simply cannot presume that the microscopic behaviour of agents found to reproduce stylized facts is in fact an accurate and relevant description of the phenomena of interest. For this reason a final proof for the validity of an agent-based model which reproduces stylized facts is only hard to be given.

On the other hand, using agent-based models is not limited to reproducing stylized facts and thereby finding *possible* explanations for emerging macroscopic patterns. While the starting point for agent-based models following the generative idea are patterns on the macro level, another strand of models in the literature is based on the possibility to perform a wide range of numerical experiments and hence analyse what emerging patterns arise based on micro specifications of the model. The focus here is on the improvement of our understanding of the dynamic processes within a complex system. ABM from this perspective acts as a laboratory for computational experiments created *in silicio* (Pyka and Fagiolo 2007, Leombruni 2002).

As Axelrod puts it: "Simulation is a third way of doing science. Like deduction, it starts with a set of explicit assumptions. But unlike deduction, it does not prove theorems. Instead, a simulation generates data that can be analysed inductively. Unlike typical induction, however, the simulated data comes from a rigorously specified set of rules rather than direct measurement of the real world." (Axelrod 1997).

Detached from the need to reproduce stylized facts, ABM can also be as a scientific method to increase our understanding of complex systems. While the starting point of ABM in a generative sense are macroscopic patterns, the starting point of ABM as an *in-silicio* laboratory is a set of specifications for the micro level, i.e. a well-grounded set of behavioural rules on the agent level. Tesfatsion (2006) distinguishes here between two different objectives of ABM. First, ABM can be used to get a *normative understanding*, evaluating whether designs proposed for economic policies, institutions, and processes will result in socially desirable system performance over time. Second, ABM allows for *qualitative insights and theory generation*. The objective here is to understand economic systems through a systematic examination of their potential dynamical behaviours under alternatively specified initial conditions. Using ABM as an *in-silicio* laboratory, however, comes at a cost. To gain wide acceptance, the model need to be validated through a broad set of empirical micro-level data.

#### 3.3 The need for verification, validation and calibration

The most common critique against agent-based models is perceived lack of robustness. To start, any modeller needs to be aware of the possibility of simple failures in the implementation of the model in a computer simulation. This *verification* of the computer program is a crucial step for the credibility of a model.

Verification simply asks whether the model does what we think it is supposed to do (Ormerod and Rosewell 2009). Verification, however, is more than just looking for bugs in the computer code. As for example Dawid and Kopel (1998) note: "we have to be aware of the fact that simulation results may crucially depend on implementation details which have hardly any economic meaning". In other words, even though the technical code might be correct, the way how the model is translated and implemented in a computer software might lead to biased results which have strong effects on the model's outcome, yet were unintended by the modeller. Especially for complex models, it is therefore necessary to make the

implementation of the agent-based model transparent for other researcher. This can either be done by providing the original simulation code or by making main procedures public, including the pseudo code in the publication.

Despite the correct implementation of the simulation model, the modeller is also often confronted with critiques questioning whether the model is an accurate representation of the real world from the perspective of the models intended applications, i.e. the model's validity (Ormerod and Rosewell 2009). Following the distinction made in section 3.2 we can differ between two ways of *validation*: i.e. the *input validation* and *output validation*. While the latter refers to the matching of model results and against acquired real-world data, the former regards ensuring that the fundamental structural, behavioural and institutional conditions incorporated in the model reproduce the main aspects of the actual system (Bianchi et al. 2008).

Without going into much detail, this issue is still under great debate and several strategies have been developed to approach the problem of validity of agent-based models, such as for example the *indirect calibration* approach and the *Werker-Brenner* approach etc. The underlying concept of these validation strategies is an elaborate multilevel approach where input and output validation is combined. Starting with model designed to reproduce stylized facts, the modeller can use the micro-specifications found to be valid to replicate the macro patterns as the starting point for a wide set of simulations experiments aiming to give further insights into the dynamical behaviours of the model (see for example Windrum et al. 2007, Ormerod and Rosewell 2009, Werker and Brenner 2004 for interesting discussions on this issue).

Although without doubt, the validation of an agent-based model is of particular interest, especially if we want to derive valid outcomes such as policy recommendations, we have to be aware that the complexity of ABM always invites for criticism. Despite any thorough validation of the model, we may be confronted with questions whether all elements of the model are necessary or if other, so far missing elements, are relevant too and hence, should be included. Second, the possibility of performing validation is restricted by the set of relevant Data available. For validation of agent-based models we need more that some widely accepted macroeconomic stylised facts. As ABM build on the actions and

interactions of heterogeneous economic actors, this approach requires a fundamentally new understanding of the behaviour of these actors.

#### 4. Conclusions

Shifting the focus from an over simplifying perception of economic systems to a more realistic one, ABM is designed to overcome the limiting possibilities of a traditional analytical framework. Central to this new approach is its exceptional perspective of economic actors, treating economic agents as heterogeneous and individual actors which build economic systems from the bottom up.

Using this method, however, comes at cost. We need to be aware of how ABM can be used to deepen our understanding of complex economic processes. Although ABM at its core makes a huge step towards a more realistic model of economic systems, one cannot expect a fully detailed picture. As with any model, agent-based model is designed as a purposeful representation of a system. Purposeful in that sense means that ABM can be used in a *generative* way, reproducing macro level pattern and hence finding *possible* explanations on the micro level, but also as an *in-silicio* test laboratory where we study the outcome of different micro-level specifications.

Especially for the latter case, despite any effort to ensure validity of our models, the complexity created within ABM invites for critiques. To encounter them, we need a better understanding of the behaviour of economic actors but also well accepted standard-models. So far, ABM cannot aim for including all possible aspects of an economy. In contrast, we need at this point to focus on a new joint understanding of basic economic processes, emerging from a profound acknowledgement of human behaviour. Building on that, we can stepwise increase the complexity of our models, gaining descriptive accuracy and hence increasing the predictive power of ABM.

In this light, it is also important to emphasize the complementary relationship of ABM with behavioural economics, offering the possibility for fruitful cross-fertilisation. In behavioural economics, instead of a general optimization approach, decision rules are frequently informed by psychological insights of human behaviour, which serve as a heuristic description of decision processes. Empirical observations confirm in many instances a deviation from a theoretically derived optimal solution. As these deviations are systematic

and not random, their explanation is of high scientific interest. In many cases, these empirical observations can be explained by behavioural economists with the application of decision heuristics for comparatively simple and artificial cases. ABM allows now for an extension of these comparatively simple and artificial cases towards models with a higher degree of complexity. With this it can be tested whether unexpected feedbacks and phase transitions resulting from the interaction of heterogeneous agents might have a strong impact on the expected results. Contrariwise, ABM finds a rich collection of decision heuristics in the behavioural economics literature which can be used to program agents' decision rules, framing a new and profound understanding of economic systems.

#### **Acknowledgements:**

This work is part of the research project VISIBLE – Virtual Simulation Lab for the Analysis of Investments in Learning and Education.

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