Artificial Multi-Agent Stock Markets: Simple Strategies, Complex Outcomes

A.O.I. Hoffmann, S.A. Delre, J.H. von Eije and W. Jager

University of Groningen
Faculty of Economics
Landleven 5
9700 AV Groningen
The Netherlands
Email: a.o.i.hoffmann@rug.nl

Reference:

Hoffmann, A. O. I., Delre, S. A., Von Eije, J. H., & Jager, W. (2006). Artificial Multi-Agent Stock Markets: Simple Strategies, Complex Outcomes. In C.Bruun (Ed.), Advances in Artificial Economics: The Economy as a Complex Dynamic System (pp. 167-176). Heidelberg: Springer Verlag.

1. Introduction

Both micro level investor behavior as well as macro level stock market dynamics are research fields that are full of "puzzles" or unresolved research questions and therefore enjoy a strong interest of scholars and practitioners alike. On a micro level, aberrances in individual investor behavior are the subject of intense debate in e.g., behavioral finance (for an introductory overview of the field, see e.g., Nofsinger 2002; Schleifer 2000; Shefrin 2002; Shiller 2005). On a macro level, the absence of (linear) autocorrelation, and the occurrence of fat tails and volatility clustering in asset returns distributions are often studied stylized facts (Cont 2001).

Methodologically, there is a great heterogeneity in the techniques used to solve the above-mentioned puzzles. Surveys, case studies, laboratory experiments, and a plethora of statistical analysis are amongst the many methods that are used in this field. A relatively recent development in finance is the use of multi-agent simulation models as a research method (LeBaron 2000, 2005). The usage of multi-agent simulation models allows researchers both to make a coupling between the before identified micro and macro levels and to get a better understanding of the complexity that is often experienced in this field.

Investor behavior and related stock market dynamics are fields par excellence to observe complexity. Often, macro level outcomes, such as crashes and bubbles, "emerge". Interaction and nonlinearity in the micro level behavior of actors may cause these emergent phenomena. Small changes in the initial situation of a model or in the behavior of one or several interacting actors may lead to completely different outcomes on a macro level. Social simulation is a particularly appropriate tool in helping to explain the interactions between the micro and macro level of this complex behavior.

2. Background

A first step when using multi-agent simulation research to solve puzzles in finance is to formalize a limited number of micro level agent rules that, in the ideal situation, represent empirically found characteristics of investors' behavior. A population of investor agents is generated by the simulation model and these agents are provided with these rules. Subsequently, a number of simulation experiments are performed and finally, the macro level results (often in the form of stock price or returns time series) of these simulation experiments are compared with data from real stock markets in order to see to what extent real-life stylized facts are replicated.

In this paper, we continue the line of research of Hoffmann, Delre, Von Eije, & Jager (2005). In that paper, the need to incorporate theories of social needs, social interactions and social networks of investors in finance research - as first introduced in Hoffmann and Jager (2005) - was argued for. The objective of the research program is to identify critical micro level factors that drive investors' behavior and to explain complex macro level phenomena that result from the aggregation and interaction of micro level investor behavior. An adapted version of the model of Day & Huang (1990) is explored, which can be seen as a simple nonlinear dynamical system. The power of this simple model resides in the fact that simple agent rules are able to generate non-linear dynamics like stock market price and returns time series. Without any news, e.g., in the form of noise, this model is able to capture a number of stylized facts that are often observed in financial markets, like volatility clustering. The interactions between fundamental and trend following agents alone is enough to generate these complex outcomes. In the next section, the model will be briefly described.

3. The simulation model

In the model, investors can follow either a more fundamentally based "rational" strategy (called the α-strategy) or a more socially based trend following strategy (called the β-strategy). The α-strategy is based on a comparison between the current market price p and a given long-run investment value u. Whenever the market price is below the long-run investment value, the α-investor buys. Whenever the market price is above the long-run investment value, the α-investor sells. When the market price equals the long-run investment value, the α-investor holds. This behavior is limited by a topping price M (set at 1.0) and a bottoming price m (set at 0.0). The β-strategy, on the other hand, suggests more socially oriented behavior. β-investors buy when they expect an upward price trend (whenever the current price p is above a given current fundamental value v) and sell when they expect a downward price trend (whenever p is below v).

The extent to which investors follow an α-strategy or a β-strategy is weighted by the parameter S_i that represents the social susceptibility of an investor i. Stock markets and stocks alike may differ to the extent that investors focus more on fundamental characteristics of a share like price/earnings ratios and beta's, or focus more on social aspects of a share like information about which shares friends, colleagues or prominent finance experts buy. Investors may change their S given the circumstances, which leads to dynamism in the strategies they use.

The above can be formalized in the following simple formula for total excess demand:

$$E_i(p) = (1 - S_i) * (u - p) + S_i * (p - v) \tag{3.1}$$

At each time step, the price will rise when there is a positive excess demand and the price will fall when there is a negative excess demand [1]. The price is calculated as:

$$p_{t+1} = \begin{cases} E_i(p_t) > 0 \rightarrow \left|E_i(p_t)\right| + p_t * (1 - \left|E_i(p_t)\right|) \\ otherwise \rightarrow p_t * (1 - \left|E_i(p_t)\right|) \end{cases} \tag{3.2}$$

Following Day and Huang (1990), we assume α and β strategies to be individual strategies. Therefore, the excess demand is also an individual indicator of how much a single investor wants to buy or to sell. However, this leads to the problem that the total excess demand, E(p) can overpass the boundary conditions 0.0 and 1.0.

$$E(p) = \sum_{i=0}^{n} E_i(p) \tag{3.3}$$

[1] This is a common way of updating the price, see e.g., Cont & Bouchaud (2000).

This leads to explosive price developments and a very limited parameter space for which useful price time series can be studied. We bounded the total excess demand between 0.0 and 1.0 using an exponential transformation (3.4).

$$E(p) = 1 - \gamma \cdot \exp(-\left|\sum_{i=0}^{n} E_i(p)\right|) \tag{3.4}$$

Here γ represents how strongly the market reacts to investors' actions. This parameter γ is comparable to the price adjustment coefficient c as used by Day and Huang (1990).

On the individual level, the behavior of the investors is driven by the parameter S. However, the behavior of investors is not the same in all circumstances. Investors can change their S according to their feelings and their fears (3.5). We formalize the changes in S as a combination of the agent's confidence coming from previous returns and fear coming from the deviation of the price from the fundamental value. The returns are derived from an estimation of how good individual investor agents have forecasted the price for the next period, better forecasts implying superior returns (3.6). Investors with higher returns are expected to feel more confident. The more the current price deviates from the fundamental value, the higher the fear of investors that the stock price developments will reverse, possibly leading to losses for these investors. Therefore, at certain moments in time, trend following investors may decide to return to a more "rational" or fundamental's based strategy. This adaptation of the model (the addition of a switching mechanism in the investors' strategy) also addresses the weak point of the standard model as identified in Hoffmann et al. (2005) and more generally in Arthur (1995). This was that the market dynamics are generated by the actions of the investors, but the cognition of the investors is never affected by the evolution of the market.

$$S_i = 1 - (confidence_i * fear)_i \tag{3.5}$$

$$confidence_i = 1 - \exp(-returns_i) \tag{3.6}$$

$$fear_i = \exp\left(\frac{-(p_t - v)^2}{\delta}\right) \tag{3.7}$$

$$returns_i = \frac{1}{(p_t - p_{forecasted})^2} \tag{3.8}$$

It should be noticed, that the only parameter that is introduced in comparison to the previous version of the model is δ. This is the individual tendency of investors to be afraid. When this tendency is higher, investors will more quickly develop feelings of fear in case the current price deviates from the fundamental value. We interpret this parameter as the speed of investors' reaction to changes in the price relative to the fundamental value. We fix δ for every investors or we distribute it uniformly (e.g., δ=[0.0, 1.0]). In the next section, a number of preliminary results - both *with* and *without* the switching mechanism - are discussed.

4. Results

In the first experiment, the influence of changing levels of trend following versus fundamental investors on the stock market dynamics is investigated.

In experiment 1.1, the level of trend following investors is uniformly distributed between 0.10 and 0.12, resulting in an average level of trend following investors of 0.11.

In experiment 1.2, the level of trend following investors is uniformly distributed between 0.1 and 0.5, resulting in an average level of trend following investors of 0.3.

In both experiments, the starting price p is 0.501, the long run investment value u and the current fundamental value v are both 0.500, and there are 100 investing agents.

The results of 500 time steps were studied [2] and it was found that with lower proportions of trend following investors (as in experiment 1.1), the standard deviation of returns is much smaller than with larger proportions of trend following investors (as in experiment 1.2). This result indicates that social interaction amongst investors may lead to an increasing level of stock market volatility, as measured by the standard deviation of the returns. This is intuitive in the sense that if an increasing number of investors rely on a social strategy to make their investment decisions, it becomes more likely that herding behavior, the corresponding stock price inflation, and increased stock market volatility occurs. Moreover, this result confirms the results of the earlier study by Hoffmann et al. (2005).

In figure 4.1 and 4.2, the returns time series of experiments 1.1 and 1.2, respectively, are plotted.

[2] As a robustness check, for every simulation experiment, at least 20 runs were performed with different initial conditions.

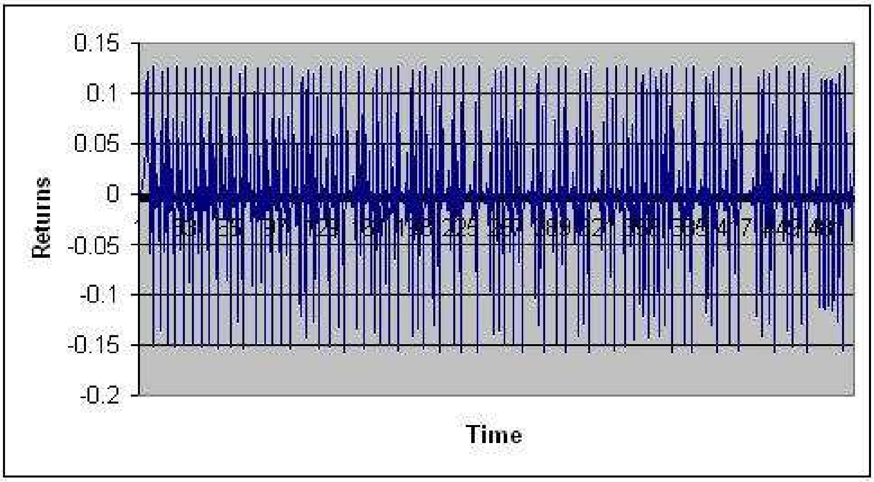

Fig. 4.1. Returns time series from experiment 1.1

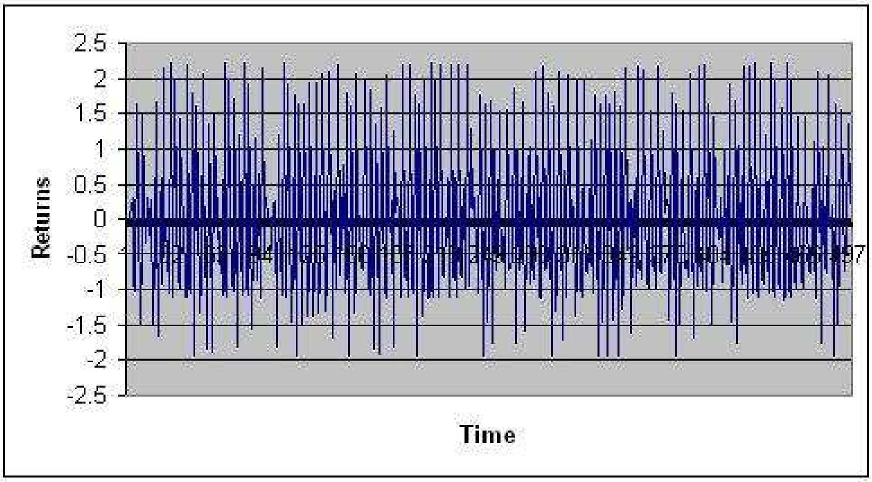

Fig. 4.2. Returns time series from experiment 1.2

In the second experiment, the influence of two different levels of the parameter δ on the stock market returns was studied. In both experiments, the initial proportion of trend following investors is 0.11, the starting price p is 0.501, the long run investment value u and the current fundamental value v are both 0.500, and there are 100 investor agents. For 500 time steps, the results were studied. In experiment 2.1, the value of δ is 0.5 and in experiment 2.2, the value of δ is 1.0.

It was found that when investors have a higher initial individual tendency to become afraid (indicated by a higher level of δ), the risk of previous periods becomes less important for the risk of today, as measured by ARCH [3] and GARCH [4] effects. The ARCH term represents the lagged squared error, while the GARCH term represents the lagged conditional variance. In tables 4.1 and 4.2, the ARCH and GARCH effects are displayed in the conditional variance equations for experiment 2.1 and 2.2, respectively. When investors react more fiercely to deviations of the current price from the current fundamental value, and therefore switch more easily from a trend following to a more fundamental or "rational" strategy, the stock markets become more stable, in the sense that there is less volatility clustering. So, the fear of future losses might limit the current stock market volatility.

Table 4.1. Conditional variance equation of experiment 2.1

	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.017435	0.024994	-0.697575	0.4854
		Variance Equation		
C	0.073749	0.012858	5.735721	0.0000
RESID(-1)^2	-0.136801	0.007226	-18.93076	0.0000
GARCH(-1)	0.760946	0.068734	11.07094	0.0000
R-squared	-0.001595	Durbin-Watson stat		2.355628

Table 4.2. Conditional variance equation of experiment 2.2

	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.002547	0.025472	-0.099991	0.9204
		Variance Equation		
C	0.157537	0.048608	3.240967	0.0012
RESID(-1)^2	-0.180182	0.012961	-13.90142	0.0000
GARCH(-1)	0.495622	0.202700	2.445101	0.0145
R-squared	-0.000020	Durbin-Watson stat		2.156982

[3] ARCH is the test for conditional heteroscedasticity as developed by Engle (1982).

[4] GARCH is the generalized model for conditional heteroscedasticity as developed independently by Bollerslev (1986) and Taylor (1986).

In the third experiment, the returns for each individual agent in the agent population as aggregated over the 500 time steps of the simulation were calculated using formula 3.8, resulting in 100 observations (one for each agent). Also, for each agent, the level of S was recorded. Scatter plots of the relationship between the level of S and the returns were made for two situations; a situation with a lower average level of S and a situation with a higher average level of S. In experiment 3.1, S was set as a uniform distribution between 0.01 and 0.21, resulting in an average level of S of 0.11. In experiment 3.2, S was set as a uniform distribution between 0.1 and 0.5, resulting in an average level of S of 0.3. This experimental design leads to the observation of the following phenomenon.

In stock markets that are dominated by "rational" investors using a fundamental strategy as in experiment 3.1, investors with higher levels of S have higher returns than investors with lower levels of S. So, in these markets it is beneficial to be a trend following investor, and these investors can be said to be "free-riding" on the fundamental investors. In figure 4.3, this relationship is plotted.

However, in markets with a higher average level of trend following investors, as in experiment 3.2, a more complex pattern emerges. In these markets, the relationship between the level of S and the individual returns follows a U-shape, as can be seen in figure 4.4. Investors with relatively low levels of S have high returns, and so do investors with relatively high levels of S. Investors with intermediate levels of S are proverbially "stuck in the middle", as they earn lower returns than these other two groups of investors. So, in this market an investor should either be a pronounced "rational" investor following a fundamental strategy or a pronounced trend following investor in order to obtain high returns. Overall, the returns are higher in experiment 3.2 than in experiment 3.1.

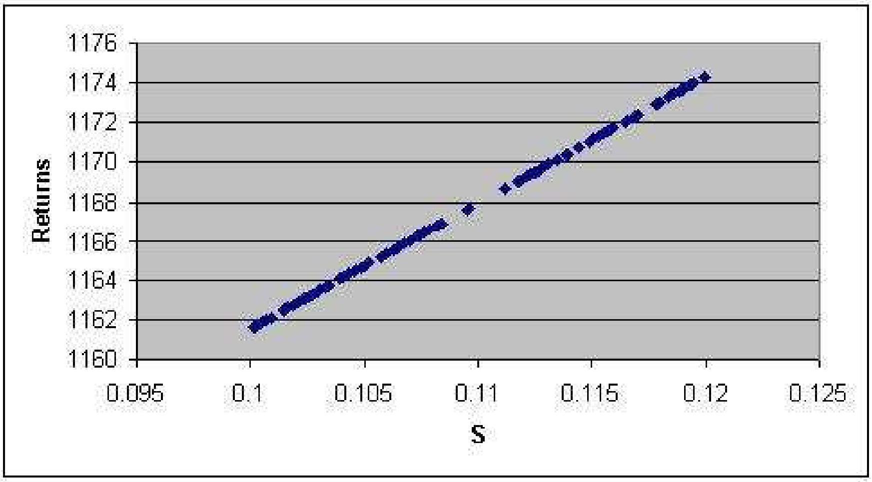

Fig. 4.3. The relationship between S and the individual returns for experiment 3.1

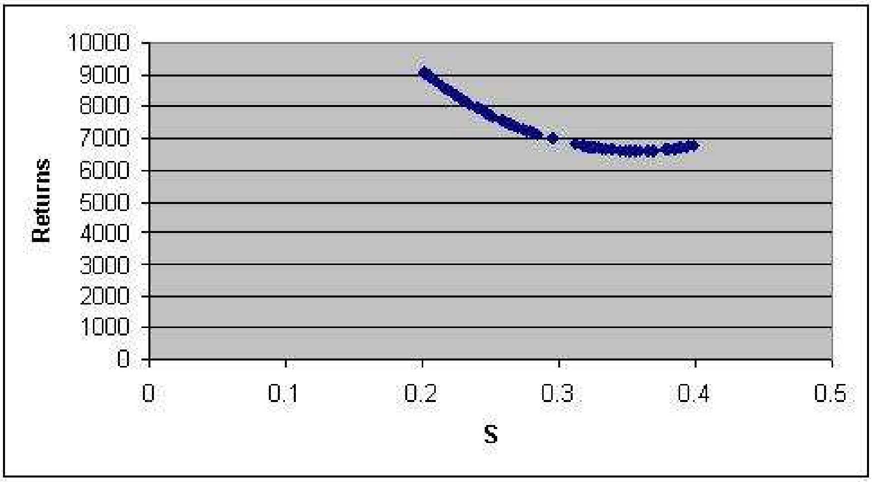

Fig. 4.4. The relationship between S and the individual returns for experiment 3.2

5. Conclusions

In this paper, it was shown how a relatively simple simulation model with simple micro level agent rules is capable of generating complex macro level outcomes. These outcomes of the simulation model, in the form of the returns time series, show a number of stylized facts that can also be observed in real returns time series. So, there is a qualitative resemblance between the model and the reality. However, due to e.g., the oversimplified nature of the simulation model, a quantitative gap between the results of the model and real returns time series remains.

In order to tighten or close this gap, it is necessary to radically rethink and restructure the current model in specific and the way artificial stock markets are built in general. This rethinking and restructuring may take the form of the research approach as it will be presented in one of our articles that is currently in preparation (Hoffmann, Jager & Von Eije 2006).

In general, this approach consists of four critical steps, that together constitute a complete empirical circle. Micro level agent rules are formalized based on empirical research, social interactions amongst micro level investor agents lead to macro level simulation results, macro level simulation results are subsequently compared to macro level real stock market results, and eventually the simulation model can be adapted according to the results of this comparison. The final objective is to build a level 3 model of a stock market as defined by Axtell and Epstein (1994).

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