

1st TSFS Finance Conference, TSFS 2013, 12-14 December 2013, Sousse, Tunisia

## **Agent-Based Modeling and Investors' Behavior Explanation of Asset Price Dynamics on Artificial Financial Markets**

Yosra Mefteh Rekik <sup>a\*</sup>, Wafik Hachicha <sup>b</sup>, Younes Boujelbene <sup>a,b</sup>

<sup>a</sup> Department of Financial and Accounting Methods, Sfax University, 3018, Tunisia

<sup>b</sup> Department of Industrial Management, Sfax University, 3018, Tunisia

---

### **Abstract**

Standard asset pricing models based on rational expectations and homogeneity have problems explaining the complex and volatile nature of financial markets. The heterogeneity in expectations can lead to market instability and complicate dynamics of prices, which are driven by endogenous market forces. In this sense, we use Agent-based computational approach and more specifically artificial Stock Market modeling to explore the market dynamics from a behavioral perspective. Our aim is to point out that the investors' irrationality explains various numbers of financial anomalies, especially the phenomena that traditional financials models have never been able to explain. We built a virtual financial market that contains three types of investors: fundamentalists, non-fundamentalist and loss adverse investors. Therefore, the difficulty of the prediction is due to several features: the complexity, the non-linearity and the dynamism of the financial market system, as well as the investor psychology. The Artificial Neural Networks learning mechanism take on the role of traders, who from their futures return expectations and place orders based on their expectations. The results of intensive analysis indicate that the existence of agents having heterogeneous beliefs and preferences has provided a better understanding of price dynamics in the financial market.

© 2014 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/3.0/>).

Selection and peer-review under responsibility of the Tunisian Society for Financial Studies (TSFS)

*Key Words:* Artificial Stock Market; Behavioral Finance; Artificial Neural Network; Agent-Based Simulation

\* Corresponding author. Tel.: +216 20647147.

E-mail address: [meftehyosra@yahoo.fr](mailto:meftehyosra@yahoo.fr)

## 1. Introduction

Economics and financial theories have been based on rational investors and on market efficiency hypothesis, which posits that market prices fully reflect all available information (Fama, 1970). In traditional models, rational use of information, their decision making is based on utility function with beliefs, calculated via optimal statistical procedures. Thus, the representative investor is an individual who acts as an expected utility maximizer and adheres to the axioms of rational choice theory. These presumptions have clearly played a crucial role in shaping the widely accepted understanding of risk, determinants of asset prices, portfolio management principles, etc. Many financial researchers have examined the core arguments of Efficient Market Hypothesis, Capital Asset Pricing Model, and Anticipation Utility Theory. For instance, Banz (1981) and Bamber (1987) found that fluctuating future stock prices has a “scale effect”. Cross (1973), French (1980), and Gibbons and Hess (1981) also found the “week effect” on the moving trend of stock prices so that investors can gain excess in returns by adapting the reverse operation strategy. Furthermore, Herbert Simon (1991) has emphasized the importance of bounded rationality, taking into account the limited ability of agents to adapt optimally, or even satisfactorily, to complex environments. Paradoxically, it is hardly rational to attempt being perfectly rational. Moreover, a large thread of literature of psychology and behavioral finance instigated by laboratory experiments of Kahneman and Tversky (1973, 1974) and Kahneman (2011) suggest that economic behavior is often better explained by simple heuristic rules and irrational biases rather than by dynamic optimization. According to the prospect theory, the investors’ psychological drives will lead their actual decision-making process to deviate from rationality.

Recently, Chiarella and He (2003) mention a growing dissatisfaction with the representative agent paradigm. Then, LeBaron (2005) and Hommes (2006) state that at the end of the 20th century finance witnessed a revolution and they notice an important paradigm shift from a representative rational agent approach towards a behavioral agent-based approach. More recently, Frijns et al. (2010) mention a demise of the EMH. Behavioral finance provides an alternative theory regarding financial markets. Based on experimental psychology literature, behavioral finance considers that cognitive biases could affect asset prices. In the field of behavioral finance, researchers set psychological biases underlying the behavioral explanations on the observed security price behavior (Kahneman and Tversky, 1979, 1982; Shefrin and Statman, 2000; Barberis and Thaler, 2002; Szyska, 2008). In fact, individuals are thought to make judgments under uncertainty because limited time and cognitive resources lead them to apply behavioral biases such as herding, loss aversion, anchoring and other behavioral biases by investors’. It seems evident that psychology plays an important role in financial markets and deserves thorough investigation. The effects of behavioral finance can be viewed as another answer to unrealistic assumptions of the Efficient Market Hypothesis. The confirmation of heterogeneity on the stock market can be founded in empirical research of Vissing-Jorgensen (2003). The author reports that there exists heterogeneity in forecasting future asset prices on the Stock Market: 50% of individual investors consider the stock market to be overvalued, 25% believe that it is fairly valued, about 15% are unsure, and less than 10% believe that it is under valued. The complexity of the financial rules governing the stock market and their confrontation with investors’ activities make the interpretation and the explication of observed global behavior very difficult to understand. According to Takahashi and Terano (2003), the investors’ decision making rules which are based on behavioral finance are much more complicated than those adopted in the traditional finance, which makes it difficult to derive asset prices analytically in behavioral finance. In this area of complex system, other recent studies report that complex phenomena emerge from interactions between micro-rules and macro-behavior (Arthur, 1997; Wolfram, 1994).

We cannot understand market outcomes through the eyes of a single representative type of rational agents. In agent-based models, the market is filled with heterogeneous, boundedly rational agents with different expectation and behaviors. This bottom-up method involves large numbers of interacting agents with the “rule of thumb” trading strategies, and the aggregation of simple interactions at the micro level may generate sophisticated structure at the macro level. In the context of financial markets, the micro-rules correspond to investors’ behavior and the macro-behavior corresponds to the fluctuation of asset prices. A relatively novel approach for studying the link between individual investors’ behavior and financial market dynamics, based on agent-based methodology, has become known as Artificial Financial Markets. These are often computational models of financial markets, and are usually composed of a number of heterogeneous and bounded rational agents, interacting through some trading mechanism,

while possibly learning and evolving. According to this approach, markets are seen as complex dynamical systems consisting of heterogeneous learning, boundedly rational heterogeneous agents (Hommes, 2006; LeBaron, 2006; Lux, 2009; Chiarella et al., 2009; LeBaron, 2012 and Viktor et al., 2013).

Both approaches, agent-based models and behavioral finance, complement each other and could be used together as agent based approach framework could serving as a useful theoretical tool for verification of findings from behavioral finance. LeBaron (2005) argues that “Agent-based technologies are well suited for testing behavioral theories” and anticipates that “The connections between agent-based approaches and behavioral approaches will probably become more intertwined as both fields progress”. Over the last 15 years, a number of computer-simulated, artificial markets have been put forward. Following the pioneering work done at the Santa Fe Institute (Le Baron et al., 1999), a large number of researchers have proposed models for artificial markets populated with heterogeneous agents endowed with learning. In the last decade, we use the behavioral multi-agent based simulation in order to complete the description of theoretical phenomena by many aspects based on the individual’s behavior and their interactions (Hoffmann et al., 2007; Henrik, 2008). This evolution shows that the multi-agent based simulation is a promising approach to study the stock market dynamics. This lead to several examples of Artificial Stock Markets in the literature, Santa Fe Institute Artificial Stock Market developed by Arthur et al. (1997), also described in (LeBaron et al., 1999; LeBaron, 2006) and the Genoa Artificial Stock Market (Marchesi et al. 2000; Cincotti et al. 2001). The complementarities of behavioral finance research and the agent-based methodology have been recognized in the literature as a nascent field of research with many opportunities ahead. Takahashi and Terano (2003), Hoffmann et al. (2007) and Mathieu et al. (2010) are counted as rare examples of agent-based papers that pursue the idea of explicit accounting for behavioral theories in financial market simulations. In Takahashi and Terano (2003), the focus is on overconfidence and loss aversion, while Hoffmann et al. (2007) focus on social dimensions of investor behavior. Barberis and Thaler (2003) and Scheinkman et al. (2004) mention overconfidence, De Grauwe and Grimaldi (2006), Boswijk et al. (2007) and Kukacha et al. (2013) suggest market sentiment, and Chiang et al. (2007) and Chiarella et al. (2003) put stress on herding behavior.

In an interesting model, LeBaron (2000) examines how investors’ heterogeneous time horizons affect evolution of the market, its convergence to the known homogenous rational expectations equilibrium and domination of different types of investors. The learning mechanism in this model is an interesting combination of the neural network technique and the evolutionary search mechanism. A much more recent method has appeared, the technological analysis, where computers are used as a tool to predict the stock movements. Technological analysis tries to model and simulate as accurately as possible the behavior of the Stock Exchanges, by different computational techniques which I will discuss as follows: Genetic Algorithms, originated by Holland (1975), are learning methods which mimic the biological process of evolution. A genetic algorithm creates ‘candidate solutions’ from a defined set of building blocks, which can be interpreted as genes. GA which helps in finding optimal parameters for technical indicators by making them evolve, by combinations and mutations, starting with a population of a given model, has established different parameters (Armano et al. 2005; Chen and Liao., 2005 and Chen, 2007). Fuzzy Logic controllers are used in combination of artificial neural networks and genetic algorithm (Gradojevic, 2006). Fuzzy neural network are also studied, and sometimes demonstrate good performances (Chang and Liu., 2006; Hachicha et al., 2011). Artificial Neural networks, invented by psychologist Rosenblatt in 1958, ANNs replicate the biological structure numerically. Here, neurons are embodied by artificial units spanning a two-dimensional network. Therefore, the intelligent appropriate technique used to satisfy this dynamics is the Neural Network (Yang, 2002). This technique has provided fruitful ideas and new tools to statistical methodology (Chen and Yen, 2002; Martinez-Jaramillo, 2007). Classifier Systems, learning classiffier systems, which were derived by Holland (1991), proposed to model economic agents and finally, they were used in the Santa Fe Artificial Stock Market. Such mechanism has been used to perform financial forecasting in works like Arthur et al. (1997), Le Baron et al. (2002) and Schulenburg and Ross (2002).

In this paper, taking into account the micro-rules and macro-behavior, we seek an explanation of the price dynamics via the complementary played by these approaches. Models of financial markets present several weaknesses in the mental representations of the reality. In fact, the interaction between the micro-structural and behavioral approach is uncertain. This serious obstacle can be avoided in experimental setting such as an agent-based artificial stock market where the information possessed by traders can be controlled. Therefore, we propose an overview of Artificial Neural Networks, regarded as a key component in many agent-based financial markets for

learning and adaptation. The contribution of this research and the advantage of computational agent-based models developing an artificial stock market can be found in the link between the modeling and the simulation of the stock market and particularly the investor behavior and his decision making.

The rest of the paper is organized as follows. In the next section, the formulation of the proposed model is introduced. In this section our basic asset pricing model, the agents' expectations formation, the walrasian trading mechanism and the Artificial Neural Networks learning in our simulation. In Section 3 results experimentations and interpretations are displayed. Section 4 contains the concluding remarks and extensions for possible the future work.

## 2. The Artificial Stock Market

Their pioneering model along the lines of Grossman and Stiglitz (1980) and Hussman (1992) was used subsequently by Arthur et al. (1997), LeBaron et al. (1999, 2002) on Santa Fe Institute Financial Market and Pajaras et al. (2005). A computer simulation of the financial market involving 100 investors was used as the model in this paper; risk assets and risk free assets are the two possible transaction models which are adopted along with other behavioral features.

Table 1. Parameterization

| Symbol         | Explanation                                       |
|----------------|---|
| M              | The number of investors (100)                     |
| N              | The number of issued stocks (100)                 |
| T              | Simulation periods (1000)                         |
| $r_f$          | The risk free interest rate (0.1)                 |
| $\bar{\alpha}$ | Risk averse ratio of the investor (0.5, constant) |
| K              | Measuring the weight of prior belief (0.8)        |
| $\rho$         | Exogenous parameter (0.95)                        |
| $\bar{d}$      | Mean of autoregressive process                    |
| $\sigma_\mu^2$ | Conditional variance of dividend process (0.0734) |
| $\theta$       | Forecast error update's parameter (0.013)         |
| f              | (6.333)   |
| g              | (16.688)  |

Models including the market interactions of many investors following such strategies are clearly hard to solve in an analytical manner. Therefore, Kim and Markowitz (1988) investigated the destabilizing potential of dynamic hedging strategies via Monte Carlo simulations of a relatively complicated model of price formation in an Artificial Financial Market. The Monte Carlo simulation is the only reasonable option to computationally examine the impact of suggested changes on the model outcomes to perform simple Monte Carlo simulations (Metropolis and Ulam, 1949; Rubinstein and Kroese, 2008; and Alfarano et al. 2010). Within this method, we repeatedly stochastically generate crucial variables using different random number generator settings and consequently run the model employing generated values. This sample represents the true distribution of model outcomes. The sufficient number of runs is, therefore, very important to obtain statistically valid and reasonably robust sample.

### 2.1. Market Structure

The literature has identified many different auction mechanisms used in agent-based artificial financial market. The first mechanism consists in creating a simple price response to the excess of demand with a simple clearing mechanism as what is in the models of Farmer and Joshi (2002), farmer (2002), Martinez-Jaramillo (2007). The second mechanism is clearing in temporary market equilibrium, where the price is determined so that the total demand equals the total number of shares in the market (Levy et al., 1995; LeBaron, 2001). The third mechanism is a more realistic one, where actual order book is simulated, and buy and sell orders are crossed using a certain well-defined procedure, as found in the models of Marchesi et al. (2000), Yang (2002), Ponta et al. (2011). One of the most common examples within this category of price formation mechanism is a double-auction market. The final mechanism is the Walrasian auctioneer, in which the equilibrium price is set so that overall demand equals overall supply, as found in the market clearing mechanism in price determination (Grossman and Stiglitz, 1980; Hussman, 1992; Pajaras et al., 2005). To simplify the model, we choose the walrasian auction, as being able to observe all the traders' demand functions, adjust prices in various markets to equilibrate aggregate supply and aggregate demand.

Therefore, all trades will be executed at a fixed price and the agents are not permitted to trade out of equilibrium.

## 2.2. Model of Traders

We assume that fundamental investors and irrational agents are myopic one-period constant absolute risk aversion (CARA). CARA is used in the models of Arthur et al. (1997), Palmer et al. (1998), Chen and Yeh (2002), Yang (2002) and Hommes (2006). They maximize an expected utility function in the following from:

$$U(W_{i,t+1}) = -e^{-\lambda W_{i,t+1}} \quad (1)$$

Where:

$W_{t+1}$  is the agent's expected wealth level for the next period

$\lambda = -\sigma^2 U(W) / \sigma U(W)$  is the degree of relative risk aversion.

In this virtual market, there are two types of assets available for traders. One is the risk free asset paid with a fixed interest  $r$  and the other is the risky asset paid at the beginning of each period, a dividend which follows an autoregressive process AR (1) as:

$dt = \bar{d} + \rho (d_{t-1} - \bar{d}) + \mu t$ . Where,  $\mu t \sim N(0, \sigma^2 \mu)$ ,  $\rho$ : exogenous parameter,  $d_{t-1}$ : dividend at  $(t-1)$  and  $\bar{d}$  is a mean of the autoregressive process.

Agents maximize their utility function with respect to their budget constraint:

$$W_{i,t+1} = x_{i,t} (p_{t+1} + d_{t+1}) - p_t (1 + r_f) (W_{i,t} - p_t x_{i,t}) \quad (2)$$

Where:

$x_{i,t}$  is the number of risky assets held by trader  $I$  at time  $t$ ,

$r_f$  is the risk free interest rate,

$d_{t+1}$  is the dividend attributed to risky assets,

$p_t$  is the price of the stock at time period  $t$ .

It is well known that under CARA utility and Gaussian distribution for forecasts, traders' desire demand,  $x_{i,t}$ , for holding shares of the risky asset is linear in the expected excess return (Grossman and Stiglitz, 1980). According to Arthur et al. (1997), the demand of shares for agent  $I$  at time  $t$  is giving by:

$$x_{i,t} = \frac{E_{i,t}(p_{t+1} + d_{t+1}) - p_t (1 + r_f)}{\lambda \cdot \sigma_{i,t,p+d}^2} \quad (3)$$

Where  $\sigma_{i,t}^2$  is the conditional variance of  $(P_{t+1} + D_{t+1})$ .

The notation  $E_{i,t}$  indicates the best forecast of agent  $i$  at time  $t$ . It is the essential elements of agent-based Artificial Stock Market.

On the basis of LeBaron's (1999) work, the price equilibrium with rational expectations of the risk asset as a linear function of dividend is expressed as follows:

$$p_t = f \cdot d_t + g \quad (4)$$

The price sequence above will be in rational expectation equilibrium and it reflects the "fundamental Value". This expectation equation is one of the crucial items that traders need to estimate. It is used as a Benchmark for our experiments under the market efficiency hypothesis. In this way, we can compare the price and obtain our simulation with theoretical price.

At equilibrium, the total demand must be equal to the total supply. Therefore, we assume that total supply equals the number of shares issued.

$$\sum_{i=1}^M x_{i,t} = N \quad (5)$$

The rational expectation equilibrium of future price and dividend are as follows:

$$E(p_{t+1} + d_{t+1}) = \rho(p_t + d_t) + (1 - \rho)[(1 + f)\bar{d} + g] \quad (6)$$

With,

$$f = \frac{\rho}{(1 + r_f - \rho)} \quad g = \frac{\bar{d}(1 + f)(1 - \rho) - \lambda \sigma_{p+d}^2}{r_f}$$

### 2.3. Model of Agent Behavior

One of the most important design issues is the modeling of agents. This issue covers heterogeneity, decision making, utility function, and learning of agent (Chen and Yeh, 1999; Grothmann, 2002; Hommes, 2006 and Othalia et al., 2013). Our virtual financial market is populated by fixed number of traders  $N$ , falling into two types of investors: fundamentals and non-fundamentals. In this analysis, we focus on the influence of the fundamentals, the herding, the anchoring investors' behavior and the loss adverse investors in financial markets.

#### 2.3.1 Fundamentalists:

We refer to investors who make investment decisions based on fundamental values as “fundamentalists”. We assume that they form expectations about the values of future prices and dividends by means of an Artificial Stock Market. The trade price is given as follows:

$$p_t = \beta \cdot \sum_{i=1}^M w_{i,t} E(p_{t+1} + d_{t+1}) - \alpha \quad (7)$$

With:

$$\beta = \frac{1}{(1 + r_f)}; w_{i,t} = \frac{\frac{1}{\sigma_{i,t,p+d}^2}}{\sum_{i=1}^M \frac{1}{\sigma_{i,t,p+d}^2}} \alpha = \beta \cdot \frac{\lambda N}{\sum_{i=1}^M \frac{1}{\sigma_{i,t,p+d}^2}}$$

#### 2.3.2 Non-Fundamentals investors

As they are sometimes called, “noise traders” represent another type of belief as introduced in the seminal paper by DeLong et al. (1990). They believe the asset price is not determined by economic fundamentals only, but it can be partially predicted using simple technical trading rules, extrapolation techniques or taking various patterns observed in the past prices into account. The reason why we should place special focus on these behavioral biases is that they are generally supposed to have strong impact on traders' behavior.

**Herding:** Herding often occurs when many people take the same action, perhaps because some imitate the actions of others in making investment (Graham, 1999). In other words, investors copy the behavior of other investors, leading to changes in their decision-making process after observing others. Investors ignore to a certain degree their private opinions and follow the market, leading to a switch from non-trading to trading (Manahov et al., 2013). The examination of herding patterns in Agent-based models is always based on short-run profitability's of individual strategies and herding is detected via the evolution of market fractions. Therefore, we introduce a concept of rather irrational 'blind' herding which is based on public information and aims to imitate traders' behavior during large stocks. In this approach, one of the trading strategies does not behave in the traditional way, but copies the behavior of the most successful trades of the previous day. According to Brock and Hommes (2006), at time  $t$  the strategy

primarily evaluates its own performance measure, then compares the performance measures of all other strategies and for the next period ( $t + 1$ ) it adjusts its beliefs about the trend parameters so that they mimic the last period's most profitable strategy.

**Anchoring:** Anchoring heuristic establishes that people often base their decision making process on elements or conditions of reference (Brav and Heaton, 2002). According to this behavioral bias, the expectation method used by investors who are subject to anchoring behavior can be expressed as follows:

$$\begin{cases} p_{t+1}^f = p_t \cdot (1 + \text{React}_{\text{Anchoring}}) \\ d_{t+1}^f = d_t \cdot (1 + \text{React}_{\text{Anchoring}}) \end{cases} \quad (8)$$

Where:

$$\text{Reaction}_{\text{Anc}} = (1 - k) + [1/5 \sum_{i=1}^5 \left[ \frac{(Pt-1+dt-1)}{(Pt-1)-i} - 1 \right]] + K [1/5 \sum_{i=1}^5 \left[ \frac{Pt-5}{Pt-5-i} - 1 \right]]$$

$K$  is the parameter measuring the weight of prior belief ( $1/2 < K < 1$ ).

Thus, the expression of the equilibrium price of the victims through anchoring agents is formalized as follows:

$$p_t = \beta \cdot \sum_{i=1}^M w_{i,t} [p_t (1 + \text{React}_{\text{ANC}}) + d_t (1 + \text{React}_{\text{ANC}})] - \alpha \quad (9)$$

$$\text{With:} \quad \beta = \frac{1}{(1 + r_f)} \quad w_{i,t} = \frac{\frac{1}{\sigma_{i,t,p+d}^2}}{\sum_{i=1}^M \frac{1}{\sigma_{i,t,p+d}^2}} \quad \alpha = \beta \cdot \frac{\lambda \cdot N}{\sum_{i=1}^M \frac{1}{\sigma_{i,t,p+d}^2}}$$

**Loss-averse investors:** We formulate the model of the investors based on Prospect Theory in accordance with the one characteristic of decision making, which is that people tend to recognize losses twice as large as gain. Like the fundamentalists, loss adverse investors use the artificial neural network to make forecasts but they may be reluctant to realize losses.

Following Khaneman and Tversky's theory, loss adverse investors tend to estimate losses twice as large as profits. Thus, we stylize the utility function involving the loss aversion feature according to Shimokawa et al. (2007) as follows:

$$U(w_{i,t+1}) = -e^{-\lambda B_{i,t} w_{Li,t+1}} \quad (10)$$

$B_{i,t}$  is a parameter related to the loss Aversion as:

$$\begin{cases} B_{i,t} = 4 & \text{if } E\{(p_{t+1} + d_{t+1}) - (p + d)_{t-1}^{\text{ref}}\} < 0 \\ B_{i,t} = 1 & \text{if } E\{(p_{t+1} + d_{t+1}) - (p + d)_{t-1}^{\text{ref}}\} \geq 0 \end{cases} \quad (11)$$

With  $(P + d)_{t-1}^{\text{ref}}$  is the reference point and determined by the average of price before  $R$  periods. In others words, the gains and the losses are measured relative to reference point set to be the average of prices and dividend before 10 periods as follows:

$$(p + d)_{t-1}^{\text{ref}} = \frac{(p_t + d_t) + (p_{t-1} + d_{t-1}) + \dots + (p_{t-R+1} + d_{t-R+1})}{R} \quad (12)$$



The expression of the equilibrium price victims' agents through loss aversion is formalized as follows:

$$p_t = \beta \cdot \sum_{i=1}^M w_{i,t} E(p_{t+1} + d_{t+1}) - \theta \quad (13)$$

With:

$$\beta = \frac{1}{(1 + r_f)}; w_{i,t} = \frac{1}{\sum_{i=1}^M \frac{\sigma_{i,t,p+d}^2}{\sigma_{i,t,p+d}^2}} \text{ et } \theta = \beta \cdot \frac{\lambda \cdot B \cdot N}{\sum_{i=1}^M \frac{1}{\sigma_{i,t,p+d}^2}}$$

#### 2.4. Modeling traders' with Artificial Neural Network

Learning is a key factor in our simulation of an Artificial Financial Market. Agent should be able to update their trading strategies in response to changing market conditions. Artificial Intelligence in general and Evolutionary Computation in particular are two of the most influential areas involved in the design of techniques and tools to perform some forms of financial forecasting.

In the current paper, we propose an overview of Artificial Neural Network principles (Rosenblatt, 1958) which are regarded as a key component in many agent-based financial market for modeling of learning and adaptation (Chen and Yen, 2002; Martinez-Jaramillo, 2007).

Artificial intelligence is the generic name attributed to the field of computer science dedicated to the development of programs that attempt to replicate human intelligence. ANN is one of the artificial intelligence techniques that has played an important role in solving problems with extreme difficult or unknown analytical solutions. The ability of neural networks to discover nonlinear relationships (Phillip et al. 1989) in input data makes them ideal for modeling nonlinear dynamic systems such as the Stock Market. Neural networks, with their remarkable ability to derive meaning from complicated or imprecise data, can be used to extract patterns and detect trends that are too complex to be noticed by either humans or other computer techniques.

The most fundamental aspect of ANNs is their use of simple interconnected processing elements, which corresponds loosely to neurons and synaptic connections in the brain. The strength of the connecting links determines the functionality of the ANN as a whole. The most common applications of ANNs involve regression and classification. Regression models attempt to estimate input-output transformation functions, while classification involves using the known inputs to determine class membership (Tsoukalas and Uhrig, 1997; Hachicha, 2011).

According to Haykin (1999), there are three basic elements in the structure of an abstract neuron with  $m$  inputs, which are illustrated in Fig. 1. (1) a set of connecting links or synapses, each is characterized by their weight of its own. Specifically, an input signal  $x_i$  to the synapse  $j$  connected to neuron  $k$  is multiplied by the synaptic weight  $w_{kj}$ ; (2) an adder  $\Sigma$ , which sums the input signals weighted by the respective synapses of the neuron, and (3) an activation function  $\phi(\cdot)$ , which limits the permissible amplitude range of the output signal to some finite value. It defines the output  $y_k$  of the neuron in terms of the induced local field, which is formed by the linear combiner output  $u_k$  and the bias  $b_k$ . This externally applied bias is used to increase or lower the net input of the activation function.

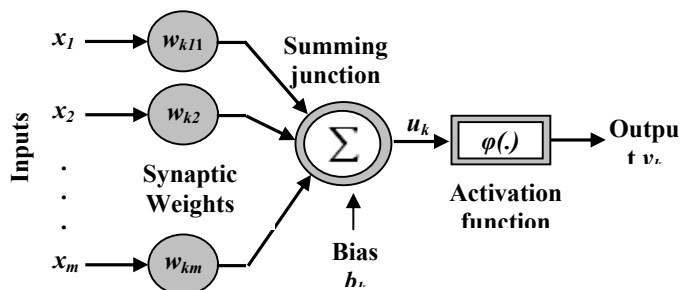


Fig. 1. Non-linear model of an abstract neuron



The performance criteria of ANN models during calibration are habitually evaluated by using statistical parameters: Root Mean Squared Error (RMSE) and correlation coefficient or efficiency criterion  $R^2$ . In addition, during validation of an ANN model, the performance criterion is evaluated using Percentage Error in deepest level fluctuation (PE). All these performance criteria are used in order to evaluate the effectiveness of an ANN and its ability to make precise predictions.

The RMSE is calculated by:

$$RMSE = \sqrt{\frac{\sum_{i=1}^N (Y_{i0} - Y_{ic})^2}{N}} \quad (14)$$

Where:

$Y_{i0}$  is the observed data,  $Y_{ic}$  the calculated data and  $N$  is the number of observations. RMSE indicates the discrepancy between the observed and calculated values. The lowest the RMSE, the more accurate the prediction is.

The  $R^2$  efficiency criterion, given by:

$$R^2 = 1 - \frac{\sum_{i=1}^N (Y_{i0} - Y_{ic})^2}{\sum_{i=1}^N Y_{i0}^2 - \frac{\sum_{i=1}^N Y_{ic}^2}{N}} \quad (15)$$

$R^2$  represents the percentage of the initial uncertainty explained by the model. The best fit between observed and calculated values, which is unlikely to occur, would have  $RMSE=0$  and  $R^2=1$ .

The PE is given by:

$$PE = \left| \frac{Y_{i0} - Y_{ic}}{Y_{i0}} \right| \times 100 \quad (16)$$

Nero-one is used to develop the Back Propagation learning algorithm. After several tests of varying the number of neurons in the hidden layer, we found that three neurons constitute the optimal structure of the hidden layer (Table. 2).

Table 2. Artificial Neural Network performance criteria.

| Number of hidden layers | RMSE            | $R^2$    | PE              |
|-------------------------|-----------------|----------|-----------------|
| 2                       | 4.70E-05        | 0.99     | 3.09E-05        |
| <b>3</b>                | <b>1.46E-05</b> | <b>1</b> | <b>9.92E-06</b> |
| 4                       | 2.45E-05        | 1        | 1.66E-05        |
| 5                       | 2.26E-04        | 1        | 1.50E-04        |

The architecture of the neural network, as shown in Graph bellow is composed of a single input layer, containing a single neuron, namely the vector  $(P_t + d_t)$ , an output layer, indicating the results forecasts  $(E(P_{t+1} + d_{t+1}))$  and three neurons of hidden layers.

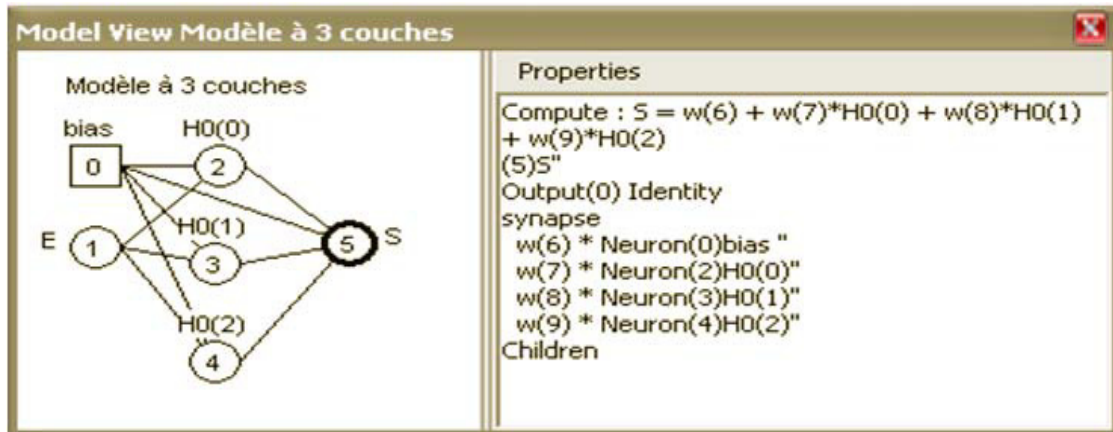


Fig. 2. Artificial Neural Network Architecture

The goal of artificial agent is to build forecasts of the future price and dividend  $E(P_{t+1} + d_{t+1})$  which will be used in their demand functions.

By means of feed-forward Artificial Neural Network, we see in Fig. 3 the expectations of future price and dividend versus rational expectations.

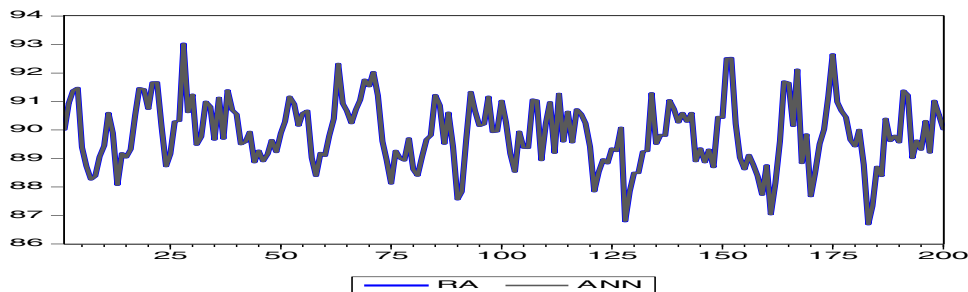


Fig. 3. Rational Vs. Artificial Neural Network Expectations

The graphical comparison made on the same graph the desired and expected results by the neural network for visual analysis. The graph in Fig. 3 illustrates the dynamic simulation of the forecast price and future dividends. To run simulation, we use the same parameter values as used by LeBaron (1999, 2002). Thus, we have simulated expectations for 1000 observations. As shown, the convergence of the Artificial Neural Network to the rational expectations occurs at quadratic mean error  $10^{-5}$ . The results show a correlation between the predictions made by the neural network and predictions under the assumption of rational expectations.

### 3. Simulation Results and interpretations

Simulations using the three agent-based models were carried out and after analyzing more than 1000 observations, we present the most interesting findings.

#### 3.1. Experiment with rational investors'

The first analyzed belief types are fundamentalists or rational traders. We analyze the price dynamics when every investor in our Artificial Stock Market is a fundamentalist who make investment decision based on fundamental values. They believe that the asset price is determined solely by economic fundamentals according to the EMH introduced in Fama (1970). These fundamental investors' are supposed to form expectations about the values of future prices and dividends by means of a feed-forward Artificial Neural Network with one input ( $P_t, d_t$ ), one hidden layer (with three neurons) and on output ( $P_{t+1}, d_{t+1}$ ).

There are two reasons for choosing this relatively simple network design. First, the single hidden layer feed-forward

network possesses the universal approximation property that it can approximate any nonlinear function to an arbitrary degree of accuracy with a suitable number of hidden units (White, 1992). Second, since the rational expectations mapping in equation (6) is a simple linear one, a more complex architecture is unnecessary.

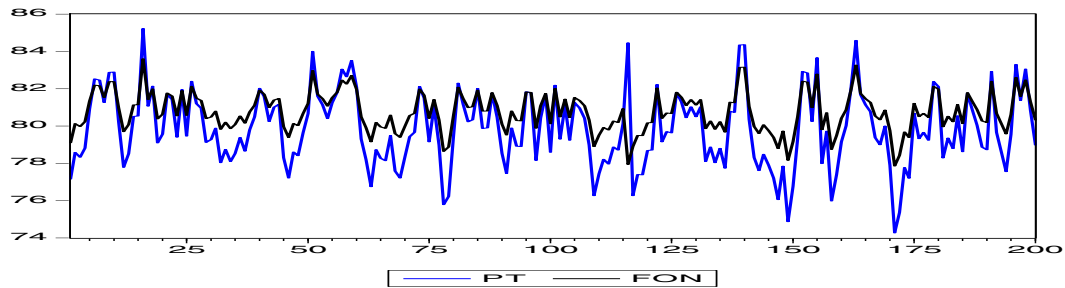


Fig. 4. Price fluctuations when 100% fundamentalist are present at the market

Fundamentalists believe that prices always converge to their fundamental values. Fig. 4 shows the history of traded price and the fundamental value. This chart indicates that when every investor is assumed to be a fundamentalist, the dynamics of the traded price diverge with the fundamental value. This result is inconsistent with the traditional financial theory where agents are assumed to use the same model, the discount of future dividends to form their expectations. Even if each investor has different anticipation, these errors are cancelled out as a whole and do not affect prices. However, the existence of efficient markets is not consistent with empirical evidence. It is also possible to test theories from behavioral finance to model and simulate of complex systems integrating greater heterogeneity among investors so as to analyze the influence of investors' behavior heterogeneity on the asset prices through the experiments in our virtual market.

### 3.2. Experiments with irrational investors'

Financial Markets are subject to uncertainty risk and ever changing events; these aspects make such markets a particularly interesting field to examine market participants' behavior and decision-market processes. In our artificial market, we introduce some irrational agents who form expectations' about the value of future prices. We consider irrational investors subject to the herding behavior, anchoring heuristic and loss adverse behavior. While we analysis the dynamics of the model where new behavioral element are introduced.

We analyze the price dynamics when every investor's in the Artificial Stock Market is a herd investor who makes investment decision based on the market leaders' actions. Fig. 5 shows that the existence of irrational investor's can explain the price dynamics through the experiments in our virtual market. The greater the tendency for investors subject to bias mimetic increases, the more difference between the price and the fundamental value is widening. According to the work of Simone Alfarano (2005, 2010), mimetic behavior described in the psychological literature lead investors to have a herd behavior that systematically affects the dynamics of asset prices.

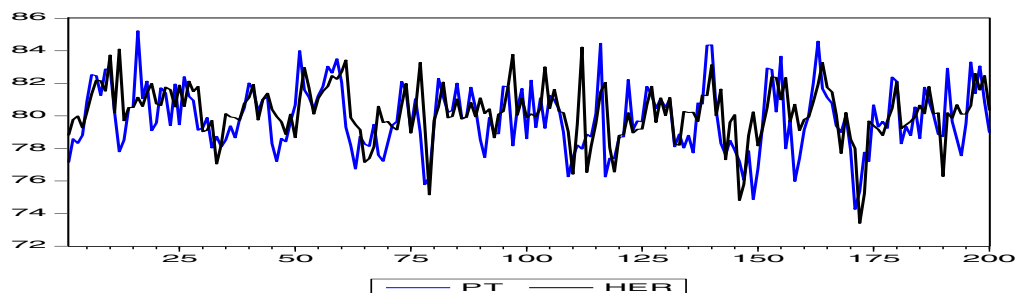


Fig. 5. Price dynamics when 100% of investors are subject to of herd behavior

We run similar simulation but including investors who are affected by anchoring bias in Fig 6. We see that the proposition of anchoring investors behavior in the market explain the price dynamics. The results of the equilibrium price in the presence of victims anchor agents are similar to those observed in the case of mimetic behavior. In other words, the equilibrium price resulting from exchange agents who are subject to the anchoring bias deviates from the fundamental value while following the same trend. According to Brav and Heaton (2002), Kaestner (2005) and Adrian and Hua (2011), this cognitive bias may influence the price formation on financial markets so that it can give rise to the phenomena of under-reaction to information received by investors.

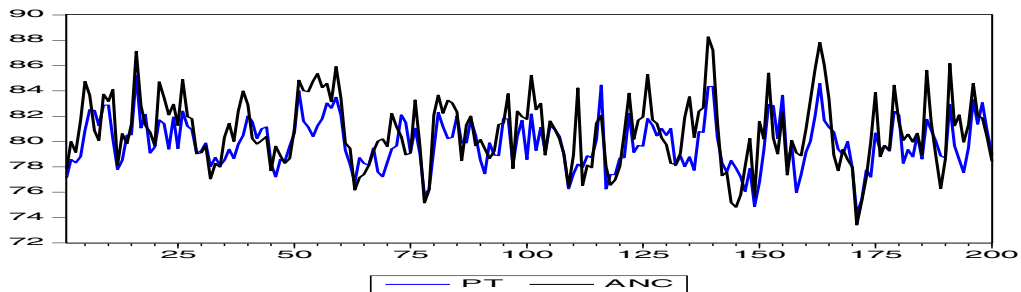


Fig. 6. Price dynamics when 100% of investors are victims to anchoring heuristic

We conduct the experiments in case the market contains investors who are loss adverse. Like the fundamental investors, loss adverse agents use Artificial Neural Network to make forecasts, but they may be reluctant to realize losses. As shown in Fig. 7, when the stock market contains this type of investors, the traded price vastly deviates from the fundamental value. This deviation shows the presence of investors' loss aversion which arises from the human tendency to avoid regret. In summary, loss aversion forced investors to adapt to changing course. These are marked by a high rigidity to change their behavior and number of opportunities, which explains the fact that evolution of prices is more stable than actually observed.

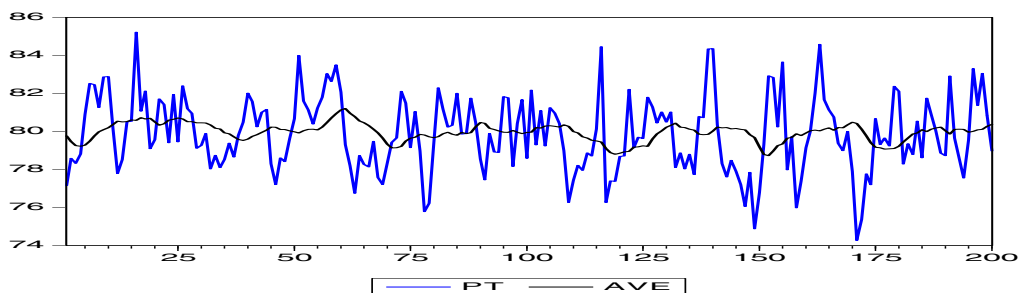


Fig. 7. Price dynamics when 100% loss Aversion behavior exists

### 3.3. Combinations of fundamentalists and irrational investors

We consider the possible combination of behavioral investors in our simulations and examine whether the combinations explain more the assets prices dynamics. We combine four winning setups and hence the following combinations are analyzed:

- Fundamentalist and herding behavior affecting the prices dynamics,
- Fundamentalist and anchoring heuristic affecting prices dynamics,
- Fundamentalist and loss aversion behavior affecting prices dynamics,
- Combination of all these three modifications.

Now, we introduce in our artificial market some irrational investors and we will comment on results of each of the three behavioral modifications individually. In the first experiment, we introduce the herding behavior which

consists of imitating the market leaders' actions.

The combination of fundamentalist and herding behavior of investors in the Market can explain the dynamics price more than a Market containing only fundamentalist's investors. This result confirms many studies that heterogeneous agent-based models have included herding behavior originating among artificial agents (Lux and Marchesi, 2000; Wagner, 2003; Chiarella et al., 2003; Alfarano et al., 2010; Yamamoto, 2011; Kukacka and Barunik, 2013 and Manahov et al., 2013).

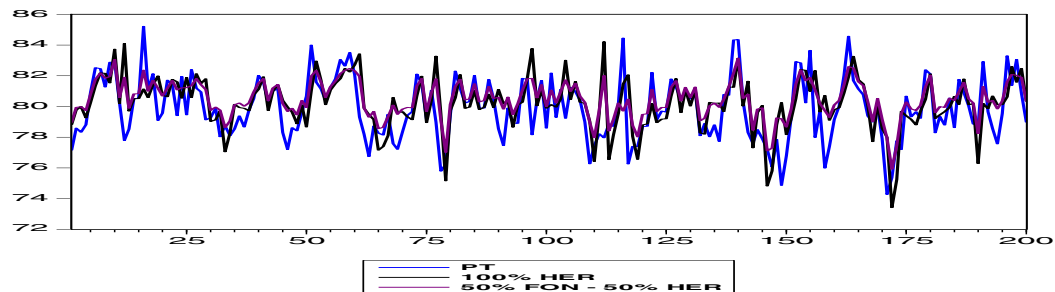


Fig. 8. Price dynamics when the market contains herding behavior and fundamentalists

In a second experiment, we introduce agents who are subject to anchoring bias. Anchoring is a heuristic bias which starts from an initial value (given by the problem formulation, or by some partial computation) and then adjusts it towards the final value. The consequence of this heuristic is that investors overestimate conjunction of events (with high individual probabilities), and underestimate a disjunction of events (with low individual probabilities).

In our simulation, Fig. 9 shows that the irrational investors who are subject to anchoring heuristic and whose behaviors generate an influence to the dynamics price in the market. In accordance with Brav and Heaton's (2002) findings, the bias anchor in human judgment, described in the psychological literature, is considered as a cognitive bias that leads individuals to over or under react to the information they receive. These reactions have incorrect information as they have a systematic effect on asset prices.

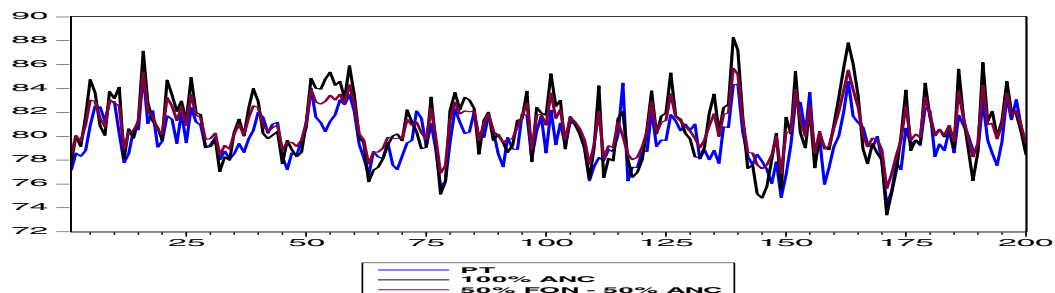


Fig. 9. Price dynamics when market contains anchoring heuristic and fundamentalists

Compared with previous experiments, the introduction of the loss aversion agents as those who form preferences does not conform to the theory of expected utility watercraft, as suggested by Kahneman and Tversky (1979) and does not appear to have a significant impact on price dynamics as shown in Fig. 10.

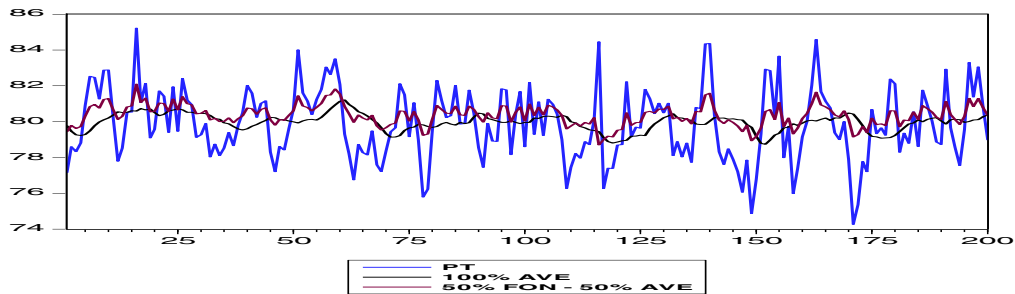


Fig. 10. Price dynamics when market contains Loss Adverse investors' and fundamentalists

Furthermore, from Table 3, the combination of several types of agents with different proportions may cause differences more or less low of asset prices and therefore we would be able to give plausible explanations for the dynamics of stock prices taking into account the heterogeneity of expectations and the irrationality of some investors.

Table 3. Result of price differentials versus degree of heterogeneity of investors

|   | Mean deviation | deviation +  | deviation -  | Deviation max |
|---|----------------|--------------|--------------|---------------|
| 100% Fondamentalists                          | 1.31%          | 1.45%        | 0.69%        | 7.70%         |
| 100% Anchoring                                | 1.87%          | 2.15%        | 1.23%        | 7.69%         |
| 100% Loss Aversion                            | 1.82%          | 1.86%        | 1.77%        | 8.08%         |
| 100% Herding                                  | 1.56%          | 1.82%        | 1.23%        | 9.55%         |
| <b>40 % F, 30 % Anc, 20 % Her et 10% Loss</b> | <b>0.72%</b>   | <b>0.82%</b> | <b>0.46%</b> | <b>3.01%</b>  |

By varying the percentage of agents in the market, we have elucidated the impact of the degree of heterogeneity of investors on the price dynamics. From Fig. 11, we maintain that the explicit consideration of irrational agents who are victims of some cognitive biases, such as herding behavior, through anchoring heuristic and Loss aversion behavior, allows a better understanding of the dynamics of stock prices. This experiment adds another layer of complexity by allowing heterogeneity among market participants.

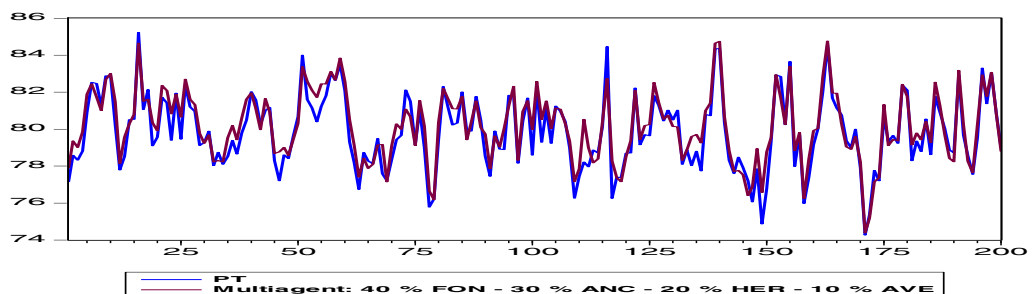


Fig. 11. Price dynamics and heterogeneous traders

A relatively novel approach for studying the link between individual investor behavior and financial market dynamics, based on agent-based modeling, has become known as Artificial Financial Markets. We postulate that the dynamics of Stock Markets are not the result of the extrapolation of a standard individual behavior but rather the consequence of the confrontations of heterogeneous investor's beliefs.

We would like to make a few final remarks about the work done in this study. Firstly, this paper can be read as an extension of the research line stressing that macroeconomic behavior is not a simple scaling-up of microeconomic behavior. More specifically, it is closely related to the agent-based computational model. This type of model allows us to trace a bottom-up path which is infeasible for conventional models built upon the device of the representative agent, and hence provides an ideal tool to show more precisely how microeconomic behavior can be quite different from macroeconomic behavior.

#### 4. Conclusion

This paper advocates the view that agent-based financial modeling that consider financial markets as complex dynamical systems consisting of interacting heterogeneous agents can potentially become a viable alternative. We discuss, at the conceptual level, the need for the proper account of market complexity, agent heterogeneity, rational traders and adaptive expectations. The literature on heterogeneous agents attempts to introduce a more realistic and empirically sound alternative to the classical representative agent model, with rational expectations, into asset pricing. Away from the fiction of the representative agent and in polarizing along a path on the analysis of the interaction between rational and irrational investors, we have elucidated the dynamics of asset prices through a multi-agent simulation by the creation of a virtual stock market and placing the cognitive mechanisms at the center of analysis. The paper focuses in particular on the Artificial Stock Market modeling and examines the impact of suggested changes on the model outcomes we rely on Monte Carlo methods.

We have built an Artificial Financial Market that contains three types of investors: fundamentalists, non-fundamentalist and loss adverse investors to allow traders' heterogeneity, irrational investors and market dynamics to financial problems. Artificial financial markets are models for studying the link between individual investor behavior and financial market dynamics. As a result of the analysis, we have found that, (i) according to the traditional financial theories, when every investor in the market is a fundamentalist, the fluctuations of the traded price deviates with the one of the fundamental value, (ii) when we introduce irrational traders, the traded price agree with the fundamental value, (iii) loss adverse investors, in our agent based simulation, don't generate a large explication from the price dynamics and (iv) we show that it makes sense to consider behavioural finance within the heterogeneous agents modelling frameworks and that both approaches can desirably complement one another. These results stipulate that heterogeneous agents, who are affected by behavioral biases help to explain the dynamics of prices in the Stock Financial Market. Agent-based markets offer an important technology for exploring conjectures about evolution out rationality in finance. They allow for computational experiments which can reveal the underling dynamics in a world of heterogeneous and learning agents. Understanding the dynamics of these markets as through experiments is necessary for building up our intuition for what is going on in real markets.

To quote from LeBaron (2000), "The field is only in its infancy, and much remains to be done". In future research, other computational techniques could be investigated for agent decision making. For instance, the Genetic Algorithm and the Fuzzy logic could test the Agent-based models using the double-auction mechanism that allows issuing real market orders.

#### REFERENCES

- Arthur, B., Holland, J., LeBaron, B., Palmer, R., Tayler, P., 1997. Asset Pricing Under Endogenous Expectations in an Artificial Stock Market, *The Economy as an Evolving Complex System II*, pp. 15-44.
- Armano, F., Marchesi, M. and Murru, A. 2005. A Hybrid Genetic Neural Architecture for Stock Indexes forecasting.
- Barberis, C. and Thaler, H. 2003. A Survey of Behavioral Finance, *Handbook of the Economics of Finance*, Vol 1, pp. 1053-1128.
- Bamber, L., 1987. Unexpected Earnings, Firm Size, and Trading Volume Around Quarterly Earnings Announcements, *The Accounting Review*, pp. 510-532.
- Banz, W., 1985. The Relationship between Return and Market Value of Common Stock, *Journal of Financial Economics*, 9, 3-18.
- Boswijk, H., Hommes, C. and Manzan, S., 2007. Behavioral Heterogeneity in Stock Prices, *Journal of Economics Dynamics and Control*, 31, pp. 1938-1970.
- Chang, P. and Liu, C. 2006. A Tsk Type Fuzzy Rule Based System for Stock Price Prediction.
- Chen, H. 2007. Computationally Intelligent Agents in Economics and Finance. Shifting the research frontier. In *Computational Intelligence Economics and Finance*, vol. II, Springer-Verlag Berlin Heidelberg. pp. 143- 176.
- Chen and Yen, C.-H. 2002. On the Emergent Properties of Artificial Stock Markets, *Journal of Economic Behavior and Organization*, 49, pp. 217-239.
- Chen and Yen. 1999. On the Consequence of Following the Herd': Evidence from the Artificial Stock Market, *International Conference On Artificial Intelligence*, pp. 388-394.



- Chen, S. and Liao, C. 2005. Behavioral finance and agent-based computational finance: toward an integrating framework, In Proceedings of 7th Joint Conference on Information Sciences, pp. 1235-1238.
- Chiang, T., Jeon, N., and Li, H., 2007. Dynamic Correlation Analysis of Financial Contagion: Evidence from Asian Markets. *Journal of International Money and Finance* 26, pp. 1206-1228.
- Chiarella, C. and X.-Z. He 2003. Heterogeneous Beliefs, Risk and Learning in a Simple Asset Pricing Model with a Market Maker, *Macroeconomic Dynamics*, 7, pp. 503-536.
- Cross, F., 1973. The Behavior of Stock Prices on Friday and Mondays, *Financial Analysts Journal*, 29, pp. 67-69.
- DeBondt, W. 1998. A Portrait of the Individual Investor, *European Economic Review*, 42, pp. 831-844.
- Frankfurter G. and McGoun, E. 2000. Market Efficiency or Behavioral Finance: The Nature of the Debate, *The Journal of Psychology and Financial Markets*, 4, pp. 200-210.
- Fama, E. 1970. Efficient Capital Markets: A Review of Theory and Empirical Work. *The Journal of Finance*, 25, 383-417.
- Farmer, J. 2002. Market Force, Ecology and Evolution, *Industrial and Corporate Change*, 11, pp. 895-953.
- Farmer and S. Joshi. 2002. The Price Dynamics of Common Trading Strategies, *Journal of Economic Behavior and Organization*, 49, 149-171.
- French, K. 1980. Stock Return and the Weekend Effect, *Journal of Financial Economics*, 8, pp. 55-69.
- Frijns, B., Lehnert, T. and Zwinkels, R. 2010. Behavioral Heterogeneity in the Option Market, *Journal of Economic Dynamics and Control*, 34, pp. 2273-2287.
- Gradojevic, N. 2006. Non-linear, Hybrid Exchange Rate Modeling and Trading Profitability in the Foreign Exchange Market, In Press, Corrected Proof.
- Graham, J. 1999. Herding Among Investment Newsletters: Theory and Evidence, *Journal of Finance* 54, pp. 237-286.
- Grothmann, R. 2002. Multi-Agent Market Modeling Based on Neural Networks, Ph.D. thesis, Faculty of Economics, University of Bremen. 17, 54, 58
- Grossman and Stiglitz, 1980. On the Impossibility of Informationally Efficiency Markets, *American Economic Review*, pp. 393-408.
- Hoffmann, W. Jager, and Eije. E. 2007. Social Simulation of Stock Markets: Taking it to the Next Level, *Journal of Artificial Societies and Social Simulation*, 10.
- Hachicha, W. 2011. A Simulation Meta Modelling Based Neural Networks for Lot-sizing Problem in MTO Sector, *International Journal of Simulation Modelling*, 10, pp. 191-203
- Hachicha, W., Ghorbel, A. 2012. A Survey of Control-Chart Pattern-Recognition Literature (1991-2010) based on a New Conceptual Classification Scheme, *Computers and Industrial Engineering*, 63, pp. 204-222.
- Haykin, S. 1999. *Neural Networks*, Prentice-Hall, New Jersey, NY.
- Henrik, A. 2008. Estimation of an Adaptive Stock Market Model with Heterogeneous agents, *Journal of Empirical Finance*, 15, pp. 342-362.
- Hommes, C. 2001. Financial Markets as Nonlinear Adaptive Evolutionary System, *Quantitative Finance*, 1, pp. 149-167.
- Hommes W.A. Brock 2006. Heterogeneous Beliefs and Routes to Chaos in a Simple Assets Pricing Model, *Journal of Economic Dynamics and Control*, 22, pp. 1235-1274.
- Hiroshi Takahashi and Takao Terano 2003, Agent-Based Approach to Investors' Behavior and Asset Price Fluctuation in Financial Markets, *Journal of Artificial Societies and Social Simulation*, 6.
- Levy, M. Levy, H. and Solomon, S. 2000. *Microscopic Simulation of Financial Markets: From Investor Behavior to Market Phenomena*, Academic press.
- Holland, J., LeBaron, B., Palmer, R. and Tylar, P. 1997b. Asset Pricing Under Endogenous Expectations in an Artificial Stock Market. *Economic Notes*, 26, pp. 297-330.
- LeBaron, B. W.B. Arthur, and R. Palmer 1999. Time Series Properties of an Artificial Stock Market, *Journal of Economic Dynamics and Control*, 23:1487-1516.
- LeBaron, B. 2001. A Builder's Guide to Agent Based Financial Markets, *Quantitative Finance*, 1, pp. 254-261.
- LeBaron, B. 2002. Building the Santa Fe Artificial Stock Market, Working Paper, Brandeis University.
- LeBaron, B. 2005. Agent-Based Computational Finance, Technical report, International Business School, Brandeis University, USA.
- LeBaron 2006. Agent-Based Computational Finance, *Handbook of Computational Economics*, 2, pp. 882-892.
- Lux, T. and Marchesi, M. 2000. Volatility Clustering in Financial Markets: A Micro simulation of Financial Agents, *International journal of theoretical and applied finance*, pp. 675-702.
- Martinez-Jaramillo, S. 2007. Artificial Financial Markets: An Agent Based Approach to Reproduce Stylized Facts and to study the Red Queen Effect, Ph.D. thesis, Centre for Computational Finance and Economic Agents, University of Essex.
- Markowitz, H. 1988. Stock Market Simulator sms1: Program Description, Baruch College Working, Paper Series 88-24.
- Mathieu, P., Beauls, B. and Brandouy, L., 2010. Simuler pour Comprendre : Une Explication des Dynamiques de Marchés financiers par les systèmes multi-agents", *Systèmes d'Information et Management*, 14, pp. 51-70.
- Palmer, R., Arthur, W., Holland, J. and LeBaron, B. 1998. An Artificial Stock Market, *Artificial Life and Robotics*, 3, pp. 27-31.
- Phillip D. Wasserman, Van Nostrand, 1989, *Neural Computing: Theory and Practice*, Van Nostrand Reinhold, New York, 1989.
- Ponta, L., Raberto, M. and Cincotti, S. 2011. A Multi-Assets Artificial Stock Market With Zero-Intelligence Traders, *A Letter Journal Exploring the Frontiers of Physics*, 93.
- Raberto M, Cincotti S, Focardi SM, Marchesi M. 2001. Agent-Based Simulation of a Financial Market, *Physica A* 319-327.
- Rubinstein, R. Y. and Kroese, D., 2008. *Simulation and the Monte Carlo Method* (2nd ed.). New York: John Wiley and Sons.
- Szyska Adam, 2008. From the Efficient Market Hypothesis to Behavioral Finance: How Investors' Psychology Change the Vision of Financial Market, Working paper Poland University of Economics.
- Shefrin, H. and M. Statman 2000. Behavioral Portfolio Theory, *Journal of financial and quantitative analysis*, 35.
- Shefrin, H. 2000. *Beyond Greed and Fear - Understanding Behavioral Finance and the Psychology of Investing*, Harvard Business School Press.
- Simon, A. 1991. Bounded Rationality and Organizational Learning, *Organization Science*, 2, pp. 125-134.
- Simon A., Lux, T., and Wanger F., 2010, Excess Volatility and Herding in an Artificial financial Market: Analytical Approach and Estimation, Munich Personal RePEc Archive.

- Takahashi Hishori and Takao Terano, 2003. Agent-Based Approach to Investor's Behavior and Asset Price Fluctuation in Financial Markets, *Journal of Artificial Societies and Social Simulation*, 6.
- Tversky, A. and D. Kahneman, 1979. Prospect Theory: An Analysis of Decisions Under Risk, *Econometrica*, 12, pp13-32.
- Tsoukalas, L.H.; Uhrig, R.E. 1997. *Fuzzy and Neural Approaches in Engineering*, New York: Wiley.
- Othalia L. Pierre, P. and Roger, N., The Emergence of Artificial Emotions from Cognitive and Neurological Processes, *Biologically Inspired Cognitive Architectures*, 4, pp. 54-68.
- Vissing-Jorgensen, A. 2003. Perspective on Behavioral Finance: Does "Irrationality" Disappear with Wealth? Evidence from Expectations and Actions, *NBER Macroeconomics Annual*, Cambridge: MIT Press.
- Yang 2002. The Efficiency of an Artificial Double Auction Stock Market With Neural Learning Agents, *Evolutionary Computation in Economics and Finance*, pp. 85-106.