Advances in Agent-based Computational Finance:

Aseet Pricing models from Heterogeneous and Interactive perspective

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***Abstract*—This brief survey gives an introduction on agent-based computational finance (ABCF), focusing on features of heterogeneity and interaction among agents. In contrast to traditional deductive asset pricing theory with strictly defined representative investors, ABCF is characteristic of multi heterogeneous agents, making their own trading decisions in a virtual market respectively and interacting with each other evolutionally. Among a vast array of potential ABCF models, Santa Fe artificial stock model (SF-ASM) and heterogeneous agent model (HAM) are supposed to be the most prominent and prevalent. We give a simple conclusion and future directions for ABCF.**

Keywords- agent-based; computational finance; heterogeneity; Asset pricing

I. INTRODUCTION

In recent decades, studies in financial market have been witnessed significant trends: the emergence and development of behavioral finance, the application of financial econometrics and computational simulations, and the evolutional economics which was originated from biology and ecology. The foundation of neoclassic paradigm, on which traditional theories such as CAPM and Black-Scholes Models[1] build their hypothesis, has experienced huge impact from the anomalies in financial markets. Since the assumptions such as complete rationality, homogeneity and perfect information are supposed to be far from the reality, agent-based computational approaches have become more and more popular and challenging.

The subjects in agent-based computational finance (ABCF) are specified as bounded rational agents, who use different heuristics or rule of thumb strategies. For several years, rational expectation is one of the two issues which academic debates focus on, while the other one is market efficiency. Psychological evidences exhibit that individuals are limited in the knowledge about their environment and in their computing abilities[2], and people make judgment and decision with biases caused by heuristic[3]. Meanwhile, bounded rationality has also been robustly supported by a great amount of empirical evidences in the financial markets.

As a result, agent-based computation finance is rooted from the concept of bounded rationality. the strategies are plain of heterogeneity since individual's behavior inevitably influenced by sorts of psychological factors like beliefs and preferences,[4] Traditional asset pricing models, especially those based on inter-temporal equilibrium framework, commonly take advantage of a top-down approach. Because of the assumption of homogeneity, traditional theory could conveniently determinate the prices after the optimal asset-holding by utility maximization of representative investors with defined preferences and unbiased believes. For example, Lucas (1978) assumed that the economy can be described by a representative investor with a standard utility function, who consumes aggregate consumption. As to agent-based economics, however, considering that individuals interact with adaptive behavior in the stock market competition, the effect of heterogeneity on stock prices dynamics are considered as a bottom-up process.[5]Thus, we need to replace the concept of representative investors[6] with individuals called "agents", to reflect the heterogeneity and complexity in the reality.

Besides the heterogeneity, however, agents are subjective to interactions under certain patterns and laws. Aggregation of simple interactions at the micro level may generate sophisticated behavior and structure at macro level (see Hommes, 2006).

Learning and adaptation, both studied typically in social sciences, are considered to be among the most important interactions in ABCF. Just as Duffy (2006) suggests, whether agents learn or adapt depends on the importance of the problem or choices that agents face. The manner in which agents learn is largely a function of the information they posses and of their cognitive abilities.

Besides learning, agent-based researchers have used a variety of different evolutionary algorithms to characterize the behavior of populations of heterogeneous, interacting agents.

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[1] Campbell (2000) gives a good survey of the traditional models.
[2] Simon (1957) define this phenomenon as "bounded rationality", and was surveyed by Sargent (1993).
[3] See Shefrin (2000).

[4] "Obviously, the world of finance, as a whole is heterogeneous. Market institutions, financial utilities, pricing algorithms are perpetually evolving and shaping a very diverse world", cited from Brandouy (2005).
[5] See Brandouy (2005).
[6] The assumption of representative investors is questioned and criticized due to its excessive simplicity, see kirman (1992).

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In agent-based computational finance, the most important evolutional algorithm is Genetic algorithm. The general field of evolutionary computation includes other methods such as evolutionary programming, evolutionary strategies, and genetic programming. Generally speaking, various types of learning and evolution bring extremely abundant features to price dynamics.

As an emerging and promising field, ABCF demonstrates particular appealing advantages to understand modern financial market by breaking the boundaries of several fields, such as finance, information science, econometrics and even ecology.[7] Besides, subject-based economics also includes another important branch, which is well known as experimental economics [8] Both computational and experimental approach are more tractable and convenient in exploring the relationship between micro behaviors and macro dynamics by sharing some identical properties such as bounded rationality and of the subjects, computational method, however. [9]

II. THE BASIC ASSUMPTIONS OF THE MODELS

In constructing an agent-based financial model the designers are faced with a large number of questions. Among them, there are two most essential ones: First, what types of agents should be designed and how would they embody the heterogeneity among them? Second, how would the agents interact with each other in an evolution process?

A. Heterogeneity

Heterogeneity is an essential concept incorporating different levels and various aspects of meanings. To extend the traditional pricing framework, Campbell (2000) induced heterogeneous constraints, income and preference to modify the SDF(Stochastic Discount Factor). Shefrin (2004) gives "sentiment" more content on heterogeneity of beliefs, risk tolerance and time discounting preference to deduce a behavioral component. Brandouy (2005) point out heterogeneity should refer to two main features: firstly, agents are not initially endowed with the same rational heuristics, and secondly, they do not necessarily share the same beliefs and preferences.

One of the most critical questions is how to describe heterogeneous preferences. Are agents simple mean-variance preferences, or absolutely without regularity? Are they constant relative risk aversion (CRRA), or constant absolute risk aversion (CARA)? Many pioneer literatures use CARA to define agents' preferences[10], while, there are still exceptional attempts. [11] Recently, more and more researchers tend to incorporate the impact of wealth distributions and dynamics into utility functions. Considering the case with CRRA utility, in Brock and Hommes (1998) and Chiarella & He (2001) 's frame work, investors' relative wealth affects asset demand and realized price. So did Anufriev and Bottazzi (2005), Chiarella, Dieci and Gardini (2002, 2005). Chiarella and He (2002) provided explicit study of how dynamics of asset pricing is affected by different risk attitudes of different types of investors. In the context of agent based artificial market, the impacts of risk preference on wealth shares dynamics and the survival of investors are studied respectively by Chen and Huang (2007a and 2007b). Obviously, it's no longer surprising to find wealth dependent and time varying preferences in the modified agent based models.

Another important issue is heterogeneous beliefs, which could be interpreted as different forms of expectations and strategies. Some earlier papers discussed the heterogeneity in information and intelligence, [12] which are more or less analyzed from the angle of incomplete information.[13] Just as Boswijk, Hommes and Manzan (2007) mentioned, disagreement in asset pricing models can be arised under two assumptions: different information and different interpretation. In the first case, asymmetric information causes heterogeneous expectations among agents. And the second assumption may mainly contain behavioral and psychological factors, such as heuristics and representativeness. The latter heterogeneity is considered much more in agent based models.

There gives many examples on heterogeneous beliefs. Emilio Barucci, Leonardo Landi (1996) define three classes of traders in the market: rational traders, feedback traders and fundamentalist traders. Brock and Hommes (1998) considers a small number simple linear belief types, such as rational agents, fundamentalists, trend extrapolators, contrarians and biased traders.[14] Kiselev, Phillips, Gabitov (2000) believe the dynamics of the model are determined by the game of 'intelligent' and 'random' traders. Cincotti, Focardi, Marchesi, Raberto (2003) study the performance of three active trading strategies (mean-variance, mean-reversion and relative chartist strategy) in the framework where the great majority of agents follows a random behavior. He and Kryzanowski (2006) assume that two groups of agents hold different beliefs about firm fundamental values, and the more sophisticated group (rational) adopts contrarian strategies against the naive group (quasis). Shimokawa, Suzuki and Misaw(2007) point out a possibility that loss-averse feature of investors explains vast number of financial stylized facts. This reflects heavy behavioral and psychological color.

B. Interaction:Learning and Evolution

In agent-based models, there exists two types of important interactions as learning and evolution.

[7] LeBaron (2006) give several reasons.

[8] A good survey is made by Duffy (2006).

[9] Directly challenging experimental methods, computational simulations tend to offer the possibility on the mental procedures of artificial intelligent agents and be more adaptive to agent interactions in complex environments to replicate the stylized facts. See Bengh (2002) and Brandouy(2005).

[10] See Routledge (1994), Arthur, Holland, LeBaron, Palmer, Tayler (1997), Lettau (1997), and Brock and Hommes (1998).

[11] See Beltratti and Margarita (1992), Levy, Levy and Solomon (1994), and Arifovic(1996).

[12] Gode and Sudder (1996) select two types of market participants profit motivated human trades and "zero intelligence" machine traders.

[13] The idea of uncertainty and information is organized in the framework of Grossman and Stigliz (1980).

14 Similar idea can be found in Chiarella and He (2002), Tetsuya Shimokawa, Kyoko Suzuki, Misaw (2007), Chang (2007),and He and Li (2007).

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There are two broad of learning processes, reinforcement learning and belief learning following Selten (1991). Reinforcement learners condition their actions on their own histories and abide by the principle which have yielded relatively high (or low) payoffs in the past are more (or less) likely to be played in subsequent periods. Reinforcement learning models have been widely used in the agent-based literature.[15] Belief-based learning models, however, assume that players have history dependent beliefs over the opponents actions, and may choose actions that myopically best responses to their beliefs.

Riechmann (2000) suggest that another dimension that can be distinguished is the level at which learning is modeled, the individual and the population level . With individual learning an agent learns exclusively on the basis of his own experience, whereas population or social learners base themselves on the experience of other players as well.

LeBaron (2005) point out setting up an evolutionary learning framework requires several preliminary steps. First, the mapping from behavioral rules into a genetic structure is important. One needs to end up with some type of object that represents the behavior manipulated by evolutionary operators. Second, in most evolutionary methods there will be a population of the previously mentioned solutions among which investors choose a better rule to substitute the bad one. The computer simulates evolution by removing some set of low fitness solutions. Third, in financial settings agents and strategies can be evolved by using wealth or utility-based fitness.

Hommes (2005) divide interactions into local ones and social ones. neither of them may lead to strong dependencies and cause large movements at the aggregate level. Follmer (1974)shows that even short range interaction may propagate through the economy and lead to aggregate uncertainty causing a breakdown of price equilibria. Kirman (1991) considers an exchange rate model where the fractions of chartists and fundamentalists are driven by the stochastic model for opinion formation.

Hommes (2005) mentioned that social interaction among individuals refers to a situation where the utility or payoff of an agent depends directly upon the choices of other individuals in their reference group, in addition to the dependence which occurs through the intermediation of markets. Schelling (1971) considers a model where individuals have preferences over their neighborhood and shows that, even when these preferences are relatively weak, it may lead to pronounced residual segregation.

In sum, heterogeneity of agents and interactions lie in the core of agent based computational system. Meanwhile, other design issues, such as information representation, benchmarks, software and hardware environment, are also need to be considered carefully.

[15] Arthur (1991,1993) was among the first economists to suggest modeling agent behavior using reinforcement-type learning algorithm. In Bendor et al. (2001) consider player's period aspiration level in their reinforcement learning models.

III. Two Frameworks of Agent-Based Models

A. *the Santa Fe Aritifical Stock Market Model*

The Santa Fe artificial stock market (SF-ASM) is one of the most adventuresome artificial market projects. It is outlined in detail in Arthur et al.(1997) and LeBaron et al.(1999). The model tries to use a classifier based system to combine a well defined economic structure in the market trading mechanisms and an inductive learning model. In this section, we give a brief outline of the market structure and a summary of some of the results.

The basic conditions of the market are given by some existing work such as Stiglitz (1980). Under assumption of one period, myopic, constant absolute risk aversion (CARA) utility, agents make a decision on their desired asset composition between a risk free bond and a risky stock with a stochastic dividend. The bond is in infinite supply and pays a constant interest rate, *r*. The dividend *d* follows a well defined stochastic process (See LeBaron, 2006) as:

$$d_t = \bar{d} + \rho(d_{t-1} - \bar{d}) + \varepsilon_t \qquad (1)$$

Where ε_t is gaussian, independent, and identically distributed, and $\rho = 0.95$ for all experiments. It is well known that under CARA utility, and gaussian distributions for dividends and prices, the demand for holding shares of the risky asset by agent *i* is given by

$$s_{t,i} = \frac{V_t\left(p_{t+1} + d_{t+1}\right) - p_t(1+r)}{\gamma\sigma^2_{t,i,p+d}} \qquad (2)$$

where P_t is the price of the risky asset at t, $\sigma^2_{t,i,p+d}$ is the conditional variance of $p+d$ at time t, for agent i, γ is the coefficient of absolute risk aversion, and $E_{t,i}$ is the expectation for agent i at time t. Assuming a fixed number of agents, N, and a number of shares equal to the number of agents gives

$$N = \sum_{i=1}^{N} s_i \qquad (3)$$

which closes the model. In this market there is a well defined linear homogeneous rational expectations equilibrium (REE) in which all traders agree on the model for forecasting future dividends, and the relation between prices and the dividend fundamental. An example of this would be

$$P_t = b + a d_t \qquad (4)$$

The parameters *a* and *b* can be easily derived from the underlying parameters of the model by simply substituting the pricing function back into the demand function, and setting it equal to 1, which is an identity and must hold for all d_t.

At this point, this is only a very simple economic framework with nothing particularly new. Where this breaks from tradition is in the formation of expectations. Individual expectations of the agents are formed using a classifier system[16]which tries to determine the relevant state of the economy, and this in turn leads to a price and dividend forecast which will go into the demand function.

In standard classfier systems, there is a determination which is the strongest rule depending on past performance, while the rule recommends an action. Here, each rule maps into a real vector of forecast parameters, $a_{i,j}, b_{i,j}, \sigma^2_{i,j}$ which the agent uses to build a conditional linear forecast as follows: (See LeBaron 2006)

$$E_{t,i,j}\left(p_{t+1}+d_{t+1}\right)=\alpha_{i,j}\left(p_t+d_t\right)+b_{i,j} \quad (5)$$

This expectation along with the variance estimate, $\sigma_{i,j}$ allows the agent to generate a demand function for shares using Eq. (2).

All matched rules are evaluated according to their accuracy in predicting price and dividends. Each rule keeps a record of its squared forecast error according to

$$\sigma^2_{t,i,j}=\beta\sigma^2_{t-1,i,j}+(1-\beta)((p_{t+1}+d_{t+1})-E_{t,i,j}(p_{t+1}+d_{t+1}))^2 \quad (6)$$

This estimate is used to share demand, and determine the strength of the forecast rules in evolution. The final important part involves the evolution of new rules. Which are generated using a genetic algorithm with uniform crossover and mutation.

Like most artificial market models, agents in SF-ASM are designed with the character of heterogeneity and interaction. In the whole experimental process, agents have different believes and preferences, showing dynamic heterogeneity. But the most important part of the SF-ASM is its implementation of learning and forecasting. This is done with a classifier forecasting system, through which traders can use, or ignore, any part of a predefined set of current information in their forecasts.

This is one of the most complex artificial markets in existence which brings both advantages and disadvantages. One thing this market does is to allow heterogeneous agents to explore a fairly wide range of possible forecasting rules. The interactions that cause trend following rules to persist are endogenous, because they are not forced to be in the market. On the other hand, the market is relatively difficult to track in terms of analytic studies. It is sometimes difficult to pin down causalities acting inside the market.

Although SF-ASM generates profound influence to the development of computational modeling based on agents,[17] some design issues are still criticized. Johnson (2002) gives an overview and critique of the software from a design perspective. As to GA, Harrald (1998) point out the traditional distinction between the phenotype and genotype in biology and doubted whether the adaptation can be directly operated in social processes. As response to this criticism, Chen and Yeh (2001) propose a new architecture rests on a mechanism called "business school" that allows for some strategy learning to occur across agents in a very natural way. Ehrentreich (2005) claims that the original SF-ASM GA mutation operators are biased. LeBaron (2006) questions Ehrentreich (2005)'s criticism, and declares that what is biased in terms of mutation and whether one has to have an unbiased mutation operator are both not clear.

B. Heterogeneous Agent Model

Brock and Hommes (1998) developed a heterogeneous agent model (HAM)[18] , in which adaptive belief systems (ABS) were introduced as a financial market application of the evolutionary selection of expectation rules. An ABS is in fact a standard discounted value asset pricing model derived from mean-variance maximization, extended to the case of heterogeneous beliefs. An ABS is in fact similar to SFI model which may be seen as a stylized, to some extent, analytically tractable version of more complicated artificial markets. A convenient feature of an ABS is that it can be formulated in terms of deviation from a benchmark fundamental. (See Hommes, 2006) Consider an asset pricing model with one risky asset and one risk free asset. Let P_t denotes the price (ex dividend) per share of the risky asset at time t, and let $\{y_t\}$ be the stochastic dividend process of the risky asset. The risky free asset is perfectly elastically supplied at gross return

$$W_{t+1}=RW_t+(p_{t+1}+y_{t+1}-Rp_t)z_t \quad (7)$$

$R>1$ .We have

For the dynamics of wealth where bold face type denotes random variables and z_t denotes the number of shares of the asset purchased at date *t*. Let E_t, V_t denote the conditional expectation and conditional variance operators, based on publicly available information set consisting of past prices and dividends. Let $E_{h,t}$, $\mathrm{V}_{h,t}$ denote the "beliefs" of investor type *h* about the conditional variance. Note that the conditional variance of wealth W_{t+1} equals z_t^2 times the conditional variance of excess return per share $p_{t+1}+y_{t+1}-Rp_t$ .We assume that beliefs about the conditional variance of excess returns are constant and the same for everyone, i.e. $V_{ht}(p_{t+1}+y_{t+1}-Rp_t)\equiv\sigma^2$ for all types h^1. Assume each

16 Classifier systems are inductive, rule-based learning systems that combine reinforcement-type learning over a set of simple logical rules. Classifier systems are perhaps best viewed as models of individual learning, akin to expert systems.

17 The framework is adopted by Chen and Yeh (2001), Tay and Linn (2001), Chen and Liao (2005) and Chen and Huang (2007ab).

18 A good survey of HAM could be found in Hommes (2006).

type is a myopic mean variance maximizer, so for type *h*, the demand for shares z_{ht} solves

$$Max_z(E_{ht}W_{t+1} - \frac{a}{2}V_{ht}[W_{t+1}]) \quad (8)$$

$$z_{ht} = E_{ht}[p_{t+1} + y_{t-1} - Rp_t]/a\sigma^2 \quad (9)$$

Where *a* denotes the risk aversion, which is assumed to be equal for all traders. Let z_{st} denote the supply of shares per investors and n_{ht} the fraction of investors of type *h* at date *t*. Equilibrium of demand and supply implies.

$$\sum n_{ht}(E_{ht}[p_{t+1} + y_{t+1} - Rp_t]/a\sigma^2) = z_{st} \quad (10)$$

Then the market clearing equilibrium price p_t can be determined by

$$Rp_t = \frac{1}{\sum_{h=1}^{H}\frac{n_{ht}}{\sigma_{ht}^2}}\left(\sum_{h=1}^{H}\frac{n_{ht}}{\sigma_{ht}^2}E_{ht}[p_{t+1} + y_{t+1}] - a_{ht}Z\right) \quad (11)$$

Where the summation is over *H* groups.

In SFI-ASM framework, the key step to build the artificial market is the approaches (may be different from RE) to form the expectation of every agent. In this process, the essential issues, such as heterogeneity, learning and evolution could be induced in varied forms. It's more or less similar to HAM. Traders nevertheless believe that in a heterogeneous world prices may deviate from their fundamental value P_t^*. It is convenient to introduce deviation from the fundamental price:

$$x_t = p_t - p_t^* \quad (12)$$

Beliefs about the future price of the risky asset are of the form

$$E_{ht}[p_{t+1}] = E_{ht}[p_{t+1}^*] + f_h(x_{t-1}, \ldots, x_{t-L}) \quad \textit{for all } h,t. \quad (13)$$

Each forecasting rule f_h represents a model of the market (e.g. a technical trading rule), according to which type h believes that prices will deviate from the fundamental price. Due to heterogeneity, forecasting rules differ between technical traders (such as feedback traders, trend extrapolators and contrarians) and sophisticated traders(such as fundamentalists or rational arbitragers).

The evolutionary part of the model, describing how beliefs are updated over time, follows the endogenous selection of forecasting rules. The fraction n_{ht} of the trader types are given by the multinomial logit probabilities of a discrete choice:[19]

$$n_{ht} = \frac{\exp(\beta U_{h,t-1})}{Z_{t-1}}, \qquad Z_{t-1} = \sum_{i=1}^{n}\exp(\beta U_{h,t-1}). \quad (14)$$

$U_{h,t-1}$ is the fitness measure of strategy h evaluated at the beginning of period t. A natural candidate for evolutionary fitness is (accumulated) realized profits, given by

$$U_{ht} = (p_t + y_t - Rp_{t-1})\frac{E_{h,t-1}[p_t + y_t - Rp_{t-1}]}{a\sigma^2} + \omega U_{h,t-1} \quad (15)$$

Where $0 \le \omega \le 1$ is the memory parameter measuring how fast past realized profits are discounted for strategy.[20]

Recently, several modifications of ABS have been studied. Chiarella and He (2001) study the asset price dynamics in heterogeneous agent framework under the constant relative risk aversion (CRRA) utility, assuming that investors' relative wealth affects asset the demand and realized asset price. Anufriev and Bottazzi (2005) characterize the type of equilibrium and their stability in an HAM model with CRRA utility and an arbitrary number of agents. Chiarella, Dieci and Gardini (2002, 2005) use CRRA utility in an ABS(adaptive belief systems) with a market maker price setting rule. Chang (2005) studies the effects of social interactions in an ABS with a Walrasian market clearing price. DeGrauwe and Grimaldi (2005a, b) recently applied the ABS framework to exchange rate modeling. A related stochastic model with heterogeneous agents and endogenous strategy switching similar to the ABS has recently been introduced in Follmer et al. (2005). He and Li (2007) use an HAM model find that agent heterogeneity, risk-adjusted trend chasing, and the interplay of noisy fundamental and demand processes and the underlying deterministic dynamics can be the source of power-law distributed fluctuations.

C. *Comparisons of the Two Frameworks*

Both SFI-ASM and HAM, which capture some stylized facts in the financial market, are very prominent frameworks in the family of agent based computational models. The two models share many similarities in paradigm, design concept and methodology, however, there are several different aspects they emphasize respectively. First, the agents in the HAM model are restricted to few types, while SFI-ASM model portrays more diversified and sophisticated trading strategies through reproduction, mutation and crossover. Second, HAM tends to be analytically tractable in mathematic form, while SFI-ASM exhibits more computational features. Last but not least, strict heterogeneity plays an essential role in HAM, with more attention paid to a complex adaptive system characteristic with nonlinear and chaotic properties.

[19] See Brock and Hommes (1997).

[20] Hommes focus on the case where there are no difference in the cost for strategies.

Differently, SFI-ASM sets up to simulated the real world and makes more effort to capture the said ecology-like evolutionary system in a nature manner.

IV. CONCLUSION AND DIRECTIONS FOR THE FUTURE

This paper gives a rough overview of the current state of research in ABCF(agent-based computational finance) along with several design principles and examples. While the standard economics and finance remains a top-down, deductive theorizing method with fully rational agents, the findings of ABCF researchers using bottom-up, bounded rational, inductive methods of behavior are attracting increasing attention. Although the ABCF models are questioned for the sensitivity to sizes, horizons and initial conditions because of too many parameters and too much freedom of the models, they remain deserve the priority in our attempt to understand the academic frontier. Nevertheless, it is important to note that this is fairly a potential field.

Firstly, ABCF models are naturally behavioral models which suited for testing behavioral theories.[21] The models answered two key questions in behavior structure, as how well do behavioral biases hold up under aggregation and which types of biases will survive in co-evolutionary struggles. Therefore, the connections between agent-based approaches and behavioral approaches will probably become more inter-wined as both fields progress.

Secondly, ABCF might take advantage of the achievements in economic laboratories. Further comparisons of different models using a variety of experimental data sets are needed. Further parallel experiments with human and artificial agents situated in the same environment are needed to better understand the external validity of agent-based models as well as to appropriately calibrate chose models.

Finally, it is worthy to note that, until now only few attempts have been made to estimate on financial data[23] and little work has been done on policy implications. Much computationally and theoretically oriented work need people exert more endeavors to application of ABCF in the real economic and financial practice. Even though it is a great challenging to many theorists and modelers, it reminds us to study aimed at a satisfying application.

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[21] Shimokawa, Suzuki, Misawa (2007) point out a possibility that loss-averse feature of investors explains vast number of financial stylized facts and plays a crucial role in price formation.

[23] An attempt is made by Boswijk, Hommes and Manza (2007).