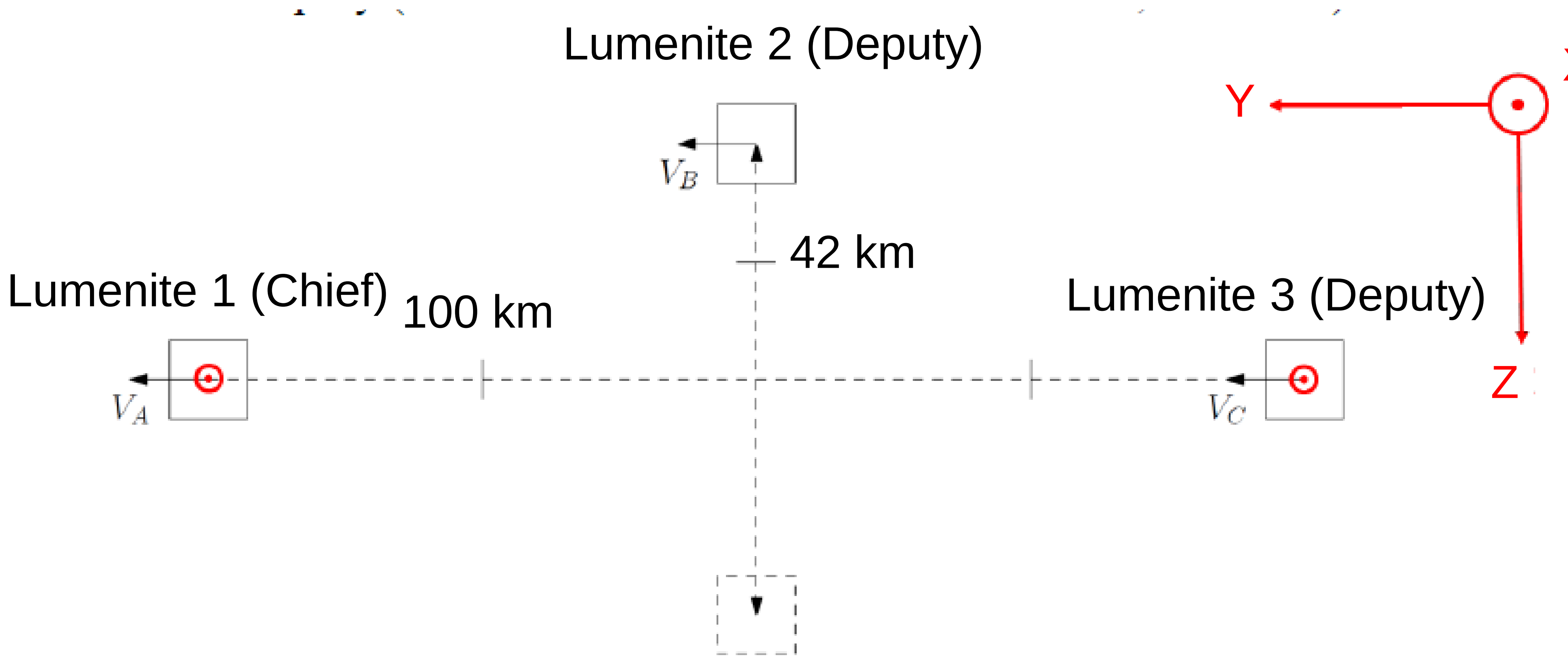
Outline

1. Formation Flying Requirements
2. Formation Flying Relative Co-Ordinates
3. Formation Flying Initialization and Reference
4. Orbital Disturbance Forces on Relative Motion
5. Propulsion System Characteristics
6. Propulsion System Parameters
7. Discrete Time Formation Flying Controller Design
8. Formation Control Law Implementation
9. Ground Control Sequence

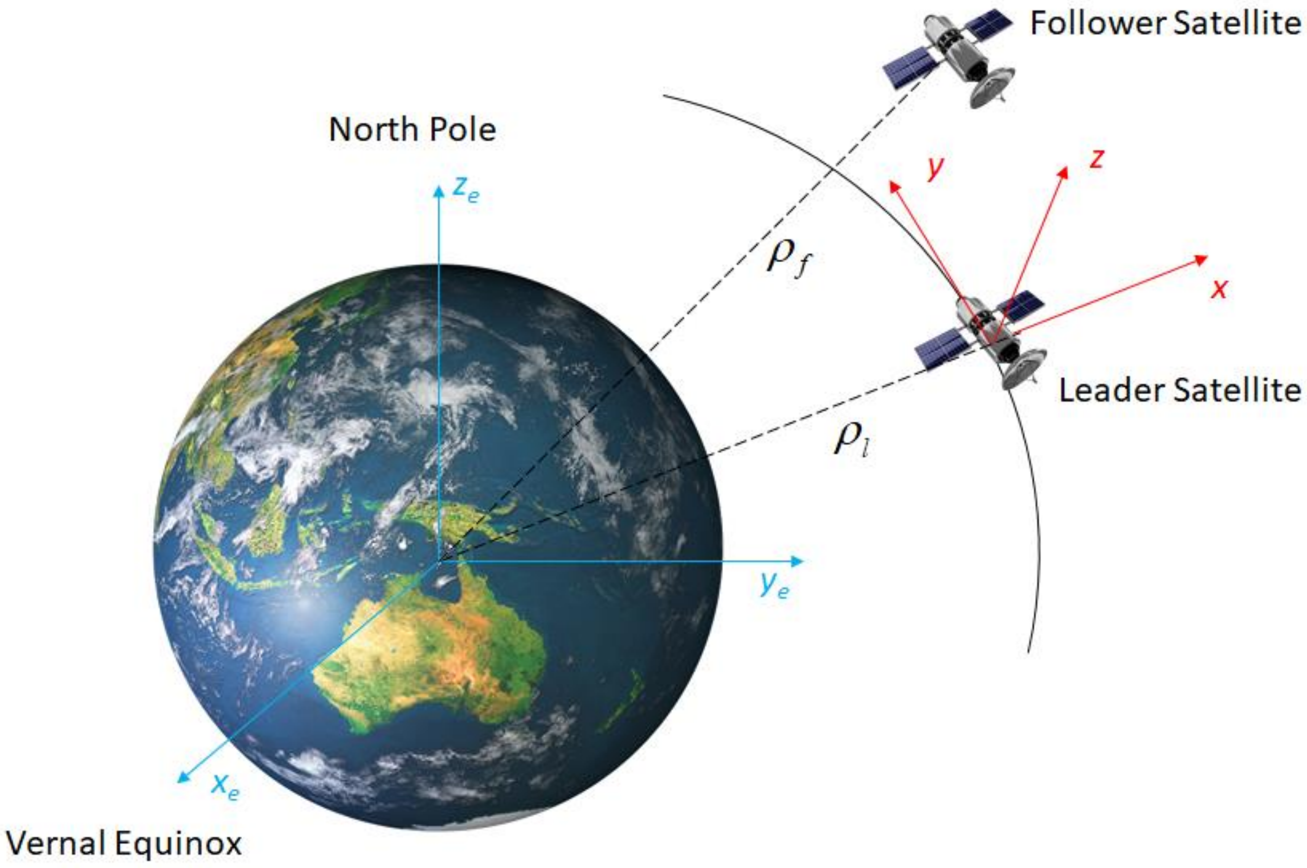
1. Formation Flying Requirements

- **All 3 satellites inserted into Target Reference Orbit : 585 km, 10 deg inclination about 2000 km each along track separation in same orbit**
- **After insertion, to achieve 100 km base line along track between each satellite**
- **Lumenite 2 to change plane to achieve +/- 42 km across track (2º RAAN Change) between Lumenite 1/3 and Lumenite 2**
- **Required formation control threshold : 5 km (1 sigma)**
- **2 Deputy (with Propulsion) Satellites and 1 Chief (w/o Propulsion) Satellite**



2. Formation Flying Relative Co-Ordinate

- Formulation and Equation of Motion with J_2 Perturbation



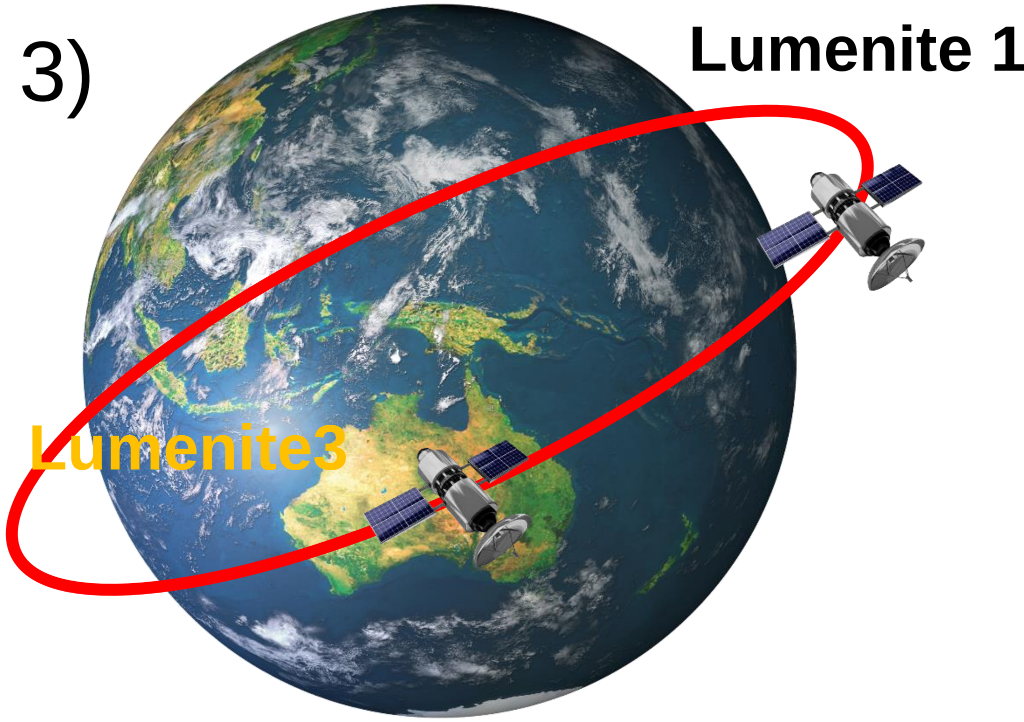
$$\ddot{x} - 2nc\dot{y} - \left(5c^2 - 2\right)n^2 x = u_x$$
$$\ddot{y} + 2nc\dot{x} = u_y$$
$$\ddot{z} + k^2 z = 2lk\cos\left(kt + \phi\right) + u_z$$

$$s = \frac{3J_2 R_e^2}{8a^2}\left[1 + 3\cos\left(2i_l\right)\right]$$
$$c = \sqrt{1+s}$$
J_2 (the second zonal harmonic coefficient of order 0)
$$= 1{,}082.6 \times 10^{-6}$$
$$k = nc + \frac{3nJ_2 R_e^2}{2a^2}\left[\cos i_f\right]^2$$
$$l = 0$$

3. Formation Flying Initialization and Reference

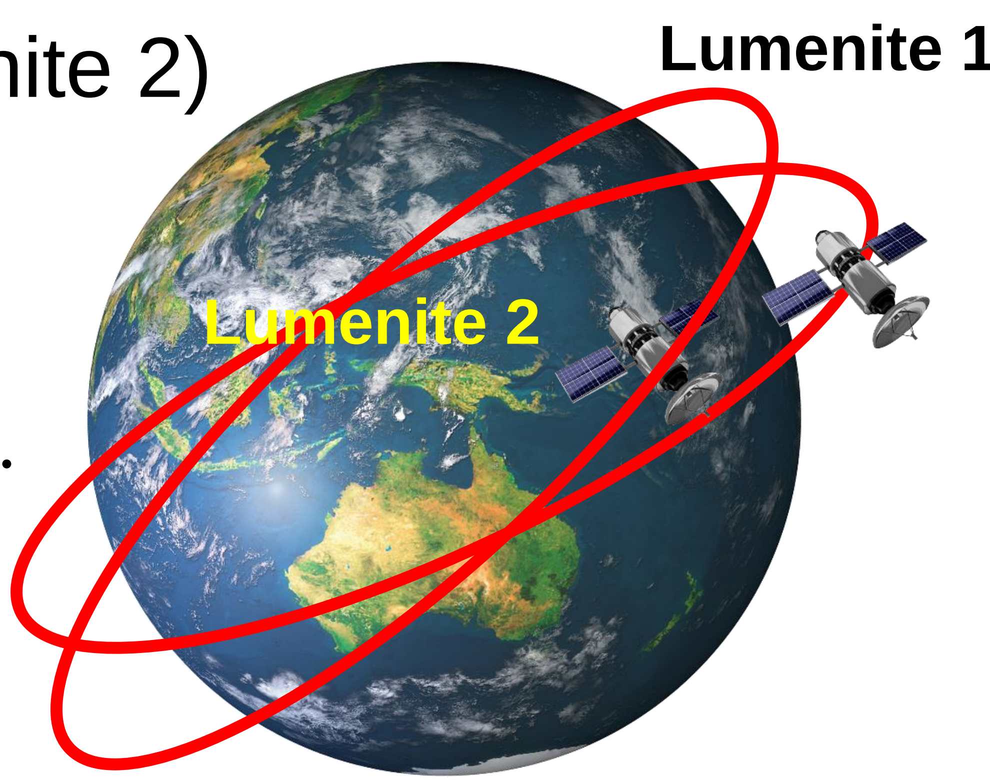
- In plane Formation (Deputy – Lumenite 3)

$$x(0)=0,\ y(0)=200\ \text{km},\ z(0)=0,$$
$$\dot{x}(0)=0,\ \dot{y}(0)=-2nx(0)\sqrt{1+s}=0,\ \dot{z}(0)=0.$$



- Cross Track Formation (Deputy – Lumenite 2)

$$x(0)=0,\ y(0)=100\ \text{km},\ z(0)=0,$$
$$\dot{x}(0)=0,\ \dot{y}(0)=-2nx(0)\sqrt{1+s}=0,\ \dot{z}(0)=45.8\ \text{m/s}.$$



Stable Periodic Relative Motion with J_2 Perturbation

- Stable relative motion with J_2 perturbation for (i) circular chief orbit (2) same inclination for Lumenite 1/3 and Lumenite 2 orbital planes (3) no orbital disturbance forces, e.g. solar radiation pressure and drag

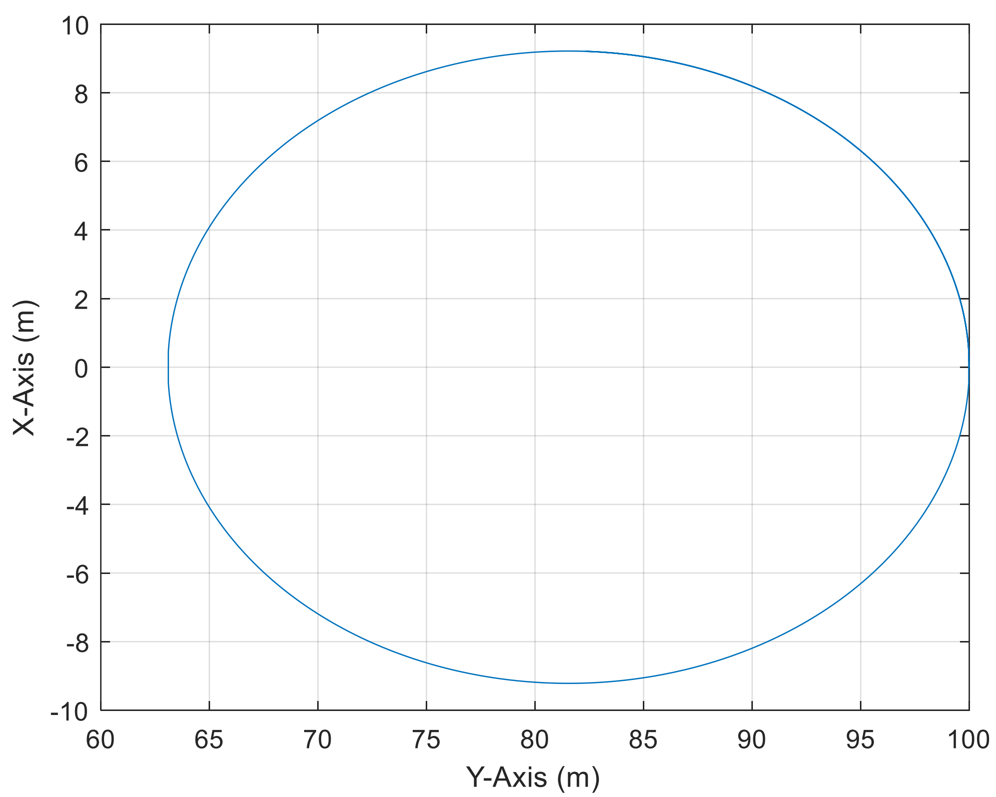
$$x(t) = x(0)\cos\left(nt\sqrt{1-s}\right) + \frac{1}{n\sqrt{1-s}}\dot{x}(0)\sin\left(nt\sqrt{1-s}\right)$$

$$y(t) = -\frac{2\sqrt{1+s}}{\sqrt{1-s}}x(0)\sin\left(nt\sqrt{1-s}\right) + \frac{2\sqrt{1+s}}{n(1-s)}\dot{x}(0)\cos\left(nt\sqrt{1-s}\right) + y(0) - \frac{2\sqrt{1+s}}{n(1-s)}\dot{x}(0)$$

$$z(t) = \frac{\dot{z}(0)}{k}\sin(kt) + z(0)\cos(kt)$$

with

$$x(0) + \frac{\dot{y}(0)}{2n\sqrt{1+s}} = 0 \Rightarrow \dot{y}(0) = -2nx(0)\sqrt{1+s}$$

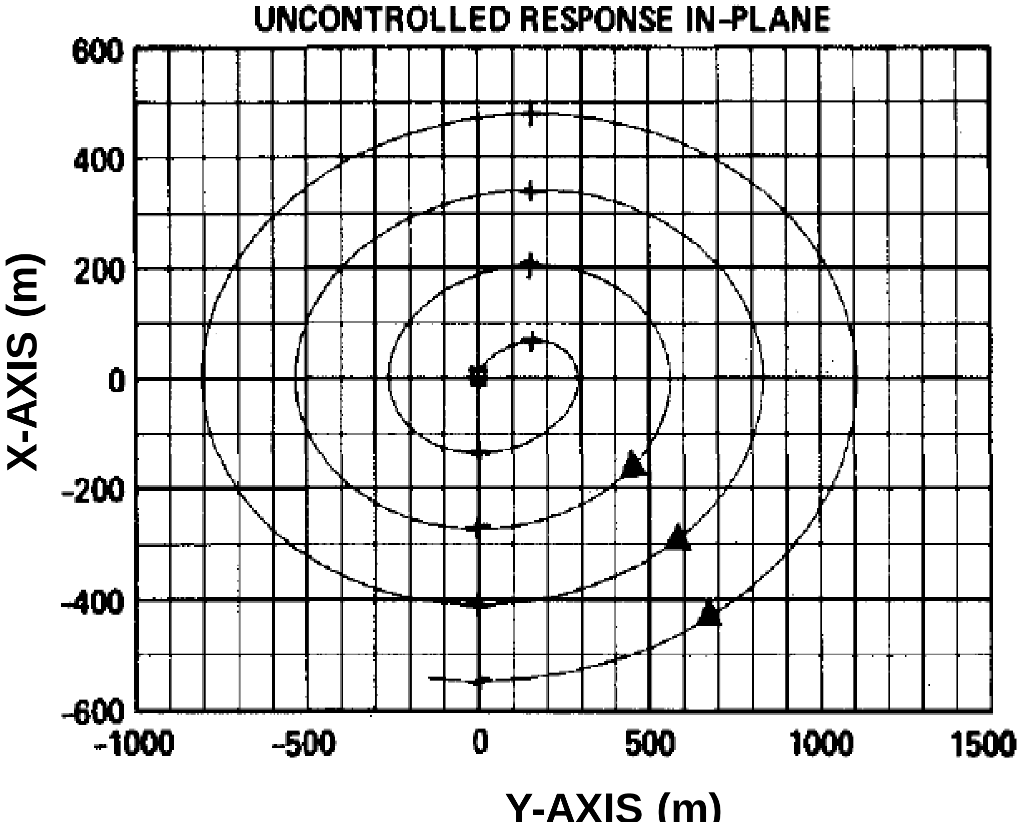


4. Orbital Disturbance Forces on Relative Motion

- Relative Motion with Orbital Disturbance Forces

$$\ddot{x} - 2n\dot{y} - 3n^2 x = d_x$$
$$\ddot{y} \quad + 2n\dot{x} = d_y$$
$$\ddot{z} \quad + n^2 z = d_z$$

- Solution with Undamped Growing In-Plane Relative Position



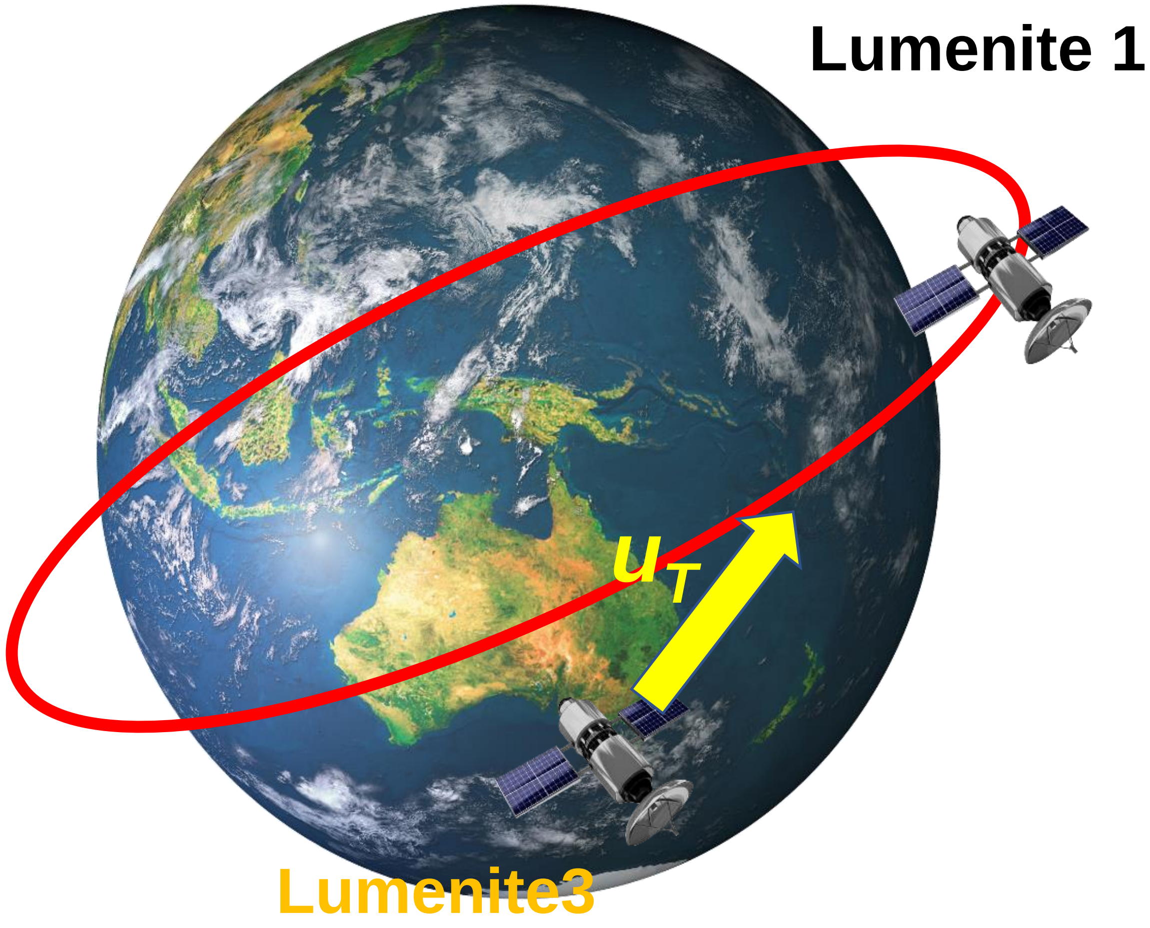
$$x = \frac{3}{2}\frac{d_y}{n^2}\left[\sin nt - nt\cos nt\right] - \frac{d_x}{n^2}\left[\frac{3}{2}nt\sin nt - 2\left(1-\cos nt\right)\right]$$

$$y = \frac{d_y}{n^2}\left[3nt\sin nt - 5\left(1-\cos nt\right)\right] + \frac{d_x}{n^2}\left[3nt - 6\sin nt + 3nt\cos nt\right]$$

$$z = \frac{d_z}{n^2}\left(1-\cos nt\right)$$

Formation Control Solution

- Convert Leader and Follower Satellite Positions from ECEF to Relative Co-Ordinate
- Compute Thruster Command u(t) in Relative Co-Ordinate
- Slew Satellite in Correct Direction $\hat{u}$
- Fire Thruster for burn time t_{on}



5. Thruster Characteristics

- Satellite Dry Mass = 17.1 kg

  **Max Propellant = 0.8 kg

  **Max satellite wet mass = 17.9 kg

- For every maneuver

  Battery recharge - 5 orbits (include tank heating)

- For control design you can use either the follow, both can assume can be done once every 5 orbits

  <u>Firing Profile 1</u>

  Firing 5 minutes

  OR

  <u>Firing Profile 2</u> (but not preferred as this requires more frequent firing)

  Firing 2 minutes

  Rest 1 minutes

  Firing 2 minutes

6. Thruster Formulas

1. Thrust Force, $\left(U_T = \dot{m}_p V_{eq}\right)$, (function of remaining propellant
   i. Available in MatLab file from Samuel
2. Specific Impulse, I_{sp}, computation
   i. $$I_{sp} = \frac{U_T}{\dot{m}_p g}$$
   ii. Also available in MatLab file from Samuel
3. Total Impulse, I_{tot}, in terms of total propellant mass m_p
   i. $$I_{tot} = U_T \cdot T_{totalburntime} = m_p g I_{sp}$$
4. Propellant Consumption
   i. $$\dot{m}_p = \frac{U_T}{g I_{sp}}$$
5. Delta Velocity
   i. $$\Delta V = I_{sp} g \ln\left(\frac{m_{rest\_of\_spacecraft} + m_{propulsion\_wetmass}}{m_{rest\_of\_spacecraft} + m_{propulsion\_drymass}}\right)$$

7. Discrete Time Formation Flying Controller Design

$$\mathrm{x} = \begin{bmatrix} x & y & z & \dot{x} & \dot{y} & \dot{z} \end{bmatrix}^T$$

$$\mathrm{u} = \begin{bmatrix} u_x & u_y & u_z \end{bmatrix}^T$$

$$\dot{\mathrm{x}} = A\mathrm{x} + B\mathrm{u}$$

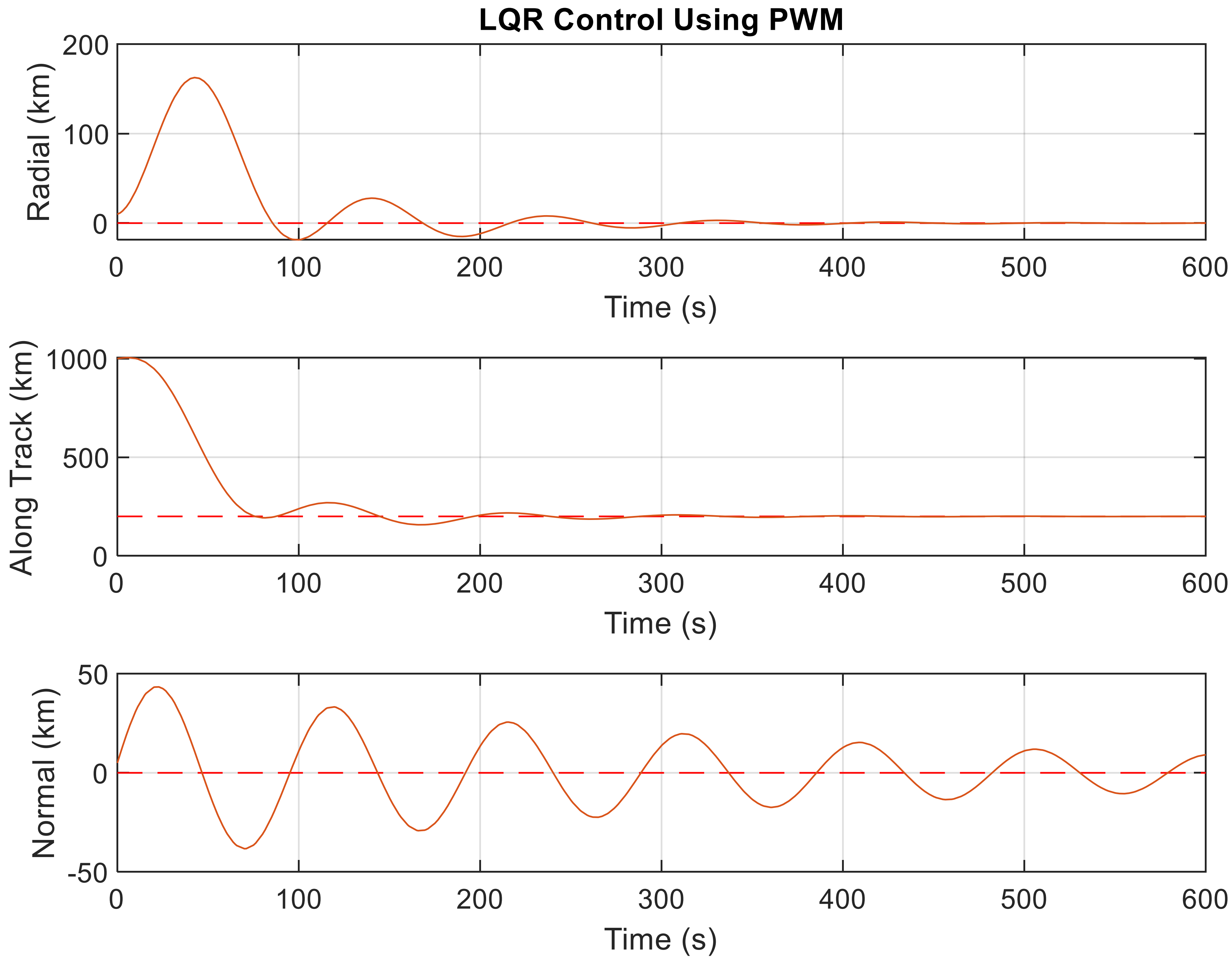
$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \left(5c^2 - 2\right)n^2 & 0 & 0 & 0 & 2nc & 0 \\ 0 & 0 & 0 & -2nc & 0 & 0 \\ 0 & 0 & -k^2 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{x}\left(k+1\right) = \Phi \mathrm{x}\left(k\right) + \Gamma \mathrm{u}\left(k\right), \qquad k = 0,1,\ldots,$$

$$\mathrm{u}\left(k\right) = \begin{cases} K\mathrm{x}\left(k\right), & k = nN,\ldots,nN+p-1 \\ 0, & k = nN+p,\ldots,nN+p+q-1 \end{cases}$$

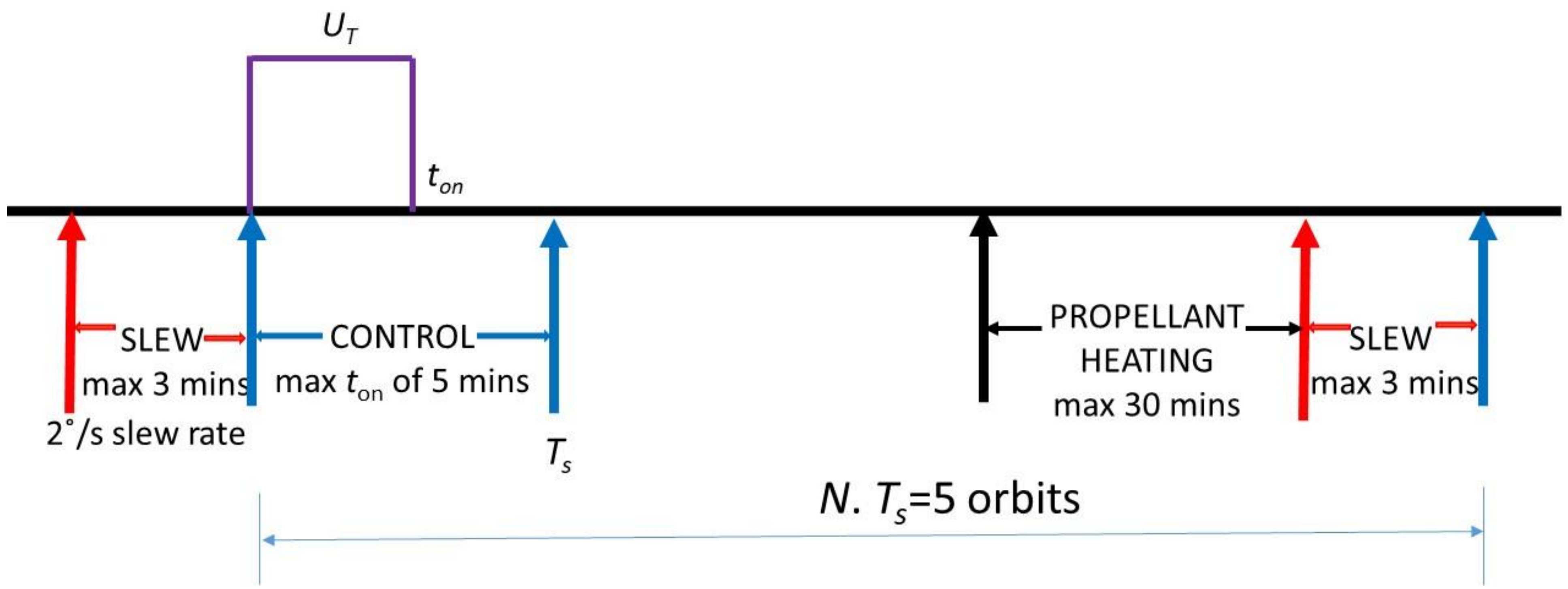
Periodic Control with 1 Firing every 5 minutes

- With 5 ms pulse sequentially – x axis time in mins not sec



8. Formation Control Law Implementation

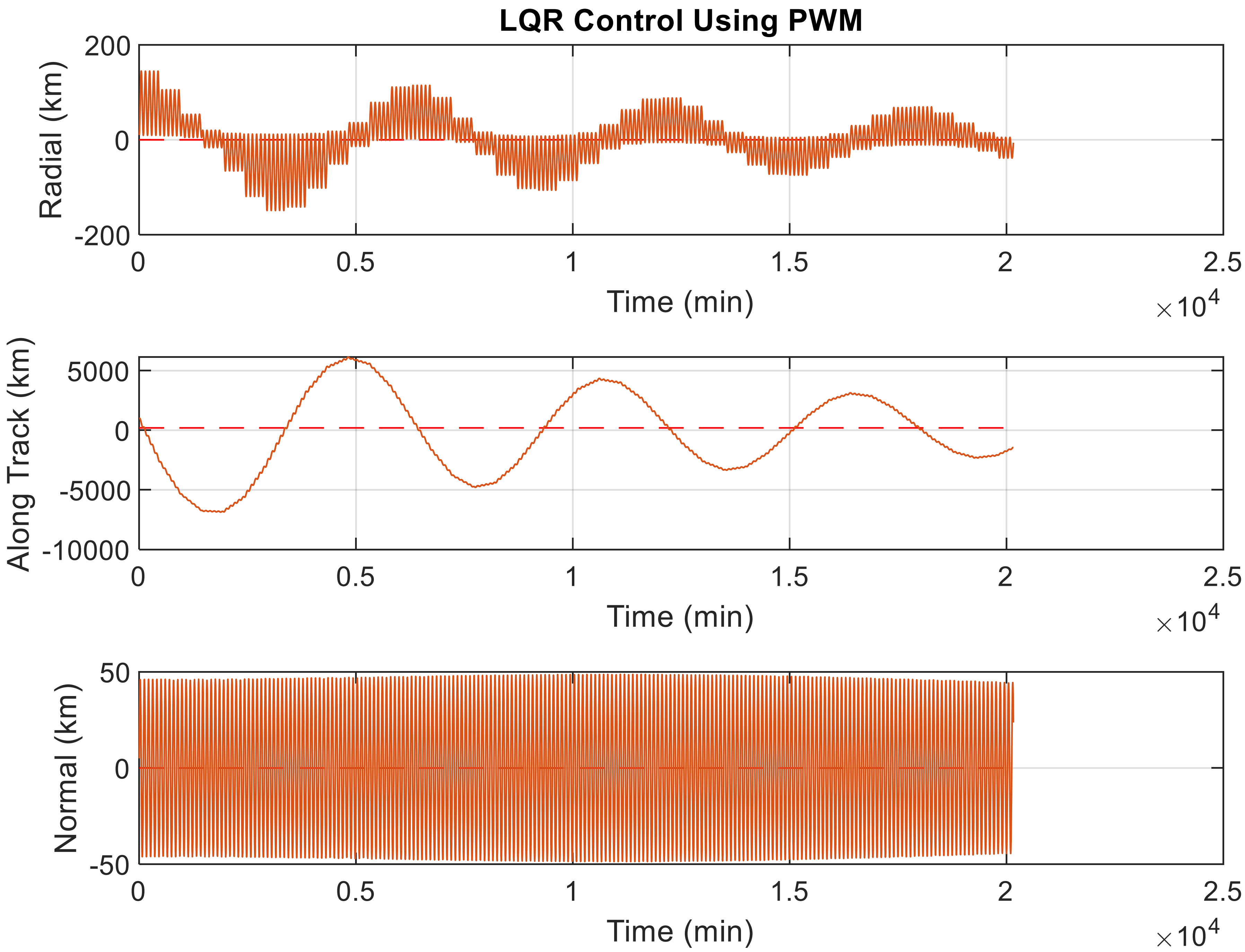
- Periodic Control



- Single Thrust Control
  - Determine u_x, u_y, u_z
  - Find corresponding thrust magnitude $|u|$ and orientation $\hat{u}$
  - Determine the thruster burn time t_{on}
- Pulse Width Modulation

$$\mathrm{u}(t) = \begin{cases} U_T \hat{u} & , \quad 0 \leq t \leq t_{on} = \dfrac{|u|}{U_T} T_s \quad , \\ 0 & , \quad t_{on} < t \leq N \cdot T_s \end{cases}$$

Periodic Control with 1 Firing Every 5 Orbits



9. Ground Control Sequence