Controllability of Financial Networks

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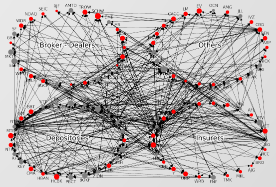
Outline

- Background: The FinancialRiskMeter (FRM)
- Brief Recap of Graph Theory
- Control Theory: Controllability and Minimum Input Set
- Computing Minimum Input Sets
- Input Data
- Application: Control Theory for FRM
- Results

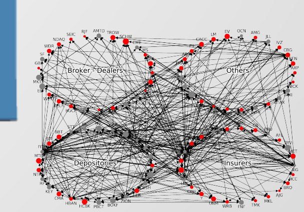


The Financial Risk Meter

http://frm.wiwi.hu-berlin.de/ FinancialRiskMeter

Tail event dependencies between 100 financial institutions and 6 macro variables - using quantile lasso regression with $\tau = 0.05$

Average penalisation parameter λ is plotted over time to obtain FRM.



Notation & Terminology

An **undirected graph** $\mathscr{G} = (\mathscr{V}, \mathscr{E})$ consists of a list of vertices $\mathscr{V} = \{1,2,\ldots,N\}$ and a set of edges $\mathscr{E} = \{(i,j),(k,l),\ldots\}$ for $i,j,k,l \in \mathscr{V}$ where

- vertices $\mathscr{V}$: node, individual, agent
- edges $\mathscr{E}$: links, connections, ties

Notation & Terminology

A **directed graph** $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of a list of vertices

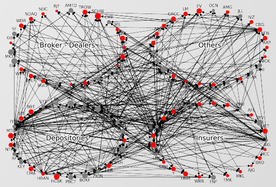
$\mathcal{V} = \{1, 2, \ldots, N\}$ and a set of **ordered edges**

$\mathcal{E} = \{(i,j), (j,i), (k,l), (l,k) \ldots\}$ for $i, j, k, l \in \mathcal{V}$

where vertices $\mathcal{V}$: node, individual, agent

and edges $\mathcal{E}$: arrows, directed edges, directed lines

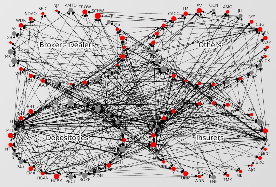
Note: Spatial projection of nodes (usually has no special meaning)



Notation & Terminology

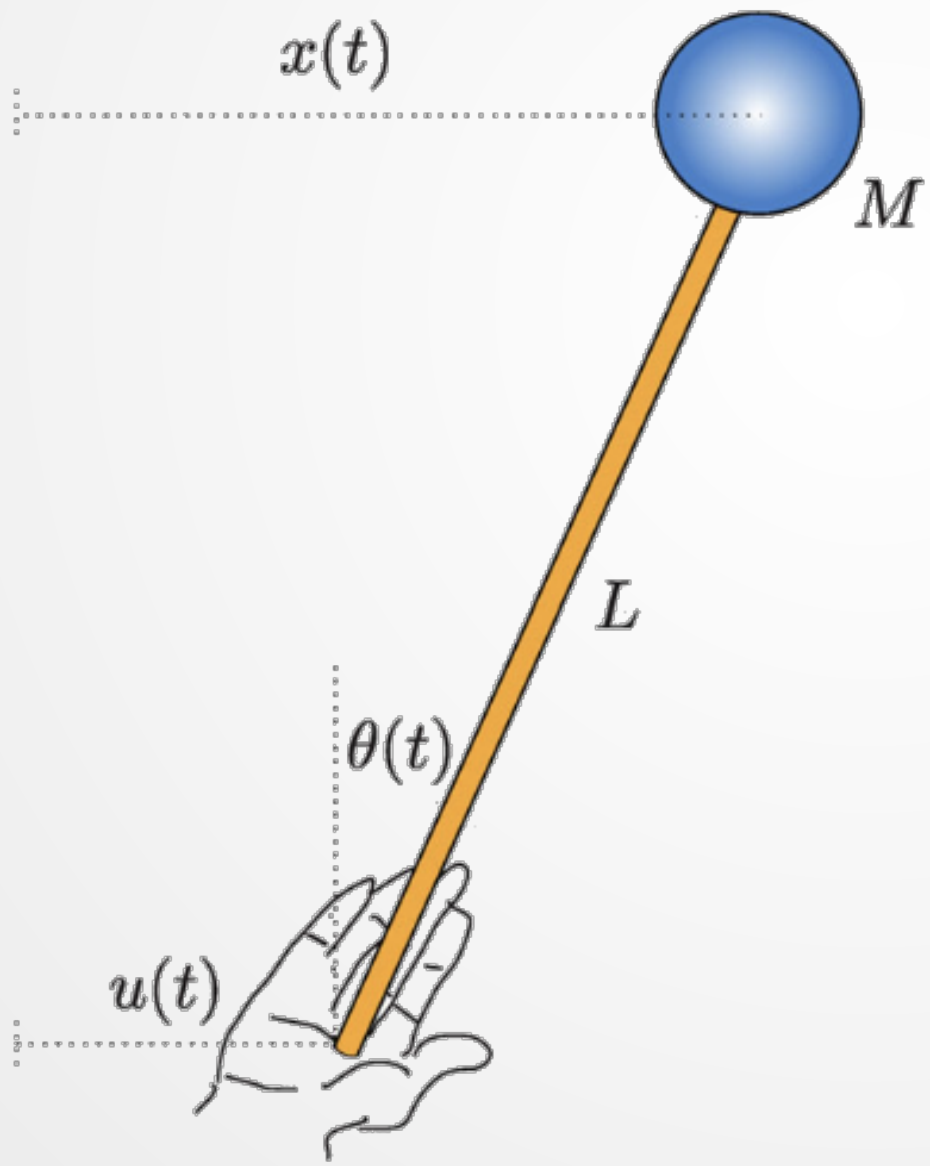
A **directed graph** could be represented by its **adjacency matrix** $A = [a_{i,j}]$ where

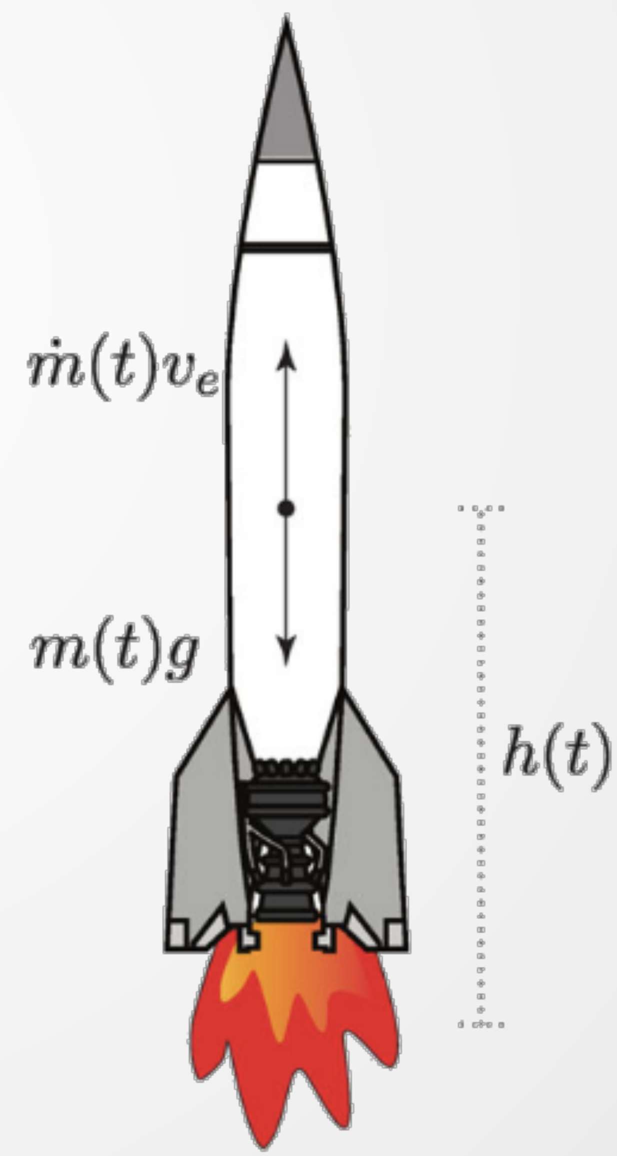
$$a_{i,j} = \begin{cases} 1, & \text{if there is an arrow from i to j} \\ 0, & \text{otherwise} \end{cases}$$



Control Theory: Controllability

A system is controllable if we can drive it from any initial state to any desired final state in finite time.





Control Theory: Controllability

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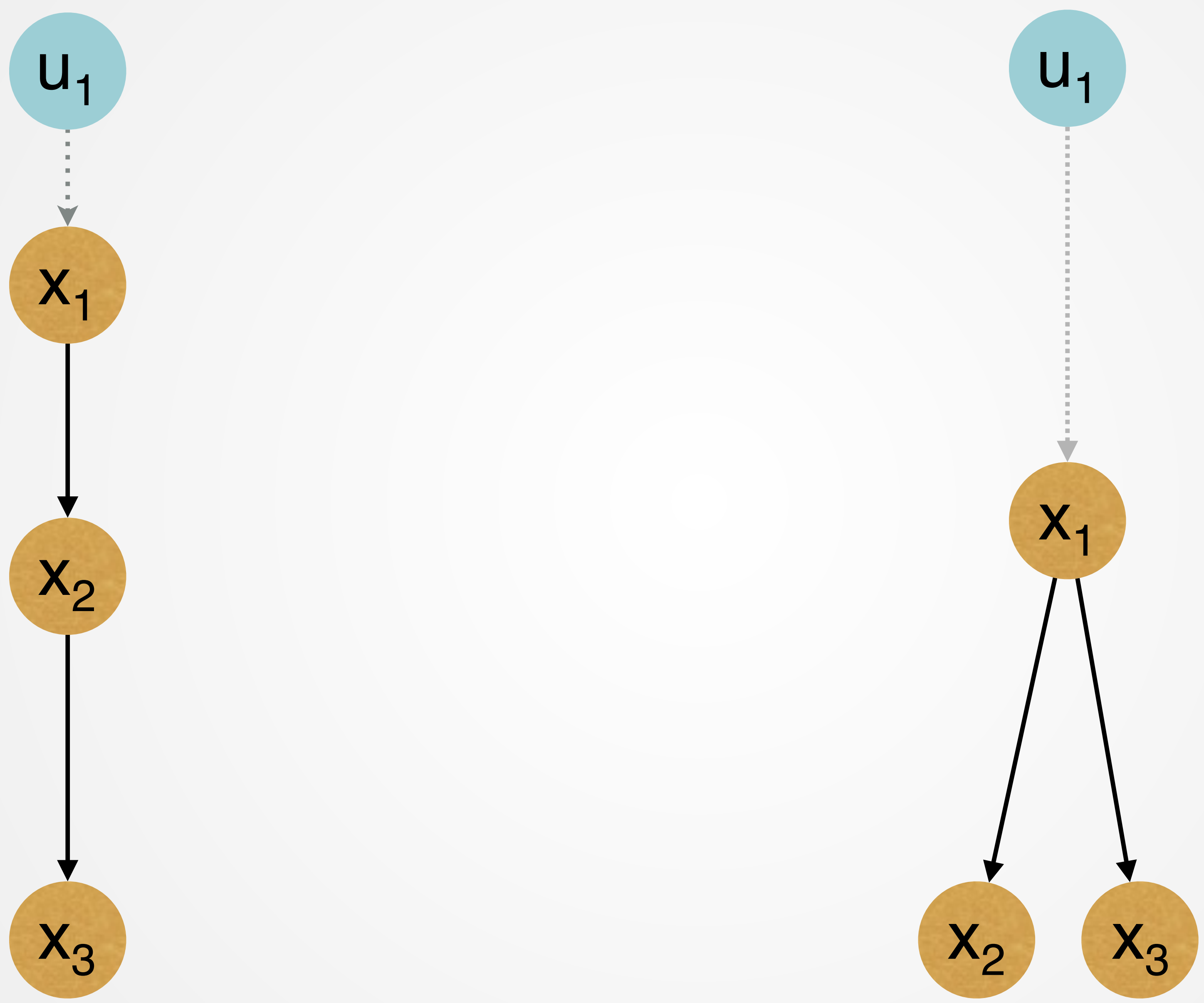
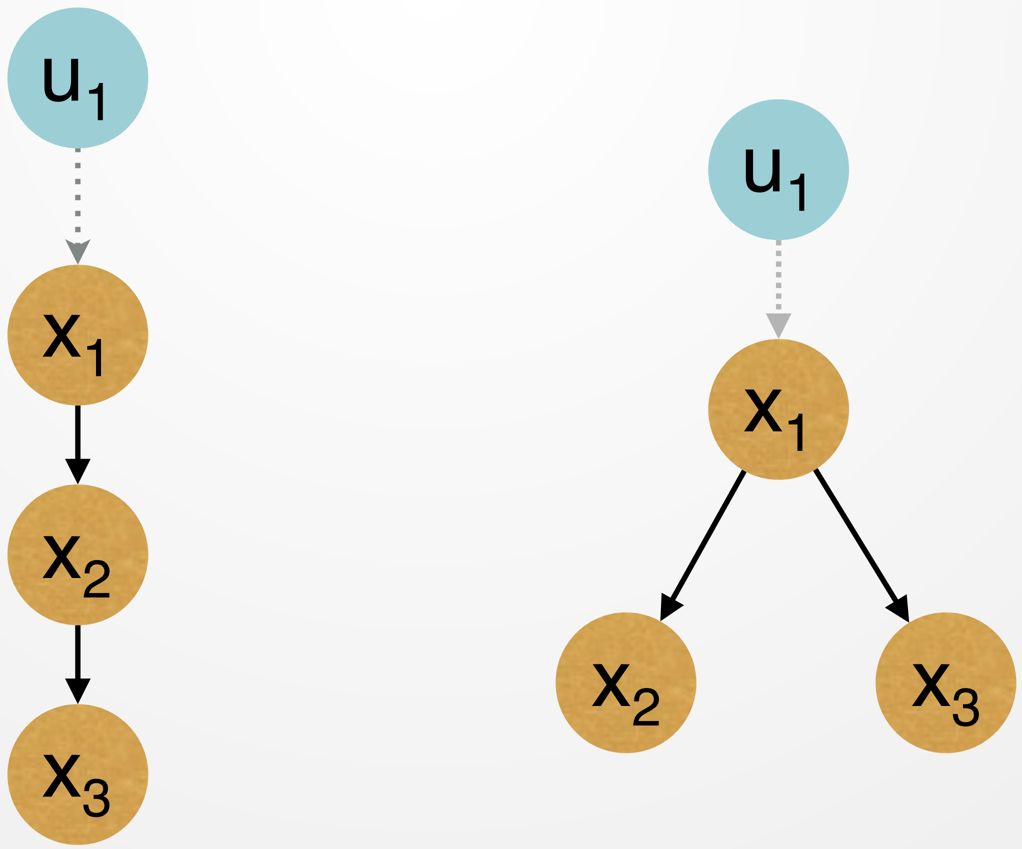


Control Theory: Minimum Input Set

A Minimum Input Set (MIS, also known as minimum driver node set) for a given graph G is a set of nodes such that

1. Graph is controllable using only nodes in MIS as input nodes
2. Cardinality of MIS in minimal, i.e. there is no alternative input set that uses fewer nodes but can still control the graph fully.



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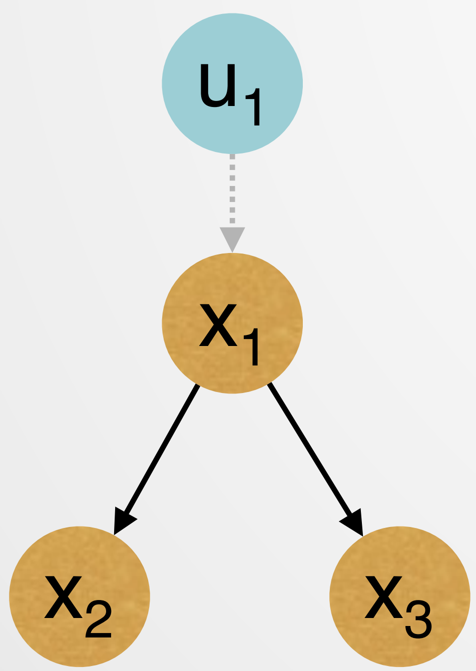
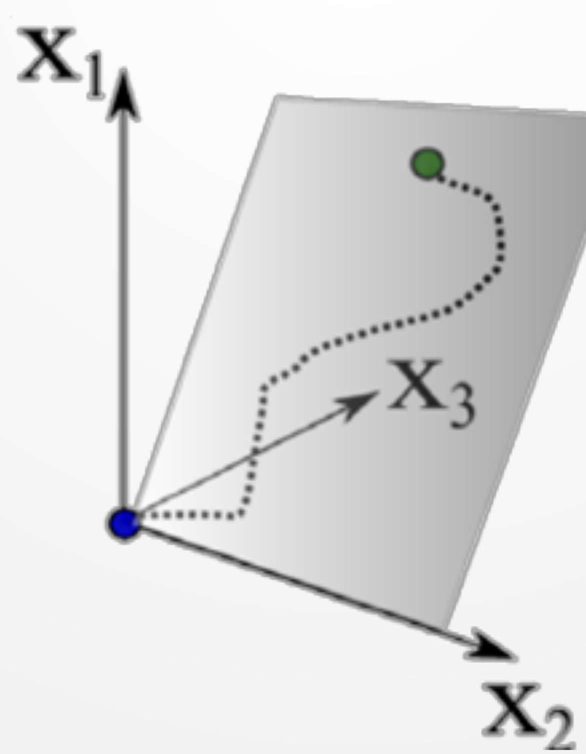
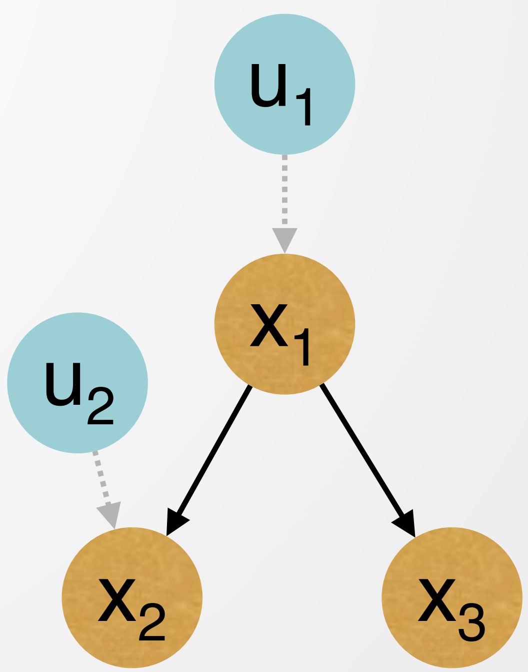




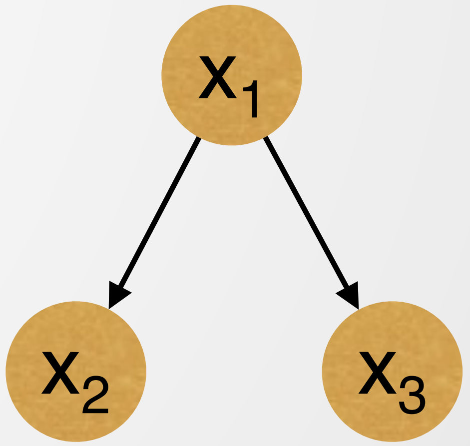


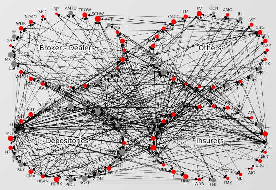
Computing Minimum Input Sets

Input: DiGraph G

Output: Minimum Input Set

1. Convert Graph to its corresponding bipartite graph B
2. Find maximal matching of B
3. Find unmatched nodes in the in set in B
4. Return: Set of unmatched nodes in the input set





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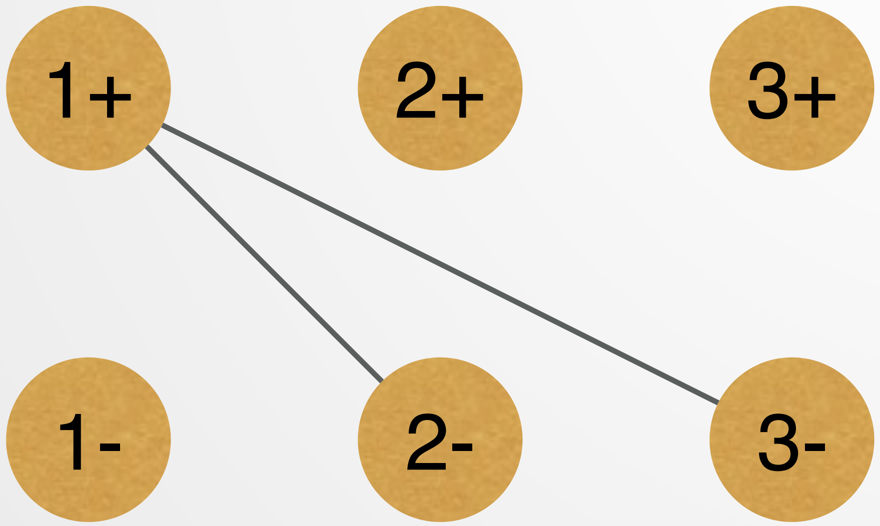

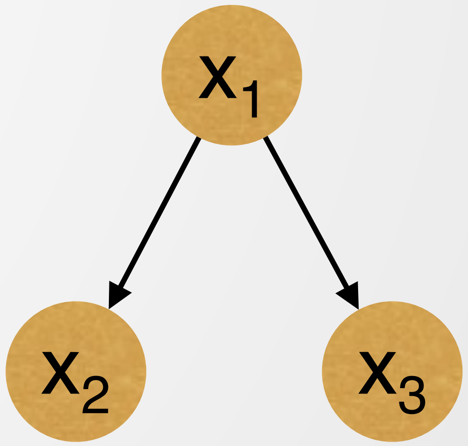

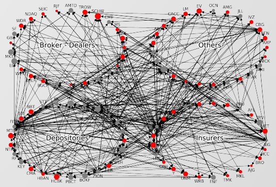

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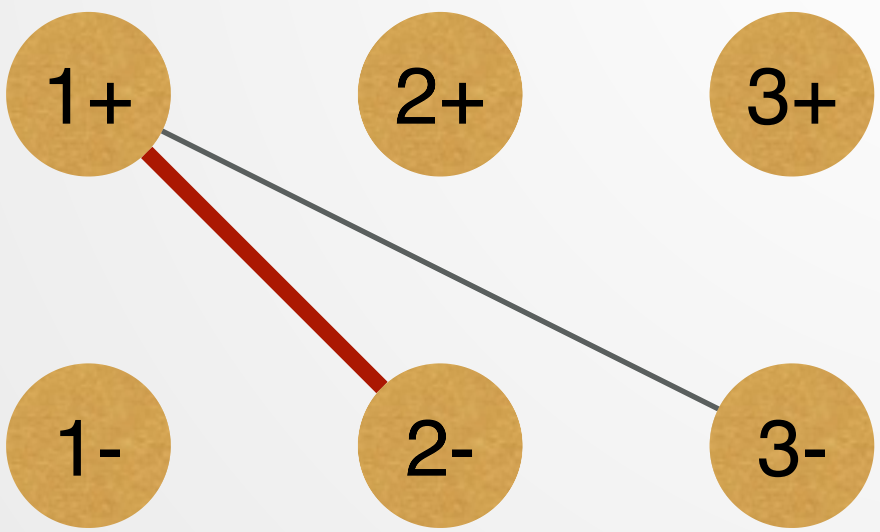

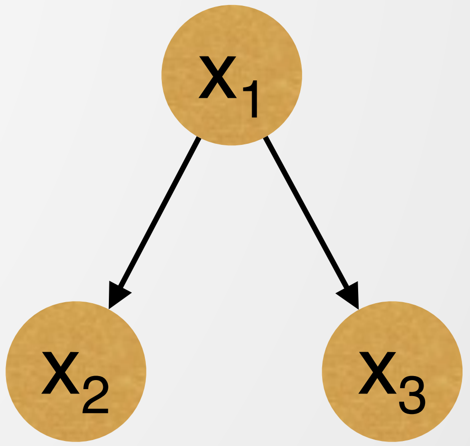

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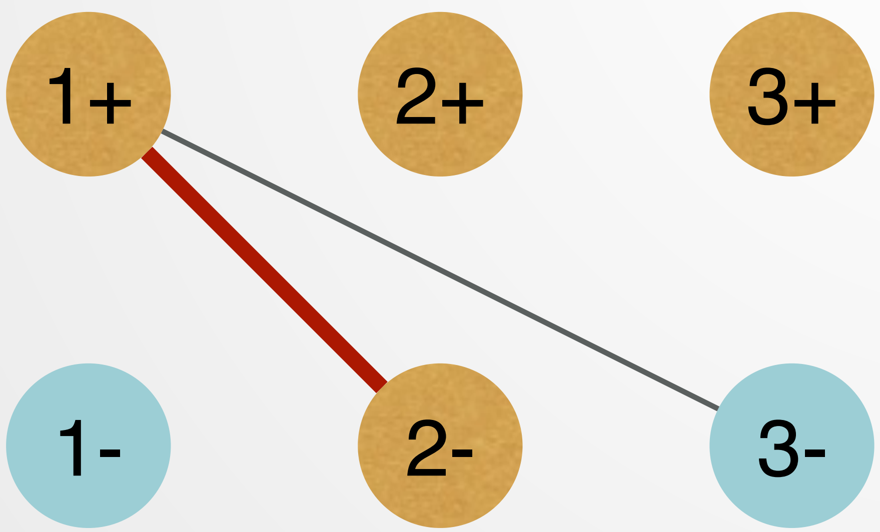

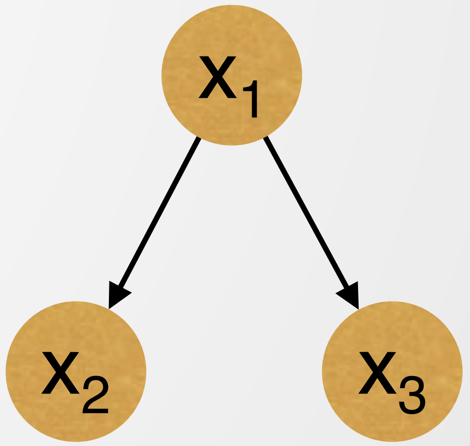

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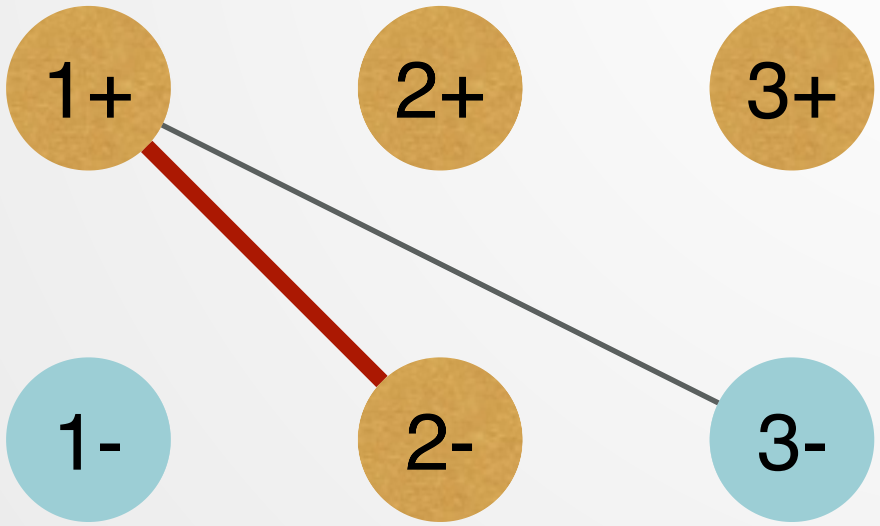

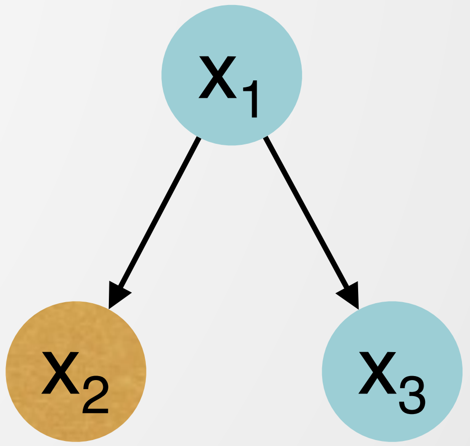

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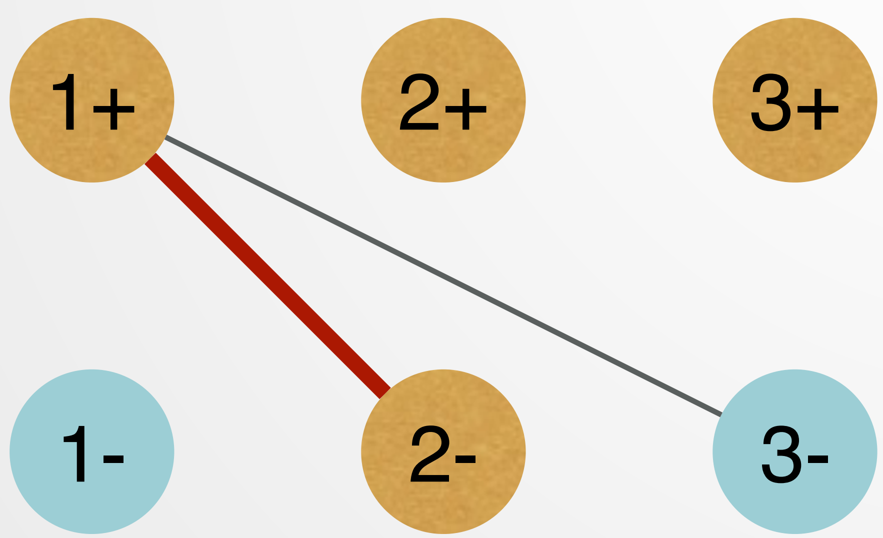

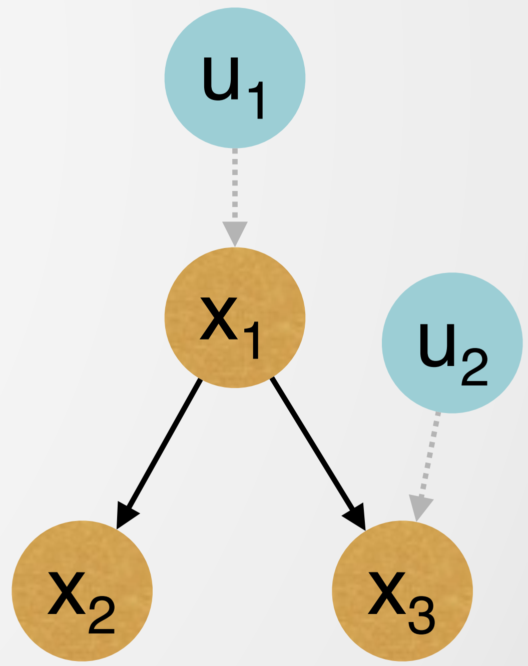

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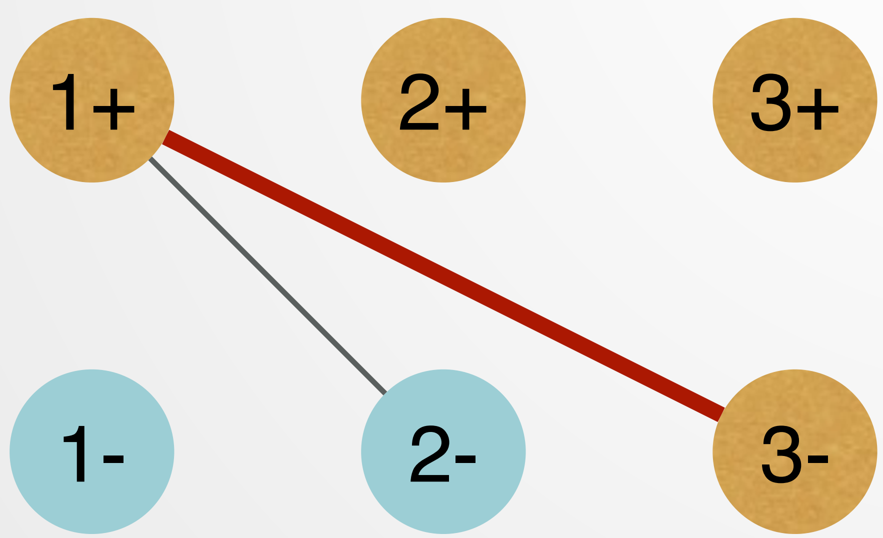
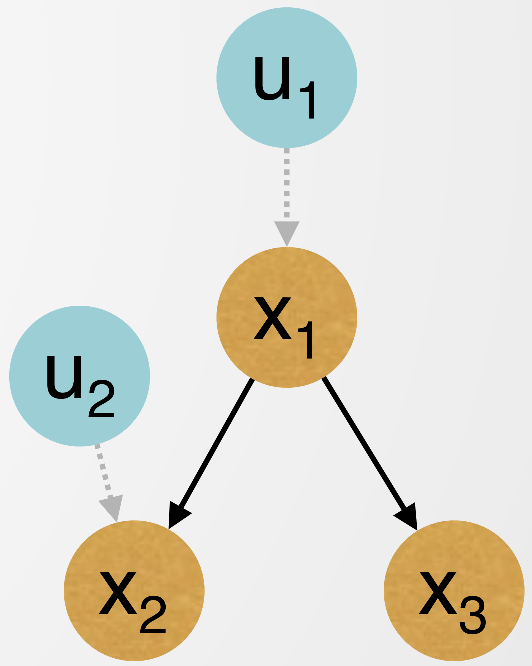




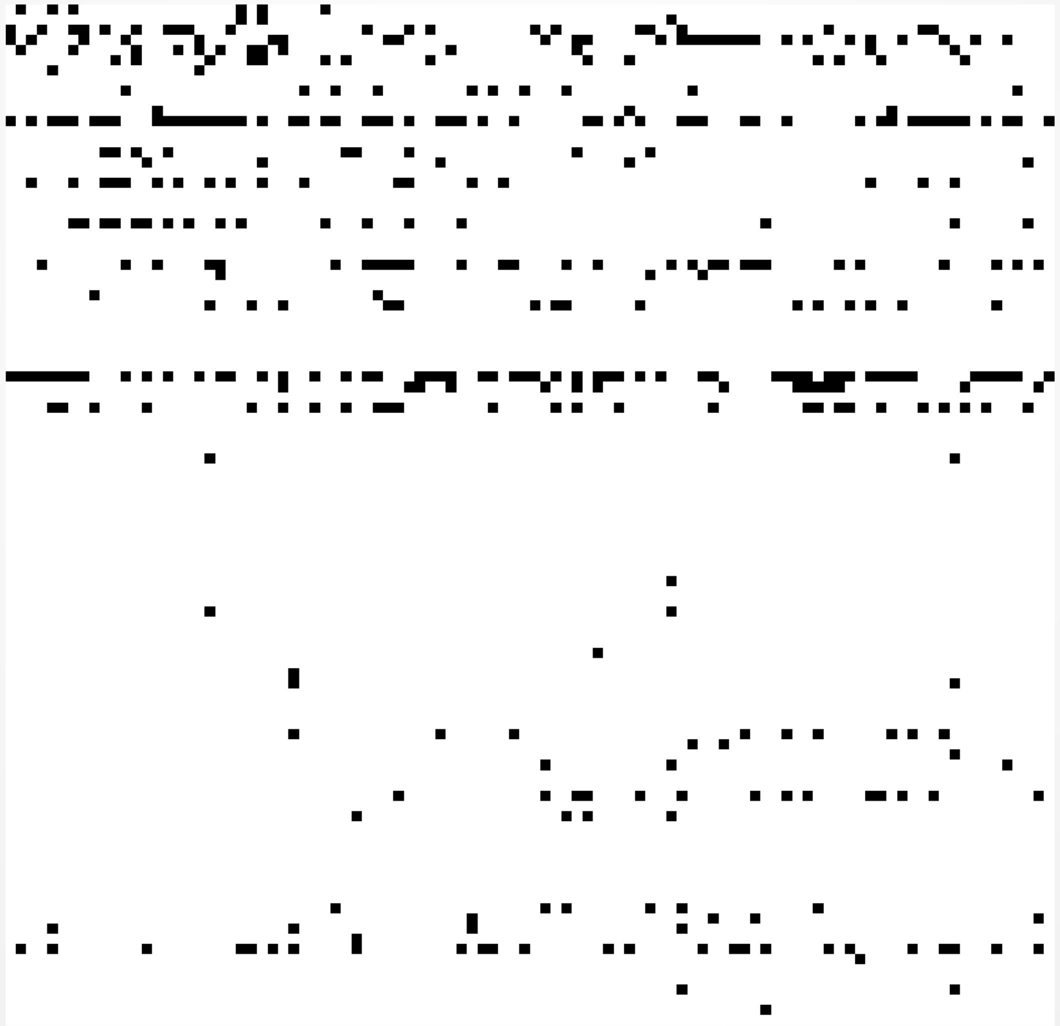
Adjacency matrix from the FRM

- At each time step, we estimate 100 models one coefficient per financial institution - here we use data from 2009-11-27
- This gives us a 100 x 100 matrix which we use as adjacency matrix

Heat map of adjacency matrix

Adjacency matrix from the FRM

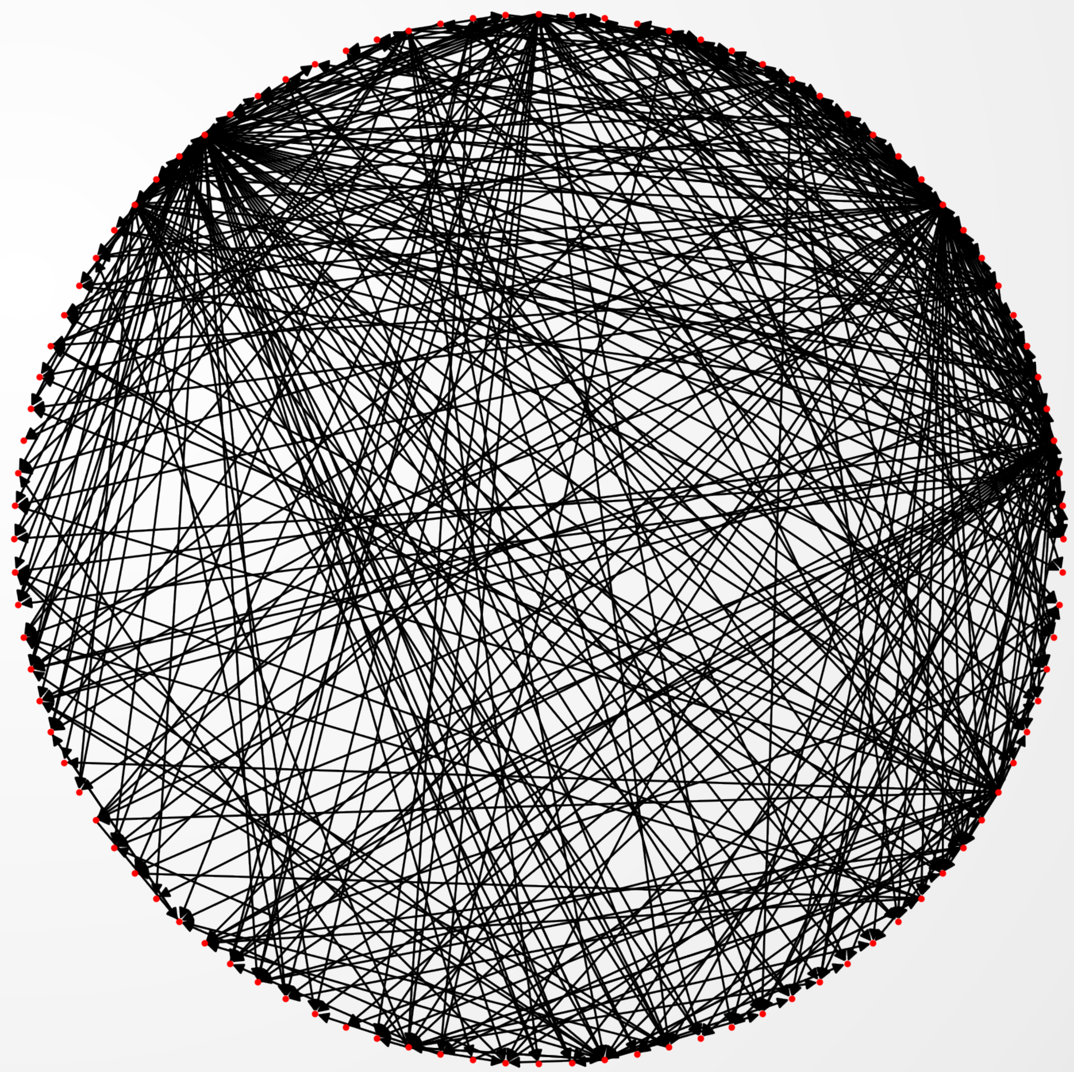
Matrix is relatively spare because Lasso forces most coefficients to 0.



Heat map of adjacency matrix

Adjacency matrix from the FRM

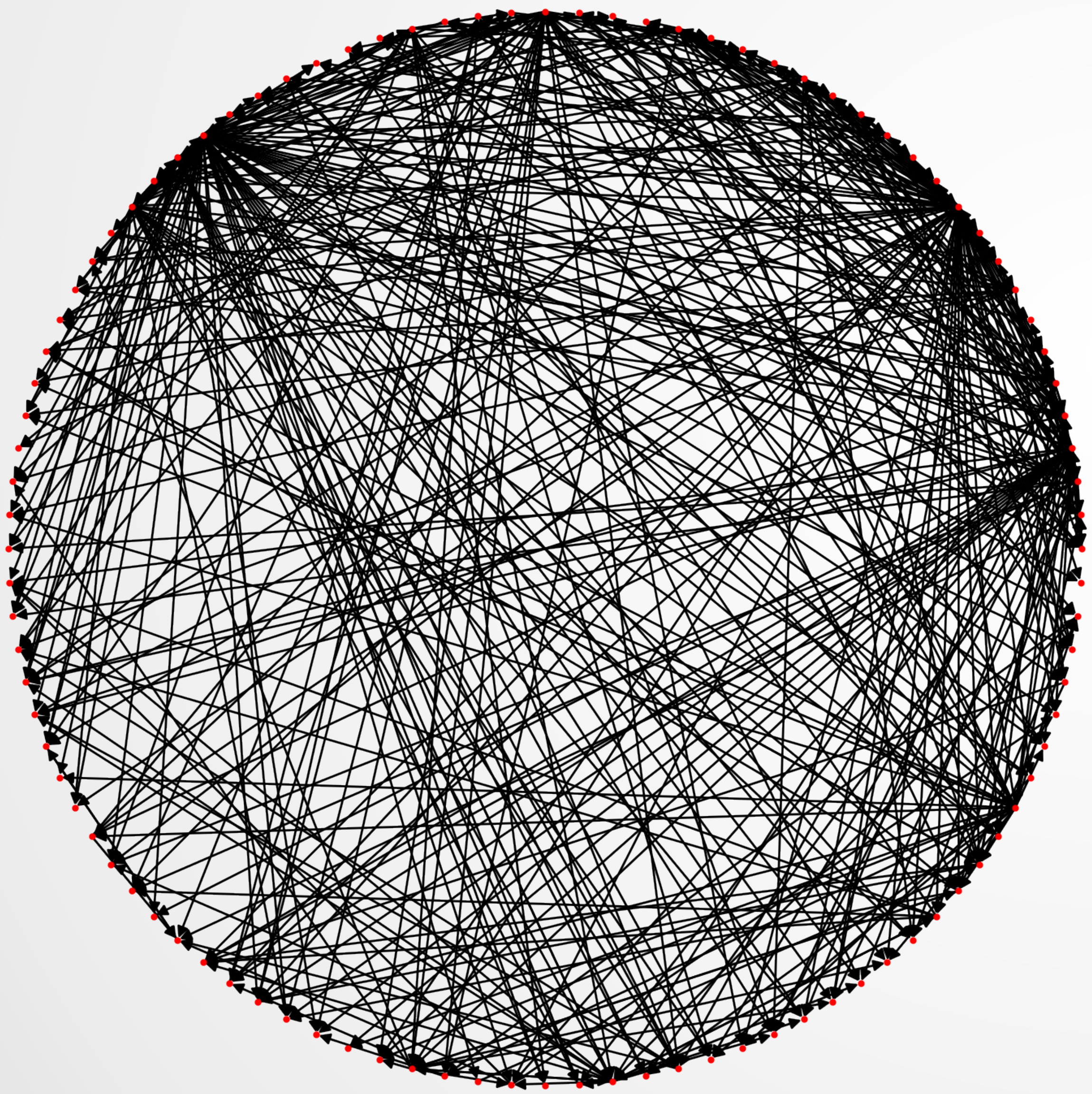
- From this adjacency matrix, we obtain a weighted DiGraph
- Even though matrix is relatively sparse, resulting graph is still too complex to be easily interpreted

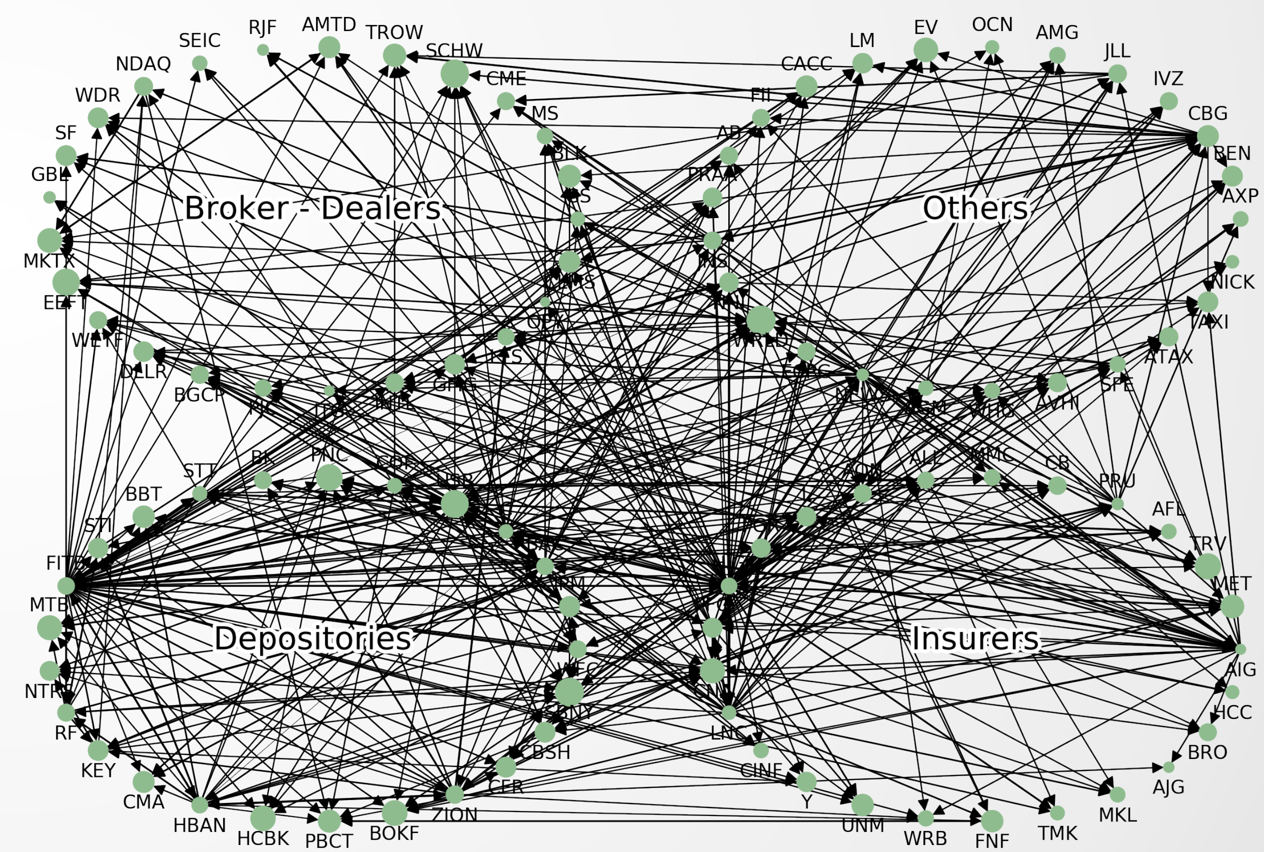


Heat map of adjacency matrix (left) and resulting graph (right)

Adjacency matrix from the FRM

- Thus, we organise institutions by sector





Resulting graph (left) and clustered graph with labels (right)

Adjacency matrix from the FRM

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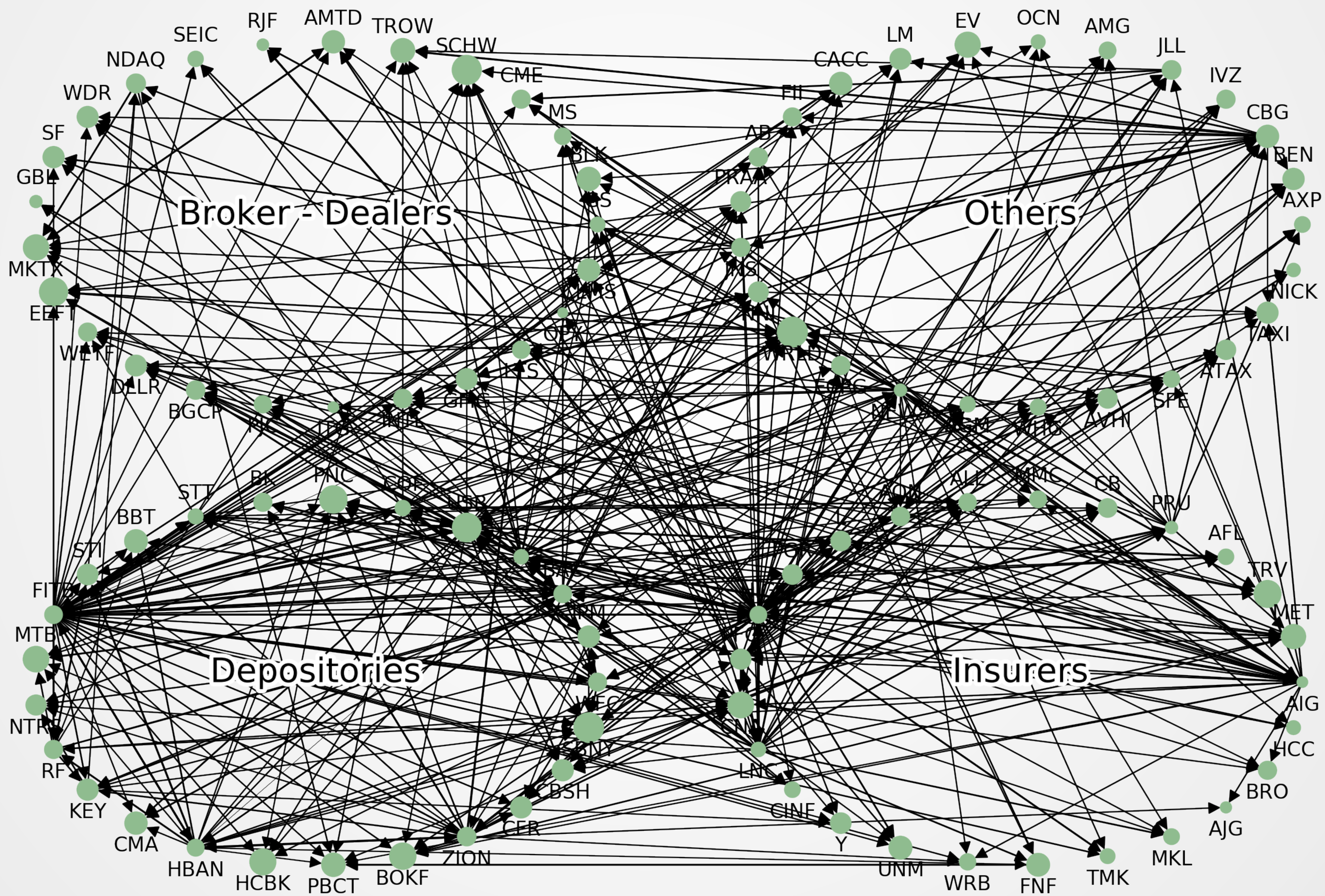
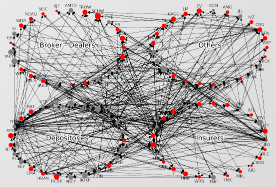
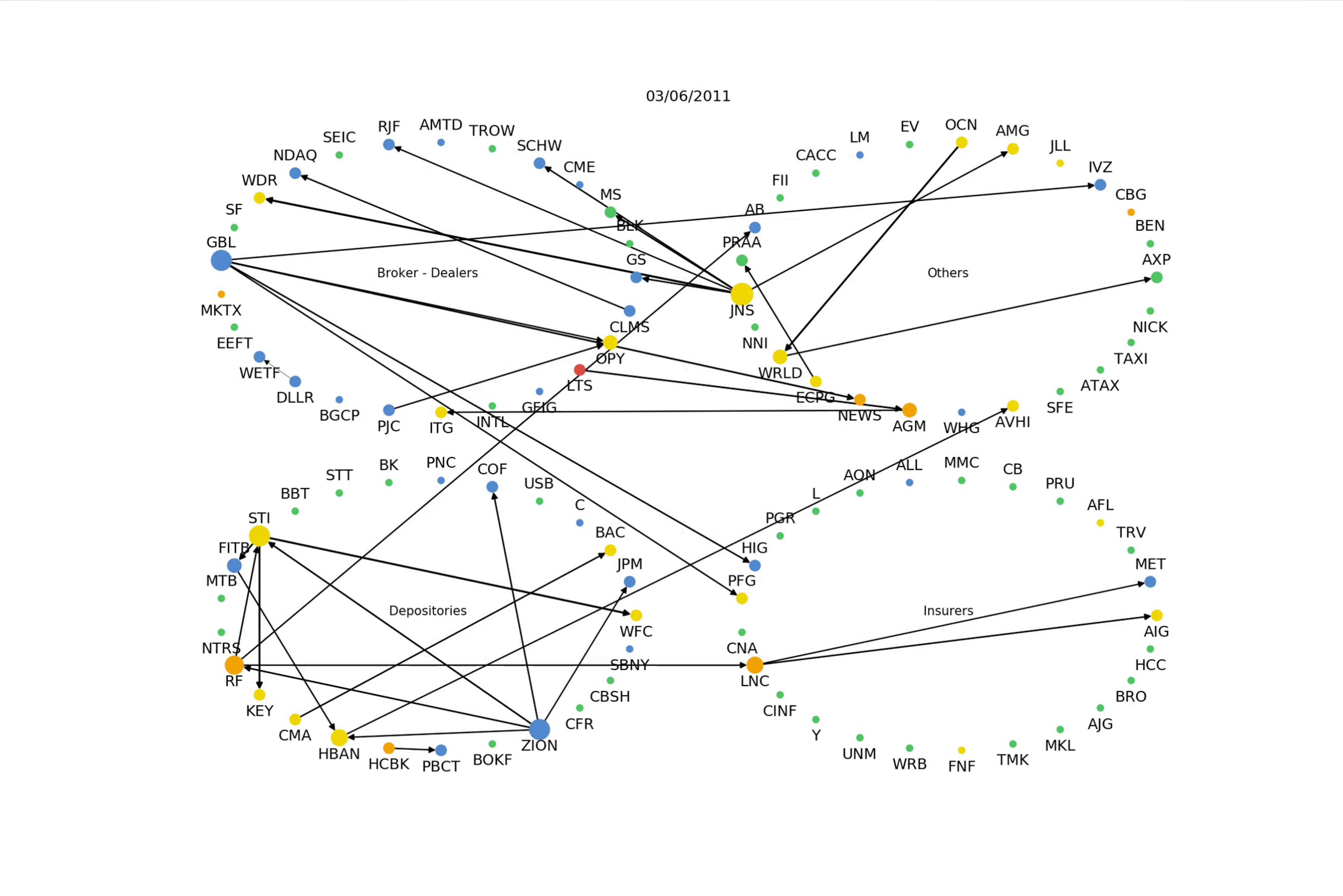
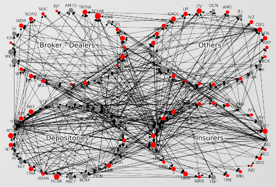


Clustered graph with labels



Excursus: FRM movie





Adjacency matrix from the FRM

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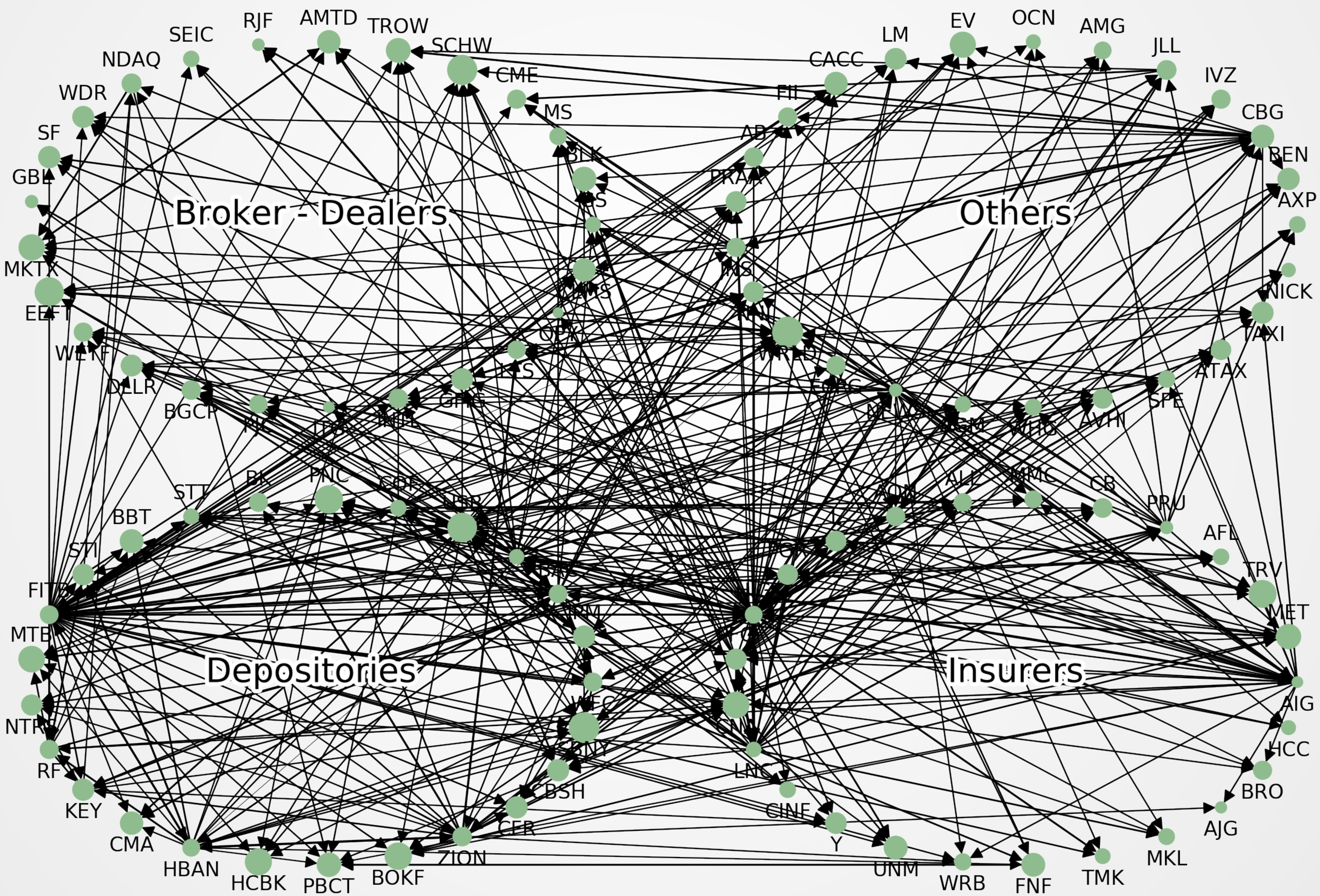
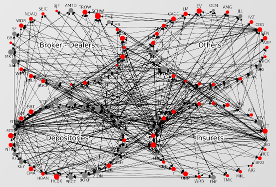


Clustered graph with labels



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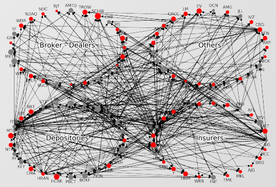
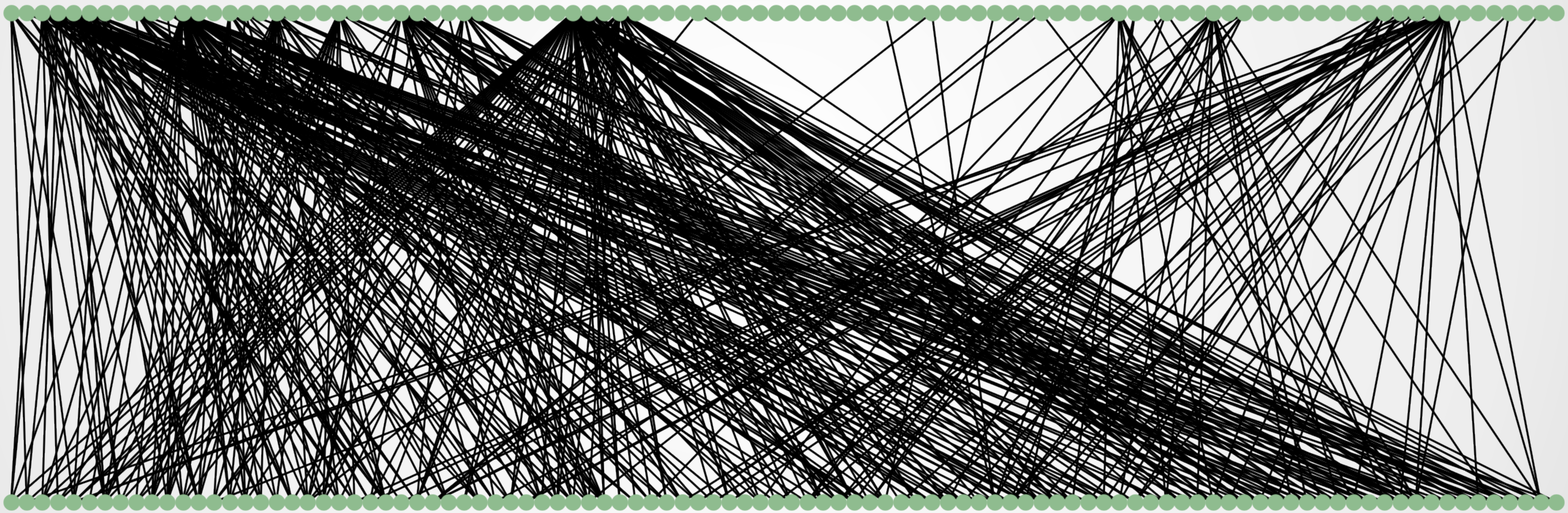
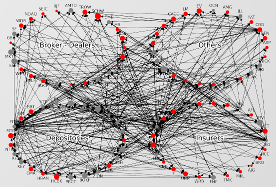


Application to FRM

- Step 1: Obtain Corresponding Bipartite Graph of the DiGraph

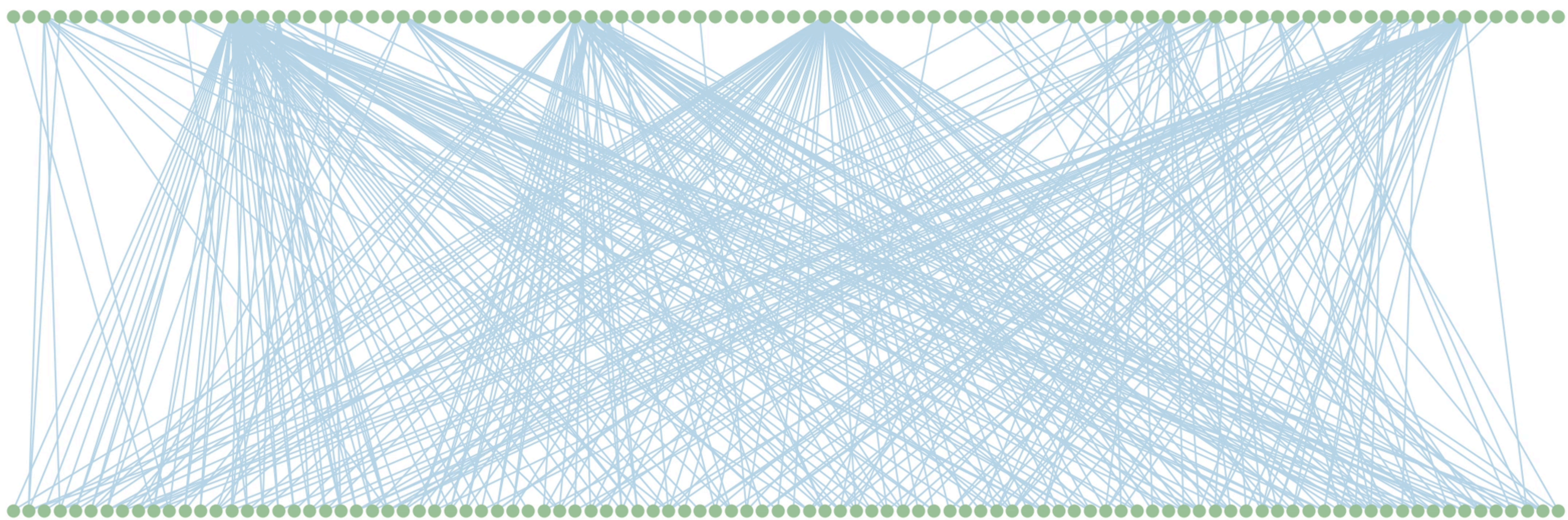


Bipartite graph, out set is on top, in set on bottom



Application to FRM

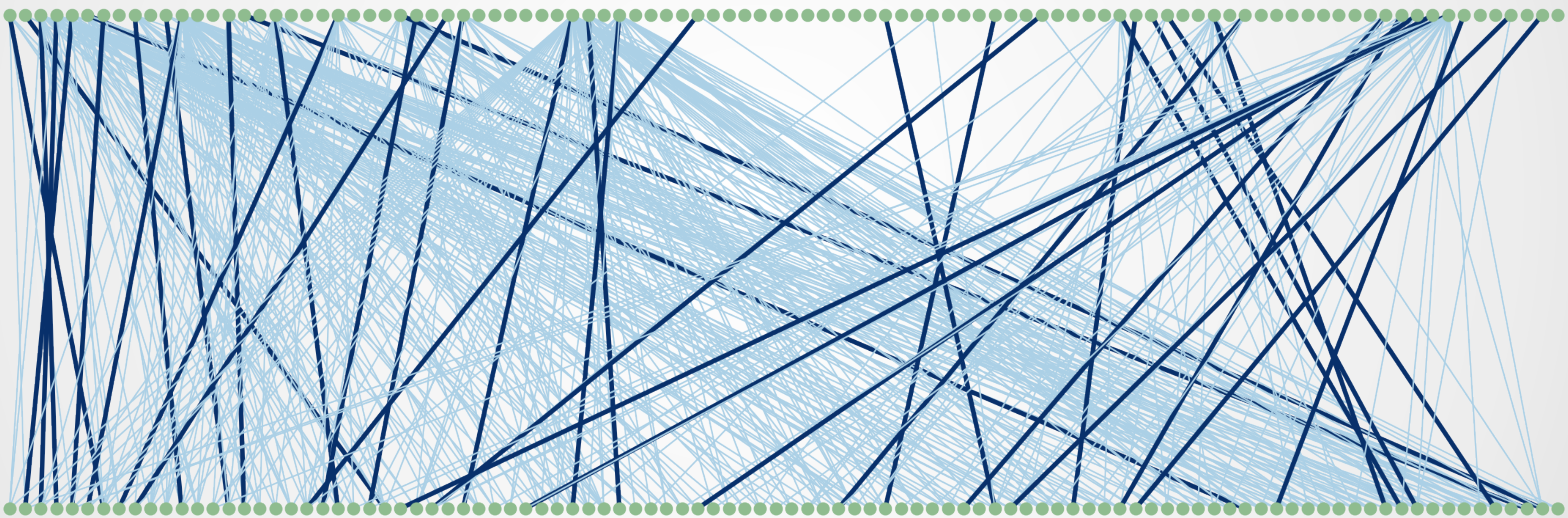
- Step 2: Find maximum matching
- We use max weight matching here



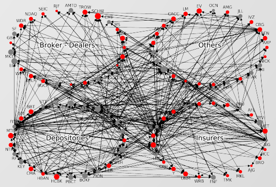
Obtaining maximum weight matching

Application to FRM

- Step 2: Find maximum matching
- We use max weight matching here

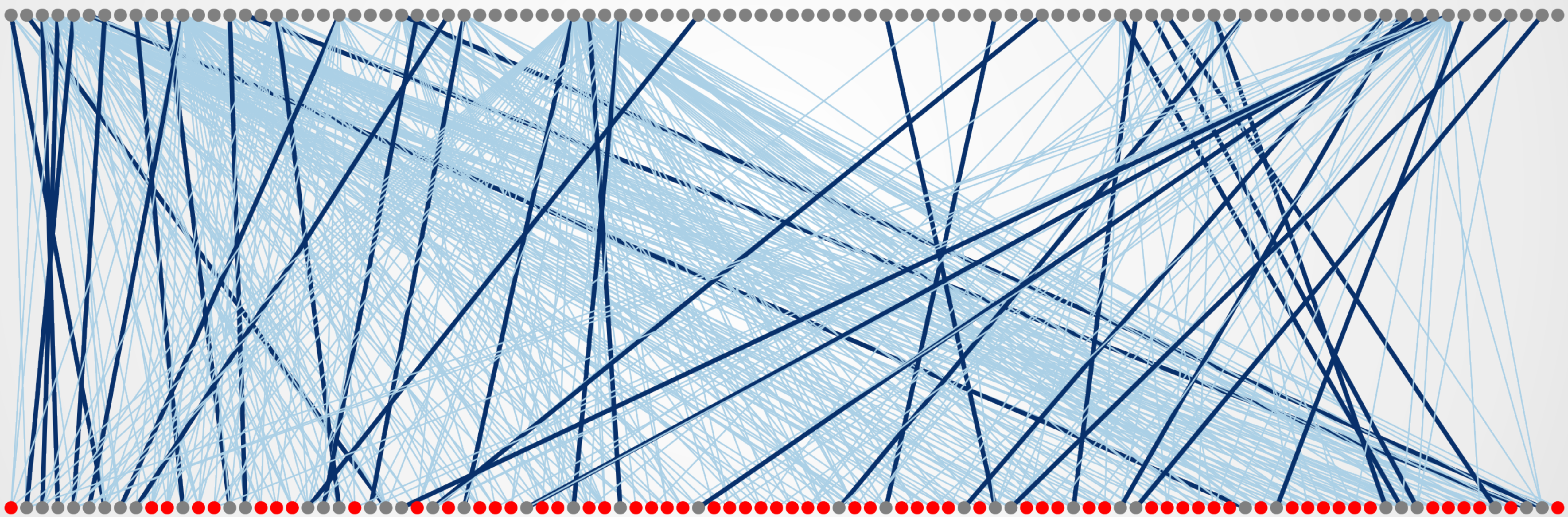


Maximum weight matching



Application to FRM

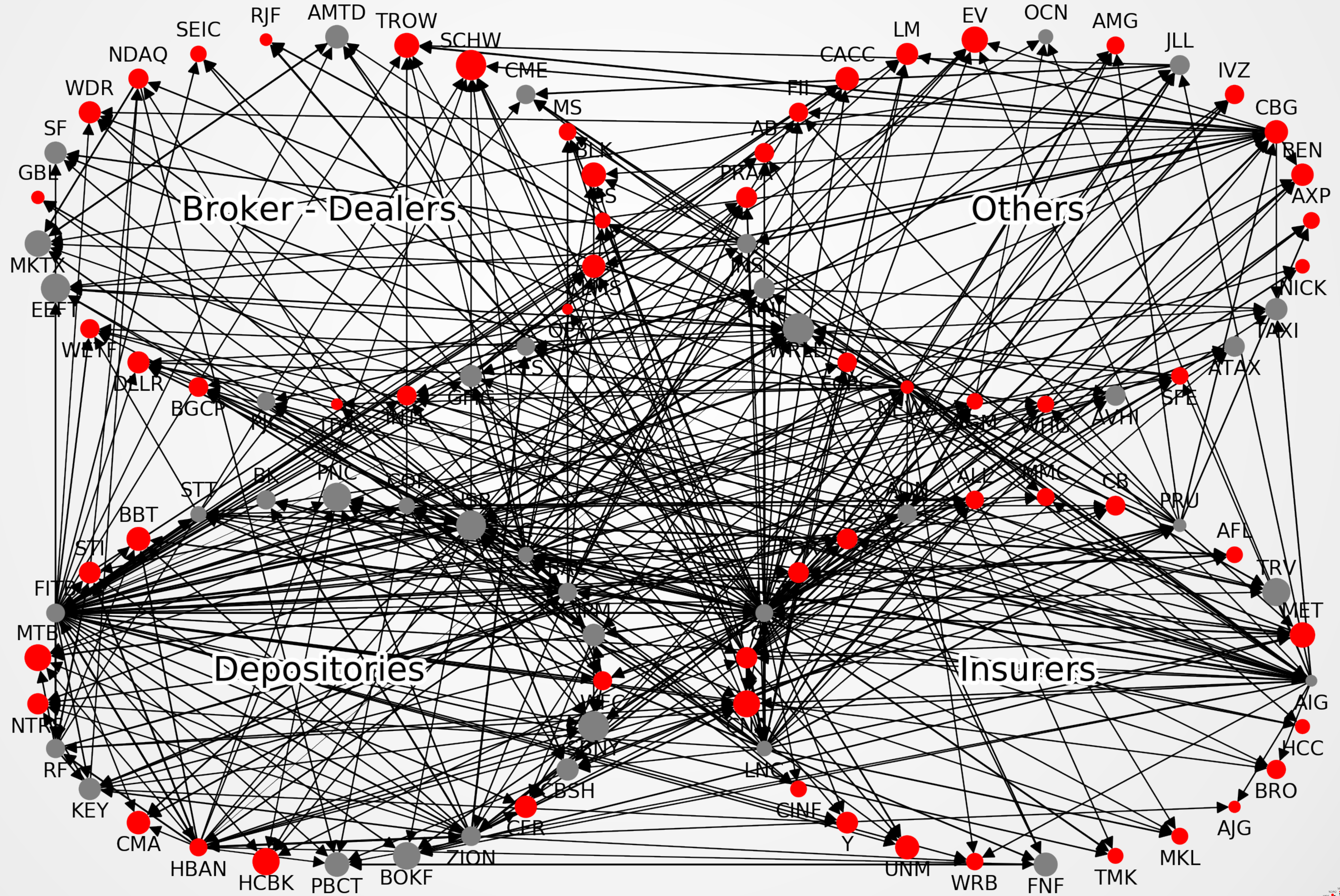
- Step 3: Find unmatched nodes of the in set which is one possible minimum input set



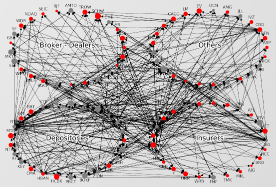
Maximum matching with unmatched nodes in in set highlighted red

Results

- Step 4: Project minimum input set on original DiGraph

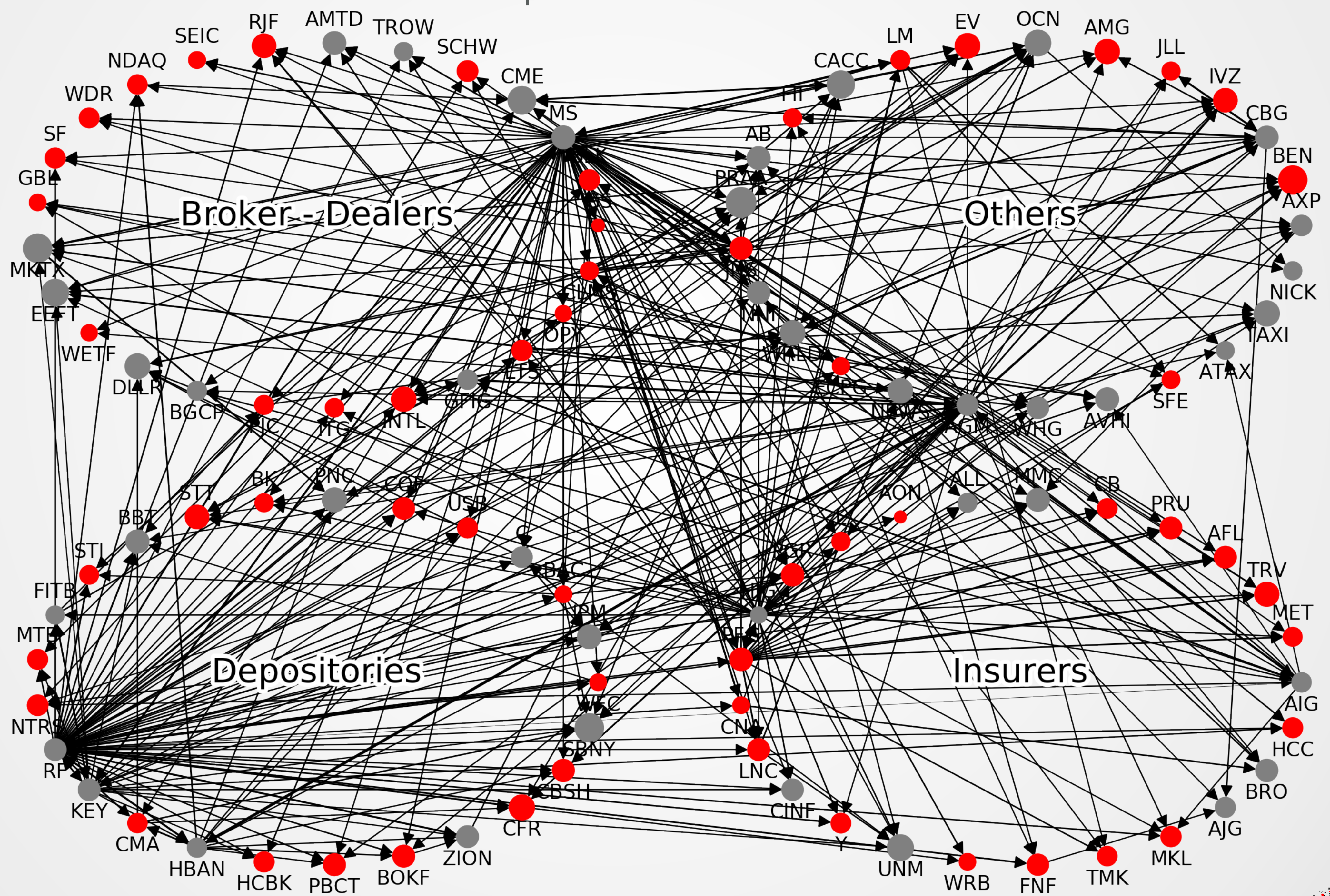


Original Graph with nodes in MIS highlighted



Results

- Result for a different time step: 2008 - 10 - 17



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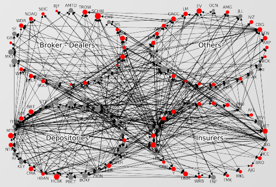
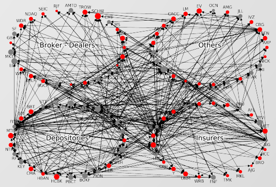


Results

- Graph is relatively sparse, thus we need a lot of input nodes to control full graph
- Possible extensions:
  - Find all possible minimum input sets and find nodes that appear in all of them
  - Do analysis over time and find nodes that are persistently in the MIS



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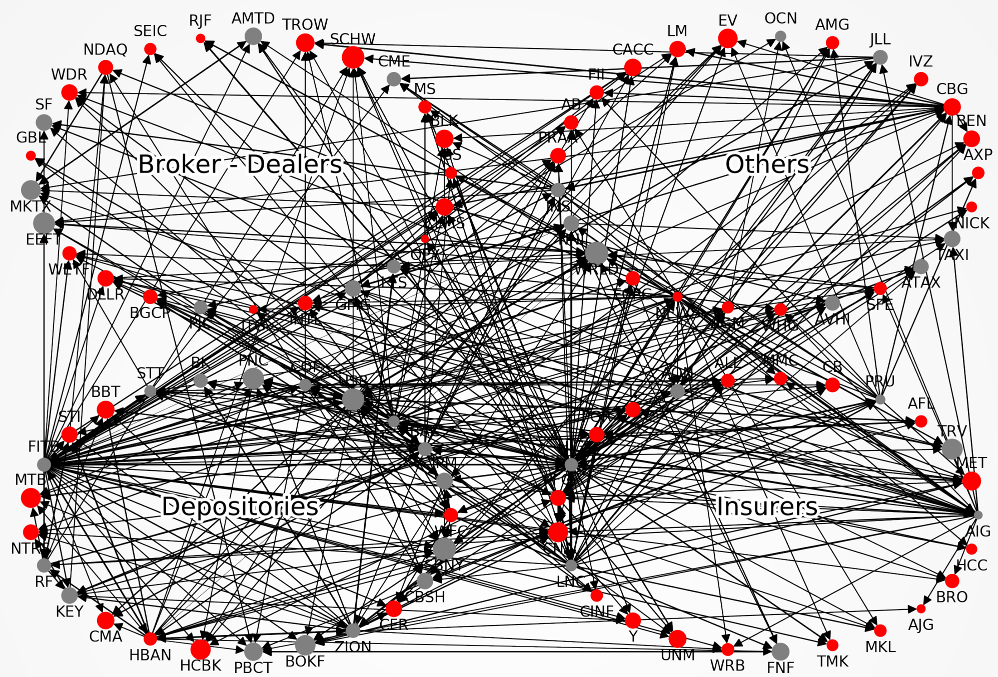
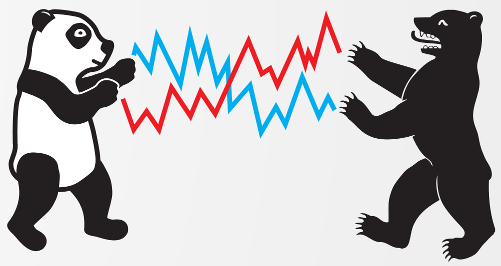
- I thank Prof. Klimm for suggesting this topic and helpful discussions

Controllability of Financial Networks





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