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B. Tech. Degree V Semester Supplementary Examination April 2018

CE/CS/EC/EE/IT/ME/SE AS 15-1501 NUMERICAL AND STATISTICAL METHODS

(2015 Scheme)

Time: 3 Hours

Maximum Marks: 60

PART A

(Answer ALL questions)

 $(10 \times 2 = 20)$

- I. (a) Prove that $\Delta \log f(x) = \log \left(1 + \frac{\Delta f(x)}{f(x)} \right)$.
 - (b) Evaluate $\int_{0}^{1} dx / (1 + x^{2})$ using Simpson's 1/3 rule taking $h = \frac{1}{4}$.
 - (c) Derive a formula to find a real root of the equation f(x) = 0 by Regula-Falsi method
 - (d) Solve $y' = y \frac{2x}{y}$, y(0) = 1 in the range $0 \le x \le 0.2$ using Euler's method. Take h = 0.1.
 - (e) Find the mean and variance of a random variable X with probability density function $f(x) = \frac{3}{2}(1-x^2), 0 < x < 1$.
 - (f) Write down the probability density function of the binomial distribution for which mean is 4 and variance 3.
 - (g) Define: (i) Level of Significance (ii) Power of the test.
 - (h) The sizes and mean of 2 independent random samples are 400, 225, 3.5 and 3 respectively. Can we conclude that the samples are drawn from the same population with SD 1.5?
 - (i) If X follows Poisson distribution with $p(X=2) = \frac{2}{3}P(X=1)$, then find P(X=3).
 - (j) A random sample of size 18 is taken from a Normal distribution. Test the hypothesis $Ho: \sigma^2 = 0.36$ against $H1: \sigma^2 > 0.36$ given $S^2 = 0.68$, $\alpha = 5\%$.

PART B

 $(4 \times 10 = 40)$

- II. (a) Apply Gauss Seidal iteration method to solve the equations 20x + y 2z = 17, 3x + 20y z = -18, 2x 3y + 20z = 25.
 - (b) Use Newton's divided difference formula to find f(7) if f(3) = 24, f(5) = 120, f(8) = 504, f(9) = 720 and f(12) = 1716.

OR ·

III. (a) Given the values

Using Lagrange's formula for interpolation, find the value of f(4).

(b) From the following table of half yearly premium for policies maturing at different ages, estimate the premium for policies maturing at ages 46 and 63.

| Age(x) | 45 | 50 | 55 | 60 | 65 |
|-------------|------|-----|-----|-----|-----|
| Premium (y) | 1150 | 960 | 830 | 740 | 680 |

IV. (a) Obtain the first and second derivatives of \sqrt{x} at x = 15 from the table.

(x) 15 17 19 21 23 25
$$\sqrt{x}$$
 3.873 4.123 4.359 4.583 4.796 5.000

(b) The velocity v(km/min) of a moped which starts from rest is given at fixed intervals of time t(min) as follows:

$$t:$$
 2 4 6 8 10 12 14 16 18 20 $v:$ 10 18 25 29 32 20 11 5 2 0

Estimate approximately the distance covered in 20 minutes.

OR

- V. (a) Solve $y' = 3x + \frac{y}{2}$ and y(0) = 1 by Taylor's series method and compute y(0.1) and y(0.2).
 - (b) Apply Runge-Kutta method of fourth order to find an approximate value of y when x = 0.2 given that dy/dx = x + y and y = 1 when x = 0.
- VI. (a) Derive the mean and variance of Binomial distribution.
 - (b) Fit a Poisson distribution to the following data and test the goodness of fit.

$$x : 0 1 2 3 4$$

 $f(x): 109 65 22 3 1$

OR

VII. (a) Fit a curve of the form $y = ae^{bx}$ to the following data by the method of least squares.

$$x:$$
 1 2 3 4 5 6 $y:$ 1.6 4.5 13.8 40.2 125 300

(b) The mean mark on a final examination was 72 and Standard deviation 9. The top 10% of the students are to receive Grade A. What is the minimum mark a student must get in order to receive Grade A, if the marks are normally distributed?

- VIII. (a) A population follows a normal distribution with mean μ and variance 9. To test $Ho: \mu = 5$ against $\mu > 7$, the test procedure is to reject Ho if $\bar{x} \ge 6$ where \bar{x} is the mean of a sample of size 16. Find the significance level and power of the test.
 - (b) Two independent random samples of sizes $n_1 = 10$, $n_2 = 7$ were observed to have sample variance $s_1^2 = 16$, $s_2^2 = 3$. Using $\alpha = 0.1$, test $Ho: \sigma_1^2 = \sigma_2^2$ against $H1: \sigma_1^2 \neq \sigma_2^2$.

OR

- IX. (a) A company claims that its light bulbs are superior to those of its main competitor. A study shows that a sample of size $n_1 = 40$ of its bulbs has a mean life time of continuous use with a SD of 27 hours, while a sample of size $n_2 = 40$ bulbs made by its main competitor has a mean life time of 638 hours of continuous use with a SD of 31 hours. Can we accept the claim of the company at 0.05 level of significance?
 - (b) A random sample of size 12 is taken from a normal population with mean μ and variance 9. Find the probability that the variance of the sample lies between 3.4 and 14.8.
