

B.Tech. Degree V Semester Examination November 2019CE/CS/EC/EE/IT/ME/SE AS 15-1501 NUMERICAL AND STATISTICAL METHODS
(2015 Scheme)

Time: 3 Hours

Maximum Marks: 60

PART A
(Answer ALL questions)

(10 × 2 = 20)

- I. (a) Develop an iteration formula to find the cube root of
- N
- using Newton-Raphson method.

(b) Prove that $\mu^2 = 1 + \frac{\sigma^2}{4}$.

- (c) From the following table, express
- y
- as a function of
- x
- .

| | | | | |
|-------|---|---|----|----|
| x : | 1 | 2 | 3 | 4 |
| y : | 3 | 6 | 11 | 18 |

(d) Prove that $hf'(a) = \Delta f(a) - \frac{1}{2} \Delta^2 f(a) + \frac{1}{3} \Delta^3 f(a) - \frac{1}{4} \Delta^4 f(a) + \dots$

- (e) Using Taylor series method compute the solution of
- $\frac{dy}{dx} = x + y$
- ,
- $y(0) = 1$
- at the point
- $x = 0.2$
- correct to 3 decimal places.

- (f) Find the mean and variance of the total number of heads obtained when an unbiased coin is tossed 3 times.

- (g) If
- x
- follows Poisson distribution such that
- $p(x=1) = p(x=2)$
- find
- $p(x=3)$
- .

- (h) Write the normal equations to fit a power curve of the form
- $y = ax^b$
- .

- (i) Explain the two types of errors that may be committed in sampling.

- (j) Define (i) critical region (ii) level of significance (iii) test statistic.

PART B

(4 × 10 = 40)

- II. (a) Find a real root of
- $x^3 + x - 1 = 0$
- , near
- $x = 1$
- correct to three decimal places by the method of false position.

- (b) Solve the following system of equations by Gauss-Seidel method.

$$10x + 2y + z = 9, \quad x + 10y - z + 22 = 0, \quad -2x + 3y + 10z = 22.$$

OR

- III. (a) From the following table of half yearly premium for policies maturing at different ages, estimate the premium for policies maturing at ages 46 and 53.

| | | | | | |
|------------------|------|-----|-----|-----|-----|
| Age (x): | 45 | 50 | 55 | 60 | 65 |
| Premium (y): | 1150 | 960 | 830 | 740 | 680 |

- (b) Use Newton's divided formula to evaluate
- $f(7)$
- if
- $f(3) = 24$
- ,
- $f(5) = 120$
- ,
- $f(8) = 504$
- ,
- $f(9) = 720$
- and
- $f(12) = 1716$
- .

(P.T.O.)

- IV. (a) Find the derivative at the middle point for the given data:

| | | | | | | | |
|----|-----|-----|-----|-----|-----|-----|-----|
| x: | 0 | 3 | 6 | 9 | 12 | 15 | 18 |
| y: | 135 | 149 | 157 | 183 | 201 | 205 | 193 |

- (b) The speed, v meters per second, of a car after it starts, is shown in the following table:

| | | | | | | | | | | | |
|----|---|-----|-------|------|------|-------|-------|-----|-----|-----|-----|
| t: | 0 | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 |
| v: | 0 | 3.6 | 10.08 | 18.9 | 21.6 | 18.54 | 10.26 | 5.4 | 4.5 | 5.4 | 9.0 |

Find the distance travelled by the car in 2 minutes.

OR

- V. (a) Use improved Euler's method to approximate y when $x = 0.1$ given

$$\frac{dy}{dx} = \frac{y-x}{y+x}, \quad y(0) = 1 \text{ by taking } h = 0.1.$$

- (b) By Runge-Kutta method of order 4, calculate the value of y for $x = 0.2$ correct to 3 decimal places when $\frac{dy}{dx} = x - 2y$, $y(0) = 0$ taking $h = 0.1$.

- VI. (a) Six dice are thrown 720 times. How many times do you expect at least 3 dice show a five or six.

- (b) Find the mean and variance of the Poisson distribution.

OR

- VII. (a) In a competitive examination, 5000 students have appeared for a paper in mathematics. Their average marks was 62 and standard deviation 12. If there are only 100 vacancies, find the minimum marks that one should score in order to get selected.

- (b) Fit a parabolic curve to the following data.

| | | | | | | | |
|----|-----|-----|-----|-----|-----|-----|-----|
| x: | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| y: | 1.1 | 1.3 | 1.6 | 2.0 | 2.7 | 3.4 | 4.1 |

- VIII. (a) An engineer making engine parts with axle diameter 0.7 inches and s.d. of 0.04 inches. A random sample of 10 parts shown a mean of 0.742 inches. Test the hypothesis $H_0: \mu = 0.7$ against $H_1: \mu \neq 0.7$ at 5% level of significance.

- (b) The means of two random samples of size 1000 and 2000 are 67.5 and 68 inches respectively. Can the samples be regarded to have been drawn from the same population with S.D 9.5 inches? Test at 5% level of significance.

OR

- IX. (a) A sample of 200 students has S.D. 5.5. Test whether the sample was taken from the above population with S.D. 5.

- (b) Two independent random sample of size $n_1 = 10$, $n_2 = 7$ when observed to have sample variances $s_1^2 = 16$, $s_2^2 = 3$ using $\alpha = 0.01$. Test $H_0: \sigma_1^2 = \sigma_2^2$ against $H_1: \sigma_1^2 \neq \sigma_2^2$.

B.Tech. Degree V Semester Examination November 2017**CE/CS/EC/EE/IT/ME/SE
AS 15-1501 NUMERICAL AND STATISTICAL METHODS
(2015 Scheme)**

Time : 3 Hours

Maximum Marks : 60

**PART A
(Answer ALL questions)**

(10 × 2 = 20)

- I. (a) Show that $\Delta^3 y_0 = y_3 - 3y_2 + 3y_1 - y_0$. (1+1)
- (b) Find the cubic polynomial $y(x)$ such that $y(0)=1, y(1)=0, y(2)=1$ and $y(3)=10$, hence find $y(4)$. ($\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$)
- (c) Find the third divided difference with arguments 2, 4, 9, 10 where $f(x) = x^3 - 2x$. (1+1)
- (d) A curve is passing through (0, 4), (2, 8), (4, 15), (6, 7) and (8, 6). Find its slope at $x=1$. ($\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$)
- (e) Develop a general polynomial for numerical integration. (1+1)
- (f) Define the following. ($\frac{1}{2} + \frac{1}{2}$)
- (i) Random variable (ii) Sample space (iii) Mathematical expectation.
- (g) Check whether $f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$ is a probability density function or not. (1+1)
- (h) The probabilities of Poisson variate taking the values 3 and 4 are equal. Calculate the probabilities of the Poisson variate taking the value 1. ($\frac{1}{2} + 1 + 1$)
- (i) Write normal equations when fitting the straight line $y = ax^2 + bx + c$. (1+1)
- (j) Define (i) Critical region (ii) Significance level and (iii) Power of the test. ($1 + \frac{1}{2} + \frac{1}{2}$)

PART B

(4 × 10 = 40)

- II. (a) By using Newton Raphson's method find the real root of $x^4 - x - 10 = 0$, correct to three decimal places. (1+2+1+1)
- (b) Solve the system of equations by Gauss-Siedel method. ($1 + 1 + 1 + 1 + 1$)
- $8x + y + z = 8, 2x + 4y + z = 4, x + 3y + 5z = 5$.

OR

- III. (a) The population of a town in the census is given below. Examine the population for the year 1896. (1+1+2+1)

| Year | 1891 | 1901 | 1911 | 1921 | 1931 |
|-------------------------|------|------|------|------|------|
| Population in thousands | 46 | 66 | 81 | 93 | 101 |

- (b) Using the following table, find $f(x)$ as a polynomial by Newton's formula (2+2+1)

| x | -1 | 0 | 3 | 6 | 7 |
|---|----|----|----|-----|------|
| y | 3 | -6 | 39 | 122 | 1611 |

- IV. (a) Find $f''(1.6)$ from the following table.

| | | | | | | | |
|---|-------|-------|-------|-------|-------|-------|--------|
| x | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 |
| y | 7.829 | 8.403 | 8.781 | 9.129 | 9.451 | 9.750 | 10.031 |

- (b) Using Simpson's $\frac{1}{3}$ rd rule, evaluate $\int_0^1 \frac{dx}{1+x^2}$, taking $h = \frac{1}{4}$.

OR

- V. (a) Consider the initial value problem $\frac{dy}{dx} = xy$, $y(0) = 1$. Find $y(0.4)$ by Euler's method.

- (b) Using Runge-Kutta method of second order, solve

$$\frac{dy}{dx} = x - 2y, y(0) = 1 \text{ at } x = 0.1, 0.2.$$

- VI. (a) Six coins are thrown simultaneously. Find the probability of getting at least four heads.

- (b) For a binomial distribution, prove that mean = np and variance = npq .

OR

- VII. (a) Prove that Poisson distribution as the limiting case of Binomial distribution.

- (b) Fit a curve $y = ae^{bx}$ for the points (1, 10), (5, 15), (7, 12), (9, 15), (12, 21).

- VIII. (a) The mean breaking strength of a certain kind of metallic rope is 160 pounds. If six pieces of ropes (randomly selected from different rolls) have a mean breaking strength of 154.3 pounds with standard deviation 6.4 pounds, test the null hypothesis $\mu < 160$ pounds at 1% l.o.s. Assume that population follows normal distribution.

- (b) The mean values of birth rate with standard deviations and sample sizes are given below by Socio-economic status. Is the mean difference in birth weight significant between Socio-economic groups?

| | High Socio-economic group | Low Socio-economic group |
|--------------------|---------------------------|--------------------------|
| Sample size | $n_1 = 15$ | $n_1 = 10$ |
| Birth weight in kg | $\bar{x} = 2.91$ | $\bar{y} = 2.26$ |
| Standard deviation | $S_1 = 0.27$ | $S_2 = 0.22$ |

OR

- IX. (a) Test the null hypothesis that $\sigma = 0.022$ inch for the diameters of certain wire rope against the alternative hypothesis $\sigma \neq 0.022$ inch. Given that a random sample of size 18 yielded $S^2 = .000324$.

- (b) The standard deviation of a sample size 15 from a normal population was found to be 7. Examine whether the hypothesis that the S.D is 7.6 is acceptable.

BTech Degree V semester Second Internal Examination October 2019

1501 (Numerical and statistical methods)

Time: 2 hours

Total Marks: 50

Part - A (Answer All questions)

(8 x 2.5 = 20)

- a) Define the following
 - i) Discrete probability distribution.
 - ii) Expectation of a random variable.
 - iii) Variance of a random variable.
- b) Obtain the probability function and mean of the total number of heads occurring in 3 tosses of an unbiased coin.
- c) X has a Poisson distribution with $P(x=2) = \frac{2}{3}P(x=1)$

Find (i) $P(x=0)$, (ii) $P(x=3)$

$m = \frac{4}{3}$

$\frac{1}{8} \quad \frac{3}{8} \quad \frac{3}{8} \quad \frac{1}{8}$ mean = $\frac{3}{2}$

$P(x=0) = 0.263$ $P(x=3) = 0.1038$
- d) Write the normal equations for fitting an exponential curve of the form $y = ae^{bx}$.
- e) Define the following
 - i) Standard error
 - ii) Significance level
 - iii) Critical region
- f) A random sample of size 36 is taken from a normal population with standard deviation 3. Find the probability that sample mean exceeds the population mean at least by one.
- g) Explain the steps adopted in testing of statistical hypothesis.
- h) A random sample of size 18 taken from a normal distribution. Test the hypothesis $H_0: \sigma^2 = 0.36$ against $H_1: \sigma^2 > 0.36$ given $s_1^2 = 0.68$ and $\alpha = 5\%$

$\chi^2 = 32.11$

$\chi^2_{\alpha} =$

Part - B (Answer Any three full questions)

(3x10 = 30)

- i) a) 6 coins are tossed simultaneously find the probability of getting
 - 1) exactly 3 heads. 2) atleast 4 heads. 3) atmost 3 heads
- b) Find the mean and variance of Poisson distribution.
- ii) a) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and variance of the distribution.

$\mu = 50, \sigma = 10$

$z_1 = -0.5$ $z_2 = 1.4$
- b) Fit a second-degree parabola to the following data

| | | | | | |
|---|---|---|----|----|----|
| X | 0 | 1 | 2 | 3 | 4 |
| Y | 1 | 5 | 10 | 22 | 38 |

$n=5$ ~~$n=10$~~ $\sum x = 10$ $\sum y = 76$ $\sum xy = 243$
 $\sum x^2 = 30$ $\sum x^3 = 100$
 $\sum xy = 243$ $\sum x^2y = 851$
 $y = 1.42 + 0.26x + 2.21x^2$

iii) a) A trucking firm is suspicious of the claim that average life of certain tyres is atleast 28000 miles. To check the claim the firm puts 40 of these tyres on its trucks and gets a mean lifetime 27473 miles with a standard deviation of 1348 miles, what can it conclude if $\alpha = 0.01$

$$Z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = -2.52 \quad Z_d = 2.33$$

Reject H_0

b) A random sample of size 16 has 53 as mean the sum of squares of the deviation from mean is 135. Can this sample be regarded as taken from a population having 56 as mean, Obtain the conclusion if $\alpha = 0.05$

iv) a) Intelligence tests were given to two groups of boys and girls produced the following results

| | Mean | S.D. | Sample size |
|-------|------|------|-------------|
| Girls | 75 | 8 | 60 |
| Boys | 73 | 10 | 100 |

Examine whether

1. Difference in means significant
2. Difference in S. Ds significant.

$$\frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = 1.39 \quad \text{accept } H_0$$

$$\frac{s_1 - s_2}{\sqrt{\frac{s_1^2}{2n_1} + \frac{s_2^2}{2n_2}}} = 1.967 \quad \text{Reject } H_0$$