9

MATHEMATICS

QUARTER 3
Week 6

CapSLET

Capsulized Self-Learning Empowerment Toolkit

Schools Division Office of Zamboanga City
Region IX, Zamboanga Peninsula
Zamboanga City

"Unido, Junto avanza con el EduKalidad Cree, junto junto puede!"

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CapSLET

Capsulized Self-Learning Empowerment Toolkit

SUBJECT & GRADE LEVEL	**MATHEMATICS9**					
QUARTER	**THIRD**	**WEEK**	**6**	**DAY**	____________ mm/dd/yyyy	
TOPIC	**Lesson 1: Illustrating Similar Polygon** **Lesson 2: Similar Triangles** **Triangle Similarity Theorem (SAS, SSS and AA Similarity Theorem)**					
LEARNING COMPETENCY	The learner illustrates similarity of polygon. M9GE-IIIg-1 The learner illustrates similarity of figures. (M9GE-IIIg -1) The learner proves the conditions for similarity of triangles. (M9GE-IIIg-h-1) 1.1 SAS similarity theorem 1.2 SSS similarity theorem 1.3 AA similarity theorem					

UNDERSTAND

Lesson 1: Illustrating Similar Polygon

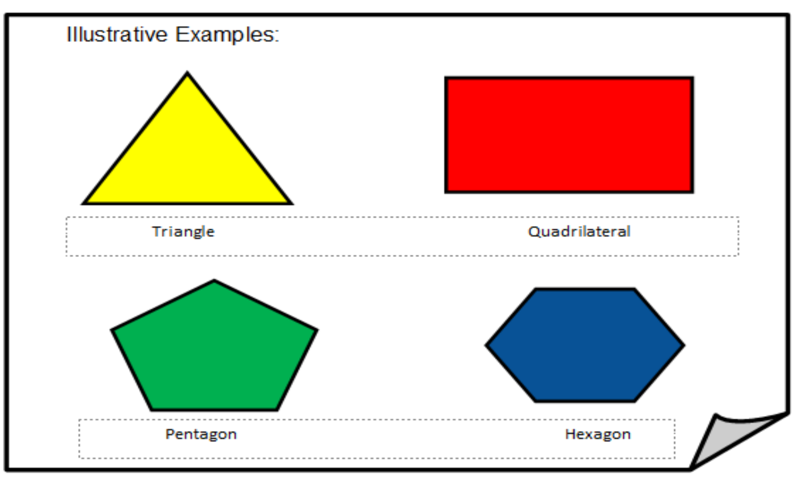
Let us recall the concept of polygon.

MATHEMATICAL CONCEPT:

Any plane figure that closes in a space using only line segments.

What is a Polygon?

- Polygon is any two-dimensional shape formed with straight line.
- Triangles, quadrilaterals, pentagons and hexagons are all examples of polygons.
- Triangle has three sides, quadrilaterals have four sides, pentagons have five sides and hexagons have six sides.



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"SIMILAR POLYGONS"

How do you identify similar polygon?

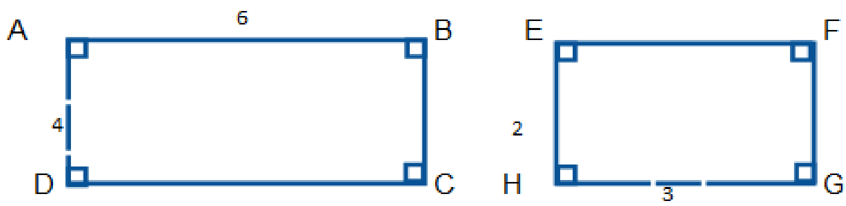
DEFINITION:

Two Polygons are similar (~) if both conditions below are satisfied.

A. Corresponding angles are congruent.

B. Lengths of the corresponding sides are in proportion.

Illustrative example 1: Consider polygons ABCD and EFGH



Check the angles and corresponding sides.

- **All the angles in a rectangle are congruent to each other with 90° .**
- **And now we need to check that the sides are proportional**

Use the cross product to show that

$$\frac{4}{6} = \frac{2}{3}$$

Check:

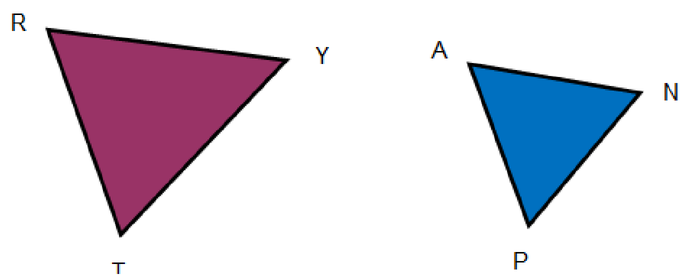
4(3) = 6(2)

12 = 12

sides are proportional

Illustrative example 2:

Similarity between two triangles.



NOTE:

ΔTRY ~ ΔPAN if and only if

1. **all pairs of corresponding angles are congruent.**
2. **all pairs of corresponding sides are proportional.**

1. Angles are congruent.

$\angle R \cong \angle A$

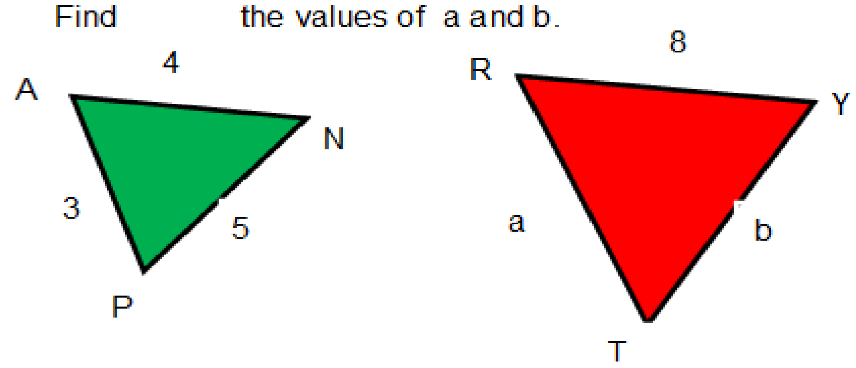
$\angle Y \cong \angle N$

$\angle T \cong \angle P$

2. Sides are proportional.

$$\frac{TR}{PA} = \frac{RY}{AN} = \frac{YT}{NP}$$

Example 3. Given that $\Delta TRY \sim \Delta PAN$.
Find the values of a and b.



Solution:

Since the corresponding angles are congruent and corresponding sides are proportional ,

Step 1 : $\frac{PA}{TR} = \frac{AN}{RY} = \frac{NP}{YT}$ Write the proportional sides of the triangle.

Step 2. $\frac{3}{a} = \frac{4}{8} = \frac{5}{b}$ Substitute the corresponding value of each sides.

Step 3. Find $\frac{4}{8} = \frac{1}{2}$ the scale factor.

Step 4. Use the cross product to find the values of a and b.

$\frac{3}{a} \times \frac{1}{2}$, $\frac{1}{2} \times \frac{5}{b}$

$3(2) = a\ (1)$ $1(b) = 2\ (5)$

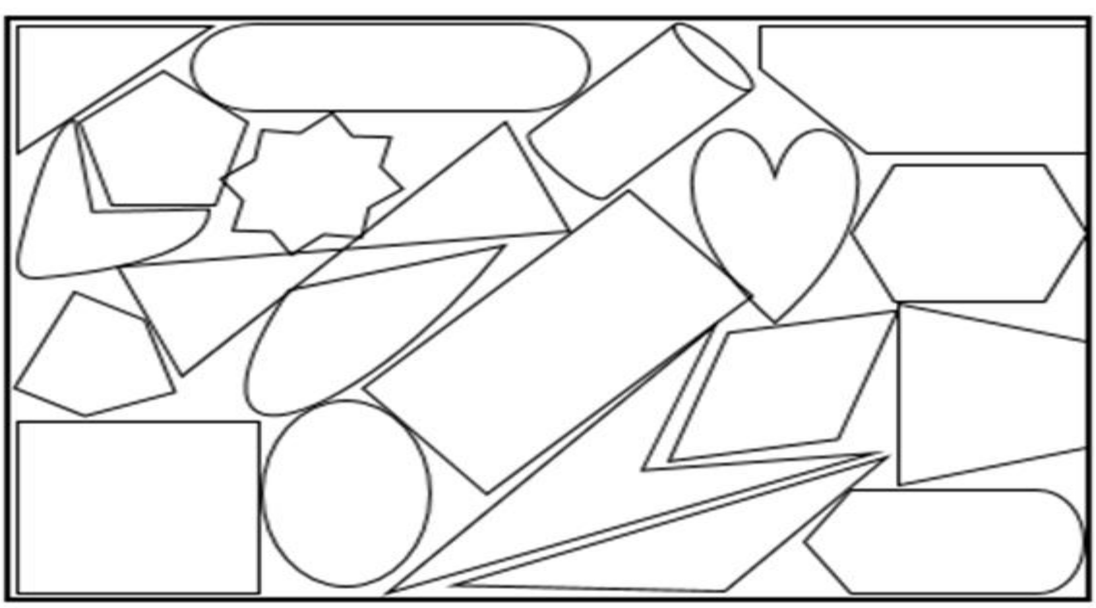
$6 = a$ or $a = 6$ $b = 10$

SAQ 1: *What do you understand about similarity of polygon?*
SAQ 2: *How does this similar polygon apply an important role in our community?*

Let's Practice! ***(Write your answer on a separate sheet.)***

ACTIVITY 1: "Which shapes are Polygon?"

Directions: Color all the polygon **BLUE;** otherwise color it **RED.**



ACTIVITY 2: Poly-made Building

Directions: Find similar polygons in the picture. Mark (*/*) for each similar polygon that you can see in each picture.

REMEMBER

Key Points

- ✓ Similar polygons are polygons with congruent corresponding angles and proportional corresponding sides.
- ✓ Two polygons are similar (~) if both conditions below are satisfied.
  a. Corresponding angles are congruent.
  b. Lengths of the corresponding sides are in proportion.

TRY

Let's see how much have you learned today!

General Directions: Study the following assessments carefully and write your answers on a separate sheet.

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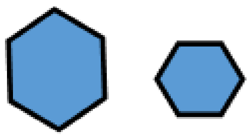
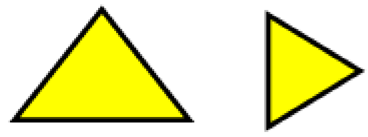
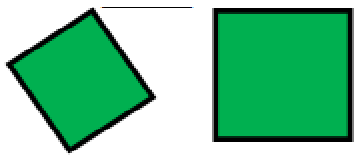
Assessment 1.

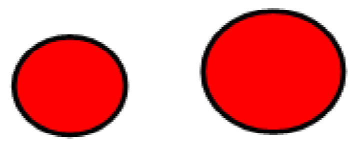
"AM I SIMILAR POLYGON?"

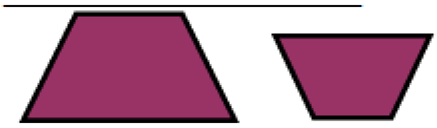
Directions: Write SP (similar polygon) or NSP (not similar polygon) for each figure below.

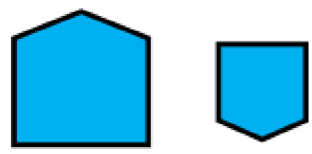
1. 

2. 

3. 

4. 

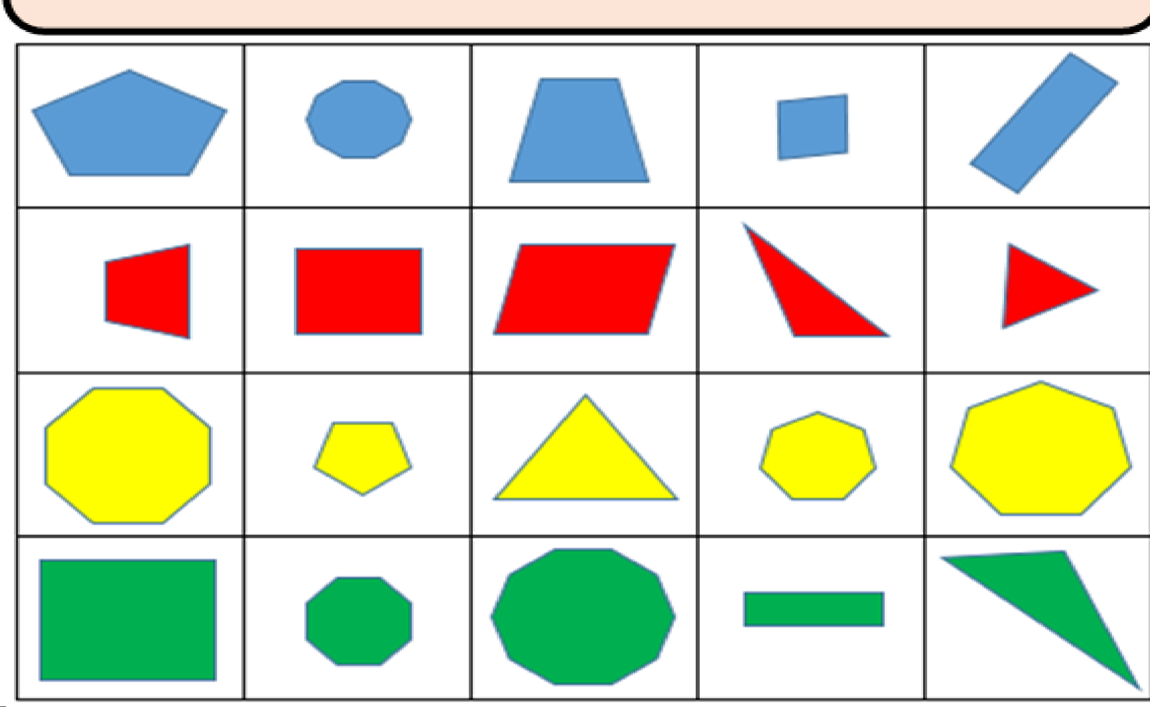
5. 

6. 

Assessment 2.

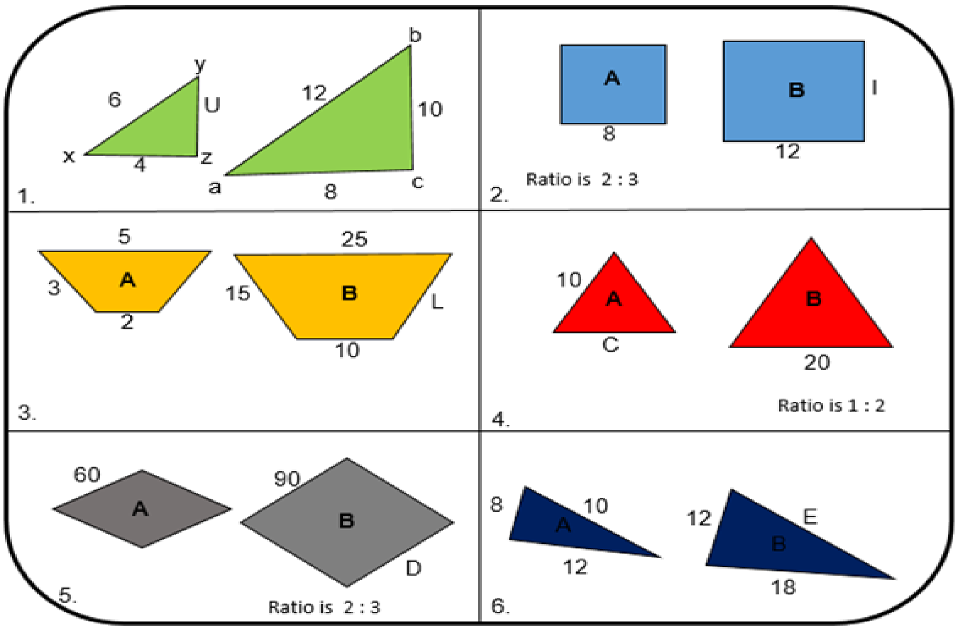
"FINDING MY SIMILAR"

Directions: Find the Similar of the polygons given below. Connect the Similar Polygon with a line.



Assessment 3. "Father of Geometry"

Directions: Each problem shows similar polygons. Use ratio and proportion to solve for the unknown side. Then, match each answer to the letter in the table below to solve the riddle.



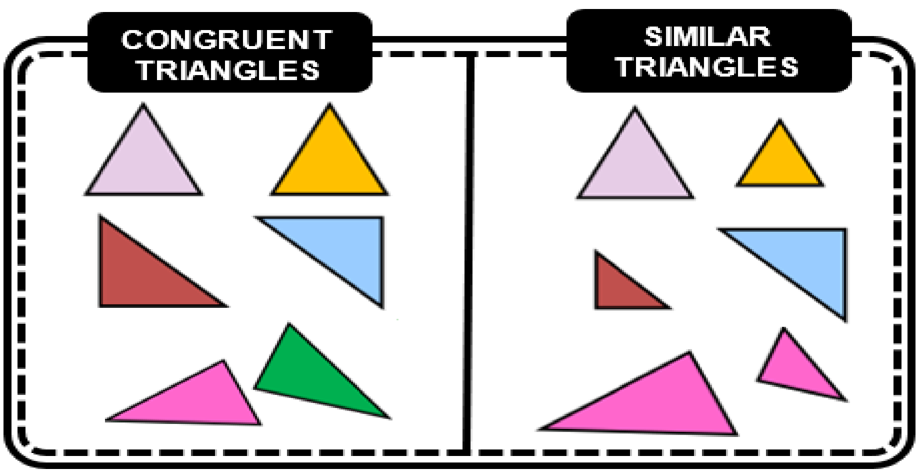
The Father of Geometry is ________________________

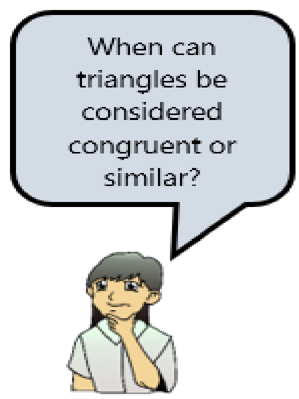
15	5	10	15	12	90

UNDERSTAND

Lesson 2: Similar Triangles

Look at the given figures below:





CONGRUENT TRIANGLES

Triangles are said to be congruent if they have the same size and shape.

SIMILAR TRIANGLES

Similar triangles are of the same shape but not of the same size. That is, if their corresponding angles are congruent and the lengths of their corresponding sides are proportional.

Note: Similarity is denoted with the symbol ~.

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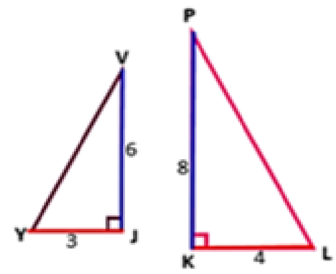
Triangle Similarity Theorem (SAS, SSS and AA Similarity Theorem)

Side-Angle-Side (SAS) Similarity Theorem

If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides that include these angles are proportional, then the two triangles are similar.

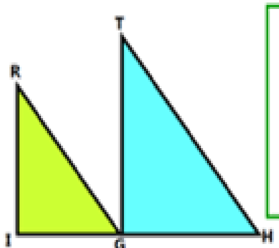
Let us take a look at the figures below to show that the two triangles are similar.

Statements	***Reasons***
$\angle J \cong \angle K$	$\angle J$ and $\angle K$ are both right angles which measure 90º.
$\frac{JY}{KL} = \frac{JV}{KP}$	The included sides are proportional. $\frac{3}{4} = \frac{6}{8}$
Then, $\Delta YJV \sim \Delta LKP$.	



Using ***SAS Similarity Theorem***, we can prove that triangles are similar. The example on the left uses **flowchart** to prove that the triangles are similar while the example on the right uses a **two-column proof**.

FLOWCHART



Given:

$\overline{RG} \parallel \overline{TH}$

$\frac{RG}{TH} = \frac{GI}{HG}$

Prove:

$\Delta RGI \sim \Delta THG$

$\overline{RG} \parallel \overline{TH}$

$\frac{RG}{TH} = \frac{GI}{HG}$

Given

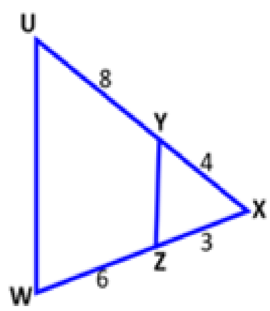
$\angle RGI \cong \angle THG$

Corresponding angles are congruent.

$\Delta RGI \sim \Delta THG$

SAS Similarity Theorem

TWO-COLUMN PROOF



Given:

$\overline{YZ} \parallel \overline{UW}$

$\frac{XY}{YU} = \frac{XZ}{ZW}$

Prove:

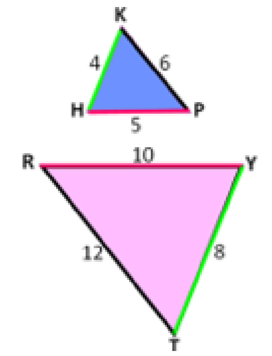
$\Delta YXZ \sim \Delta UXW$

STATEMENTS	**REASONS**
1. $\overline{RG} \parallel \overline{TH}$ $\frac{XY}{YU} = \frac{XZ}{ZW}$	Given
2. $\angle X \cong \angle X$	Reflexive Property
3. $\Delta YXZ \sim \Delta UXW$	SAS Similarity Theorem

Side-Side-Side (SSS) Similarity Theorem

If the corresponding sides of two triangles are proportional, then the two triangles are similar.

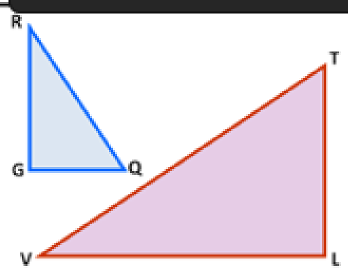
Let us take a look at the figures below to show that the two triangles are similar using SSS Similarity Theorem.



Statements	Reasons
$\frac{KP}{TR} = \frac{PH}{RY} = \frac{KH}{YT}$	The three corresponding sides are proportional. $\frac{6}{12} = \frac{5}{10} = \frac{4}{8}$
Then, $\Delta KPH \sim \Delta TRY$.	

Here are some examples on how to prove that triangles are similar using SSS Similarity Theorem. The example below(right) uses flowchart while the one on the left uses a two-column proof to prove that the two triangles are similar.

FLOWCHART



Given: $\frac{RG}{VL} = \frac{GQ}{LT} = \frac{RQ}{VT}$

Prove: $\Delta RGQ \sim \Delta VLT$

Proof:

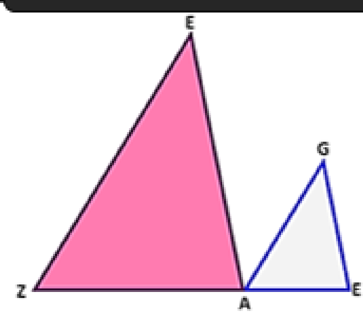
$\frac{RG}{VL} = \frac{GQ}{LT} = \frac{RQ}{VT}$

Given

$\Delta RGQ \sim \Delta VLT$

SSS Similarity Theorem

TWO-COLUMN PROOF



Given: $\frac{EZ}{GA} = \frac{EA}{GE} = \frac{AZ}{EA}$

Prove: $\Delta EZA \sim \Delta GAE$

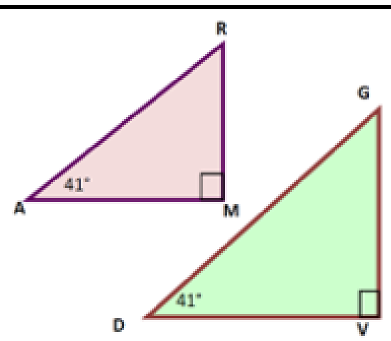
Proof:

STATEMENTS	REASONS
1. $\frac{EZ}{GA} = \frac{EA}{GE} = \frac{AZ}{EA}$	Given
2. $\Delta EZA \sim \Delta GAE$	SAS Similarity Theorem

Angle-Angle (AA) Similarity Theorem

If two angles of the triangle are congruent to two angles of another triangle, then the two triangles are similar.

Let us take a look at the figures below to show that the two triangles are similar using AA Similarity Theorem.



Statements	Reasons
$\angle A \cong \angle D$	Both angles measure 41º.
$\angle M \cong \angle V$	Both angles are right angles and measure 90º.
Since two pairs of corresponding angles are congruent, then $\Delta RMA \sim \Delta GVD$.	

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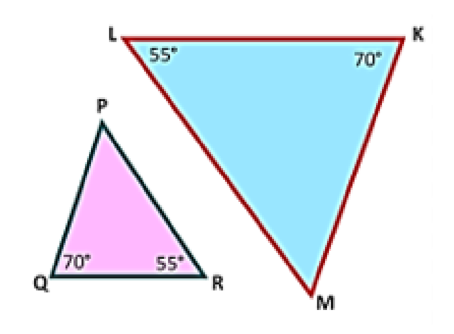
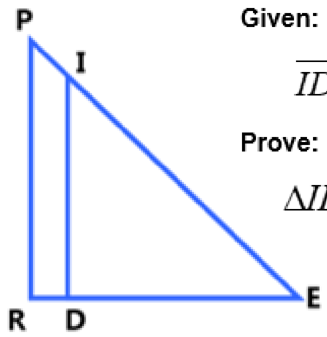


Statements	Reasons
$\angle R \cong \angle L$	Both angles measure 55º.
$\angle Q \cong \angle K$	Both angles measure 70º.
Since two pairs of corresponding angles are congruent, then $\Delta RQP \sim \Delta LKM$.	

Here are some examples on how to prove that triangles are similar using AA Similarity Theorem.

EXAMPLE 1.



Given:

$\overline{ID} \parallel \overline{PR}$

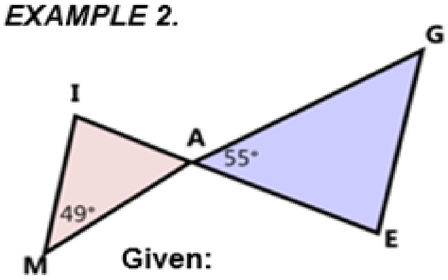
Prove:

$\Delta IDE \sim \Delta PRE$

Proof:

STATEMENTS	REASONS
1. $\overline{ID} \parallel \overline{PR}$	Given
2. $\angle IDE \cong \angle PRE$	Corresponding angles are congruent.
3. $\angle E \cong \angle E$	Reflexive Property
3. $\Delta IDE \sim \Delta PRE$	AA Similarity Theorem

EXAMPLE 2.



Given:

$\overline{IM} \parallel \overline{EG}$

Prove:

$\Delta IMA \sim \Delta EGA$

Proof:

STATEMENTS	REASONS
1. $\overline{IM} \parallel \overline{EG}$	Given
2. $\angle IMA \cong \angle EGA$	Alternate interior angles are congruent.
3. $\angle IAM \cong \angle EAG$	Vertical angles are congruent.
4. $\Delta IMA \sim \Delta EGA$	AA Similarity Theorem

SAQ 1: *How do you know if the triangles are similar?*
SAQ 2: *What is the importance of Triangle Similarity Theorems (SAS, SSS, AA)?*

Let's Practice! ***(Write your answer on separate sheets.)***

ACTIVITY 1. Let's Prove It!

A. Directions. Complete the two-column proof. Shade the correct reason to prove that the triangles are similar using the SAS Similarity Theorem.

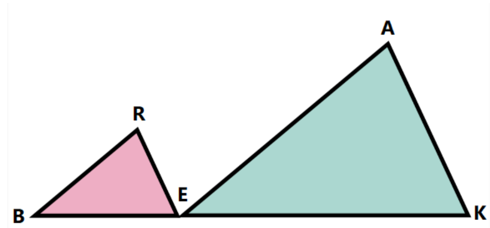
Given:

$\overline{RE} \parallel \overline{AK}$

$\frac{RE}{AK} = \frac{EB}{KE}$

Prove:

$\Delta REB \sim \Delta AKE$



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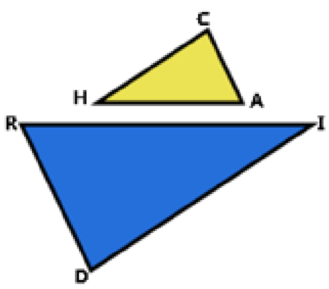
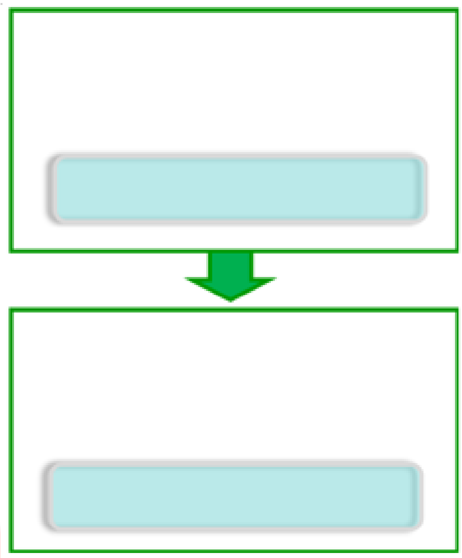
STATEMENTS	REASONS
1. $\overline{RE} \parallel \overline{AK}$ $\frac{RE}{AK} = \frac{EB}{KE}$	Definition of Parallel Lines. Definition of Proportionality. Given
2. $\angle REB \cong \angle AKE$	Vertical angles are congruent. Alternate interior angles are congruent. Corresponding angles are congruent.
3. $\Delta REB \sim \Delta AKE$	SAS Similarity Theorem SSS Similarity Theorem AA Similarity Theorem

B. Directions. Supply the statements and reasons to complete the flowchart to prove that the triangles are similar using SSS Similarity Theorem.

Given:

$$\frac{CA}{DR} = \frac{CH}{DI} = \frac{HA}{IR}$$





Prove:

$\Delta CAH \sim \Delta DRI$

CHOICES:

$\Delta CAH \sim \Delta DRI$	$\frac{CA}{DR} = \frac{CH}{DI} = \frac{HA}{IR}$
SSS Similarity Theorem	Given

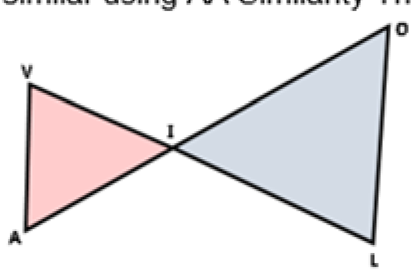
C. *Directions.* Prove that the triangles are similar. Given the statements, rearrange the given reason to prove that the triangles are similar using AA Similarity Theorem.

Given:

$\overline{AV} \parallel \overline{OL}$

Prove:

$\Delta AIV \sim \Delta OIL$



STATEMENTS	REASONS
1. $\overline{AV} \parallel \overline{OL}$	
2. ∠VAI≅∠LOI	
3. ∠AIV≅∠OIL	
4. ΔAIV~ΔOIL	

Choices:

- Alternate interior angles are congruent.
- AA Similarity Theorem
- Vertical angles are congruent.
- Given

REMEMBER

Key Points

- **Similar triangles** are of the same shape but not of the same size. That is, if their corresponding angles are congruent and the lengths of their corresponding sides are proportional.
- **Side-Angle-Side (SAS) Similarity Theorem**
  If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides that include these angles are proportional, then the two triangles are similar.
- **Side-Side-Side (SAS) Similarity Theorem**
  If the corresponding sides of two triangles are proportional, then the two triangles are similar.
- **Angle-Angle (AA) Similarity Theorem**
  If two angles of the triangle are congruent to two angles of another triangle, then the two triangles are similar.

TRY

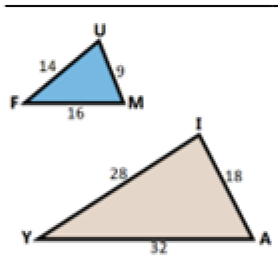
Let's see how much you have learned today!

General Directions: Study the following assessments carefully and write your answers on separate sheets.

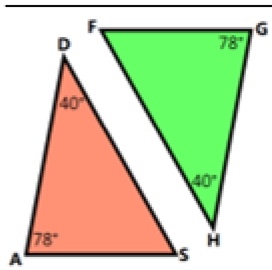
Assessment 1.

Directions: Determine what similarity theorem will be used to prove that the triangles are similar. If no similarity theorem is applicable, write not similar.

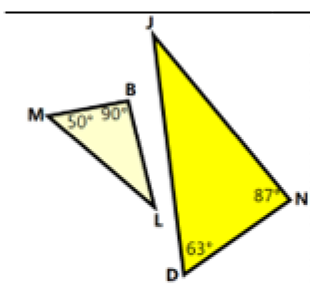
1. ____________________



2. ____________________



3. ____________________



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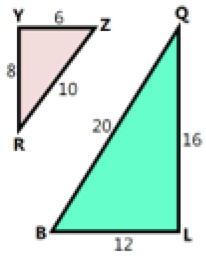
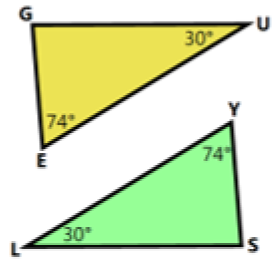
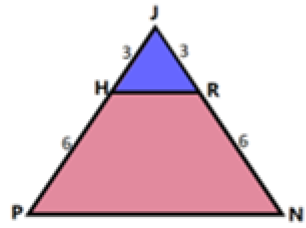
4. ____________________



5. ____________________



6. ____________________



Assessment 2.

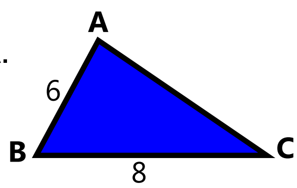
Directions: Read and understand each statement carefully. Choose the letter of the best answer.

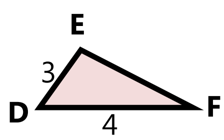
1. When are triangles said to be similar?
   A. If the angles are congruent.
   B. If the corresponding sides are proportional
   C. Both A and B

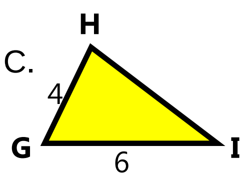
2. Which similarity theorem states that "If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides that include these angles are proportional, then the two triangles are similar."?
   A. SAS Similarity Theorem B. SSS Similarity Theorem C. AA Similarity Theorem

3. If the three corresponding sides of two triangles are proportional, what type of similarity theorem is being illustrated?
   A. SAS Similarity Theorem B. SSS Similarity Theorem C. AA Similarity Theorem

4. Given the figure below, which of the following is not similar to each other?

A. 

B. 

C. 

5. What theorem will be used to show that ΔMKA ~ ΔRKE?
   A. SAS Similarity Theorem B. SSS Similarity Theorem C. AA Similarity Theorem

For number 6 and 7, refer to *Figure 1.*

6. Given that ∠B ≅ ∠P, Which of the following statements is necessary to prove that the triangles are similar?
   A. $\frac{AB}{NP}$ B. $\frac{BL}{PI}$ C. $\frac{AB}{NP} = \frac{BL}{PI}$

7. What theorem will be used to prove that ΔABL ~ Δ NPI?
   A. SAS Similarity Theorem
   B. SSS Similarity Theorem
   C. AA Similarity Theorem

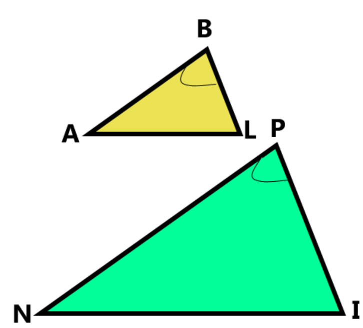


Figure 1

For number 8, 9 and 10, refer to Figure 2.

8. Given that $\overline{CW} \parallel \overline{NR}$, what reason will support ∠CWO ≅ ∠NRO?
   A. Alternate interior angles are congruent.
   B. Vertical angles are congruent.
   C. Corresponding angles are congruent.

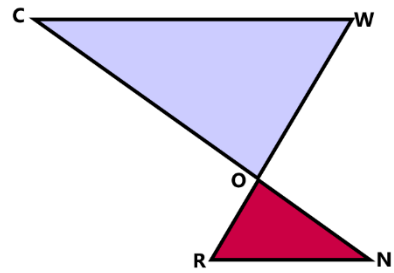


Figure 2

9. Which statement will justify that ∠COW ≅ ∠NOR?
   A. Alternate interior angles are congruent.
   B. Vertical angles are congruent.
   C. Corresponding angles are congruent.

10. Which theorem will prove that ∆COW ~ ∆NOR?
   A. SAS Similarity Theorem B. SSS Similarity Theorem C. AA Similarity Theorem

Assessment 3.

Directions: Solve for the missing measure. Find your answer on the next column and notice the letter before it. Write this letter in the box located at the bottom of this page to answer the mystery question.

If ∆**MNO ~** ∆**XYZ**, find the missing measure.

1. If MN = 15, XY = 20, MO = 30, XZ = 40, and *m*∠M = 78, what is *m*∠X?
2. If *m*∠N = *m*∠M = *m*∠X =50, what is *m*∠Y?
3. If MN = 4, XY = 10, NO = 10, YZ = 25, and MO = 2, what is XZ?
4. If *m*∠M = 50, *m*∠X = 50, *m*∠O = 70, what is *m*∠Z?
5. If MO = 3, XZ = 12, NO = 2, YZ = 8, XY = 4, what is MN?
6. If *m*∠M = 105, *m*∠X = 105, MO = 8, XZ = 2, XY = 1, what is MN?

C	4
D	5
E	16
I	78
L	70
N	130
U	50

Who is the Father of Geometry?

5	2	6	4	1	3

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