Eqn # 1: $z_1 = x_1 \cdot w_1 + b_1$

Eqn # 2: $a_1 = f(z_1) = \frac{1}{1+e^{-z_1}}$

Eqn # 3: $z_2 = a_1 \cdot w_2 + b_2$

Eqn # 4: $a_2 = f(z_2) = \frac{1}{1+e^{-z_2}}$

Eqn # 5: $E = \frac{1}{2}(T - a_2)^2$

1. $\frac{\partial E}{\partial w_2} = \frac{\partial E}{\partial a_2} \cdot \frac{\partial a_2}{\partial z_2} \cdot \frac{\partial z_2}{\partial w_2}$

- $E = \frac{1}{2}(T - a_2)^2$

  $\frac{\partial E}{\partial a_2} = 2 \cdot \frac{1}{2}(T - a_2) \cdot (-1) = -(T - a_2)$
- $a_2 = \frac{1}{1+e^{-z_2}} = (1 + e^{-z_2})^{-1}$

  $\frac{\partial a_2}{\partial z_2} = -1 \cdot (1 + e^{-z_2})^{-2} \cdot e^{-z_2} \cdot (-1)$

  $= \frac{e^{-z_2}}{(1 + e^{-z_2})^{-2}} = (a_2)^2 \frac{1 - a_2}{a_2} = a_2(1 - a_2)$
- $z_2 = a_1 \cdot w_2 + b_2$

  $\frac{\partial z_2}{\partial w_2} = a_1$

$\frac{\partial E}{\partial w_2} = -(T - a_2) \cdot a_2(1 - a_2) \cdot a_1$

2. $\frac{\partial E}{\partial b_2} = \frac{\partial E}{\partial a_2} \cdot \frac{\partial a_2}{\partial z_2} \cdot \frac{\partial z_2}{\partial b_2}$

- $E = \frac{1}{2}(T - a_2)^2$

  $\frac{\partial E}{\partial a_2} = 2 \cdot \frac{1}{2}(T - a_2) \cdot (-1) = -(T - a_2)$
- $a_2 = \frac{1}{1 + e^{-z_2}} = (1 + e^{-z_2})^{-1}$

  $\frac{\partial a_2}{\partial z_2} = -1 \cdot (1 + e^{-z_2})^{-2} \cdot e^{-z_2} \cdot (-1)$

  $= \frac{e^{-z_2}}{(1 + e^{-z_2})^{-2}} = (a_2)^2 \cdot \frac{1 - a_2}{a_2} = a_2(1 - a_2)$
- $z_2 = a_1 \cdot w_2 + b_2$

  $\frac{\partial z_2}{\partial b_2} = 1$

$\frac{\partial E}{\partial b_2} = -(T - a_2) \cdot a_2(1 - a_2) \cdot 1$

3. $\frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial a_2} \cdot \frac{\partial a_2}{\partial z_2} \cdot \frac{\partial z_2}{\partial a_1} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_1}$

- $E = \frac{1}{2}(T - a_2)^2$

  $\frac{\partial E}{\partial a_2} = -(T - a_2)$

- $a_2 = \frac{1}{1+e^{-z_2}} = (1 + e^{-z_2})^{-1}$

  $\frac{\partial a_2}{\partial z_2} = a_2(1 - a_2)$

- $z_2 = a_1 \cdot w_2 + b_2$

  $\frac{\partial z_2}{\partial a_1} = w_2$

- $a_1 = \frac{1}{1+e^{-z_1}} = (1 + e^{-z_1})^{-1}$

  $\frac{\partial a_1}{\partial z_1} = a_1(1 - a_1)$

- $z_1 = x_1 \cdot w_1 + b_1$

  $\frac{\partial z_1}{\partial w_1} = x_1$

$$\frac{\partial E}{\partial w_1} = -(T - a_2) \cdot a_2(1 - a_2) \cdot w_2 \cdot a_1(1 - a_1) \cdot x_1$$

4. $\frac{\partial E}{\partial b_1} = \frac{\partial E}{\partial a_2} \cdot \frac{\partial a_2}{\partial z_2} \cdot \frac{\partial z_2}{\partial a_1} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z_1}{\partial b_1}$

- $E = \frac{1}{2}(T - a_2)^2$

  $\frac{\partial E}{\partial a_2} = -(T - a_2)$

- $a_2 = \frac{1}{1+e^{-z_2}} = (1 + e^{-z_2})^{-1}$

  $\frac{\partial a_2}{\partial z_2} = a_2(1 - a_2)$

- $z_2 = a_1 \cdot w_2 + b_2$

  $\frac{\partial z_2}{\partial a_1} = w_2$

- $a_1 = \frac{1}{1+e^{-z_1}} = (1 + e^{-z_1})^{-1}$

  $\frac{\partial a_1}{\partial z_1} = a_1(1 - a_1)$

- $z_1 = x_1 \cdot w_1 + b_1$

  $\frac{\partial z_1}{\partial b_1} = 1$

$$\frac{\partial E}{\partial b_1} = -(T - a_2) \cdot a_2(1 - a_2) \cdot w_2 \cdot a_1(1 - a_1) \cdot 1$$