Que 1.

1) If the two regression coefficients are $-8/15$ and $-5/6$ then the corelation coefficient is

$\Rightarrow$ $r = \sqrt{\left(\frac{-8^4}{15_3}\right)\left(\frac{-5}{6_3}\right)}$

$= \sqrt{\frac{4}{9}} = \pm\frac{2}{3}$

$= -\frac{2}{3}$

2) A and B are independent events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ then $P(A \cup B)$

$\Rightarrow$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) . P(B)$

$= 1/2 \times 1/3$

$= 1/6$

$P(A \cup B) = \frac{1}{2} + \frac{1}{3} + \frac{1}{6}$

$= \frac{2}{3}$

3) Using Gauss elimination method the solution of system of equations $x + 2y + z = 4$, $-3y + 2z = -3$, $-7y - 2z = -6$

$\Rightarrow$ $x + 2y + z = 4$ — ①

$-3y + 2z = -3$ — ②

$-7y - 2z = -6$ — ③

Add ② + ③

$-10y = -9$, $y = \frac{9}{10}$.

$$-3y + 2z = -3$$

$$-3 \times \frac{9}{10} + 2z = -3.$$

$$-\frac{27}{10} + 2z = -3$$

$$2z = -3 + 27/10$$

$$2z = -0.3$$

$$z = \frac{-0.3}{2} = \frac{-3}{20}.$$

Substitute in ①

$$x = 4 - 2y - z$$

$$= 4 - 2 \times 9/10 + 3/20$$

$$= 4 - 33/20$$

$$x = \frac{47}{20}$$

$$\therefore \quad x = \frac{47}{20}, \quad y = \frac{9}{10}, \quad z = \frac{-3}{20}. \quad \text{(ii)}$$

4) If a curve passing through (0,0), (2,4), (4,8) is given by $y = y_0 + u\Delta y_0$ then y at $x = 1$ is given by $(x = x_0 + uh)$

⇒

x	y	Δy_0	$\Delta^2 y_0$
0	0		
		4	
2	4		0
		4	
4	8		

5) The range of correlation coefficient 'r' for bivariate data is

$\Rightarrow$ iii) $-1 \le r \le 1$

6) If x_0, x_1 are two initial approximations to the root of $f(x) = 0$, by secant method next approximation x_2 is given by

$\rightarrow$ i) $x_2 = x_1 - \frac{(x_1 - x_0)}{(f_1 - f_0)} \times f_1$

Que 2) a) The first four moments of distribution about the value 4 are -1.5, 17, -30 and 108 respectively. Obtain the first four central moments about mean, β_1 and β_2.

$\rightarrow$ ~~Fit a straight line of the form $y = a + bx$ using least squares method to the following data~~

~~x~~	~~0~~	~~1~~	~~2~~	~~3~~	~~4~~
~~y~~	~~2~~	~~1~~	~~4~~	~~7~~	~~10~~

$\mu_1' = -1.5$

$\mu_2' = 17$

$\mu_3' = -30$

$\mu_4' = 108$

$\mu_1 = 0$

$\mu_2 = \mu_2' - (\mu_1')^2$

$= 17 - (-1.5)^2$

$= 14.75$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$

$= -30 - 3(17)(-1.5) + 2(-1.5)^3$

$= 39.75$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$$
$$= 108 - 4(-30)(-1.5) + 6(17)(-1.5)^2 - 3(-1.5)^4$$
$$= 142.3125$$

∴ coefficient of skewness

$$\beta_1 = \frac{\mu_3^2}{\mu_2^2} = \cancel{\frac{(-30)^2}{(17)^2}} = \cancel{3.114} \quad \frac{(39.75)^2}{(14.75)^2} = 7.2625$$

$$\beta_2 = \frac{\mu_4}{(\mu_2)^2} = \frac{142.3125}{(14.75)^2} \doteq 0.6541$$

$\boxed{\beta_1 = 7.2625}$ $\boxed{\beta_2 = 0.6541}$

Arithmetic Mean $= \mu_1' + A$
$= -1.5 + 4$
$= \boxed{2.5}$

b) Fit a straight line of the form $y = a + bx$ using least squares method to the following data

x	0	1	2	3	4
y	-2	1	4	7	10

→ First, we prepare the table for fitting straight line, for that we need Σx, Σy, Σxy, Σx^2

x	y	xy	x^2
0	-2	0	0
1	1	1	1
2	4	8	4
3	7	21	9
4	10	40	16

$\Sigma x = 10$

$\Sigma y = 20$

$\Sigma xy = 70$

$\Sigma x^2 = 30$

$\Sigma y = na + b\Sigma x$

$\Sigma xy = a\Sigma x + b\Sigma x^2$

$20 = 5a + 10b$ ① — $\times 2$

$70 = 10a + 30b$ ②

$40 = 10a + 20b$

$- \; 70 = 10a + 30b$

$-30 = -10b$

$b = \cancel{+1/3}\ 30/10$

$b = 3$.

$20 = 5a + 30$

$-10 = 5a$

$a = -2$.

$y = a + bx$

$\boxed{y = -2 + 3x}$

c) Two regression lines of a bivariate data are $3x + 2y = 26$ and $6x + y = 31$. Find the mean values of x and y. Also, determine the correlation coefficient between x and y.

⇒ given $3x + 2y = 26$ — ① $\times 2$

$6x + y = 31$ — ②

$6x + 4y = 52$

$- \; 6x + y = 31$

$3y = 21$

$21 = 3y$, $\underline{\underline{y = 7}}$

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$3x + 2y = 26$

$3x + 2\times 7 = 26$

$3x = 26 - \cancel{21}\ 14$

$x = 4$

$\bar{x}$ = mean of $x = 4$ and $\bar{y}$ = mean of $y = 7$.

$\boxed{\begin{array}{l}\text{Mean of } x = 4 \\ \text{Mean of } y = 7\end{array}}$

$$Y = \frac{-3}{2}x + \frac{26}{2}$$

$$Y = byx \cdot X + a$$

$$byx = -\frac{3}{2}$$

$$X = \frac{-1}{6}Y + \frac{31}{6} = Y \cdot bxy + a$$

$$bxy = \frac{-1}{6}$$

$$r = \sqrt{bxy \cdot byx}$$

$$= \sqrt{\frac{-1}{6} \times \frac{-3}{2}} = \sqrt{\frac{1}{4}} = \frac{1}{2} = \pm 0.5$$

Since both bxy and byx are -ve.

$\boxed{\therefore\ r = -0.5}$

Que 3) a) Calculate the coefficient of variation for the data given as follows. 36, 15, 25, 10 and 14.

$\Rightarrow$ $N = 5$

$\Sigma x = 36 + 15 + 25 + 10 + 14$

$= 100$

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Mean $= \frac{\sum x}{n} = \frac{100}{5} = 20 = \bar{x}$

x	$x = (X - \bar{X})$	$x^2 = (X - \bar{X})^2$
36	16	256
15	-5	25
25	5	25
10	-10	100
14	-6	36

$\sum x^2 = 442$

$\sigma = \sqrt{\frac{\sum x^2}{n}}$

$= \sqrt{\frac{442}{5}}$

$= \sqrt{88.4}$

$\sigma = 9.4$

coefficient of variation $= \frac{\sigma}{\bar{X}} \times 100$

$= \frac{9.4}{20} \times 100$

$= \underline{\underline{47}}$

b) Fit a second degree parabola of the form $y = a + bx + cx^2$ using squares method to the following data.

x	0	1	2	3
y	2	1	6	17

$\Rightarrow$ To fit the 2^{nd} degree polynomial by least squares method we have to solve followings eq^n

$\sum y = c\sum x^2 + b\sum x + na$

$\sum xy = c\sum x^3 + b\sum x^2 + a\sum x$

$$\Sigma x^2 y = c\Sigma x^4 + b\Sigma x^3 + a\Sigma x^2$$

x	y	x^2	x^3	x^4	xy	x^2y
0	2	0	0	0	0	0
1	1	1	1	1	1	1
2	6	4	8	16	12	24
3	17	9	27	81	51	153

$\Sigma x = 6$

$\Sigma y = 26$

$\Sigma x^2 = 14$

$\Sigma x^3 = 36$

$\Sigma x^4 = 95$

$\Sigma xy = 64$

$\Sigma x^2 y = 178$

eq^n ① $\Rightarrow$ $26 = 14c + 6b + 4a$ — $\times 6$

eq^n ② $\Rightarrow$ $64 = 36c + 14b + 6a$ — $\times 4$

eq^n ③ $\Rightarrow$ $178 = 95c + 36b + 14a$

$$156 = 84c + 36b + 24a$$
$$-\ \underline{256 = 144c + 56b + 24a}$$
$$-100 = -60c - 20b$$

$$10 = 6c - 2b$$

$$\therefore c = \frac{10+2b}{6}$$

$$26 = \frac{14}{6}(10+2b) + 6b + 4a$$

$$156 = 140 + 28b + 6b + 4a$$

$$16 = 34b + 4a. \quad — ③ \times 6$$

$$64 = \frac{\cancel{36}^{6}}{\cancel{6}}(10+2b) + 14b + 6a$$

$$\frac{32}{3} = 10 + 16b + 6a.$$

$$2/3 = 16b + 6a \quad \text{—} \quad (4) \quad \times 4$$

$$96 = 204b + 24a$$
$$-\ 8/3 = 64b + 24a$$
$$\cancel{\frac{280}{3}} = \cancel{140b}$$

$$\boxed{b = 2/3}$$

$$2/3 = 16 \times \frac{2}{3} + 6a$$

$$2/3 - 32/3 = 6a$$
$$-10 = 6a$$
$$\boxed{a = -5/3}$$

$$c = \frac{10 + 2b}{6}$$

$$c = \frac{10 + 4/3}{6} = \frac{30 + 4/3}{6} = \frac{34/3}{6/1} = \frac{17}{9} \qquad \boxed{c = \frac{17}{9}}$$

c) Find the carelation coefficient between the variables population density (x) and death rates (y) as given in the following data:

x	200	400	500	700	300
y	12	18	16	21	10

⇒

x	y	xy	x^2	y^2
200	12	2400	40000	144
400	18	7200	160000	324
500	16	8000	250000	256
700	21	14700	490000	441
300	10	3000	90000	100

$\Sigma xy = 35300$

$\Sigma x^2 = 1030000$

$\Sigma x = 2100$

$\Sigma y = 77$

$\Sigma y^2 = 1265$

$$r(x,y) = \frac{cov(x,y)}{6x\,6y}$$

$$cov(x,y) = \frac{\Sigma xy}{n} - \frac{\Sigma x}{n} \cdot \frac{\Sigma y}{n}$$

$$= \frac{35300}{5} - \frac{2100}{5} \cdot \frac{77}{5}$$

$$= \cancel{25280}\ 7060 - 6468$$

$$= 592$$

$$6x = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

$$= \sqrt{\frac{1030000}{5} - \left(\frac{2100}{5}\right)^2} = \sqrt{206000 - 176400} = \sqrt{29600}$$

$$= 172.04$$

$$6y = \sqrt{\frac{\Sigma y^2}{n} - \left(\frac{\Sigma y}{n}\right)^2} = \sqrt{\frac{1265}{5} - \left(\frac{77}{5}\right)^2} = \sqrt{253 - 237.16}$$

$$= \sqrt{15.84} = \underline{\underline{3.9799}}$$

$$r = \frac{cov(x,y)}{\sigma x \, \sigma y}$$

$$= \frac{592}{172.04 \times 3.9799}$$

$$\boxed{r = 0.864}$$

Que 4) a) Find the expected value of the sum of the faces obtained when two fair dice are tossed simultaneously.

→ $X = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$$P(x) = \left\{\frac{1}{36}, \frac{2}{36}, \frac{3}{36}, \frac{4}{36}, \frac{5}{36}\right\}$$

$$E(x) = \sum x_i \,(p(x) = x)$$

$$= 2 \times \frac{1}{36} + 3 \times \frac{2}{36} + 4 \times \frac{3}{36} + 5 \times \frac{4}{36} + 6 \times \frac{5}{36} + 7 \times \frac{6}{36} +$$

$$8 \times \frac{5}{36} + 9 \times \frac{4}{36} + 10 \times \frac{3}{36} + 11 \times \frac{2}{36} + 12 \times \frac{1}{36}$$

$$= \frac{2 + 6 + 12 + 20 + 30 + 42 + 40 + 36 + 30 + 22 + 12}{36}$$

$$= \frac{252}{36} = 7$$

$$\boxed{E(x) = 7}$$

b) An unbiased coin is tossed five times. Find the probability of observing at least four heads.

→ No. of possible outcomes $= 2^5 = 32$

Getting atleast 4 heads is combination of getting 4 and 5 heads.

No. of ways for getting 4 heads is 5.

No. of ways for getting 5 heads is 1.

Probability of getting at least 4 heads is

$= \frac{5+1}{32} = \boxed{\frac{3}{16}}$

c) In a sample of 1000 cases, the mean score in a certain examination is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find the expected number of students scoring between 12 & 15 (both inclusive).

[Given : $Z_1 = 0.4$, $A_1 = 0.1554$; $Z_2 = 0.8$, $A_2 = 0.2881$]

→ given $\mu = 14$

$\sigma = 2.5$

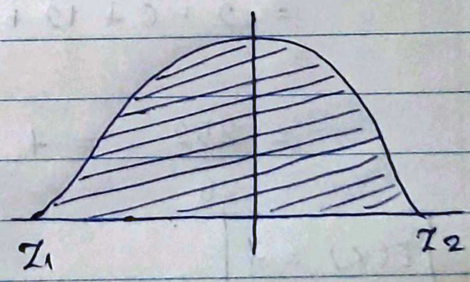
Also,

$Z_1 = 0.4$

$Z_2 = 0.8$

Also given, A_1 = area corresponding to $Z_1 = 0.1554$

$A_2 = 0.2881$



$P(12 \leq x \leq 15)$

$= A_1 + A_2$

$= 0.1554 + 0.2881$

$= 0.4435$

So, for 1000 cases =

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Que 5) a) A riddle is given to three students to solve independently. The individual probabilities of the riddle being solved by the three students are 0.3, 0.4 and 0.5 respectively. Find the probability that the riddle gets solved.

$\Longrightarrow$ $P(A) = 0.3$, $P(B) = 0.4$, $P(C) = 0.5$

$\therefore$ P(problem is not solved) = P(none solve the problem)

$$= P(\bar{A}) \times P(\bar{B}) \times P(\bar{C})$$
$$= (1 - P(A)) \times (1 - P(B)) \times (1 - P(C))$$
$$= (1-0.3) \times (1-0.4) \times (1-0.5)$$
$$= 0.7 \times 0.6 \times 0.5$$
$$= 0.21$$
$$= \frac{21}{100}$$

$\therefore$ P(problem is solved) = 1 - P(problem is not solved)

$$= 1 - 0.21$$
$$= \boxed{0.79}$$

b) On an average, there are two printing mistakes on a page of a book. Using Poisson distribution, find the probability that a randomly selected page from the book has at least one printing mistake.

$\Longrightarrow$ Let r be the number of typing mistakes is a poisson variable with the parameter $\lambda = 2$.

$$p(r) = \frac{e^{-\lambda} \cdot \lambda^{r}}{r!}$$

$$= \frac{e^{-2} \cdot 2^{r}}{r!}$$

i) P(2 mistakes) = $P(r = 2)$

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$= \frac{e^{-2} \cdot 2^{r}}{2!}$

$= \frac{0.1353 \times 4}{2}$

$= \underline{\underline{0.2706}}$.

ii) P(at least 1 mistake)

$= 1 - P(r < 1)$

$= 1 - P(r = 0)$

$= 1 - \frac{e^{-2} \cdot 2^{0}}{1}$

$= 1 - e^{-2}$

$= \underline{\underline{0.8646}}$

c) In a mouse breeding experiment, a geneticist has obtained 172 brown mice with pink eyes, 60 brown mice with brown eyes, 62 white mice with pink eyes and 26 white mice with brown eyes. Theory predicts that these types of mice should be obtained in the ratios 9 : 3 : 3 : 1. Test the compatibility of the data with theory, using 5% level of significance. [Given $\chi^2_{tab} = 7.815$]

⇒ Let Ho = the mice with pink eyes, brown eyes with ratio 9 : 3 : 3 : 1

expected percentage are

$9 + 3 + 3 + 1 = 16$.

$\frac{9}{16} \times 320 = 180$, $\frac{3}{16} \times 320 = 60$, $\frac{1}{16} \times \cancel{320}^{20} = 20$.

observed mice (Oi)	172	60	62	26
expected mice (ei)	180	60	60	20

So, to test Ho,

$$\chi^2_{k-p-1} = \sum_{k=1}^{4} \frac{(oi-ei)^2}{ei}$$

$$= \frac{(172-180)^2}{180} + \frac{(60-60)^2}{60} + \frac{(60-62)^2}{62} + \frac{(26-20)^2}{20}$$

$$= \frac{16}{45} + 0 + \frac{2}{31} + \frac{9}{5}$$

$$= 2.2200$$

here, $k = 4$, $p = 0$

but the tabular value is $\chi^2_{tab} = 7.815$

$$\Rightarrow \chi^2_3 < \chi^2_{tab}$$

so, we accept Ho at 5% level of significance.

Que6) a) Find a root of the equation $x^4 + 2x^3 - x - 1 = 0$, lying in the interval $[0, 1]$ using the bisection method at the end of fifth iteration.

$\Rightarrow$ let $f(x) = x^4 + 2x^3 - x - 1$

$f(0) = -1$

$f(1) = 1 + 2 - 1 = 2$

1st iteration,

$$x_1 = \frac{0+1}{2} = 0.5$$

$$f(x_1) = (0.5)^4 + 2(0.5)^3 - (0.5) - 1$$

$$= -1.1875 \Rightarrow -ve$$

root lies bet^ 0.5 & 1.

2nd iteration

$$x_2 = \frac{0.5+1}{2} = 0.75$$

$$f(x_2) = (0.75)^4 + 2(0.75)^3 - (0.75) - 1$$

$$= -0.5898 \Rightarrow -ve$$

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root lies bet^ 0.75 & 1

3rd iteration

$$x_3 = \frac{0.75+1}{2} = 0.875$$

$$f(x_3) = (0.875)^4 + 2(0.875)^3 - 0.875 - 1$$
$$= 0.05 = +ve.$$

root lies between 0.75 & 0.875.

$$x_4 = \frac{0.75 + 0.875}{2} = 0.8125 \quad \text{— 4th iteration}$$

$$f(x_4) = (0.8125)^4 + 2(0.8125)^3 - 0.8125 - 1$$
$$= -0.3039 = -ve$$

root lies between 0.8125 and 0.875

5th iteration

$$x_5 = \cancel{0.8125+} \frac{0.875 + 0.8125}{2} = 0.84375$$

$$f(x_5) = (0.84375)^4 + 2(0.84375)^3 - 0.84375 - 1$$
$$= -0.642 = -ve$$

root lies between 0.84375 and 0.875

$$x_6 = \frac{0.84375 + 0.875}{2} = 0.859375$$

6th iteration.

$$= |0.859375 - 0.844375| = 0.015622 < 0.02$$

Thus, if permissible error is 0.02 then root at the end of 6th iteration gives required accuracy.

$$n \geq \frac{\log(1-0) - \log(0.0005)}{\log 2}$$

$$\geq 10.965$$

$$\simeq 11.$$

i.e. n = 11 iterations are required to achieve the degree of accuracy.

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b) Obtain the real root of the equation $x^3-4x-9=0$ by applying Newton Raphson method at the end of third iteration.

⇒ let $f(x) = x^3-4x-9$

to find interval

$f(0) = -9 = -ve$

$f(1) = 1-4-9 = -12 = -ve$

$f(2) = -ve$

$f(3) = +ve$

here

$f(2) = -ve$

$f(3) = +ve$

Root lies between 2 and 3

1^{st} iteration

$$x_1 = \frac{2+3}{2} = \frac{5}{2} = 2.5$$

Now $f(x_1) = (2.5)^3 - 4(2.5) - 9$

$f(x_1) = -3.375 = -ve$

root lies between 2.5 & 3

2^{nd} iteration

$$x_2 = \frac{2.5+3}{2} = 2.75$$

$x_2 = 2.75$

$f(x_2) = (2.75)^3 - 4(2.75) - 9 = +ve$

$f(x_2) = +ve$

root lies between 2.5 & 2.75

3^{rd} iteration

$$x_3 = \frac{2.5+2.75}{2} = 2.625$$

$f(x_3) = -ve.$

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root lies betn 2.625 and 2.75

$$x_4 = \frac{2.625 + 2.75}{2} = 2.6875$$

$f(x_4) = -ve$

root lies between 2.6875 & 2.75

$$x_5 = \frac{2.6875 + 2.75}{2} = 2.71875$$

$f(x_5) = +ve$

So, the root of eqn $f(x) = 0$ is $\underline{\underline{2.71875}}$.

c) Solve by Gauss - Seidal method, the system of equations

$$10x_1 + x_2 + x_3 = 12$$
$$2x_1 + 10x_2 + x_3 = 13$$
$$2x_1 + 2x_2 + 10x_3 = 14$$

⇒ Given,

$$10x_1 + x_2 + x_3 = 12$$
$$2x_1 + 10x_2 + x_3 = 13$$
$$2x_1 + 2x_2 + 10x_3 = 14$$

$$x_1 = \frac{1}{10}[12 - x_2 - x_3]$$

$$x_2 = \frac{1}{10}[13 - 2x_1 - x_3]$$

$$x_3 = \frac{1}{10}[14 - 2x_1 - 2x_2]$$

For 1^{st} iteration,

take $x_2 = 0$, $x_3 = 0$

$x_1 = 12/10 = 1.2$

$x_1 = 1.2$, $x_3 = 0$, $x_2 = 1.06$

$x_1 = 1.2$, $x_2 = 1.06$, $x_3 = 0.948$

For 2^{nd} iteration, $x_2 = 1.06$, $x_3 = 0.948$

$x_1 = \frac{1}{10}[12 - 1.06 - 0.948] = 0.9992$

$x_2 = 0.992$, $x_3 = 0.948$ gives $x_2 = 1.0668$

$x_1 = 0.992$, $x_2 = 1.0068$ gives $x_3 = 1.0002$

For 3^{rd} iteration, $x_2 = 1.0668$, $x_3 = 1.002$

$x_1 = 0.993$, $x_3 = 1.0002$, gives $x_2 = 1.00$

$x_1 = 0.993$, $x_2 = 1.00$ gives $x_3 = 1.00$

Result,

$x_1 = 1.00$, $x_2 = 1.00$, $x_3 = 1.00$

que 7) a) Solve by Gauss elimination method, the system of equations:

$$2x_1 + x_2 + x_3 = 10, \quad 3x_1 + 2x_2 + 3x_3 = 18,$$
$$x_1 + 4x_2 + 9x_3 = 16$$

⇒ given,

$$2x_1 + x_2 + x_3 = 10 \quad \text{--- (1)}$$
$$3x_1 + 2x_2 + 3x_3 = 18 \quad \text{--- (2)}$$
$$x_1 + 4x_2 + 9x_3 = 16 \quad \text{--- (3)}$$

the matrix form is,

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 18 \\ 16 \end{bmatrix}$$

Augmented matrix is

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{array}\right]$$

$$\xrightarrow{R_{13}} \left[\begin{array}{ccc|c} 1 & 4 & 9 & 16 \\ 3 & 2 & 3 & 18 \\ 2 & 1 & 1 & 10 \end{array}\right]$$

$$\xrightarrow[R_3 - 2R_2]{R_2 - 3R_3} \left[\begin{array}{ccc|c} 1 & 4 & 9 & 16 \\ 0 & -1 & 0 & \cancel{12}\,9 \\ 0 & -3 & -5 & -26 \end{array}\right]$$

$$\xrightarrow{R_3 - 3R_2} \left[\begin{array}{ccc|c} 1 & 4 & 9 & 16 \\ 0 & -1 & 0 & \cancel{12}\,9 \\ 0 & 0 & -\cancel{8}1 & \cancel{10}\,-5 \end{array}\right]$$

$R_1 \longrightarrow x_1 + 4x_2 + 9x_3 = 16$

$R_2 \longrightarrow -x_2 = 9$

$x_2 = -9$

$R_3 \longrightarrow -x_3 = -5$

$x_3 = 5$

$x_1 + 4(-9) + 9(5) = 16$

$x_1 - 36 + 45 = 16$

$x_1 = 7$

$\therefore \ \underline{\underline{x_1 = 7}}, \ \underline{\underline{x_2 = -9}}, \ \underline{\underline{x_3 = 5}}$.

b) Solve by Jacobi's iteration method, the system of equatio

$20x_1 + x_2 - 2x_3 = 17$, $3x_1 + 20x_2 - x_3 = -18$

$2x_1 - 3x_2 + 20x_3 = 25$.

$\Rightarrow$ consider given system,

$20x_1 + x_2 - 2x_3 = 17$

$3x_1 + 20x_2 - x_3 = -18$

$2x_1 - 3x_2 + 20x_3 = 25$

from ① eqⁿ

$$x = \frac{1}{20}[17 - x_2 + 2x_3]$$

from ② eqⁿ

$$x_2 = \frac{1}{20}[-18 - 3x_1 + x_3]$$

from ③ eqⁿ

$$x_3 = \frac{1}{20}[25 - 2x_1 + 3x_2]$$

1st iteration

let $x_1 = x_2 = x_3 = 0$

$$x_1^{(1)} = \frac{1}{20}[17 - 0 - 0] = \frac{17}{20}$$

$$x_1^{(1)} = 0.85$$

$$x_2^{(1)} = \frac{1}{20}[-18 - 0 - 0] = \frac{-18}{10}$$

$$x_3^{(1)} = \frac{1}{20}[25 - 0 - 0] = \frac{25}{20}$$

2nd iteration

$$x_1^{(2)} = \frac{1}{20}[17 - (-0.9) + 2(1.25)]$$

$$x_1^{(2)} = 1.02$$

$$x_2^{(2)} = 1/20[-18 - 3(0.85) + 1.25]$$

$$x_2^{(2)} = -0.965$$

$$x_3^{(2)} = +1.03$$

No. of iteration	x_1	x_2	x_3
1	0.85	-0.9	1.25
2	1.02	-0.965	1.03
3	1.0025	-1.0015	1.00325
4	1.0004	-0.999	0.999
5	0.99985	-1.00011	1.00111
6	1.000	-0.9999	1.00

c) Find a real root of the equation $x^3 - 2x - 5 = 0$ by the method of false position at the end of fourth iteration.

⇒ Here, first we find interval of the root

$f(x) = x^3 - 2x - 5 = 0$

$f(0) = -5$

$f(1) = -6$

$f(2) = -1$

$f(3) = 16$

this implies that interval of root is 2, 3.

a	f(a)	b	f(b)	c	f(c)
2	-1	3	16	2.0588	-0.3903
2.0588	-0.3908	3	16	2.0812	-0.149
2.0812	-0.149	3	16	2.0896	-0.05515
2.0896	-0.055	3	16	2.0926	-0.0206
2.0926	-0.0209	3	16	2.0938	-0.0564

1st iteration

$$c = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{2(16) - 3(-1)}{16 - (-1)}$$

$c = 2.0588$

$f(c) = (2.0588)^3 - 2(2.0588) - 5$

$f(c) = 0.3908$

2nd iteration

$$c = \frac{(2.0588)(16) - 3(-0.3908)}{16 - (0.3903)}$$

$c = 2.0812$

$f(c) = -0.149$

3^{rd} iteration -

$$c = \frac{2.0812(16) - 3(-0.149)}{16 - (-0.149)} = 2.0896$$

$f(c) = (2.0896)^3 - 2(2.0896) - 5$
$= -0.05511$

4^{th} iteration.

$c = 2.0926$

$f(c) = -0.0206$

5^{th} iteration -

$c = 2.0938$

$f(c) = -8.38 \times 10^{-3}$

Que 8) a) Using Newton's forward interpolation formula, find y at $x = 8$ from the data :

x	0	5	10	15	20	25
y	7	11	14	18	24	32

⟹ Here the value of $x = 8$ which belongs to first half of intervals.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
0	7					
		4				
5	11		-1			
		3		2		
10	14		1		-1	
		4		1		0
15	18		2		-1	
		6		0		
20	24		2			
		8				
25	32					

$h = 5$, $x_0 = 0$, $\cancel{x} \; u = \frac{x-0}{5} = \frac{x}{5}$

$$y = f(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$+ \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0$$

$$= 7 + \cancel{8}u.4 + \frac{u(u-1)(-1)}{2} + \frac{u(u-1)(u-2)(2)}{6} +$$

$$\frac{u(u-1)(u-2)(u-3)(-1)}{24}$$

$$= 7 + \frac{4x}{5} + \left(\frac{x}{5}\right)\left(\frac{x}{5}-1\right)(-1)\times\frac{1}{2} + \frac{(x/5)(x/5-1)(x/5-2)\cancel{2}}{\cancel{6}\,3}$$

$$+ \frac{(x/5)(x/5-1)(x/5-2)(x/5-3)(-1)}{24}$$

$$= 7 + \frac{4x}{5} - \frac{1}{50}(x^2-5x) + \frac{1}{375}(x^3-15x^2+50x) +$$

$$\frac{-1}{1875}(x^4-30x^3+275x^2-750x)$$

$$= 7 + \frac{4(8)}{5} - \frac{1}{50}(8^2-5(8)) + \frac{1}{375}(8^3-15(8)^2+50(8)) +$$

$$\frac{-1}{1875}(8^4-30(8)^3+275(8)^2-750(8))$$

$$= 7 + 6.4 - 0.48 + 1.20533 - 0.1792$$

$$= \underline{\underline{13.94613}}$$

b) Evaluate $\int_1^2 \frac{dx}{x^2}$ using simpson's $\frac{1}{3}^{rd}$ rule. $(h = 0.25)$

$\Rightarrow$ here $h = 0.25$ and $f(x) = 1/x^2$.

so, table is

x $\}$ ~~0.1~~ ~~0.25~~

y

x	1	1.25	1.50	1.75	2.00
y	1	0.64	0.444	0.3265	0.25
	y_0	y_1	y_2	y_3	y_4

By Simpson's $1/3^{rd}$ rule,

$$I = \frac{h}{3}\left[(y_0 + y_4) + 2(y_2) + 4(y_1 + y_3)\right]$$

$$= \frac{0.25}{3}\left[(1 + 0.25) + 2(0.444) + 4(0.64 + 0.3265)\right]$$

$$= \underline{0.50033}$$

c) Use Euler's method to solve $\frac{dy}{dx} = 1 + xy$, $y(0) = 1$. Tabulate values of y for $x = 0$ to $x = 0.3$. $(h = 0.1)$

⇒ given $f(x, y) = 1 + xy$

$x_0 = 0$, $y_0 = 1$, $h = 0.1$

by Euler's method,

$$y_1 = y_0 + h f(x_0, y_0)$$
$$= 1 + 0.1 f(0, 1)$$
$$= 1 + (0.1)(1 + 0)$$
$$y_1 = 1.1$$

$$y_2 = y_1 + h f(x_1, y_1)$$
$$= 1.1 + (0.1) f(0.1, 1.1)$$
$$= 1.211$$

$$y_3 = y_2 + h f(x_2, y_2)$$
$$= 1.211 + (0.1) f(0.2, 1.211)$$
$$y_3 = 1.3352$$

$$y_4 = y_3 + h f(x_3, y_3)$$
$$= 1.3352 + (0.1) f(0.3, 1.3352)$$
$$= 1.4752$$

x	0	0.1	0.2	0.3	0.4
y	1	1.1	1.211	1.3352	1.4752.

Que 9) a) Use Runge-kutta method of fourth-order to solve $\frac{dy}{dx} = x + y^2$, $y(0) = 1$ at $x = 0.1$ with $h = 0.1$.

→ given $x_0 = 0$, $y_0 = 1$, $h = 0.1$

$f(x, y) = x + y^2$

$y_1 = y(0.1)$

$k_1 = hf(x_0, y_0) = 0.1(0, 1) = 0.1$

$k_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$

$= 0.1(f(0 + 0.05, 1 + \frac{0.1}{2})$

$= 0.1 \; f(0.05, 1.05)$

$= 0.1 \times 1.1525$

$= 0.11525.$

$k_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2})$

$= 0.1 \; f(0.05, 1.057)$

$= 0.1167$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= 0.1 \; f(0.1, 1.1167)$

$= 0.1 \times 1.347$

$= 0.134$

$k = \frac{1}{6}(k_1 + k_2 + k_3 + k_4) = \frac{1}{6}(0.1 + 0.11525 + 0.1167 + 0.134)$

$= 0.077$

$y_1 = y(0.1) = y_0 + k = 1 + 0.077 = \underline{1.077}$

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b) Use modified Euler's method to find $y(0.1)$, given $\frac{dy}{dx} = 1 + xy$, $y(0) = 1$ and $h = 0.1$

$\Rightarrow$ given $f(x, y) = 1 + xy$

$x_0 = 0$, $y_0 = 1$, $h = 0.1$

by Euler's method

$y_1 = y_0 + hf(x_0, y_0)$

$= 1 + (0.1) f(0,1)$

$= 1 + (0.1)(1+0)$

$y_1 = 1.1$

$y_2 = y_1 + hf(x_1, y_2)$

$= 1.1 + (0.1) f(0.1, 1.1)$

$= 1.211$

$y_3 = y_2 + hf(x_2, y_3)$

$= 1.211 + (0.1) f(0.2, 1.211)$

$= 1.3352$

$y_4 = y_3 + hf(x_3, y_3)$

$= 1.3352 + (0.1) f(0.3, 1.3352)$

$= 1.4752$

x	0	0.1	0.2	0.3	0.4
y	1	1.1	1.211	1.3352	1.4752

c) Using Newton's backward difference formula, find the value of $\sqrt{155}$ from the data :

x	150	152	154	156
$y = \sqrt{x}$	12.247	12.329	12.410	12.490

$\Rightarrow$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
150	12.247			
		0.082		
152	12.329		-0.001	
		0.081		0
154	12.410		-0.001	
		0.080		
156	12.490			

here $h = 2$, $x_3 = 156$, $x = 155$, $n = 3$

$$u = \frac{x - x_n}{h} = \frac{155 - 156}{2} = -0.5$$

Using Newton's backward interpolation formula,

$$y = y_3 + u \nabla y_3 + \frac{u(u+1)}{2!} \nabla^2 y_3 + \frac{u(u+1)(u+2)}{3!} \nabla^3 y_3$$

$$= 12.490 + (-0.5)(0.800) + \left[\frac{(-0.5)(-0.5+1)(-0.001)}{2!}\right] + \left[\frac{(-0.5)(-0.5+1)(-0.5+2)(-0.001)}{3!}\right]$$

$$= 12.490 - 0.4 - 1.25 \times 10^{-4}$$

$$= 12.4496$$

$\therefore$ By Newton's backward interpolation formula,

$$\sqrt{x} = \sqrt{155} = \underline{\underline{12.4496}}$$