Rainbow

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Discrete Math

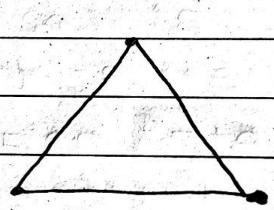
Unit 4

Graph solved Question

NOV_DEC_2022

Q3)

a) - A simple graph is unweighted, undirected graph with no self loops and parallel edges.
~~A simple graph is said to be opposite of complete graph.~~



- It is not possible to draw a simple graph, with 4 vertices and 7 edges.
- Because in simple graph, each edge connect two distinct vertices, and having 7 edges with 4 vertices will violate the rule that each vertex can have maximum degree of 3. This violates the handshaking lemma, which state sum of degree of all vertices is twice the no of edges.

$\frac{n(n-1)}{2} = \frac{4(3)}{2} = \frac{12}{2} = 6$

- In this case there is 4 vertices to find maximum edges there is formula

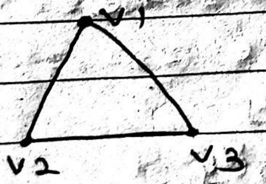
$$\frac{n \cdot (n-1)}{2} = \frac{4(4-1)}{2}$$

$$= \frac{12}{2}$$

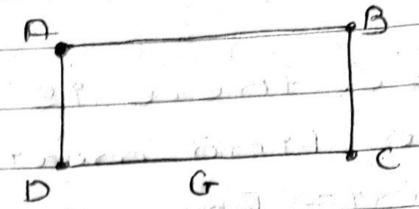
$$= 6$$

∴ 6 ~~e~~ maximum edges can be there with four vertices.

(i) complete graph:
- A graph G is called complete graph if every vertex in G is connected ~~to~~ with every other vertex.



(ii) Regular Graph:
A Graph G is said to be regular graph if degree of each vertex is same.



d(A) = 2

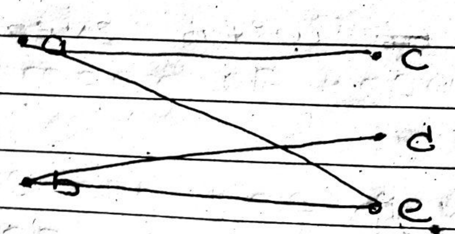
d(B) = 2

d(C) = 2

d(d) = 2

iii) <u>Bipartite</u> <u>graph</u>:-

A Graph (G) is said to be Bipartite graph if its vertex set can be partition into two set as V_1 and V_2 such that no vertex in same partition can be adjacent.



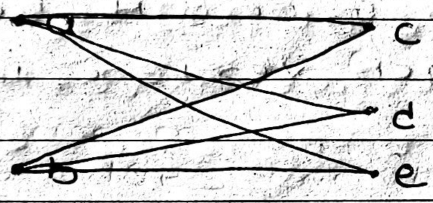
$V_1 = \{a, b\}$

$V_2 = \{c, d, e\}$

iv) <u>Complete</u> <u>Bipartite</u> <u>graph</u>:-

~~It is gra~~

A graph G is said to complete Bipartite graph whose vertex set can be partition into two subset of m and n each vertex of subset m is uniquely connect with each vertex of subset n.



$m = \{a, b\}$
$N = \{c, d, e\}$

v) Path and circuit :-

- In graph a path is a sequence of vertices where consecutive vertices are covered by edges is called path.

- A circuit is a closed path, it start and ends with same vertex.

Path :- A - B - C - D
circuit :- A - B - C - D - A

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Q3 c)

① In Graph G and H both are having same no of vertices and edges 7 vertices and 7 edges.

② In graph G and H both are having each vertex with degree 2

③ In graph G and H both are having minimum circuit as 7.

④ So it is isomorphic.

Q4)
b)

(i) Number of edges :-

Graph has 9 vertices of degree 2, 2, 2, 3, 3, 4, 4, 5

$V = 9$

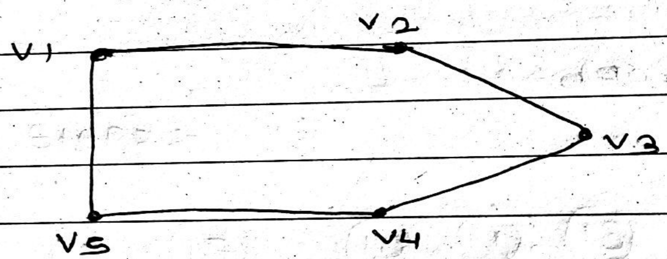
$\therefore$ Total degree $= 2+2+2+3+3+4+4+5$
$= 28$

$e = \frac{\text{Total degree}}{2} = \frac{28}{2} = 14$ edge

May_June 2023

Q3)

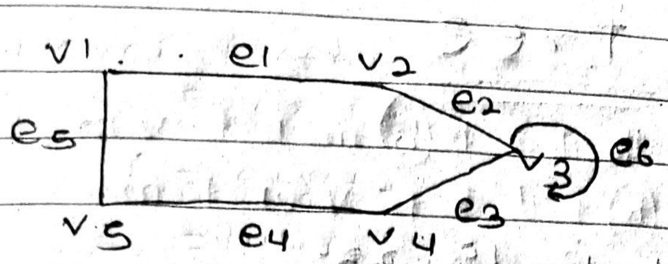
b) In discreate mathematics, an adjacency matrix represent connection between vertices in graph. It's a square matrix where entry A[i][j] is 1 if there is edge between vertices i and j, and 0 otherwise.



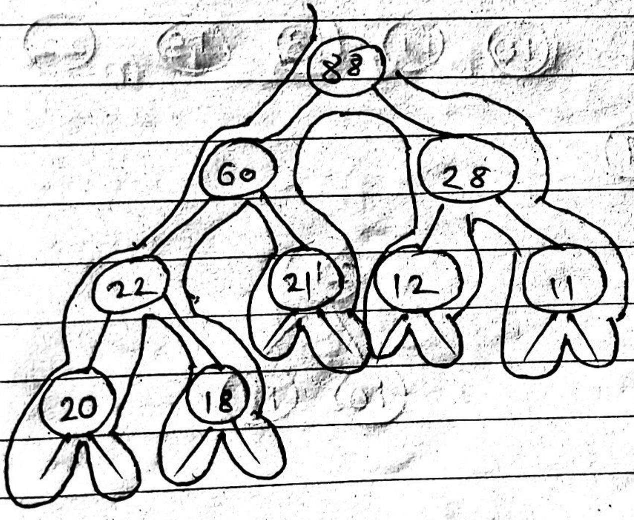
$$
\begin{array}{c|c}
 & \begin{array}{ccccc} V1 & V2 & V3 & V4 & V5 \end{array} \\
\begin{array}{c} V1 \\ V2 \\ V3 \\ V4 \\ V5 \end{array} &
\begin{bmatrix}
0 & 1 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 0
\end{bmatrix}
\end{array}
$$

Incidence matrix :-

→ an incidence matrix describes relationship between vertices and edges.
Entry M[i][j] is 1 if vertex is incident, 2 if it is self loop, 0 otherwise.



	e1	e2	e3	e4	e5	e6
v1	1	0	0	0	1	0
v2	1	1	0	0	0	0
v3	0	1	1	0	0	2
v4	0	0	1	1	0	0
v5	0	0	0	1	1	0

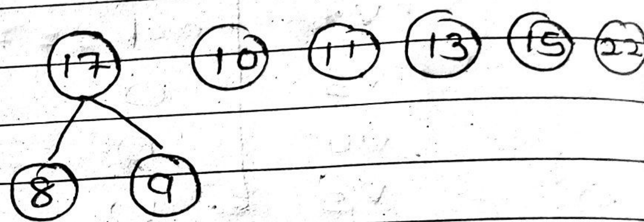


1 Preoader - 88, 60, 22, 20, 18, 21, 28, 12, 11,
2 Inoader - 20, 22, 18, 60, 21, 88, 12, 28, 11
3 postorder - 20, 18, 22, 21, 60, 12, 11, 28, 88

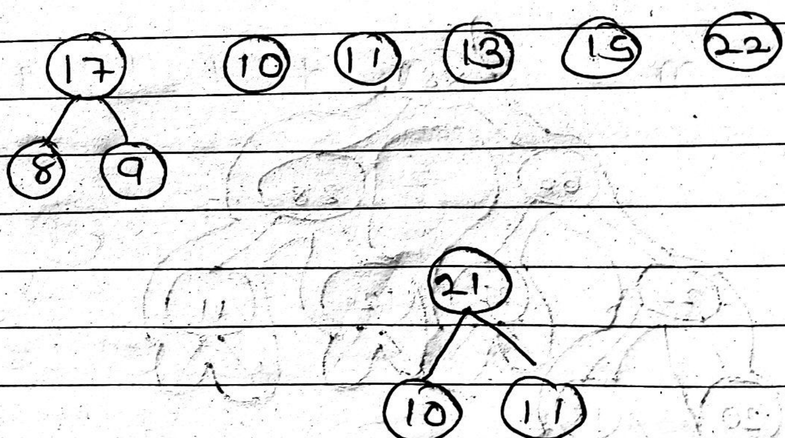
Tree

① 8, 9, 10, 11, 13, 15, 22.

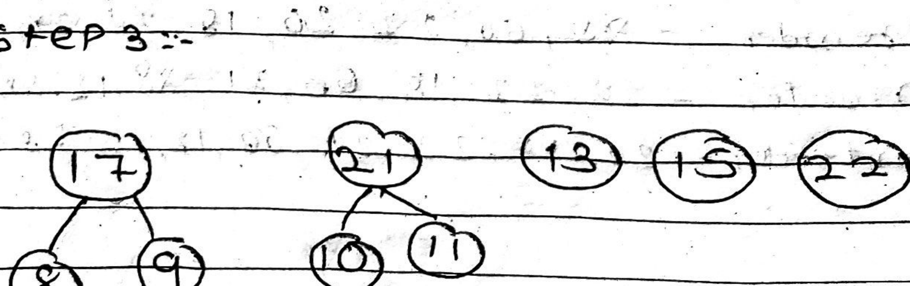
Step 1 :-



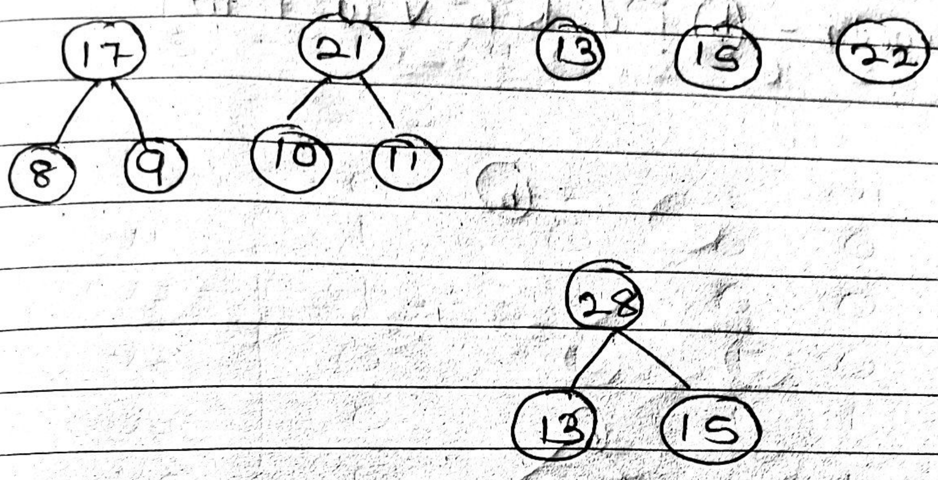
Step 2 :-



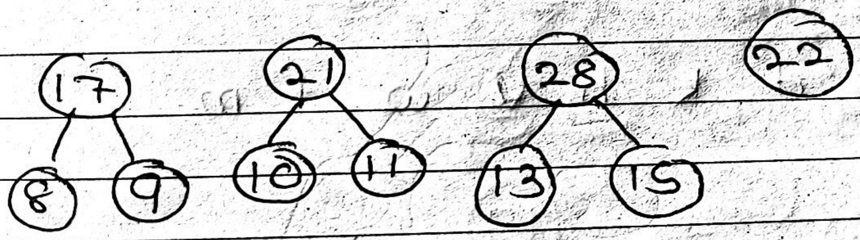
Step 3 :-



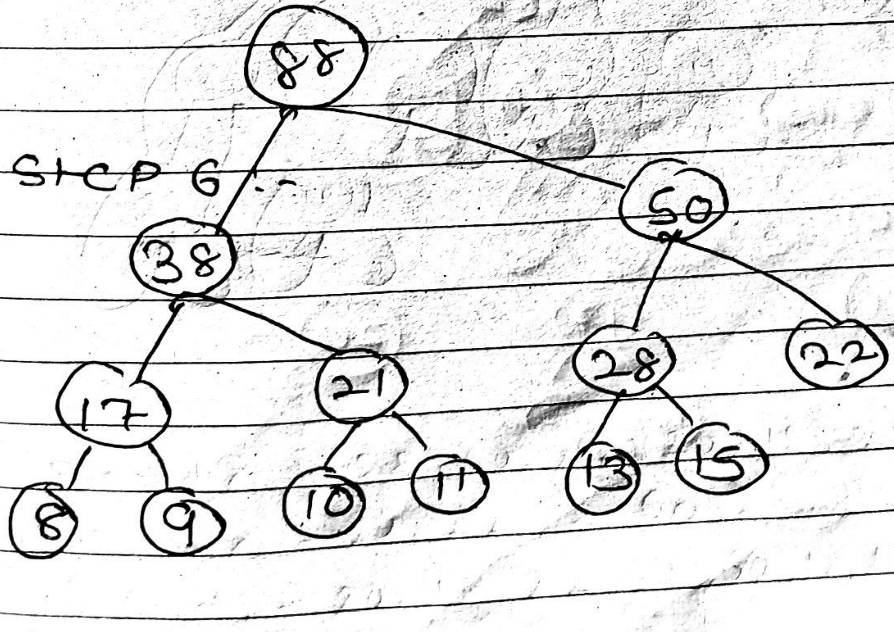
Step 4 :-



Step 5 :-



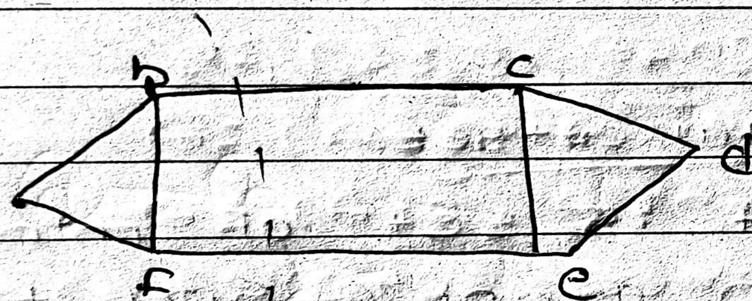
Step 6 :-



1 Cut set:-

- A cut set is set of nodes in graph whose removal disconnect the remaining nodes.

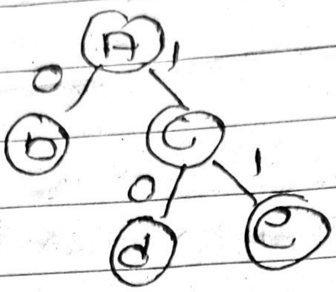
2



2

② Prefix code :-

-



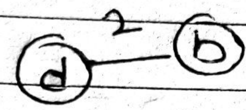
b = 0
d = 10
e = 11

③ Fundomental cutset :-

- cut set is set of minimal edges in graph G such that, when broken, the whole graph breaks into exactly two parts.

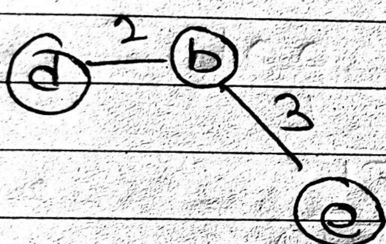
(*) Prism

(1)

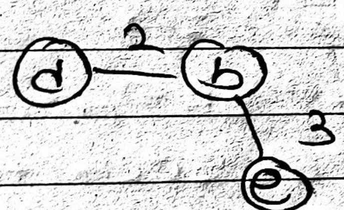


(d,b) & e,4

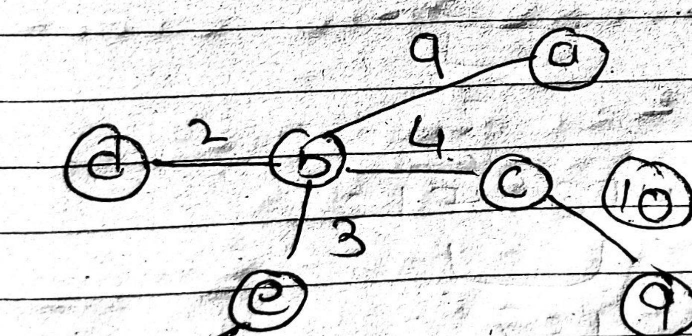
(2)



(3)



(4)



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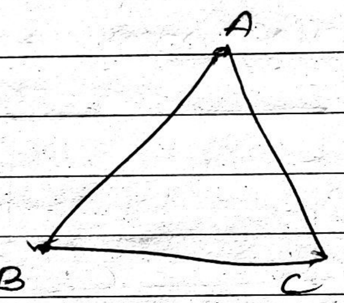
* compute graph

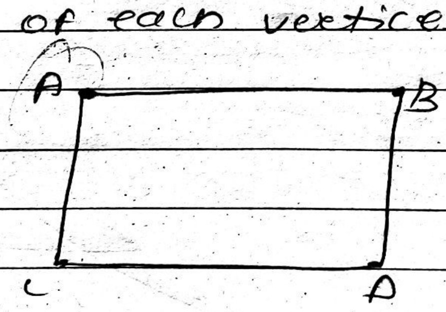
each vertex should be connected to other vertices

A graph G is said to compute graph if every vertex ~~of graph~~ in G is connected with every other vertex



* **Regular graph:-**

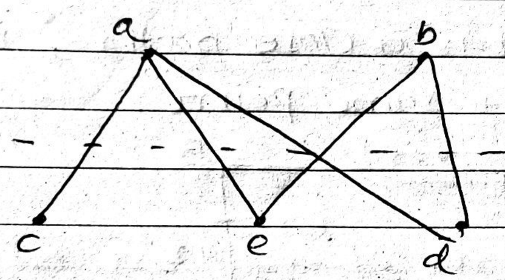
A graph G is said to be regular graph if degree of each vertices is same.



d(A) = 2
d(B) = 2
d(C) = 2
d(D) = 2

* Bipartite graph.

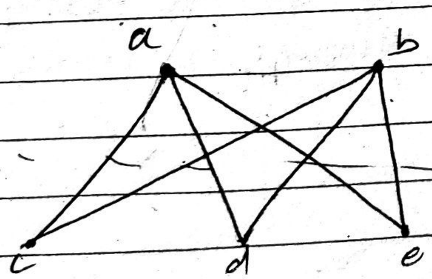
A graph G is said to be bipartite graph if its vertex set can be partition into two sets as V_1 & V_2 such that no vertices in the same pantition can be adjacent.



$V_1 = \{a, b\}$
$V_2 = \{c, e, d\}$

* complete bipartite graph

A graph G is said to be complete bipartite graph whose vertex set can be partition into two subset of m and n such that each vertex of subset m is join to every vertex of subset n by unique edges.

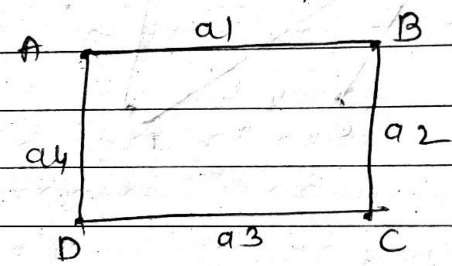


$m = \{ab\}$
$n = \{c, d, e\}$

* path & circuit

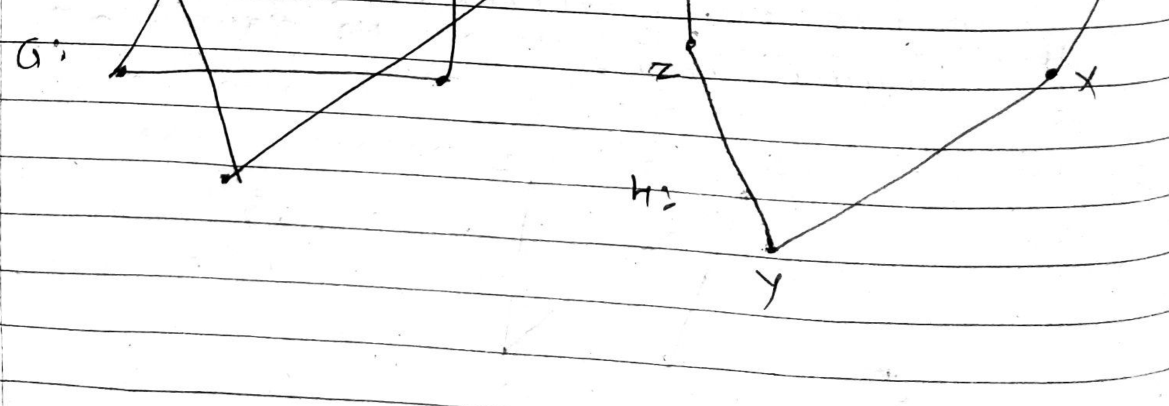
In a graph
Path is sequence of vertices ~~covered by~~ where consecutive vertices are covered by edges.

circuit: It is a close path, it starts and ends at same point.



path = A - B - C - D
circuit :- A - B - C - D - A

* Isomorphic :-



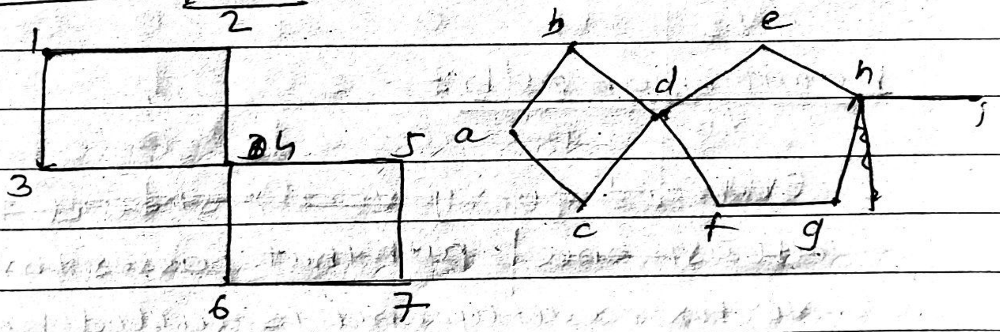
(i) In Graph G and H both are having 7 vertices and 7 edges

(ii) Both graph G and H having each vertex of degree 2.

(iii) In Graph G and H both are having minimum length of circuit as 7.

(iv) Therefore graph G and H are isomorphic to each other.

Eulerian path



A graph is Euler path if and only if there are at most two vertices with {0 or 2} odd degree.

- Graph G_1 has zero vertices with odd degree and G_2 has two vertices h and i with odd degree.

$\therefore$ both G_1 and G_2 are Euler path.

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Eulier circuit:

* A graph is Euler circuit if and only if degree of every vertex is even

∴ Graph G_1 and G_2 both are not Eulier circuit

~~Hamiltonian path~~:-

sum of degree ~~of adjacent pair should be greater than vertices minus one~~ ~~non~~ ~~then~~ it is Hamiltonianpath.

Hamiltonian path:-

~~Each and every vertex should be covered atleast once~~ path that passes through every vertex exactly once is called hamitonian path.

Hamiltonian circuit:-

In graph G_1 we should start and end with same vertex by visiting every vertex exactly at once, then it is Hamiltonian circuit.

4b]

The graph has 9 vertices of degrees
2, 2, 2, 3, 3, 3, 4, 4, 5.

$v = 9$

Total degree $= 2+2+2+3+3+3+4+4+5$
$= 28$

No. of edges $e = \frac{\text{Total degree}}{2} = \frac{28}{2} = 14$

Euler's Formula :-
e = no. of edges
v = no. of vertices

No. of faces $\boxed{f = e - v + 2}$ *
$= 14 - 9 + 2$
$= 7$

$\left[\frac{n(n-1)}{2}\right] \rightarrow$ maximum edges

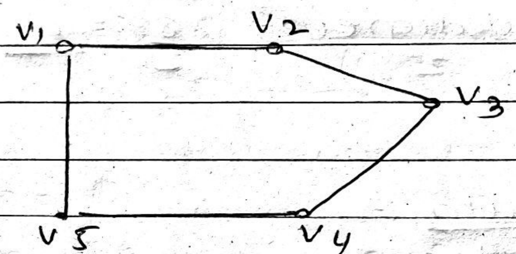
$\boxed{\Sigma(d) = 2m}$ a handshake dem

m → edges

* Adjacency matrix:-

- Adjacency matrix represents connections between vertices in a graph. It's a square matrix where entry $A[i][j]$ is 1, if there is an edge between vertices i and j, and 0 otherewise.

eg-

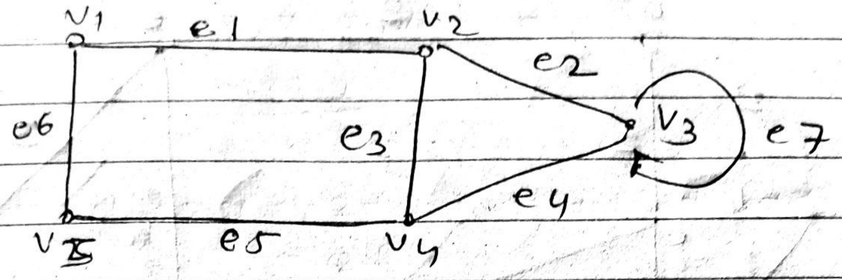


$$
\begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{array}
\begin{array}{c} \begin{array}{ccccc} V_1 & V_2 & V_3 & V_4 & V_5 \end{array} \\
\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}
$$

* Incidence Matrix:

- An incidence matrix describes relationship between vertices and edges.
- Its a rectangular matrix with row corresponding to vertices and column to edges. Entry $m[i][j]$ is 1 if vertex i is incident to edge j, and 0 otherwise and 2 if it has self loop.

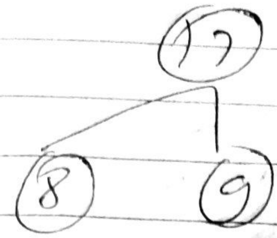
eg-



$v_1, v_2 \ldots v_5 \Rightarrow$ vertices
$e_1, e_2 \ldots e_7 \Rightarrow$ edges.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7
v_1	1	0	0	0	0	1	0
v_2	1	1	1	0	0	0	0
v_3	0	1	0	1	0	0	2
v_4	0	0	1	1	1	0	0
v_5	0	0	0	0	1	1	0

* Halfman Greedy tree
8, 9, 10, 11, 13, 15, 22

1


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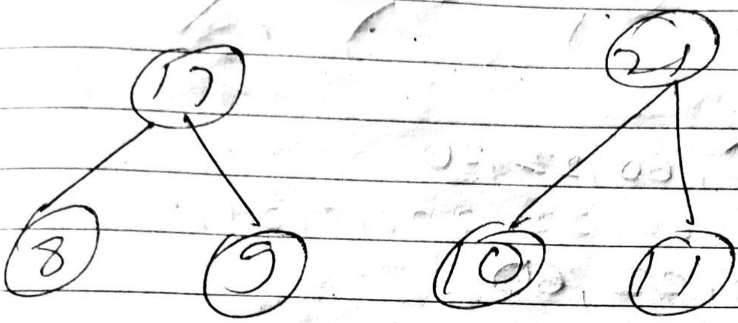

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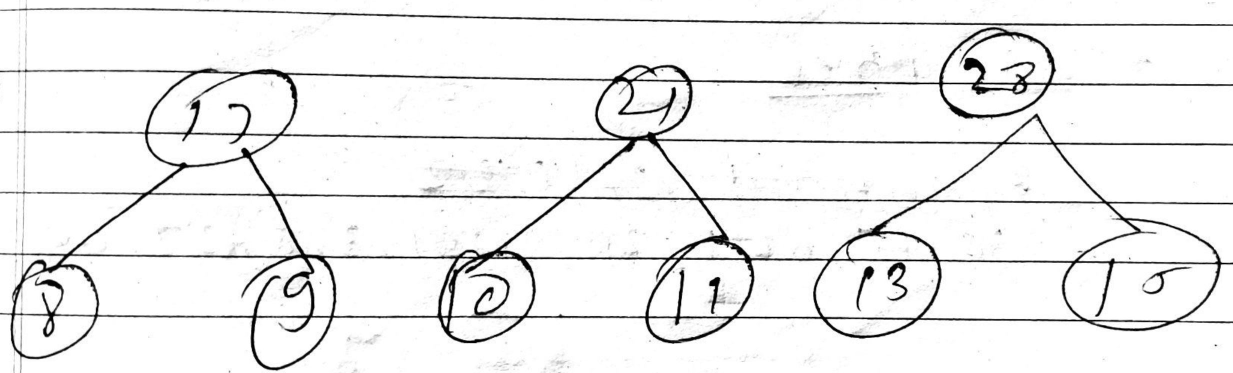

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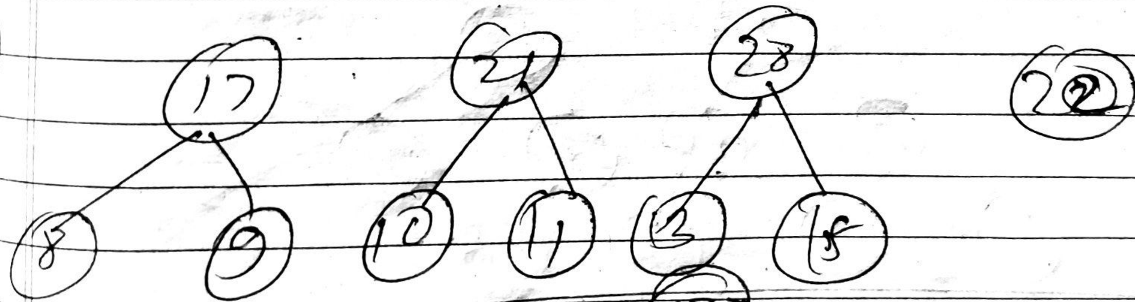

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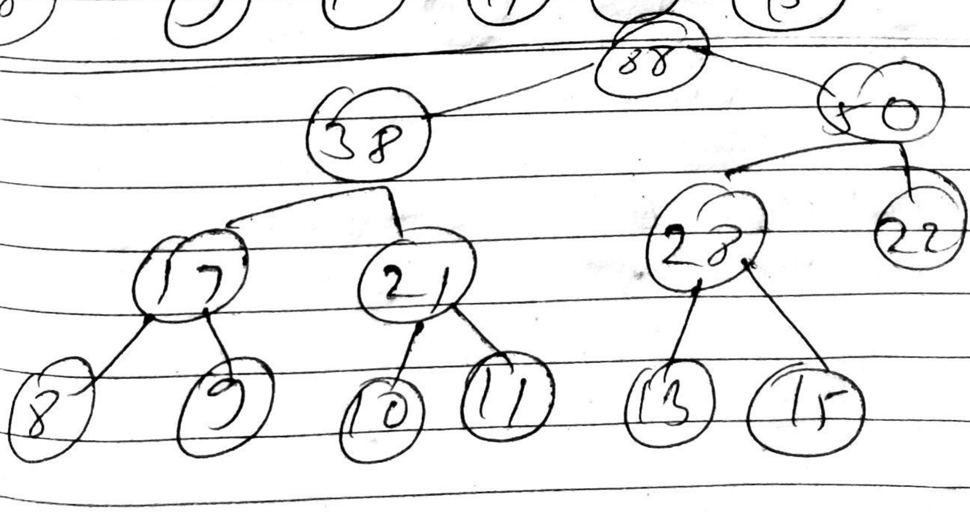
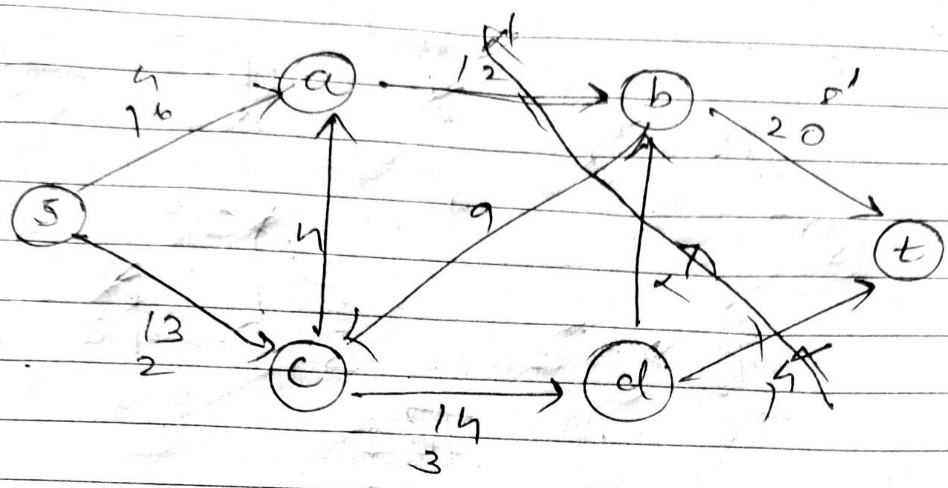


* Maximum & minimum cut



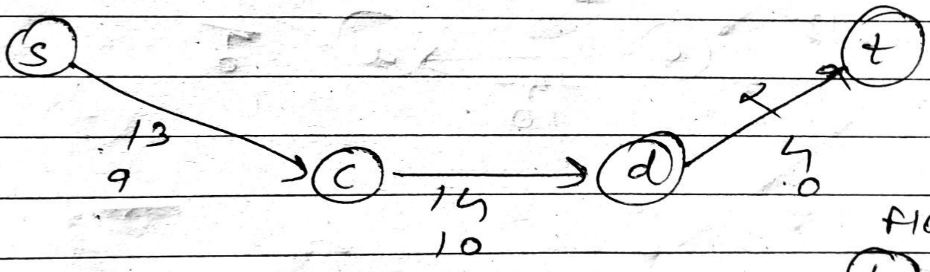
(1)

$s \xrightarrow{16} a \xrightarrow{12} b \xrightarrow{20} t$

FLOW ⇒ 12

$s \xrightarrow{4} a \xrightarrow{0} b \xrightarrow{8} t$

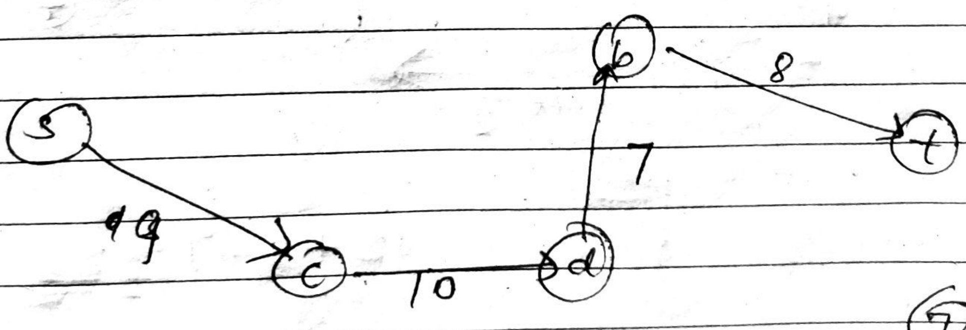
(2)



FLOW ⇒ 4

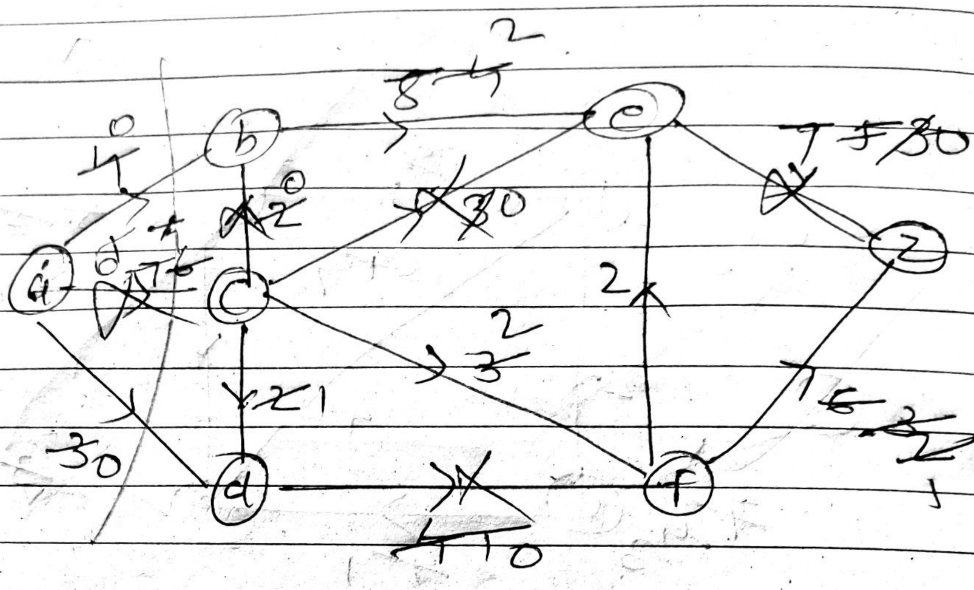
$s \xrightarrow{9} c \xrightarrow{10} d \xrightarrow{0} t$

(3)

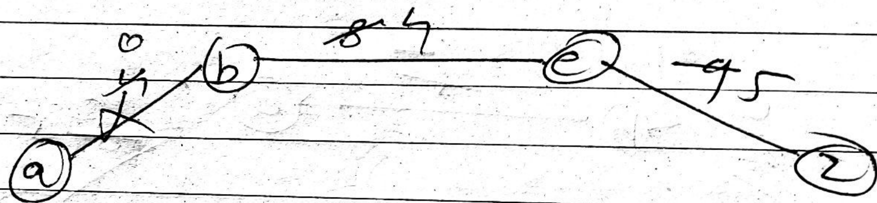


FLOW ⇒ 7

$s \xrightarrow{2} c \xrightarrow{3} d \xrightarrow{0} b \xrightarrow{1} t$

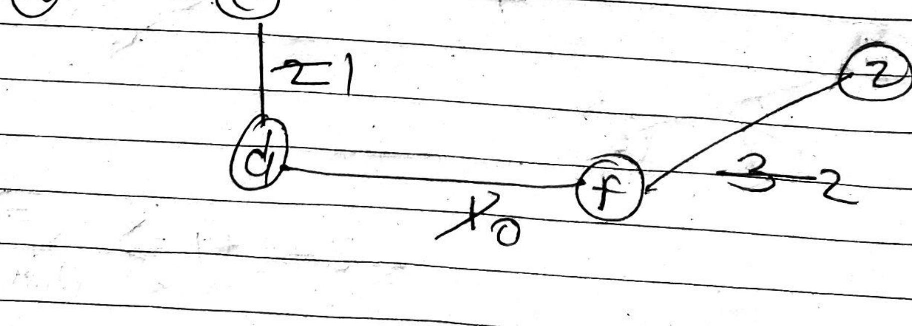


5



3

7



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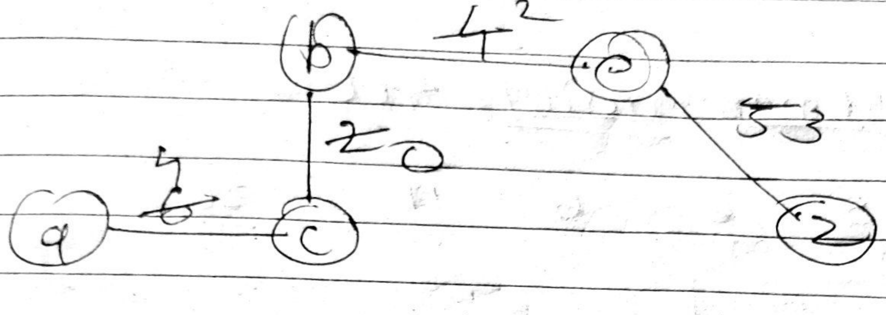

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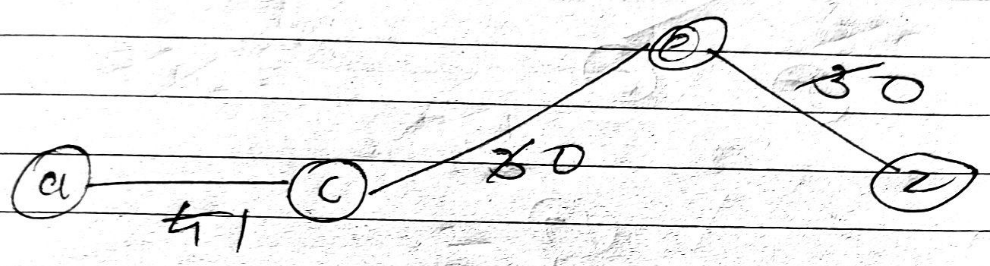
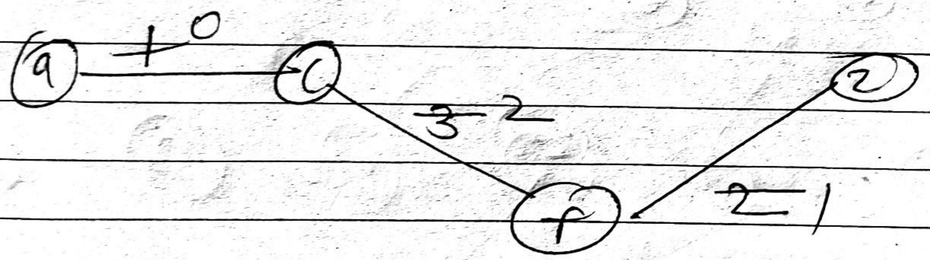


1



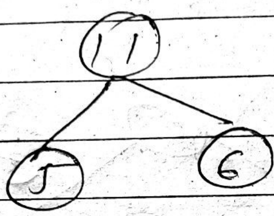
$4+3+1+2+3+1 = 14$

maximum cut

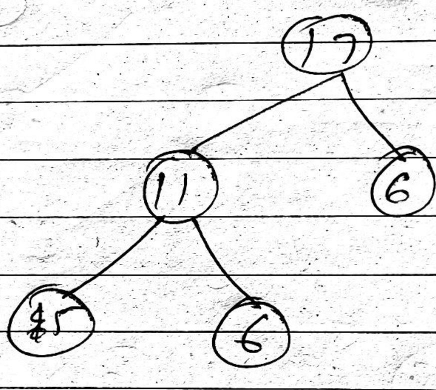
Halfman greedy tree

5 6 6 11 20

(1)



(2)



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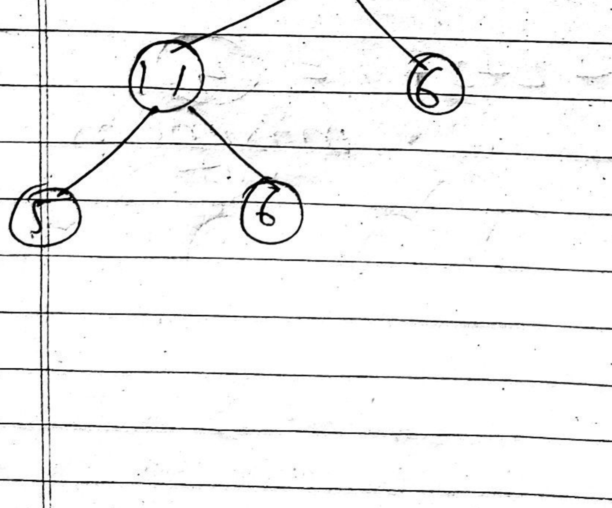
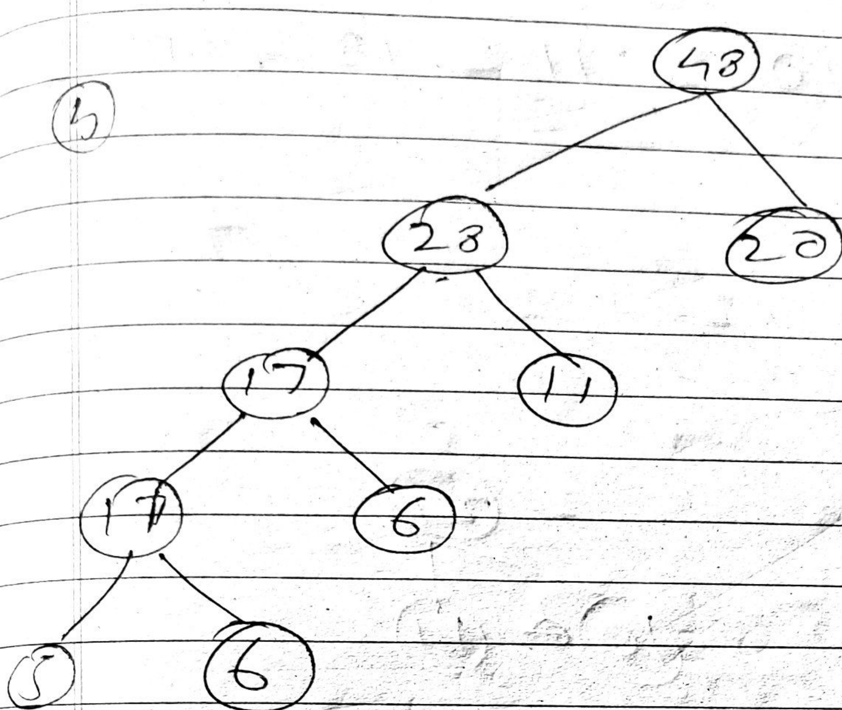


(b)



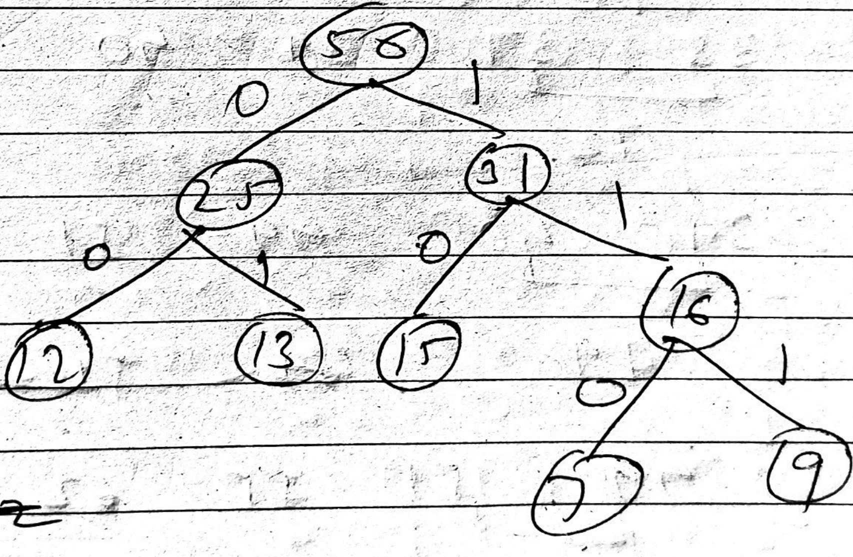
12 15 7 13 9

$\underbrace{7 \quad 9}_{16}$ 12 13 15

$\underbrace{12 \quad 13}_{25}$ 15 16

$\underbrace{15 \quad 16}_{31}$ 25

$\underbrace{25 \quad 31}_{56}$



~~1 22~~

12 = 00

13 = 01

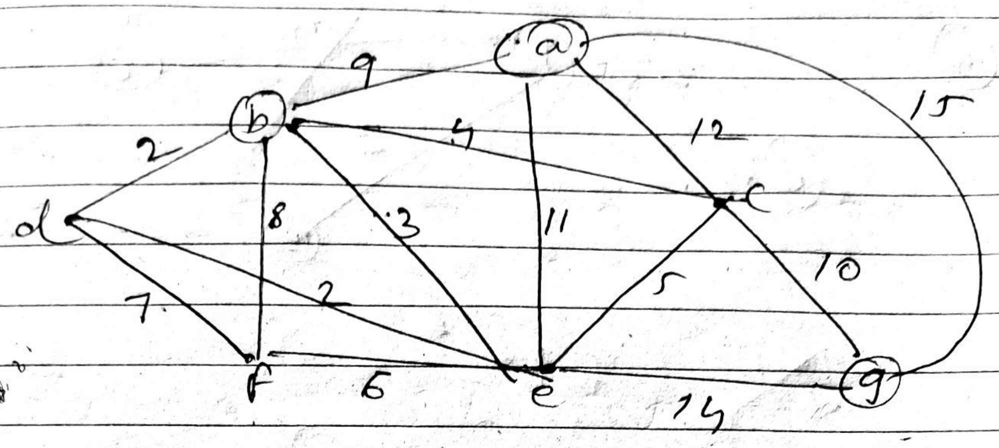
15 = 10

~~16 = 11~~

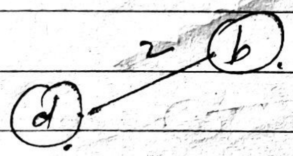
7 = 110

$\boxed{\text{prefix code}}$

Prim's Algorithm



(1)



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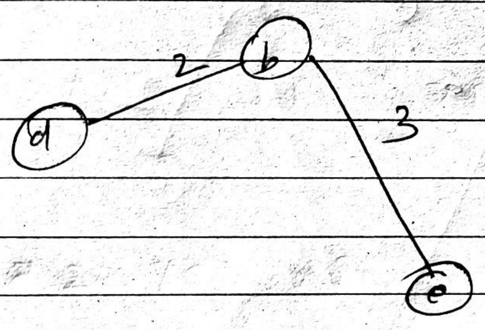

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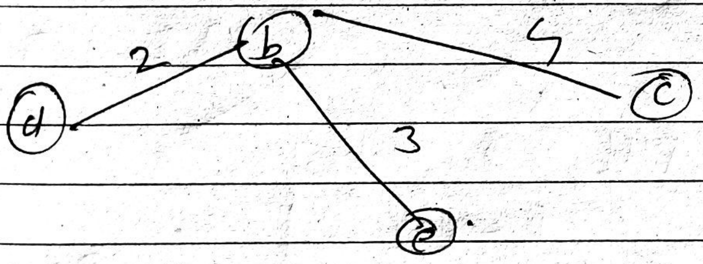

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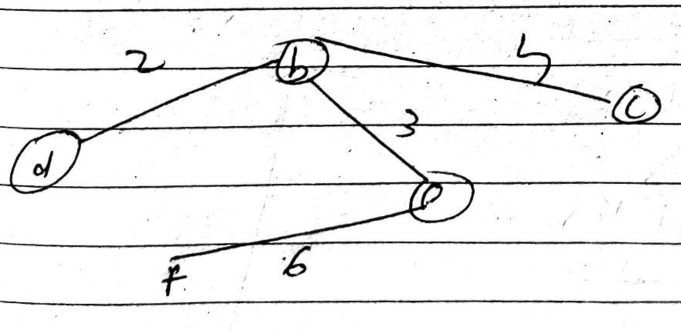

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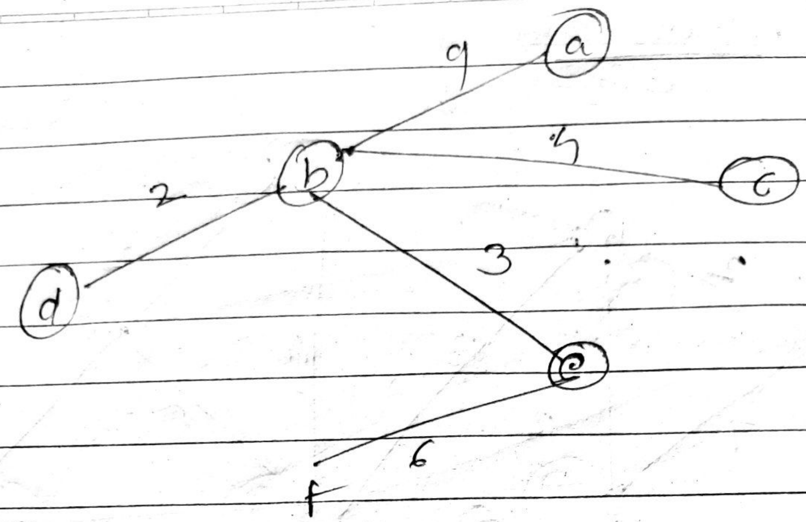

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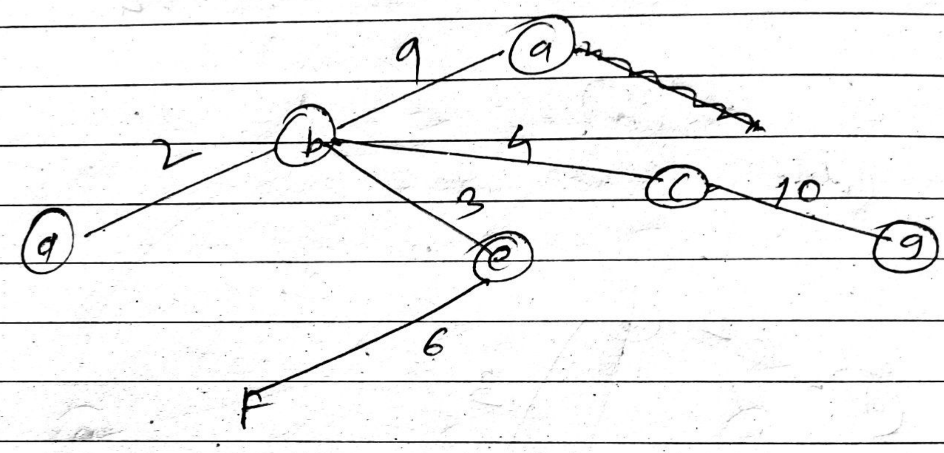
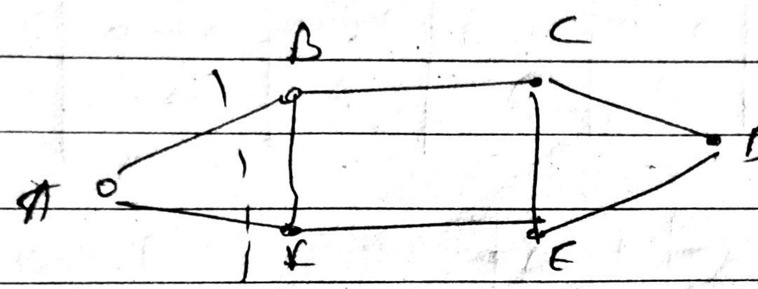
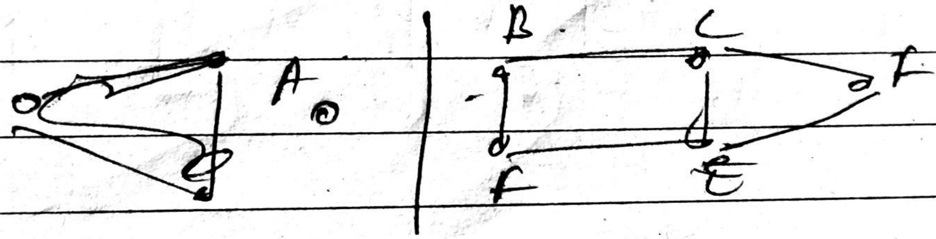


$2 + 9 + 4 + 3 + 6 + 10$

$\Rightarrow \underline{\underline{34}}$

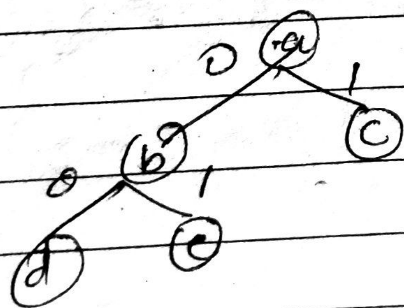
* A cutset is set of nodes in graph whose removal disconnect remaining nodes





* Prefix code :-

sequence of bit assign to characters in binary tree assign 0 bit to left sub tree and one bit to right sub tree.



d ⇒ 00

e = 01

c = 1

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* Binary Search tree :-

BST is a binary tree which has the following properties.

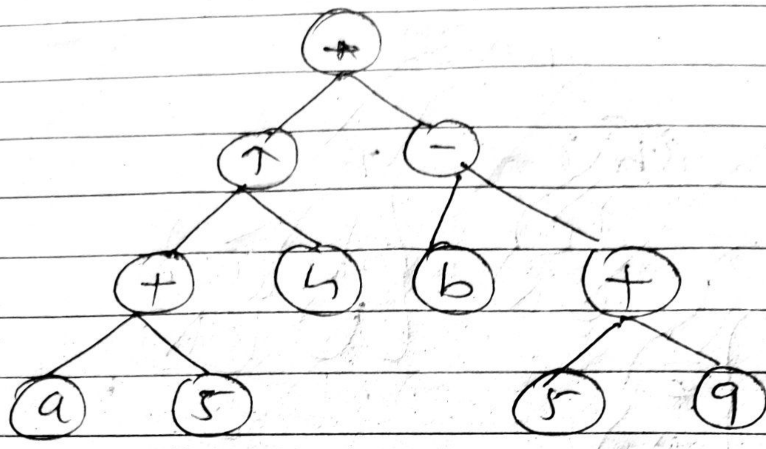
(i) Each node has a value

(ii) A total order is defined on these values

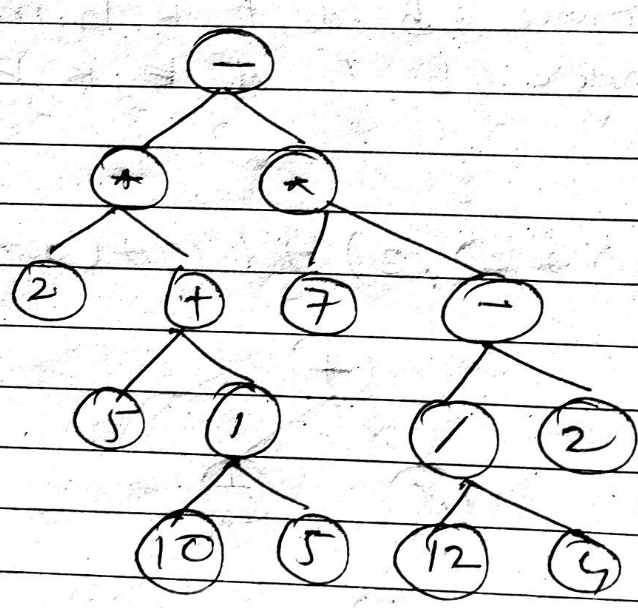
(iii) The left subtree of a node contains only values less than the node values

(iv) The right subtree of a node contains only values greater than or equal to the nodes values

$((a+5)\uparrow 4) * (b-(5+9))$



$((a+5)\uparrow 4) * (b-(5+9))$



$(((10/5)+5)*2) - (7*((12/4)-2))$

$((2+5)*2) - (7*(3-2))$

$14 - (7)$

$\underline{\underline{7}}$

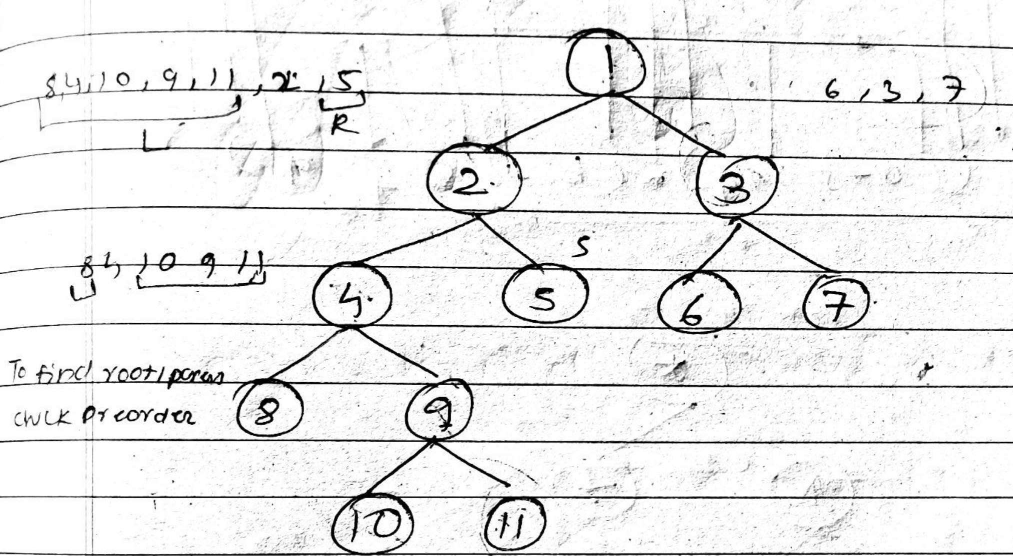
Binary tree

scan left to right

Preorder :- 1 2 4 8 9 10 11 5 3 6 7

Inorder : 8 4 10 9 11 2 5 1 6 3 7

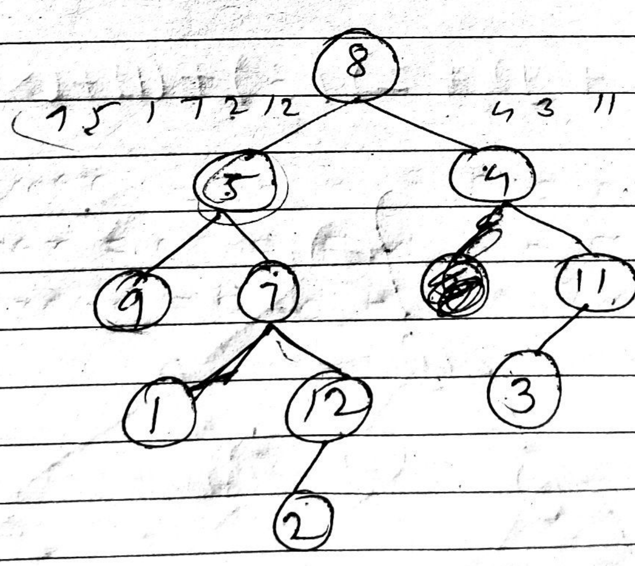
(i) find root from preorder



scan right left

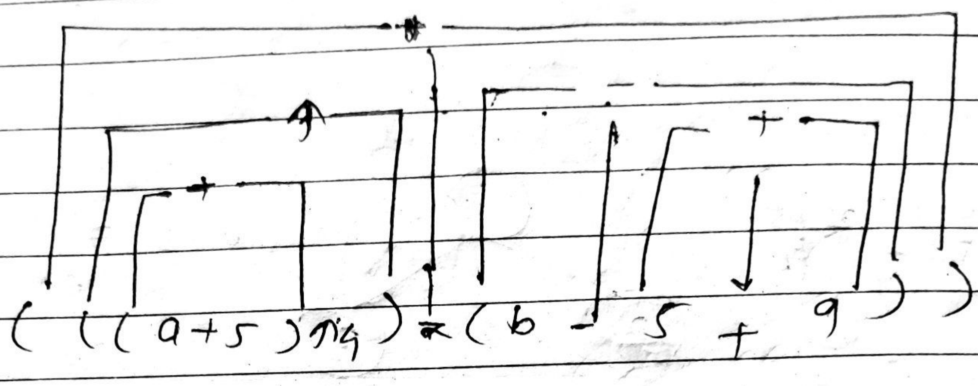
LR Root postorder :- 9 1 2 12 7 5 3 11 4 8 root

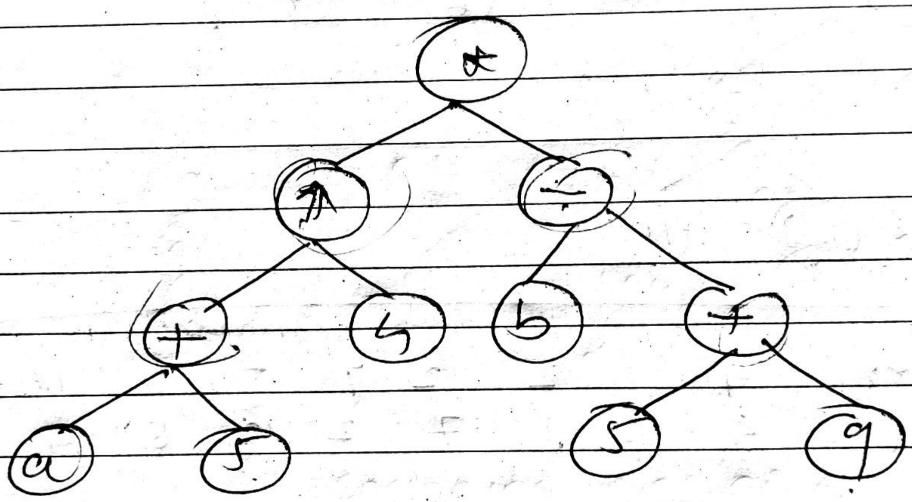
L Root R Inorder :- 9 5 1 7 2 12 8 4 3 11



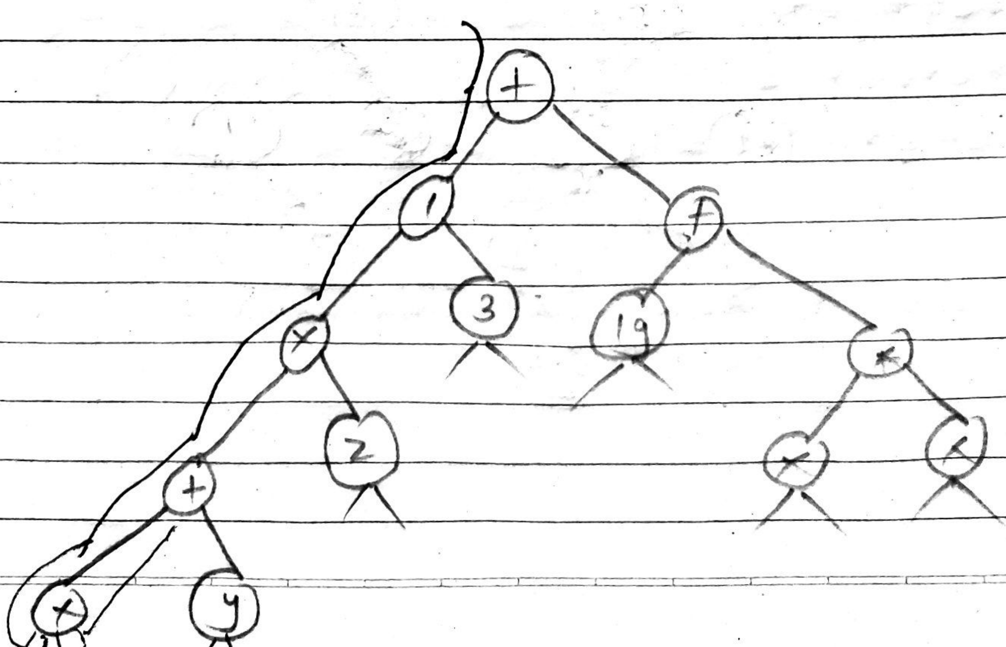
represent expression using binary tree

* $((a+5) \uparrow 4) * (b - (5+9))$

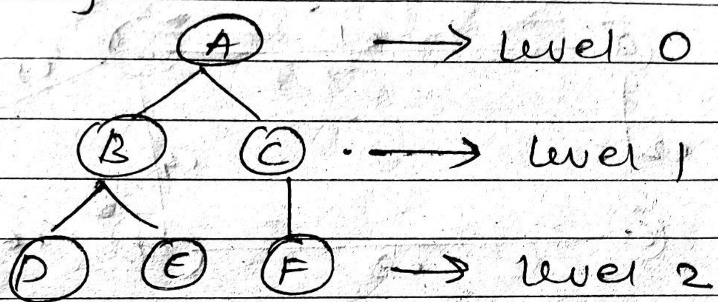




* $(((x+y) * 2) / 3) + (19 + (x * x))$



* Level of trees :-
- Level of tree is represented by no. of stages from root node to leaf node
- On each node there are some nodes available.
- ~~Generally~~



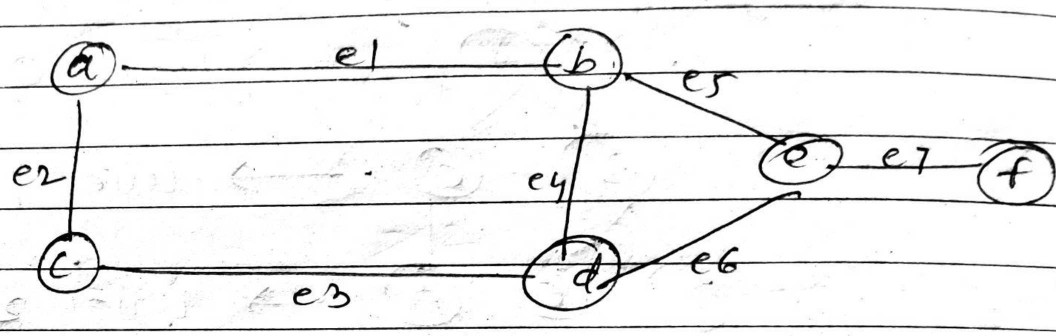
* Depth of node :-
- Depth of node is no. of edges from node to root node of the tree
- We find depth of all leaf nodes

* Height of trees :-
- height of tree is the no. of edges ~~from~~ on the longest downward path betⁿ the root node and leaf node.

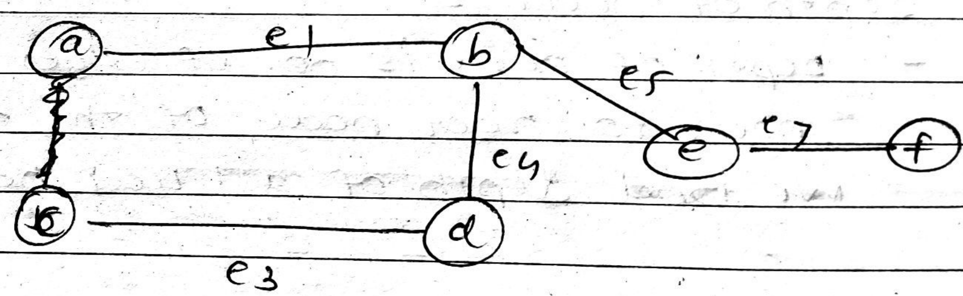
* Forest :-
- It is a collection of disjoint trees.
- forest is a collection of acyclic graph which is not connected.

NO cycle

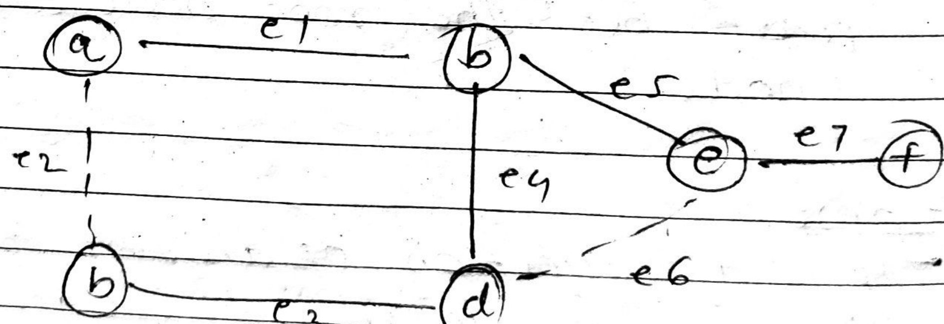
- Fundamental circuit :-
  - when a circuit is formed by adding chord to spanning tree i.e. $(T + e)$, then it is called as fundamental circuit.



Graph G

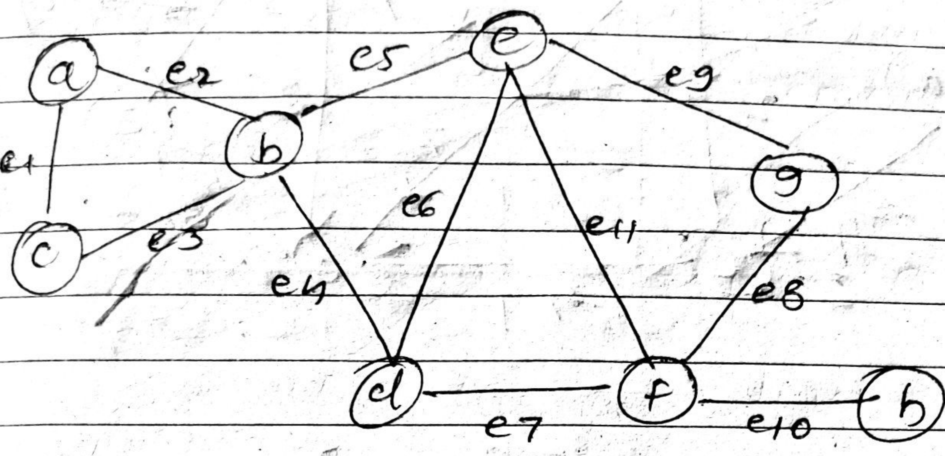


Spanning tree of G

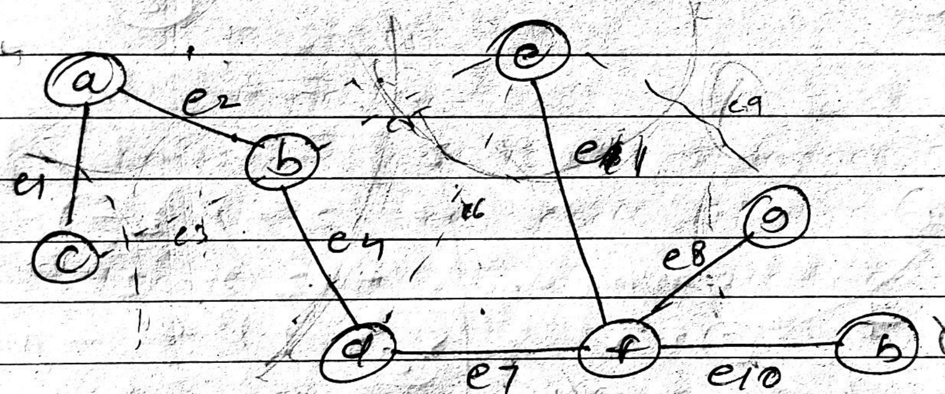


spanning tree with fundamental circuit.

* find fundametal cutset



Graph g



spanning tree of graph G

Branches	F cutsets
e1	{e1, e3}
e2	{e2, e3}
e4	{e4, e5}
e7	{e7, e5, e6}
e10	{e10}
e8	{e8, e9}
e11	{e11, e9, e6, e5}

{1 branch & multiple chords}

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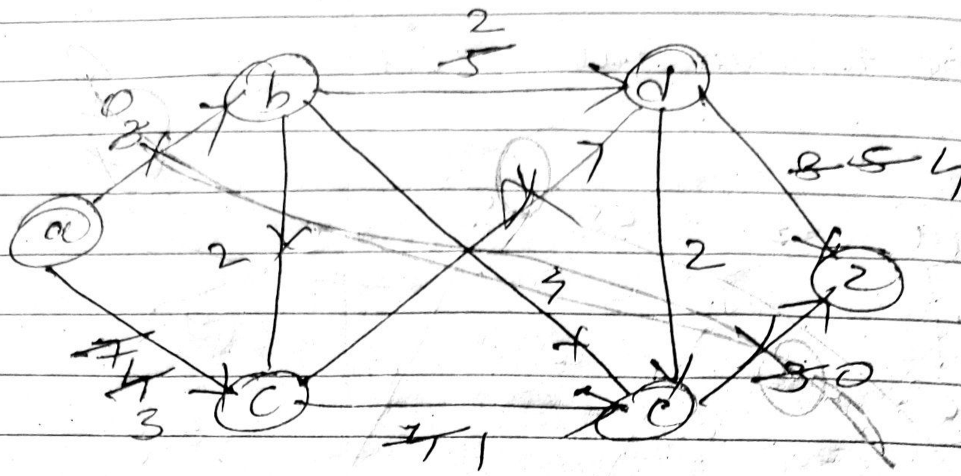
* Maximum & minimum cut



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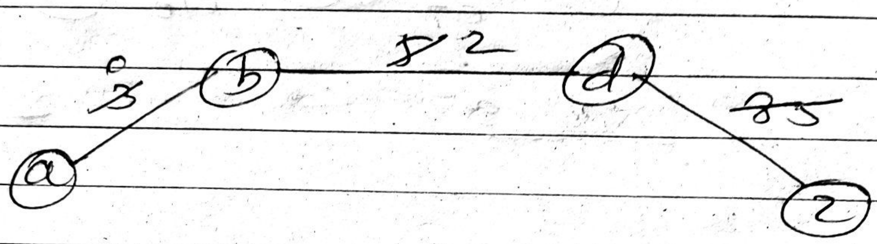

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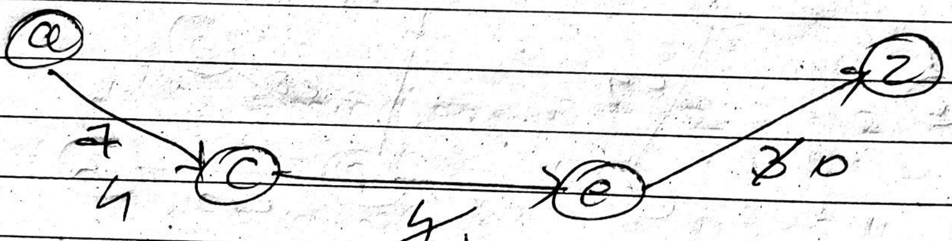
(2)



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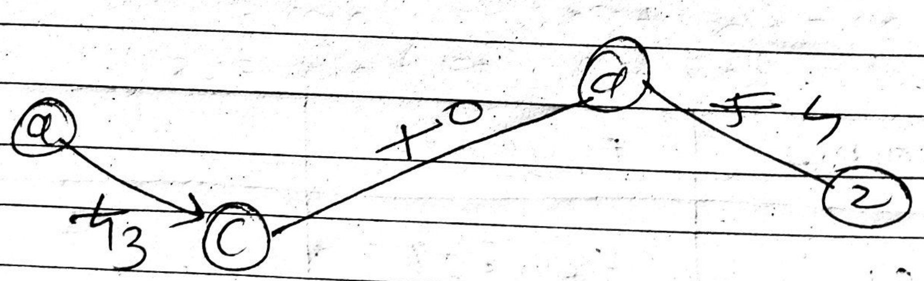
(3)



(1)

maximum flow = 3 + 3 + 1
= 7

~~Minim~~ ~~Minimum cut~~ = 7

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* Properties of binary operation

① Closure law } Algebraic structure
② Associative law } Semi groups } monoids } groups } Abelian groups
③ Indentity law
④ Inverse law
⑤ Commutative law

① Closure property :-

$\boxed{\{\forall a, b \in A \quad a * b \in A}$ A - no empty set

$\forall$ all

eg. $\boxed{2+1=3}$

② Associative property :-

$\boxed{(a * b) * c = a * (b * c)}$

eg. $(2 * 1) * 3 = 2 * (1 * 3)$

$\boxed{6 = 6}$

③ Identity property :-

$\boxed{a * e = e * a = a}$

eg. $3 \times 1 = 1 \times 3 = 3$

④ Inverse property :-

$a * a^{-1} = e$

⑤ Commutative property :-

$a * b = b * a$

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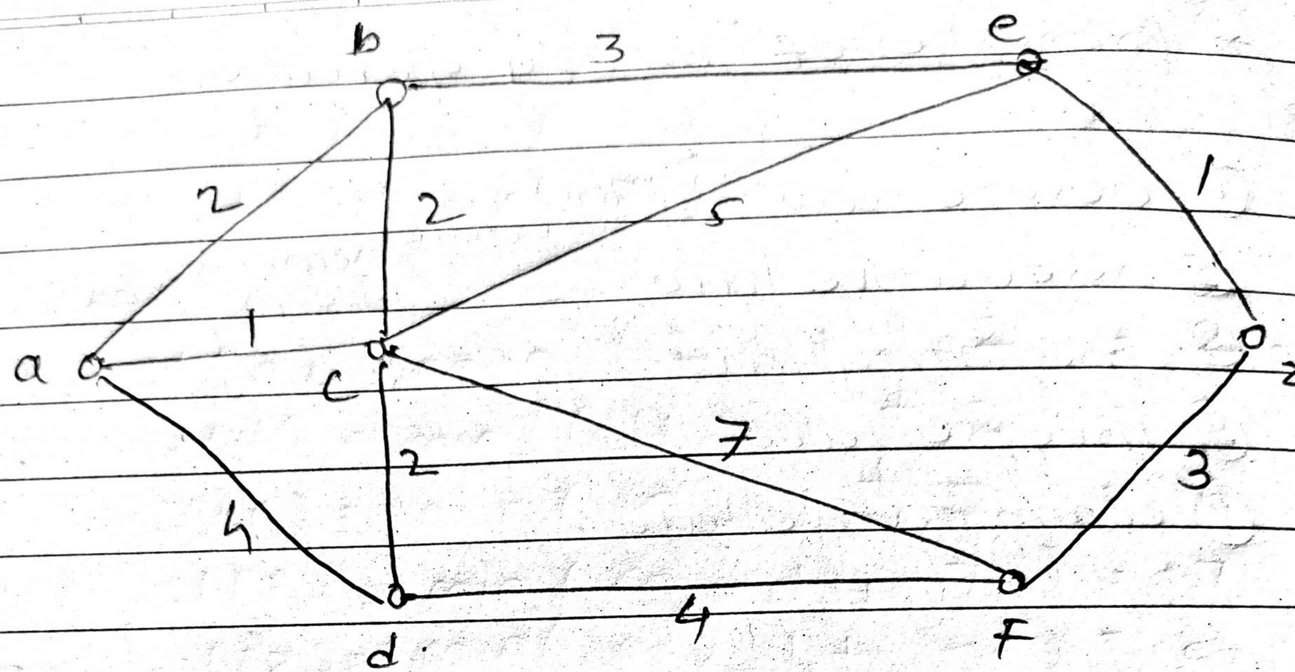
(1)



1] $P = \{\phi\}$
$T = \{a, b, c, d, e, f, z\}$

2] $P = \{a\}$
$T = \{b, c, d, e, F, z\}$

$l(b) = \min(\infty, 0 + 2) = 2$
$l(c) = \min(\infty, 0 + 1) = 1$ — min
$l(d) = \min(\infty, 0 + 4) = 4$
$l(e) = \min(\infty, 0 + \infty) = \infty$
$l(F) = \min(\infty, 0 + \infty) = \infty$
$l(z) = \min(\infty, 0 + \infty) = \infty$

3] $P = \{a, b\}$

3] $P = \{a, c\}$
$T = \{b, d, e, f, z\}$

$l(b) = \min(2, 1+2) = 2$ - min
$l(d) = \min(4, 1+2) = 3$
$l(e) = \min(\infty, 1+5) = 6$
$l(f) = \min(\infty, 1+7) = 7$
$l(z) = \min(\infty, 1+\infty) = \infty$

4] $P = \{a, c, b\}$
$T = \{d, e, f, z\}$

$l(d) = \min(3, 2+\infty) = 3$ - min
$l(e) = \min(6, 2+3) = 5$
$l(f) = \min(7, 2+\infty) = 7$
$l(z) = \min(\infty, 2+\infty) = \infty$

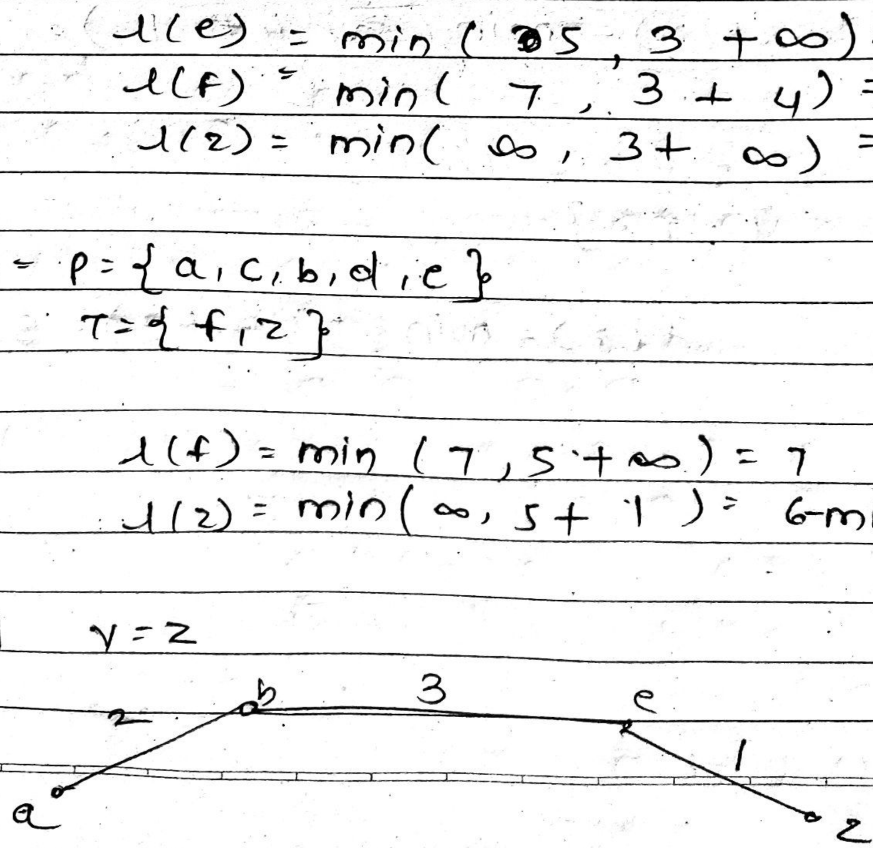
5] $P = \{a, c, b, d\}$
$T = \{e, f, z\}$

$l(e) = \min(5, 3+\infty) = 5$ - min
$l(f) = \min(7, 3+4) = 7$
$l(z) = \min(\infty, 3+\infty) = \infty$

6] $P = \{a, c, b, d, e\}$
$T = \{f, z\}$

$l(f) = \min(7, 5+\infty) = 7$
$l(z) = \min(\infty, 5+1) = 6$-min

7] $v = z$



Abelian group

① Let G be set of all non-zero elements

→ Given

(i) $(a * b) = ab/2 \in G$ as $ab \neq 0$

$\therefore$ It satisfies closure property.

LHS=RHS (ii) Associative property.

$$(a*b) = \frac{ab}{2}$$

$$(a*b)*c = a*(b*c)$$

LHS

$(a*b)*c$

$= \frac{ab}{2} * c$

$= \frac{abc}{4}$

RHS

$a*(b*c)$

$\Rightarrow a * \frac{bc}{2}$

$\Rightarrow \frac{abc}{2\times 2} \Rightarrow \frac{abc}{4}$

$\therefore$ $*$ is associative in G.

find identity element (iii) Identity property

$a \times e = a$ $\quad$ $e \times a = a$

$\frac{ae}{2} = a$ $\quad$ $\frac{ea}{2} = a$

$e = 2$ $\quad$ $e = 2$

$\therefore$ 2 is identity element in G

fraction (iii) Inverse property

$$a * a^{-1} = e$$

$$a * a^{-1} = \frac{aa^{-1}}{2} = e$$

$$2 = \frac{aa^{-1}}{2}$$

$$aa^{-1} = 4$$

$$\boxed{a^{-1} = \frac{4}{a}}$$

$\therefore$ Inverse of a is $\frac{4}{a}$, $\forall a \in G$.

(iv) Commutative property:-

$$a * b = b * a$$

$$\frac{ab}{2} = \frac{ba}{2}$$

$\therefore$ LHS = RHS

$\therefore$ It satisfies commutative property.

Thus, $(G, *)$ is an abelian group.

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Ring

The structure $(R, +, \times)$ consisting of non-void set R and two binary composition, denoted by + and × or. Is said to be ring. if the following axioms satisfies:-

(i) $(R, +)$ is an abelian group with identity 0.

(ii) $(R, \cdot)$ is semigroup means it

(iii)

* Types of Ring.

1) Commutative Ring
- $(R, +, \cdot)$
- $a \cdot b = b \cdot a$

ii) Ring with unity.
- $(R, +, \cdot)$
- if it has multiplicative identity
- $\therefore\ a \cdot e = a = e a$
- $\boxed{e = 1}$

iii) Ring with zero divisor
- ~~$a \cdot b \neq 0$~~ $a \cdot b = 0$
- if $a \neq 0$ & $b \neq 0$
- $5 *_3 6 = \frac{5 \times 6}{3} = \frac{30}{3} = 10$

Remider. = 0

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iv) Ring without zero divisor.

$a \cdot b \neq 0$

$a \neq 0$, $b \neq 0$

* Integral Domain
- $R(+, \cdot)$
-

i) Ring → commutative Ring

$a \cdot b = b \cdot a$

ii) Ring → nonzero Divisor Ring w.r.t • operator

$a \cdot b \neq 0$

$a \neq 0$ $b \neq 0$

iii) Ring → should be Distributive w.r.t (+)

- $a * (b+c) = a*b + a*c$
- $(b+c)*a = b*a + c*a$

* Field
- $R(+, \cdot)$
-

(i) $(\cdot)$ operator should be commutative Ring

- $a \cdot b = b \cdot a$

(ii) $(\cdot)$ operator should be Distributive over addition (+)

- $a*(b+c) = a*b + a*c$
- $(b+c)*a$ $b*a + c*a$

(iii) Non zero Divisor Ring w.r.t $(\cdot)$ operator $a \cdot b \neq 0$

$a \neq 0$ $b \neq 0$

division Ring = multiplication Inverse

(iv) Non zero element they should have multiplicative inverse.

Field = Integral Domain + divison Ring

* Define

1] Field :-
A commutative ring with unity in which every non zero element possesses their multiplicative inverse, is called as field
A field is an integral domain.
eg. - $(R, +, \cdot)$, $(Q, +, \cdot)$ are fields
- $(Z, +, \cdot)$ is integral domain but not field.

2] Ring :-
The structure $(R, +, \cdot)$ consisting of non-empty set R and two binary composition denoted by + and x or . is said to be ring if the following axioms satisfies :-
(i) $(R, +)$ is an abelian group
(ii) $(R, \cdot)$ is a semigroup
(iii) For any 3 elements a, b, c ∈ R the left distributive law $a \cdot (b + c) = a \cdot b + a \cdot c$
and the right distributive property $(b + c) \cdot a = ab + c \cdot a$ holds.

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3] Ring Homomorphism :-

- Let $(R, +, *)$ and $(S, +, *)$ be two rings.
- A function $\phi: R \to S$ is called a ring homomorphism.

4] Integral Domain :-
- A commutative ring with zero divisors is called as integral domain

eg.
1. $(R, +, \cdot)$ $(Z, +, \cdot)$ are integral domains.
2. $(Z_4, +, \cdot)$ is ring with zero divisors, $\therefore$ It is not integrate domain.

5] Ring with Unity :-
- A ring $(R, +, \cdot)$ is called as ring with unity if $\forall a \in R$, $\exists$ $1 \in R$, such that $a \cdot 1 = 1 \cdot a = a$

eg.
$(Z, +, \cdot)$ is commutative ring with unity.

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COMBINATION

(1) $^{10}C_3$

$$^nC_r = \frac{n!}{r!(n-r)!}$$

$$= \frac{10!}{3!(7)!}$$

$$= \frac{10 \times \cancel{9}^{3} \times \cancel{8}^{4} \times \cancel{7!}}{\cancel{3} \times \cancel{2} \times 1 \times \cancel{7!}}$$

$$= \underline{120}$$

$^{12}C_4$

$$^nC_r = \frac{n!}{r!(n-r)!}$$

$$= \frac{12!}{4!(8!)}$$

$$= \frac{\cancel{12} \times 11 \times \cancel{10}^{5} \times 9}{\cancel{2} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$$

$$= 55 \times 9$$

$$= \underline{495}$$

<u>59,400</u> ways

Q. A commitee including 3 boys and 4 girls is to be formed form a group of 10 boys and 12 girls. How many different commitees can be formed from the group.

Q. committe of 5 people is to be formed from group of 4 men and 7 women. How many possible committees can be formed, if atleast 3 women are on committee.

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(2) 5 people committee

(i) 3 women selected in $^7C_3$ ways
2 men selected in $^4C_2$ ways

Total no. of ways $= {}^7C_3 \times {}^4C_2$

$= \frac{7 \times \cancel{6}^{2} \times 5}{\cancel{3} \times \cancel{2} \times 1} \times \frac{\cancel{4}^{2} \times 3}{\cancel{2} \times 1}$

$= 35 \times 6$

$= \underline{\underline{210 \text{ ways}}}$

(ii) 4 women selected in $^7C_4$ ways
1 men selected in $^4C_1$ ways

Total no. of ways $= {}^7C_4 \times {}^4C_1$

$= \frac{7 \times \cancel{6} \times 5 \times \cancel{4}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1} \times \frac{4}{1}$

$= 35 \times 4$

$= 140$ ways

(iii) 5 women selected in $^7C_5$ ways
0 men selected in $^4C_0$ ways

Total no. of ways $= \frac{7 \times 6 \times \cancel{5} \times \cancel{4} \times \cancel{3}}{1 \times 2 \times \cancel{3} \times \cancel{4} \times \cancel{5}} \times 1$

$= 21$ ways

committe formed $= 210 + 140 + 21$

$= \underline{371}$ ways

* In a certain country, the car number plate is formed by 4 digits from the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 followed by 3 letters from the alphabet. How many number plates can be formed if neither the digits nor the letters are repeated.

⇒ letter

26 25 24

$26 \times 25 \times 24$

digits

9 8 7 6

$9 \times 8 \times 7 \times 6$

$\Rightarrow 26 \times 25 \times 24 \times 9 \times 8 \times 7 \times 6$

$= 47,17,44,00.$

* How many letters with or without meaning, can be formed out of the letters of word 'LOGARITHMS' if repetition of letters is not allowed)

⇒ 10 9 8 7

$\Rightarrow 10 \times 9 \times 8 \times 7.$

$\Rightarrow 5040$

* How many 6 digits odd numbers greater than 6,00,000 can be formed from the digits 5, 6, 7, 8, 9 and 0

(i) if repetition is allowed.

4	6	6	6	6	3

$4 \times 6 \times 6 \times 6 \times 6 \times 3$

$= \underline{15,552}$

(ii) if repetition is not allowed

4	4	3	2	1	3

$\Rightarrow 4 \times 4 \times 3 \times 2 \times 1 \times 3$

$\Rightarrow \underline{288}$

PROBABILITY THEORY

* A box contains 4 red, 3 white and 2 blue balls. Three balls are drawn at random. Find out number of ways of selecting the ball of different colors.

⇒

~~Case 1~~ :

$$^4C_1 \times {}^3C_1 \times {}^2C_1$$

$$4 \times 3 \times 2$$

$$\underline{\underline{24}}$$

Sample space :-

$$^9C_3 = \frac{\cancel{9}^{3} \times \cancel{8}^{4} \times 7}{1 \times \cancel{2} \times \cancel{3}} = 84$$

so required probability $= \frac{\cancel{24}^{\cancel{12}^{6}}}{\cancel{84}_{\cancel{42}_{21}}} = \frac{6}{21}$

(2) 6 white, 5 black

(i) Two must white

$$^6C_2 \times {}^5C_2 \qquad \frac{\cancel{6}^{3} \times 5}{1 \times \cancel{2}} \times \frac{5 \times \cancel{4}^{2}}{\cancel{2} \times 1}$$

$$15 \times 10 = \underline{\underline{150}}$$

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* The company has 10 members on its board of directors. In how many ways can they elect a president, a vicepresident, a secretary and treasure.

⇒

10 9 8 7

$\cancel{{}^{10}C_4}$

$\cancel{\dfrac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1}}$

~~210 ways~~

$${}^{10}P_4 = 10 \times 9 \times 8 \times 7$$
$$= \underline{5040 \text{ ways}}$$

* **Permutation :-**

Type 1 :

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

Type 2 :

$$^{n}P_{r} = \frac{n!}{r_1!\, r_2!\, r_3! \cdots r_n!}$$

Type 3 :

$$n^{r}$$

* **Combination :**

~~ncr~~ $$^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

* "Engineering"

$r_1 (n) = 3$ $\quad n = 11$

$r_2 (g) = 2$

$r_3 (E) = 3$

$r_4 (i) = 2$

$$^{n}P_{r} = \frac{11!}{3!\,3!\,2!\,2!}$$

* combination

how many way's we can select 6 Ball out of 10 ball's.

$$^{10}C_{\cancel{6}} = \frac{10!}{6!\,(10-\cancel{6})!}$$

$$= \frac{10!}{6!\,(4!)}$$