* Method - 1

Bisection method

1) Use Bisection method to find root of the equation $f(x) = x^4 + 2x^3 - x - 1 = 0$ lying the interval $[0, 1]$ at the end of sixth iteration.

→ Given, $x^4 + 2x^3 - x - 1 = 0$.

$f(x) = x^4 + 2x^3 - x - 1$

$f(0) = -1 < 0$

$f(1) = 1 > 0$

① $x_1 = \frac{a+b}{2} = \frac{0+1}{2} = 0.5$

$f(0.5) = (0.5)^4 + 2(0.5)^3 - 0.5 - 1$

$= -1.1875 < 0$

Root lies between 0.5 and 1

② $x_2 = \frac{a+b}{2} = \frac{0.5+1}{2} = 0.75$

$f(0.75) = (0.75)^4 + 2(0.75)^3 - 0.75 - 1$

$= -0.5898 < 0$

Root lies between 0.75 and 1

③ $x_3 = \frac{a+b}{2} = \frac{0.75+1}{2} = 0.875$

$f(0.875) = 0.875^4 + 2(0.875)^3 - 0.875 - 1$

$= 0.0510 > 0$.

Root lies betn 0.75 and 0.875.

④ $x_4 = \frac{a+b}{2} = \frac{0.75 + 0.875}{2} = 0.8125$

$f(0.8125) = 0.8125^4 + 2(0.8125)^3 - 0.8125 - 1$

$= -0.3039 < 0$

Root lies between 0.8125 and 0.875

(5) $x_5 = \frac{a+b}{2} = \frac{0.8125 + 0.875}{2} = 0.84375$

$f(0.84375) = 0.84375^4 + 2(0.84375)^3 - 0.84375 - 1$

$= -0.1355 < 0.$

Root lies between 0.84375 and 0.875

(6) $x_6 = \frac{a+b}{2} = \frac{0.84375 + 0.875}{2} = 0.8593.$

$f(0.8593) = 0.8593^4 + 2(0.8593)^3 - 0.8593 - 1$

$= -0.0450 < 0$

Root lies between 0.8593 and 0.875

$\boxed{\therefore \text{ cooreed values of iteration is } 0.8593 \text{ and } 0.875}$

2] Using Bisection method find the cube root of 100 upto sixth iteration.

→ let $\sqrt[3]{100} = x$

$100 = x^3$

$f(x) = x^3 - 100 = 0$

put.

$f(0) = -100.$

$f(1) = -99.$

$f(2) = -92.$

$f(3) = -73.$

$f(4) = -36. < 0.$

$f(5) = 25 > 0$

Root lies between 4 and 5

(1) $x_1 = \frac{a+b}{2} = \frac{4+5}{2} = 4.5$

$f(4.5) = 4.5^3 - 100$

$= -8.875 < 0.$

Root lies between 4.5 and 5.

(2) $x_2 = \frac{a+b}{2} = \frac{4.5+5}{2} = 4.75$

$f(4.75) = 4.75^3 - 100$
$= 7.1718 > 0.$

Root lies between 4.5 and 4.75.

(3) $x_3 = \frac{a+b}{2} = \frac{4.5+4.75}{2} = 4.625$

$f(4.625) = 4.625^3 - 100$
$= -1.0683 < 0.$

Root lies between 4.625 and 4.75

(4) $x_4 = \frac{a+b}{2} = \frac{4.625+4.75}{2} = 4.6875$

$f(4.6875) = 4.6875^3 - 100$
$= 2.9968 > 0$

Root lies betn 4.625 and 4.6875

(5) $x_5 = \frac{a+b}{2} = \frac{4.625+4.6875}{2} = 4.65625$

$f(4.65625) = 4.65625^3 - 100$
$= 0.9505 > 0$

Root lies betn 4.625 and 4.65625

(6) $x_6 = \frac{a+b}{2} = \frac{4.625+4.65625}{2} = 4.6406.$

$f(4.6406) = 4.6406^3 - 100$
$= -0.0622 < 0$

Root lies between 4.6406 and 4.65625

$\boxed{\text{Correct value of iteration } 4.6406 \text{ and } 4.65625}$

5] $x^3 - 2x - 5 = 0$ with 5 steps.

→ ~~Let~~ Given

$$f(x) = x^3 - 2x - 5$$

put $f(0) = -5$

$f(1) = -6$.

$f(2) = -1 < 0$

$f(3) = 16. > 0$

Root lies between 2 and 3.

① $x_1 = \dfrac{a+b}{2} = \dfrac{2+3}{2} = 2.5$

$f(2.5) = 2.5^3 - 2(2.5) - 5$

$= 5.625 > 0$

Root lies betn 2 and 2.5

② $x_2 = \dfrac{a+b}{2} = \dfrac{2+2.5}{2} = 2.25$

$f(2.25) = 2.25^3 - 2(2.25) - 5$

$= 1.8906 > 0$

Root lies between 2 and 2.25

③ $x_3 = \dfrac{a+b}{2} = \dfrac{2+2.25}{2} = 2.125$

$f(2.125) = 2.125^3 - 2(2.125) - 5$

$= 0.3457 > 0$

Root lies between. 2 and 2.125

④ $x_4 = \dfrac{a+b}{2} = \dfrac{2+2.125}{2} = 2.0625$

$f(2.0625) = 2.0625^3 - 2(2.0625) - 5$

$= -0.3513 < 0$

Root lies between 2.0625 and 2.125

(5) $x_5 = \frac{a+b}{2} = \frac{2.0625 + 2.125}{2} = 2.09375$

$f(2.09375) = 2.09375^3 - 2(2.09375) - 5$

$= -0.008 < 0$

Root lies between 2.09375 and 2.125

$\boxed{\text{Correct value of iteration is 2.09375 and 2.125}}$

4) $x \log_{10} x = 1.2$ upto 5th iteration.

→ Given

$f(x) = x \log_{10} x - 1.2$

$f(0)$ = does not exist

$f(1) = -1.2$

$f(2) = -0.5979 < 0$

$f(3) = 0.2313 > 0$

Root lies between 2 and 3.

(1) $x_1 = \frac{a+b}{2} = \frac{2+3}{2} = 2.5$

$f(2.5) = 2.5 \log_{10} 2.5 - 1.2.$

$= -0.2051 < 0.$

Root lies between 2.5 and 3

(2) $x_2 = \frac{a+b}{2} = \frac{2.5+3}{2} = 2.75.$

$f(2.75) = 2.75 \log_{10} 2.75 - 1.2.$

$= 0.008 > 0.$

Root lies between 2.5 and 2.75

(3) $x_3 = \frac{a+b}{2} = \frac{2.5 + 2.75}{2} = 2.625$

$f(2.625) = 2.625 \log_{10} 2.625 - 1.2$

$= -0.099 < 0$

Root lies betn 2.625 and 2.75

④ $x_4 = \frac{a+b}{2} = \frac{2.625 + 2.75}{2} = 2.6875$

$f(2.6875) = 2.6875 \log_{10} 2.6875 - 1.2$

$= -0.0461 < 0$

Root lies betn 2.6875 and 2.75

⑤ $x_5 = \frac{a+b}{2} = \frac{2.6875 + 2.75}{2} = 2.71875$

$f(2.71875) = 2.71875 \log_{10} 2.71875 - 1.2$

$= -0.0190 < 0$

Root lies between 2.71875 and 2.75

$\boxed{\therefore \text{correct value of iteration is } 2.71875 \text{ and } 2.75}$

*Method - 2.

formula: $x_2 = x_1 - \frac{(x_1 - x_0)}{f(x_1) - f(x_0)} \times f(x_1)$

or

$= \frac{a.f(b) - b\,f(a)}{f(b) - f(a)}$

Regula falssi method

1] use Regula falsi method to find a real root of the eq^n. $e^x - 4x = 0$. upto 3 decimal places.

→ Given

$f(x) = e^x - 4x$

$f(0) = e^0 - 4(0) = 1 > 0$ $x_0 = 0$ $f(x_0) = 1$

$f(1) = e^1 - 4(1) = -1.2817 < 0$ $x_1 = 1$ $f(x_1) = -1.2817$

Root lies between 0 and 1

① $\cancel{x_1} = \cancel{a+b}$

$$x_2 = x_1 - \frac{(x_1 - x_0)}{f(x_1) - f(x_0)} \times f(x_1)$$

$$= 1 - \frac{(1 - 0)}{(-1.2817) - (1)} \times -1.2817$$

$$= 0.4382$$

~~Root lies between~~

$f(0.4382) = e^{0.4382} - 4(0.4382) = -0.2028 < 0$.

$f(x_2) = -0.2028$ and $x_2 = 0.4382$.

Root lies between 0 and 0.4382.

② $$x_3 = x_2 - \frac{(x_2 - x_0)}{f(x_2) - f(x_0)} \times f(x_2)$$

$$= 0.4382 - \left(\frac{0.4382 - 0}{(-0.2028) - 1}\right) \times -0.2028.$$

$$= 0.3643$$

$f(x_3) = f(0.3643) = e^{0.3643} - 4(0.3643) = -0.0176$

Root lies between 0 and 0.3643.

2] $x^3 - 2x - 5 = 0$ use Regula falsi method.

→ Given

$$f(x) = x^3 - 2x - 5.$$

put

$$f(0) = -5$$
$$f(1) = -6$$
$$f(2) = \boxed{-1}$$
$$f(3) = \boxed{16}$$

$a = 2$ $\quad f(a) = -1$

$b = 3$ $\quad f(b) = 16$

Root lies betn 2 and 3.

① $$x_1 = \frac{a \cdot f(b) - b\,f(a)}{f(b) - f(a)}$$

$$= \frac{2(16) - 3(-1)}{16 - (-1)}$$

$$= 2.0588$$

$$f(2.0588) = 2.0588^3 - 2(2.0588) - 5$$
$$= -0.3910 < 0.$$

Root lies betn 2.0588 and 3.

$a = 2.0588$ $\quad f(a) = -0.3910$ | $b = 3$ $\quad f(b) = 16$

② $$x_2 = \frac{a \cdot f(b) - b\,f(a)}{f(b) - f(a)}$$

$$= \frac{2.0588(16) - 3(-0.3910)}{16 - (-0.3910)}$$

$$= 2.0812$$

$$f(2.0812) = 2.0812^3 - 2(2.0812) - 5$$
$$= -0.1479.$$

Root lies betn 2.0812 and 3

$a = 2.0812$ $\quad f(a) = -0.1479$

$b = 3$ $\quad f(b) = 16$

* Method - 3

Gauss elimination method

1. $4x_1 + x_2 + x_3 = 4$
   $x_1 + 4x_2 - 2x_3 = 4$
   $3x_1 + 2x_2 - 4x_3 = 6$

→ Given

$4x_1 + x_2 + x_3 = 4$ —— ①
$x_1 + 4x_2 - 2x_3 = 4$ —— ②
$3x_1 + 2x_2 - 4x_3 = 6$ —— ③

Substract 4 × ② by ①

$$\begin{array}{r} \cancel{4x_1} + 16x_2 - 8x_3 = 16 \\ - \quad \cancel{4x_1} + x_2 + x_3 = 4 \\ - \quad - \quad - \\ \hline 15x_2 - 9x_3 = 12 \end{array}$$ —— ④

substract 4 × ③ by 3 × ①

$$\begin{array}{r} \cancel{12x_1} + 8x_2 - 16x_3 = 24 \\ - \quad \cancel{12x_1} + 3x_2 + 3x_3 = 12 \\ - \quad - \quad - \\ \hline 5x_2 - 19x_3 = 12 \end{array}$$ —— ⑤

Substract ④ − ⑤ × 3

$$\begin{array}{r} 15\cancel{x_2} - 9x_3 = 12 \\ 15\cancel{x_2} - 57x_3 = 36 \\ - \quad + \quad - \\ \hline 48x_3 = -24 \end{array}$$

$x_3 = \frac{-24}{48} = \frac{-1}{2}$

$\boxed{x_3 = -\frac{1}{2}}$

put x_3 in ⑤

$5x_2 - 19(-\frac{1}{2}) = 12$

$5x_2 + \frac{19}{2} = 12$

$5x_2 = 12 - \frac{19}{2}$

$5x_2 = \frac{5}{2}$

$x_2 = \frac{5}{5 \times 2}$ $\boxed{x_2 = \frac{1}{2}}$

put $x_3 = -\frac{1}{2}$, $x_2 = \frac{1}{2}$ in ②

$x_1 + 4(\frac{1}{2}) - 2(-\frac{1}{2}) = 4$

$x_1 + 2 + 1 = 4$

$x_1 + 3 = 4$

$x_1 = 4 - 3$

$\boxed{x_1 = 1}$

2] $x_1 + 4x_2 - x_3 = -5$

$x_1 + x_2 - 6x_3 = -12$

$3x_1 - x_2 - x_3 = 4$

→ Given

$x_1 + 4x_2 - x_3 = -5$ —— ①

$x_1 + x_2 - 6x_3 = -12$ —— ②

$3x_1 - x_2 - x_3 = 4$ —— ③

substreact ② − ①

$$\begin{array}{rrrrl} \cancel{x_1} & + x_2 & - 6x_3 & = & -12 \\ - \cancel{x_1} & \underset{-}{+ 4x_2} & \underset{+}{- x_3} & = & \underset{+}{-5} \\ \hline & -3x_2 & - 5x_3 & = & -7 \quad \text{—— ④} \end{array}$$

substreact ③ − ① × 3.

$$\begin{array}{rrrrl} 3\cancel{x_1} & - x_2 & - x_3 & = & 4 \\ - 3\cancel{x_1} & \underset{-}{+ 12x_2} & \underset{+}{- 3x_3} & = & \underset{+}{-15} \\ \hline & -13x_2 & + 2x_3 & = & 19 \quad \text{—— ⑤} \end{array}$$

~~Sub~~

substract eqn ④ × 13 − eqn ⑤ × 3

$$\begin{array}{rrrl} -39\cancel{x_2} & - 65x_3 & = & -91 \\ \underset{+}{-} 39\cancel{x_2} & \underset{-}{+ 6x_3} & = & \underset{-}{57} \\ \hline & -71x_3 & = & -148. \end{array}$$

$x_3 = \frac{-148}{-71}$

$\boxed{x_3 = \frac{148}{71}}$

put x_3 in ④

$-3x_2 - 5\left(\frac{148}{71}\right) = -7$

$-3x_2 - \frac{740}{71} = -7$

$-3x_2 = -7 + \frac{740}{71}$

$-3x_2 = \frac{243}{71}$

$x_2 = \frac{243}{71 \times 3}$

$\boxed{x_2 = -\frac{81}{71}}$

put $x_3 = \frac{148}{71}$ & $x_2 = -\frac{81}{71}$ in ①

$x_1 + 4\left(-\frac{81}{71}\right) - \frac{148}{71} = -5$

$x_1 - \frac{472}{71} = -5$

$x_1 = -5 + \frac{472}{71}$

$\boxed{x_1 = \frac{117}{71}}$

8] $2x_1 + x_2 + x_3 = 10$ —— ①

$3x_1 + 2x_2 + 3x_3 = 18$ —— ②

$x_1 + 4x_2 + 9x_3 = 16$ —— ③

→

sub ② × 2 — ① × 3

$$\cancel{6x_1} + 4x_2 + 6x_3 = 36$$
$$- \quad \cancel{6x_1} \underset{-}{+} 3x_2 \underset{-}{+} 3x_3 = \underset{-}{30}$$
$$x_2 + 3x_3 = 6 \quad —— ④$$

sub ① × 2 — ①

$$\cancel{2x_1} + 8x_2 + 18x_3 = 32$$
$$- \quad \cancel{2x_1} \underset{-}{+} x_2 \underset{-}{+} x_3 = \underset{-}{10}$$
$$7x_2 + 17x_3 = 22 \quad —— ⑤$$

sub ④ × 7 — ⑤

$$\cancel{7x_2} + 21x_3 = 42$$
$$- \quad \cancel{7x_2} \underset{-}{+} 17x_3 = \underset{-}{22}$$
$$4x_3 = 20$$
$$x_3 = \frac{\cancel{20}\,5}{\cancel{4}\,1} \quad \boxed{x_3 = 5}$$

put $x_3 = 5$ in ④

$x_2 + 3x_3 = 6.$

$x_2 + 3(5) = 6$

$x_2 + 15 = 6$

$x_2 = 6 - 15$

$\boxed{x_2 = -9}$

put $x_3 = 5$ & $x_2 = -9$ in ①

$2x_1 + x_2 + x_3 = 10$

$2x_1 + (-9) + 5 = 10$

$2x_1 - 4 = 10$

$2x_1 = 10 + 4$

$2x_1 = 14$

$x_1 = \frac{\cancel{14}\,7}{\cancel{2}\,1}$

$\boxed{x_1 = 7}$

4. $10x + y + 2z = 13$
$2x + 3y + 10z = 15$
$3x + 10y + z = 14$

→ Given,

$$10x + y + 2z = 13 \quad —(1)$$
$$2x + 3y + 10z = 15 \quad —(2)$$
$$3x + 10y + z = 14 \quad —(3)$$

Substract (2) × 5 − (1)

$$\cancel{10x} + 15y + 50z = 75$$
$$-\ \cancel{10x} + y + 2z = 13$$
$$14y + 48z = 62 \quad —(4)$$

Substract (3) × 10 − (1) × 3

$$\cancel{30x} + 100y + 10z = 140$$
$$-\ \cancel{30x} + 3y + 6z = 39$$
$$97y + 4z = 101 \quad —(5)$$

Sub (4) × 97 − (5) × 14

$$\cancel{1358y} + 4656z = 6014$$
$$-\ \cancel{1358y} + 56z = 1414$$
$$4600z = 4600$$
$$z = \frac{\cancel{4600}}{\cancel{4600}}, \quad \boxed{z = 1}$$

put $z = 1$ in (4)

$$14y + 48(1) = 62$$
$$14y + 48 = 62$$
$$14y = 62 - 48$$
$$14y = 14$$
$$y = \frac{\cancel{14}}{\cancel{14}}$$
$$\boxed{y = 1}$$

put $z = 1$ & $y = 1$ in (1)

$$10x + 1 + 2(1) = 13$$
$$10x + 3 = 13$$
$$10x = 13 - 3$$
$$10x = 10$$
$$x = \frac{\cancel{10}}{\cancel{10}}$$
$$\boxed{x = 1}$$

5] $5x + 2y + z = 12$

$x + 4y + 2z = 15$

$x + 2y + 5z = 20$

→ Given,

$5x + 2y + z = 12$ —— ①

$x + 4y + 2z = 15$ —— ②

$x + 2y + 5z = 20$ —— ③

substract ② × 5 − ①

$$\begin{array}{r} \cancel{5x} + 20y + 10z = 75 \\ - \cancel{5x} + 2y + z = 12 \\ \hline 18y + 9z = 63 \end{array}$$ —— ④

substract ③ × 5 − ①

$$\begin{array}{r} \cancel{5x} + 10y + 25z = 100 \\ - \cancel{5x} + 2y + z = 12 \\ \hline 8y + 24z = 88 \end{array}$$ —— ⑤

substract eqⁿ ④ × 8 − ⑤ × 18

$$\begin{array}{r} \cancel{144y} + 72z = 504 \\ - \cancel{144y} + 432z = 1584 \\ \hline -360z = -1080 \end{array}$$

$z = \frac{-\cancel{1080}\ 3}{-\cancel{360}\ 1}$

$\boxed{z = 3}$

put $z = 3$ in ⑤

$8y + 24z = 88$

$8y + 24(3) = 88$

$8y + 72 = 88$

$8y = 88 - 72$

$8y = 16$

$y = \frac{\cancel{16}\ 2}{\cancel{8}\ 1}$

$\boxed{y = 2}$

put $z = 3$ & $y = 2$ in ②

$x + 4y + 2z = 15$

$x + 4(2) + 2(3) = 15$

$x + 8 + 6 = 15$

$x + 14 = 15$

$x = 15 - 14$

$\boxed{x = 1}$

∴ $x = 1$ $y = 2$ $z = 3$

* Method - 4

Gauss Jordon method

1] $x_1 + 2x_2 + x_3 = 8.$

$2x_1 + 3x_2 + 4x_3 = 20$

$4x_1 + 3x_2 + 2x_3 = 16.$

→ Given,

$x_1 + 2x_2 + x_3 = 8$ — ①

$2x_1 + 3x_2 + 4x_3 = 20$ — ②

$4x_1 + 3x_2 + 2x_3 = 16$ — ③

eq^n ① × 2 − ②

$\cancel{2x_1} + 4x_2 + 2x_3 = 16$

$- \cancel{2x_1} + 3x_2 + 4x_3 = 20$

$x_2 - 2x_3 = -4$ — ④

eq^n ① × 4 − ③

$\cancel{4x_1} + 8x_2 + 4x_3 = 32$

$- \cancel{4x_1} + 3x_2 + 2x_3 = 16$

$5x_2 + 2x_3 = 16$ — ⑤

Substract ④ × 5 − ⑤

$\cancel{5x_2} - 10x_3 = -20$

$- \cancel{5x_2} + 2x_3 = 16$

$-12x_3 = -36$

$x_3 = \frac{-36}{-12}$ $\boxed{x_3 = 3}$

put $x_3 = 3$ in ④

$x_2 - 2x_3 = -4$

$x_2 - 2(3) = -4$

$x_2 - 6 = -4$

$x_2 = -4 + 6$

$\boxed{x_2 = 2}$

put $x_3 = 3$ & $x_2 = 2$ in ①

$x_1 + 2x_2 + x_3 = 8$

$x_1 + 2(2) + 3 = 8$

$x_1 + 7 = 8$

$x_1 = 8 - 7$

$\boxed{x_1 = 1}$

$\boxed{x_1 = 1 \quad x_2 = 2 \quad x_3 = 3}$

2] $10x + y + z = 12$

$x + 10y + z = 12$

$x + y + 10z = 12$

$\longrightarrow$ Given,

$10x + y + z = 12$ — ①

$x + 10y + z = 12$ — ②

$x + y + 10z = 12$ — ③

eqⁿ ① − ② × 10

$\cancel{10x} + y + z = 12$

$-\ \cancel{10x} + 100y + 10z = 120$

$-99y - 9z = -108$ — ④

eqⁿ ① − ③ × 10

$\cancel{10x} + y + z = 12$

$-\ \cancel{10x} + 10y + 100z = 120$

$-9y - 99z = -108$ — ⑤

④ × 9 − ⑤ × 99.

$-\cancel{891y} - 81z = -972$ ~~1620~~

$-\cancel{891y} - 9801z = $ ~~1620~~ -10692

$+ \quad\quad + \quad\quad +$

$9720z = $ ~~9702~~ 9720

$z = \dfrac{\cancel{9720}}{\cancel{9720}}\ 1$

$\boxed{z = 1}$

put $z = 1$ in ④

$-99y - 9z = -108$

$-99y - 9(1) = -108$

$-99y = -108 + 9$

$-99y = -99$

$y = \dfrac{\cancel{-99}}{\cancel{-99}}\ 1$

$\boxed{y = 1}$

put $y = 1$ in ②

$x + 10y + z = 12$

$x + 10(1) + 1 = 12$

$x + 11 = 12$

$x = 12 - 11$

$\boxed{x = 1}$

$\boxed{x = 1, \ y = 1 \ \ z = 1}$

2] $4x_1 + 2x_2 + 14x_3 = 14$

$2x_1 + 17x_2 - 5x_3 = -101$

$14x_1 - 5x_2 + 83x_3 = 155$

* Method-5

Cholesky method

→ let.

$AX = B$

$$A = \begin{bmatrix} 4 & 2 & 14 \\ 2 & 17 & -5 \\ 14 & -5 & 83 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad B = \begin{bmatrix} 14 \\ -101 \\ 155 \end{bmatrix}$$

$A = LL^T$

$$\begin{bmatrix} 4 & 2 & 14 \\ 2 & 17 & -5 \\ 14 & -5 & 83 \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} L_{11} & L_{21} & L_{31} \\ 0 & L_{22} & L_{32} \\ 0 & 0 & L_{33} \end{bmatrix}$$

$L_{11}^2 = 4$

$\boxed{L_{11} = 2}$

$L_{11} L_{21} = 2$

$2 L_{21} = 2$

$L_{21} = \frac{\cancel{2}1}{\cancel{2}1}$

$\boxed{L_{21} = 1}$

$L_{11} L_{31} = 14$

$2 L_{31} = 14$

$L_{31} = \frac{\cancel{14}7}{\cancel{2}1}$

$\boxed{L_{31} = 7}$

$L_{21}^2 + L_{22}^2 = 17$

$1^2 + L_{22}^2 = 17$

$L_{22}^2 = 17 - 1$

$L_{22}^2 = 16$

$\boxed{L_{22} = 4}$

$L_{21} L_{31} + L_{22} L_{32} = -5$

$(1 \times 7) + 4 L_{32} = -5$

$7 + 4 L_{32} = -5$

$4 L_{32} = -5 - 7$

$4 L_{32} = -12$

$L_{32} = \frac{-\cancel{12}3}{\cancel{4}1}$

$\boxed{L_{32} = -3}$

$L_{31}^2 + L_{32}^2 + L_{33}^2 = 83$

$7^2 + (-3)^2 + L_{33}^2 = 83$

$49 + 9 + L_{33}^2 = 83$

$58 + L_{33}^2 = 83$

$L_{33}^2 = 83 - 58$

$L_{33}^2 = 25$

$\boxed{L_{33} = 5}$

$$L = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 4 & 0 \\ 7 & -3 & 5 \end{bmatrix}$$

$$L \underline{L^T X} = B$$

$$L^T X = Z \qquad Z = \{ Z_1, Z_2, Z_3 \}$$

$$LZ = B.$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 1 & 4 & 0 \\ 7 & -3 & 5 \end{bmatrix} \begin{bmatrix} Z^1 \\ Z^2 \\ Z^3 \end{bmatrix} = \begin{bmatrix} 14 \\ -101 \\ 155 \end{bmatrix}$$

$$2Z^1 = 14$$

$$Z^1 = \frac{\cancel{14}\,7}{\cancel{2}\,1}$$

$$\boxed{Z_1 = 7}$$

$$Z_1 + 4Z_2 = -101$$

$$7 + 4Z_2 = -101$$

$$4Z_2 = -101 - 7$$

$$4Z_2 = -108$$

$$\boxed{Z_2 = -27}$$

$$7Z_1 + (-3\ Z_2) + 5Z_3 = 155$$

$$7(7) + (-3(-27)) + 5Z_3 = 155$$

$$49 + 81 + 5Z_3 = 155$$

$$130 + 5Z_3 = 155$$

$$5Z_3 = 155 - 130$$

$$5Z_3 = 25$$

$$Z_3 = \frac{\cancel{25}\,5}{\cancel{5}\,1} \qquad \boxed{Z_3 = 5}$$

$L^T x = z$

$$\begin{bmatrix} 2 & 1 & 7 \\ 0 & 4 & -3 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -27 \\ 5 \end{bmatrix}$$

$5x_3 = 5$

$x_3 = \frac{\cancel{5}^{1}}{\cancel{5}}$

$\boxed{x_3 = 1}$

$4x_2 + (-3x_3) = -27$

$4x_2 + (-3(1)) = -27$

$4x_2 - 3 = -27$

$4x_2 = -27 + 3$

$4x_2 = -24$

$x_2 = \frac{-\cancel{24}^{6}}{\cancel{4}_{1}}$

$\boxed{x_2 = -6}$

$2x_1 + x_2 + 7x_3 = 5$

$2x_1 + (-6) + 7(1) = 5$

$2x_1 - 6 + 7 = 5$

$2x_1 + 1 = 5$

$2x = 5 - 1$

$2x = 4$

$x = \frac{\cancel{4}^{2}}{\cancel{2}_{1}}$

$\boxed{x = 2}$

* method - 6.

Rungee - kutta method.

1] solve $\frac{dy}{dx} = y - x$ $x_0 = 0.2$ $y(0) = 2$ $h_0 = 0.1$

2nd order.

→ Given $\frac{dy}{dx} = y - x$

$f(x, y) = y - x$

$x_0 = 0$ $y_0 = 2$ $h_0 = 0.1$

$k_1 = h f(x_0 y_0) = h(y_0 - x_0) = 0.1(2 - 0)$

$k = 0.2.$

$k_2 = hf(x_0 + h, y_0 + k_1)$

$= h[(y_0 + k_1) - (x_0 + h)]$

$= 0.1[(2 + 0.2) - (0 + 0.1)]$

$= 0.1[2.2 - 0.1]$

$= 0.21$

$k = \frac{k_1 + k_2}{2} = \frac{0.2 + 0.21}{2}$

$k = 0.205$

Sol^n $y_{x_0 + h} = y_0 + k$

$y_{0+0.1} = 2 + 0.205$

$\boxed{y_{0.1} = 2.205}$ ← y_0

$x_0 = 0.1$ $y_0 = 2.205$ $h = 0.1$

$$k_1 = hf(x_0, y_0)$$
$$= h(y_0 - x_0)$$
$$= 0.1(2.205 - 0.1)$$
$$= 0.2105$$

$$k_2 = hf((x_0 + h), (y_0 + k_1))$$
$$= h((y_0 + k_1) - (x_0 + h))$$
$$= 0.1[(2.205 + 0.2105) - (0.1 + 0.1)]$$
$$= 0.22155$$

$$k = \frac{k_1 + k_2}{2} = \frac{0.2105 + 0.22155}{2}$$
$$= 0.2160.$$

$$y_{(x_0 + h)} = \underline{y_0 + k}$$
$$y_{(0.1 + 0.1)} = 2.205 + 0.2160$$
$$y_{(0.2)} = 2.421$$

2] Using fourth order Rangee kutta.
solve $\frac{dy}{dx} = \sqrt{x+y}$ $y(0) = 1$ to find y at
$x = 0.2$ taking $h = 0.2$.

$\longrightarrow$ Given

$$\frac{dy}{dx} = \sqrt{x+y}$$

$$f(x,y) = \sqrt{x+y}.$$

$x_0 = 0$ $y_0 = 1$ $h = 0.2$.

$$k_1 = hf(x_0 y_0)$$
$$= h(\sqrt{x_0 + y_0})$$
$$= 0.2(\sqrt{0+1})$$
$$\boxed{k_1 = 0.2.}$$

$$k_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{hk_1}{2})$$
$$= h\left(\sqrt{\left(x_0 + \frac{h}{2}\right) + \left(y_0 + \frac{k_1}{2}\right)}\right)$$
$$= 0.2\left(\sqrt{\left(0 + \frac{0.2}{2}\right) + \left(1 + \frac{0.2}{2}\right)}\right)$$
$$= 0.2\sqrt{0.1 + 1.1}$$
$$= 0.2\sqrt{1.2}.$$
$$\boxed{k_2 = 0.2190}$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$
$$= h\left(\sqrt{\left(x_0 + \frac{h}{2}\right) + \left(y_0 + \frac{k_2}{2}\right)}\right)$$
$$= 0.2\left(\sqrt{\left(0 + \frac{0.2}{2}\right) + \left(1 + \frac{0.2190}{2}\right)}\right)$$
$$= 0.2\sqrt{1.2095}$$
$$\boxed{k_3 = 0.2199}$$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= h\left(\sqrt{(x_0+h) + (y_0+k_3)}\right)$

$= 0.2\left(\sqrt{(0+0.2) + (1+0.2199)}\right)$

$= 0.2\sqrt{1.4199}$

$\boxed{k_4 = 0.2383}$

$k = \dfrac{k_1 + 2k_2 + 2k_3 + k_4}{6}$

$= \dfrac{0.2 + (2(0.2190)) + (2(0.2199)) + 0.2383}{6}$

$k = 0.21935$

$y_{x_0+h} = y_0 + k$

$y_{0+0.2} = 1 + 0.21935$

$\boxed{y_{0.2} = 1.21935}$

3] 4^{th} order solve $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ at $x = 0.2$

$h = 0.2$.

$\rightarrow$ Given

$$\frac{dy}{dx} = \frac{y-x}{y+x}$$

$$f(x,y) = \frac{y - x}{y+x}$$

$x_0 = 0$, $y_0 = 1$, $x = 0.2$, $h = 0.2$

$$k_1 = hf(x_0\ y_0)$$

$$= h\left(\frac{y_0 - x_0}{y_0 + x_0}\right)$$

$$= 0.2\left(\frac{1 - 0}{1+0}\right)$$

$$\boxed{k_1 = 0.2}$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$= h\left[\frac{\left(y_0 + \frac{k_1}{2}\right) - \left(x_0 + \frac{h}{2}\right)}{\left(y_0 + \frac{k_1}{2}\right) - \left(x_0 + \frac{h}{2}\right)}\right)$$

$$= 0.2\left[\frac{\left(1 + \frac{0.2}{2}\right) - \left(0 + \frac{0.2}{2}\right)}{\left(1 + \frac{0.2}{2}\right) + \left(0 + \frac{0.2}{2}\right)}\right]$$

$$= 0.2\left[\frac{1}{1.2}\right]$$

$$= 0.166$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$= h\left[\frac{\left(y_0 + \frac{k_2}{2}\right) - \left(x_0 + \frac{h}{2}\right)}{\left(y_0 + \frac{k_2}{2}\right) - \left(x_0 + \frac{h}{2}\right)}\right]$$

$$= 0.2\left[\frac{\left(1 + \frac{0.166}{2}\right) - \left(0 + \frac{0.2}{2}\right)}{\left(1 + \frac{0.166}{2}\right) + \left(0 + \frac{0.2}{2}\right)}\right]$$

$$= 0.2\left[\frac{0.983}{1.183}\right]$$

$$\boxed{k_3 = 0.1661}$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$

$$= h\left[\frac{(y_0 + k_3) - (x_0 + h)}{(y_0 + k_3) + (x_0 + h)}\right]$$

$$= 0.2\left[\frac{(1 + 0.1661) - (0 + 0.2)}{(1 + 0.1661) + (0 + 0.2)}\right]$$

$$= 0.2\left[\frac{0.9661}{1.3661}\right]$$

$$= 0.1414.$$

$$k = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$$

$$= \frac{0.2 + (2(0.166) + (2(0.1661)) + 0.1414}{6}$$

$$= \frac{1.0056}{6}$$

$$\boxed{k = 0.1676}$$

$$y_{x_0+h} = y_0 + k$$

$$y_{0+0.2} = 1 + 0.1676$$

$$\boxed{y_{0.2} = 1.1676}$$

4] Solve example using Rangee kutta method

$$\frac{dy}{dx} = xy \quad , y(1) = 2 \quad \text{at } x = 1.2, \text{ with } h = 0.2$$

→ Given,

$$\frac{dy}{dx} = xy$$

$$f(x, y) = xy$$

$$x_0 = 1 \quad y_0 = 2 \quad x = 1.2 \quad h = 0.2$$

$$k_1 = hf(x_0, y_0)$$

$$= h(x_0 y_0)$$

$$= 0.2(1 * 2)$$

$$\boxed{k_1 = 0.4}$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$= h\left[\left(x_0 + \frac{h}{2}\right) * \left(y_0 + \frac{k_1}{2}\right)\right]$$

$$= 0.2\left[\left(1 + \frac{0.2}{2}\right) * \left(2 + \frac{0.4}{2}\right)\right]$$

$$= 0.2\left[1.1 * 2.2\right]$$

$$= 0.2\left[2.42\right]$$

$$\boxed{k_2 = 0.484}$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$= 0.2\left[\left(1 + \frac{0.2}{2}\right) * \left(2 + \frac{0.484}{2}\right)\right]$$

$$= 0.2\left[1.1 * 2.242\right]$$

$$= 0.2\left[2.4662\right]$$

$$\boxed{k_3 = 0.49324}$$

$$k_4 = hf(x_0+h,\ y_0+k_3)$$

$$= h\left[(x_0+h) * (y_0+k_3)\right]$$

$$= 0.2\left[(1+0.2) * (2+0.49324)\right]$$

$$= 0.2\left[2.991888\right]$$

$$\boxed{k_4 = 0.5983}$$

$$k = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$$

$$= \frac{0.4 + (2(0.484)) + (2(0.49324)) + 0.5983}{6}$$

$$= \frac{2.95278}{6}$$

$$= 0.49213.$$

sol^n is.

$$y_{x_0+h} = y_0 + k$$

$$y_{1+0.2} = 2 + 0.49213$$

$$\boxed{y_{1.2} = 2.49213}$$

* Method - 7

$1+2-5$ $8+4-5$ $12-5$

Newton - Raphson Method.

1] Find the real root of $x^3 + 2x - 5 = 0$ end 5th iteration.

→ Given,

$$f(x) = x^3 + 2x - 5$$
$$f'(x) = 3x^2 + 2$$
$$f''(x) = 6x$$

$$f(0) = -5$$
$$f(1) = -2 < 0$$
$$f(2) = 7 > 0$$

Root lies between 1 & 2.

$$x_0 = \frac{1+2}{2} = 1.5$$

① $$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1.5 - \frac{f(1.5)}{f'(1.5)}$$

$$= 1.5 - \frac{1.375}{8.75}$$

$$= 1.3428$$

② $$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.3428 - \frac{f(1.3428)}{f'(1.3428)}$$

$$= 1.3428 - \frac{0.1068}{7.4093}$$

$$= 1.3283.$$

(3) $x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)}$

$= 1.3283 - \dfrac{f(1.3283)}{f'(1.3283)}$

$= 1.3283 - \dfrac{0.0002}{7.2931}$

$\boxed{x_3 = 1.3282}$

(4) $x_4 = x_3 - \dfrac{f(x_3)}{f'(x_3)}$

$= 1.3282 - \dfrac{-0.0005}{7.2923}$

$= 1.3282$

(5) $x_5 = x_4 - \dfrac{f(x_4)}{f(x_3)}$

$= 1.3282 - \dfrac{-0.0005}{7.2923}$

$= 1.3282$

* method - 8

Gauss Sedial method.

i] $2x + 15y + z = 33$ — (1)

$13x + 2y + 3z = 46$ — (2)

$x - y + 10z = 25$ — (3)

→ $x = \dfrac{\cancel{2y}\ 46 - 2y - 3z}{13}$ — (4)

$y = \dfrac{33 - 2x - z}{15}$ — (5)

$z = \dfrac{25 - x + y}{10}$ — (6)

Iteration 1 :

put $\cancel{x=0}$, $y = 0$, $z = 0$ in (4)

$x_1 = \dfrac{46 - 2(0) - 3(0)}{13}$

$\boxed{x_1 = 3.5384}$

put $x = 3.5384$ $z = 0$ in (5)

$y_1 = \dfrac{33 - 2(3.5384) - 0}{15}$

$\boxed{y_1 = 1.7282}$

put $x = 3.5384$ $y = 1.7282$ in (6)

$z_1 = \dfrac{25 - 3.5384 + 1.7282}{10}$

$\boxed{z_1 = 2.3189.}$

Iteration - 2.

put $y = 1.7282$ $z = 2.3189$ in (4)

$$x = \frac{46 - 2(1.7282) - 3(2.3189)}{13}$$

$\boxed{x = 2.7374}$

put $x = 2.7374$ $z = 2.3189$ in (5)

$$y = \frac{33 - 2(2.7374) - 2.3189}{15}$$

$\boxed{y = 1.6804}$

put $x = 2.7374$ $y = 1.6804$ in (6)

$$z = \frac{25 - 2.7374 + 1.6804}{10}$$

$\boxed{z = 2.3943}$

Iteration - 3

put $y = 1.6804$ $z = 2.3943$ in (4)

$$x = \frac{46 - 2(1.6804) - 3(2.3943)}{13}$$

$\boxed{x = 2.7274}$

put $x = 2.7274$ $z = 2.3943$ in (5)

$$y = \frac{33 - 2(2.7274) - 2.3943}{15}$$

$\boxed{y = 1.6767}$

put $x = 2.7274$ $y = 1.6767$ in (6)

$$z = \frac{25 - 2.7274 + 1.6767}{10}$$

$\boxed{z = 2.3949}$

Iteration - 4

put $y = 1.6767$ $z = 2.3949$ in (4)

$$x = \frac{46 - 2(1.6767) - 3(2.3949)}{13}$$

$$\boxed{x = 2.7278.}$$

put $x = 2.7278$ $z = 2.3949$ in (5)

$$y = \frac{33 - 2(2.7278) - 2.3949}{15}$$

P $\boxed{y = 1.6766.}$

put $x = 2.7278$ $y = 1.6766$ in (6)

$$z = \frac{25 - 2.7278 + 1.6766}{10}$$

$$\boxed{z = 2.3948.}$$

*method - 9.

Jacobi's iteration method.

1] $20x_1 + x_2 - 2x_3 = 17$ —— ①

$3x_1 + 20x_2 - x_3 = -18$ —— ②

$2x_1 - 3x_2 + 20x_3 = 25$ —— ③

→ from ① ② ③ we get the eqⁿ

$$x_1 = \frac{17 - x_2 + 2x_3}{20} \quad —— ④$$

$$x_2 = \frac{-18 - 3x_1 + x_3}{20} \quad —— ⑤$$

$$x_3 = \frac{25 - 2x_1 + 3x_2}{20} \quad —— ⑥$$

Iteration - 1

put $x_1 = 0$ $x_2 = 0$ $x_3 = 0$ in ④ ⑤ & ⑥ we get x, y, z.

$$x = \frac{17 - 0 + 2(0)}{20}$$

$\boxed{x = 0.85}$

$$x_2 = \frac{-18 - 3(0) + 0}{20}$$

$\boxed{x_2 = -0.9}$

$$x_3 = \frac{25 - 2(0) + 3(0)}{20}$$

$\boxed{x_3 = 1.25}$

Iteration - 2

put $x_1 = 0.85$, $\overset{x_2}{y} = -0.9$ $\overset{x_3}{z} = 1.25$ we get x, y, z value

$$x_1 = \frac{17 - (-0.9) + 1.25}{20}$$

$$\boxed{x_1 = 0.9575}$$

$$x_2 = \frac{-18 - 3(0.85) + 1.25}{20}$$

$$\boxed{x_2 = -0.965}$$

$$x_3 = \frac{25 - 2(0.85) + 3(-0.9)}{20}$$

$$\boxed{x_3 = 1.03.}$$

Iteration - 3.

put $x_1 = 0.9575$ $x_2 = -0.965$ $x_3 = 1.03$ we get x_1, x_2, x_3.

$$x_1 = \frac{17 - (-0.965) + 1.03}{20}$$

$$\boxed{x_1 = 0.94970}$$

$$x_2 = \frac{-18 - 3(0.9575) + 1.03}{20}$$

$$\boxed{x_2 = -0.9921}$$

$$x_3 = \frac{25 - 2(0.9575) + 3(-0.965)}{20}$$

$$\boxed{x_3 = 1.0095}$$