Engineering Mathematics - III

Computer / IT Engineering

Solutions (Date - 20/6/2023)

Q1. (a) $\bar{x} = 2, \bar{y} = -3$; $b_{xy} = -0.11$; $y = 0$

Regression equation of x on y. we get

$$(x - \bar{x}) = b_{xy} (y - \bar{y})$$

$$(x - 2) = -0.11 (0 - (-3))$$

$$x - 2 = -0.11 (3) = -0.33$$

$$x = -0.33 + 2$$

$$\boxed{x = 1.77}$$

$\boxed{\text{Solution :- iv}}$

(b) $f(x) = \begin{cases} 1/4 ; & -2 \leq x \leq 2 \\ 0 ; & \text{otherwise} \end{cases}$

$$\therefore P(x \leq 1) = \int_{-\infty}^{1} f(x)\, dx$$

$$= \int_{-\infty}^{-2} 0\, dx + \int_{-2}^{1} 1/4\, dx$$

$$= \frac{1}{4} (x)_{-2}^{1} = \frac{1}{4} [1 - (-2)]$$

$\boxed{\text{solution: iv}}$

$$\boxed{P(x \leq 1) = 3/4}$$

(c) Given

x	0	1	2
y	4	0	6

Lagrange's polynomial is $y = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x)$

$$y = 4\left[\frac{(x-1)(x-2)}{(0-1)(0-2)}\right] + 0[\quad] + 6\left[\frac{(x-0)(x-1)}{(2-0)(2-1)}\right]$$

$$= 4\left[\frac{x^2-2x-x+2}{(-1)(-2)}\right] + 6\left[\frac{x^2-x}{2(1)}\right]$$

$$= 2[x^2-3x+2] + 3[x^2-x]$$

$$\boxed{y = 5x^2 - 9x + 4}$$

$\boxed{\text{Solution : (iii)}}$

(d)

$x + \frac{1}{4}y + \frac{1}{4}z = 1$; —(1)

$\frac{15}{4}y - \frac{9}{4}z = 3$; —(2)

$\frac{5}{4}y - \frac{19}{4}z = 3$; —(3)

Eq(3) $- \frac{1}{3}$ Eq(2), we get

$x + \frac{1}{4}y + \frac{1}{4}z = 1$

$\frac{15}{4}y - \frac{9}{4}z = 3$

$-\frac{16}{4}z = 2$

$\Rightarrow z = \cancel{\frac{(2 \times 4)}{22}}$; $\cancel{\boxed{z = \frac{-4}{11}}}$

$z = \frac{-8}{16}$ $\boxed{z = \frac{-1}{2}}$

$\Rightarrow 15y - 9\left(-\frac{1}{2}\right) = 12$

$30y + 9 = 24$

$y = \frac{15}{30} \Rightarrow \boxed{y = \frac{1}{2}}$

$\frac{5}{4} - \frac{1}{3}\left(\frac{15}{4}\right)$

$-\frac{19}{4} - \frac{1}{3}\left(-\frac{9}{4}\right)$

$-\frac{22}{4}$

$3 - \frac{1}{3}(9)$

$4x + y + z = 4$

$15y - 9z = 12$

$\cancel{5y - 19z = 12} \times 3$

$15y -$

$x = 1 - \frac{1}{8} + \frac{1}{8}$

$\boxed{x = 1}$

$\boxed{\text{Solution: (iv)}}$

(e) Given $\mu_2 = 16$; $\mu_4 = 162$

$$\beta_2 = \frac{\mu_4}{(\mu_2)^2} = \frac{162}{(16)^2} = \frac{162}{256} = 0.6328$$

$\boxed{\text{Solution (ii)}}$

(f) Given $[a, b]$; $x_0 = \frac{a+b}{2}$ $\boxed{\text{Sol: (iii)}}$

Q 2 (a) No. of students $= 5$;

x	$(x-\bar{x})$	$(x-\bar{x})^2$
46	-9	81
34	-21	441
52	-3	9
78	23	529
65	10	100
		$\Sigma(x-\bar{x})^2 = 1160$

$\Sigma x = 275$; $n = 5$

$\therefore \bar{x} = \frac{275}{5} = 55$

$\boxed{\bar{x} = 55}$

$$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$$

$$\sigma = \sqrt{\frac{1160}{5}}$$

$$\sigma = \sqrt{232}$$

$\therefore \boxed{\sigma = 15.23154}$

b) fit a law of the form $y = ap + b$ by least square method for the data,

p	100	120	140	160	180	200
y	0.9	1.1	1.2	1.4	1.6	1.7

→

p	y	p^2	py
100	0.9	10,000	90
120	1.1	14,400	132
140	1.2	19,600	168
160	1.4	25,600	224
180	1.6	32,400	288
200	1.7	40,000	340
$\Sigma p = 900$	$\Sigma y = 7.9$	$\Sigma p^2 =$ 142,000	$\Sigma py =$ 1242

To get $y = ap + b$; solve following eqn's :

$a.\Sigma p + bn = \Sigma y$ — ① and.

$a.\Sigma p^2 + b.\Sigma p = \Sigma py$ — ②

$\therefore$ $900a + 6b = 7.9$ — ③ ×150

$142000a + 900b = 1242$ — ④

$$\begin{array}{r} + \; 135000a + 900b = 1185 \\ \underline{-142000a \mp 900b = -1242} \\ -7000a = -57 \end{array}$$

$a = \dfrac{-57}{-7000}$ $= 8.142857143 \times 10^{-3}$ OR $= 0.008142857143$

$$\boxed{\begin{array}{l} a = 8.142857143 \times 10^{-3} \\ a = 0.008142857143 \end{array}}$$ OR

&

$(900 \times 0.008142) + 6b = 7.9$

$7.3278 + 6b = 7.9$

$6b = 7.9 - 7.3278$

$6b = 0.5722$

$b = \dfrac{0.5722}{6}$

$\boxed{b = 0.09536666}$

$$\boxed{\begin{array}{l} \therefore y = ap + b \\ y = 0.008142p + 0.095366 \end{array}}$$

(c) Given:- $9x+y=\lambda$ and $4x+y=\mu$ & $\bar{x}=2$
$\bar{y}=-3$.

As both Regression lines pass through mean point $(\bar{x},\bar{y})$, we have

$9\bar{x}+\bar{y}=\lambda$ and $4\bar{x}+\bar{y}=\mu$

$9(2)+(-3)=\lambda$ $\quad$ $4(2)+(-3)=\mu$

$\boxed{\lambda=15}$ $\quad$ $\boxed{\mu=5}$

∴ Regression Lines becomes
$9x+y=15$ and $4x+y=5$

Let $9x+y=15$, be the regression line of x on y, i.e
$(x-\bar{x})=r\frac{\sigma_x}{\sigma_y}(y-\bar{y})$
$(x-\bar{x})=b_{xy}(y-\bar{y})$

∴ we have

$9x=15-y$

$x=\frac{15}{9}-\frac{y}{9}$

$x-\bar{x}=\frac{15}{9}-\frac{y}{9}-\bar{x}$

$(x-2)=\frac{15}{9}-2-\frac{y}{9}$

$(x-2)=\frac{-3}{9}-\frac{y}{9}$

i.e $(x-2)=-\frac{1}{9}(y+3)$ —— (A)

Comparing (A) with Regression lines x on y, we get

$\boxed{b_{xy}=\frac{-1}{9}}$;

Similarly let $4x+y=5$, be regression line of y on x. i.e $y-\bar{y}=r\frac{\sigma_y}{\sigma_x}(x-\bar{x})$ OR

$y-\bar{y}=b_{yx}(x-\bar{x})$

$4x+y=5$

$y=5-4x$

$y-\bar{y}=5-4x-\bar{y}$

$y-(-3)=5-4x-(-3)$

$(y+3)=8-4x$

$(y+3)=-4(x-2)$ —— (B)

ie comparing (B) with Regression line y on x weget

$\boxed{b_{yx}=-4}$

But $r^2=b_{xy}\times b_{yx}=-\frac{1}{9}\times -4$

$r^2=\frac{4}{9}$

OR $r=\pm\frac{2}{3}$

But $\boxed{r=\frac{-2}{3}}$; ∵ both b_{xy} & b_{yx} are -ve.

Q3. (a) Given :- $A=5$; $\mu_1'=\cancel{5}\,2$, $\mu_2'=20$; $\mu_3'=40$

$\mu_4'=50$.

To find; $\mu_1, \mu_2, \mu_3, \mu_4$ and β_1, β_2

$\boxed{\mu_1=0}$ always

$\mu_2=\mu_2'-(\mu_1')^2=20-(\cancel{5}\,2)^2=\cancel{20-25}=20-4$

$\boxed{\cancel{\mu_2=-5}}$ $\boxed{\mu_2=16}$

$\mu_3=\mu_3'-3\mu_2'\mu_1'+2(\mu_1')^3$

$=40-3(20)(\cancel{5}\,2)+2(\cancel{5}\,2)^3=40-120+16$

$=40-300+\cancel{}250$

$\boxed{\mu_3=-64}$

$\boxed{\cancel{\mu_3=-10}}$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$

$= (50) - 4(40)(2) + 6(20)(2)^2 - 3(2)^4$

$= 50 - 320 + 480 - 48$

$\boxed{\mu_4 = 162}$

$\beta_1 = \frac{(\mu_3)^2}{(\mu_2)^3} = \frac{(-64)^2}{(16)^3} = \frac{4^2 \times 4^2 \times 4^2}{4^3 \times 4^3} = \boxed{1 = \beta_1}$

$\beta_2 = \frac{\mu_4}{(\mu_2)^2} = \frac{162}{(16)^2} = \frac{162}{256} = \boxed{0.6328 = \beta_2}$

(b) Given

x	y	x^2	x^3	x^4	xy	x^2y
0	2	0	0	0	0	0
1	2	1	1	1	2	2
2	4	4	8	16	8	16
3	8	9	27	81	24	72

$\Sigma xy = 34$

$\Sigma x^2y = 90$

$n = 4$

$\Sigma x = 6$; $\Sigma y = 16$; $\Sigma x^2 = 14$; $\Sigma x^3 = 36$; $\Sigma x^4 = 98$;

To get $y = ax^2 + bx + c$; we need to solve

$a\Sigma x^2 + b\Sigma x + cn = \Sigma y$ — (1)

$a\Sigma x^3 + b\Sigma x^2 + c\Sigma x = \Sigma xy$ — (2)

$a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2 = \Sigma x^2y$ — (3) ; we get

~~$14x^2 +$~~

$14a + 6b + 4c = 16$

$36a + 14b + 6c = 34$

$98a + 36b + 14c = 90$

$\Rightarrow a = 1,\ b = -1,\ c = 2$

$\Rightarrow \boxed{y = x^2 - x + 2}$

ⓒ Given:- $n = 10$; $\Sigma x = 40$; $\Sigma x^2 = 190$; $\Sigma y^2 = 200$.
$\Sigma xy = 150$; $\Sigma y = 40$

$$\text{coefficient of correlation } (r) = \frac{[\Sigma xy] - \frac{1}{n} \bar{x}\bar{y}}{\sqrt{\Sigma x^2 - \frac{1}{n}\bar{x}^2} \sqrt{\Sigma y^2 - \frac{1}{n}\bar{y}^2}}$$

$\therefore \bar{x} = \frac{\Sigma x}{n} = \frac{40}{10} = 4$ & $(\bar{x})^2 = 16$

$\bar{y} = \frac{\Sigma y}{n} = \frac{40}{10} = 4$ & $(\bar{y})^2 = 16$

$$\therefore r = \frac{(150) - \left(\frac{4 \times 4}{10}\right)}{\sqrt{190 - \frac{16}{10}} \sqrt{200 - \frac{16}{10}}}$$

$$r = \frac{148.4}{\sqrt{188.4 \times 198.4}} = \frac{148.4}{\sqrt{37378.56}}$$

$$r = \frac{148.4}{193.3353563}$$

$$r = 0.767578152 \approx \boxed{0.767 = r}$$

Q4. ⓐ Given:-

Bag 1: 2W and 3R

Bag 2: 4W and 5R

~~to find~~: Event A := Drawing a red Ball from B_1 & putting in B_2

$$P(A) = \frac{3}{5}$$

Similarly Event (B) = drawing white Ball from B_1

$$P(B) = \frac{2}{5}$$

After putting one ball in B_2; ~~ueg~~ total no. of Balls in Bag 2 after Event A/B is '10'

$\therefore$ P(getting 1R ball out of Bag 2)

$= P(1R \text{ after } A) + P(1R \text{ after } B)$

$= \left(\frac{6}{10} \times \frac{3}{5}\right) + \left(\frac{5}{10} \times \frac{2}{5}\right)$

$= \frac{18}{50} + \frac{10}{50}$

$= \frac{28}{50} = \frac{56}{100}$

$= \underline{\underline{0.56}}$

(b) Given:- Expected no. of matches India won out of 5 is 3

$\Rightarrow P(\text{india won a match}) = \frac{3}{5} = p$

$\Rightarrow P(\text{india loss a match}) = \frac{2}{5} = q$ Here $p+q=1$

$\therefore$ P(india wins the 5-match series.)

$= P(\text{india won all 5 match}) + P(\text{india won 4 \& lost 1})$
$+ P(\text{india won 3, lost 2})$

$= \cancel{B(5,}$

$= {}^5C_5 (p)^5 (q)^0 + {}^5C_4 (p)^4 (q)^1 + {}^5C_3 (p)^3 (q)^2$

$= \left(\frac{3}{5}\right)^5 + 5\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right) + 10\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^2$

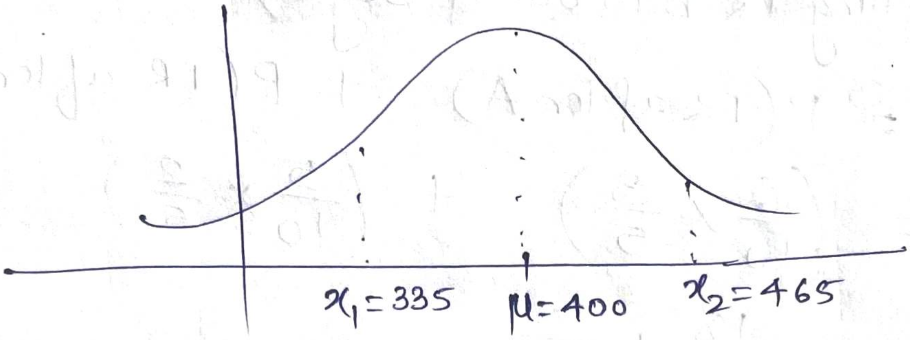
$= \left(\frac{1}{5}\right)^5 \left[3^5 + 3^4.5^1.2 + 10.3^3.2^2\right]$

$= \frac{3^3}{5^5}\left[9 + 30 + 40\right] = \frac{27 \times 79}{3125} = \frac{2133}{3125}$

P(india won 5-match series) $= \boxed{0.68256}$ Ans

(c) Given :- μ = mean = 400, σ = S.D = 50

To find :- No. of items out of 2000 b/w 335 to 465 hr



for $x_1 = 335$

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{335 - 400}{50} = \frac{-65}{50} = -1.3 \approx 1.3$$

$A_1 = 0.4032$ = Area b/w x_1 and μ

Similarly for $x_2 = 465$

$$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{465 - 400}{50} = \frac{65}{50} = 1.3$$

$A_2 = 0.4032$ = Area b/w x_2 and μ

$\therefore$ Prob. of items b/w x_1 & x_2 = $A_1 + A_2$

$= 0.4032 + 0.4032$

$P(x_1 \leq x \leq x_2) = 0.8064$

$\therefore$ No. of items out of ~~b/w~~ 2000 b/w x_1 & x_2

is $= P(x_1 \leq x \leq x_2) \times 2000$

$= 0.8064 \times 2000$

$= 1612.8$

$\boxed{\approx 1613 \text{ (approx)}}$

Q5. (a) If a fair coin is been tossed 3 times

no. of possible out comes = $2 \times 2 \times 2 = 8$

OR $|\text{sample space}| = 8$

let $P(0 \text{ Head}) = \frac{1}{8}$

$P(1 \text{ Head}) = \frac{3}{8}$

$P(2 \text{ Head}) = \frac{3}{8}$

$P(3 \text{ Head}) = \frac{1}{8}$

$\therefore$ Mathematical Expectation $E(x) = \sum_{i=0}^{n} x_i P(x_i)$

$$\therefore E(\text{Head}) = \sum_{i=0}^{3} x_i P(x_i)$$

$$= \left(0 \times \frac{1}{8}\right) + \left(1 \times \frac{3}{8}\right) + \left(2 \times \frac{3}{8}\right) + \left(3 \times \frac{1}{8}\right)$$

$$= 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8}$$

$$= \frac{12}{8} = \frac{3}{2}$$

OR $\boxed{E(\text{Head}) = 1.5}$

$\therefore$ Expected value of number of Heads in 3 toss is equal to 1.5

(b) Given:- $\lambda = 180/hr \Rightarrow \lambda = 3$ car/min

To find:- Prob of atleast 2 car pass in one min.

$\Rightarrow$ P(atleast 2 car) $= P(x \geq 2)$

$$= 1 - [P(x=0) + P(x=1)]$$

$$= 1 - \left[\frac{e^{-3}(3)^0}{0!} + \frac{e^{-3}(3)^1}{1!}\right]$$

$$= 1 - [e^{-3} + e^{-3}(3)]$$

$$= 1 - \left[\frac{4}{e^3}\right]$$

$$= 1 - 0.1991$$

$$= 0.80085 \approx 0.8$$

P(atleast 2 car pass one min) $\approx \boxed{0.8}$

(c) Given:- Let Ho: The proportions of blood type O, A, B & AB are in ratio 49 : 38 : 9 : 4

observed in 200 people is Blood type O, A, B and AB, 88, 80, 22 and 10, whereas, expected frequencies should be in ratio 49 : 38 : 9 : 4

i.e $49x + 38x + 9x + 4x = 200$

$100x = 200$

OR $\boxed{x = 2}$

$\Rightarrow$ $49x = 98$
$38x = 76$
$9x = 18$
$4x = 8$

$\therefore$

observed freq	expected frequency
88	98
80	76
22	18
10	8

Here $n = 4$

$\therefore \chi^2_{n-1} = \chi^2_3 = \sum_{i=1}^{n} \frac{(o_f - e_f)^2}{e_f}$

$= \frac{(88-98)^2}{98} + \frac{(80-76)^2}{76} + \frac{(22-18)^2}{18} + \frac{(10-8)^2}{8}$

$= \frac{100}{98} + \frac{16}{76} + \frac{16}{18} + \frac{4}{8}$

$\boxed{\chi^2_3 = 2.6198}$

Given $\chi^2_{3;0.05} = 7.815$

As
$\therefore \chi^2_3 < \chi^2_{3;0.05}$

$\therefore$ we accept H_0 at 5% level of significance

$\therefore$ The ~~data~~ observed data of blood type is accordance with the ratio.

Q6.

(a). Given:- $x^4 + 2x^3 - x - 1 = 0$; $x_0 = 0$, $x_1 = 1$

To find:- A root between $[0, 1]$

Solution:- let $f(x) = x^4 + 2x^3 - x - 1$

$\boxed{f(0) = -1}$ and $f(1) = (1)^4 + 2(1)^3 - (1) - 1$

$= 1 + 2 - 2 =$

$\boxed{f(1) = 1}$

$\therefore$ Root lies b/w $x = 0$ & $x = 1$, ($\because f(0)$ & $f(1)$ are of opposite sign)

1st iteration:- using Bisection method

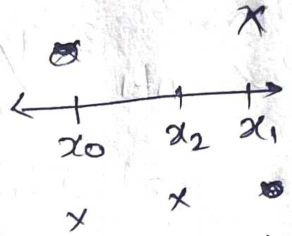
OR 1st step:-

$x_2 = \frac{x_0 + x_1}{2} = \frac{0+1}{2} = 0.5$

$\boxed{x_2 = 0.5}$

$f(x_2) = (0.5)^4 + 2(0.5)^3 - (0.5) - 1$

$= 0.0625 + 0.25 - 1.5$



$\boxed{f(x_2) = -1.1875}$, i.e. $-ve$

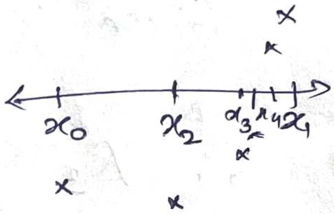
$\therefore$ Root lies b/w x_1 and x_2, $\therefore$ Using Bisection method

$\boxed{\text{2nd iteration}}$/step :- $x_3 = \dfrac{x_1 + x_2}{2}$

$x_3 = \dfrac{1 + 0.5}{2} = \dfrac{1.5}{2} = \boxed{0.75 = x_3}$

$f(x_3) = (0.75)^4 + 2(0.75)^3 - (0.75) - 1$

$= (0.3164) + (0.84375) - 1.75$



$\boxed{f(x_3) = -0.58985}$, i.e. $-ve$ / $f(x_3) < 0$

$\Rightarrow$ Root lies b/w x_1 and x_3

$\therefore$ Using Bisection method.

$\boxed{\text{3rd step}}$: $x_4 = \dfrac{x_1 + x_3}{2} = \dfrac{1 + 0.75}{2} = \boxed{0.875 = x_4}$

$f(x_4) = (0.875)^4 + 2(0.875)^3 - (0.875) - 1$

$= (0.5862) + 1.3399 - 1.875$

$\boxed{f(x_4) = 0.05104375}$, i.e. $+ve$, $f(x_4) > 0$

$\therefore$ Root lies between x_3 & x_4

$\therefore$ Using Bisection method, we get

$\boxed{\text{4th step}}$: $x_5 = \dfrac{x_3 + x_4}{2} = \dfrac{0.75 + 0.875}{2} = \boxed{0.8125 = x_5}$

$f(x_5) = (0.8125)^4 + 2(0.8125)^3 - (0.8125) - 1$

$= 0.4358 + 1.07275 - 1.8125$

$\boxed{f(x_5) = -0.30395}$, i.e. $-ve$

Here $f(x_5) < 0$

$\Rightarrow$ Root lies b/w x_4 & x_5

Using Bisection method, we get

~~x_6~~ Fifth step/iteration

$$x_6 = \frac{x_4 + x_5}{2} = \frac{0.875 + 0.8125}{2} = 0.84375$$

$\therefore \boxed{x_6 = 0.84375}$

$\therefore$ The root ~~a~~ at the end of fifth iteration is 0.84375

(b). Given:- $x^3 + 2x - 5 = 0$

To find :- A real root ~~afte~~ after 5th iteration

Sol.:- Let $f(x) = x^3 + 2x - 5$

check for $x=0$; $f(0) = (0)^3 + 2(0) - 5 = -5$

for $x=1$, $f(1) = (1)^3 + 2(1) - 5 = 1+2-5 = -2$

$x=2$, $f(2) = (2)^3 + 2(2) - 5 = 8+4-5 = 7$

$\therefore$ Here $f(1) \times f(2) < 0$ OR $f(1)$ & $f(2)$ has opposite sign

$\therefore$ Root lies b/w $x=1$ and $x=2$

$f'(x) = 3x^2 + 2$ and $f''(x) = 6x$

check for x_0 (the starting point)

$f(1) \cdot f''(1) = (-2) \times 6(1) = -12 < 0$

$f(2) \cdot f''(2) = (7) \times 6(2) = 19 > 0$

$\Rightarrow \boxed{x_0 = 1}$ $\therefore$ $f(1) \cdot f''(1)$ is -ve

Step 1:- Using Newton Raphson

$x^3+2x-5 \quad 3x^2+2$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1 - \frac{-2}{5}$$

$$= 1 + \frac{2}{5}$$

$$\boxed{x_1 = \frac{7}{5}} \boxed{= 1.4}$$

Step 2:- Using N.R, we get

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \left(\frac{7}{5}\right) - \frac{f(7/5)}{f'(7/5)}$$

$$x_2 = \left(\frac{7}{5}\right) - \frac{0.544}{7.88} = 1.4 - 0.069$$

$$\boxed{x_2 = 1.331}$$

Step 3:- Using N.R, we get

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.331 - \frac{f(1.331)}{f'(1.331)}$$

$$= 1.331 - \frac{0.01994769}{7.314683}$$

$$= 1.331 - 0.002727$$

$$\boxed{x_3 = 1.328273} \approx \boxed{1.328 = x_3}$$

Step 4:- Using NR, we get

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = (1.328) - \frac{f(1.328)}{f'(1.328)}$$

$$= 1.328 - \frac{(-0.00196)}{7.29}$$

$$= 1.328 + 0.0002689$$

$\boxed{x_4 = 1.3283}$ approximately similar to x_3

Step 5:- Using N.R method, we get

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)}$$

$$= (1.3283) - \frac{f(1.3283)}{f'(1.3283)}$$

$$= 1.3283 - \frac{0.00022714}{7.29314267}$$

$$= 1.3283 - 0.000031144$$

$$x_5 = 1.32826885 \approx 1.3283$$

$$\boxed{x_5 = 1.3283}$$

$\therefore$ Root after 5th iteration is 1.3283

(c) Given:- $20x_1 + x_2 - 2x_3 = 17$ — (1)

$3x_1 + 20x_2 - x_3 = -18$ — (2)

$2x_1 - 3x_2 + 20x_3 = 25$ — (3)

To find:- Solution using Gauss - Seidel [Already Diagonal Dominent]

Sol:- Find ~~x~~ eq. of x_1, x_2 & x_3 from eq (1), (2), (3)

we get $x_1 = \frac{1}{20}[17 - x_2 + 2x_3]$ — (4)

$x_2 = \frac{1}{20}[-18 - 3x_1 + x_3]$ — (5)

$x_3 = \frac{1}{20}[25 - 2x_1 + 3x_2]$ — (6)

Step 1:- Assume $x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0$

$\therefore x_1^{(1)} = \frac{1}{20}[17 - x_2^{(0)} + 2x_3^{(0)}]$

$= \frac{1}{20}[17 - 0 + 0] = \frac{17}{20} = \boxed{0.85 = x_1^{(1)}}$

$x_2^{(1)} = \frac{1}{20}[-18 - 3x_1^{(1)} + x_3^{(0)}]$

$= \frac{1}{20}[-18 - 3(0.85) + 0] = \boxed{-1.0275 = x_2^{(1)}}$

$x_3^{(1)} = \frac{1}{20}[25 - 2x_1^{(1)} + 3x_2^{(1)}]$

$= \frac{1}{20}[25 - 2(0.85) + 3(-1.0275)]$

$\boxed{x_3^{(1)} = 1.0109}$

Step 2:- Values are $x_1^{(1)} = 0.85$, $x_2^{(1)} = -1.0275$ &

$x_3^{(1)} = 1.0109$

$\therefore x_1^{(2)} = \frac{1}{20}[17 - x_2^{(1)} + 2x_3^{(1)}]$

$= \frac{1}{20}[17 - (-1.0275) + 2(1.0109)]$

$\boxed{x_1^{(2)} = 1.0025}$

Similarly

$x_2^{(2)} = \frac{1}{20}[-18 - 3(1.0025) - (1.0109)]$

$\boxed{x_2^{(2)} = -0.9998}$

$x_3^{(2)} = \frac{1}{20}[25 - 2(1.0025) + 3(-0.9998)]$

$\boxed{x_3^{(2)} = 0.9998}$

Step 3 :- values be $x_1^{(2)} = 1.0025$, $x_2^{(2)} = -0.9998$
$x_3^{(2)} = 0.9998$

$\therefore x_1^{(3)} = \frac{1}{20}[17 - (-0.9998) + 2(0.9998)]$

$\boxed{x_1^{(3)} = 1}$

$x_2^{(3)} = \frac{1}{20}[-18 - 3(1) + (0.9998)]$

$\boxed{x_2^{(3)} = -1}$

$x_3^{(3)} = \frac{1}{20}[25 - 2(1) + 3(-1)]$

$\boxed{x_3^{(3)} = 1}$

$\therefore$ solution of given system after 3 iteration is

$\boxed{x_1 = 1}$, $\boxed{x_2 = 1}$ & $\boxed{x_3 = 1}$

Q7. (a) Given:- $2x_1 + x_2 + x_3 = 10$ — (1)
$3x_1 + 2x_2 + 3x_3 = 18$ — (2)
$x_1 + 4x_2 + 9x_3 = 16$ — (3)

To find:- solution using Gauss - Elimination.

Sol:- After rearranging eq. of the system, we get

$x_1 + 4x_2 + 9x_3 = 16$ —(1)

$3x_1 + 2x_2 + 3x_3 = 18$ —(2)

$2x_1 + x_2 + x_3 = 10$ —(3)

Apply Eq. ② − 3 Eq① &

Eq. ③ − ~~2~~ Eq① , we get

$$x_1 + 4x_2 + 9x_3 = 16 \quad -①$$

$$0x_1 - 10x_2 - 24x_3 = -30 \quad -④$$

$$0x_1 - 7x_2 - 17x_3 = -22 \quad -⑤$$

Divide Eq④ by −10 and Eq⑤ by −1, we get

$$x_1 + 4x_2 + 9x_3 = 16 \quad -①$$

$$x_2 + 2.4x_3 = 3 \quad -④$$

$$7x_2 + 17x_3 = 22 \quad -⑤$$

Apply Eq⑤ − 7 Eq④ , we get

$$x_1 + 4x_2 + 9x_3 = 16 \quad -①$$

$$1x_2 + \frac{24}{10}x_3 = 3 \quad -④$$

$$0x_2 + \frac{2}{10}x_3 = 1 \quad -⑥$$

From eq⑥, we get $\frac{2}{10}x_3 = 1 \Rightarrow x_3 = \frac{10}{2} \Rightarrow \boxed{x_3 = 5}$

From eq④, we get $x_2 + \frac{24}{10}x_3 = 3$

$$x_2 + \left(\frac{24}{10} \times 5\right) = 3$$

$$x_2 = 3 - 12 \Rightarrow \boxed{x_2 = -9}$$

From eq① , we get $x_1 + 4[-9] + 9[5] = 16$

$$x_1 = 16 - 45 + 36$$

$$\boxed{x_1 = 7}$$

Hence solved.

(b) Given:- $4x_1 + 2x_2 + x_3 = 14$ —— (1)

$x_1 + 5x_2 - x_3 = 10$ —— (2)

$x_1 + x_2 + 8x_3 = 20$ —— (3)

To find:- Solve using Jacobi's iteration

Sol:- Already Diagonal Dominent,

$\therefore$ find x_1, x_2, x_3 from eq (1), (2) & (3).

$$x_1 = \frac{1}{4}\left[14 - 2x_2 - x_3\right]$$

$$x_2 = \frac{1}{5}\left[10 - x_1 + x_3\right]$$

$$x_3 = \frac{1}{8}\left[20 - x_1 - x_2\right]$$

$\boxed{\text{step 1}}$:- Assume $x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0$, ~~we~~

$$\therefore x_1^{(1)} = \frac{1}{4}\left[14 - 2x_2^{(0)} - x_3^{(0)}\right] = \frac{14}{4} = \frac{7}{2} = \boxed{3.5 = x_1^{(1)}}$$

$$x_2^{(1)} = \frac{1}{5}\left[10 - x_1^{(0)} + x_3^{(0)}\right] = \frac{10}{5} = \boxed{2 = x_2^{(1)}}$$

$$x_3^{(1)} = \frac{1}{8}\left[20 - x_1^{(0)} - x_2^{(0)}\right] = \frac{20}{8} = \boxed{2.5 = x_3^{(1)}}$$

$\boxed{\text{step 2}}$:- values are $\boxed{x_1^{(1)} = 3.5}$, $\boxed{x_2^{(1)} = 2}$, $\boxed{x_3^{(1)} = 2.5}$

$$\therefore x_1^{(2)} = \frac{1}{4}\left[14 - 2x_2^{(1)} - x_3^{(1)}\right] = \frac{1}{4}\left[14 - 2(2) - 2.5\right]$$

$$= \frac{1}{4}\left[7.5\right] = \boxed{1.875 = x_1^{(2)}}$$

$$x_2^{(2)} = \frac{1}{5}\left[10 - x_1^{(1)} + x_3^{(1)}\right] = \frac{1}{5}\left[10 - 3.5 + 2.5\right]$$

$$= \frac{9}{5} = \boxed{1.8 = x_2^{(2)}}$$

$$x_3^{(2)} = \frac{1}{8}\left[20 - x_1^{(1)} - x_2^{(1)}\right] = \frac{1}{8}\left[20 - 3.5 - 2\right]$$

$$\boxed{x_3^{(2)} = 1.8125}$$

$\boxed{\text{step 3}}$

$$x_1^{(3)} = \frac{1}{4}\left[14 - 2x_2^{(2)} - x_3^{(2)}\right]$$

$$= \frac{1}{4}\left[14 - 2(1.8) - (1.8125)\right]$$

$$= \frac{8.5875}{4}$$

$$\boxed{x_1^{(3)} = 2.146875} \approx \boxed{x_1^{(3)} = 2.15}$$

$$x_2^{(3)} = \frac{1}{5}\left[10 - x_1^{(2)} + x_3^{(2)}\right]$$

$$= \frac{1}{5}\left[10 - 1.875 + 1.8125\right]$$

$$= \frac{9.9375}{5} = \boxed{1.9875 = x_2^{(3)}}$$

$$x_3^{(3)} = \frac{1}{8}\left[20 - x_1^{(2)} - x_2^{(2)}\right]$$

$$= \frac{1}{8}\left[20 - 1.875 - 1.8\right] = \frac{16.325}{8}$$

$$\therefore \boxed{x_3^{(3)} = 2.04}$$

~~Hence Solved~~

$\therefore$ Solution of the system after 3 iteration is

$x_1 = 2.15$, $x_2 = 1.9875$, $x_3 = 2.04$

Hence solved

(c) Given:- $e^x - 4x = 0$

To find:- A real solution, using Regula-falsi

Sol:- Let $f(x) = e^x - 4x$

check for $x=0$; $f(0) = e^0 - 4(0) = \boxed{1 = f(0)}$

$x = 1$; $f(1) = e^1 - 4(1) = \boxed{-1.28 = f(1)}$

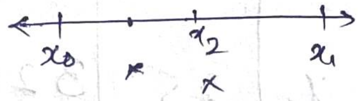
Here, $f(0) \times f(1) < 0$ (OR) $f(0)$ & $f(1)$ are of opposite sign

$\therefore$ Root lie between $x_0 = 0$ and $x_1 = 1$

Step 1 $\therefore$ $x_2 = x_1 - \left[\dfrac{x_1 - x_0}{f(x_1) - f(x_0)} \times f(x_1)\right]$

$x_2 = 1 - \left[\dfrac{1-0}{-1.28 - 1} \times (-1.28)\right]$

$\boxed{x_2 = 0.439}$



$f(x_2) = f(0.439) = -0.2048 < 0$

$\therefore$ Root lie b/w x_0 and x_2

Step 2 $x_3 = x_2 - \left[\dfrac{x_2 - x_0}{f(x_2) - f(x_0)} \times f(x_2)\right]$

$= 0.439 - \left[\dfrac{0.439 - 0}{-0.2048 - 1} \times (-0.2048)\right]$

$\boxed{x_3 = 0.3643}$

$f(x_3) = f(0.3643) = -0.0177 < 0$

$\therefore$ Root lie b/w x_0 and x_3.

Step 3 $x_4 = x_3 - \left[\dfrac{x_3 - x_0}{f(x_3) - f(x_0)} \times f(x_3)\right]$

$= 0.3643 - \left[\dfrac{0.3643 - 0}{-0.0177 - 1} \times (-0.0177)\right]$

$\boxed{x_4 = 0.35796}$

continue till max 5 steps

Q8. (a)

Sol:- ~~Given~~:- Forward Difference Table is as follows:-

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y_0$
x_0	0	$7 = y_0$					
			$4 = \Delta y_0$				
x_1	~~8~~ 5	$11 = y_1$		$-1 = \Delta^2 y_0$			
			$3 = \Delta y_1$		$2 = \Delta^3 y_0$		
x_2	10	$14 = y_2$		$1 = \Delta^2 y_1$		$-1 = \Delta^4 y_0$	
			$4 = \Delta y_2$		$1 = \Delta^3 y_1$		$0 = \Delta^5 y_0$
x_3	15	$18 = y_3$		$2 = \Delta^2 y_2$		$-1 = \Delta^4 y_1$	
			$6 = \Delta y_3$		$0 = \Delta^3 y_2$		
x_4	20	$24 = y_4$		$2 = \Delta^2 y_3$			
			$8 = \Delta y_4$				
x_5	25	$32 = y_5$					

Here, $\boxed{x_0 = 0}$; $h = x_1 - x_0 = 5 - 0 = \boxed{5 = h}$

and $\boxed{u = \frac{x - x_0}{h}} = \frac{x-0}{5} = \left(\frac{x}{5}\right) = u$; $\boxed{u = \frac{x}{5}}$

$\therefore$ By Newtons Forward difference interpolation formula, we get

$$y = y_0 + u\,\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$+ \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + 0 \quad \text{——} (*)$$

we want y at $x = 8 \Rightarrow \boxed{u = \frac{8}{5}}$

$\therefore$ (*) Becomes

$$y = y_0 + \left(\frac{8}{5}\right)\times 4 + \frac{1}{2}\left(\frac{8}{5}\right)\left(\frac{3}{5}\right)(-1) + \frac{1}{6}\left(\frac{8}{5}\right)\left(\frac{3}{5}\right)\left(\frac{-2}{5}\right)(2)$$

at $x=8$

$$+ \frac{1}{24}\left(\frac{8}{5}\right)\left(\frac{3}{5}\right)\left(\frac{-2}{5}\right)\left(\frac{-7}{5}\right)(1) + 0$$

$y_{at\, x=8} = (7) + (6.4) + (-0.48) + (-0.128) + (-0.0224)$

$= 12.7696$

$\therefore$ y at $x=8$ is 12.77 (approx)

(b) Given:- $\int_0^1 \frac{1}{x^2+1}\,dx$; $h = 0.2$

Sol:- Here $y = \frac{1}{x^2+1}$; $x_0 = 0$; $x_1 = x_0 + h = 0 + 0.2 = 0.2$ and so on.

$\therefore y_0 = \frac{1}{x_0^2+1} = \frac{1}{0+1} = 1$;

$y_1 = \frac{1}{(0.2)^2+1} = \frac{1}{0.04+1} = \frac{1}{1.04} = 0.96154$

$y_2 = \frac{1}{(0.4)^2+1} = \frac{1}{1.16} = 0.862$

$y_3 = \frac{1}{(0.6)^2+1} = \frac{1}{1.36} = 0.7353$

$y_4 = \frac{1}{(0.8)^2+1} = \frac{1}{1.64} = 0.61$

$y_5 = \frac{1}{1^2+1} = \frac{1}{2} = 0.5$

Using $\frac{1}{3}$rd simpson rule, we get

$$\int_{x_0}^{x_5} y\,dx = \frac{1}{3} \times h\left[(y_0+y_5) + 2(y_2+y_4) + 4(y_1+y_3)\right]$$

$$= \left(\frac{1}{3} \times 0.2\right)\left[(1+0.5) + 2(0.862+0.61) + 4(0.96154+0.7353)\right]$$

$$= \frac{2}{30}\left[1.5 + 2.944 + 6.78736\right]$$

$$= \frac{2}{30} \times 11.23136$$

$$\boxed{\int_0^1 \frac{1}{x^2+1}\,dx = 0.74875733 \approx 0.75}$$ Hence Solved.

(c) Given:- $\frac{dy}{dx} = x+y$, $y(0)=1$;, $h=0.1$

Sol: To find:- y for $x=0$ to $x=0.3$,

Sol:- Here $f(x,y) = \frac{dy}{dx} = x+y$;

$x_0 = 0$; $y_0 = 1$, $h=0.1$ & $x_1 = 0.1$, $x_2 = 0.2$, $x_3 = 0.3$

∴ Using Euler formula

$$y_1 = y_0 + h \cdot f(x_0, y_0) \quad \text{—— (1)}$$

$$f(x_0, y_0) = x_0 + y_0 = 0+1 = 1$$

$$\therefore y_1 = 1 + [0.1 \times 1] = 1 + 0.1 = \boxed{1.1 = y_1} \text{ at } x_1$$

$$f(x_1, y_1) = x_1 + y_1 = 0.1 + 1.1 = 1.2$$

∴ Using Euler formula.

$$y_2 = y_1 + h\, f(x_1, y_1)$$

$$y_2 = 1.1 + [0.1 \times 1.2]$$

$$= 1.1 + 0.12$$

$$\boxed{y_2 = 1.22} \text{ at } x_2$$

$$f(x_2, y_2) = x_2 + y_2 = 0.2 + 1.22 = 1.42$$

$$\therefore y_3 = y_2 + h\, f(x_2, y_2)$$

$$= 1.22 + [0.1 \times 1.42]$$

$$= 1.22 + 0.142$$

$$\boxed{y_3 = 1.362}$$

∴ Tabulating the values as follows

x	$x_0 = 0$	$x_1 = 0.1$	$x_2 = 0.2$	$x_3 = 0.3$
y	$y_0 = 1$	$y_1 = 1.1$	$y_2 = 1.22$	$y_3 = 1.362$

Q9.

(a) Given :- $\frac{dy}{dx} = xy$, $y(1) = 2$, $h = 0.2$

To find :- y at $x = 1.2$, i.e y_1 at $x_1 = 1.2$

Sol :- Here $f(x,y) = xy$; $x_0 = 1$, $y_0 = 2$, $h = 0.2$

$\therefore x_1 = x_0 + h = 1 + 0.2 = 1.2 = x_1$

Using R.K 4th order, we have

$y_1 = y_0 + K$; where $K = \frac{K_1 + 2K_2 + 2K_3 + K_4}{6}$

$\therefore K_1 = h \cdot f(x_0, y_0)$

$= 0.2 \times x_0 y_0$

$= 0.2 \times 1 \times 2$

$\boxed{K_1 = 0.4}$

$K_2 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$

$= 0.2 \left[f\left(1 + \frac{0.2}{2}, 2 + \frac{0.4}{2}\right)\right]$

$= 0.2 \left[f(1 + 0.1, 2 + 0.2)\right]$

$= 0.2 \left[f(1.1, 2.2)\right]$

$= 0.2 \times 1.1 \times 2.2$

$\boxed{K_2 = 0.484}$

$K_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$

$= 0.2 \, f(1.1, 2 + 0.242)$

$= 0.2 \times 1.1 \times 2.242$

$\boxed{K_3 = 0.49324}$

$K_4 = h\, f(x_1, y_0 + K_3)$

$= 0.2\, f(1.2, 2.49324)$

$= 0.2 \times 1.2 \times 2.49324$

$\boxed{K_4 = 0.5984}$

$\therefore K = \dfrac{K_1 + 2K_2 + 2K_3 + K_4}{6}$

$= \dfrac{0.4 + (2 \times 0.484) + (2 \times 0.49324) + (0.5984)}{6}$

$= \dfrac{0.4 + 0.968 + 0.98648 + 0.5984}{6}$

$= \dfrac{2.95288}{6}$

$\boxed{K = 0.49214667} \approx 0.49215$

$\therefore y_1 = y_0 + K$

$= 2 + (0.49215)$

$\boxed{y_1 = 2.49215}$ at $x = 1.2$

Hence solved

(b) Given:- $\dfrac{dy}{dx} + xy^2 = 0 \Rightarrow \dfrac{dy}{dx} = -xy^2$

$y(0) = 2$, $h = 0.2$

To find:- y_1 at $x_1 = 0.2$

Sol:- Here $f(x,y) = -xy^2$, $x_0 = 0$, $y_0 = 2$, $h = 0.2$

$\therefore f(x_0, y_0) = -0 \times (2)^2 = 0$

$\therefore$ Using Euler method

$y_1 = y_0 + h\, f(x_0, y_0)$

$$y_1 = 2 + [0.2 \times 0] = 2$$

$$\therefore \boxed{y_1^{(0)} = 2}$$

$$\therefore y_1^{(1)} = y_0 + h\left[\frac{f(x_0, y_0) + f(x_1, y_1^{(0)})}{2}\right]$$

$$= 2 + \frac{0.2}{2}\left[0 + (-0.2 \times (2)^2)\right]$$

$$= 2 + 0.1[-0.8]$$

$$= 2 - 0.08$$

$$\boxed{y_1^{(1)} = 1.92}$$

$$\therefore f(x_1, y_1^{(1)}) = -x_1 \cdot (y_1^{(1)})^2 = -0.2 \times (1.92)^2$$

$$= -0.73728$$

$$\therefore y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$= 2 + 0.1\left[0 + (-0.73728)\right]$$

$$= 2 - 0.073728$$

$$\boxed{y_1^{(2)} = 1.926272}$$

After two iterations $y_1^{(2)} = y_1$

$\therefore y_1$ at $x = 0.2$ is 1.926272

Hence Solved

(c) Given

Backward Difference Table:-

	x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
x_0	150	$12.247 = y_0$			
			$0.082 = \nabla y_1$		
x_1	152	$12.329 = y_1$		$-0.001 = \nabla^2 y_2$	
			$0.081 = \nabla y_2$		$0 = \nabla^3 y_3$
x_2	154	$12.410 = y_2$		$-0.001 = \nabla^2 y_3$	
			$0.08 = \nabla y_3$		
x_3	156	$12.490 = y_3$			

Here $\boxed{x_0 = 150}$, $h = x_1 - x_0 = 152 - 150 = \boxed{2 = h}$, $x_n = x_3 = 156$

and $u = \frac{x - x_n}{h} = \frac{x - x_3}{h} = \boxed{\frac{x-156}{2} = u}$

To find y at $x = 155$ as $y = \sqrt{x}$

$u = \frac{155-156}{2} = \frac{-1}{2} = -0.5$

By using newtons Backward Difference interpolation formula, we get

$$y = y_3 + u\nabla y_3 + \frac{u(u+1)}{2!}\nabla^2 y_3 + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_3$$

$$y = (12.490) + \left(\frac{x-156}{2}\right).0.08 + \left(\frac{x-156}{2}\right)\left(\frac{x-156}{2}+1\right)\times\frac{1}{2}\times(-0.001)$$

$$+ 0$$

$$y_{at\,x=155} = (12.490) + (-0.5\times 0.08) + \left(\frac{-0.5\times 0.5}{2}\times(-0.001)\right)$$

$$= (12.490) - 0.04 + 0.000125$$

$$y_{at\,x=155} = 12.450125 \approx 12.450$$

i.e $\boxed{y = 12.450}$ at $x = 155$

Hence Solved.