Understanding Sparsity

Friday, June 11, 2021 11:21 AM

Lasso sparsity

$\lambda \uparrow \quad w \to 0$

Ridge

alpha	age	sex	bmi	bp	s1	s2	s3	s4	s5	s6
0.0000	-9.160885	-205.462260	516.684624	340.627341	-895.543609	561.214533	153.884786	126.734316	861.121400	52.419828
0.0001	-9.118336	-205.337133	516.880570	340.556792	-883.415291	551.553259	148.578680	125.355917	856.480254	52.467627
0.0010	-8.763583	-204.321125	518.371729	339.975385	-787.690766	475.274718	106.786540	114.632063	819.739542	52.872100
0.0100	-6.401088	-198.669767	522.048548	336.348363	-383.709187	152.663678	-66.060583	75.611090	659.869402	55.828128
0.1000	6.642753	-172.242166	485.523872	314.682122	-72.939323	-80.590053	-174.466515	83.616653	484.363285	73.584154
1.0000	42.242217	-57.305508	282.170831	198.061386	14.363544	-22.551274	-136.930053	102.023193	260.104308	98.552274
10.0000	21.174004	1.659796	63.659772	48.493240	18.421492	12.875448	-38.915435	38.842464	61.612405	35.505355
100.0000	2.858979	0.629452	7.540604	5.849997	2.710879	2.142134	-4.834047	5.108223	7.448466	4.576129
1000.0000	0.295726	0.069290	0.769004	0.597829	0.282900	0.225936	-0.495607	0.527031	0.761497	0.471029
10000.0000	0.029674	0.006995	0.077054	0.059915	0.028412	0.022715	-0.049686	0.052870	0.076321	0.047241

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0.0000	-9.160885	-205.462260	516.684624	340.627341	-895.543596	561.214523	153.884780	126.734314	861.121395	52.419828
0.0001	-9.071288	-205.337332	516.780313	340.539730	-888.652320	555.952271	150.585260	125.453044	858.639860	52.379002

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0.0010	-8.264924	-204.213177	517.641106	339.751339	-826.653342	508.609613	120.899583	113.924518	836.314382	52.011583
0.0100	-1.361404	-192.944226	526.348511	332.649058	-430.205495	191.277876	-44.048113	68.990747	688.384976	47.939528
0.1000	0.000000	-113.976046	526.737112	292.635423	-82.691928	-0.000000	-152.691332	0.000000	551.077200	7.169852
1.0000	0.000000	0.000000	363.882636	27.278420	0.000000	0.000000	-0.000000	0.000000	336.135971	0.000000
10.0000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	-0.000000	0.000000	0.000000	0.000000
100.0000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	-0.000000	0.000000	0.000000	0.000000
1000.0000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	-0.000000	0.000000	0.000000	0.000000
10000.0000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	-0.000000	0.000000	0.000000	0.000000

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10.0000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	-0.000000	0.000000	0.000000	0.000000
100.0000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	-0.000000	0.000000	0.000000	0.000000
1000.0000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	-0.000000	0.000000	0.000000	0.000000
10000.0000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	-0.000000	0.000000	0.000000	0.000000

$$m = \frac{\sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^{n} (x_i - \bar{x})^2 + \lambda}$$

Simple
x | y

$$y = mx + b$$

$$b = \bar{y} - m\bar{x}$$

$\bar{y} \rightarrow \text{mean}(y)$

$\bar{x} \rightarrow \text{mean}(x)$

$$m = \frac{\sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

$$b = \bar{y} - m\bar{x}$$

$\bar{y} \rightarrow \text{mean}(y)$

$\bar{x} \rightarrow \text{mean}(x)$

$$m = \frac{\sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

$b = \bar{y} - m\bar{x}$ $\quad m = ?$

$$L = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 + \lambda |m|$$

$$\frac{d}{dm} \sum_{i=1}^{n} (y_i - mx_i - \bar{y} + m\bar{x})^2 + 2\lambda m$$

$$\frac{dL}{dm} = \sum_{i=1}^{n} (y_i - mx_i - \bar{y} + m\bar{x})^2 + 2\lambda |m| = 2 \sum (y_i - mx_i - \bar{y} + m\bar{x})(-x_i + \bar{x}) + 2\lambda = 0$$

$$L = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 + \lambda |m|$$

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$$\frac{dL}{dm} = \sum_{i=1}^{n} (y_i - mx_i - \bar{y} + m\bar{x})^2 + 2\lambda |m| = 2\sum (y_i - mx_i - \bar{y} + m\bar{x})(-x_i + \bar{x}) + 2\lambda = 0$$

$$-2\sum [(y_i - \bar{y}) - m(x_i - \bar{x})](x_i - \bar{x}) + 2\lambda = 0$$

$$-\sum [(y_i - \bar{y})(x_i - \bar{x}) - m(x_i - \bar{x})^2] + \lambda = 0$$

$$-\sum (y_i - \bar{y})(x_i - \bar{x}) + m\sum (x_i - \bar{x})^2 + \lambda = 0$$

$$m\sum (x_i - \bar{x})^2 = \sum (y_i - \bar{y})(x_i - \bar{x}) - \lambda$$

$$m = \frac{\sum (y_i - \bar{y})(x_i - \bar{x}) - \lambda}{\sum (x_i - \bar{x})^2}$$

$$-\sum [(y_i-\bar{y})(x_i-\bar{x}) - m(x_i-\bar{x})]$$

$$-\sum (y_i-\bar{y})(x_i-\bar{x}) + m\sum (x_i-\bar{x})^2 + \lambda = 0$$

$$\sum (x_i-\bar{x})^2$$

Lasso coeff

for $m>0$

$$m = \frac{\sum (y_i-\bar{y})(x_i-\bar{x}) - \lambda}{\sum (x_i-\bar{x})^2}$$

for $m=0$

$$m = \frac{\sum (y_i-\bar{y})(x_i-\bar{x})}{\sum (x_i-\bar{x})^2}$$

for $m<0$

$$m = \frac{\sum (y_i-\bar{y})(x_i-\bar{x}) + \lambda}{\sum (x_i-\bar{x})^2}$$

Lasso

for $m > 0$

$$m = \frac{\sum (y_i - \bar{y})(x_i - \bar{x}) - \lambda}{\sum (x_i - \bar{x})^2}$$

($\lambda > 0$)

for $m = 0$

$$m = \frac{\sum (y_i - \bar{y})(x_i - \bar{x})}{\sum (x_i - \bar{x})^2}$$

for $m < 0$

$$m = \frac{\sum (y_i - \bar{y})(x_i - \bar{x}) + \lambda}{\sum (x_i - \bar{x})^2}$$

$m \rightarrow$ 2, $\frac{9}{5}$, 1, 0, -1, 5 $\rightarrow$ 0

$$m = \frac{(yx) - \lambda}{x^2}$$

$\lambda \uparrow$

$y_{x=100}$

$x^2 = 50$

$\lambda = 100$

$m = 0$

$\lambda > 100$

$m = -1$

150

150

$$m = \frac{100 - \lambda}{50}$$

$\lambda = 0$ | $\lambda = 10$

$m = 2$ | $m = \frac{9}{5}$

$\lambda = 50$ $m = 1$

$$m = \frac{yx + \lambda}{x_2} = \frac{100 + \lambda}{50}$$

$$= \frac{100 + 150}{50} =$$

$m = 5$

$m = \frac{100 - \lambda}{50}$ $m = 2$ | $m = \frac{9}{5}$ $\lambda = 50$ $m = 1$

$m = -1$ 150

$= \frac{100 + 150}{50} =$ $m = 5$

I

$m < 0$ $\lambda > 0$

$$m = \frac{\sum (y_i - \bar{y})(x_i - \bar{x}) + \lambda}{\sum (x_i - \bar{x})^2}$$

(−ve)

$m = -\frac{100 - \lambda}{50}$

$= \frac{-100 - 150}{50} = -5$

$m = -\frac{100 + \lambda}{50}$

$\lambda = 0$ $m = -2$

$\lambda = 50$ $m = -1$

$\lambda = 100$ $m = 0 \rightarrow 1$

$\lambda = 150$

$\lambda = 150$, $m = -5$

1) 0→
2) stop

$m = 1$

$m = \frac{-100 + \lambda}{50}$

$\lambda = 0 \quad m = -2$

$\lambda = 50 \quad m = -1$

$\lambda = 100 \quad m = 0 \rightarrow 1$

$\lambda = 150$

$m = 1$

$\lambda = 150, \; m = -5$

1) 0 →
2) stop

$$m = \frac{\sum (y_i - \bar{y})(x_i - \bar{x})}{\sum (x_i - \bar{x})^2 + \lambda}$$

0.1 → 0 ✗

$\lambda \rightarrow$ Ridge $\lambda \ll 0$, $\lambda = 0$

Lasso → $\lambda \rightarrow$ numerator

$\lambda = 10000000$

Denominator