Plan of Attack

16 June 2023 19:46

Crash course on Probability
→ for data science

2 hours
- 1st Session → Basic Terms
- 2nd Session → Advanced topics
  - → Naive Bayes
  - → Logistic Reg

→ 4 hours → ML / DS

Terminology

15 June 2023 18:20

5 terms

tossing a coin → RE

H, T

1. Random Experiment

   An experiment is called random experiment if it satisfies the following two conditions:

   (i) It has more than one possible outcome.

   (ii) It is not possible to predict the outcome in advance

die → getting a number < 3

{1, 2} → {1, 2, 3, 4, 5, 6}

2. Trial

   Trial refers to a single execution of a random experiment. Each trial produces an outcome.

1 → H, T

H → outcome

1 → T → outcome

3. Outcome

   Outcome refers to a single possible result of a trial.

1 → H, T   {H, T}

4. Sample Space

   Sample Space of a random experiment is the set of all possible outcomes that can occur. Generally, one random experiment will have one set of sample space.

SS → {1, 2, 3, 4, 5, 6}

5. Event

   Event is a specific set of outcomes from a random experiment or process. Essentially, it's a subset of the sample space. An event can include a single outcome, or it can include multiple outcomes. One random experiments can have multiple events.

→ 1 → H   {H, T}

getting a head {H}

die roll → A → getting an odd {1, 3, 5}

B → getting an even {2, 4, 6}

{1, 2, 3, 4, 5, 6}

Examples

16 June 2023 14:51

1. Rolling a die
2. Tossing a coin twice
3. Titanic example

→ 3

1) RE → rolling a die
2) Trial → rolling the die once
3) Outcome → {3}
4) SS → {1, 2, 3, 4, 5, 6}
5) Event
   → getting a 3 → {3}
   → getting a number > 4 → {5, 6}
   → getting a number odd → {1, 3, 5}

2

RE → tossing the coin twice

trial → tossing the coin twice (once)

outcome → {H, T}

→ SS → {(H, H), (H, T), (T, H), (T, T)}

Event → getting 2 heads {(H, H)}

getting atleast 1 head {(H, H), (H, T), (T, H)}

3 out

1, 2, 3

1 passen

Titanic → 891 passengers → Pclass

RE → randomly drawing out a passenger

trial → and finding its Pclass

Outcome → {1}

SS → {1, 2, 3}

Event → A → the passenger is from Pclass = 2 {2}

B → not from Pclass = 2 {1, 3}

Types of Events

16 June 2023 08:29

2, 4, 6

{2} <2 {1}

>3 {4, 5, 6}

1. Simple Event: Also known as an elementary event, a simple event is an event that consists of exactly one outcome.

   For example, when rolling a fair six-sided die, getting a 3 is a simple event.

2. Compound Event: A compound event consists of two or more simple events.

   For example, when rolling a die, the event "rolling an odd number" is a compound event because it consists of three simple events: rolling a 1, rolling a 3, or rolling a 5.

3. Independent Events: Two events are independent if the occurrence of one event does not affect the probability of the occurrence of the other event.

   For example, if you flip a coin and roll a die, the outcome of the coin flip does not affect the outcome of the die roll.

   spade

4. Dependent Events: Events are dependent if the occurrence of one event does affect the probability of the occurrence of the other event.

   For example, if you draw two cards from a deck without replacement, the outcome of the first draw affects the outcome of the second draw because there are fewer cards left in the deck.

   52

   $P(\cancel{Ace}) = \frac{13}{52}$

   $P(\text{spade}) = \frac{}{51}$

5. Mutually Exclusive Events: Two events are mutually exclusive (or disjoint) if they cannot both occur at the same time.

   For example, when rolling a die, the events "roll a 2" and "roll a 4" are mutually exclusive because a single roll of the die cannot result in both a 2 and a 4.

   rollin → odd
   → even

6. Exhaustive Events: A set of events is exhaustive if at least one of the events must occur when the experiment is performed.

   For example, when rolling a die, the events "roll an even number" and "roll an odd number" are exhaustive because one or the other must occur on any roll.

   7 → impossible

   $\boxed{1 \leq x \leq 6}$

7. Impossible event and Certain Event

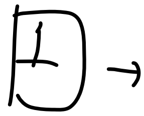
What is Probability

15 June 2023 18:14

In simplest terms, probability is a measure of the likelihood that a particular event will occur. It is a fundamental concept in statistics and is used to make predictions and informed decisions in a wide range of disciplines, including science, engineering, medicine, economics, and social sciences.

Probability is usually expressed as a number between 0 and 1, inclusive:

- A probability of 0 means that an event will not happen.
- A probability of 1 means that an event will certainly happen.
- A probability of 0.5 means that an event will happen half the time (or that it is as likely to happen as not to happen).



Empirical Probability Vs Theoretical Probability

15 June 2023 18:15

\# trials

10 → 100 Passenger → 70% M 30% F

3 males $P(M) = 0.3$ → $P(M) = 0.7$

Empirical Probability

Empirical probability, also known as experimental probability, is a probability measure that is based on observed data, rather than theoretical assumptions. It's calculated as the ratio of the number of times a particular event occurs to the total number of trials.

H → 55

Trials → 100

$P(H) = \frac{55}{100}$

A. Suppose that, in our 100 tosses, we get heads 55 times and tails 45 times. What is the empirical probability of getting a head

B. Let's say you have a bag with 50 marbles. Out of these 50 marbles, 20 are red, 15 are blue, and 15 are green. You start to draw marbles one at a time, replacing the marble back into the bag after each draw. After 200 draws, you find that you've drawn a red marble 80 times, a blue marble 70 times, and a green marble 50 times. What is the empirical probability of getting a red marble?

20 red

$\frac{80}{200}$

30 die

Theoretical Probability

Theoretical (or classical) probability is used when each outcome in a sample space is equally likely to occur. If we denote an event of interest as Event A, we calculate the theoretical probability of that event as:

Theoretical Probability of Event A = Number of Favourable Outcomes (that is, outcomes in Event A) / Total Number of Outcomes in the Sample Space

{1, 2, 3, 4, 5, 6}

roll → 3

$\frac{1}{6}$ → P(3)

A. Consider a scenario of tossing a fair coin 3 times. Find the probability of getting exactly 2 heads.

B. Consider a scenario of rolling 2 dice. What is the probability of getting a sum = 7

{H, T}

$\frac{1}{2}$ ← P(H)

No. of trials → infinite coin toss

empirical prob → theoritical

10 → 3 → 0.3 → 0.5

100 → 45H → 0.45

1000 → 470 → 0.47

Random Variable → misleading → function

15 June 2023 18:16

In the context of probability theory, a random variable is a function that maps the outcomes of a random process (known as the sample space) to a set of real numbers.

real number

Discrete RV
Continuous RV

$X = \{1, 0\}$
H T

Input: The input to the function is an outcome from the sample space of a random process.

input → logic → output

Output: The output of the function is a real number that we assign to each possible outcome.

rolling a die

$Y = \{1,2,3,4,5,6\}$ SS

1 2 → Y → 1 2

{1, 2, 3, 4, 5, 6}

H / T → input → RV logic? → output ? 1 0

↳ random experim
tossing a coin

→ SS = {H, T}

The transformation from input to output in the function of a random variable is determined by how we choose to define the random variable.
And the choice of how to define a random variable often depends on the specific aspects of the random process (or event) that you're interested in studying.

(1,2)
(1,1)
input → RV logic → output → real number as output
2 3 4

outcomes from SS → input
event → logic

sample space
↳ item → sum
outcome

RE → Rolling 2 dice together

A → getting a sum of 7

items 36

{3, 4} ← outcome

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) . . . . . (2,6)
. . . . . . (6,6)

$X = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

PMF →

3 4 X
5 3

RE → rolling 2 dice

A → where the num on die 1 > num on die 2 &

SS = {(1,1) (1,2) .
4,5 , 5,4 (1,1)

(1,1)
input → logic → output

1-1 0
1-2 -1

SS = { (1,1) (1,2) ...
4,5 , 5,4 (6,6)

input → logic → Output

(1,2)

(5,4)

5-4

num#die 1 − num die 2

* random → +ve → event

0 → same

-ve → not fam

output → 1.5

1.32

Discrete ←

Random var

Discrete RV

Continuous RV

RE → cgpa → 0-10

{0-10}

{0-10} → logic → contin. den

8.35

8.35

Probability Distribution of a Random Variable (Discrete)

15 June 2023 18:16

A probability distribution is a list of all of the possible outcomes of a random variable along with their corresponding probability values.

1) Toss
2) 1 dice
3) 2 dice

X	1	0
P(X)	1/2	1/2

Probability dist of a random var

$P(X=1) = 1/2$ $P(X=2)$

Toss a coin
{H, T}
→ RV → X = {1, 0}

rolling a dice
SS = {1, 2, 3, 4, 5, 6}
→ RV → {1, 2, 3, 4, 5, 6}
X

X	1	2	3	4	5	6
P(X)	1/6	1/6	1/6	1/6	1/6	1/6

Probability dist of random variable

(input)
sample space

output

rolling 2 dice
SS = 36 items
↓
RV → sum

1 2

(a,b)	1	2	3	4	5	6
1	(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)
2	(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
3	(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
4	(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
5	(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
6	(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$P(X=2)$

X

X = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	1/36	1/18	1/12	1/9	5/36	1/6	5/36	1/9	1/12	1/18	1/36

Probability Dist Function

100 dice roll

100 → 600

contin RV

X → P(X) → $\boxed{y = f(x)}$

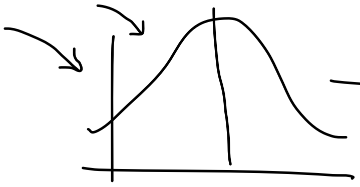
P(X) → $\boxed{y = x^2 + 5x + 3}$

$X \rightarrow P(X)$ $\boxed{y = f(x)}$ $P(X) \rightarrow$ $\boxed{y = x^2 + 5x + 3}$

$f(x) \leftarrow y$

PDF

mathematical $X = 5/36$



→ a lot of thing X

PDF / PMF

↓ contin

discre

↳ normal

PDF → distribut

Density (PDF (continous)

mass (PMF) (discrete)

Mean of a Random Variable

15 June 2023 18:17

The mean of a random variable, often called the expected value is essentially the average outcome of a random process that is repeated many times. More technically, it's a weighted average of the possible outcomes of the random variable, where each outcome is weighted by its probability of occurrence.

$\frac{21}{6} = \boxed{3.5}$

$\boxed{\text{Mean of } X}$ → multiple trials (0 → 1000 dic → avg → mean (X)

$\frac{3+4+2+4 \dots}{1000} = \boxed{}$

rolling a die

$1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6}$ mean

X	1	2	3	4	5	6	= 3.5
p(x)	1/6	1/6	1/6	1/6	1/6	1/6	

$\frac{1}{6}$ $\frac{1}{2}$

$5 \ 3 \ 4 \ 5 \ 3 \ 3 = \frac{5+3+4+5+3+3}{6} = \boxed{\frac{23}{6}}$

$\frac{2}{6} = \frac{1}{3}$

$\frac{2(5) + 1(4) + 3(3)}{6} = \frac{2}{6}(5) + \frac{1}{6}(4) + \frac{3}{6}(3)$ → mean

$= \boxed{\frac{1}{3}(5) + \frac{1}{6}(4) + \frac{1}{2}(3)} =$

mean

$$\boxed{E[X] = \sum_{i=1}^{n} x_i \, p(x_i)}$$

Variance of a Random Variable

15 June 2023 18:17

The variance of a random variable is a statistical measurement that describes how much individual observations in a group differ from the mean (expected value).

rolling

X 1 2 3 4 5 6

$\boxed{E[X]} = 3.5$

$Var(X)$

normal variance → $Var(X)$

$$var = \sum_{i=1}^{n} \frac{\boxed{(x_i - \bar{x})^2}}{n-1}$$

→ mean

2, 4, 5, 6, 7 . . . 10

6

variance → spread

$\boxed{\text{squared dist}}$ mean

avg

$(1-3.5)^2 + (2-3.5)^2$

$(3-3.5)^2$

$\boxed{\text{mean}}$ → Expect

$\boxed{(X - E[X])^2}$

6mm

$Var(X) = E[(X - E[X])^2]$

$$\boxed{Var(X) = E[(X - E[X])^2]}$$

and given random variable X

$$Var(E) = E[(X^2 + (E[X])^2 - 2XE[X])]$$

$E[X+Y] = E[X] + E[Y]$

$E[XY] = E[X]\,E[Y]$

$$= E[X^2] + E[(E[X])^2] - E[2XE[X]]$$

$$= E[X^2] + E[(E[X])^2] - \boxed{E[2]}\,E[X]\,E[E[X]]$$

$E[2] \rightarrow 2$

$\boxed{2 \times 1}$

$E[X] \rightarrow$ number → constant

$E[X]$

rand → 1, 2, 3

$$= E[X^2] + E[(E[X])^2] - 2E[X]E[X]$$

$$= E[X^2] + E[(E[X])^2] - 2(E[X])^2$$

$$E[X^2] + (E[X])^2 - 2(E[X])^2$$

$$= E[X^2] + (E[X])^2 - 2(E[X])^2$$

$$\boxed{Var(x) = E[X^2] - (E[X])^2}$$ → ✓

cont
discrete

$$\boxed{Var(X) = E[(X - E[X])^2]}$$ ✓