Recap

03 June 2023 10:22

PCA

→ Eigen values

→ Eigen decomposition → Matrix decomposition / Factorization

$\boxed{\text{SVD} \to \text{PCA}}$

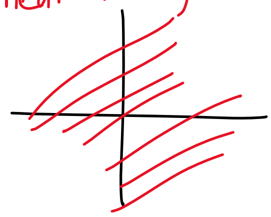
$\to \boxed{\text{SVD}} \leftarrow$ imp

SVD → difficult

Non-square Matrix (rectangular)

03 June 2023 10:23

linear transform



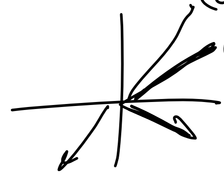
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$\boxed{2 \times 2}$ → square $\qquad$ 2×3

$\boxed{\text{Non-square}}$ matrices → {linear transformation}

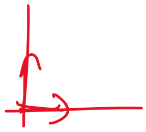
$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$2 \times 2 \qquad 2 \times 1 \rightarrow 2 \times 1$



$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

2×3



$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

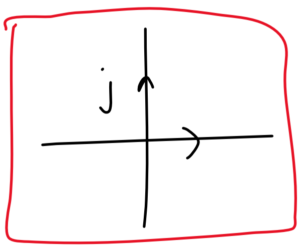
(1,0) $\hat{i}$ $\qquad$ $\hat{j}$ (0,1)

$\hat{i}$

(0,1)

$$\begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 3 & 1 \end{bmatrix}$$

(0,0,1)
3d



$\hat{i} \quad \hat{j}$

$\underbrace{(1,0)}_{2d} \rightarrow (1,2,3)$ 3d coord

3×2 ↑ output space ↑ input space

2d ↓ 3d

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \rightarrow \boxed{2 \times 1}$$

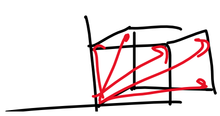
2×3 ↑ output (2d) ↑ input (3d) $\qquad$ 3×1

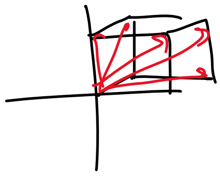
4×2 ↑ output (4d) ↑ input (2d)

$m \times n$ ↑ output ↑ input

$\boxed{m \times n}$ → m dim, → n dim

2×3





Rectangular Diagonal Matrix ← 2 transformation

05 June 2023 14:05

A matrix that would be diagonal if it were square, but instead is rectangular due to extra rows or columns of zeros.

$$\begin{bmatrix} a & 0 & \boxed{\begin{matrix} 0 \\ b \end{matrix}} \\ 0 & b & \end{bmatrix} \qquad \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \qquad \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

rect diag matrix

square non-zero items

$$\begin{bmatrix} a & 0 \\ 0 & b \\ \boxed{0} & \boxed{0} \end{bmatrix} \rightarrow \text{diag matrix}$$

2 transformation

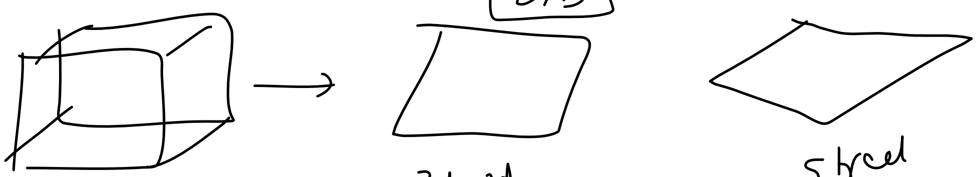
$$\underbrace{\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \end{bmatrix}}_{2\times3} \longrightarrow \overbrace{\begin{bmatrix} \boxed{a} & 0 \\ 0 & \boxed{b} \end{bmatrix}}^{B}_{2\times2} \begin{array}{ccc} x & y & z \\ \hline \end{array} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}_{2\times3} \rightarrow \boxed{2\times3}$$

(2) (1)

non-square

rect diag → linear

$\boxed{2\times3}$



3d → 2d

shred

2×3

3×2

$$\begin{bmatrix} a & 0 \\ 0 & b \\ 0 & 0 \end{bmatrix}_{3\times2} \rightarrow \text{linear transformation}$$

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}_{2\times2} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}_{3\times2}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}_{3\times2} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}_{2\times2} \rightarrow \boxed{4\times2}$$

2d

(b) a

3×2

(b) 3×2 2×2 a

What is SVD

Singular value decomp

03 June 2023 10:23

$$A = V \lambda V^{-1}$$

square matrix

SVD is a matrix decomposition/factorization method that decomposes a matrix into three other matrices. Given a matrix A, the singular value decomposition of A is usually written as:

$\boxed{A = U\Sigma V^T}$ $\rightarrow$ $\boxed{A} = \underbrace{U \Sigma V^T}$

Here:

- U and V are orthogonal matrices. U is the left singular vectors and V is the right singular vectors.
- Σ is a diagonal matrix containing what we call the singular values.

Applications of SVD

05 June 2023 05:56

1. Machine Learning and Data Science: SVD is used in Principal Component Analysis (PCA), a technique for dimensionality reduction. This is helpful when dealing with high-dimensional data. It's also used in various recommendation systems, for example in collaborative filtering which is used in Netflix movie recommendation.

2. Natural Language Processing (NLP): SVD is used in Latent Semantic Analysis (LSA), a technique for extracting the underlying meaning (semantic information) from textual data. LSA uses SVD to reduce the dimensionality of a term-document matrix, which helps identify relationships between terms and documents.

3. Computer Vision: In computer vision, SVD is used in image compression. By keeping only the largest singular values and corresponding singular vectors, we can represent an image using less data without losing too much information.

4. Signal Processing: SVD is used to separate useful signals from noise. This is useful in applications like mobile communications and audio signal processing.

5. Numerical Linear Algebra: SVD is used for matrix inversion and solving systems of linear equations. It is often a numerically stable way to solve ill-conditioned systems.

6. Psychometrics: In psychology and education, SVD is used in the construction and scoring of psychological and educational tests, where it is often important to extract underlying latent traits.

7. Bioinformatics: SVD and related techniques are often used to analyze gene expression data, where it is important to identify the underlying patterns of gene activity.

8. Quantum Computing: SVD is also used in quantum state tomography to understand the state of a quantum system.

→ Linear Algebra

↓

SVD

SVD The Equation

03 June 2023 10:23

$$\boxed{A = U \Sigma V^T}$$

any matrix

definite

Orthogonal matrix

rectangular diagonal

orthogonal

$$\underline{A} = \underline{U} \Sigma \underline{V}^T$$

$(m\times n)$ $(m\times m)$ $(m\times n)$ $(n\times n)$

$m \times n$

$\boxed{m\times n}$

$$\rightarrow \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

$90^\circ \rightarrow$

$ac + bd = 0$

$\sqrt{a^2} + \sqrt{b^2} = 1$

$U \rightarrow ?$ $\Sigma \rightarrow ?$ $V \rightarrow ?$

Eigen decomposition

$$A = V \Lambda V^{-1}$$

$(n\times n)$ $(n\times n)$ $(n\times n)$ $(n\times n)$

$$\rightarrow A = V \Lambda V^{-1}$$

Symmetric

orthogonal

diagonal

Relationship with Eigen Decomposition

03 June 2023 10:39

$\sqrt{a^2}\sqrt{b^2}$

$$\boxed{A = U \Sigma V^T} \leftarrow a \quad b \qquad A = V \Lambda V^{-1}$$

$(m \times n)$ non-square → square

symmetric

$V^{-1} = V^T \quad \boxed{VV^{-1}} = I$

$$\begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

3x2 2x3 3x3 square

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 1 & 1 \end{bmatrix}$$

2x3 3x2

$$\begin{bmatrix} 6 & 1 \\ 1 & 1 \end{bmatrix}$$

$\boxed{2 \times 2}$ square

symmetric

$V \rightarrow$

$$AA^T = U\Sigma V^T (U \Sigma V^T)^T$$

A^TA

$$= U\Sigma V^T V \Sigma^T U^T$$

$$= U \Sigma \Sigma^T U^T$$

$$\boxed{AA^T = UXU^T}$$

where $X = \Sigma \Sigma^T$

$VV^T = I$

$V^T = V^{-1}$

$$A^TA = (U\Sigma V^T)^T U \Sigma V^T$$

$$= V \Sigma^T U^T U \Sigma V^T$$

$$= V \Sigma^T \Sigma V^T =$$

$$\boxed{A^TA = VYV^T}$$

where $Y = \Sigma^T \Sigma$

$U^{-1} = U^T$

eigen values

$$\boxed{AA^T} = U \boxed{X} U^T \rightarrow X = \Sigma \Sigma^T$$

$$A^TA = V \boxed{Y} V^T \rightarrow Y = \Sigma^T \Sigma \qquad A = U$$

eigen values

symmetric

eigen

$U \rightarrow$ matrix whose cols contain eigenvectors of AA^T

left

... cols contain

eigenvectors of AA^T

left

$A \to U$ (singular vector) $V \to$ matrix whose cols contain eigenvectors of A^TA

$A \to V$ (singular vectors

right

$$A = U \Sigma V^T$$

(2×3) 2×2 2×3 (3×3)

$$X = \Sigma \Sigma^T$$

$$Y = \Sigma^T \Sigma$$

$$\Sigma = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a^2 & 0 \\ 0 & b^2 \end{bmatrix} \to a^2\ b^2$$

eigenvalues AA^T

$$Y = \begin{bmatrix} a & 0 \\ 0 & b \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \end{bmatrix} = \begin{bmatrix} a^2 & 0 & 0 \\ 0 & b^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \to a^2 b^2 0$$

eigen value A^TA

$X, Y \to \sqrt{a^2}\ \sqrt{b^2} \to a\quad b \to$ SVD

$\begin{pmatrix} AA^T \\ A^TA \end{pmatrix} \to a^2 b^2 \to \sqrt{a^2}\ \sqrt{b^2} \to a, b \to$ SVD

$\to$ singular value $\to$ sqrt(eigen value)

$$A = U \Sigma V^T$$

left singular

$(a, b) \to \sqrt{a^2}\ \sqrt{b^2}$

right sing (A^TA)

singular vecor (AA^T)

(a, b) → $\sqrt{a^2}\ \sqrt{b^2}$

singular values

sqrt(eigenvalues) ↳ A^TA AA^T

$AA^T \rightarrow a^2\ b^2$

$A^TA \rightarrow a^2\ b^2$ → eigen value

$\Sigma \rightarrow \sqrt{a^2}\sqrt{b^2}$ → singular values

$a^2\ b^2$ eigen

$A = U \Sigma V^T$

$U \rightarrow AA^T$

$V \rightarrow A^TA$

$\Sigma \rightarrow \sqrt{a^2}\ \sqrt{b^2}$

$\downarrow \quad \downarrow$

$a \quad b$

singular value

Geometric Intuition

03 June 2023 10:23

$\boxed{A} = \underbrace{U \Sigma V^T}$ → geometric intuition

matrix (m×n)

$\boxed{A} = \boxed{V} \Lambda \boxed{V^{-1}}$

inverse of V

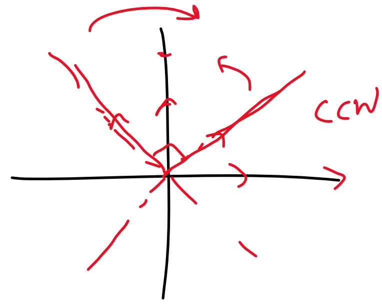
symmetric

Orthogonal

diagonal

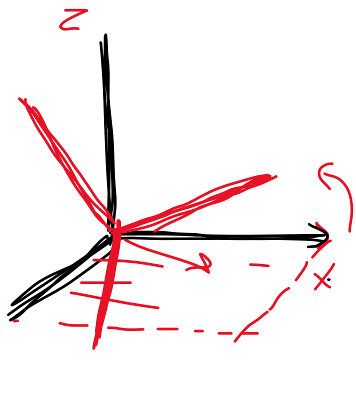
[2×2

$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$



$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

3d space



$A = U \boxed{\Sigma} \boxed{V^T}$

m×n

(2×3)

2d output

3d (input

$\boxed{V^T}$

eigen vector

U U^T

orthogonal

$\boxed{V} \to \boxed{A^T A}$ → 3×3

3d

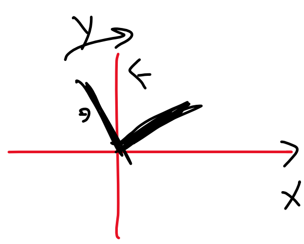
3d animal

$U \to \boxed{AA^T}$

$\Sigma = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \end{bmatrix}$

→ $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

Σ2 Σ1



V^T V^T

A → (ccw) rotation input space

n>m (m×n) → dim reduced

m>n 2×3 → stretching

→ rotation (cw)

$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 1 \end{bmatrix}$

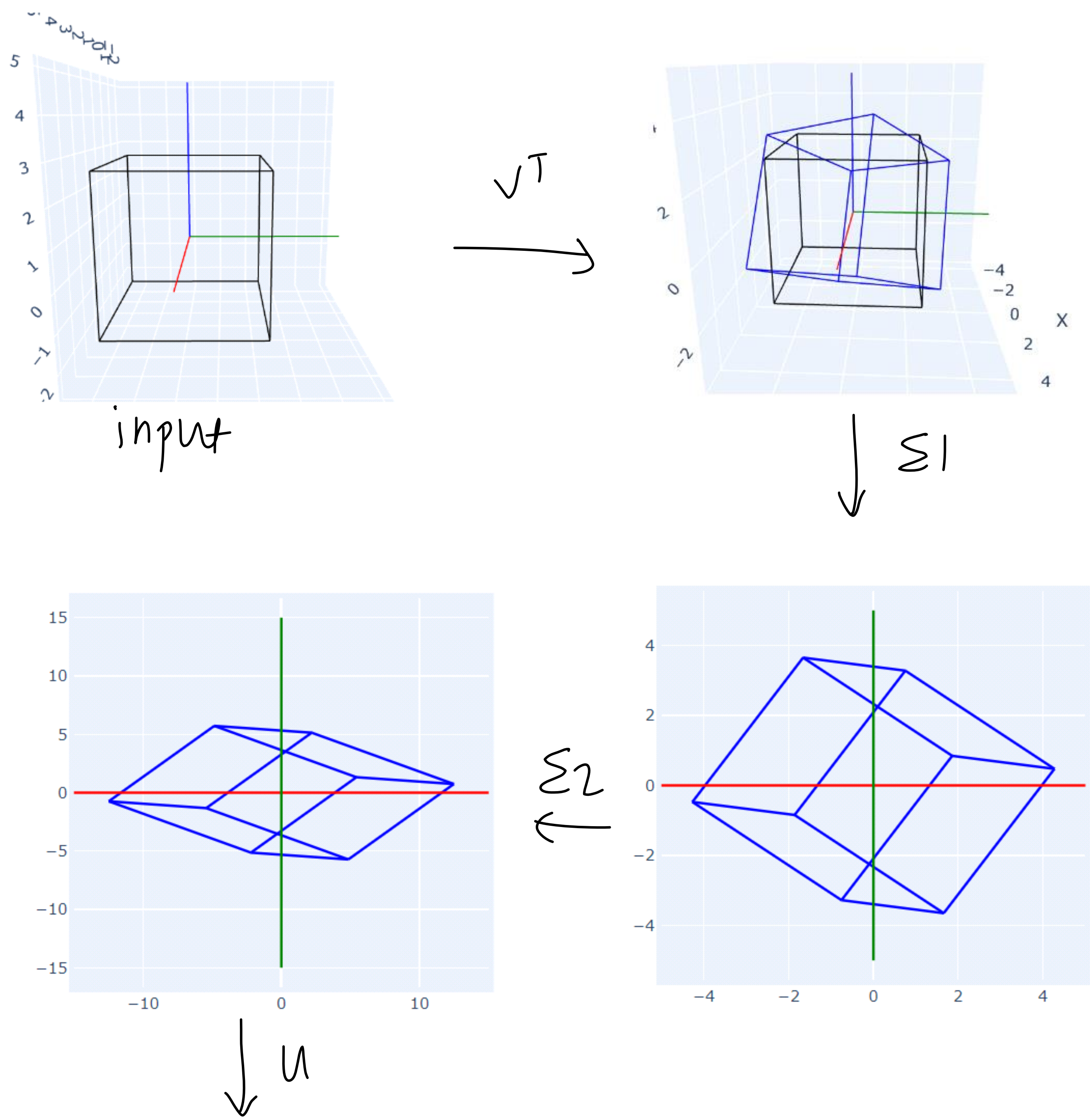
2x3

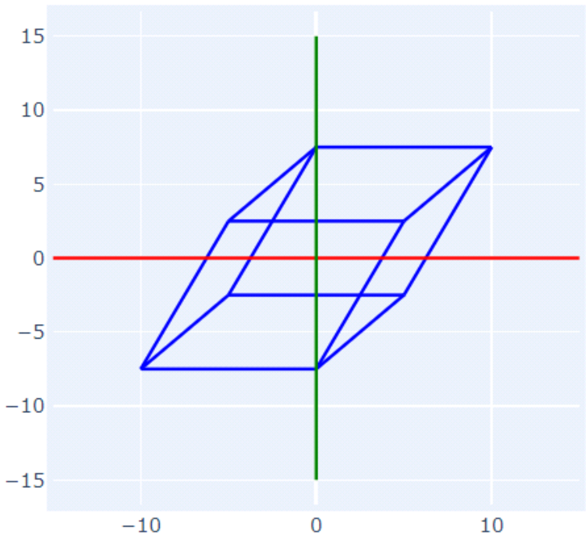
$\rightarrow$ rotation (CW) 2×3

$$A = U \Sigma V^T$$

Demo 1 [2x3]

05 June 2023 14:59

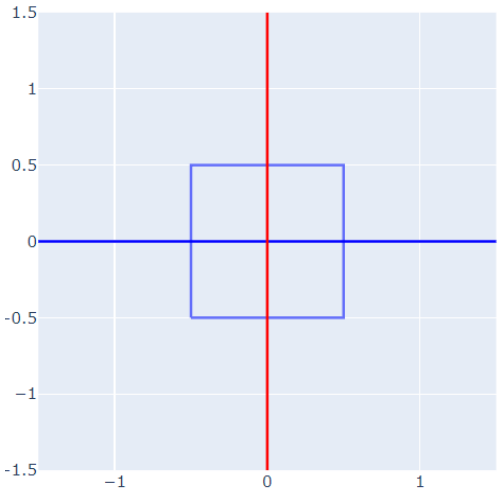



15
10
5
0
−5
−10
−15
−10
0
10

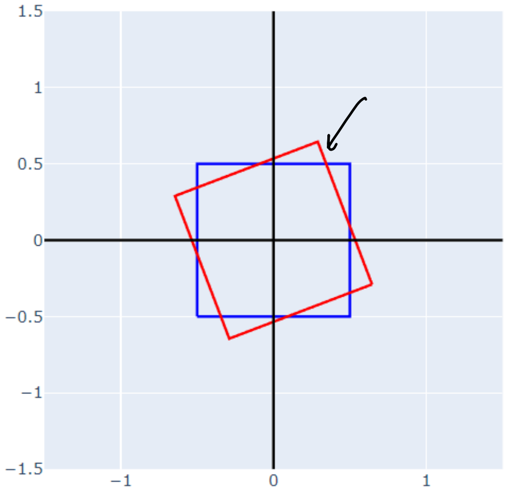
Demo 2 [3x2]

05 June 2023 15:50

$\boxed{3 \times 2}$



VT



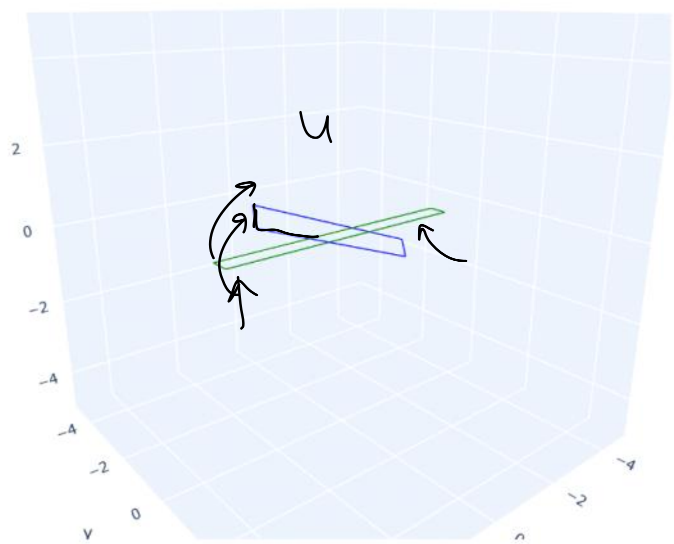
$A = U \Sigma V^T$

U

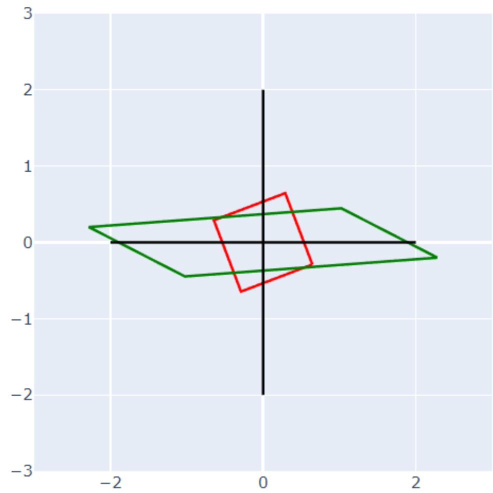
$\Sigma 1$ and $\Sigma 2$

scaling

dim add



$\Sigma 2$



$$\boxed{A = U \Sigma V^T}$$ → Special case of eigen decomposition

2x2

2d 2d

rotation

$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ ← scaling

How to Calculate SVD

05 June 2023 08:03

$A = U \Sigma V^T$

$\boxed{AA^T} = U \Lambda U^T$

$A^T A = V \Lambda V^T$

np.linalg.svd(A)

$\boxed{U, S, V\_t}$

SVD in PCA

03 June 2023 10:23

PCA → principal component

↳ eigen decomposi

faster (large

X cgpa | iq | 10th marks → $\boxed{\text{cov matrix}}$ → X

$\boxed{3 \times 3}$

↓

eigen decomposition

→ SVD → $\boxed{pc}$ → sklearn

iris $\boxed{X \to (150, 4)}$

↳ cov → (4, 4)

SL | SW | PL | PW

COV(X) → 1) mean center all the cols

2) → $\boxed{\dfrac{X^T X}{n-1}}$ = cov matrix

$\boxed{\dfrac{X_c^T X_c}{150-1}}$ $X \to X_c$ (150, 4)

$$\boxed{\operatorname{COV}(X) = \frac{X_c^T \cdot X_c}{n-1}}$$

$\boxed{X_c} = U \Sigma \boxed{V}^T$ → SVD

eigenvector

eigen values

$$\boxed{X_c^T X_c = V \Sigma V^T} \rightarrow \text{eigen decom}$$

COV

$V \rightarrow$ eigen vector of $X_c^T X_c \rightarrow$ covarian

covariance eigen

$X \rightarrow X^T X$

COV