02 June 2023 19:59

Eigen decomp

PCA variants

Kernel PCA

Incremental PCA

Randomized PCA

+ SVD

Some Special Matrices

01 June 2023 16:32

A^{100} $\begin{bmatrix} a & b \\ 0 & b \end{bmatrix}$ $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

1. Diagonal Matrix -

   A diagonal matrix is a type of square matrix where the entries outside the main diagonal are all zero; the main diagonal is from the top left to the bottom right of the square matrix.

   a. Powers: The nth power of a diagonal matrix (where n is a non-negative integer) can be obtained by raising each diagonal element to the power of n.
   b. Eigenvalues: The eigenvalues of a diagonal matrix are just the values on the diagonal. The corresponding eigenvectors are the standard basis vectors.
   c. Multiplication by a Vector: When a diagonal matrix multiplies a vector, it scales each component of the vector by the corresponding element on the diagonal.
   d. Matrix Multiplication: The product of two diagonal matrices is just the diagonal matrix with the corresponding elements on the diagonals multiplied.

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ linear transformation

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ A^{100}

$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$A^{100} = AAAA \ldots A$ 100 times

$A = \begin{bmatrix} 5 & 0 \\ 0 & 6 \end{bmatrix}$ $A^{100} = \begin{bmatrix} 5^{100} & 0 \\ 0 & 6^{100} \end{bmatrix}$

$A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \rightarrow \lambda = a \quad \lambda = b$

2. Orthogonal Matrix

   An orthogonal matrix is a square matrix whose columns and rows are orthogonal unit vectors (i.e., orthonormal vectors), meaning that they are all of unit length and are at right angles to each other.

   Perfect rotation, no scaling or shearing. $\rightarrow \boxed{A^T = A^{-1}}$

   a. Inverse Equals Transpose: The transpose of an orthogonal matrix equals its inverse, i.e., A^T = A^(-1). This property makes calculations with orthogonal matrices computationally efficient.

3. Symmetric Matrix → transfor $\boxed{A = A^T}$ $\begin{bmatrix} a & c \\ c & b \end{bmatrix} \rightarrow \begin{bmatrix} a & c \\ c & b \end{bmatrix}$

   A symmetric matrix is a type of square matrix that is equal to its own transpose. In other words, if you swap its rows with columns, you get the same matrix.

   a. Real Eigenvalues: The eigenvalues of a real symmetric matrix are always real, not complex.
   b. Orthogonal Eigenvectors: For a real symmetric matrix, the eigenvectors corresponding to different eigenvalues are always orthogonal to each other. If the eigenvalues are distinct, you can even choose an orthonormal basis of eigenvectors.

(2,2) $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$ (4,8)

i 2x2

$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ $\begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix}$ $\rightarrow \begin{bmatrix} ac & 0 \\ 0 & bd \end{bmatrix}$

$\rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ orthogonal

90°

$\boxed{ab + cd = 0}$

$\rightarrow \sqrt{a^2+c^2} = 1 \quad \sqrt{b^2+d^2} = 1$

orthogonal 30°

$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \rightarrow$ orthogonal $\theta \rightarrow 30°$

identity $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$

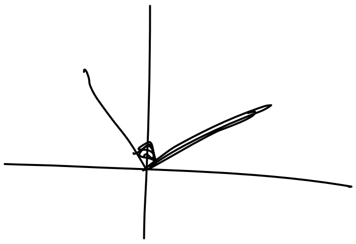
$\frac{\sqrt{3}}{2}\frac{1}{2} - \frac{1}{2}\frac{\sqrt{3}}{2} = 0$

$\sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} =$

$\sqrt{\frac{3}{4}+\frac{1}{4}} = \sqrt{\frac{4}{4}} = 1$

$\boxed{\theta =}$ $\boxed{30°}$

cgpa | iq $\rightarrow \begin{bmatrix} var(cgpa) & cov(cgpa, iq) \\ cov(cgpa, iq) & var(iq) \end{bmatrix}$

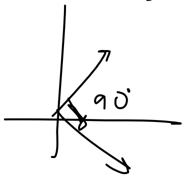


$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$\underbrace{ab}+ cd = 0$

orthon E-V

symmtric



Matrix Composition

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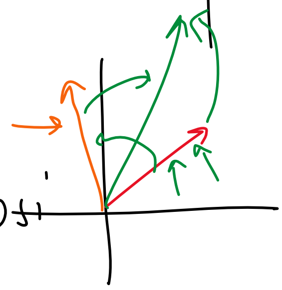
Matrix composition

$$\boxed{A\ B\ C}$$

$$D = A\ B\ C$$

complex

$$\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e & f \\ g & h \end{bmatrix}\right) = \begin{bmatrix} k & l \\ m & n \end{bmatrix}$$



$ABC = D$ composi

$D = ABC$ decomposi'n

Matrix Decomposition

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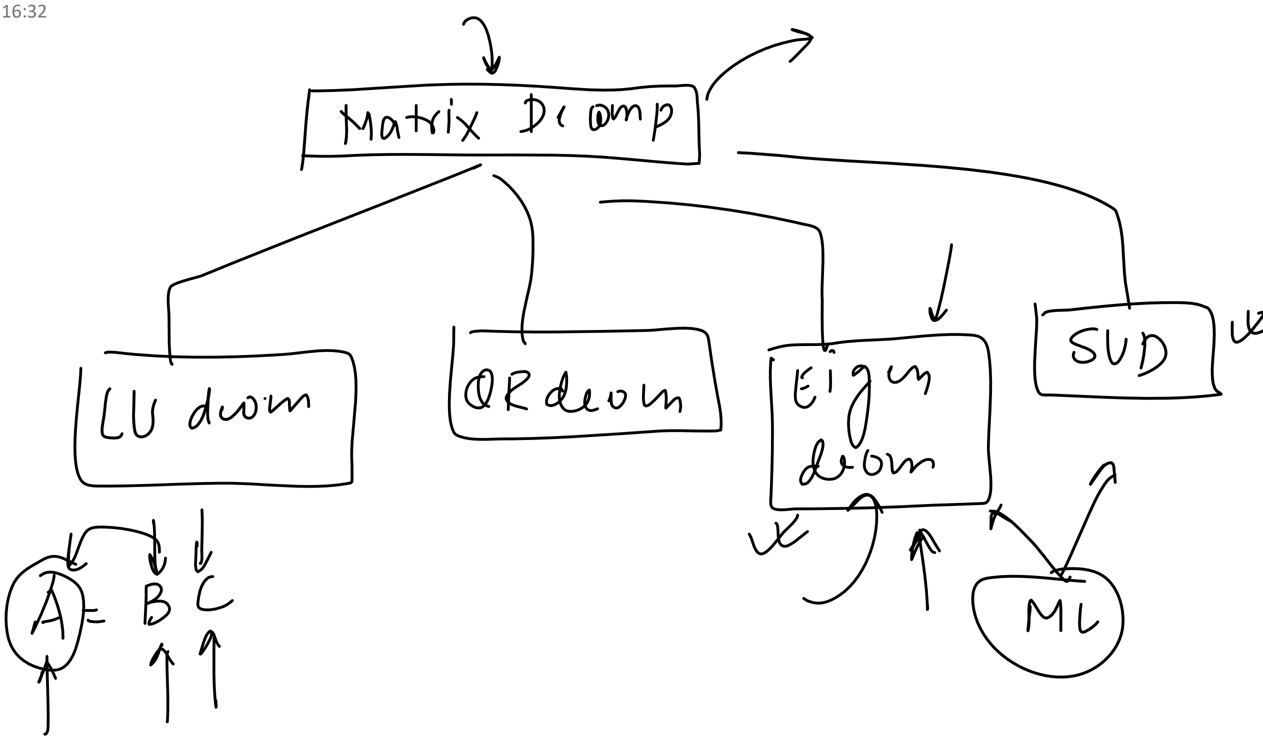

Eigen Decomposition

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Λ

$A = V \Lambda V^{-1}$ matrices

The eigen decomposition of a matrix A is given by the equation:

$\boxed{A = V\Lambda V^{-1}}$ $A = V\Lambda V^{-1}$ symmetric

Where:

- V is a matrix whose columns are the eigenvectors of A
- Λ is a diagonal matrix whose entries are the eigenvalues of A
- V^{-1} is the inverse of V

Assuming

1. Square matrix: Eigen decomposition is only defined for square matrices
2. Diagonalizability: For a n*n matrix it should have n linearly independent eigen vectors.

(2x2) → 2 eigenvector (2)

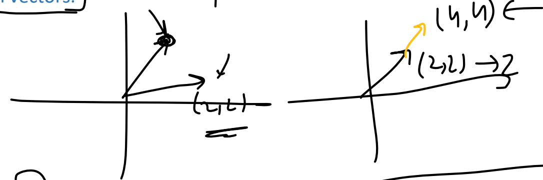
1 , 0

$\boxed{2\times 2}$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\lambda_1 \; \lambda_2$ $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$

$\boxed{A = V\Lambda V^{-1}}$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \end{bmatrix}^{-1}$



(4,4)

(1,1)

(2,2) → 2

$\boxed{A\vec{v} = \lambda \vec{v}}$

evalue evector

$\boxed{\begin{matrix} A\vec{v}_1 = \lambda_1 \vec{v}_1 \\ A\vec{v}_2 = \lambda_2 \vec{v}_2 \end{matrix}}$

$\vec{v}_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$

$v_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$

$V = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 \end{bmatrix}$ → $\boxed{AV = V\Lambda}$

$V = \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \end{bmatrix}$ $\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

$\vec{v}_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$

$A\vec{v}_1 = \lambda_1 \vec{v}_1$ $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $A\vec{v}_2 = \lambda_2 \vec{v}_2$

$\underset{2\times 2}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}} = \lambda_1 \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$

$\begin{bmatrix} ax_1 + by_1 \\ cx_1 + dy_1 \end{bmatrix} = \begin{bmatrix} \lambda x_1 \\ \lambda y_1 \end{bmatrix}$ → $\left\{ \begin{matrix} ax_1 + by_1 = \lambda_1 x_1 \\ cx_1 + dy_1 = \lambda_1 y_1 \\ ax_2 + by_2 = \lambda_2 x_2 \\ cx_2 + dy_2 = \lambda_2 y_2 \end{matrix} \right\}$

$AV = V\Lambda$

$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} x_1 & x_2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda \end{bmatrix}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$ax_1 + by_1 = \lambda_1 x_1$

$$\begin{bmatrix} ax_1 + by_1 & ax_2 + by_2 \\ cx_1 + dy_1 & cx_2 + dy_2 \end{bmatrix} = \begin{bmatrix} \lambda_1 x_1 & \lambda_2 x_2 \\ \lambda_1 y_1 & \lambda_2 y_2 \end{bmatrix}$$

$A\vec{v}_1 = \lambda_1 \vec{v}_1$

$A\vec{v}_2 = \lambda_2 \vec{v}_2$ $\longrightarrow$ $AV = V\Lambda$

$\boxed{A = V \Lambda V^{-1}}$ $\longrightarrow$ Eigen decompon

eigen vectors

diag (eigen value)

Eigen Decomposition of Symmetric Matrix

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$A \rightarrow \begin{bmatrix} a & c \\ c & b \end{bmatrix}$ symmetric $\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

diagonal
ortho
symmetric

$\boxed{A = V \Lambda V^{-1}} \rightarrow$ [spectral decomposition]

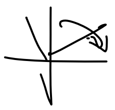
eigen vector (A)

$V \rightarrow$ orthogonal $\Lambda \rightarrow$ diagonal

symmetr $\rightarrow$ ortho (diag) orth)

$\boxed{COV} \rightarrow$ symm

V^{-1}



$\boxed{A} = V \Lambda V^{-1}$

A $\downarrow$ linear tran

V $\uparrow$ rotates

Λ $\rightarrow$ scales

V^{-1} $\rightarrow$ rotation

$A \rightarrow$ symmetric

$\downarrow$

linear

$\downarrow$

rotate scale rotate

Advantages of Eigen Decomposition

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→ ml → pca
→ physics
→ signal theory
→ quantum

$$\boxed{\begin{matrix} A \\ n \times n \end{matrix}} = \boxed{A^{1000}}$$

$A \times A \times A \ldots$

$A^{100} = V \lambda V^{-1} \underbrace{V}_{} \Lambda \underbrace{V^{-1} V}_{} \lambda V^{-1} \ldots$

$= V \boxed{\Lambda^{100}} V^{-1} \rightarrow$ 2 mul.

↳ diagonal

$A \rightarrow$ diagon

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

Kernel PCA

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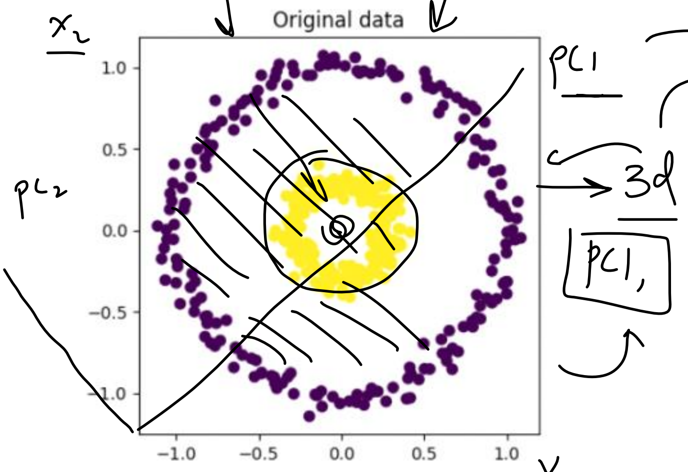
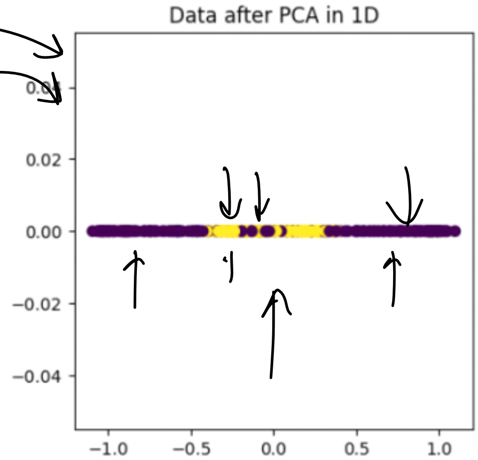
SVM → Kernel trick

concentric circles





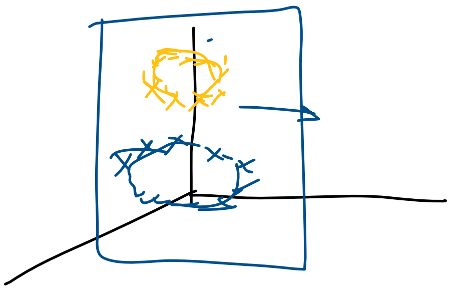
PCA fails

Kernel PCA

mathematical funct → kernel → 2d → 3d

$\boxed{y = e^{-x^2}}$ → rbf

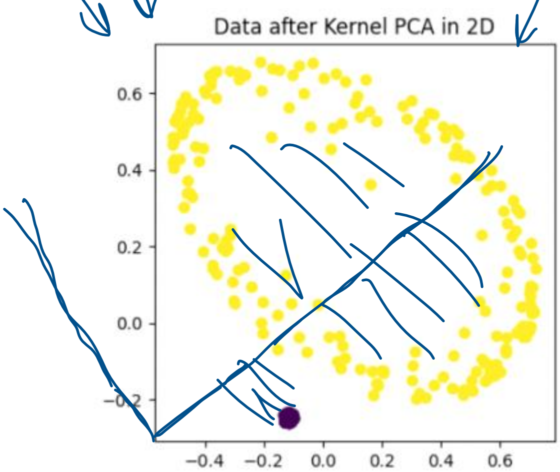
PCA assumes that the principal components are a linear combination of the original features. It can't handle complex polynomial relationships between features.

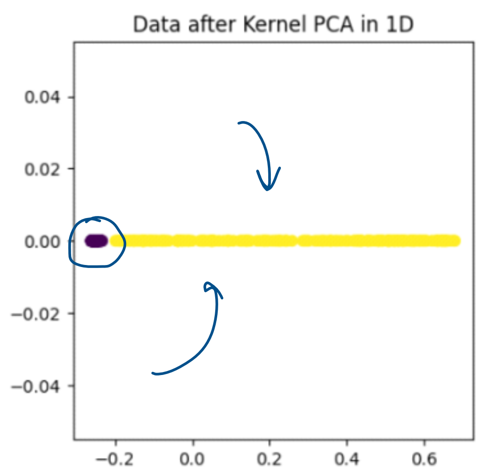


← Kernel PCA

Kernel SVM

3d - 2d





$\overbrace{x_1 \quad x_2}^{2}$ y

$\boxed{X = 400, 2}$ →

400 | X = 400, 2 → 1 cols

PC1 → best sep

$e^{-(\text{gamma} * \text{distance}^2)} = e^{-\gamma x^2}$

gamma

distance similarity matrix

high

(400, 2) → 2d

(400, 400) → 400 d

row1 → euclidean

row2
row
row400

400x400

$$\begin{array}{c|ccccc} & r_1 & r_2 & r_3 & \cdots & r_{400} \\ \hline r_1 & \circ & \circ & \circ & \circ & \circ \\ r_2 & \circ & \circ & \circ & \circ & \circ \\ \vdots & & & & & \\ r_{400} & & & & & \end{array}$$

400 → 400

distance square

| 400X400 |

→ | Symmetric | → eigen decomposit.

400 d → 400 eigenvect.

2 → 1 →