Recap

14 April 2023 19:28

Linear Algebra

Vectors

Matrix

Revision (2)

Computation

Intuition

ML

DL

12th board

Next

Linear Regress

Feature

Stats

Calculus

next week

What are Matrices

14 April 2023 14:48

- What are Matrices
- Order of a matrix
- Notation
- Uses and Application Areas

A matrix is a rectangular array of numbers, symbols, or expressions arranged in rows and columns. The numbers, symbols, or expressions are called the elements of the matrix.

$$\boxed{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

rows and cols

element

$\boxed{2 \times 3}$

$\boxed{\text{tool}}$

2×2

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

order of matrix

rows X # col's

$\boxed{\text{Shape}}$ ← $\boxed{\text{2D}}$

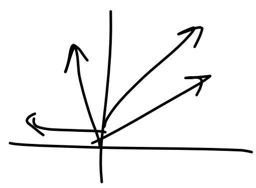
1. Linear Systems: Matrices can be used to represent and solve systems of linear equations. A system of linear equations can be written in matrix form as Ax = b, where A is the matrix of coefficients, x is the column vector of unknowns, and b is the column vector of constants. Methods such as Gaussian elimination, LU decomposition, and matrix inversion can be employed to find the solutions to the system.
2. Linear Transformations: Matrices are used to represent linear transformations between vector spaces. A matrix can define a linear transformation that maps vectors from one space to another while preserving the operations of vector addition and scalar multiplication. For example, rotation, scaling, and reflection transformations in geometry can be represented using matrices.
3. Eigenvalues and Eigenvectors: Matrices are used in the study of eigenvalues and eigenvectors, which are essential in various applications such as differential equations, stability analysis, and diagonalization of matrices. An eigenvalue-eigenvector pair (λ, v) of a square matrix A satisfies the equation Av = λv.
4. Graph Theory: In graph theory, matrices can be used to represent graphs through adjacency matrices, incidence matrices, and Laplacian matrices. These matrix representations provide a convenient way to analyze the properties of graphs and perform operations on them.
5. Markov Chains: Matrices are used in the study of Markov chains, which are stochastic processes that undergo transitions from one state to another according to certain probabilistic rules. Transition matrices describe the probabilities of transitioning between different states in a Markov chain and can be used to analyze the long-term behavior of the system.
6. Computer Graphics: Matrices are used extensively in computer graphics to represent transformations such as translation, rotation, scaling, and projection. These transformations are applied to 2D or 3D models to manipulate their position, orientation, and size in a virtual environment.
7. Control Theory: In control theory, matrices are used to represent and analyze linear systems, such as state-space models and transfer functions. The use of matrices in control theory allows for the design and analysis of control strategies for complex systems.

$$\boxed{\begin{aligned} x + 2y &= 5 \\ 5x + 3y &= 6 \end{aligned}}$$

xy ex

vector/point

coordi. + more



7. Control Theory: In control theory, matrices are used to represent and analyze linear systems, such as state-space models and transfer functions. The use of matrices in control theory allows for the design and analysis of control strategies for complex systems.
8. Optimization: In optimization problems, matrices can be used to represent constraints, objectives, and variables. Techniques such as linear programming, quadratic programming, and semidefinite programming rely on matrices and matrix operations to find optimal solutions.

Types of Matrices

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- Row Matrix
- Col Matrix
- Square matrix(diagonal) and Non-square Matrix
- Diagonal and scalar Matrix
- Identity Matrix
- Zero Matrix

$\begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 5 \end{bmatrix}$ ← normal

special

1) row matrix → row vector

$\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}_{1\times 4}$

row

2) Col matrix

$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}_{4\times 1}$

3) Square #rows = #cols

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$ a_{11}, a_{22}

$\boxed{i=j}$

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times 3}$ a_{11}, a_{22}, a_{33}

i = row

j = cols

4) Diagonal → square

$i=j \neq 0$

non dia

$i \neq j = 0$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3\times 3}$

5) Scalar matrix → diagonal matrix

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$

$\boxed{3\times 3}$

6) Identity $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow$ order $\rightarrow$ 3

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow$ order 2

7) Zero $\rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Matrix Equality

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2×2 2×2

$$A = B$$

1) order should be same ✓

2) $\boxed{A_{ij} = B_{ij}}$

$$\underset{2\times 2}{A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}} \neq \underset{2\times 2}{B = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}}$$

Scalar Operation

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single 2 , 3 , −5 → add → multipy

- → Scalar Addition
- → Scalar Multiplication
- → Negative of a Matrix
- → Rules

  K(A+B) = KA+KB

$\boxed{K+A} =$ $K=2$ $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

→ $K+A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

→ $KA = 2\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$

negative

$A = -A$

$A = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}$ $K = -1$

$KA = -1 \times \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix}$

$\boxed{-A}$

$\boxed{A, B}$ K

$K(A+B) = \underbrace{KA}_{mat} + \underbrace{KB}_{mat}$

Matrix Addition and Subtraction

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- Matrix Addition
- Matrix Subtraction

Rules
- -> same order
- -> A+B=B+A
- -> (A+B)+C = A+(B+C)
- -> Additive Identity
- -> Additive Inverse

→ add →
→ sub
→ multi
→ div

criteria

A B → order same

$\boxed{1+5}$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

A (2x2) B (2x2)

$$\begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$ $\boxed{2\times 2}$

100 x 200 100 x 200

→ $\boxed{A+B = B+A}$

order

Assosiative

$(A+B)+C = A+(B+C)$

subtraction

A - B

scalar multi / matrix add

$\boxed{A + (-1)B}$ negative of B

$-B$

1) Additive identity

2) Additive Inverse

$A + \boxed{X} = A$

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \boxed{X} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

2x2

$$X = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

zero matrix

$A - \boxed{-A}$

additive Inverse

$A - \boxed{X} = 0$

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - X = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

additive
Inverse

$$\boxed{-A} \quad \begin{bmatrix} -2 & -3 \\ -4 & -5 \end{bmatrix}$$

Matrix Multiplication

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Multiplying Matrix Rules
-> A.B!=B.A
-> (AB)C = A(BC)
-> A(B+C)=AB+AC
-> Multiplicative Identity

$$\underset{A}{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}} \times \underset{B}{\begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix}} = \begin{bmatrix} & \end{bmatrix}$$

matrix

$2 \times \boxed{2} \longleftrightarrow \boxed{2} \times 2$

$\boxed{2 \times 2}$

geometric

$1 \times 3 \neq 1 \times 3$

$\boxed{A \cdot B \neq B \cdot A}$

$$\begin{bmatrix} \underset{A}{2 \times 3} & \underset{B}{3 \times 3} \\ B & A \\ 3 \times 3 & 2 \times 3 \end{bmatrix}$$

$2 \times 3 = 3 \times 3$

$\boxed{2 \times 3}$

Assosiative

$(AB)C = A(BC)$

$\boxed{A(B+C)} = \boxed{AB + AC}$

row vector

col vector

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} =$$

$2 \times \boxed{3}$ $\quad$ $\boxed{3} \times 3$

$$\begin{bmatrix} 30 & 36 & 42 \\ 66 & 71 & 100 \end{bmatrix}$$

$\boxed{\text{multiplicative}}$ $\boxed{\text{identity}}$

$\boxed{AX = A}$ $\quad$ $\boxed{I}$ $\quad$ $\boxed{AI = A}$

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \boxed{X} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

A

$\underline{4}$

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

Transpose of a Matrix

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solving eqn

Rules

1) $(A^T)^T = A$
2) $(A+B)^T = A^T + B^T$
3) $(AB)^T = B^T \cdot A^T$

→ Matrix Transpose
→ Symmetric Matrix
→ Skew Symmetric
Rules
-> A^T^T = A
-> (A+B)^T=A^T + A^T
-> (AB)^T=B^TA^T

$(A^T)^T = A$

$(m \times n)^T$

$(n \times m)^T = \boxed{m \times n}$

Transpose

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$A^T \quad A'$

$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

$\boxed{B^T} = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

$(A+B)^T = A^T + B^T$

$C = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 4 \end{bmatrix}_{2\times 3}$

$C^T = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 4 & 4 \end{bmatrix}_{3\times 2}$

$m \times n = n \times m$

$\boxed{(AB)^T = B^T \cdot A^T}$

geometrical

Symmetric matrix

$\boxed{A = A^T}$

$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

Skew symmetric

$\boxed{A^T = -A}$

Determinant

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2, 3, −11, −15

calculate inv(A)

A^{-1}

What is Determinant
1*1 -> 2*2->Det(3*3)
Rules
-> det(A) = det(A')
Singular Matrix

The determinant is a $\boxed{\text{scalar value}}$ computed from a square matrix (a matrix with the same number of rows and columns) that carries important information about the matrix. It has several uses in linear algebra, including determining the invertibility of a matrix, finding the solution to systems of linear equations, and calculating the volume scaling factor for linear transformations.

$$\boxed{AX = B}$$

$$\underset{A}{\begin{bmatrix} 1 & 2 \\ 6 & 5 \end{bmatrix}} \underset{\boxed{X}}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{B}{\begin{bmatrix} 4 \\ 11 \end{bmatrix}}$$

$$X = \frac{\boxed{B}}{\boxed{A}} \cdot \boxed{A^{-1}}$$

division X multiplication

$$X = B \cdot \boxed{A^{-1}}$$

$$\frac{1}{A} = A^{-1}$$

$$\boxed{A^{1}} = \boxed{A}$$

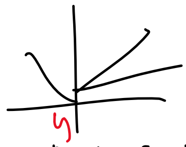
A^{-1}

$\boxed{A}$ → given it is invertible → $\boxed{\text{square}}$ → can't invertible

inverted

$\boxed{\text{square}}$ → inverse

non-square → $\boxed{\text{pseudoinverse}}$



$$\boxed{A^{-1}} = \frac{1}{\boxed{\det(A)}} \boxed{\text{Adj}(A)}$$

1 Determinant → 2 Minor → 3 Cofactor → 4 Adjoint

$\boxed{\text{Inverse}}$

$\boxed{1\times1}$ $\boxed{2\times2}$ $\boxed{3\times3}$

$A = [1]$ 1x1

$\det(A) = \boxed{1}$

$$\boxed{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\det(A) = \Delta = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1\times4 - 3\times2$$

$$= \boxed{-2}$$

$$B = \begin{bmatrix} -2 & 5 \\ -6 & 7 \end{bmatrix}$$

$-14 - (-30)$ ✓

$\begin{bmatrix} -6 & 7 \end{bmatrix}$

$\boxed{\det(B)} = \Delta = \begin{vmatrix} -2 & 5 \\ -6 & 7 \end{vmatrix} = -14 - (-30)$
$= 30 - 14 = \boxed{16}$ ✓

How to decide if inv of a matrix is possible

A A^{-1} $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\det(A) = 0$
↓
singular

1) square ✓

→ 2) $\det(A) \neq 0$

non-singular
matrices inverse

$A \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\boxed{ad - bc = 0}$ inverse ✗

$\boxed{1 \times 1}$

$\boxed{3 \times 3}$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ 3×3

2×2

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\det(A) = \Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$

b_1 a_{12} $1+2=3$

$\boxed{13} = 4$ $(-1)^3 = -1$

$(-1)^4 = 1$

$= +1 \times \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} \boxed{-} 2 \times \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3 \times \begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix}$

$= -3 + 12 - 9 = 0$ $\det(A) = 0 \rightarrow$ singular matrix
↓
inverse ✗

in $A^{-1} = \dfrac{1}{\det(A)} \operatorname{adj}(A) \leftarrow$

$\det(A) = 0$

cofactor → minor

Minor

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Minor of an element aij of a Determinant is the determinant obtained by deleting its ith row and jth col. It is denoted by Mij

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \det(A) = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$M_{11} = a_{22}$ $\qquad$ $M_{12} = a_{21}$

$M_{21} = a_{12}$ $\qquad$ $M_{22} = a_{11}$

$$A = \begin{bmatrix} a_{11} & a_{12} & \boxed{a_{13}} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\boxed{M_{11}} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = a_{22}a_{33} - a_{23}a_{32} \qquad M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$= a_{21}a_{33} - a_{23}a_{31}$$

Cofactor

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Cofactor of an element of aij of a determinant is defined by

Aij = (-1)^i+jMij where Mij is the minor of aij

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad M$$

Det = Sum of the product of elements of any row(or col) with Their corresponding cofactors.

$$A_{11} = (-1)^2 M_{11} \leftarrow A_{21} = (-1)^{2+1} M_{21} = -M_{21}$$

$i=1 \quad j=1$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & \boxed{a_{32}} & a_{33} \end{bmatrix}$$

$$A_{32} = (-1)^{3+2} M_{32}$$

$$\det = \boxed{a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}}$$

Adjoint

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$$\boxed{A \cdot A^{-1} = I} \qquad \boxed{A^{-1}}$$

The adjugate of a matrix, also known as the classical adjoint, is a matrix formed by replacing each element in the original matrix with its corresponding cofactor and then taking the transpose of the resulting matrix. The adjugate of matrix A is denoted as adj(A).

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$\frac{5}{7} \quad \boxed{5 \times 7^{-1}} \longrightarrow \operatorname{adj}(A) = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

Inverse of Matrix

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(AB)^-1 = B^-1A^-1

An inverse matrix is a matrix that, when multiplied by the original matrix, results in the identity matrix. The inverse matrix is defined only for square matrices (matrices with the same number of rows and columns) and not all square matrices have an inverse.

A matrix is invertible (has an inverse) if and only if it is non-singular, meaning its determinant is non-zero. If the determinant of A is zero, A is called a singular matrix, and it does not have an inverse.

Inverse matrices play a crucial role in linear algebra and have many applications, such as solving systems of linear equations, finding the solution to a matrix equation, and performing various matrix operations. There are several methods for finding the inverse of a matrix, including Gaussian elimination, the adjugate method, and LU decomposition.

$$A \cdot A^{-1} = A^{-1} \cdot A = I$$

$$\boxed{A \cdot A^{-1} = I}$$

$$AX = B$$

$$A^{-1}AX = A^{-1}B$$

$$IX = A^{-1}B$$

$$\boxed{X = A^{-1}B}$$

$$\boxed{A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)}$$

$\neq 0$ non-singular

invertible

$$\boxed{(AB)^{-1} = B^{-1}A^{-1}}$$

Solving a system of linear equations

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$$\begin{bmatrix} x + y = 5 \\ 4x + 3y = 15 \end{bmatrix} \quad \boxed{x, y}$$

$$\left\{ \begin{matrix} \text{computation} \\ \text{intuition} \end{matrix} \right\}$$

$$2\times1 \rightarrow \begin{bmatrix} 1 & 1 \\ 4 & 3 \end{bmatrix}_{2\times2} \begin{bmatrix} x \\ y \end{bmatrix}_{2\times1} = \begin{bmatrix} 5 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} x+y \\ 4x+3y \end{bmatrix} = \begin{bmatrix} 5 \\ 15 \end{bmatrix} \Rightarrow \boxed{\begin{matrix} x+y=5 \\ 4x+3y=15 \end{matrix}}$$

$$\underset{A}{\begin{bmatrix} 1 & 1 \\ 4 & 3 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{B}{\begin{bmatrix} 5 \\ 15 \end{bmatrix}} \qquad \boxed{A^{-1} \cdot B}$$

$$\boxed{AX = B} \leftarrow \quad \underbrace{A^{-1}A}X = A^{-1}B$$

$$IX = A^{-1}B$$

$$\rightarrow X = \boxed{A^{-1}B}$$

$$A = \begin{bmatrix} 1 & \boxed{1} \\ 4 & 3 \end{bmatrix} \qquad 3 - 4 = \boxed{-1} \neq 0$$

$$(-1)^{1+1}3 \quad (-1)^{2}\times 1$$

$$(-1)^3 4$$

$$(-1)^3 1$$

$$A^{-1} = \frac{1}{\det(A)} \mathrm{adj}(A) \qquad \begin{bmatrix} 3 & -4 \\ -1 & 1 \end{bmatrix}^T =$$

$$\mathrm{adj} \begin{bmatrix} 3 & -1 \\ -4 & 1 \end{bmatrix}$$

$$= \frac{1}{-1} \begin{bmatrix} 3 & -1 \\ -4 & 1 \end{bmatrix}$$

$$A^{-1} = \underset{2\times2}{\begin{bmatrix} -3 & 1 \\ 4 & -1 \end{bmatrix}} \cdot \underset{2\times1}{\begin{bmatrix} 5 \\ 15 \end{bmatrix}}$$

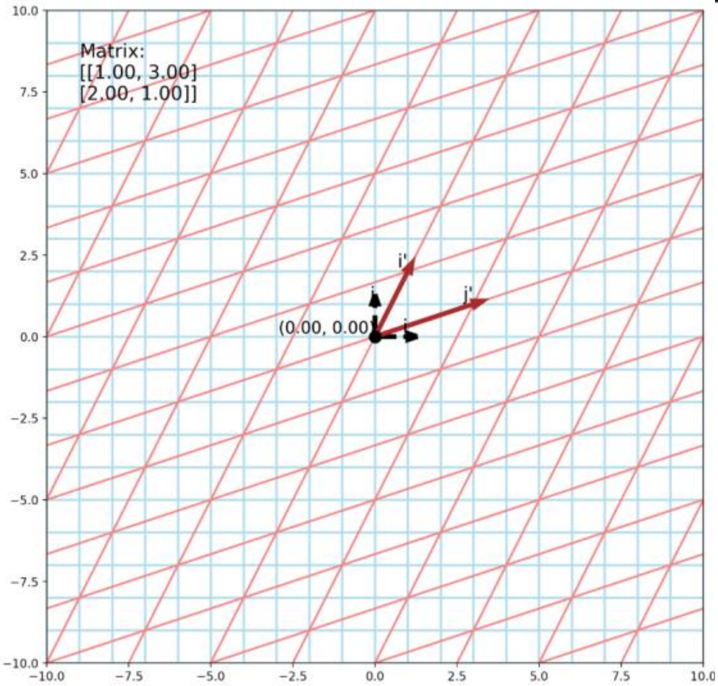
$$= \begin{bmatrix} 0 \\ \end{bmatrix} = \begin{bmatrix} x \\ \end{bmatrix} = \boxed{\begin{matrix} x = 0 \\ y = 5 \end{matrix}}$$

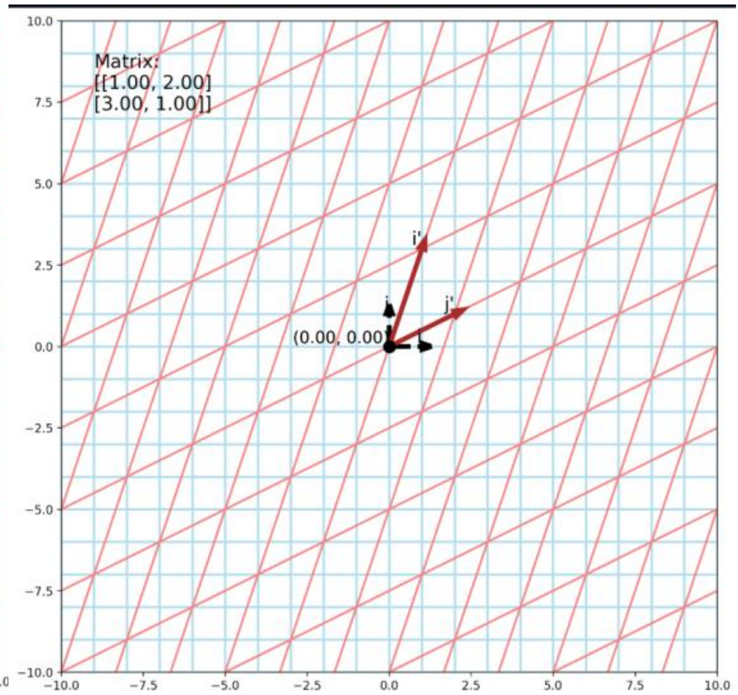
$$= \begin{bmatrix} 0 \\ 5 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \; - \; - \end{bmatrix}$$

$$x + 3y + 4z = 5$$
$$6x + 4y + z = 16$$
$$2x + 2y + 2z = 11$$

$$\underbrace{\begin{bmatrix} 1 & 3 & 4 \\ 6 & 4 & 1 \\ 2 & 2 & 2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 5 \\ 16 \\ 11 \end{bmatrix}}_{B}$$

$$\boxed{AX = B} \qquad \boxed{X = A^{-1}B}$$


Matrix:
[[1.00, 3.00]
[2.00, 1.00]]
i'
j'
(0.00, 0.00)


Matrix:
[[1.00, 2.00]
[3.00, 1.00]]
i'
j'
(0.00, 0.00)