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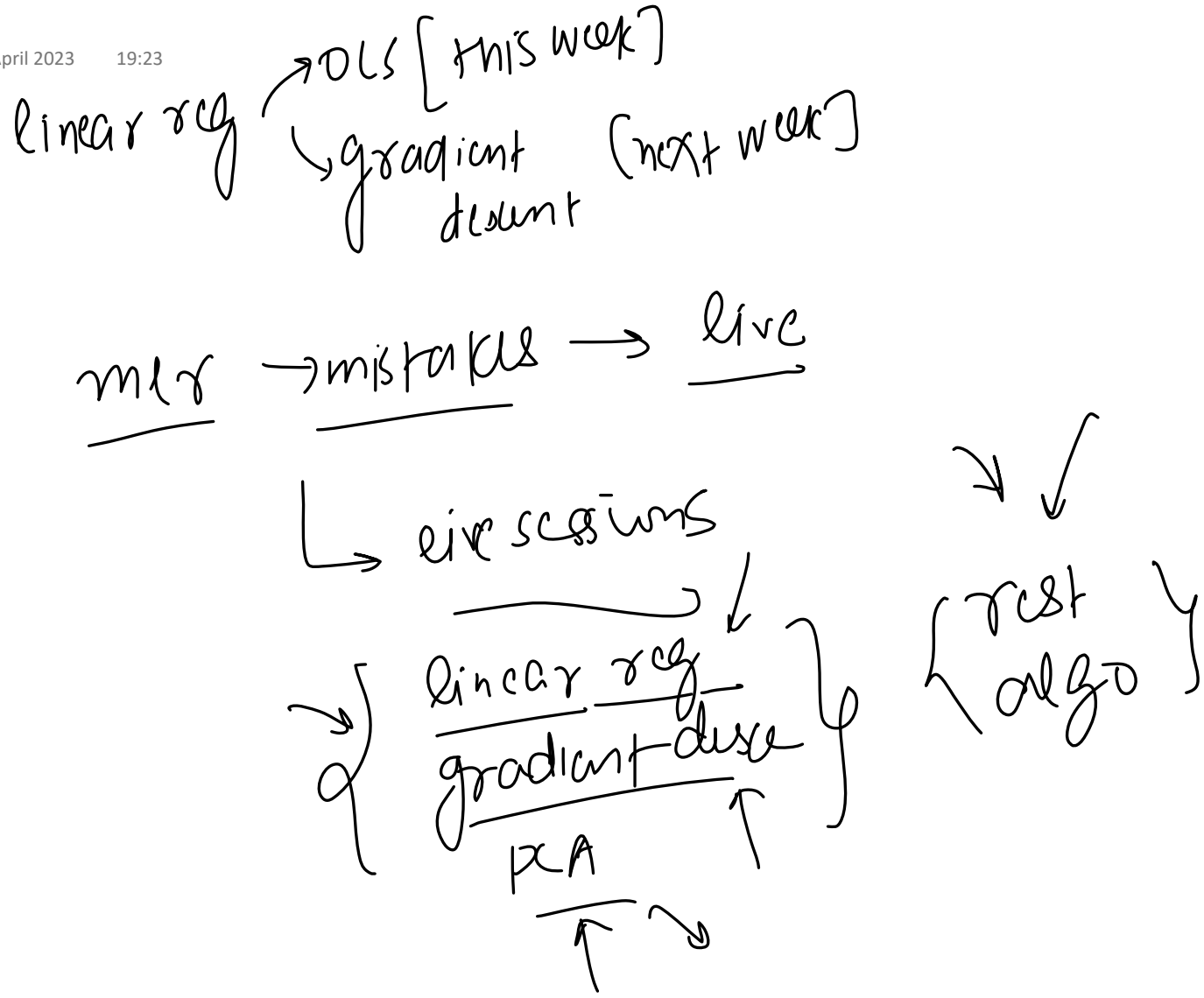

Recap

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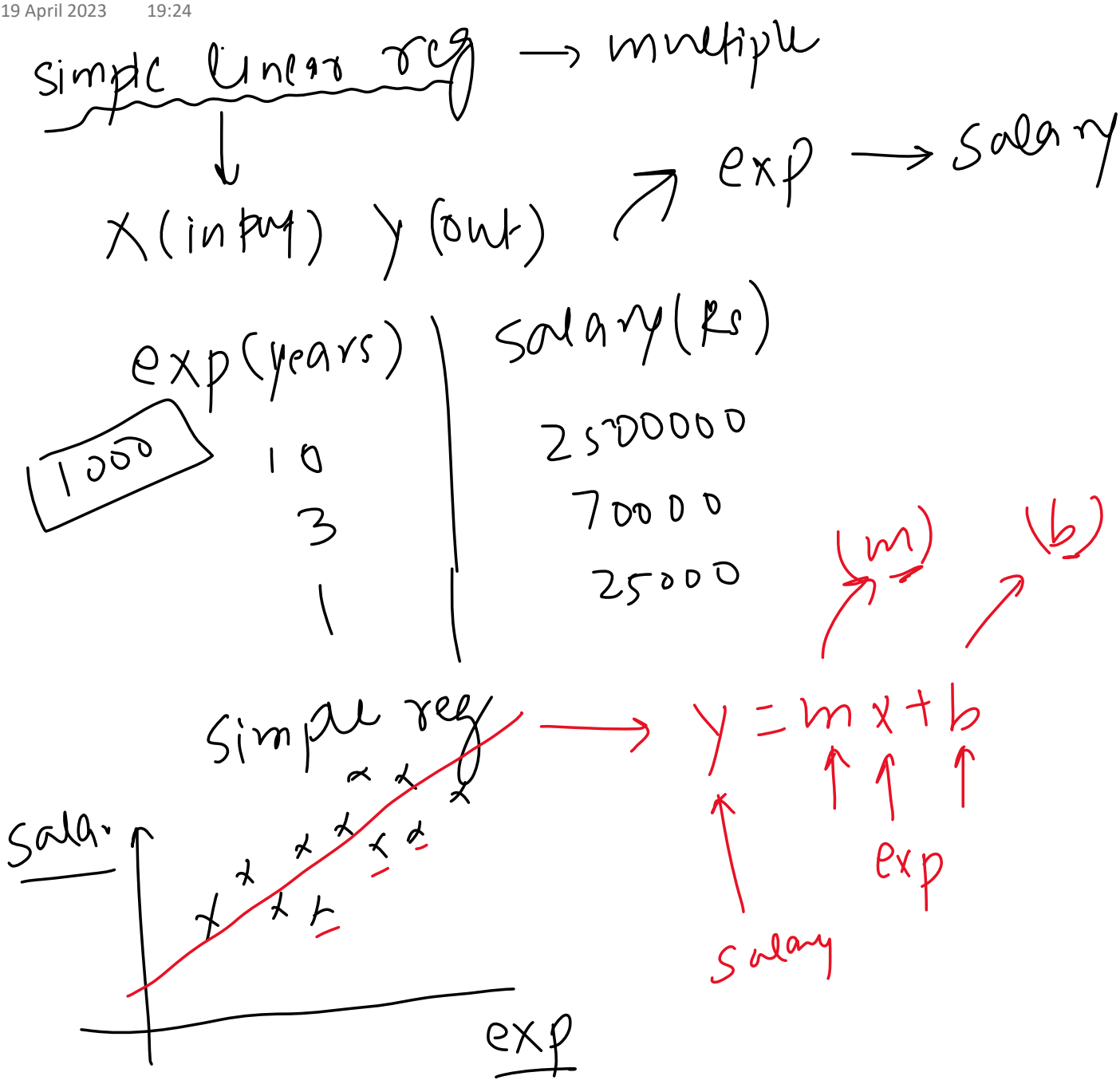

What is Multiple Linear Regression

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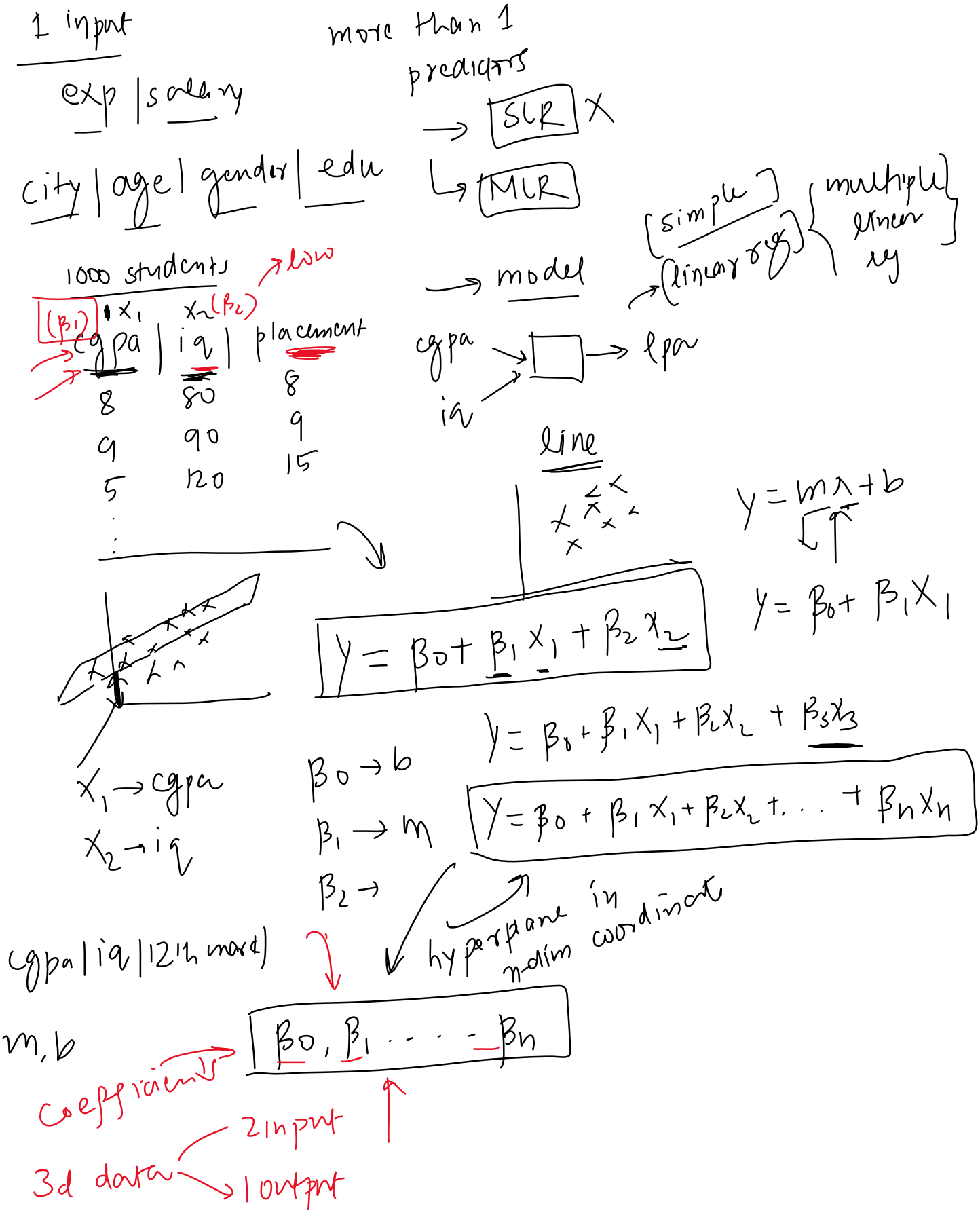

Python Code

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3) m cols → n students

x_1 cgpa	x_2 iq	y placement
8 x_{11}	80 x_{12}	8
7 x_{21}	70 x_{22}	7
5 x_{31}	120 x_{32}	15

predict

β_0 (β_1) (β_2)

$$\hat{y}_1 = \beta_0 + \beta_1 x_{11} + \beta_2 x_{12} + \beta_3 x_{13} + \beta_4 x_{14} + \ldots + \beta_m x_{1m}$$
$$\hat{y}_2 = \beta_0 + \beta_1 x_{21} + \beta_2 x_{22} + \ldots + \beta_m x_{2m}$$
$$\hat{y}_3 = \beta_0 + \beta_1 x_{31} + \beta_2 x_{32} + \ldots + \beta_m x_{3m}$$
$$\hat{y}_n = \beta_0 + \beta_1 x_{n1} + \beta_2 x_{n2} + \ldots + \beta_m x_{nm}$$

$y_1 = 8 \quad y_2 = 7 \quad y_3 = 15$

$\hat{y}_1 = ? \quad \hat{y}_2 = ? \quad \hat{y}_3 = ?$

$$\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \end{bmatrix} = \begin{bmatrix} \beta_0 + \beta_1 8 + \beta_2 80 \\ \beta_0 + \beta_1 7 + \beta_2 70 \\ \beta_0 + \beta_1 5 + \beta_2 120 \end{bmatrix}$$

$$\hat{y} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_n \end{bmatrix} = \begin{bmatrix} \beta_0 + \beta_1 x_{11} + \beta_2 x_{12} + \beta_3 x_{13} + \beta_4 x_{14} + \ldots + \beta_m x_{1m} \\ \beta_0 + \beta_1 x_{21} + \beta_2 x_{22} + \ldots + \beta_m x_{2m} \\ \beta_0 + \beta_1 x_{31} + \beta_2 x_{32} + \ldots + \beta_m x_{3m} \\ \vdots \\ \beta_0 + \beta_1 x_{n1} + \beta_2 x_{n2} + \ldots + \beta_m x_{nm} \end{bmatrix}$$

$n \times 1$

$$= \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1m} \\ 1 & x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & & & & \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_m \end{bmatrix}$$

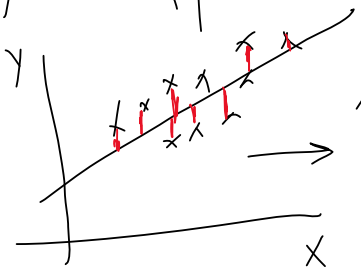
$(n \times 1)$

$$\hat{y} = X\beta$$

$n \times (m+1) \leftrightarrow (m+1) \times 1$

$n \times 1$

$$\hat{y} = X\beta$$



$d_1^2 + d_2^2 + d_3^2 + \ldots d_n^2$

$$E = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \quad \min$$

minimize

matrix form

matrix form

$$\underline{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}_{n \times 1} \qquad \underline{\hat{y}} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_n \end{bmatrix}_{n \times 1}$$

$$e = y - \hat{y} = \begin{bmatrix} y_1 - \hat{y}_1 \\ y_2 - \hat{y}_2 \end{bmatrix}$$

$$e = y - \hat{y} = \begin{bmatrix} \vdots \\ y_2 - \hat{y}_2 \\ \vdots \\ y_n - \hat{y}_n \end{bmatrix}_{n\times 1}$$

$$e^T e = \begin{bmatrix} y_1 - \hat{y}_1 & y_2 - \hat{y}_2 & \cdots & y_n - \hat{y}_n \end{bmatrix}_{1\times n} \begin{bmatrix} y_1 - \hat{y}_1 \\ y_2 - \hat{y}_2 \\ \vdots \\ y_n - \hat{y}_n \end{bmatrix}_{n\times 1} =$$

$$e^T e = (y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2 + \cdots + (y_n - \hat{y}_n)^2 \quad (1\times 1)$$

$$= \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$E = e^T e$$

$$E = (y - \hat{y})^T (y - \hat{y}) = (y^T - \hat{y}^T)(y - \hat{y})$$

$$E = y^T y - y^T \hat{y} - \hat{y}^T y + \hat{y}^T \hat{y}$$

$$E = y^T y - 2y^T \hat{y} + \hat{y}^T \hat{y}$$

eq (3) $\hat{y} = X\beta$

$y = f(x) \rightarrow x$

$$E = y^T y - 2y^T X\beta + (X\beta)^T (X\beta)$$

$$E = y^T y - 2y^T X\beta + \beta^T X^T X\beta$$

eq (4)

$y \rightarrow$ data output

$x \rightarrow$ data input

$f(x) = x^2 \rightarrow 16, 4$

$\frac{d}{d\beta}\left(\beta^T X^T X \beta\right)$

$E(\beta) =$ find such value of β matrix for which E is min

$\frac{dE}{d\beta} = 0$

$\frac{dE}{d\beta} = 0 - 2y^T X + 2\beta^T X^T X = 0$

$2\beta^T X^T X = 2y^T X$

$\beta^T X^T X = y^T X$

$A^T = A$

matrix diff

A is symmetric

$\beta^T X^T X (X^T X)^{-1} = y^T X (X^T X)^{-1}$

$\frac{d}{d\beta}\left(\beta^T X^T X \beta\right)$

$\frac{d}{d\beta}\beta^T X^T X \beta$ A is symmetric

$\downarrow$

$x^T A x \quad 2x^T A$

$= 2\beta^T X^T X$

$(X^T X)^T = X^T X$

$A^T = A$

$\beta^T X^T X (X^T X)^{-1} = y^T X (X^T X)^{-1}$

$\beta^T I = y^T X (X^T X)^{-1}$

$\beta^T = y^T X (X^T X)^{-1}$

$(\beta^T)^T = [y^T X (X^T X)^{-1}]^T$

$A \quad B$

$\beta = [(X^T X)^{-1}]^T (y^T X)^T$

$\beta = [(X^T X)^{-1}]^T X^T y$

$\beta = (X^T X)^{-1} X^T y$ eqn 5

OLS

β = values $(m+1 \times 1)$

$(X^T X)^{-1}$

$(m+1) \times 1$

$(X^T X)^{-1}$ symmetric

$[(X^T X)^{-1}]^T = (X^T X)^{-1}$

$X^T X = A$

$A A^{-1} = I$

$(A A^{-1})^T = I^T$

$(A^{-1})^T A^T = I$

$(A^{-1})^T A = I$

$(A^{-1})^T A A^{-1} = I A^{-1}$

$(A^{-1})^T I = A^{-1}$

$(A^{-1})^T = A^{-1}$

$(X^T X)^T = X^T X$

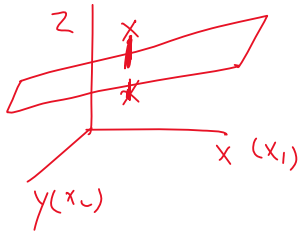
$A^T = A$

global minima

$m+1 \times n \quad n \times (m+1)$

$(m+1) \times (m+1) \quad (m+1) \times n$

$(m+1) \times n \quad n \times 1$



exp | 12th marks | salary

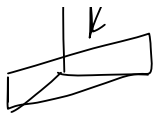
$\beta_1 \quad \beta_2$

salary $= \exp \times \beta_1 + 12^{th} \text{ V} \beta_2 + \beta_0$

0 D

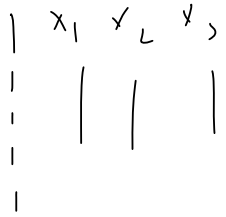
$\beta_0 = 0$

$(X^T X)^{-1} X^T y$



$\beta_0 = 0$

$$(X^T X)^{-1} X^T Y$$



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$y^T \hat{y} = \hat{y}^T y$

$Y = A \quad \hat{Y} = B$

$A^T B = B^T A$

$(A^T B)^T = B^T A$

$(AB)^T = B^T A^T$

$(A^T)^T = A$

$A^T B = (A^T B)^T$

$C = C^T$

$A^T B = C$

symmetric matrix

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \leftarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\hat{y} = X\beta$

$n \times (m+1)$

$A^T B$ is symmetric

$y^T \hat{y} = y^T X \beta$

$1 \times 1 \rightarrow$ Scalar

$(1) \quad [2]$

$A^T B$ is sym

$(1 \times n) \quad n \times (m+1) \quad (m+1) \times 1$

$1 \times n \quad n \times 1$

Code From Scratch

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Problem with OLS solution

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