Recap

21 June 2023 16:30

Terms

- exp
- trial
- Outcome
- sample
- Events

Types of probability

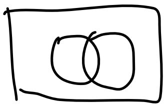
- Joint
- Marginal
- Conditional

Bayes Theorem

Venn Diagrams in Probability

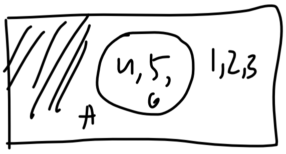
20 June 2023 15:07

Set Theory

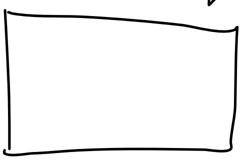


→ Probability

$\{1,2,3,4,5,6\}$



$U(1,2,3,4,5,6)$



U (sample set)

$P(U) = 1$

→ rolling a die

→ Event A → getting a num ≥ 4

$P(A) = \frac{3}{6} = \frac{1}{2}$

$P(A)' = \frac{1}{2}$

$P(U) - P(A)$

prob

A → num ≥ 4 (4,5,6)

B → even num (2,4,6)

$\{1,2,3,4,5,6\}$

$P(A \cap B) = \frac{2}{6}$

$P(U) = 1$

$P(A) = \frac{3}{6} = \frac{1}{2}$ $P(B) = \frac{1}{2}$

$P(A \cup B)' = \frac{2}{6}$

$P(A \cup B) = \frac{4}{6}$

$P(U) - P(A \cup B)$

$1 - \frac{2}{3} = \boxed{\frac{1}{3}}$

$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$

$= \frac{1}{3} + \frac{1}{3} -$

Contingency Tables in Probability

20 June 2023 15:07

experiment → rolling a die

$\boxed{A \rightarrow \geq 4}$ X

$\boxed{B \rightarrow \text{even number}}$ Y

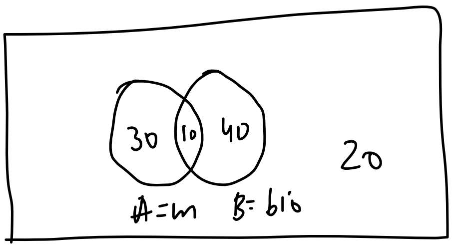
	even	not even
≥ 4	2	1
< 4	1	2

100 students

40 → only bio
30 → only maths
10 → both

	math	not math
bio	10	40
not bio	30	20

venn



Joint Probability

15 June 2023 18:22

random var

Let's say we have two random variables X and Y. The joint probability of X and Y, denoted as P(X = x, Y = y), is the probability that X takes the value x and Y takes the value y at the same time.

titanic

random var X and Y

$X=1$
$X=2$
$X=3$

$X \rightarrow \{1,2,3\}$

Let X be a random variable associated with the Pclass of a passenger
Let Y be a random variable associated with the survival status of a passenger

contingency table

$Y \rightarrow \{0,1\}$

$Y=0$
$Y=1$

X

Pclass	1	2	3
Survived			
0	80	97	372
1	136	87	119

/891

$X=1 \quad Y=1$
$X=2 \quad Y=1$
$X=3 \quad Y=1$

$P(X=1, Y=0)$

$\boxed{\frac{80}{891}}$ ← total passengers

Pclass	1	2	3
Survived			
0	0.089787	0.108866	0.417508
1	0.152637	0.097643	0.133558

= 1

$P(X=1, Y=0)$

Joint Probability Distribution

1	2	. . .	6
1/6	1/6		1/6

→ prob

$X=1 \ Y=0 \quad X=2 \ Y=0 \ . \ . \ . \ . \ . \ . \ . \quad X=3 \ Y=1$

→ p(x) —— p(x) —— —— ——

Marginal Probability/ Simple / Unconditional

15 June 2023 21:15

Marginal probability refers to the probability of an event occurring irrespective of the outcome of some other event. When dealing with random variables, the marginal probability of a random variable is simply the probability of that variable taking a certain value, regardless of the values of other variables.

Let X be a random variable associated with the Pclass of a passenger
Let Y be a random variable associated with the survival status of a passenger

Pclass	1	2	3	All
Survived				
0	80	97	372	549
1	136	87	119	342
All	216	184	491	891

/891 /891 /891

891 891 891

marginal prob

X

Pclass	1	2	3	All
Survived				
0	0.089787	0.108866	0.417508	0.616162
1	0.152637	0.097643	0.133558	0.383838
All	0.242424	0.206510	0.551066	1.000000

Y

y = 0, 1

$P(X)$ $Y = 0, 1$

$P(Y=0) = 0.61$
$P(Y=1 = 0.38$

$P(Y)$ $X = 1, 2, 3$

marginal prob dist of Y

Marginal Probability Distribution
Also known as unconditional probability P(A)

$P(X=1) = 0.24$
$P(X=2 = 0.20$
$P(X=3 = 0.55$

$\rightarrow$ 1

marginal prob dist of X

Conditional Probability

15 June 2023 18:15

A ← B   P(A|B)

P(A|B)

P(B|A)

Conditional probability is a measure of the probability of an event occurring, given that another event has already occurred. If the event of interest is A and event B has already occurred, the conditional probability of A given B is usually written as P(A|B).

Three unbiased coins are tossed. What is the conditional probability that at least two coins show heads, given that at least one coin shows heads?

{HHH, HHT, HTH, THH, HHT, THT, TTH, ~~TTT~~}

→ 7 outcomes

A → at least 2 heads

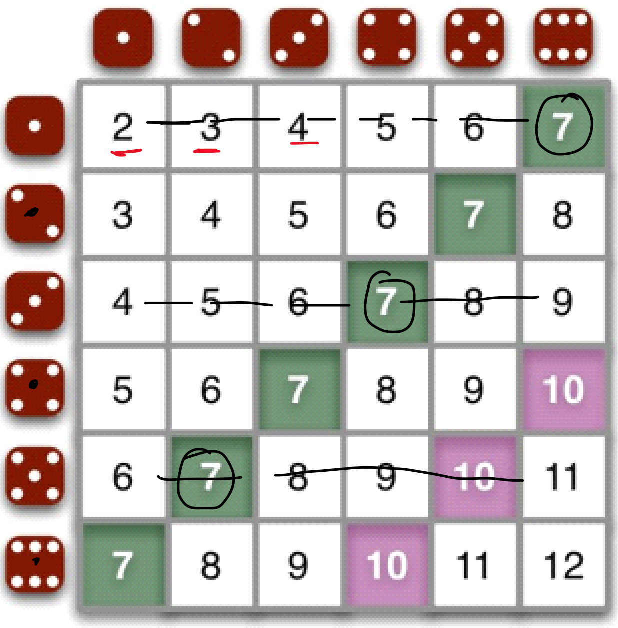
B → at least 1 head

$\frac{4}{7} = P(A|B)$

Two fair six-sided dice are rolled. What is the conditional probability that the sum of the numbers rolled is 7, given that the first die shows an odd number?

die2

die1



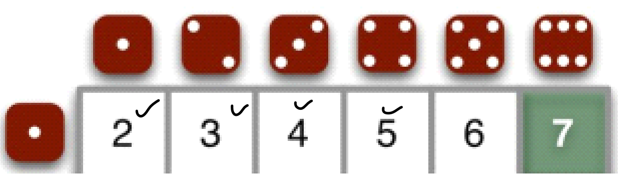
A → sum = 7

B → die1 → odd

$P(A|B) = \frac{3}{18} = \frac{1}{6}$

Two fair six-sided dice are rolled, denoted as D1 and D2. What is the conditional probability that D1 equals 2, given that the sum of D1 and D2 is less than or equal to 5?

D2



$\boxed{A \to D1 = 2}$

$B \to D1 + D2 \leq 5$

2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11
7	8	9	10	11	12

D1

$B \to D1 + D2 \leq 5$

$$\boxed{P(A|B) = \frac{3}{10}}$$

Formula for Conditional Probability

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$$

$P(A \cap B) \to$ joint prob A and B

$P(B) \to$ marginal prob of B

X

y

Pclass	1	2	3
Survived			
0	80	97	372
1	136	87	119

/891

Pclass	1	2	3
Survived			
0	0.089787	0.108866	0.417508
1	0.152637	0.097643	0.133558

→ joint prob

P=3

$$P(Y=0 \mid X=3) = \frac{P(Y=0 \cap X=3)}{P(X)} \to = \frac{0.41}{0.54} = \boxed{0.75}$$

$$P(Y=0 \mid X=2) = \frac{0.10}{0.19} = 0.52 \to$$

PCla

$$\boxed{\frac{0.08}{0.23}} = 0.34$$

$$P(X=3 \mid Y=0)$$

Intuition behind the Conditional Probability Formula

20 June 2023 20:06

The intuition behind the formula for conditional probability, P(A | B) = P(A ∩ B) / P(B), is based on the concept of reducing our sample space.

The denominator, P(B), is the probability of event B occurring. When we say we want to know the probability of A given B, we're effectively saying that B has occurred and therefore B is our new "universe" or sample space. So we're not considering cases when B didn't occur anymore, and we're normalizing by the probability of B.

The numerator, P(A ∩ B), is the joint probability of A and B, meaning both A and B occur. So in the context of our new universe where B has occurred, P(A ∩ B) represents the cases where A also occurs.

By dividing the joint probability P(A ∩ B) by the probability of B (P(B)), we effectively find the proportion of times that A occurs given that B has occurred.

In summary, the conditional probability of A given B is just the joint probability of A and B happening (A and B together in the "universe" where everything happens), normalized by the probability of B (the new "universe" where only B happens).

$$\boxed{P(A|B)} = \frac{P(A \cap B)}{\boxed{P(B)}}$$

definition

$\boxed{B} \rightarrow \boxed{SS}$ $\boxed{SS}$ ✗

$A \cap B$

$\boxed{B}$

$A \cap B$

$$\boxed{\frac{A}{B}}$$

Independent Vs Mutually Exclusive Events

15 June 2023 18:16

Independent events are events where the occurrence of one event does not affect the occurrence of another.

Examples

1. Flipping a coin and rolling a die
2. Drawing a card with replacement

Dependent events are events where the occurrence of one event does affect the occurrence of another.

$\boxed{51}$ $\frac{4}{51}$

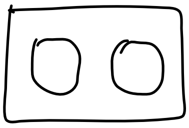
$\rightarrow P(A|B) = \frac{P(A \cap B)}{P(B)}$ → independent

Examples

1. Drawing a card without ~~with~~ replacement

Mutually Exclusive Events

Mutually exclusive events are events that cannot both occur at the same time. In other words, if one event occurs, the other cannot.



Examples

1. Flipping a coin → $\boxed{H, T}$ →
2. Rolling a die

↳ even / odd

$\boxed{P(A \cap B) = 0}$ ↳ 0

$P(A|B) = 0$

$\rightarrow P(A|B) = \frac{P(A \cap B)}{P(B)}$ ←

	If statistically independent	If mutually exclusive
$P(A \mid B) =$	$P(A)$	0
$P(B \mid A) =$	$P(B)$	0
$P(A \cap B) =$	$P(A)P(B)$	0

independent events

$\rightarrow P(A|B) = P(A)$

$\rightarrow P(B|A) = P(B)$

$\rightarrow \boxed{P(A \cap B) = P(A)P(B)}$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)\cancel{P(B)}}{\cancel{P(B)}}$

Bayes Theorem

15 June 2023 18:16

→ Bayesian ~~Statistics~~ ←

Bayes' Theorem is a fundamental concept in the field of probability and statistics that describes how to update the probabilities of hypotheses when given evidence. It's used in a wide variety of fields, including machine learning, statistics, and game theory.

The theorem is named after Thomas Bayes, who introduced an early version of the rule in his work on probability theory. Bayes' Theorem provides a way to revise existing predictions or theories (update probabilities) given new evidence.

$$\boxed{P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}}$$

$$\rightarrow \underbrace{P(A|B)}_{\text{posterior}} = \underbrace{P(A)}_{\text{prior}} \times \frac{\overbrace{P(B|A)}^{\text{likelihood}}}{\underbrace{P(B)}_{\text{marginal/evidence}}}$$

Mathematical Proof

conditional Prob

$$P(A|B) = \frac{P(A\cap B)}{P(B)} \qquad A\cap B = B\cap A$$

$$P(B|A) = \frac{P(B\cap A)}{P(A)} \Rightarrow P(B\cap A) = P(A\cap B) = P(B|A)\,P(A)$$

$$\boxed{P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}} \rightarrow \text{Bayes Theorem}$$

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

gender		survived
M	→	0
M	→	0
F	→	1
F	→	0
M	→	1

→ {M} → {0,1}

$$P(0|M) = \frac{P(M|0)\,P(0)}{P(M)} \quad P(M|0) \to 2/3,\; P(0) \to 3/5,\; P(M) \to 3/5 \quad = 2/3$$

$$P(1|M) = \frac{P(M|1)\,P(1)}{} \quad P(1) \to 2/5 \quad \frac{1}{2}$$

$$\boxed{P(0|M)} = \frac{2}{3} \checkmark \rightarrow \text{dead} > \text{alive}$$

$$\boxed{P(1|M)} = \frac{1}{3} \checkmark$$

$$\frac{\frac{1}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{5}}}{3/\cancel{5}} = \frac{1}{3}$$

$$p(1|M) = \frac{P(M|1)\,P(1)}{p(M)}$$

$P(M|1) \rightarrow 1/2$, $P(1) \rightarrow 2/5$, $p(M) \rightarrow 3/5$