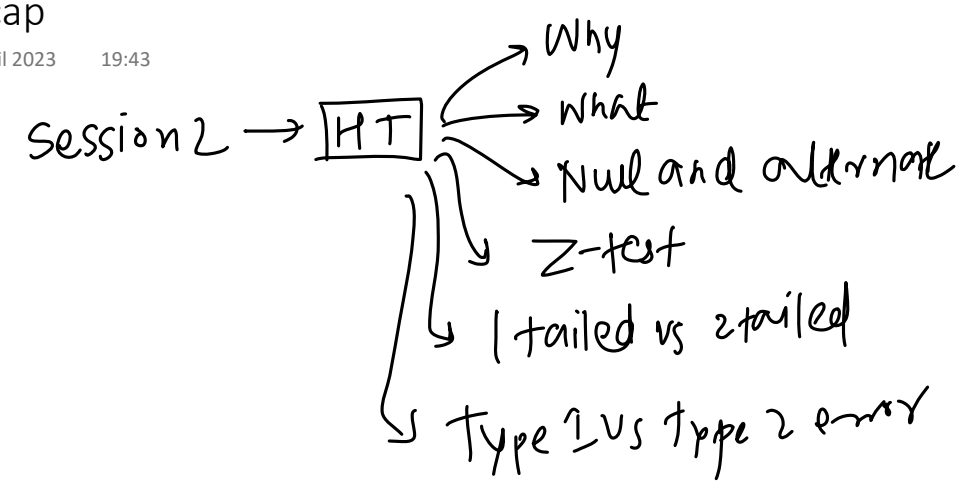
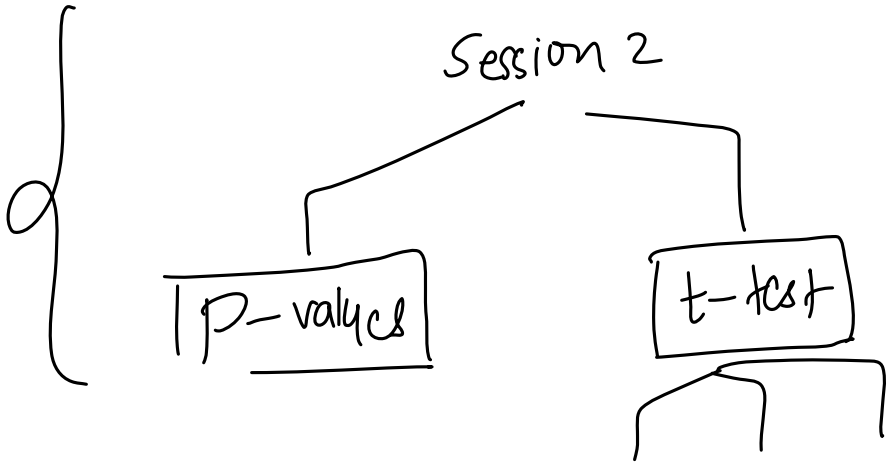
Recap

06 April 2023 19:43



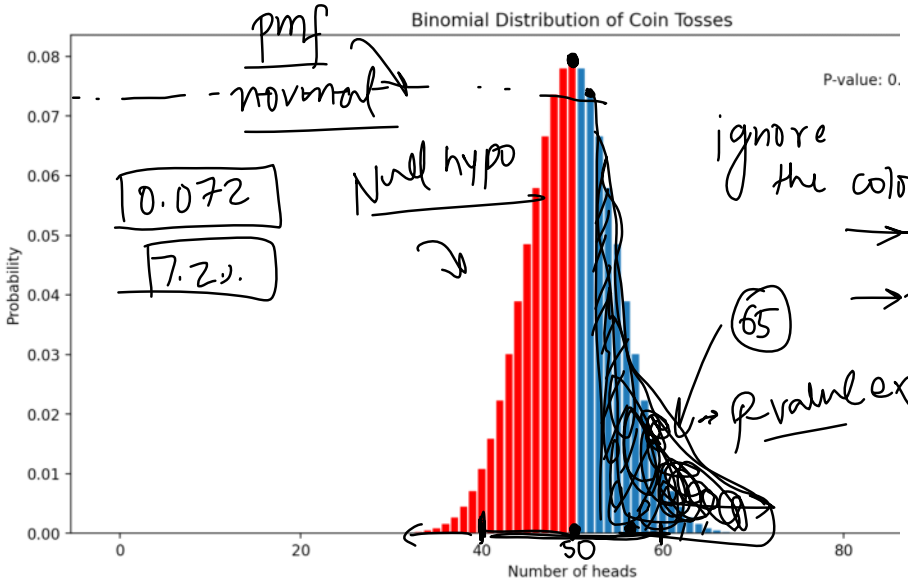
significance level (α)



P-value

06 April 2023 06:48

P-value is the probability of getting a sample as or more extreme(having more evidence against H0) than our own sample given the Null Hypothesis(H0) is true.



In simple words p-value is a measure of the strength of the evidence against the Null Hypothesis that is provided by our sample data.

Interpreting p-value

06 April 2023 08:25

With significance value

$\alpha = 0.05 / 0.01$ → p-value $\leq \alpha$ → reject your H_0

Without significance value → α → 0.05

1. Very small p-values (e.g., $p < 0.01$) indicate strong evidence against the null hypothesis, suggesting that the observed effect or difference is unlikely to have occurred by chance alone.
2. Small p-values (e.g., $0.01 \leq p < 0.05$) indicate moderate evidence against the null hypothesis, suggesting that the observed effect or difference is less likely to have occurred by chance alone.
3. Large p-values (e.g., $0.05 \leq p < 0.1$) indicate weak evidence against the null hypothesis, suggesting that the observed effect or difference might have occurred by chance alone, but there is still some level of uncertainty.
4. Very large p-values (e.g., $p \geq 0.1$) indicate weak or no evidence against the null hypothesis, suggesting that the observed effect or difference is likely to have occurred by chance alone.

P-value in context of Z-test

06 April 2023 07:08

rejection region approach p-value approach

Suppose a company is evaluating the impact of a new training program on the productivity of its employees. The company has data on the average productivity of its employees before implementing the training program. The average productivity was 50 units per day. After implementing the training program, the company measures the productivity of a random sample of 30 employees. The sample has an average productivity of 53 units per day and the pop std is 4. The company wants to know if the new training program has significantly increased productivity.

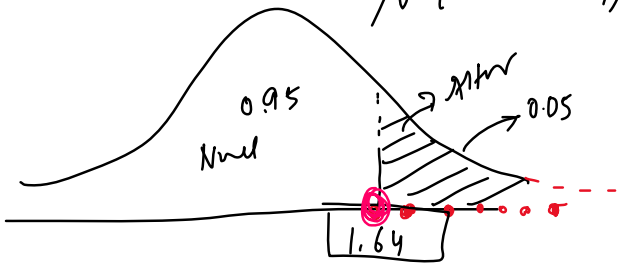
$\mu = 50$ $n = 30$ $\bar{x} = 53$

$\sigma = 4$ $\alpha = 0.05$

$H_0: \mu = 50$

$H_a: \mu > 50$

$$\text{Z-stat} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{53 - 50}{4/\sqrt{30}} = \frac{3 \times \sqrt{30}}{4} = 4.10 \quad / \quad 1.7$$



reject

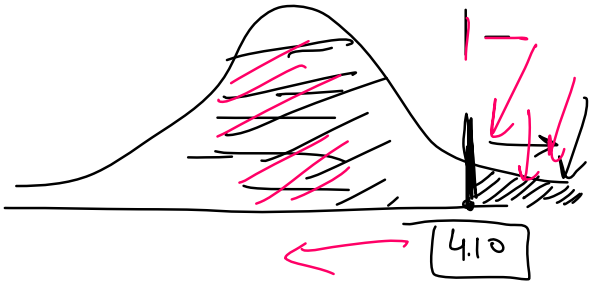
$1 - 0.95$

0.05

p-value

$Z = 4.10$

p-value → critical point



$0.999 = 0.0001$

p-value < 0.05

reject Null hypo

Suppose a snack food company claims that their Lays wafer packets contain an average weight of 50 grams per packet. To verify this claim, a consumer watchdog organization decides to test a random sample of Lays wafer packets. The organization wants to determine whether the actual average weight differs significantly from the claimed 50 grams. The organization collects a random sample of 40 Lays wafer packets and measures their weights. They find that the sample has an average weight of 49 grams, with a pop standard deviation of 5 grams.

$\mu = 50$ $n = 40$ $\bar{x} = 49$

$\sigma = 5$ $\alpha = 0.05$

$H_0: \mu = 50$

$H_a: \mu \neq 50$

p-value ↙ 2 tailed $0.206 > 0.05$

$$Z = \frac{49 - 50}{5/\sqrt{40}} = \frac{-\sqrt{40}}{5} = -1.26$$

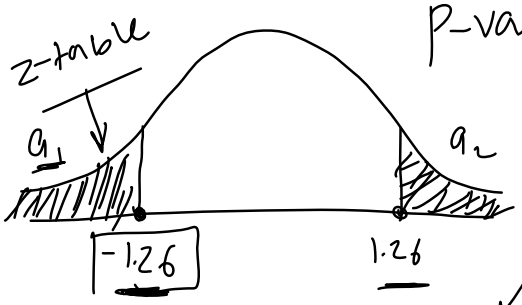
p-val 0.206

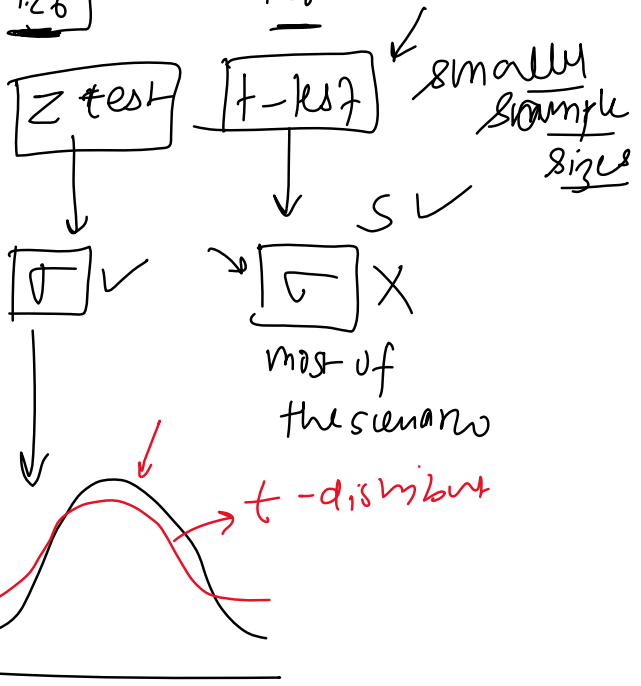
p-value $= a_1 + a_2$

$s/\sqrt{40}$ s

p-value = $a_1 + a_2$ | p-val | 0.206 |

$a_1 = 0.103 \times 2$





T-tests

06 April 2023 14:14

A t-test is a statistical test used in hypothesis testing to compare the means of two samples or to compare a sample mean to a known population mean. The t-test is based on the t-distribution, which is used when the population standard deviation is unknown and the sample size is small.

There are three main types of t-tests:

1 sample → $\bar{X}$ → μ

One-sample t-test: The one-sample t-test is used to compare the mean of a single sample to a known population mean. The null hypothesis states that there is no significant difference between the sample mean and the population mean, while the alternative hypothesis states that there is a significant difference.

→ σ X

1 class → test A → popl

→ test B → pop

Independent two-sample t-test: The independent two-sample t-test is used to compare the means of two independent samples. The null hypothesis states that there is no significant difference between the means of the two samples, while the alternative hypothesis states that there is a significant difference.

Paired t-test (dependent two-sample t-test): The paired t-test is used to compare the means of two samples that are dependent or paired, such as pre-test and post-test scores for the same group of subjects or measurements taken on the same subjects under two different conditions. The null hypothesis states that there is no significant difference between the means of the paired differences, while the alternative hypothesis states that there is a significant difference.

Single Sample t-test

06 April 2023 14:14

A one-sample t-test checks whether a sample mean differs from the population mean.

Assumptions for a single sample t-test

1. Normality - Population from which the sample is drawn is normally distributed
2. Independence - The observations in the sample must be independent, which means that the value of one observation should not influence the value of another observation.
3. Random Sampling - The sample must be a random and representative subset of the population.
4. Unknown population std - The population std is not known.

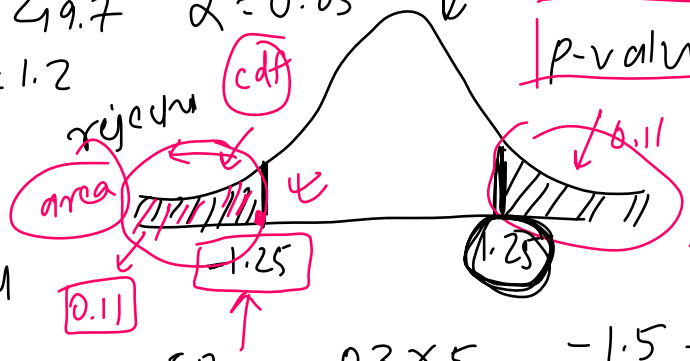
Suppose a manufacturer claims that the average weight of their new chocolate bars is 50 grams, we highly doubt that and want to check this so we drew out a sample of 25 chocolate bars and measured their weight, the sample mean came out to be 49.7 grams and the sample std deviation was 1.2 grams. Consider the significance level to be 0.05

$\mu = 50 \quad n = 25 \quad \bar{x} = 49.7 \quad \alpha = 0.05 \quad s = 1.2$

$H_0: \mu = 50$

$H_a: \mu \neq 50$

assuming it is normal



p-value = 0.22 > 0.05

Null reject ✗

$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{49.7 - 50}{1.2/\sqrt{25}} = \frac{-0.3 \times 5}{1.2} = \frac{-1.5}{1.2} = -1.25$

$df = n - 1 = 24$

$H_0: \mu = 35$

$H_a: \mu < 35$

25 sample age → normal

Shapiro-wilk test

$\mu = 35$ ↓ σ ✗ → t-test

$\bar{x}, s, \alpha = 0.05$

p-value < 0.05 not normal

p-value > 0.05 normal

Python Case Study 1

06 April 2023 17:27

Independent 2 sample t-test

06 April 2023 14:15

An independent two-sample t-test, also known as an unpaired t-test, is a statistical method used to compare the means of two independent groups to determine if there is a significant difference between them.

Assumptions for the test:

1. Independence of observations: The two samples must be independent, meaning there is no relationship between the observations in one group and the observations in the other group. The subjects in the two groups should be selected randomly and independently.
2. Normality: The data in each of the two groups should be approximately normally distributed. The t-test is considered robust to mild violations of normality, especially when the sample sizes are large (typically $n \geq 30$) and the sample sizes of the two groups are similar. If the data is highly skewed or has substantial outliers, consider using a non-parametric test, such as the Mann-Whitney U test.
3. Equal variances (Homoscedasticity): The variances of the two populations should be approximately equal. This assumption can be checked using F-test for equality of variances. If this assumption is not met, you can use Welch's t-test, which does not require equal variances.
4. Random sampling: The data should be collected using a random sampling method from the respective populations. This ensures that the sample is representative of the population and reduces the risk of selection bias.

Suppose a website owner claims that there is no difference in the average time spent on their website between desktop and mobile users. To test this claim, we collect data from 30 desktop users and 30 mobile users regarding the time spent on the website in minutes. The sample statistics are as follows:

desktop users = [12, 15, 18, 16, 20, 17, 14, 22, 19, 21, 23, 18, 25, 17, 16, 24, 20, 19, 22, 18, 15, 14, 23, 16, 12, 21, 19, 17, 20, 14]

mobile_users = [10, 12, 14, 13, 16, 15, 11, 17, 14, 16, 18, 14, 20, 15, 14, 19, 16, 15, 17, 14, 12, 11, 18, 15, 10, 16, 15, 13, 16, 11]

Desktop users:

- Sample size (n1): 30
- Sample mean (mean1): 18.5 minutes
- Sample standard deviation (std_dev1): 3.5 minutes

Mobile users:

- Sample size (n2): 30
- Sample mean (mean2): 14.3 minutes
- Sample standard deviation (std_dev2): 2.7 minutes

We will use a significance level (α) of 0.05 for the hypothesis test.

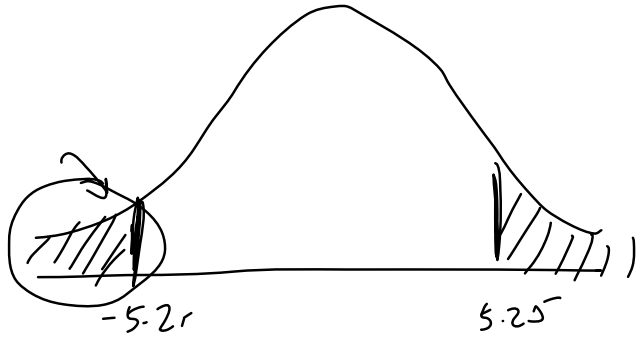
$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

t-statistic

$$t = \frac{18.5 - 14.3}{} = \frac{4.2}{} = 5.25$$

$$t = \frac{\quad}{\sqrt{\frac{(3.5)^2}{30} + \frac{(2.7)^2}{30}}} = \frac{4.2}{\sqrt{\frac{19.54}{30}}} = \boxed{5.45}$$



$n-1$

$n_1 + n_2 - 2$

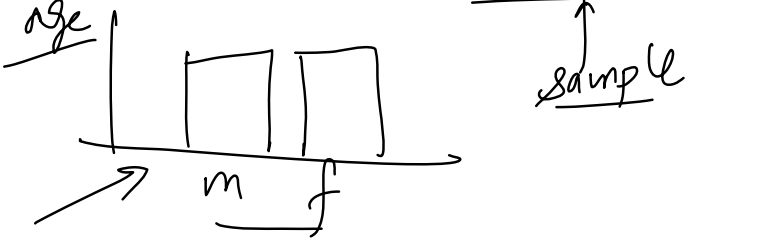
$30+30-2 = 58$

$\mu_m > \mu_f$

$\mu_m = \mu_f$

age and gender

pop



$H_0: \mu_m = \mu_f$

$H_1: \mu_m > \mu_f$

$\alpha = 0.05$

Python Case Study 2

06 April 2023 17:27

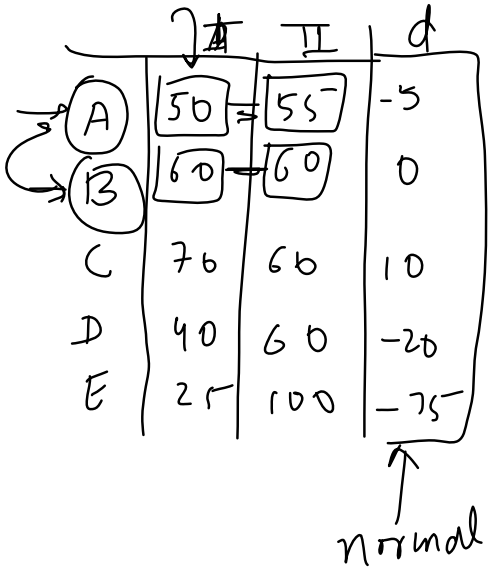
Paired 2 sample t-test

06 April 2023 14:21

A paired two-sample t-test, also known as a dependent or paired-samples t-test, is a statistical test used to compare the means of two related or dependent groups.

Common scenarios where a paired two-sample t-test is used include:

1. Before-and-after studies: Comparing the performance of a group before and after an intervention or treatment.
2. Matched or correlated groups: Comparing the performance of two groups that are matched or correlated in some way, such as siblings or pairs of individuals with similar characteristics.



Assumptions

1. Paired observations: The two sets of observations must be related or paired in some way, such as before-and-after measurements on the same subjects or observations from matched or correlated groups.
2. Normality: The differences between the paired observations should be approximately normally distributed. This assumption can be checked using graphical methods (e.g., histograms, Q-Q plots) or statistical tests for normality (e.g., Shapiro-Wilk test). Note that the t-test is generally robust to moderate violations of this assumption when the sample size is large.
3. Independence of pairs: Each pair of observations should be independent of other pairs. In other words, the outcome of one pair should not affect the outcome of another pair. This assumption is generally satisfied by appropriate study design and random sampling.

Let's assume that a fitness center is evaluating the effectiveness of a new 8-week weight loss program. They enroll 15 participants in the program and measure their weights before and after the program. The goal is to test whether the new weight loss program leads to a significant reduction in the participants' weight.

$\mu_{dif} = \mu_{bef} - \mu_{aft} = 0$

$H_0: \mu_{before} = \mu_{after}$

$H_1: \mu_{before} > \mu_{after}$

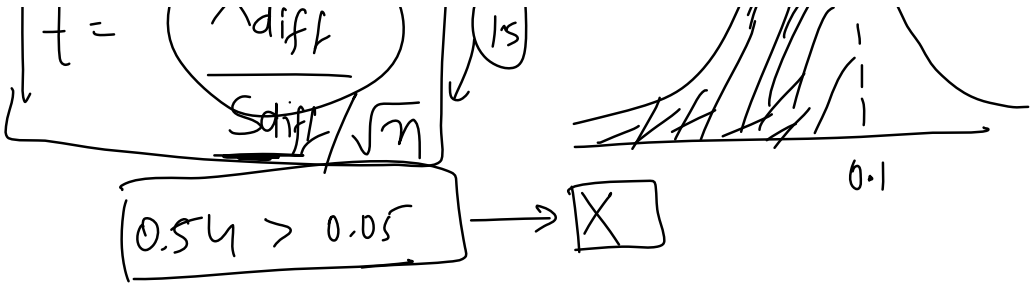
Before the program:
[80, 92, 75, 68, 85, 78, 73, 90, 70, 88, 76, 84, 82, 77, 91]

After the program:
[78, 93, 81, 67, 88, 76, 74, 91, 69, 88, 77, 81, 80, 79, 88]

Significance level (α) = 0.05

> 0.05

$$t = \frac{\bar{x}_{diff}}{\quad} \quad (15)$$



$0.54 > 0.05$ → X