Recap

15 April 2023 18:04

Stats → prob

Linear Alg

vectors

matrix

matrices → intuition → vis tools

3blue1brown

essence of linear algebra

simplic

Uses in ML/DL

Basis Vector

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vectors → $[2\ 3\ 4]$

dimension

$[-3\ 2]$

$[2\ 4]$

2d vectors

$[2\ 3]$

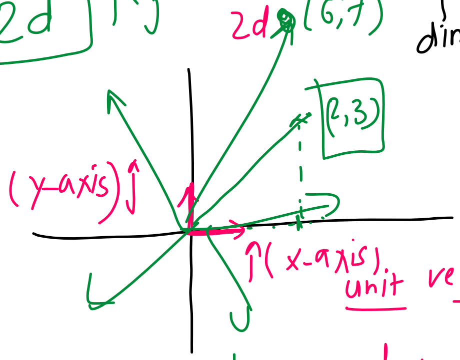
$[6\ 7]$

set scalars

$\boxed{6\hat{i} + 7\hat{j}}$

$\boxed{2d}$ $\hat{i}\ \hat{j}$

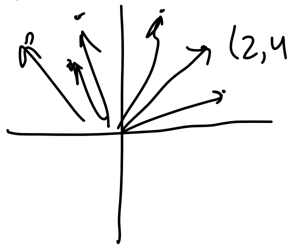
2d (6,7)



unit vectors

dist from origin 1

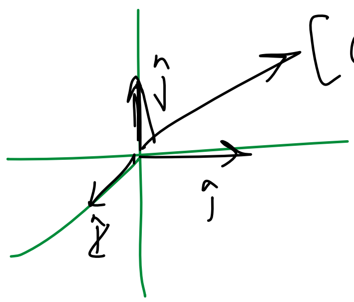
$2\hat{i} + 3\hat{j}$



(2,4) → 2d vector
2d point

$[3\ 4]$

$\boxed{3\hat{i} + 4\hat{j}}$ → linear combination



$[a\ b\ c]$

$\boxed{a\hat{i} + b\hat{j} + c\hat{k}}$

Linear Transformations

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matrices → linear transformation

$x = 2$

$f(x) = x^2 \rightarrow 4$

each input 1 output

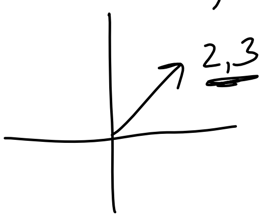
$\begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} \rightarrow$ $\boxed{\text{linear}}$ $\boxed{\text{transfor}}$

x

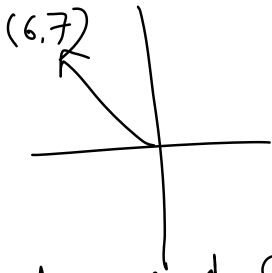
input → $f(x)$ → output

vector as input $\left(\begin{bmatrix} \text{transform matrix} \end{bmatrix} \right)$ → another vector

$\begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$



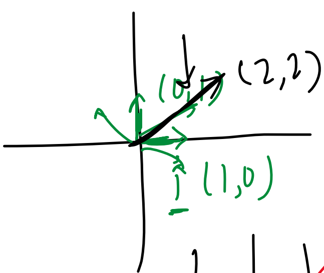
→

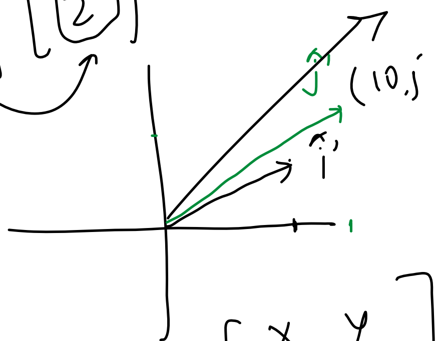


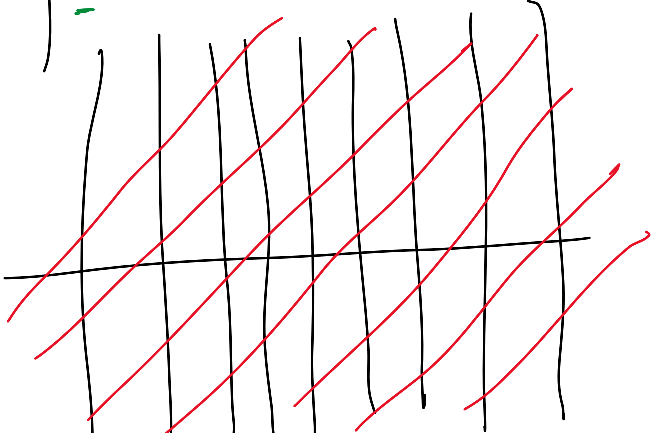
$(2,3) \rightarrow \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} \rightarrow (6,7)$

multi vector and a matrix

$x \quad \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ $(10,5)$





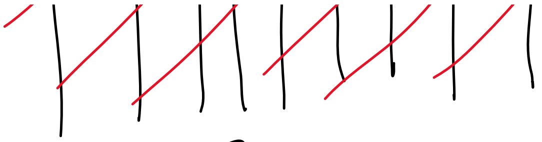


$\begin{bmatrix} x & y \\ z & w \end{bmatrix}$

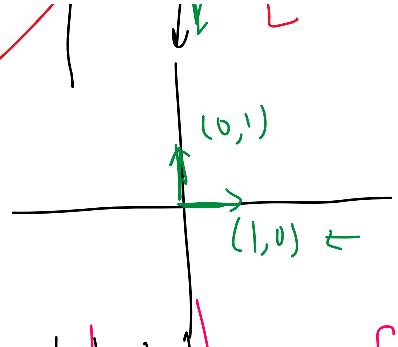
identity

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ identity

$\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$



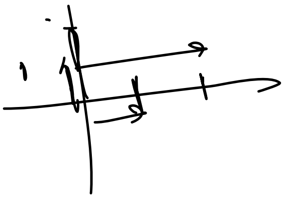
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

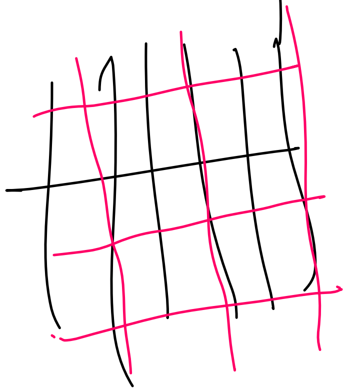


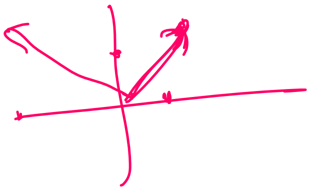
$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$[1, 0]$ $(0, 1]$

$$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$







$$\begin{bmatrix} 2 & -2 \\ -2 & 4 \end{bmatrix}$$

Linear Transformation in 3d

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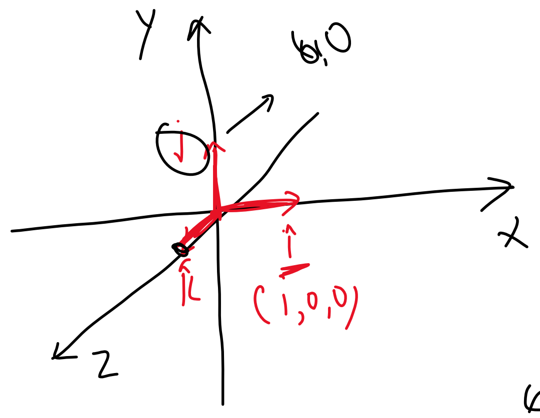

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$[1, 4, 7]$ $\qquad$ $[2, 5, 8]$

$(3, 6, 9)$

linear

Matrix Multiplication as Composition

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$$\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}\cdot\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} \qquad \begin{bmatrix}- & -\\- & -\end{bmatrix}$$

$2 \times \boxed{2}$ c $\qquad \boxed{2} \times 2$ r

2x2

$$\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$$

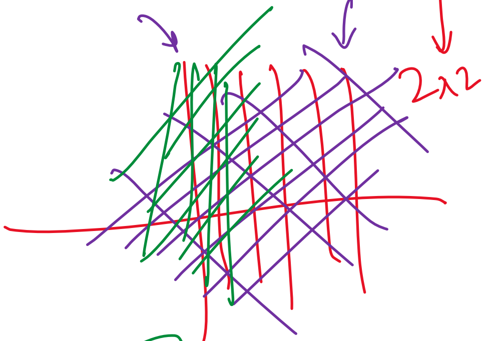
A $\qquad$ B

A.B $\begin{bmatrix}K & Y\\Z & W\end{bmatrix}$ transform

transformation

A.B

A.B



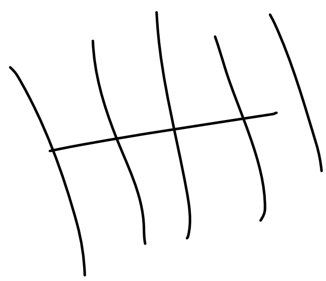
A.B

red → B
purple → A → green

red → A.B → green

$$A = \begin{bmatrix}1 & 0\\0 & 2\end{bmatrix} \qquad B = \begin{bmatrix}2 & 1\\1 & 5\end{bmatrix}$$

A.B → [ ]



A.B.C 3rd matrix coordinate space

C → AB → C''

C → B → C' → A → C''

composition of transformation

A.B $\qquad$ A.B intuition

$$A.B\begin{bmatrix}X\\Y\end{bmatrix} \qquad A.B\begin{bmatrix}X\\Y\end{bmatrix}$$

$$A.\begin{bmatrix}x'\\y'\end{bmatrix} = \begin{bmatrix}x''\\y''\end{bmatrix}$$

$$\left[y \right]$$

$$A \cdot \begin{bmatrix} x' \\ y' \end{bmatrix} = \left[y'' \right]$$

Test of Commutative Law

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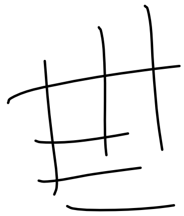
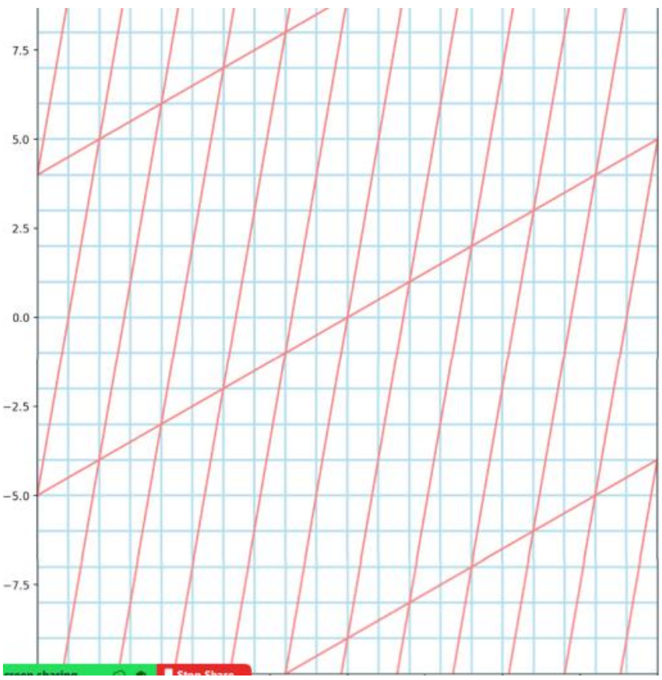
$A \cdot B \neq B \cdot A$

$A(BC) = (AB)C$ ✓

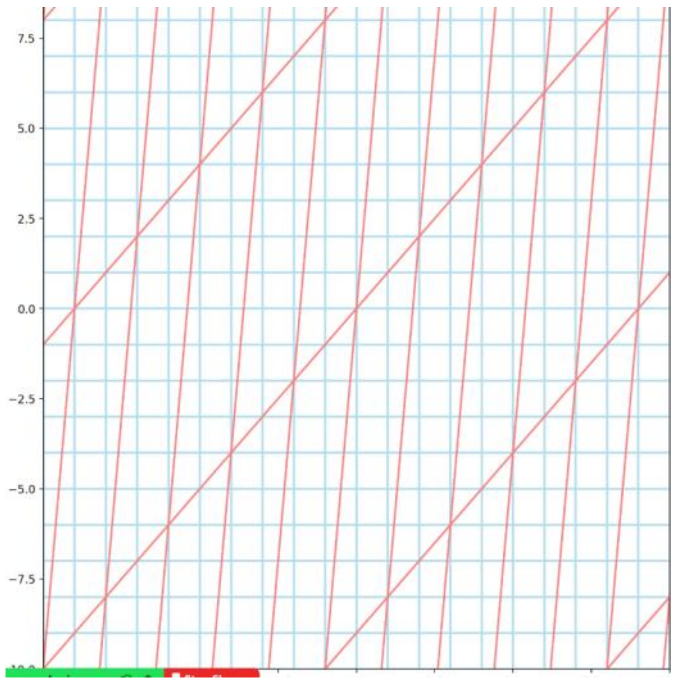


$$\begin{tikzcd} C \arrow[r] & A \arrow[r] & C' \arrow[d] \\ & & B \arrow[d] \\ & & \boxed{C''} \end{tikzcd}$$

$$\begin{tikzcd} C \arrow[r] & B \arrow[r] & C' \arrow[d] \\ & & A \arrow[d] \\ & & \boxed{C''} \end{tikzcd}$$



B·A → A first then B



A·B → B first then A

Determinant

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1x1 2x2 3x3

$\Delta = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = 10 - 12 = \boxed{-2}$

2

non-square

transform

d

Why determinant is possible only for square matrix?

The interpretation of the determinant as a scaling factor is only meaningful for square matrices because the input and output spaces must have the same dimension for this concept to be applicable.

What does it mean to have a negative determinant?

What happens when a matrix is singular? det = 0

$\begin{array}{cc} \hat{i} & \hat{j} \\ \end{array}$

$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = A$

area 1

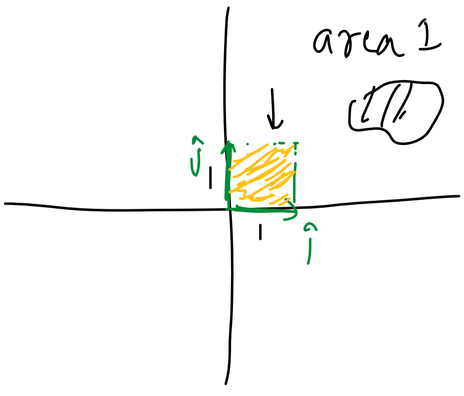
area' = 4

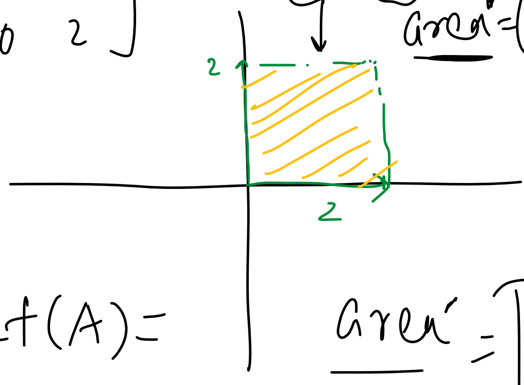
2d - 2d

2d → 1d

3d - 2d

1d - point 0d

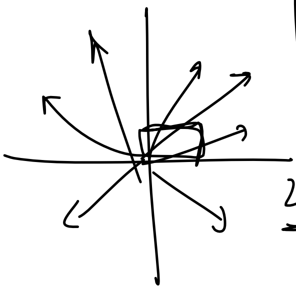


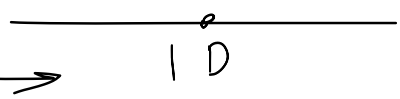


$\det(A) = \dfrac{\text{area}'}{\text{area}} = \boxed{\dfrac{4}{1}} = 4$

$\begin{vmatrix} 2 & 4 \\ 2 & 4 \end{vmatrix} = 2 \cdot 4 - 2 \cdot 4 = \boxed{0} \rightarrow \boxed{\text{inverse}}$

$\begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}$





Inverse

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$\boxed{A \longrightarrow A^{-1}}$

$AA^{-1} = \boxed{I}$

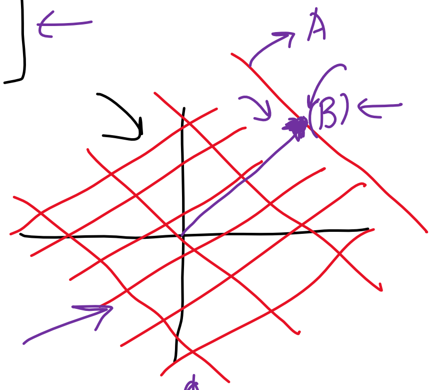
Why A.A^-1 = I ?
Why inverse is possible for square matrix only

$\boxed{A^{-1} \cdot B} \longrightarrow \boxed{x, y}$

$x + 2y = 5$

$3x + 5y = 6$

$$\underbrace{\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\boxed{X}} = \underbrace{\begin{bmatrix} 5 \\ 6 \end{bmatrix}}_{B}$$



$A^{-1}A = I$

$\boxed{AX = B}$

some vector $\boxed{X}$

$\boxed{A}X = B$

$A^{-1}AX = A^{-1}B$

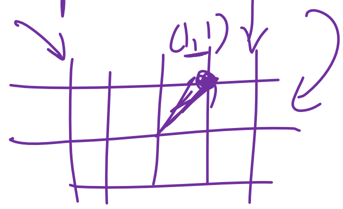
$IX = A^{-1}B$

$\boxed{X = A^{-1}B}$

reverting the transfor

$x = (1,1)$

revert $\rightarrow$ (A)



$A\boxed{A^{-1}} = I$

transformation for non square matrix?

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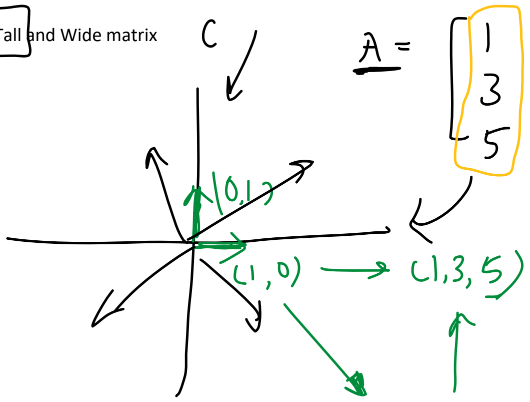
Square matrices (n × n) represent linear transformations where the domain and codomain vector spaces have the same dimensions, i.e., T: V → V. In these cases, the transformation maps a vector space onto itself. Non-square matrices can also represent linear transformations between vector spaces with different dimensions.

square matrices

Tall and Wide matrix



$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}_{3\times 2}$

row > cols

tall matrix

$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \rightarrow$ transformations

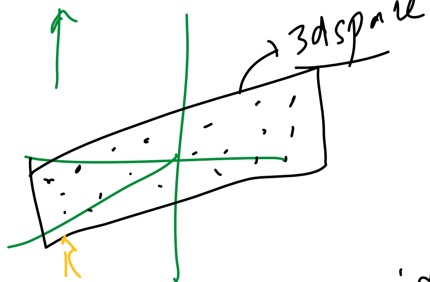
Yes

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix}$

$(1,0) \rightarrow (1,3,5)$

$(0,1) \rightarrow (2,4,6)$

3dspace



$\begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \end{array}$

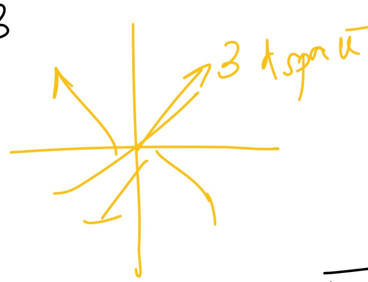
$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3}$

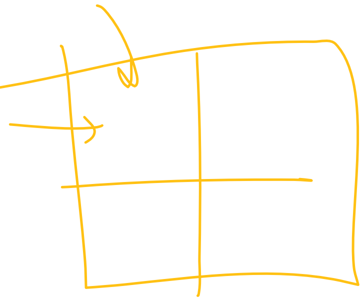
→ wide

row < cols

square

$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ → same dimension

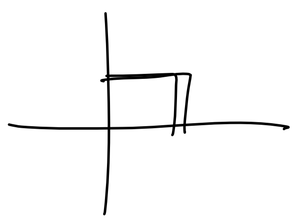




$|\det(A)|$

non-square

(det)



Why only square matrix has inverse

non-square inve

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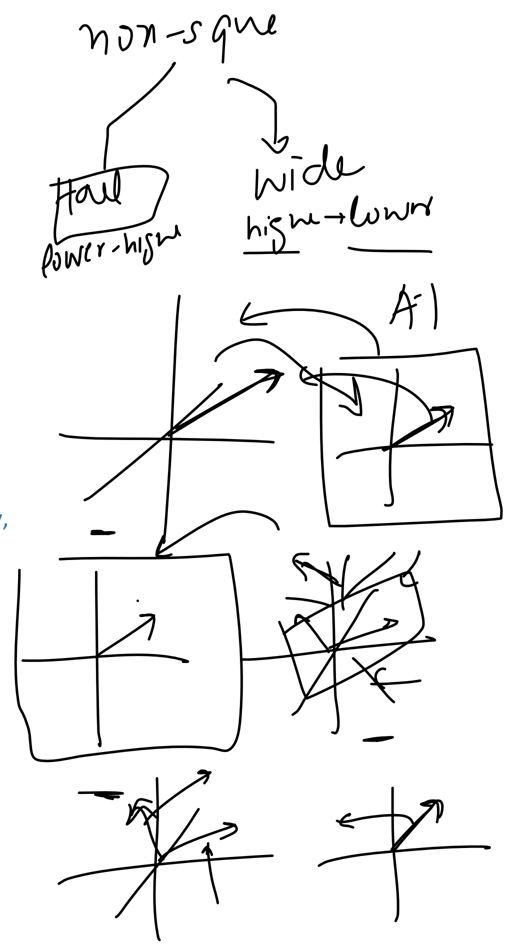
An inverse is possible only for square matrices because it is related to the concept of a matrix being a bijective linear transformation, which implies both injectivity (one-to-one) and surjectivity (onto). A square matrix represents a linear transformation between vector spaces of the same dimension, where the domain and codomain are the same. When a square matrix is invertible, its linear transformation is bijective, meaning that it has a unique inverse transformation.



Let's consider why non-square matrices cannot have inverses:

1. If a matrix A has more rows than columns (m > n), i.e., a tall matrix, the linear transformation it represents is from a lower-dimensional space to a higher-dimensional space. In this case, the transformation is generally not surjective (onto), as there are output vectors in the higher-dimensional space that have no corresponding input vector. Consequently, there is no inverse transformation that can map every output vector back to an input vector.

2. If a matrix A has more columns than rows (m < n), i.e., a wide matrix, the linear transformation it represents is from a higher-dimensional space to a lower-dimensional space (dimension reduction). In this case, the transformation is generally not injective (one-to-one), as multiple input vectors can map to the same output vector. Consequently, there is no unique inverse transformation that can map each output vector back to a unique input vector.

Again, the inverse of a matrix is possible only for square matrices because these matrices represent linear transformations between vector spaces of the same dimension. Only in these cases can a matrix potentially satisfy the conditions of being a bijective transformation, i.e., both injective and surjective, which allows the existence of a unique inverse transformation. However, not all square matrices have inverses; only those that are non-singular (with a non-zero determinant) have an inverse.

Why inverse is possible for non-singular matrices only

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→ square

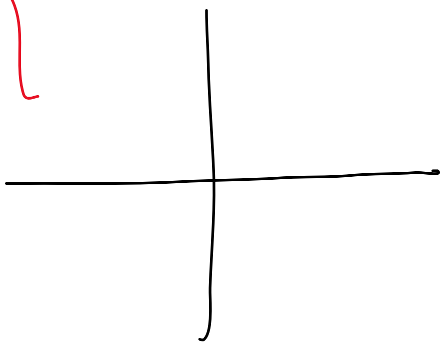
→ non-singular → $\det(A) \neq 0$ ←

$\det(A) = 0$ singular

higher - lower 2D → 1D

↓

non-square matrix



Data matrix -> representation

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Matrix multiplication

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Hadamard product

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The Hadamard product, also known as the element-wise product or Schur product, is a binary operation that takes two matrices of the same dimensions and produces a third matrix where each element is the product of the corresponding elements of the input matrices. Specifically, given two matrices A and B of the same size $m \times n$, their Hadamard product C is also an $m \times n$ matrix, where each element is defined as:

$C[i, j] = A[i, j] * B[i, j]$

for all $i = 1, ..., m$ and $j = 1, ..., n$.

https://ezyang.github.io/convolution-visualizer/