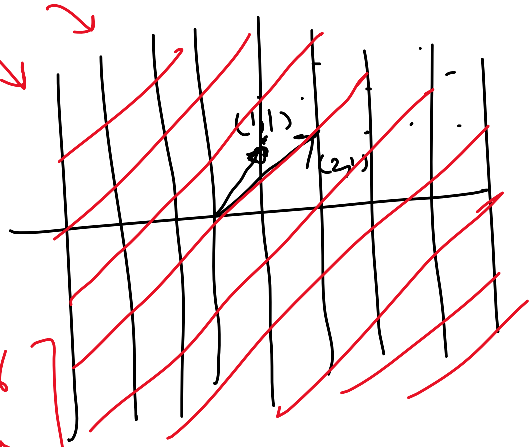
What are Matrices

30 May 2023 18:33

$$\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} \rightarrow \text{Linear transform}$$

$$\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$$

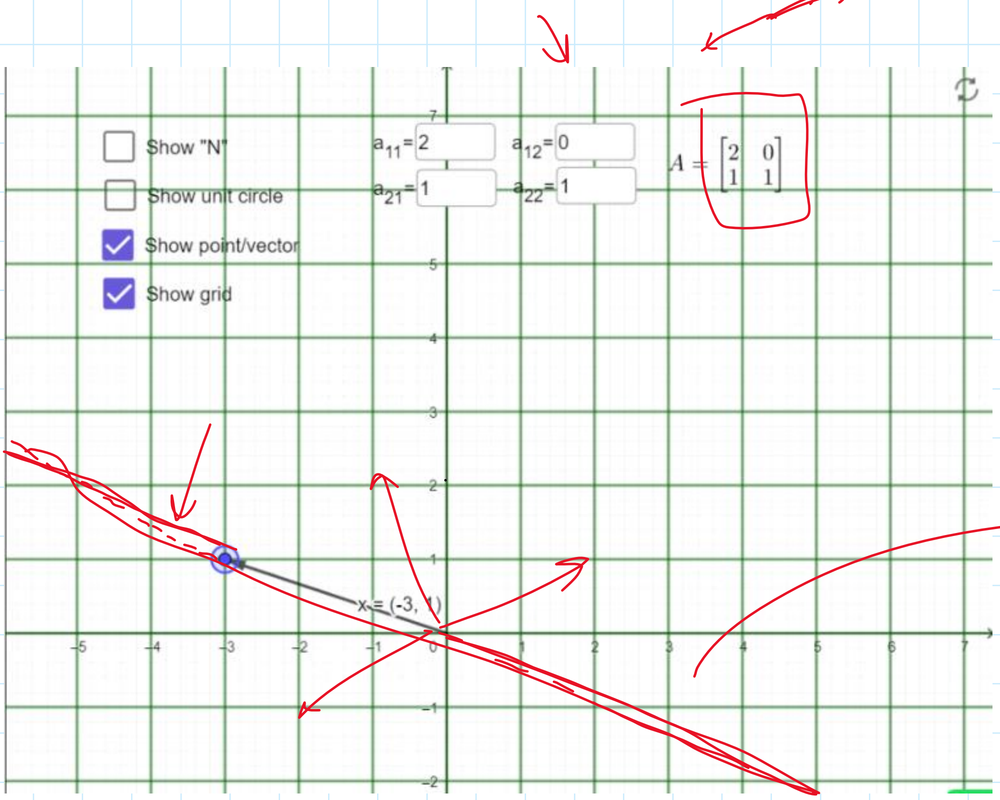


2-dim

What are Eigen Vectors and Eigen Values

30 May 2023 10:34

$$\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$



Eigen vectors

Eigen value

$\vec{V} = \lambda \vec{V}$ (2,1) →(4,1)

scalar

$\boxed{\lambda = 2}$

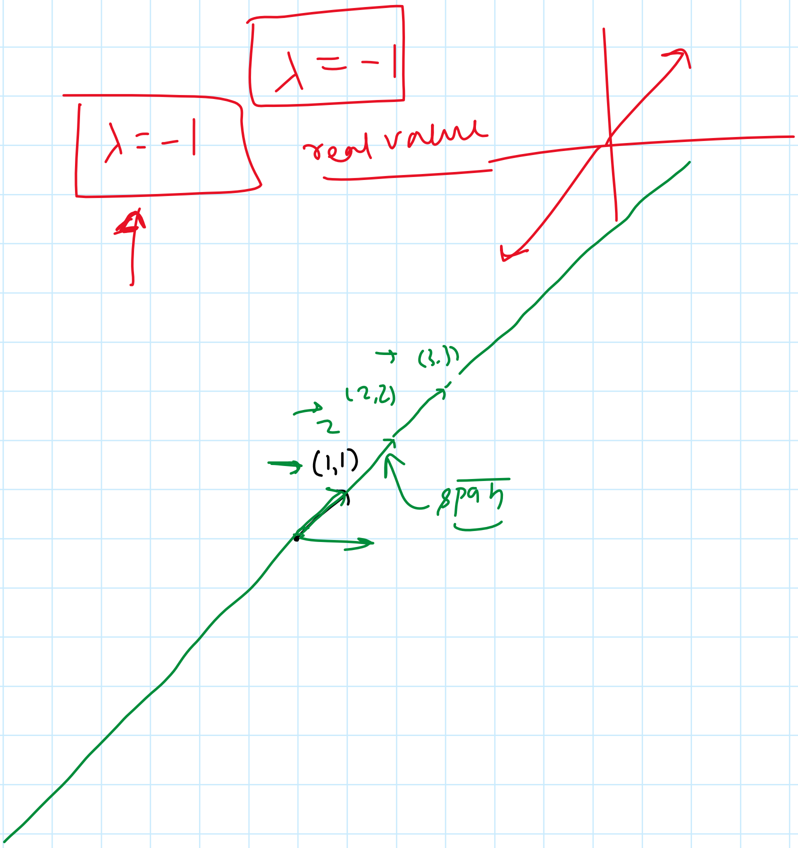
span of vector

$\begin{bmatrix} 3 \\ 6 \end{bmatrix}$ $\begin{bmatrix} 6 \\ 12 \end{bmatrix}$ $\begin{bmatrix} 1.5 \\ 3 \end{bmatrix}$ $\lambda = 0.5$

$\boxed{\lambda = -1}$

$\boxed{\lambda = -1}$

real value



Intuition - axis of rotation

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How to calculate Eigen Vectors and Eigen Values

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$$A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$$

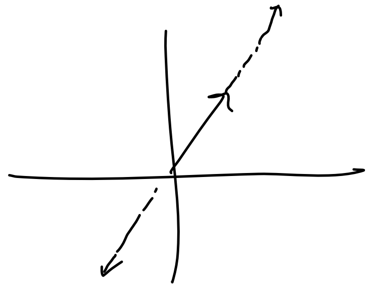
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\boxed{A\vec{x} = \lambda \vec{x}}$$

$A \rightarrow$ matrix

$\vec{x} \rightarrow$ vector

$\lambda \rightarrow$ scalar



$$I A \vec{x} = \lambda I \vec{x}$$

$$A\vec{x} - \lambda I \vec{x} = 0$$

$$\boxed{A - \lambda I}$$

$\downarrow$ matrix

$\vec{x}$ vector

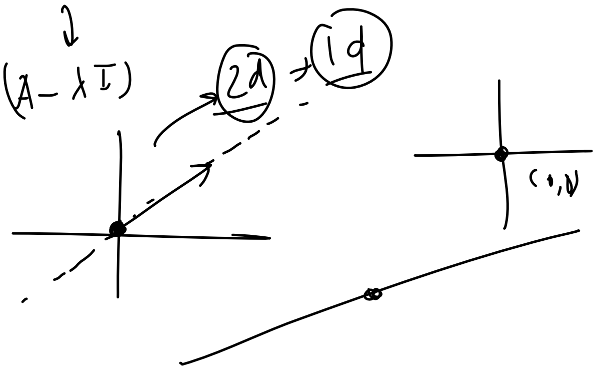
$$\boxed{(A - \lambda I)\vec{x}} = \underline{0}$$

non-invertible

$\hookrightarrow$ determinant $= 0$

zero vector

$(A - \lambda I)$



non invertible $\rightarrow$ $\det(A - \lambda I) = 0$

$\det(A) = 0$

$$\det\left(\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) = 0$$

$$\det\left(\begin{bmatrix} 2-\lambda & 3 \\ 1 & 1-\lambda \end{bmatrix} \right) = 0$$

$$\downarrow$$

$$(2-\lambda)(1-\lambda) + 3 = 0$$

$$2 - 2\lambda - \lambda + \lambda^2 + 3 = 0$$

$$\boxed{\lambda^2 - 3\lambda + 5 = 0} \leftarrow \text{eigen vectors}$$

$$\det\left(\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right) = 0$$

$$\det\left(\begin{bmatrix} 2-\lambda & 3 \\ 0 & 1-\lambda \end{bmatrix}\right) = 0$$

$$\boxed{\lambda = 2 \quad \lambda = 1}$$

$$(2-\lambda)(1-\lambda) = 0$$

eigen values

$$2 - 2\lambda - \lambda + \lambda^2 = 0$$

$$\boxed{\lambda = 2 \quad \lambda = 1}$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$(A - \lambda I)\vec{V} = 0$$

$$\left(\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\boxed{3x_2 = 0}$$

$$0x_1 + 3x_2 = 0$$

$$\boxed{x_2 = 0}$$

$$0x_1 - x_2 = 0$$

$$\boxed{1, 0} \leftarrow 2, 0, (3, 0)$$

$$x_2 = 0, \quad x_1 = 1$$

$$\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$v \qquad 2v$

$$\boxed{\lambda = 2}$$

$$(A - \lambda I)\vec{V} = 0$$

$(A - \lambda I)\vec{v} = 0$

$$\left(\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ \underline{0} & \underline{0} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(1) $[-3, 1]$

$[-6, 2]$

$\lambda = 1$ $\quad x_1 + 3x_2 = 0$

$x_1 = -3x_2$

$$\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \end{bmatrix} = \begin{bmatrix} -6+3 \\ 0+1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

$\boxed{\lambda = 1}$

$\boxed{A\vec{v} = 0}$

$\begin{bmatrix} 1 & 3 \\ 4 & 1 \end{bmatrix} \rightarrow$ eigenvec

$A\vec{v} = \underline{1}\,\vec{v}$

$\boxed{A\vec{v} = 0}$

$3d \rightarrow 2d$

$2d \rightarrow 1d$

$\boxed{n = 1} \rightarrow$ $\quad 784 \rightarrow n$ $\quad \boxed{n = 2}$

$n = 2 \rightarrow$ $\quad \rightarrow 784 \leftarrow 782$

Properties

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1. Sum of Eigenvalues: The sum of all the eigenvalues of a matrix is equal to its trace (the sum of the diagonal elements of the matrix). This holds true regardless of whether the matrix is square or not.

2. Product of Eigenvalues: The product of all the eigenvalues of a matrix is equal to its determinant. This also holds for square matrices.

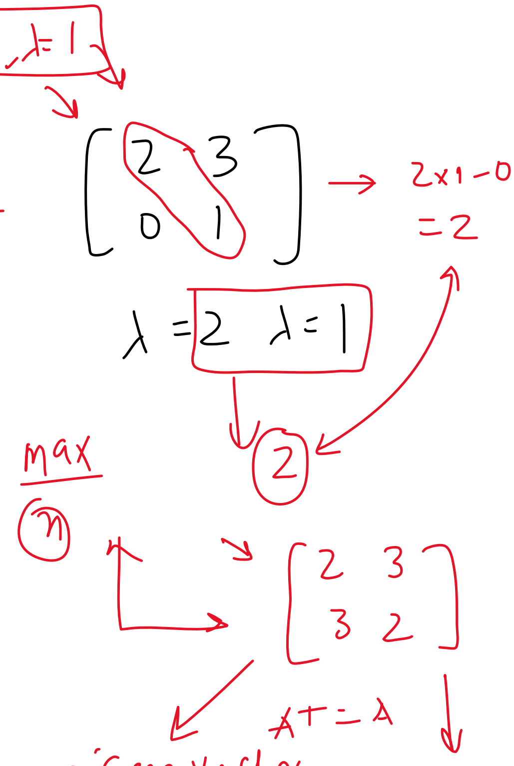
3. Eigenvectors corresponding to different eigenvalues are orthogonal: If a matrix A is symmetric (i.e., A = A^T) the eigenvectors corresponding to distinct eigenvalues are orthogonal to each other.

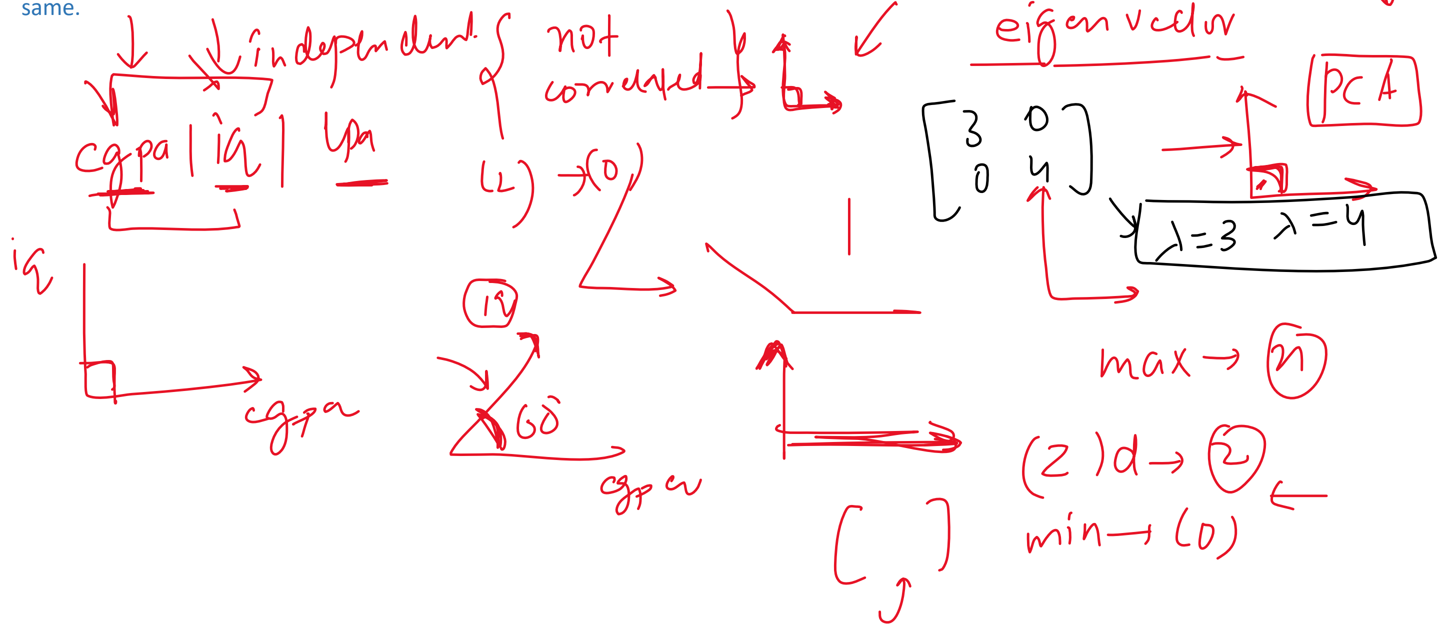
4. Eigenvalue of a Identity Matrix: For an identity matrix, the eigenvalues are all 1, regardless of the dimension of the matrix.

5. Eigenvalue of a Scalar Multiple: If B is a matrix obtained by multiplying a scalar c to a matrix A (i.e., B = cA), then the eigenvalues of B are just the eigenvalues of A each multiplied by c.

6. Eigenvalues of a Diagonal Matrix: For a diagonal matrix, the eigenvalues are the diagonal elements themselves.

7. Eigenvalues of a Transposed Matrix: The eigenvalues of a matrix and its transpose are the same.





Eigen Vectors in PCA

30 May 2023 19:26

cgpa | iq | lpa

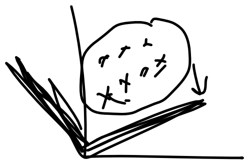
$n=2$

feature extraction

I → Id

covariance matrix

grey



PCA variants

2x2

n x n

$$\begin{bmatrix} Var_{cgpa} & cov_{(iq, cgpa)} \\ cov_{(iq, cgpa)} & var_{iq} \end{bmatrix}$$

symmetric

orthogonal ← eigen vector

features capture

↓

max variance