

Testing of Hypothesis

Hypothesis:

Quantitative Statement about population.

Null Hypothesis:- It is a claim or Statement about population Parameter that is assumed to be true until it is declared to be false. (H_0)

Alternative Hypothesis:- Any Hypothesis which is complementary to null hypothesis. (Research Hypothesis) (H_1)

$H_0: \mu = 500\text{ml}$
(left) $H_1: \mu \neq 500\text{ml}$ [two-tailed test]
or
(Right) $H_1: \mu < 500\text{ml}$ [one-tailed]
or
 $H_2: \mu > 500\text{ml}$ [one-tailed]

$H_2 \neq$ ✓
 $H_2 <$ validity of a claim
 $H_1 >$ Testing of Research hypothesis

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computation of Test statistic

Z-test, t-test, χ^2 -test, F-test.

Z-test:- $n \geq 30$ S.D. is known

$$Z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} \sim N(0, 1)$$

$$\sigma_{\bar{X}} = \text{S.E. of mean} = \frac{\sigma}{\sqrt{n}}$$

$$\sigma_{\bar{X}} = \text{S.E. of mean} = \frac{S}{\sqrt{n}}$$

✓ $n < 30$ S.D. unknown

t-test

$$t = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}}$$

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(left) or $H_1: \mu < 500\text{ml}$ [one-tailed]

(Right) or $H_1: \mu > 500\text{ml}$ [one-tailed]

computation of Test Statistic

Possible outcomes

Reject H_0

do not reject H_0

Level of Significance (α)

$\alpha \rightarrow$ prob. of error in accepting/rejecting H_0

$\alpha \rightarrow$ level of risk in rejecting a correct H_0

$$\alpha = 5\% \text{ or } 1\%$$

Level of confidence $\rightarrow C$

$$C = 1 - \alpha$$

$$C = 1 - 0.05 = 0.95 \text{ or } 95\%$$

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computation of Test statistic

Possible outcomes

Reject H_0

do not reject H_0

Level of Significance (LOS)

LOS \rightarrow prob. of error in (α) accepting/rejecting H_0

LOS (α) \rightarrow level of risk in
0.05 rejecting a correct H_0
 $\alpha = 5\%$ or 1%

Level of confidence $\rightarrow C$

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$$C = 1 - 0.05 = 0.95 \text{ or } 95\%$$

Testing of Hypothesis

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Quantitative Statement about population.

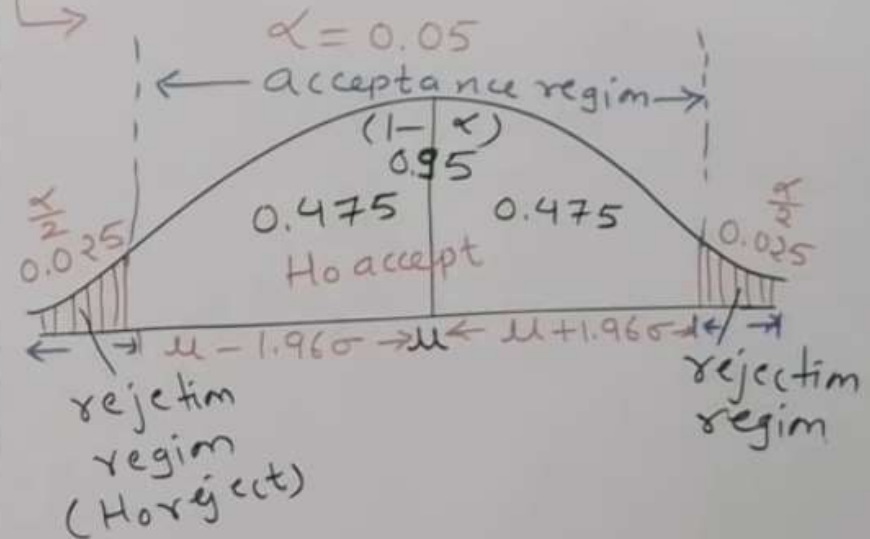
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computation of Test statistic

- ① Two-tailed ($H_1: \neq$)
- ② one-tailed ($H_1: > \text{ or } <$)



Testing of Hypothesis

Hypothesis:

Quantitative Statement about population.

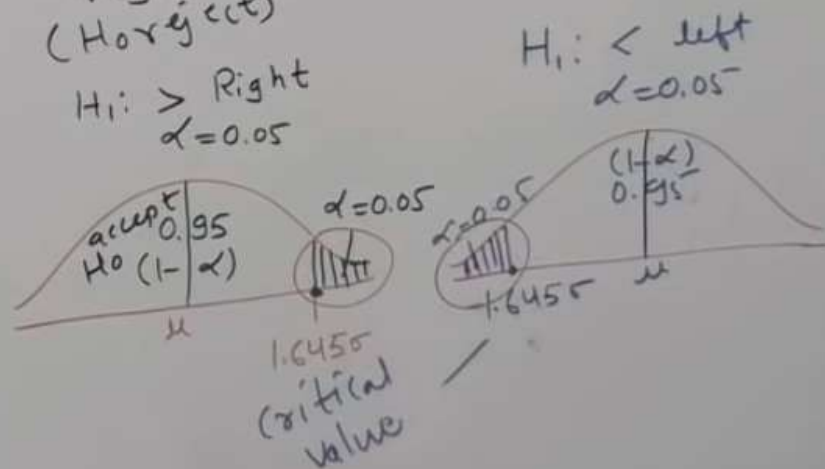
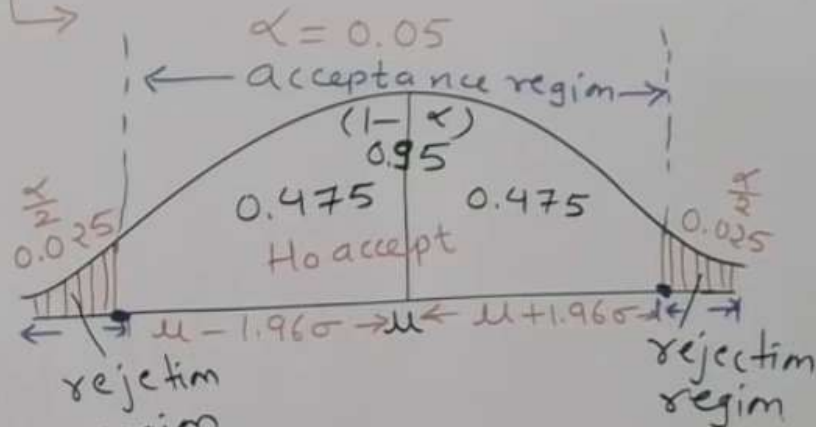
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computation of Test statistic

- ① Two-tailed ($H_1: \neq$)
- ② one-tailed ($H_1: > \text{or } <$)



Testing of Hypothesis

Decision:-

If $|Z| < Z_{\alpha}$
 Cal. value of Z tab. value of Z at α level of sig.
 (we accept H_0)

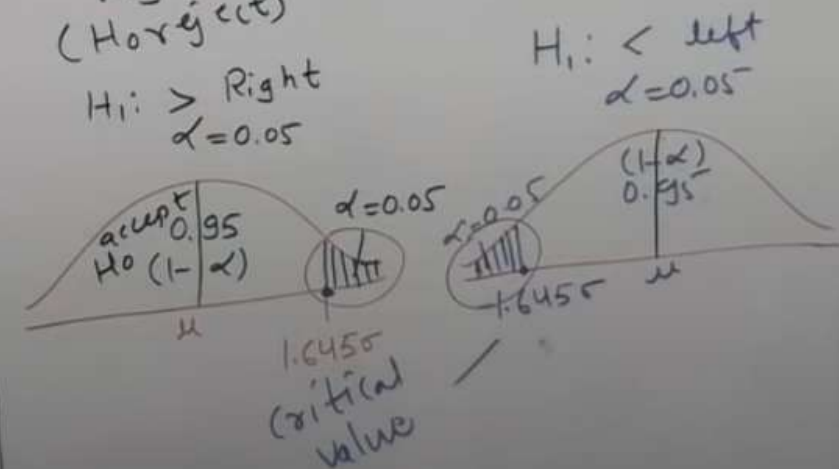
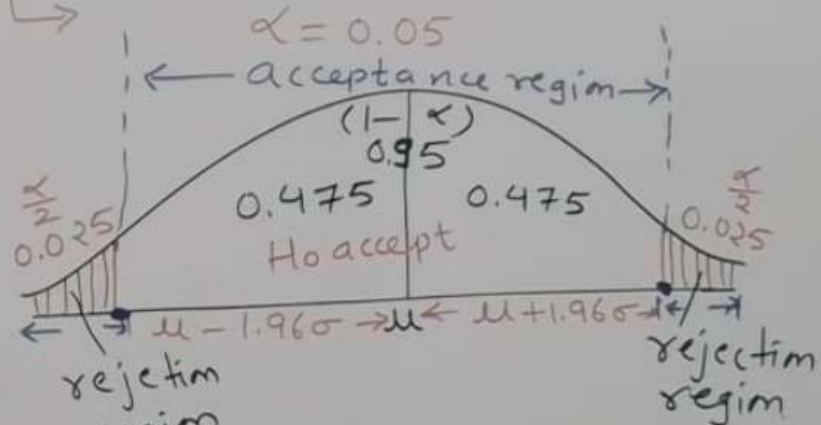
If $|Z| > Z_{\alpha}$
 reject H_0
 (diff. b/w sample
 Statistic and population
 parameter is significant)

Alternative Hypothesis:- Any Hypothesis
 which is complementary to null
 hypothesis. (Research Hypothesis)
 (H_1)

$H_0: \mu = 500\text{ml}$
 $H_1: \mu \neq 500\text{ml}$ [two-tailed test]
 (left) or $H_1: \mu < 500\text{ml}$ [one-tailed]
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computation of Test statistic

- ① Two-tailed ($H_1: \neq$)
- ② one-tailed ($H_1: > \text{or } <$)



Testing of Hypothesis of a Single Mean [LARGE SAMPLE]

(Significance)

1. Set H_0 and H_1 .

2. If Sample size $n \geq 30$
then Test statistics

$$Z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} \sim \text{SND}(0, 1)$$

where $\sigma_{\bar{X}}$ = Standard error of mean

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} \quad [\text{Population S.D. is known}]$$

$$\sigma_{\bar{X}} = \frac{S}{\sqrt{n}} \quad [\text{Population S.D. is not known}]$$

3. Set level of significance α

4. Find critical value Z_α of Z
at α level of significance from
Area Under Normal curve.

5. Decision

If Calculated $|Z| > Z_\alpha$
reject H_0
diff. is significant.

If calculated $|Z| < Z_\alpha$
accept H_0
diff. is due to sampling
fluctuations

Critical value (Z_α)

LOS (α) \rightarrow	1%	5%	10%
Two-Tailed Test	$ Z_\alpha = 2.58$	$ Z_\alpha = 1.96$	$ Z_\alpha = 1.645$
Right-Tailed Test	$Z_\alpha = 2.33$	$Z_\alpha = 1.645$	$Z_\alpha = 1.28$
Left Tailed Test	$Z_\alpha = -2.33$	$Z_\alpha = -1.645$	$Z_\alpha = -1.28$

A sample of size 400 was drawn and the sample mean was found to be 99. Test whether this sample would have come from a Normal population with mean 100 and standard deviation 8 at 5% level of significance.

Testing of Hypothesis of a Single Mean [LARGE SAMPLE]

(Significance)

Q.1 Given, $\bar{X} = 99$ $\mu = 100$

$\sigma = 8$ (S.D. known), $n = 400$ [large sample]

Null Hypothesis, H_0 : The sample has come from the population whose mean, μ is 100

$H_0: \mu = 100$

$H_1: \mu \neq 100$ [Two-tailed test]

Test Statistic

$$Z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}}$$

$$\text{where } \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{8}{\sqrt{400}} = \frac{8}{20} = \frac{2}{5}$$

$$Z = \frac{99 - 100}{\frac{2}{5}} = -\frac{5}{2}$$

$$Z = -2.5$$

$$|Z| = 2.5$$

Level of significance, $\alpha = 0.05$ or 5%

Critical value of $Z_{\alpha} = 1.96$ at 5%.

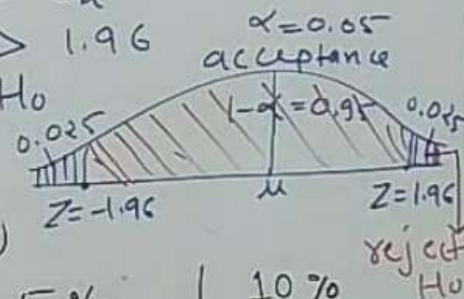
LOS and for two-tailed test.

Decision:-

$$|Z| > Z_{\alpha}$$

$$2.5 > 1.96$$

reject H_0



Critical value (Z_{α})

LOS (α)	1%	5%	10%
Two-Tailed Test	$ Z_{\alpha} = 2.58$	$ Z_{\alpha} = 1.96$	$ Z_{\alpha} = 1.645$
Right-Tailed Test	$Z_{\alpha} = 2.33$	$Z_{\alpha} = 1.645$	$Z_{\alpha} = 1.28$
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The mean life time of a sample of 400 fluorescent light tube produced by a company is found to be 1570 hours with a standard deviation of 150 hours. Test the hypothesis that the mean lifetime of the bulbs produced by the company is 1600 hours against the alternative hypothesis that it is greater than 1600 hours at 1% level of significance.

Testing of Hypothesis of a Single Mean [LARGE SAMPLE] (Significance)

Q.2 Given, $\bar{X} = 1570$ $\mu = 1600$

$S = 150$, $n = 400$ [large]
(pop'n S.D. unknown)

Null Hypothesis, H_0 : Sample has
come from the pop'n with mean
life = 1600 hrs

$H_0: \mu = 1600$ hrs

$H_1: \mu > 1600$ hrs
[Right-tailed test]

Test Statistic

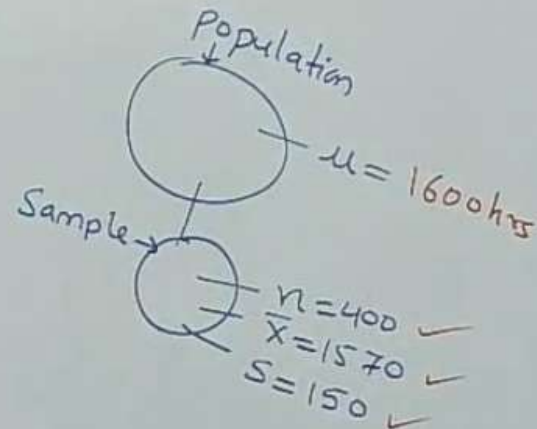
$$Z = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}}$$

where $\frac{S}{\sqrt{n}} = \frac{150}{\sqrt{400}} = \frac{150}{20} = 7.5$

$$Z = \frac{1570 - 1600}{7.5} = \frac{-30}{7.5}$$

$$Z = -4$$

$|Z| = 4$



Critical value (Z_α)

LOS \rightarrow	1%	5%	10%
Two-Tailed Test	$ Z_\alpha = 2.58$	$ Z_\alpha = 1.96$	$ Z_\alpha = 1.645$
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Testing of Hypothesis of a Single Mean [LARGE SAMPLE]

(Significance)

Q.2 Given, $\bar{X} = 1570$ $\mu = 1600$

$S = 150$, $n = 400$ [large]
(pop'n S.D. unknown)

Null Hypothesis, H_0 : Sample has come from the pop'n with mean life = 1600 hrs

$H_0: \mu = 1600$ hrs

$H_1: \mu > 1600$ hrs
[Right-tailed test]

Test Statistic

$$Z = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}}$$

where $\frac{S}{\sqrt{n}} = \frac{150}{\sqrt{400}} = \frac{150}{20} = 7.5$

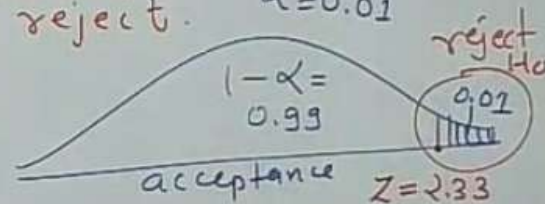
$$Z = \frac{1570 - 1600}{7.5} = \frac{-30}{7.5}$$

$$Z = -4 \quad \boxed{|Z| = 4} \quad \checkmark$$

Level of sig., $\alpha = 1\% \text{ or } 0.001$

Critical value of $Z_\alpha = 2.33$
at 1% Level of sig. and for right tailed test.

Decision $\rightarrow |Z| > Z_\alpha$
Calculated H_0 reject. $\alpha = 0.01$



Critical value (Z_α)

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Testing of Hypothesis [Type I & Type II error, Power of Test]

When we are testing Null Hypothesis (H_0) against Alternative Hypothesis (H_1) there are four possibilities.

- 1) H_0 accepted when H_0 is true. [correct]
- 2) H_0 rejected when H_0 is true. [Type I error]
- 3) H_0 accepted when H_0 is false. [Type II error]
- 4) H_0 rejected when H_0 is false. [correct]

Type I error

✓ $\alpha = P(\text{Type I error})$

$$\alpha = P(\text{reject } H_0 | H_0 \text{ is true})$$

$$\alpha = 5\%, 1\%, 20\%$$

Type II error

$$\beta = P(\text{Type II error})$$

$$\beta = P(\text{accept } H_0 | H_1 \text{ is true})$$

H_0 : medicine is curing
↓ the disease.
correct
(H_0 reject) (Type I)

H_0 : medicine is not curing
↓ the disease.
(false)
(H_0 accept) (Type II)

Testing of Hypothesis [Type I & Type II error, Power of Test]

When we are testing Null Hypothesis (H_0) against Alternative Hypothesis (H_1) there are four possibilities.

- 1) H_0 accepted when H_0 is true. [correct]
- 2) H_0 rejected when H_0 is true. [Type I error]
- 3) H_0 accepted when H_0 is false. [Type II error]
- 4) H_0 rejected when H_0 is false. [correct]

Type I error

✓ $\alpha = P(\text{Type I error})$

$$\alpha = P(\text{reject } H_0 | H_0 \text{ is true})$$

$$\alpha = 5\%, 1\%, 20\%$$

Type II error

$$\beta = P(\text{Type II error})$$

$$\beta = P(\text{accept } H_0 | H_1 \text{ is true})$$

β = minimize

β = accept H_0 when H_0 is false.

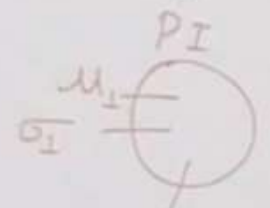
$1 - \beta$ = reject H_0 when H_0 is false
[as large as possible]

$1 - \beta$ (near 1)
test is working quite well.

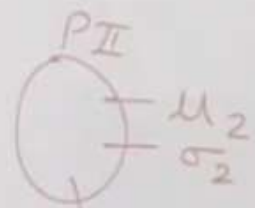
$1 - \beta$ (near 0)
test is not working well.

$1 - \beta$ = power of test.

Testing of Hypothesis of Difference of Two Sample means [LARGE SAMPLE]



Sample I
 $n_1 \geq 30$
 \bar{x}_1



Sample II
 $n_2 \geq 30$
 \bar{x}_2

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2 \text{ or } \mu_1 > \mu_2 \text{ or } \mu_1 < \mu_2$$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma_{\bar{x}_1 - \bar{x}_2}}$$

where

$\sigma_{\bar{x}_1 - \bar{x}_2}$ = standard error of mean

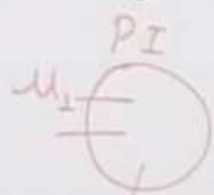
(I) population s.d. is known

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

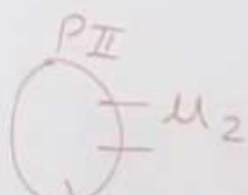
Critical value (Z_α)

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Testing of Hypothesis of Difference of Two Sample means [LARGE SAMPLE]



Sample I
 $n_1 \geq 30$
 \bar{x}_1
 s_1



Sample II
 $n_2 \geq 30$
 \bar{x}_2
 s_2

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2 \text{ or } \mu_1 > \mu_2 \text{ or } \mu_1 < \mu_2$$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma_{\bar{x}_1 - \bar{x}_2}}$$

where

$\sigma_{\bar{x}_1 - \bar{x}_2}$ = standard error of mean

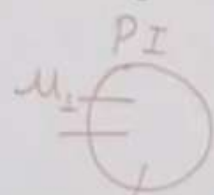
(II) population s.d. is unknown

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Critical value (Z_α)

LOS (α) \rightarrow	1%	5%	10%
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Testing of Hypothesis of Difference of Two Sample means [LARGE SAMPLE]

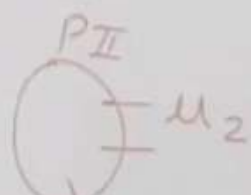


Sample I

$$n_1 \geq 30$$

$$\bar{x}_1$$

$$s_1$$

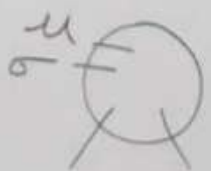


Sample II

$$n_2 \geq 30$$

$$\bar{x}_2$$

$$s_2$$



I

$$n_1$$

$$\bar{x}_1$$

II

$$n_2$$

$$\bar{x}_2$$

(III) Poplⁿ S.D. is common

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

Critical value, Z_α

$|Z| > Z_\alpha$ Ho reject

$|Z| < Z_\alpha$ Ho accept.

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2 \text{ or } \mu_1 > \mu_2 \text{ or } \mu_1 < \mu_2$$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma_{\bar{x}_1 - \bar{x}_2}}$$

where

$\sigma_{\bar{x}_1 - \bar{x}_2}$ = Standard error of mean

(II) Population S.D. is unknown

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

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