# MEASURES OF DISPERSION

| Series A | Series B | Series C |
|---|---|---|
| 100 | 102 | 1 |
| 100 | 103 | 489 |
| 100 | 100 | 5 |
| 100 | 99 | 3 |
| 100 | 98 | 2 |

no dispersion

less dispersion or more uniform

more dispersion or more variation or less uniform

$\Sigma x_A = 500$

$N_A = 5$

$\bar{X}_A = \frac{500}{5} = 100$

$\Sigma X_B = 500$

$N_B = 5$

$\bar{X}_B = \frac{500}{5} = 100$

$\Sigma X_C = 500$

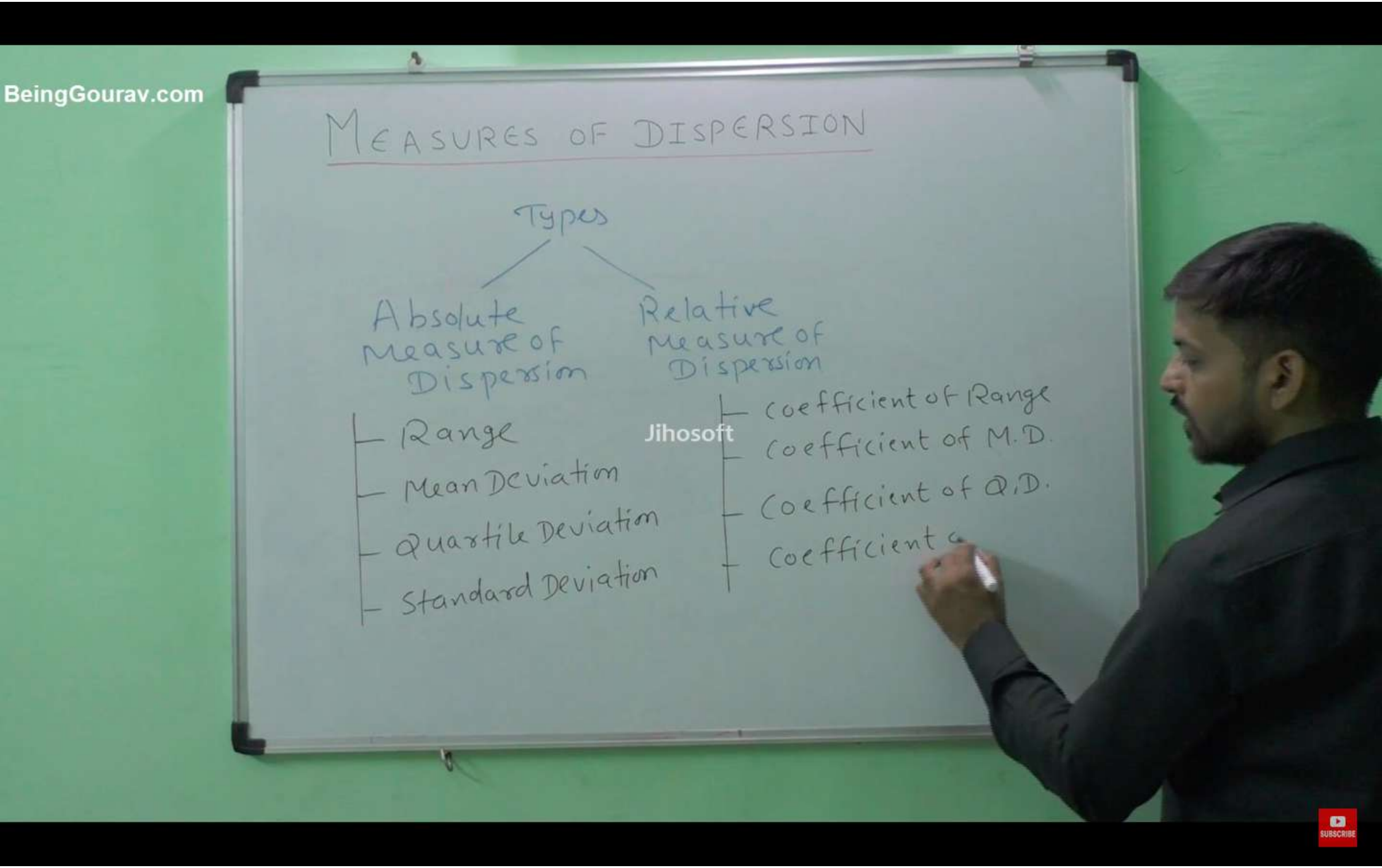
$N_C = 5$

$\bar{X}_C = \frac{500}{5} = 100$

# MEASURES OF DISPERSION

Types

- **Absolute Measure of Dispersion** — depends on units of Variable.
- **Relative Measure of Dispersion** — Independent of units of Variables



# MEASURES OF DISPERSION

## STANDARD DEVIATION (S.D.)

set of values $x_i$

$x_1$
$x_2$
$\vdots$
$x_n$

$\Sigma x_i$

→ deviation of set of values from its mean ($\bar{x}$)

Positive square root of sum of squared



$$\text{mean}, \bar{x} = \frac{\Sigma x_i}{n}$$

$$\text{S.D.} = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n}}$$

# MEASURES OF DISPERSION

## STANDARD DEVIATION (S.D.)

### ① For Individual series

$$S.D. = \sigma = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n}}$$ } Direct method

$$S.D. = \sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2}$$ } Shortcut method



$d = x_i - A$ $A$ = Assumed mean

$$S.D. = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n}}$$

# MEASURES OF DISPERSION

## STANDARD DEVIATION (S.D.)

### ① For Individual series

$$S.D. = \sigma = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n}}$$ } Direct method

Find S.D.

| $x$ | $x - \bar{x}$ $(x-8)$ | $(x-\bar{x})^2$ $(x-8)^2$ |
|---|---|---|
| 3 | −5 | 25 |
| 4 | −4 | 16 |
| 9 | 1 | 1 |
| 11 | 3 | 9 |
| 13 | 5 | 25 |
| 6 | −2 | 4 |
| 8 | 0 | 0 |
| 10 | 2 | 4 |
| $\Sigma x = 64$ | | $\Sigma (x-\bar{x})^2 = 84$ |

$$\text{mean}, \bar{x} = \frac{\Sigma x}{n} = \frac{64}{8} = 8$$



$$S.D. = \sigma = \sqrt{\frac{84}{8}}$$

$$\sigma = \sqrt{10.5}$$

$$\sigma = 3.24 \text{ Ans}$$

# MEASURES OF DISPERSION

## STANDARD DEVIATION (S.D.)

① For Discrete series

ⓐ $S.D. = \sigma = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{N}}$ } Direct method

$\Sigma f = N$ = Total of all frequencies

ⓑ $S.D. = \sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$ } Shortcut method

$d = X - A$

A = Assumed mean

# MEASURES OF DISPERSION

## STANDARD DEVIATION (S.D.)

③ For Continuous series

ⓐ $S.D. = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$ } shortcut method

$d$ = mid-value ($m$) − $A$

ⓑ $S.D. = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} \times i$ } step-deviation method

$i$ = class interval

$d = \frac{(m - A)}{i}$

# MEASURES OF DISPERSION

## STANDARD DEVIATION (S.D.)

find S.D.

| class | f | midvalue (m) | $d=(m-A)/i$ <br> $d=(m-35)/10$ | $d^2$ | fd | $fd^2$ |
|---|---|---|---|---|---|---|
| 0-10 | 15 | 5 | -3 | 9 | -45 | 135 |
| 10-20 | 15 | 15 | -2 | 4 | -30 | 60 |
| 20-30 | 23 | 25 | -1 | 1 | -23 | 23 |
| 30-40 | 22 | 35 | 0 | 0 | 0 | 0 |
| 40-50 | 25 | 45 | 1 | 1 | 25 | 25 |
| 50-60 | 10 | 55 | 2 | 4 | 20 | 40 |
| 60-70 | 5 | 65 | 3 | 9 | 15 | 45 |
| 70-80 | 10 | 75 | 4 | 16 | 40 | 160 |
| | $\Sigma f = N = 125$ | | | | $\Sigma fd = 2$ | $\Sigma fd^2 = 488$ |

$$S.D. = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} \times i$$

$$= \sqrt{\frac{488}{125} - \left(\frac{2}{125}\right)^2} \times 10$$

$S.D. = 19.76$ Ans

# Combined Standard Deviation

$$\sigma_{12} = \sqrt{\frac{N_1(\sigma_1^2 + d_1^2) + N_2(\sigma_2^2 + d_2^2)}{N_1 + N_2}}$$

Combined mean.

$$d_1 = |\bar{X}_1 - \bar{X}_{12}| \qquad \text{where } \bar{X}_{12} = \frac{N_1\bar{X}_1 + N_2\bar{X}_2}{N_1 + N_2}$$

$$d_2 = |\bar{X}_2 - \bar{X}_{12}|$$

$\sigma_1$ = S.D. of I$^{st}$ group

$\sigma_2$ = S.D. of II$^{nd}$ group



Variance = $(S.D.)^2$

$= \sigma^2$ ---> (1)

Q. An algebraic test was given to 400 high school children of whom 150 were boys and 250 were girls

$N_1 = 150$ $\quad N_2 = 250$

$\bar{X}_1 = 72$ $\quad \bar{X}_2 = 73$

$\sigma_1 = 7$ $\quad \sigma_2 = 6.4$

find mean and S.D. of combined group.

# Combined Standard Deviation

$$\sigma_{12} = \sqrt{\frac{N_1(\sigma_1^2 + d_1^2) + N_2(\sigma_2^2 + d_2^2)}{N_1 + N_2}}$$

Combined mean.

Soln:- $$\bar{X}_{12} = \frac{N_1\bar{X}_1 + N_2\bar{X}_2}{N_1 + N_2}$$

$$\bar{X}_{12} = \frac{(150)(72) + (250)(73)}{150 + 250}$$



$$\bar{X}_{12} = \frac{10800 + 18250}{400}$$

$$\boxed{\bar{X}_{12} = 72.625}$$ Ans

Q. An algebraic test was given to 400 high school children of whom 150 were boys and 250 were girls

$N_1 = 150$ $N_2 = 250$

$\bar{X}_1 = 72$ $\bar{X}_2 = 73$

$\sigma_1 = 7$ $\sigma_2 = 6.4$

find mean and S.D. of combined group.

# Combined Standard Deviation

$$\sigma_{12} = \sqrt{\frac{N_1(\sigma_1^2 + d_1^2) + N_2(\sigma_2^2 + d_2^2)}{N_1 + N_2}}$$

$$\sigma_{12} = \sqrt{\frac{150(49 + 0.390625) + 250(40.96 + 0.140625)}{150 + 250}}$$

$$\sigma_{12} = \sqrt{\frac{7408.59375 + 10295.15625}{400}}$$

$$\sigma_{12} = \sqrt{\frac{17683.75}{400}} = 6.65$$ Ans

$d_1 = |\bar{X}_1 - \bar{X}_{12}|$

$d_1 = |72 - 72.625|$

$d_1 = 0.625$

$d_2 = |\bar{X}_2 - \bar{X}_{12}|$

$d_2 = |73 - 72.625|$

$d_2 = 0.375$

$\bar{X}_{12} = 72.625$ Ans



Q. An algebraic test was given to 400 high school children of whom 150 were boys and 250 were girls

$N_1 = 150$ $N_2 = 250$

$\bar{X}_1 = 72$ $\bar{X}_2 = 73$

$\sigma_1 = 7$ $\sigma_2 = 6.4$

find mean and S.D. of combined group.

# Variance and coefficient of Variation (C.V.)

Q. An analysis of monthly wage gives the following Results

| | firm A | firm B |
|---|---|---|
| No. of workers | | |
| Average daily wages | | |
| Variance of Distribution of wages | | |

(a) which firm Pay larger wage bill ?
(b) In which firm A or B is there greater variability in individual wages?

$$\text{Variance} = (\text{S.D.})^2 = \sigma^2$$

$$\text{C.V.} = \frac{\sigma}{\bar{X}} \times 100$$

↓

Relative measure of Dispersion

Karl Pearson

$\sigma$ = S.D.
$\bar{X}$ = mean

| C.V. less | C.V. more |
|---|---|
| more consistent | less consistent |
| more Homogeneous | less Homogeneous |
| more stable | less stable |
| less variation | more variation |

# Variance and coefficient of Variation (C.V.)

Q. An analysis of monthly wage gives the following Results

| | firm A | firm B |
|---|---|---|
| No. of workers | $N_1 = 500$ | $N_2 = 600$ |
| Average daily wages | Rs. 186 $\bar{x}_1 =$ | Rs. 175 $\bar{x}_2 =$ |
| Variance of Distribution of wages | $\sigma_1^2 = 81$ | $\sigma_2^2 = 100$ |
| | $\sigma_1 = \sqrt{81} = 9$ | $\sigma_2 = \sqrt{100} = 10$ |

(a) which firm Pay larger wage bill ?

(b) In which firm A or B is there greater variability in individual wages?

(a) firm A

$$\text{Average daily wage} = \frac{\text{Total wage Paid}}{\text{No. of workers}}$$

$$186 = \frac{T.W.P.}{500}$$

$$(TW.P)_A = 93000/-$$

firm B

$$\text{Average daily wage} = \frac{T.W.P.}{\text{No. of workers}}$$

$$175 = \frac{T.W.P.}{600}$$

$$T.W.P. = 1,05,000 \quad B$$

firm B has larger wage bill.

## Variance and coefficient of Variation (C.V.)

Q. An analysis of monthly wage gives the following Results

| | firm A | firm B |
|---|---|---|
| No. of workers | $N_1 = 500$ | $N_2 = 600$ |
| Average daily wages | Rs. 186 $\bar{X}_1 =$ | Rs. 175 $\bar{X}_2 =$ ✓ |
| Variance of Distribution of wages | $\sigma_1^2 = 81$ | $\sigma_2^2 = 100$ |
| | $\sigma_1 = \sqrt{81} = 9$ ✓ | $\sigma_2 = \sqrt{100} = 10$ |

(a) which firm Pay larger wage bill ?

(b) In which firm A or B is there greater variability in individual wages?

(b) firm A

$$C.V._A = \frac{\sigma_1}{\bar{X}_1} \times 100 = \frac{9}{186} \times 100$$

$$C.V._A = 4.84 \quad -(1)$$

firm B

$$C.V._B = \frac{\sigma_2}{\bar{X}_2} \times 100$$

$$= \frac{10}{175} \times 100$$

$$C.V._B = 5.71 \quad -(2)$$

firm B has greater Variability in Individual wage.

# Variance and coefficient of Variation (C.V.)

Q. find which team A or B is more consistent in its performance:

| No. of Goal Scored in a math | Number of football match Played Team A | Team B |
|---|---|---|
| 0 | 27 | 17 |
| 1 | 9 | 9 |
| 2 | 8 | 6 |
| 3 | 5 | 5 |
| 4 | 4 | 3 |

Assumed mean, A = 2

$$C.V_A = \frac{\sigma_A}{\bar{X}_A} \times 100$$

$$\sigma_A = S.D_A = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2}$$

$$\bar{X}_A = A + \frac{\Sigma fd}{\Sigma f}$$

$$d = X - A$$

Assumed mean

Sol$^n$ → For Team A

| X | f | d = X − A, d = X − 2 | $d^2$ | fd | $fd^2$ |
|---|---|---|---|---|---|
| 0 | 27 | −2 | 4 | −54 | 108 |
| 1 | 9 | −1 | 1 | −9 | 9 |
| 2 | 8 | 0 | 0 | 0 | 0 |
| 3 | 5 | 1 | 1 | 5 | 5 |
| 4 | 4 | 2 | 4 | 8 | 16 |
| | | | | $\Sigma fd = -50$ | $\Sigma fd^2 = 138$ |

# Variance and coefficient of Variation (C.V.)

Q. find which team A or B is more consistent in its performance:

| No. of Goal Scored in a math | Number of football match Played Team A | Team B |
|---|---|---|
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| 1 | 9 | 9 |
| 2 | 8 | 6 |
| 3 | 5 | 5 |
| 4 | 4 | 3 |

Assumed mean, A = 2

Soln → For Team A

| X | f | d = X − A, d = X − 2 | $d^2$ | fd | $fd^2$ |
|---|---|---|---|---|---|
| 0 | 27 | −2 | 4 | −54 | 108 |
| 1 | 9 | −1 | 1 | −9 | 9 |
| 2 | 8 | 0 | 0 | 0 | 0 |
| 3 | 5 | 1 | 1 | 5 | 5 |
| 4 | 4 | 2 | 4 | 8 | 16 |
| | $\Sigma f = 53$ | | | $\Sigma fd = -50$ | $\Sigma fd^2 = 138$ |

$\bar{X}_A = 2 - 0.94$

$\boxed{\bar{X}_A = 1.06}$ ✓

$C.V_A = \frac{\sigma_A}{\bar{X}_A} \times 100$

$\sigma_A = S.D_A = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2}$

$\sigma_A = \sqrt{\frac{138}{53} - \left(\frac{-50}{53}\right)^2}$

$\sigma_A = \sqrt{2.60 - (-0.94)^2}$

$\sigma_A = \sqrt{2.60 - 0.8836}$

$\sigma_A = \sqrt{1.7164}$

$\boxed{\sigma_A = 1.31}$ ✓

$\bar{X}_A = A + \frac{\Sigma fd}{\Sigma f}$ (shortcut)

$\bar{X}_A = 2 + \left(\frac{-50}{53}\right)$

# Variance and coefficient of Variation (C.V.)

Q. find which team A or B is more consistent in its performance:

| No. of Goal Scored in a math | Number of football match Played Team A | Team B |
|---|---|---|
| 0 | 27 | 17 |
| 1 | 9 | 9 |
| 2 | 8 | 6 |
| 3 | 5 | 5 |
| 4 | 4 | 3 |

Assumed mean, A = 2

Soln → For Team A

| x | f | $d = x - A$, $d = x - 2$ | $d^2$ | fd | $fd^2$ |
|---|---|---|---|---|---|
| 0 | 27 | −2 | 4 | −54 | 108 |
| 1 | 9 | −1 | 1 | −9 | 9 |
| 2 | 8 | 0 | 0 | 0 | 0 |
| 3 | 5 | 1 | 1 | 5 | 5 |
| 4 | 4 | 2 | 4 | 8 | 16 |
| | $\Sigma f = 53$ | | | $\Sigma fd = -50$ | $\Sigma fd^2 = 138$ |

$\bar{X}_A = 2 - 0.94$

$\bar{X}_A = 1.06$ ✓

$\sigma_A = \sqrt{1.7164}$

$\sigma_A = 1.31$ ✓

$\bar{X}_A = A + \frac{\Sigma fd}{\Sigma f}$ (shortcut)

$\bar{X}_A = 2 + \left(\frac{-50}{53}\right)$

$C.V_A = \frac{\sigma_A}{\bar{X}_A} \times 100$

$C.V._A = \frac{1.31}{1.06} \times 100$

$= \frac{131}{1.06}$

$C.V._A = 123.6\%$ ← (1)

# Variance and coefficient of Variation (C.V.)

Q. find which team A or B is more consistent in its performance:

| No. of Goal Scored in a math | Number of football match Played Team A | Team B |
|---|---|---|
| 0 | 27 | 17 |
| 1 | 9 | 9 |
| 2 | 8 | 6 |
| 3 | 5 | 5 |
| 4 | 4 | 3 |

Assumed mean, A = 2

$$C.V_A = \frac{\sigma_A}{\bar{X}_A} \times 100$$

$$C.V._A = \frac{1.31}{1.06} \times 100$$

$$= \frac{131}{1.06}$$

$$\boxed{C.V._A = 123.6\%}$$ ①

Sol$^n$ → For Team B

| X | f | d = X − A, d = X − 2 | $d^2$ | fd | $fd^2$ |
|---|---|---|---|---|---|
| 0 | 17 | −2 | 4 | −34 | 68 |
| 1 | 9 | −1 | 1 | −9 | 9 |
| 2 | 6 | 0 | 0 | 0 | 0 |
| 3 | 5 | 1 | 1 | 5 | 5 |
| 4 | 3 | 2 | 4 | 6 | 12 |
| | $\Sigma f = N = 40$ | | | $\Sigma fd = -32$ | $\Sigma fd^2 = 94$ |

$$C.V._B = \frac{\sigma_B}{\bar{X}_B} \times 100$$

$$\sigma_B = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2}$$

$$\sigma_B = \sqrt{\frac{94}{40} - \left(\frac{-32}{40}\right)^2}$$

$$\sigma_B = \sqrt{2.35 - (-0.8)^2}$$

$$\sigma_B = \sqrt{2.35 - 0.64}$$

$$\sigma_B = \sqrt{1.71}$$

$$\boxed{\sigma_B = 1.308}$$ ✓✓