# Student- t distribution

W. S. Grosset

## Conditions:

1) The Sample Size is Small ($n < 30$)
2) Population S.D. is Unknown.
3) The population from which Sample are taken is Normally distributed.

## P.d.f. of t-distribution

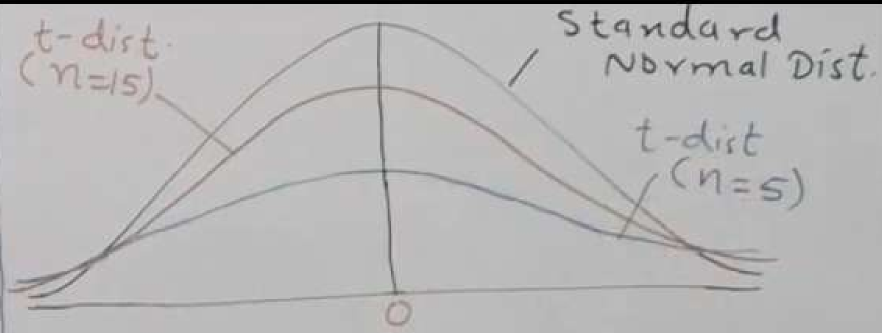
$$f(t) = C\left(1 + \frac{t^2}{\nu}\right)^{-(\nu+1)/2}$$

$\nu$ = d.f. = $n-1$, $n$ = Sample size

C = Constant

## Properties of t-distribution

1) The Value of t-Variable ranges from $-\infty$ to $\infty$
2) t-distribution is symmetrical about its mean, mean = 0
3) Variance $= \frac{n}{n-2}$, $n > 2$ var. > 1




4) t-distribution height is flatter at centre and higher in tails. (more dispersion) than S.N.D.
5) For each $\nu$ d.f., there is different t-dist. curve.

| D.f. | Area in 0.10 | 0.05 | 0.02 | 0.01 |
|---|---|---|---|---|
| 1 | 6.314 | 12.706 | 31.821 | 63.657 |
| 2 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | 2.353 | 3.182 | 4.541 | 5.841 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 8 | 1.860 | 2.306 | 2.896 | 3.355 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $\infty$ | 1.645 | 1.960 | 2.326 | 2.576 |

# Student-t distribution

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## Conditions:

1) The Sample Size is Small ($n < 30$)
2) Population S.D. is unknown
3). The population from which Sample are taken is Normally distributed

| $n \geq 30$ | $n < 30$ |
|---|---|
| popl$^n$ S.D. known | popl$^n$ S.D. unknown |
| $Z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}}$ | $t = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}}$ |
| $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$ — constant | $\sigma_{\bar{x}} = \dfrac{s}{\sqrt{n}}$ — variable |
| S.E. of mean | |

## Properties of t-distribution

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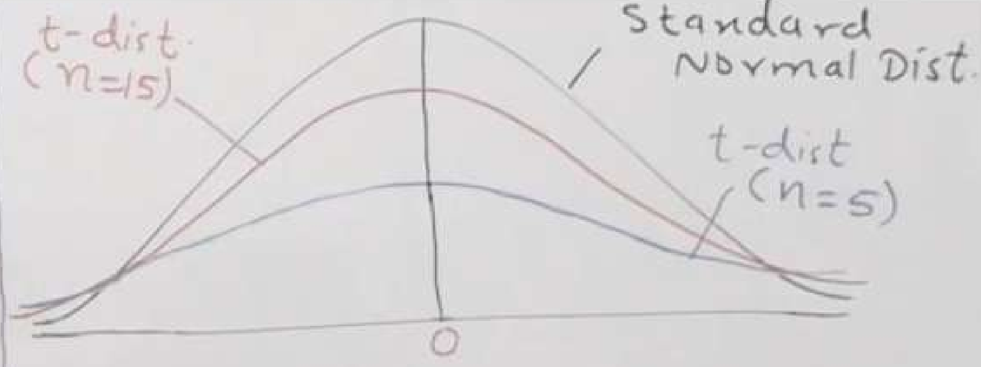


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Area in

| D.f. | 0.10 | 0.05 | 0.02 | 0.01 |
|---|---|---|---|---|
| 1 | 6.314 | 12.706 | 31.821 | 63.657 |
| 2 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | 2.353 | [illegible] | 4.541 | 5.841 |
| ⋮ | ⋮ | | ⋮ | ⋮ |
| 8 | 1.860 | [illegible] | [illegible]896 | 3.355 |
| ⋮ | ⋮ | | | ⋮ |
| $\infty$ | 1.645 | 1.9[illegible] | [illegible] | 2.576 |

# Student-t distribution

W.S. Gosset

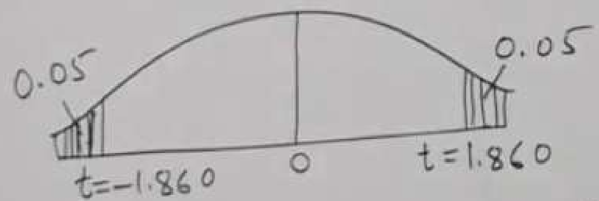
Conditions:

1) The Sample size is Small ($n<30$)
2) Population S.D. is unknown.
3). The population from which Sample are taken is Normally distributed

$H_0: \mu = \mu_1$

$H_1: \mu \neq \mu_1$ (two tailed)

$\alpha = 0.10$




$t_{8(df),\,0.10}$ for two-tailed test

$= 1.860$

$H_0: \mu = \mu_1$

$H_1: \mu > \mu_1$ (Right tailed)

$t_{8(d.f),\,0.05}$ for one-tailed test

$= 1.860$

Decision:-

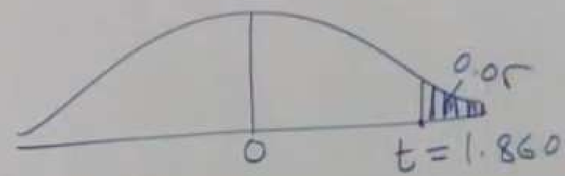
Cal. $|t|$ at certain d.f. and at certain LOS

tab. t

Cal. $|t|$ > tab. t ; reject $H_0$

Cal. $|t|$ < tab. t ; accept $H_0$

$\alpha = 0.05$




area in both tails

| D.f. | two tail → 0.10<br>one-tailed → 0.05 | 0.05<br>0.025 | 0.02<br>0.01 | 0.01<br>0.005 |
|---|---|---|---|---|
| 1 | 6.314 | 12.706 | 31.821 | 63.657 |
| 2 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | 2.353 | 3.182 | 4.541 | 5.841 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
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| ∞ | 1.645 | 1.960 | 2.326 | 2.576 |

# Application of t-test

## ① To test Significance of mean of random Sample [n < 30]

1) Set Null Hypothesis

$H_0: \bar{X} = \mu$

Set Alternative Hypothesis

$H_1: \bar{X} \neq \mu$ [two-tailed]

$\bar{X} > \mu$, $\bar{X} < \mu$ [one-tailed]

2) Test Statistics

$$t = \frac{\bar{X} - \mu}{\sigma_{\bar{x}}}$$

$$t = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \quad \text{or} \quad t = \frac{\bar{X} - \mu}{S}\sqrt{n}$$

$\bar{X}$ = sample mean

$\mu$ = population mean

$n$ = sample size

$S$ = sample S.D.

$$S = \sqrt{\frac{\Sigma(X - \bar{X})^2}{n-1}}$$

d.f. $\nu = n - 1$

Level of significance [LOS]

$\alpha = 0.05$ or $0.01$

Decision

Cal. $|t|$ tab. $t$

at certain d.f. and at certain LOS $\alpha$

① Cal. $|t|$ > tab $t$

$H_0$ reject

Cal. $|t|$ < tab $t$

$H_0$ accept

# Application of t-test

① To test significance of mean of random sample [n < 30]

Confidence limit for $\mu$

$$\bar{x} \pm \frac{s}{\sqrt{n}} t_{0.05}$$ [confidence limit at 5 % level of sig. or 95 % confidence level]

$$\bar{x} \pm \frac{s}{\sqrt{n}} t_{0.01}$$ [ 99 % C.L.

**Ten individuals are chosen at random from a population and their Heights are found to be in inches 63,63,64, 65, 66, 69, 69, 70, 70, 71. Discuss the proposal that mean height in the universe is 65 inches.**

# Application of $t$-test

## ① To test Significance of mean of random Sample $[n<30]$

**Q.1** Given, $n = 10$ [Small sample]

$H_0$: The mean height in the universe is 65 inches

$H_0: \mu = 65$ inches

$H_1: \mu \neq 65$ inches [Two-tailed test]

Calculation of Sample mean & Sample Standard deviation

| $X$ | $X-\bar{X}$ $(X-67)$ | $(X-\bar{X})^2$ |
|---|---|---|
| 63 | -4 | 16 |
| 63 | -4 | 16 |
| 64 | -3 | 9 |
| 65 | -2 | 4 |
| 66 | -1 | 1 |
| 69 | 2 | 4 |
| 69 | 2 | 4 |
| 70 | 3 | 9 |
| 70 | 3 | 9 |
| 71 | 4 | 16 |
| $\Sigma X = 670$ | | $\Sigma(X-\bar{X})^2 = 88$ |

Sample mean, $\bar{X} = \frac{\Sigma X}{n} = \frac{670}{10} = 67$

Sample S.D., $S = \sqrt{\frac{\Sigma(X-\bar{X})^2}{n-1}} = \sqrt{\frac{88}{10-1}}$

$S = \sqrt{9.77} \Rightarrow \boxed{S = 3.126}$

Test statistic

$t = \frac{\bar{X}-\mu}{S}\sqrt{n} \Rightarrow t = \frac{67-65}{3.126}\sqrt{10}$

$t = \frac{2 \times 3.162}{3.126} = 2.02$

d.f., $\nu = n-1 = 10-1 = 9$

Level of significance, $\alpha = 0.05$

tabulated $t$ at 9 d.f. and at $\alpha = 0.05$ LOS for two tailed test, $t = 2.262$

# Application of t-test

## ① To test Significance of mean of random Sample [n < 30]

**Q.1** Given, $n = 10$ [Small sample]

$H_0$: The mean height in the universe is 65 inches

$H_0: \mu = 65$ inches

$H_1: \mu \neq 65$ inches [Two-tailed test]

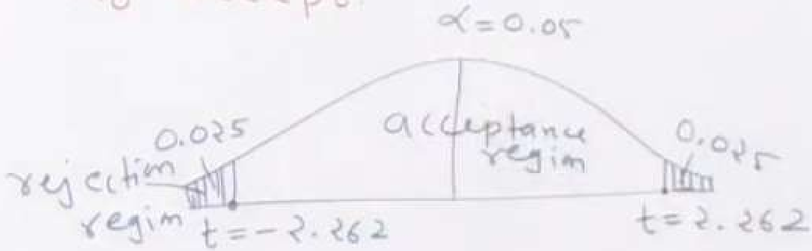
Calculation of Sample mean & Sample Standard deviation

| $X$ | $X - \bar{X}$ $(x - 67)$ | $(X - \bar{X})^2$ |
|---|---|---|
| 63 | −4 | 16 |
| 63 | −4 | 16 |
| 64 | −3 | 9 |
| 65 | −2 | 4 |
| 66 | −1 | 1 |
| 69 | 2 | 4 |
| 69 | 2 | 4 |
| 70 | 3 | 9 |
| 70 | 3 | 9 |
| 71 | 4 | 16 |
| $\Sigma X = 670$ | | $\Sigma(X - \bar{X})^2 = 88$ |

Cal. $|t|$ < tab $t$

$2.02 < 2.262$

$H_0$ accept.




Test statistic

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d.f., $\nu = n - 1 = 10 - 1 = 9$

Level of significance, $\alpha = 0.05$

tabulated $t$ at 9 d.f. and at $\alpha = 0.05$ LOS for two tailed test, $t = 2.262$

**A shop manufacturing company was distributed a particular type of brand through a large number of retail shops. Before a healthy advertising campaign, the mean sales per week, per shop was 140 dozen. After the campaign a sample of 26 shops was taken and the mean sales were found to be 147 dozens with standard deviation 16.can you consider the advertisement effective?**

# Application of t-test

① To test Significance of mean of random Sample [$n < 30$]

**Q.2** Given, $n = 26$ [Small sample]

$H_0$ : Adv. is not effective

$H_0 : \mu = 140$ dozens

$H_1 : \mu > 140$ dozens [one-tailed]

[Adv. is effective]

$\bar{X} = 147, \quad S = 16, \quad n = 26$

test statistic

$$t = \frac{\bar{X} - \mu}{S}\sqrt{n}$$

$$t = \frac{147 - 140}{16} \times \sqrt{26}$$

$$t = \frac{7}{16} \times 5.099$$

$$|t| = \frac{35.693}{16} = 2.23$$

d.f, $\nu = n - 1 = 26 - 1 = 25$

Level of significance, $\alpha = 0.05$

tabulated t at 25 d.f. and

$\alpha = 0.05$ LOS, $t = 1.708$

for one-tailed test

Decision :-

cal $|t|$ > tab t

$2.23 > 1.708$

$H_0$ reject

Adv. is effective

$\alpha = 0.05$

| d.f. | 0.10 |
|---|---|
| 1 | |
| 25 | → |

**A random sample of size 16 and 53 as a mean. The sum of squares of the deviation taken from mean is 135. Can this sample be regarded as taken from population having 56 as a mean. Obtained 95% and 99% confidence limit of the mean of population.**

# Application of t-test

① To test Significance of mean of random Sample [$n < 30$]

**Q.3** Given, $n = 16$ [small sample]

$H_0: \mu = 56$

$H_1: \mu \neq 56$ [two tailed]

$n = 16, \quad \bar{X} = 53,$

$\Sigma(X - \bar{X})^2 = 135$

$\therefore S = \sqrt{\frac{\Sigma(X-\bar{X})^2}{n-1}} = \sqrt{\frac{135}{16-1}} = \sqrt{9}$

$S = 3$

Test statistics

$t = \frac{\bar{X} - \mu}{S}\sqrt{n} \Rightarrow t = \frac{53-56}{3} \times \sqrt{16}$

$|t| = \frac{|-3|}{3} \times 4 \Rightarrow |t| = 4$

d.f, $\nu = n - 1 = 16 - 1 = 15$

Level of significance, $\alpha = 0.05$

tabulated t at 15 d.f. and $\alpha = 0.05$ Los, $t = 2.131$

**Decision** cal. $|t|$ > tab. t

$4 > 2.131$

$H_0$ reject.

Confidence limit at 95%

$\bar{X} \pm \frac{S}{\sqrt{n}} \times t_{0.05}$

$53 \pm \frac{3}{\sqrt{16}} \times 2.131$

$53 \pm \frac{3}{4} \times 2.131$

$53 \pm 1.6$

[51.4 to 54.6]

# Application of $t$-test

## ② To test Significance of difference between mean of two Sample [Independent Sample]

$H_0: \mu_{x_1} = \mu_{x_2}$

$H_1: \mu_{x_1} \neq \mu_{x_2}$

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sigma_{\bar{x}}}$$

$$t = \frac{\bar{X}_1 - \bar{X}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$t = \frac{\bar{X}_1 - \bar{X}_2}{S} \times \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

Popln I: $\mu_1$, $\sigma_1^2$

Sample I: $\bar{X}_1$, $n_1 < 30$

Popln II: $\mu_2$, $\sigma_2^2$

Sample II: $\bar{X}_2$, $n_2 < 30$

$\sigma_1^2 = \sigma_2^2 = \sigma^2$

$S$ = Combined S.D.

$$S = \sqrt{\frac{\Sigma(X_1 - \bar{X}_1)^2 + \Sigma(X_2 - \bar{X}_2)^2}{n_1 + n_2 - 2}}$$

d.f., $\nu = n_1 + n_2 - 2$

**Example 16·10.** *Below are given the gain in weights (in kgs.) of pigs fed on two diets A and B.* 9

*Gain in weight*

Diet A : 25, 32, 30, 34, 24, 14, 32, 24, 30, 31, 35, 25

Diet B : 44, 34, 22, 10, 47, 31, 40, 30, 32, 35, 18, 21, 35, 29, 22

*Test, if the two diets differ significantly as regards their effect on increase in weight.*

# Application of $t$-test

② To test Significance of difference between mean of two Sample [Independent Sample]

**Q.1** $H_0$: There is no significance difference between the mean increase in weight due to diet A & B

$H_0: \bar{x}_1 = \bar{x}_2$

$H_1: \bar{x}_2 \neq \bar{x}_2$ [two tailed]

$$\bar{x}_1 = \frac{\Sigma x_1}{n_1} = \frac{336}{12} = 28$$

$$\bar{x}_2 = \frac{\Sigma x_2}{n_2} = \frac{450}{15} = 30$$

Combined sample S.D.

$$S = \sqrt{\frac{\Sigma(x_1-\bar{x}_1)^2 + \Sigma(x_2-\bar{x}_2)^2}{n_1+n_2-2}}$$

$$S = \sqrt{\frac{380+1410}{12+15-2}} = \sqrt{\frac{1790}{25}}$$

$$S = \sqrt{71.6} \Rightarrow \boxed{S = 8.46}$$

| Diet A | | | Diet B | | |
|---|---|---|---|---|---|
| $X_1$ | $(x_1-\bar{x}_1)$ $(x_1-28)$ | $(x_1-\bar{x}_1)^2$ | $X_2$ | $(x_2-\bar{x}_2)$ $(x_2-30)$ | $(x_2-\bar{x}_2)^2$ |
| 25 | −3 | 9 | 44 | 14 | 196 |
| 32 | 4 | 16 | 34 | 4 | 16 |
| 30 | 2 | 4 | 22 | −8 | 64 |
| 34 | 6 | 36 | 10 | −20 | 400 |
| 24 | −4 | 16 | 47 | 17 | 289 |
| 14 | −14 | 196 | 31 | 1 | 1 |
| 32 | 4 | 16 | 40 | 10 | 100 |
| 24 | −4 | 16 | 30 | 0 | 0 |
| 30 | 2 | 4 | 32 | 2 | 4 |
| 31 | 3 | 9 | 35 | 5 | 25 |
| 35 | 7 | 49 | 18 | −12 | 144 |
| 25 | −3 | 9 | 21 | −9 | 81 |
| | | | 35 | 5 | 25 |
| | | | 29 | −1 | 1 |
| | | | 22 | −8 | 64 |
| $\Sigma x_1 = 336$ | | $\Sigma(x_1-\bar{x}_1)^2 = 380$ | $\Sigma x_2 = 450$ | | $\Sigma(x_2-\bar{x}_2)^2 = 1410$ |

# Application of t-test

## (2) To test Significance of difference between mean of two Sample [Independent Sample]

**Q.1** $H_0$: There is no significance difference between the mean increase in weight due to diet A & B

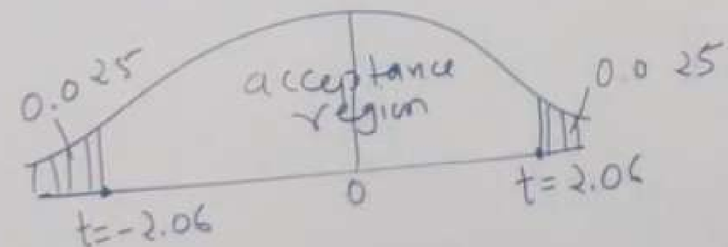
$H_0: \bar{x}_1 = \bar{x}_2$

$H_1: \bar{x}_2 \neq \bar{x}_2$ [two tailed]

$$\bar{x}_1 = \frac{\Sigma x_1}{n_1} = \frac{336}{12} = 28$$

$$\bar{x}_2 = \frac{\Sigma x_2}{n_2} = \frac{450}{15} = 30$$

$\alpha = 0.05$




test statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S}\sqrt{\frac{n_1 n_2}{n_1+n_2}}$$

$$|t| = \frac{|28-30|}{8.46}\sqrt{\frac{12\times15}{12+15}}$$

$$|t| = \frac{2}{8.46}\sqrt{\frac{180}{27}} = \frac{2}{8.46}\times 2.58$$

$$\boxed{|t| = 0.61}$$

d.f., $\nu = n_1 + n_2 - 2 = 12+15-2 = 25$

Level of siginificance, $\alpha = 0.05$

tabulated t at 25 d.f and 0.05 LOS and for two tailed test, $t = 2.06$

Decision:- Cal $|t|$ < tabulated t

$0.61 < 2.06$

$H_0$ accept

# Application of t-test

## (2) To test Significance of difference between mean of two Sample [Independent Sample]

**Q1** $H_0$: There is no significance difference between the mean increase in weight due to diet A & B

$H_0: \bar{x}_1 = \bar{x}_2$

$H_1: \bar{x}_2 \neq \bar{x}_2$ [two tailed]

$$\bar{x}_1 = \frac{\Sigma x_1}{n_1} = \frac{336}{12} = 28$$

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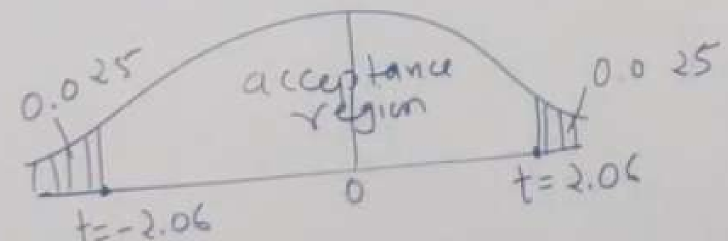
$\alpha = 0.05$




**test statistic**

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S}\sqrt{\frac{n_1 n_2}{n_1+n_2}}$$

$$|t| = \frac{|28-30|}{8.46}\sqrt{\frac{12\times 15}{12+15}}$$

$$|t| = \frac{2}{8.46}\sqrt{\frac{180}{27}} = \frac{2}{8.46} \times 2.58$$

$$\boxed{|t| = 0.61}$$

d.f., $\nu = n_1 + n_2 - 2 = 12+15-2 = 25$

Level of siginificance, $\alpha = 0.05$

tabulated t at 25 d.f and 0.05 LOS and for two tailed test, $t = 2.06$

**Decision:-** Cal $|t|$ < tabulated t

$0.61 < 2.06$

$H_0$ accept

# Application of t-test

③ To test Significance of difference between mean of two Sample [dependent Sample]

Paired Sample

$\boxed{n_1 = n_2 = n}$  $n < 30$

$$t = \frac{\bar{d}}{S/\sqrt{n}} \text{ or } \frac{\bar{d}}{S}\sqrt{n}$$

$d$ = increment = $\underset{\text{Sample I}}{x_1} - \underset{\text{Sample II}}{x_2}$

$H_0: \mu_{x_1} = \mu_{x_2}$

$H_1: \mu_{x_2} \neq \mu_{x_2}$ (Two tailed)

$H_1: \mu_{x_1} > \mu_{x_2}$ (one-tailed)

$H_1: \mu_{x_2} < \mu_{x_2}$ ( " )

$\bar{d}$ = mean of increment

$$S = \text{Sample S.D.} = \sqrt{\frac{\Sigma(d-\bar{d})^2}{n-1}}$$

Cal $|t|$  tab.

d.f. $\nu = n - 1$

An I.Q. test was administered to 5 persons before and after they were trained. The results are given below:

| Candidate | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| I.Q. before training | 110 | 120 | 123 | 132 | 125 |
| I.Q. after training | 120 | 118 | 125 | 136 | 121 |

Test whether there is any change in I.Q. after the training