RANDOM VARIABLE

Real value of random experiment is called Random variable.

Ex: Toss two coin simultaneously

$S = \{HH, HT, TH, TT\}$

r.v.

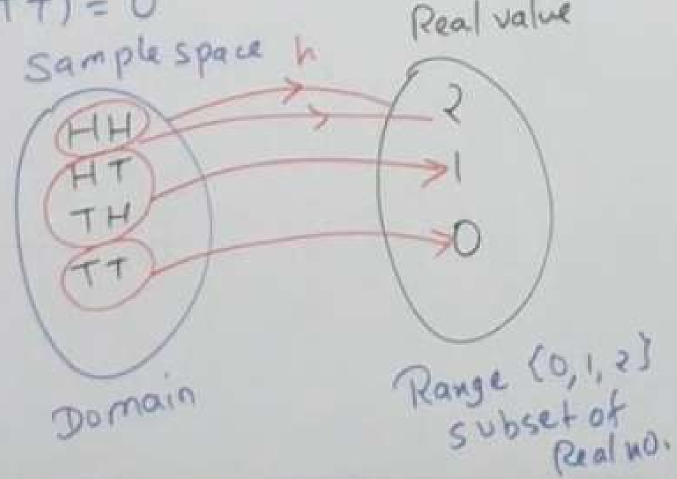
$X =$ no. of heads

$X(HH) = 2$

$X(HT) = 1$

$X(TH) = 1$

$X(TT) = 0$



Types

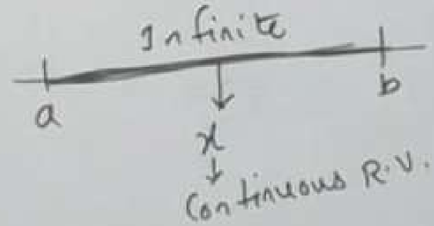
Discrete Random variable

finite or infinite countable.

Continuous Random variable

$a \leq X \leq b$

weight
temperature
Height
Age



Normal Distribution

↓ Continuous Prob. dist

$a < X < b$

→ mean

P.d.f.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \leftarrow$$

$-\infty < x < \infty$

σ → S.D.

mean $= \mu$

Variance $= \sigma^2$

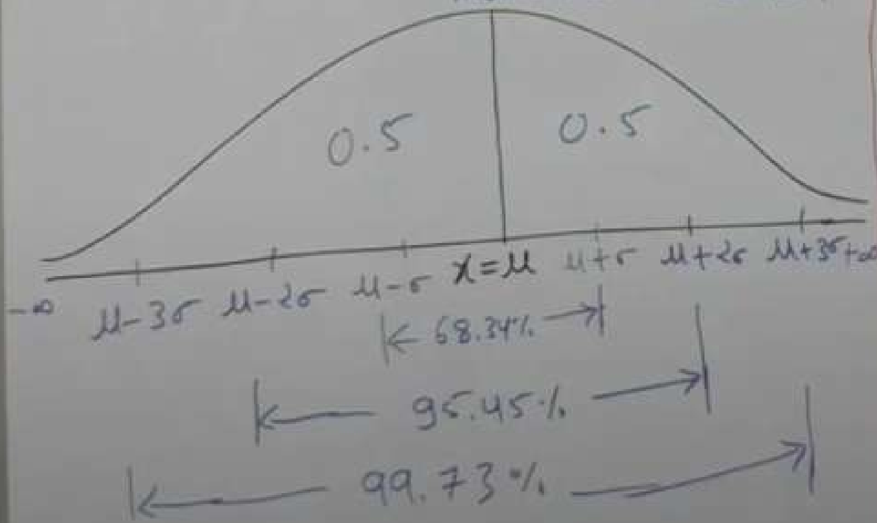
$-\infty < \mu < \infty$

$\sigma^2 > 0$

$\boxed{X \sim N(\mu, \sigma^2)}$

Total area under curve = 1

mean = mode = median



Standard Normal distribution

$Z =$ SNV, $Z = \frac{X-\mu}{\sigma}$

mean = 0, variance = 1

P.d.f

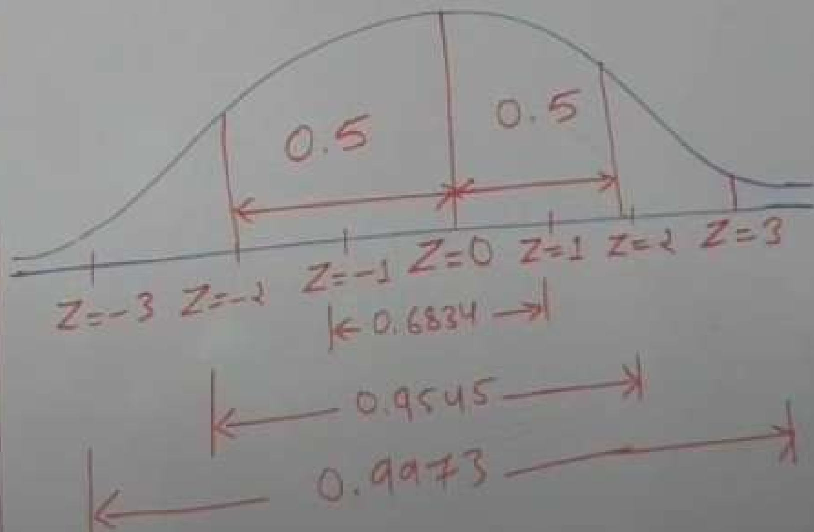
$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} z^2}$$

$-\infty < Z < \infty$

$\mu = $ mean $= 0$

$\sigma^2 = $ Var $= 1$

$Z \sim$ S.N.D $(0, 1)$



Normal Distribution

① $P(a<X<b)$

$X \sim N(\mu, \sigma^2)$

mean, var.

② Convert X into Z (S.N.V.)

By using,

$$Z = \frac{X-\mu}{\sigma} \quad —①$$

③ Put $X=a$ in ①, $Z = \frac{a-\mu}{\sigma}$

Put $X=b$ in ①, $Z = \frac{b-\mu}{\sigma}$

$$P(a<X<b) = P\left(\frac{a-\mu}{\sigma} < Z < \frac{b-\mu}{\sigma}\right)$$

Normal dist table.

Standard Normal distribution

$Z =$ SNV, $Z = \frac{X-\mu}{\sigma}$

mean $=0$, variance $=1$

P.d.f

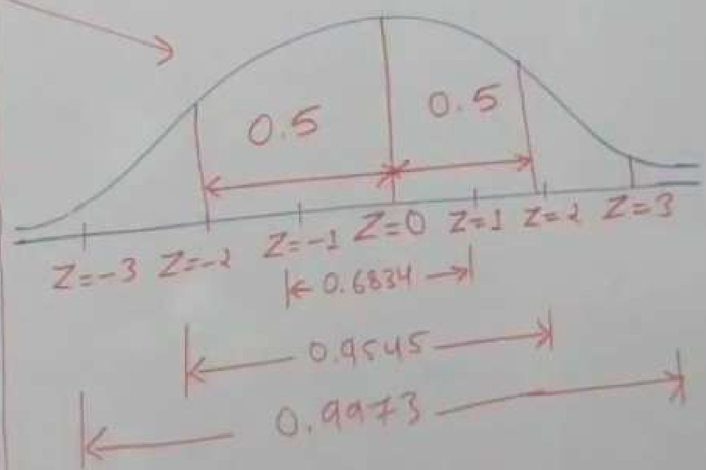
$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

$-\infty < Z < \infty$

$\mu =$ mean $= 0$

$\sigma^2 =$ Var $= 1$

$Z \sim$ S.N.D $(0, 1)$



Normal Distribution

Q.1. If X is Normal dist. variate with mean = 30 and S.D = 5 then find
(i) $P(26 \le X \le 40)$ (ii) $P(X \ge 45)$

Sol^n:- Given, mean, $\mu = 30$
S.D., $\sigma = 5$

$$\because Z = \frac{X-\mu}{\sigma}$$

$$Z = \frac{X-30}{5} \quad \text{— (1)}$$

(i) find $P(26 \le X \le 40)$

Put $x = 26$ in (1), $Z = \frac{26-30}{5}$

$$Z = -\frac{4}{5} = -0.8$$
(Area is on left of Z=0)

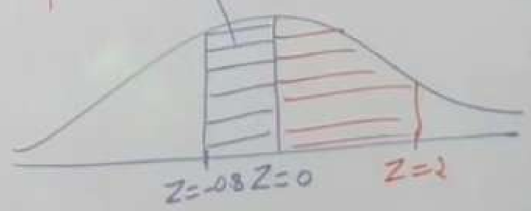
Area b/w $(Z=0 \text{ to } Z=0.8) = 0.2881$
(from Standard Normal table)

Put $x = 40$ in (1),

$$Z = \frac{40-30}{5} = 2$$
(Area is on Right of Z=0)

Area b/w $(Z=0 \text{ to } Z=2)$
$= 0.4772$

$P(26 \le X \le 40) = P(-0.8 < Z < 2)$
= (Area b/w Z=0 to Z=−0.8)
+ (Area b/w Z=0 to Z=2)
$= 0.2881 + 0.4772$
$= 0.7653$



Normal Distribution

Q.1 If X is Normal dist. variate with mean = 30 and S.D = 5 then find

(i) $P(26 \le X \le 40)$ (ii) $P(X \ge 45)$

Soln:- Given, mean, $\mu = 30$

S.D., $\sigma = 5$

$$\therefore Z = \frac{X-\mu}{\sigma}$$

$$Z = \frac{X-30}{5} \quad \text{—①}$$

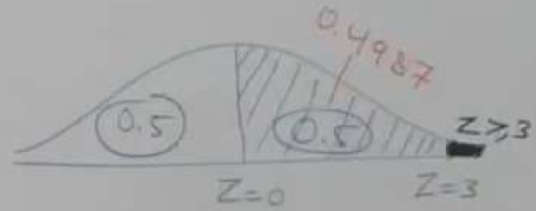
(ii) $P(X \ge 45)$

put $x = 45$ in ①

$$Z = \frac{45-30}{5} = 3$$

Area on Right of $z=0$

Area b/w $(Z=0$ to $Z=3) = 0.4987$

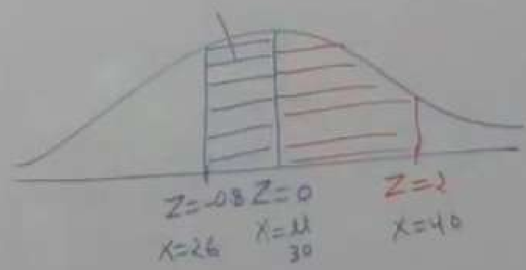


$P(X \ge 45) = P(Z \ge 3)$

$= (\text{Area of } Z \ge 3)$

$= (\text{Total area on Right of } z=0) - (\text{Area b/w } Z=0 \text{ to } Z=3)$

$P(X \ge 45) = 0.5 - 0.4987 = 0.0013$



Normal Distribution

Q.2 If X is Normal dist. variate

mean = 68.22 cm, variance = 10.8 cm (height)

total children = 1000 (given)

$P(X \geq 72) = ?$

mean, $\mu = 68.22$ cm

Variance, $\sigma^2 = 10.8$ cm

$\therefore \sigma = \sqrt{10.8} = 3.286$

$P(X \geq 72) = ?$

$\because Z = \frac{X-\mu}{\sigma} = \frac{X - 68.22}{3.286}$ —①

Put X = 72 in ①

$Z = \frac{72 - 68.22}{3.286} = 1.15$ (Area is on right of Z = 0)

Area b/w (Z = 0 to Z = 1.15) $= \boxed{0.3749}$

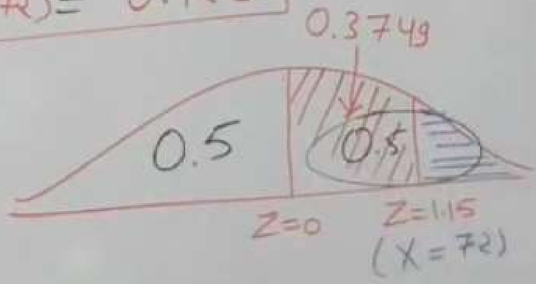
$P(X \geq 72) = P(Z \geq 1.15)$

= (Area of $Z \geq 1.15$)

= (Total area on Right of Z = 0) − (Area b/w Z = 0 to Z = 1.15)

= 0.5 − 0.3749

$\boxed{P(X \geq 72) = 0.1251}$



Normal Distribution

Q.2 If X is Normal dist. variate

mean = 68.22 cm, variance = 10.8 cm (height)

total children = 1000 (given)

$P(X \geq 72) = ?$

mean, $\mu = 68.22$ cm

Variance, $\sigma^2 = 10.8$ cm

$\therefore \sigma = \sqrt{10.8} = 3.286$

$P(X \geq 72) = ?$

$\because Z = \frac{X-\mu}{\sigma} = \frac{X - 68.22}{3.286}$ —①

Put X = 72 in ①

$Z = \frac{72 - 68.22}{3.286} = 1.15$ (Area is on right of Z=0)

Area b/w (Z = 0 to Z = 1.15)

= 0.3749

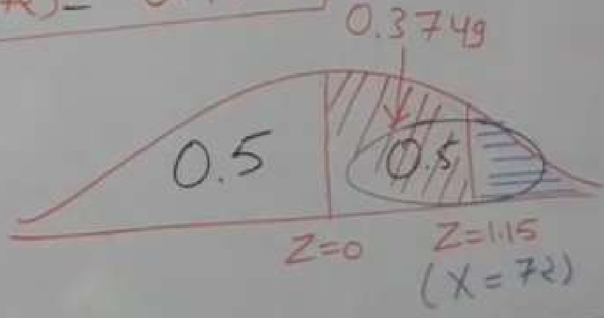
$P(X \geq 72) = P(Z \geq 1.15)$

= (Area of $Z \geq 1.15$)

= (Total area on Right of Z = 0) − (Area b/w Z = 0 to Z = 1.15)

= 0.5 − 0.3749

$\boxed{P(X \geq 72) = 0.1251}$



Expected no. of children having height $\geq$ 72 cm is

= 1000 × 0.1251

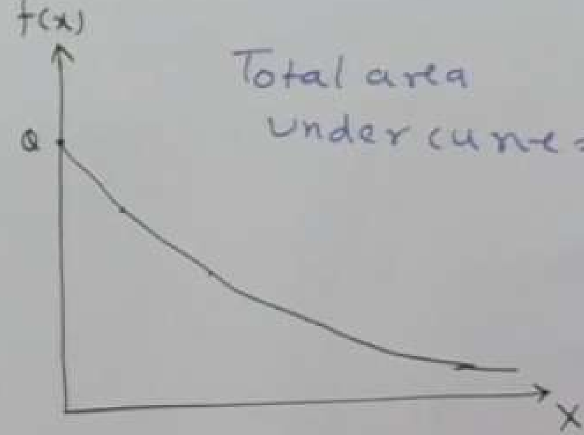
= 125.1 ≃ 125 children

Exponential Distribution

A random Variable X is said to have an exponential distribution with parameter $\theta > 0$ if its PDF is given by

$$f(x,\theta) = \begin{cases} \theta e^{-\theta x} & ; 0 \le x < \infty \\ 0 & ; \text{otherwise} \end{cases}$$

$X \sim exp(\theta)$



Total area under curve = 1

Graph

$X = x$:	0	1	2	---	∞
$f(x)$:	θ	$\theta e^{-\theta}$	$\theta e^{-2\theta}$	-	0
when $\theta = 1$	1	e^{-1}	e^{-2}	----	0

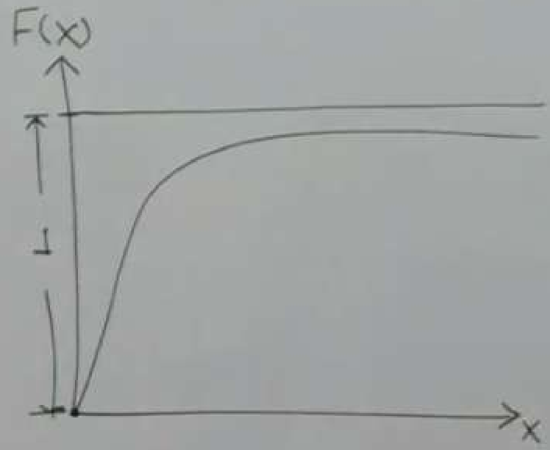
Exponential Distribution

A random Variable X is said to have an exponential distribution with parameter $\theta > 0$ if its PDF is given by

$$f(x,\theta) = \begin{cases} \theta e^{-\theta x} & ; 0 \le x < \infty \\ 0 & ; \text{otherwise} \end{cases}$$

Cumulative Distribution Function

$X = x$:	0	1	2	---	∞
$F(x)$:	0	$1-e^{-\theta}$	$1-e^{-2\theta}$	----	1
when $\theta = 1$	0,	$1-e^{-1}$,	$1-e^{-2}$,	---	1



CDF

$$F(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-\theta x} & 0 \le x < \infty \\ 1 & x = \infty \end{cases}$$