Testing of Hypothesis of Difference of Two Sample means [LARGE SAMPLE]

Q.1 Intelligence test of two groups of boys and girls gave the following results

	Mean	S.D.	Sample Size (n)
Girls	$\bar{x}_1 = 75$	$S_1 = 15$	$n_1 = 150$
Boys	$\bar{x}_2 = 70$	$S_2 = 20$	$n_2 = 250$

Is there a significant difference between mean score of boys & girls at 1% level of significance?

Null Hypothesis, H_0: There is no significance difference between mean scores of boys and girls.

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$ [Two-Tailed Test]

Test statistic

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma_{\bar{x}_1 - \bar{x}_2}}$$

where $\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{225}{150} + \frac{400}{250}}$$

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{1.5 + 1.6} = 1.761$$

$$Z = \frac{75 - 70}{1.761} = 2.84$$

Critical value (Z_α)

LOS (α) →	1%	5%	10%
Two-Tailed Test	$\lvert Z_\alpha \rvert = 2.58$	$\lvert Z_\alpha \rvert = 1.96$	$\lvert Z_\alpha \rvert = 1.645$
Right-Tailed Test	$Z_\alpha = 2.33$	$Z_\alpha = 1.645$	$Z_\alpha = 1.28$
Left Tailed Test	$Z_\alpha = -2.33$	$Z_\alpha = -1.645$	$Z_\alpha = -1.28$

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$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$ [Two-Tailed Test]

Test statistic

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma_{\bar{x}_1 - \bar{x}_2}}$$

$|z| = 2.84$

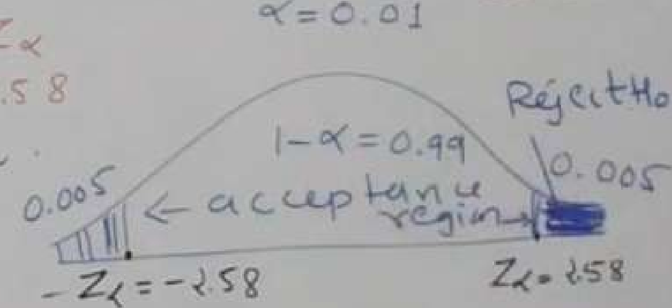
$\alpha = 0.01$ or 1% Level of sig.

Critical value, $z_\alpha = 2.58$ at 1% LOS and for Two-tailed Test

$|z| > z_\alpha$

$2.84 > 2.58$

H_0 reject.



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Testing of Hypothesis of Difference of Two Sample means [LARGE SAMPLE]

Q.2 The mean of two single large samples of 1000 & 2000 members are 67.5 inches and 68 inches respectively. Can the samples be regarded as taken from same population of standard deviation 2.5 inches? (Test at 5% LOS)

$n_1 = 1000 \qquad \bar{x}_1 = 67.5$

$n_2 = 2000 \qquad \bar{x}_2 = 68$

H_0: The sample has taken from same population having S.D $\sigma = 2.5$.

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$ (Two Tailed Test)

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma_{\bar{x}_1 - \bar{x}_2}} = \frac{67.5 - 68}{0.09675}$$

$$Z = -5.16$$

$$\boxed{|Z| = 5.16}$$

where $\sigma_{\bar{x}_1 - \bar{x}_2}$ = S.E. of difference of two mean

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$\sigma_{\bar{x}_1 - \bar{x}_2} = 2.5 \sqrt{\frac{1}{1000} + \frac{1}{2000}}$$

$$= 2.5 \times \sqrt{\frac{3}{2000}} = 2.5 \times 0.0387$$

$$\sigma_{\bar{x}_1 - \bar{x}_2} = 0.09675$$

Critical value (Z_α)

LOS(α) →	1%	5%	10%
Two-Tailed Test ✓	$\lvert Z_\alpha \rvert = 2.58$ ✓	$\lvert Z_\alpha \rvert = 1.96$	$\lvert Z_\alpha \rvert = 1.645$
Right-Tailed Test	$Z_\alpha = 2.33$	$Z_\alpha = 1.645$	$Z_\alpha = 1.28$
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Testing of Hypothesis of Difference of Two [LA… S… Sample means

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$Z = -5.16$

$\boxed{|Z| = 5.16}$

LOS, $\alpha = 5\%$

Critical value, $Z_\alpha = 1.96$

5% LOS and for two test

$|Z| > Z_\alpha$

$5.16 > 1.96$

H_0 reject

Critical value (Z_α)

LOS(α) →	1%	5%
Two-Tailed Test ✓	$\lvert Z_\alpha \rvert = 2.58$ ✓	$\lvert Z_\alpha \rvert = 1.96$
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Testing of Hypothesis for the Difference of Standard Deviations [Large Sample]

P-I: S.D = σ_1

P-II: S.D. = σ_2

S-I
n_1
$s_1 = S.D$

S-II
n_2
$s_2 = S.D$

$H_0: \sigma_1 = \sigma_2$

$H_1: \sigma_1 \neq \sigma_2$ or $\sigma_1 > \sigma_2$ or $\sigma_1 < \sigma_2$

$$Z = \frac{s_1 - s_2}{\sigma_{s_1 - s_2}}$$

$\sigma_{s_1-s_2}$ = S.E. of diff. of sample S.D.

(a) If popl^n S.D. is known

$$\sigma_{s_1-s_2} = \sqrt{\frac{\sigma_1^2}{2n_1} + \frac{\sigma_2^2}{2n_2}}$$

(b) If popl^n S.D. is unknown

$$\sigma_{s_1-s_2} = \sqrt{\frac{s_1^2}{2n_1} + \frac{s_2^2}{2n_2}}$$

Cal $|Z|$ > tab Z_α
H₀ reject.

Cal $|Z|$ < tab Z_α
H₀ accept

	Critical value (Z_α)		
LOS(α) →	1%	5%	10%
Two Tailed	$\lvert Z_\alpha \rvert = 2.58$	$\lvert Z_\alpha \rvert = 1.96$	$\lvert Z_\alpha \rvert = 1.645$
Right Tailed	$Z_\alpha = 2.33$	$Z_\alpha = 1.645$	$Z_\alpha = 1.28$
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Example 14·30. *Random samples drawn from two countries gave the following data relating to the heights of adult males :*

	Country A	Country B
Mean height (in inches)	67·42	67·25
Standard deviation (in inches)	2·58	2·50
Number in samples	1,000	1,200

(i) Is the difference between the means significant ?

(ii) Is the difference between the standard deviations significant ?

Testing of Hypothesis for the Difference of Standard Deviations [Large Sample

Q.1

$n_1 = 1000 \quad \bar{x}_1 = 67.42, \quad s_1 = 2.58$

$n_2 = 1200 \quad \bar{x}_2 = 67.25, \quad s_2 = 2.50$

(ii) $H_0: \sigma_1 = \sigma_2$. There is no siginificance diff. b/w two S.D.

$H_1: \sigma_1 \neq \sigma_2$ (two-tailed)

$$Z = \frac{s_1 - s_2}{\sigma_{s_1 - s_2}}$$

$$\sigma_{s_1-s_2} = \sqrt{\frac{s_1^2}{2n_1} + \frac{s_2^2}{2n_2}}$$

$$\sigma_{s_1-s_2} = \sqrt{\frac{(2.58)^2}{2\times 1000} + \frac{(2.50)^2}{2\times 1200}}$$

$$\sigma_{s_1-s_2} = 0.07746$$

$$\therefore Z = \frac{2.58 - 2.50}{0.07746}$$

Cal. $|Z| = 1.03$

LOS, $\alpha = 5\%$ or 0.05

tab. $Z = Z_{0.05} = 1.96$ for two-tailed test

Cal $|Z|$ < tab. Z

$1.03 < 1.96$

H_0 accept.

Critical value (Z_α)			
LOS(α) →	1%	5% ✓	10%
Two Tailed	$\lvert Z_\alpha\rvert = 2.58$	$\lvert Z_\alpha\rvert = 1.96$ ✓	$\lvert Z_\alpha\rvert = 1.645$
Right Tailed	$Z_\alpha = 2.33$	$Z_\alpha = 1.645$	$Z_\alpha = 1.28$
Left Tailed	$Z_\alpha = -2.33$	$Z_\alpha = -1.645$	$Z_\alpha = -1.28$

Sampling of Attributes

Test of Significance for Single Proportion

Population

one class → Possessing Attribute A

Second class → Not Possessing Attribute A

Presence of attribute in sampled unit = Success

Absence of attribute in sampled unit = failure

A Sample of 'n' observation is identified with a Series of 'n' independent Bernoulli trials with constant probability P of success in each trial.

The probability of 'x' success in 'n' trial is given by Binomial distribution

$$p(x) = {}^{n}C_{x} P^{x} Q^{1-x}; \quad x = 0, \ldots n$$

mean $= nP$

Variance $= nPQ$

Sampling of Attributes

Test of Significance for Single Proportion

If X = no. of success in 'n' independent trials with probability P of Success for each trial then

$E(X) = nP$ $V(X) = nPQ$

mean   variance

$Q = 1 - P$

We know that

for large n,

Binomial distribution tends to Normal distribution.

$\therefore$ for large n,

$X \sim N(nP, nPQ)$

P — Population

Sample size $= n$ — p — X = no. of person Possessing Attribute A

$Var(aX) = a^2 var(X)$

Then,

Proportion of Success in Sample, $p = \frac{X}{n}$

$$\therefore E(p) = E\left(\frac{X}{n}\right) = \frac{1}{n} E(X) = \frac{1}{\cancel{n}} \cancel{n} P$$

p is unbiased estimate of P

$$V(p) = V\left(\frac{X}{n}\right) = \frac{1}{n^2} V(X) = \frac{1}{n^{\cancel{2}}} \times \cancel{n} PQ$$

$$V(X) = \frac{PQ}{n}$$

Sampling of Attributes

Test of Significance for Single Proportion

$S.D.(p) = \sqrt{\frac{PQ}{n}}$

Sample Proportion p follow $N\left(P, \sqrt{\frac{PQ}{n}}\right)$

$$Z = \frac{\bar{x} - \mu}{S.E.(\bar{x})}$$

test statistic for large n in case of proportion (single)

$$Z = \frac{p - P}{S.E.(p)}$$

$$Z = \frac{p - P}{S.E.(p)} \sim SND(0,1)$$

① Set $H_0 : P = P_0$

$H_1 : P \neq P_0$ (two-tailed)

$P > P_0$ (one right tailed)

$P < P_0$ (one left tailed)

② $S.E.(p) = \sqrt{\frac{PQ}{n}}$ (Population proportion is known)

$S.E.(p) = \sqrt{\frac{pq}{n}}$ (Population Proportion is not known)

③ test statistic

$$Z = \frac{p - P}{S.E.(p)} \sim SND(0,1)$$

A wholesaler in apples claims that only 4% of the apples supplied by him are defective. A random sample of 600 apples contained 36 defective apples. Test the claim of wholesaler at 5% level of significance.

Sampling of Attributes

Test of Significance for Single Proportion

Q.1 Given, $n = 600$

X = no. of defective apple = 36

P = proportion of defective apple in population

$P = 4\%$ or 0.04

$Q = 1 - P = 1 - 0.04 = 0.96$

Sample Proportion, $p = \frac{X}{n} = \frac{36}{600}$

$p = 0.06$

$H_0: P = 0.04$

$H_1: P > 0.04$ (Right tailed)

Test statistic

$$Z = \frac{p - P}{S.E.(p)}$$

where $S.E.(p) = \sqrt{\frac{PQ}{n}} = \sqrt{\frac{0.04 \times 0.96}{600}}$

$$\boxed{S.E.(p) = 0.08}$$

$\therefore Z = \frac{0.06 - 0.04}{0.08} = 2.5$

LOS, $\alpha = 5\%$

Critical value of $Z_\alpha = 1.645$

Critical value (Z_α)	Level of significance (α) 1 %	5 %	10 %
Two-Tailed	$\|Z_\alpha\| = 2.58$	$\|Z_\alpha\| = 1.96$	$\|Z_\alpha\| = 1.645$
Right Tailed	$Z_\alpha = 2.33$	$Z_\alpha = 1.645$	$Z_\alpha = 1.28$
Left Tailed	$Z_\alpha = -2.33$	$Z_\alpha = -1.645$	$Z_\alpha = -1.28$

Sampling of Attributes

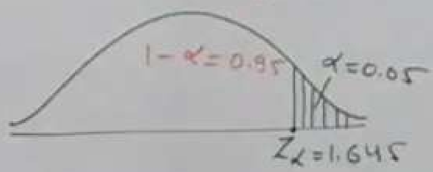
Test of Significance for Single Proportion

Decision

Cal. Z $>$ Critical Z

$2.5 > 1.645$

H_0 reject.



$p = 0.06$

$H_0 : P = 0.04$

$H_1 : P > 0.04$ (Right tailed)

Test statistic

$$Z = \frac{p - P}{S.E.(p)} \sim SND(0,1)$$

where $S.E.(p) = \sqrt{\frac{PQ}{n}} = \sqrt{\frac{0.04 \times 0.96}{600}}$

$$\boxed{S.E.(p) = 0.08}$$

$$\therefore Z = \frac{0.06 - 0.04}{0.08} = 2.5$$

LOS, $\alpha = 5\%$

Critical value of $Z_\alpha = 1.645$

Critical value (Z_α)	Level of significance (α) 1 %	5 %	10 %
Two-Tailed	$\lvert Z_\alpha \rvert = 2.58$	$\lvert Z_\alpha \rvert = 1.96$	$\lvert Z_\alpha \rvert = 1.645$
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In a random sample of 400 persons from a large population, 120 are females. Can it be said that males and females are in the ratio 5 : 3 in the population? Use 10% level of significance.

Sampling of Attributes

Test of Significance for Single Proportion

Q2 $n = 400$, $X = 120$

P = population Proportion of female $= \frac{3}{5+3} = 0.375$

$Q = 1 - P = 1 - 0.375 = 0.625$

p = Sample proportion of female $= \frac{X}{n} = \frac{120}{400} = 0.30$

$H_0: P = 0.375$

$H_1: P \neq 0.375$ (two tailed) ✓

$$Z = \frac{p - P}{S.E.(p)} \sim N(0,1)$$

where $S.E.(p) = \sqrt{\frac{PQ}{n}}$

$$S.E.(p) = \sqrt{\frac{0.375 \times 0.625}{400}}$$

$S.E.(p) = 0.024$

$\therefore |Z|_{Cal} = \frac{0.30 - 0.375}{0.024} = |-3.125| = 3.125$

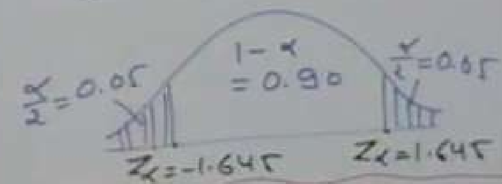
LOS, $\alpha = 10\%$

Critical value: $Z_\alpha = 1.645$

$|Z| > |Z_\alpha|$

$3.125 > 1.645$

H_0 reject.



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A die is thrown 9000 times and a throw 3 or 4 is observed 3240 times. Check whether the die cannot be regarded as an unbiased one. Use 5% level of significance.

Sampling of Attributes

Test of Significance for Single Proportion

Q3 $n = 9000$, $X = 3240$

P = Probability of getting 3 or 4 $= \frac{1}{6} + \frac{1}{6} = \frac{1}{3} = 0.333$

$Q = 1 - P = 1 - \frac{1}{3} = \frac{2}{3}$

sample proportion

$$p = \frac{X}{n} = \frac{3240}{9000} = 0.36$$

$H_0: P = \frac{1}{3}$

$H_1: P \neq \frac{1}{3}$ (two tailed)

$$Z = \frac{p - P}{S.E(p)}$$

where $S.E.(p) = \sqrt{\frac{PQ}{n}}$

$$S.E(p) = \sqrt{\frac{1}{3} \times \frac{2}{3} \times \frac{1}{9000}} = 0.005$$

Cal. $Z = \frac{0.36 - 0.333}{0.005} = 5.4$

LoS. $\alpha = 5\%$

Critical value of $Z_\alpha = 1.96$

Cal Z > Critical Z_α

$5.4 > 1.96$

H_0 reject

Critical value (Z_α)	Level of significance (α) 1 %	5 %	10 %
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