Chi-Square (χ^2) Test [goodness of fit for Binomial Distribution]

Q.1

Null Hypothesis, H_0 = The Probability of male and female birth are equal

H_0: Prob. of male birth, $p = \frac{1}{2} = q$

We know that

Binomial Distribution

$P(X=x) = {}^nC_x \, p^x q^{n-x}$; $x = 0, 1, 2, \ldots, n$

Let x = no. of male birth = 0, 1, 2, 3, 4

$P(X=x) = {}^4C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x}$; $x = 0,1,2,3,4$

$P(X=x) = {}^4C_x \left(\frac{1}{2}\right)^4$; $x = 0,1,2,3,4$

Expected frequencies

$E(X=x) = N \times P(X=x)$

$E(X=x) = 800 \times {}^4C_x \times \left(\frac{1}{2}\right)^4$; $x = 0,1,2,3,4$

$$\boxed{E(X=x) = 50 \times {}^4C_x \ ; \ x = 0,1,2,3,4}$$

$E(X=0) = 50 \times {}^4C_0 = 50 \times 1 = 50$

$E(X=1) = 50 \times {}^4C_1 = 50 \times 4 = 200$

$E(X=2) = 50 \times {}^4C_2 = 50 \times \frac{4 \times 3}{2 \times 1} = 300$

$E(X=3) = 50 \times {}^4C_3 = 50 \times \frac{4 \times 3 \times 2}{3 \times 2 \times 1} = 200$

$E(X=4) = 50 \times {}^4C_4 = 50 \times 1 = 50$

Calculation of χ^2

x	f_o	f_e	$(f_o - f_e)$	$(f_o - f_e)^2$	$\frac{(f_o - f_e)^2}{f_e}$
0	32	50	−18	324	324/50 = 6.48
1	178	200	−22	484	484/200 = 2.42
2	290	300	−10	100	100/300 = 0.33
3	236	200	36	1296	1296/200 = 6.48
4	64	50	14	196	196/50 = 3.92
	Σf_o = 800	Σf_e = 800			$\Sigma \frac{(f_o - f_e)^2}{f_e} = 19.63$

$\chi^2 = \Sigma \left[\frac{(f_o - f_e)^2}{f_e}\right] = 19.63$

d.f., $\nu = n - 1 = 5 - 1 = 4$

no. of freq

Calculated $\chi^2_{0.05, 4} = 19.63$

$$\boxed{\text{tabulated } \chi^2_{0.05, 4} = 9.488}$$

cal χ^2 > tab χ^2

19.63 > 9.488

H_0 reject

A set of 5 coins is tossed 3200 times and the number of heads appearing each time is noted. The results are given below :

No. of Heads	0	1	2	3	4	5
Frequency	80	570	1100	900	500	50

Test the hypothesis that the coins are unbiased.

Chi-Square (χ^2) Test [goodness of fit for Binomial Distribution]

Q2

Null Hypothesis, H_0 = The coins are unbiased

Prob. of head in single toss, $p = \frac{1}{2}$

$\therefore q = \frac{1}{2}$

We know that Binomial Distribution

$P(X=x) = {}^nc_x\, p^x q^{n-x}$; $x = 0, 1, 2, \dots, n$

Let x = no. of male birth $= 0, 1, 2, 3, 4, 5$

$P(X=x) = {}^5c_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{5-x}$; $x = 0, 1, 2, 3, 4, 5$

$P(X=x) = {}^5c_x \left(\frac{1}{2}\right)^5$; $x = 0, 1, 2, 3, 4, 5$

Expected frequencies

$E(X=x) = N \times P(X=x)$

$E(X=x) = 3200 \times {}^4c_x \times \left(\frac{1}{2}\right)^5$; $x = 0, 1, 2, 3, 4, 5$

$\boxed{E(X=x) = 100 \times {}^4c_x \quad ; x = 0, 1, 2, 3, 4, 5}$

$E(X=0) =$

$E(X=1) =$

$E(X=2) =$

$E(X=3) =$

$E(X=4) =$

$E(X=5) =$

Calculation of χ^2

x	f_o	f_e	$(f_o - f_e)$	$(f_o - f_e)^2$	$\frac{(f_o-f_e)^2}{f_e}$
0	80				
1	570				
2	1100				
3	900				
4	500				
5	50				

$\chi^2 = \Sigma \left[\frac{(f_o - f_e)^2}{f_e}\right] =$

d.f., $\nu = n - 1 = 6 - 1 = 5$ no. of freq.

Calculated $\chi^2_{0.05,\,5} =$

$\boxed{\text{tabulated } \chi^2_{0.05,\,5} = 11.07}$

Example 16. *In the accounting department of a bank 100 accounts are selected at random and examined for errors. The following results have beer obtained.*

No. of Errors :	*0*	*1*	*2*	*3*	*4*	*5*	*6*
No. of Accounts :	*36*	*40*	*19*	*2*	*0*	*2*	*1*

Does this information verify that the errors are distributed according to Poisson probability law ?

Chi-Square (χ^2) Test [goodness of fit for Poisson Distribution]

Q.1

Null Hypothesis, H_0 = The error are distributed according to Poisson Probability Law.

We know that

Poisson Distribution

$$P(X=x) = \frac{e^{-m} m^x}{x!} \; ; \; x = 0, 1, 2, \ldots$$

$$P(X=x) = \frac{e^{-1} 1^x}{x!} \; ; \; x = 0, 1, 2, 3, 4, 5, 6$$

Expected frequencies

$$E(X=x) = N \times P(X=x)$$

$$E(X=x) = 100 \times \frac{e^{-1} 1^x}{x!} \; ; \; x = 0, 1, 2, 3, 4, 5, 6$$

$$E(X=x) = 36.787 \times \frac{1^x}{x!} \; ; \; x = 0, 1, 2, 3, 4, 5, 6$$

$$E(X=0) = 36.787 \times \frac{1^0}{0!} = 36.787$$

$$E(X=1) = 36.787 \times \frac{1^1}{1!} = 36.787$$

$$E(X=2) = 36.787 \times \frac{1^2}{2!} = 18.39$$

mean, $\bar{x} = m = \frac{\Sigma fx}{\Sigma f} = \frac{100}{100} = 1$

$e = 2.71828$

Calculation of χ^2

x	f_0	fx	f_e	$(f_0 - f_e)$	$(f_0 - f_e)^2$	$(f_0 - f_e)^2/f_e$
0	36	0	37	−1		
1	40	40	37	3		
2	19	38	18	1		
3	2	6	6			
4	0	0	1.53			
5	2	10	0.3			
6	1	6	0.051			
	$\Sigma f_0 = 100$	$\Sigma fx = 100$				

$$\chi^2 = \Sigma\left[\frac{(f_0 - f_e)^2}{f_e}\right] =$$

no. of freq

d.f., $\nu = n -$

Calculated $\chi^2_{0.05}$, =

$\boxed{\text{tabulated } \chi^2_{0.05}, \; =}$

Chi-Square (χ^2) Test [goodness of fit for Poisson Distribution]

Q.1

Null Hypothesis, H_0 = The error are distributed according to Poisson Probability Law.

We know that

Poisson Distribution

$$P(X=x) = \frac{e^{-m} m^x}{x!} \; ; \; x = 0, 1, 2, \ldots$$

$$P(X=x) = \frac{e^{-1} 1^x}{x!} \; ; \; x = 0, 1, 2, 3, 4, 5, 6$$

Expected frequencies

$$E(X=x) = N \times P(X=x)$$

$$E(X=x) = 100 \times \frac{e^{-1} 1^x}{x!} \; ; \; x = 0,1,2,3,4,5,6$$

$$E(X=x) = 36.787 \times \frac{1^x}{x!} \; ; \; x = 0,1,2,3,4,5,6$$

$$E(X=0) = 36.787 \times \frac{1^0}{0!} = 36.787 \simeq 37$$

$$E(X=1) = 36.787 \times \frac{1^1}{1!} = 36.787 \simeq 37$$

$$E(X=2) = 36.787 \times \frac{1^2}{2!} = 18.39 \sim 18$$

$$E(X=3) = 36.787 \times \frac{1^3}{3!} = \frac{36.787}{6} = 6.13 \simeq 6$$

$$E(X=4) = 36.787 \times \frac{1^4}{4!} = \frac{36.787}{24} = 1.53$$

$$E(X=5) = 36.787 \times \frac{1^5}{5!} = \frac{36.787}{120} = 0.3$$

$$E(X=6) = 36.787 \times \frac{1^6}{6!} = \frac{36.787}{720} = 0.051$$

Calculation of χ^2

x	f_o	fx	f_e	$(f_o - f_e)$	$(f_o - f_e)^2$	$(f_o - f_e)^2/f_e$
0	36	0	37	−1	1	1/37 = 0.027
1	40	40	37	3	9	9/37 = 0.243
2	19	38	18	1	1	1/18 = 0.055
3	2 } 5	6	6 } 7.8 ≈ 8	−3	9	9/8 = 1.125
4	0	0	1.53			
5	2	10	0.3			
6	1	6	0.051			
	Σf_o = 100	Σfx = 100				$\Sigma \frac{(f_o - f_e)^2}{f_e}$ = 1.45

$$\chi^2 = \Sigma\left[\frac{(f_o - f_e)^2}{f_e}\right] = 1.45$$

no. of freq

d.f., $\nu = n - 2 = 7 - 2 = 5 - 3 = 2$

Calculated $\chi^2_{0.05, 2} = 1.45$

$$\boxed{\text{tabulated } \chi^2_{0.05, 2} = 5.991}$$

Cal χ^2 < tab χ^2

1.45 < 5.991

Ho accept

4. A sample analysis of examination results of 500 students was made . it was found that 220 students had failed, 170 had secures a third class, 90 were placed in second class, and 20 got a first class. Are these figures commensurate with the general examination results which is the ratio of 4:3:2:1 for the various categories respectively. (For 3 d.f. the value of $\chi^2_{0.05} = 7.815$).

Chi-Square (χ^2) Test [goodness of fit

Q.1

Null Hypothesis, H_0 = Data support the theory.

$$\frac{F}{4} = \frac{III}{3} = \frac{II}{2} = \frac{I}{1} = \frac{F+III+II+I}{4+3+2+1} = \frac{500}{10}$$

Under Null Hypothesis, expected frequencies are given by

$$E(F) = E(220) = \frac{4 \times 50\cancel{0}}{1\cancel{0}} = 200$$

$$E(III) = E(170) = \frac{3 \times 50\cancel{0}}{1\cancel{0}} = 150$$

$$E(II) = E(90) = \frac{2 \times 50\cancel{0}}{1\cancel{0}} = 100$$

$$E(I) = E(20) = \frac{1 \times 50\cancel{0}}{1\cancel{0}} = 50$$

f_o	f_e	$f_o - f_e$	$(f_o - f_e)^2$	$(f_o - f_e)^2/f_e$
220	200	20	400	400/200 = 2
170	150	−20	400	400/150 = 2.66
90	100	−10	100	100/100 = 1
20	50	−30	900	900/50 = 18
$\Sigma f_o = 500$	$\Sigma f_e = 500$			$\Sigma \frac{(f_o - f_e)^2}{f_e} = 23.66$

$$\chi^2 = \Sigma\left[\frac{(f_o - f_e)^2}{f_e}\right] = 23.66$$

d.f., $\nu = n - 1 = 4 - 1 = 3$

Calculated $\chi^2_{0.05,3} = 23.66$

Tabulated $\chi^2_{0.05,3} = 7.81$

cal χ^2 > tab χ^2

23.66 > 7.81

H_0 reject

Chi-Square test [one tailed & two tailed test for Single Variance]

Chi-Square test is a non-parametric test.

For testing variability of a population

we Assume population is Normally distributed.

Parametric test

test statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

$n =$ sample size

$s^2 =$ sample variance

$\sigma^2 =$ population variance.

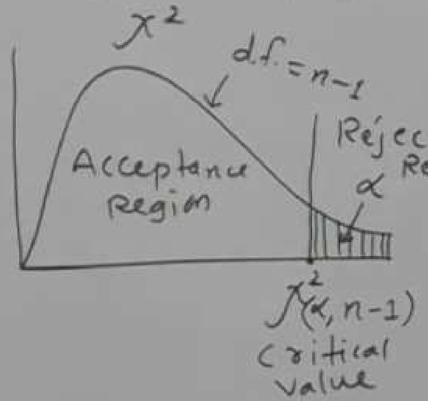
$H_0: \sigma^2 = \sigma_0^2$

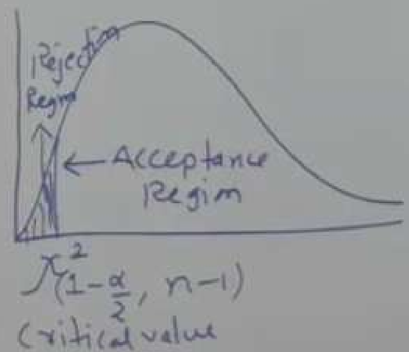
$H_1: \sigma^2 > \sigma_0^2$ (Right tail or upper tail)

or $H_1: \sigma^2 < \sigma_0^2$ (Left tail or Lower tail)

or $H_1: \sigma^2 \neq \sigma_0^2$ (two tailed)

$\boxed{LoS = \alpha}$





Area in right tail = 1 - Area in left tail

Chi-Square test [one tailed & two tailed test for single variance]

chi-square test is a non-parametric test.

For testing variability of a population

we Assume population is Normally distributed.

Parametric test

test statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

n = sample size

s^2 = sample variance

σ^2 = population variance.

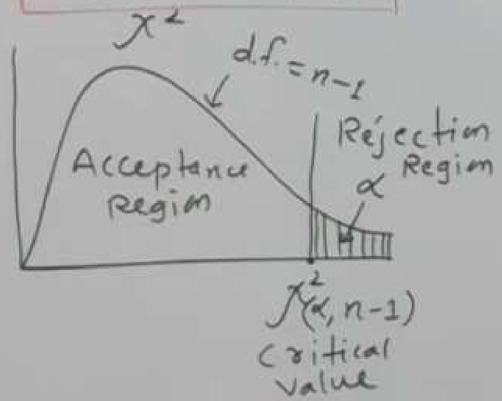
$H_0: \sigma^2 = \sigma_0^2$

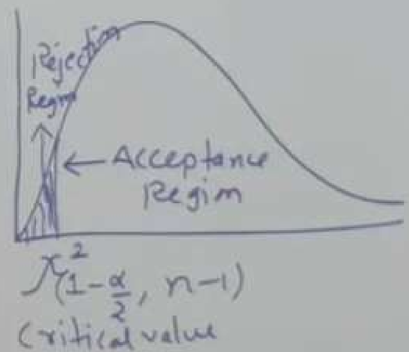
$H_1: \sigma^2 > \sigma_0^2$ (Right tail or upper tail)

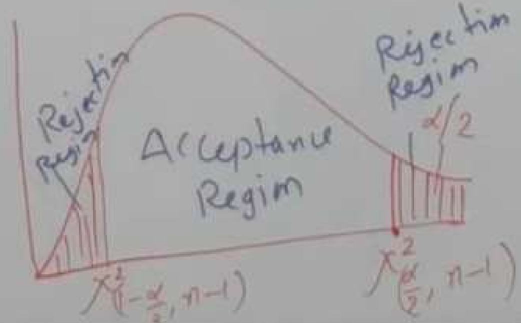
or $H_1: \sigma^2 < \sigma_0^2$ (Left tail or Lower tail)

or $\boxed{H_1: \sigma^2 \neq \sigma_0^2}$ (two tailed)

$\boxed{\text{LOS} = \alpha}$







EXAMPLE 31. It is assumed that the monthly realization of VAT in a state is normally distributed with a variance of Rs. 150 crores. A sample of 20 months was taken and the variance turned out to be Rs. 170 crores. Test at 5% level of significance whether the variance is significantly different than Rs. 150 crores?

Chi-Square test [one tailed & two tailed test for Single Variance]

Q.1

Here, $n = 20$

Level of significance, $\alpha = 0.05$

$S^2 = 170$

$H_0: \sigma^2 = 150$

$H_1: \sigma^2 \neq 150$ [two tailed test]

test statistic

$$\chi^2 = \frac{(n-1)\, s^2}{\sigma^2}$$

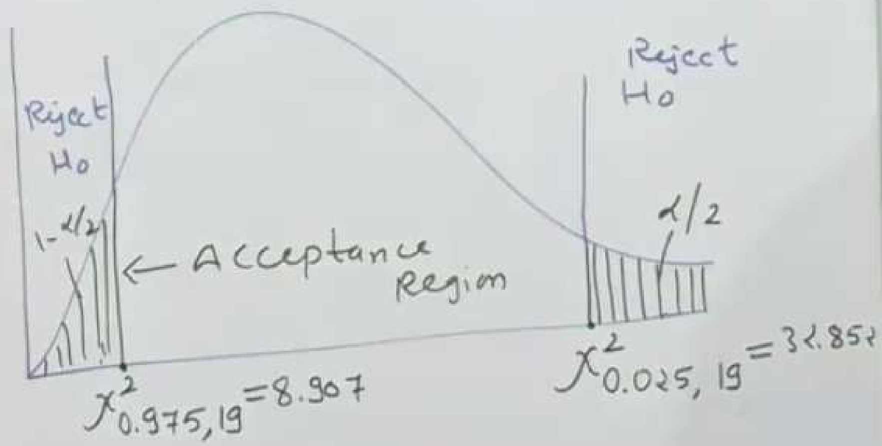
$$\chi^2 = \frac{(20-1) \times 170}{150} = 21.533$$

$$d.f. = n-1 = 20-1 = 19$$

$$\boxed{\text{Cal. } \chi^2_{0.05,\,19} = 21.533}$$

$$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025, \quad 1 - \frac{\alpha}{2} = 1 - 0.025 = 0.975$$

do not reject H_0



EXAMPLE 30. A factory owner agrees to purchase the product of a brand, if the items do not have variance more than 5 mm^2 in their length. To make sure, the owner selects a sample of 9 items from the lot. The length of each item is measured. Their values in mm are as follows:

185, 183, 187, 183, 182, 185, 183, 182, 186.

Test whether the factory owner should purchase the product. Use 5% level of significance.

Chi-Square test [one tailed & two tailed test for single variance]

Q.2

x	$(x-184)$ $(x-\bar{x})$	$(x-\bar{x})^2$
185	1	1
183	−1	1
187	3	9
183	−1	1
182	−2	4
185	1	1
183	−1	1
182	−2	4
186	2	4
$\Sigma x = 1656$		$\Sigma(x-\bar{x})^2 = 26$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{1656}{9} = 184$$

$H_0: \sigma^2 = 5$

$H_1: \sigma^2 > 5$ (Right tailed upper)

$$s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1}$$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{26}{5} = 5.2$$

LOS, $\alpha = 0.05$

d.f. $= n-1 = 9-1 = 8$

$$\boxed{\text{Calculated } \chi^2_{0.05,8} = 5.2}$$

do not reject H_0