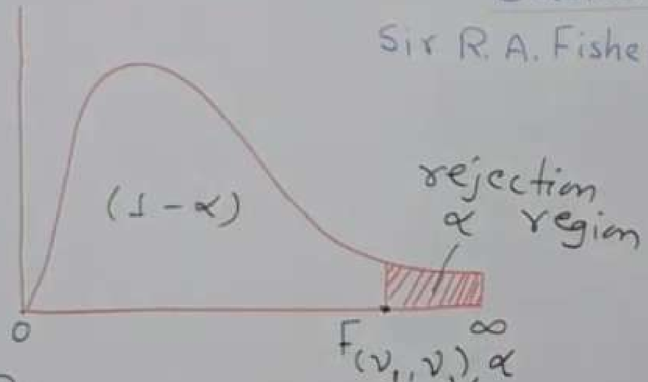
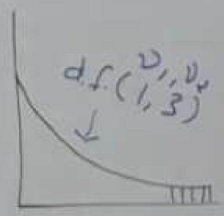
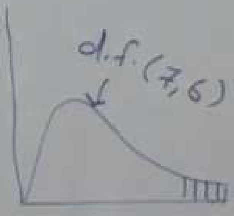
# F-Distribution

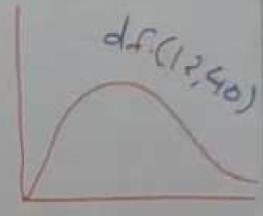
## Basics & Properties

Sir R. A. Fisher.




## Properties

① Values of F ranges from 0 to $\infty$

② It has two parameter. $\nu_1$ and $\nu_2$, the no. of two d.f.

③ Continuous distribution & never touches x-axis.

④ For different pairs of d.f. $\nu_1$ and $\nu_2$, there will be different F-distribution.

⑤ Positively Skewed and as d.f. increases curves become symmetric.

$$F_{(\nu_1, \nu_2)} = \frac{S_1^2}{S_2^2} \qquad S_1^2 > S_2^2$$

we know that

$$\frac{\nu_1 S_1^2}{\sigma_1^2} \sim \chi_1^2 \quad \text{where } \nu_1 = n_1 - 1$$

$$\frac{\nu_2 S_2^2}{\sigma_2^2} \sim \chi_2^2 \quad \text{where } \nu_2 = n_2 - 1$$

The ratio $$\frac{\frac{S_1^2}{\sigma_1^2}}{\frac{S_2^2}{\sigma_2^2}} = \frac{\chi_1^2/\nu_1}{\chi_2^2/\nu_2} = F_{(\nu_1, \nu_2)}$$

under $H_0: \sigma_1^2 = \sigma_2^2$ then

# F-Distribution

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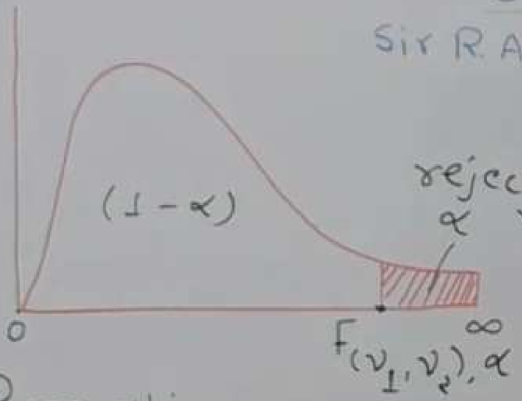



### Properties

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③ Continuous distribution & never touches x-axis.

④ For different pairs of d.f. $\nu_1$ and $\nu_2$, there will be different F-distribution.

⑤ Positively Skewed and as d.f. increases curves become symmetric.

$$\text{mean} = \frac{\nu_2}{\nu_2 - 2} \; ; \; (\nu_2 > 2)$$

mean does not exist if $\nu_2 \leq 2$

$$\text{variance} = \frac{2\nu_2^2(\nu_1 + \nu_2 - 2)}{\nu_1(\nu_1 - 2)^2(\nu_2 - 4)} \; ; \; \nu_2 > 4$$

# F-test [Introduction & Formula]

**Population I**
Varianu = $\sigma_1^2$

**Population II**
Variance = $\sigma_2^2$

↓

**Sample I**
Size = $n_1$
Varianu = $s_1^2$

**Sample II**
Size = $n_2$
Variance = $s_2^2$

**Objective:-**

1) To test significance of equality between two population variance.
2) To test significance of equality among three or more means.

Null Hypothesis $H_0$: $\sigma_1^2 = \sigma_2^2$

Alternative Hypothesis $H_1$: $\sigma_1^2 > \sigma_2^2$

## F-test Statistics

$$F_{(\nu_1, \nu_2)} = \frac{s_1^2}{s_2^2} \quad ; \quad s_1^2 > s_2^2$$

Cal. $F_{\alpha, \nu_1, \nu_2}$ > tab. $F_{\alpha, \nu_1, \nu_2}$

$H_0$ reject.

Cal. F < tab F

$H_0$ accept.

| $\nu_2 \downarrow$ \ $\nu_1 \rightarrow$ | 5 | 6 | 7 | 8 | 9 | 10 - - - - |
|---|---|---|---|---|---|---|
| 5 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.72 |
| 6 | | | | | | |
| 7 | | | | | | |
| 8 | | | | | | |
| 9 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 |
| ⋮ | | | | | | |

# F-test [Introduction & Formula]

Population I
Variance $= \sigma_1^2$
↓
Sample I
Size $= n_1$
Variance $= s_1^2$

Population II
Variance $= \sigma_2^2$
↓
Sample II
Size $= n_2$
Variance $= s_2^2$

Objective:-

1) To test significance of equality between two population variance.
2) To test significance of equality among three or more means.

Null Hypothesis $H_0: \sigma_1^2 = \sigma_2^2$

Alternative Hypothesis $H_1: \sigma_1^2 > \sigma_2^2$

## F-test Statistics

$$F_{(\nu_1, \nu_2)} = \frac{s_1^2}{s_2^2} \; ; \; s_1^2 > s_2^2$$

$$s_1^2 = \frac{\Sigma (x_1 - \bar{x}_1)^2}{n_1 - 1}$$

$$s_2^2 = \frac{\Sigma (x_2 - \bar{x}_2)^2}{n_2 - 1}$$

$\nu_1 = d.f. = n_1 - 1$

$\nu_2 = d.f. = n_2 - 1$

| $\nu_2 \downarrow$ \ $\nu_1 \rightarrow$ | 5 | 6 | 7 | 8 | 9 | 10 ---- |
|---|---|---|---|---|---|---|
| 5 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.72 |
| 6 | | | | | | |
| 7 | | | | | | |
| 8 | | | | | | |
| 9 | 3.48 | 3.37 | 3.29 | 3.23 | | 3.14 |
| ⋮ | | | | | | |

1. In a test given to two groups of students drawn from two normal populations, the marks obtained were as follows:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Group A | 18 | 20 | 36 | 50 | 49 | 36 | 34 | 49 | 41 |
| Group B | 29 | 28 | 26 | 35 | 30 | 44 | 46 | | |

Examine at 5% level of significance, whether two populations have the same variance.

# F-test [Numerical Problems]

Q1

Null Hypothesis, $H_0 : \sigma_1^2 = \sigma_2^2$

Alternative Hypothesis, $H_1 : \sigma_2^2 \neq \sigma_2^2$

(Two-tailed)

## Calculation of test statistic

| Group A | | | Group B | | |
|---|---|---|---|---|---|
| $X_1$ | $(X_1-\bar{X}_1)$ $(X_1-37)$ | $(X_1-\bar{X}_1)^2$ | $X_2$ | $X_2-\bar{X}_2$ $(X_2-34)$ | $(X_2-\bar{X}_2)^2$ |
| 18 | −19 | 361 | 29 | −5 | 25 |
| 20 | −17 | 289 | 28 | −6 | 36 |
| 36 | −1 | 1 | 26 | −8 | 64 |
| 50 | 13 | 169 | 35 | 1 | 1 |
| 49 | 12 | 144 | 30 | −4 | 16 |
| 36 | −1 | 1 | 44 | 10 | 100 |
| 34 | −3 | 9 | 46 | 12 | 144 |
| 49 | 12 | 144 | | | |
| 41 | 4 | 16 | | | |
| $\Sigma X_1 = 333$ | | $\Sigma(X_1-\bar{X}_1)^2 = 1134$ | $\Sigma X_2 = 238$ | | $\Sigma(X_2-\bar{X}_2)^2 = 386$ |

$$\bar{X}_1 = \frac{\Sigma X_1}{n_1} = \frac{333}{9} = 37$$

$$\bar{X}_2 = \frac{\Sigma X_2}{n_2} = \frac{238}{7} = 34$$

$$\text{Variance, } s_1^2 = \frac{\Sigma(X_1-\bar{X}_1)^2}{n_1-1} = \frac{1134}{9-1}$$

$$\boxed{S_1^2 = 141.75} \quad \text{d.f., } \nu_1 = n_1 - 1 = 8$$

$$\text{Variance, } s_2^2 = \frac{\Sigma(X_2-\bar{X}_2)^2}{n_2-1} = \frac{386}{7-1}$$

$$\boxed{S_2^2 = 64.33} \quad \text{d.f., } \nu_2 = n_2 - 1 = 6$$

### F-test

$$F_{(8,6)} = \frac{S_1^2}{S_2^2} = \frac{141.75}{64.33} = 2.203$$

LOS, $\alpha = 0.05$, Cal. $F_{0.05(8,6)} = 2.203$

Tabulated $F_{\alpha/2\,(8,6)} = 5.60$

0.025

Cal F < tab F

Ho accept.

# F-test [Numerical Problems]

Q2 In a laboratory experiment samples gave the following results:

| Sample | Size | Sample mean | Sum of square of deviations from mean |
|---|---|---|---|
| 1 | $n_1 = 10$ | $\bar{x}_1 = 15$ | $90 = \Sigma(X_1 - \bar{X}_1)^2$ |
| 2 | $n_2 = 13$ | $\bar{x}_2 = 14$ | $108 = \Sigma(X_2 - \bar{X}_2)^2$ |

Test equality of sample variance at 5% Level of significance.

Solⁿ: $H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 \neq \sigma_2^2$ (two-tailed)

Variance of Iˢᵗ sample

$$s_1^2 = \frac{\Sigma(X_1 - \bar{X}_1)^2}{n_1 - 1} = \frac{90}{10-1} = 10$$

$$\text{d.f.}, \nu_1 = n_1 - 1 = 10 - 1 = 9$$

Variance of IIⁿᵈ sample

$$s_2^2 = \frac{\Sigma(X_2 - \bar{X}_2)^2}{n_2 - 1} = \frac{108}{13-1} = 9$$

$$\text{d.f.}, \nu_2 = n_2 - 1 = 12$$

F-test

$$F_{(9,12)} = \frac{s_1^2}{s_2^2} = \frac{10}{9} = 1.11$$

LOS, $\alpha = 0.05$

Cal. $F_{0.05(9,12)} = 1.11$

Tab. value of $F_{0.025(9,12)} = 3.44$

two-tailed $\alpha/2$

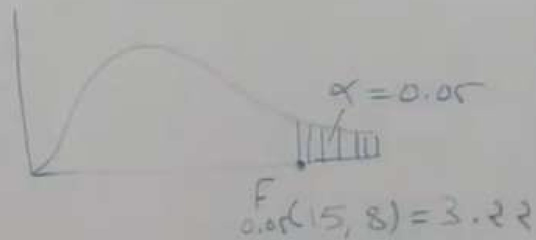
Cal F < tab F

Ho accept.

# F-test [Numerical Problems]

Q3 Following results were obtained from two samples each drawn from two different populations A and B

| | | |
|---|---|---|
| Population | A | B |
| Sample | I | II |
| Sample size | $n_1 = 16$ | $n_2 = 9$ |
| Sample S.D | $S_1 = 3$ | $S_2 = 2$ |




Test the hypothesis that variance of Brand A is more than Brand B (Use $\alpha = 0.05$)

$H_0 : \sigma_A^2 = \sigma_B^2$

$H_1 : \sigma_A^2 > \sigma_B^2$ (Right tailed) one

Variance of sample I

$S_1^2 = 3^2 = 9$

$d.f = n_1 - 1 = 16 - 1 = 15$

Variance of sample II

$S_2^2 = 2^2 = 4$

$d.f = n_2 - 1 = 9 - 1 = 8$

F - test

$$F_{(15,8)} = \frac{S_1^2}{S_2^2} = \frac{9}{4} = 2.25$$

$$\boxed{\text{Cal } F_{0.05(15,8)} = 2.25}$$

Tab. $F_{0.05}(15,8) = 3.22$

Cal F $<$ tab. F

Ho accept