DC transients

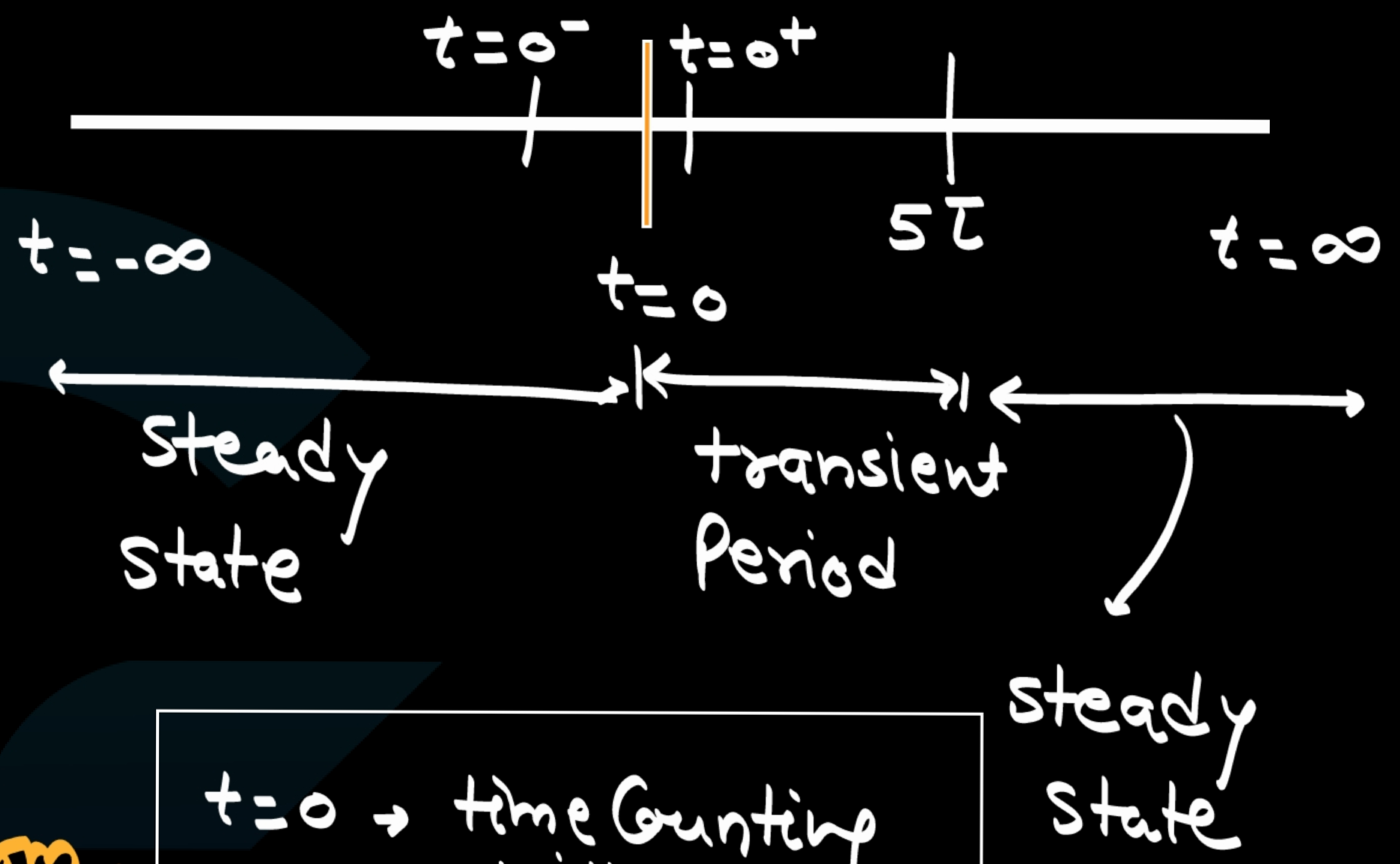
Transients present in any circuit due to change in source magnitude or load elements, and it contains energy storing elements.

Energy storing Element → Inductor → mag. form

Capacitor → Electrostatic form.

In Pure resistive circuit → Transients are not present.

Any energy storing element must present in circuit



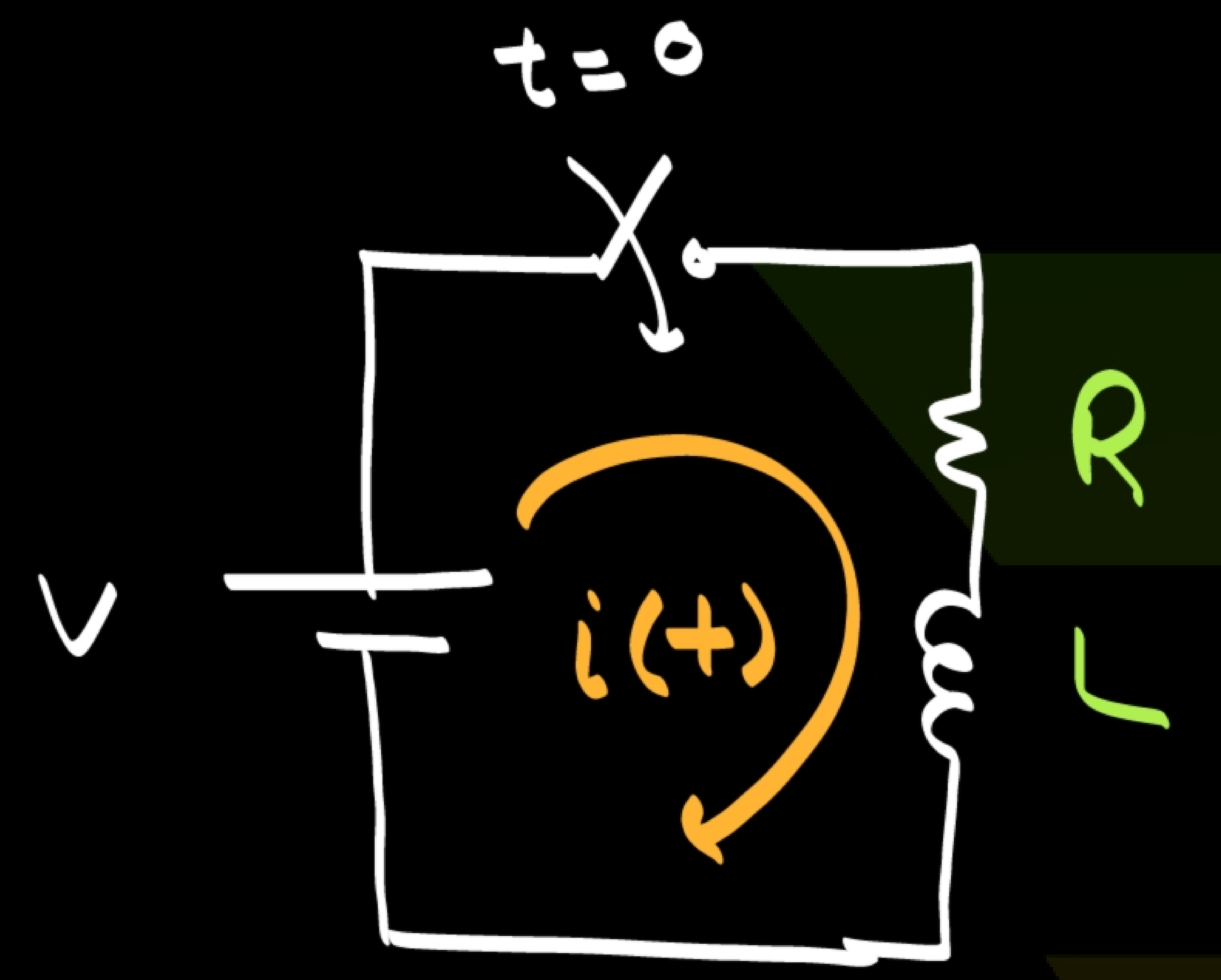
$t=0$ → time Counting initiate

$t=0^+$ → just after

$t=0^-$ → just before.

Case study

Case - I



$t = 0^-$, $i(t) = 0$ Amp

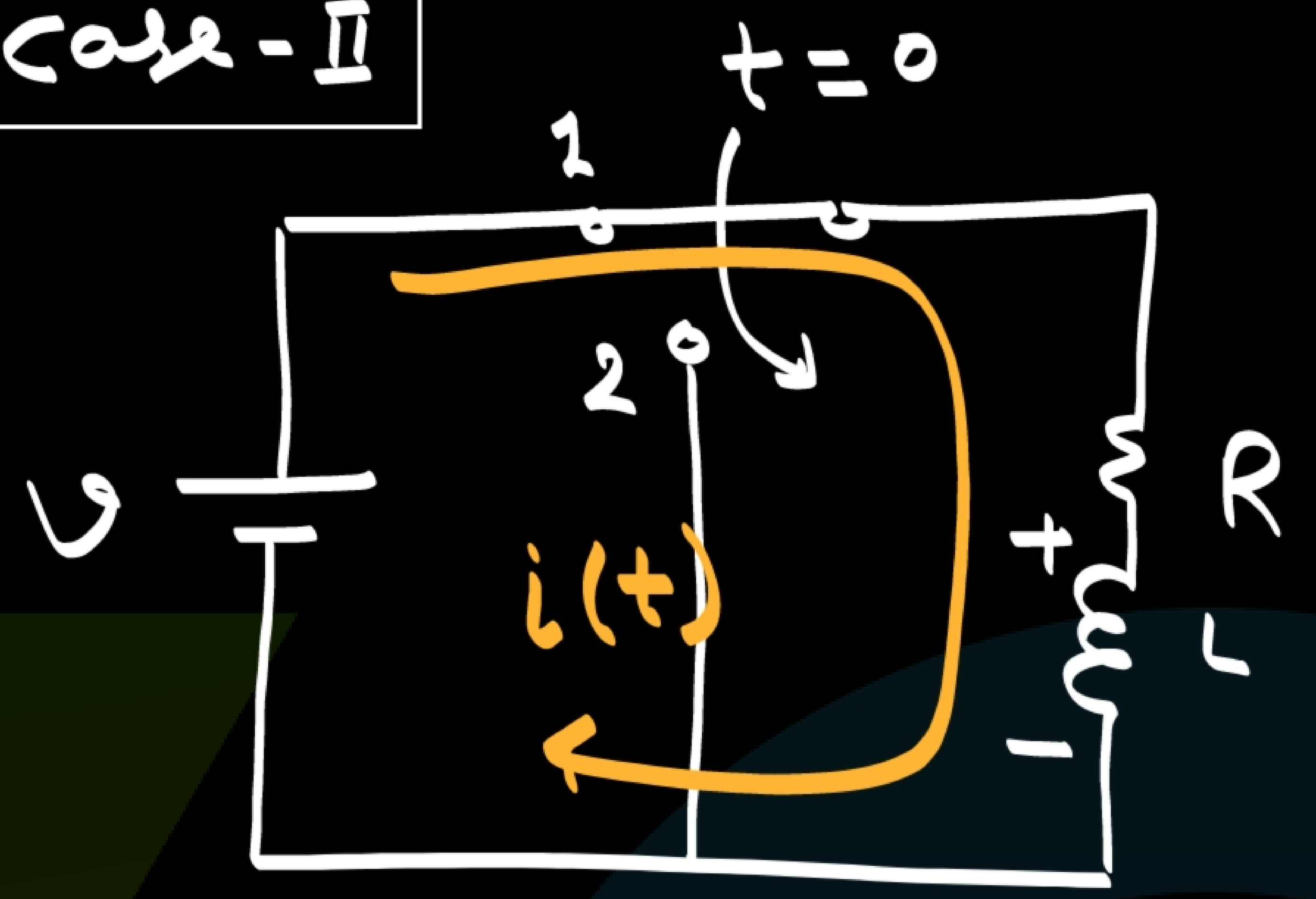
$t = 0^+$, $i(t) = 0$ Amp.

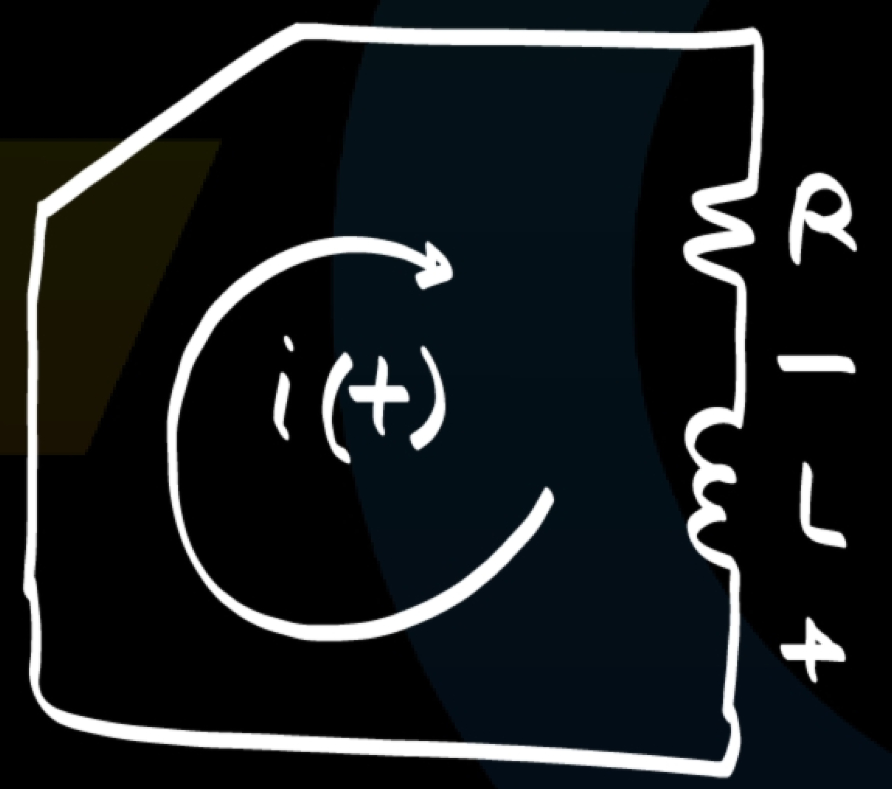
Inductor → open circuit

$t = \infty$, $i(t) = \frac{V}{R}$ Ampere

Inductor → short circuit.

Case - II





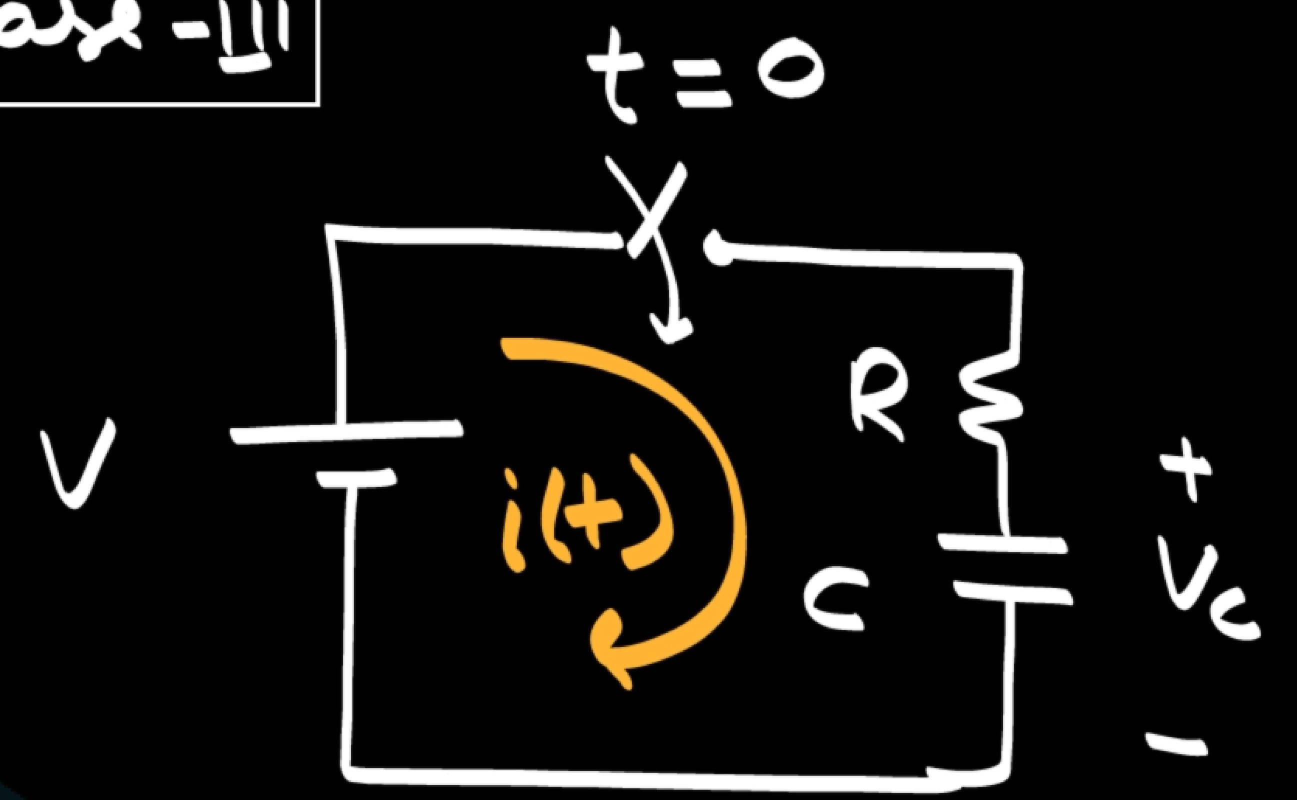
$t = 0^-$, $i(t) = I_0$ ✓

$t = 0^+$, $i(t) = I_0$ ✓

Inductor → Current Source

$t = \infty$, $i(t) = 0$ Amp. ✓

Case - III



At $t = 0^-$, $V_c(t) = 0$ volt

At $t = 0^+$, $V_c(t) = 0$ volt

Capacitor → short circuit.

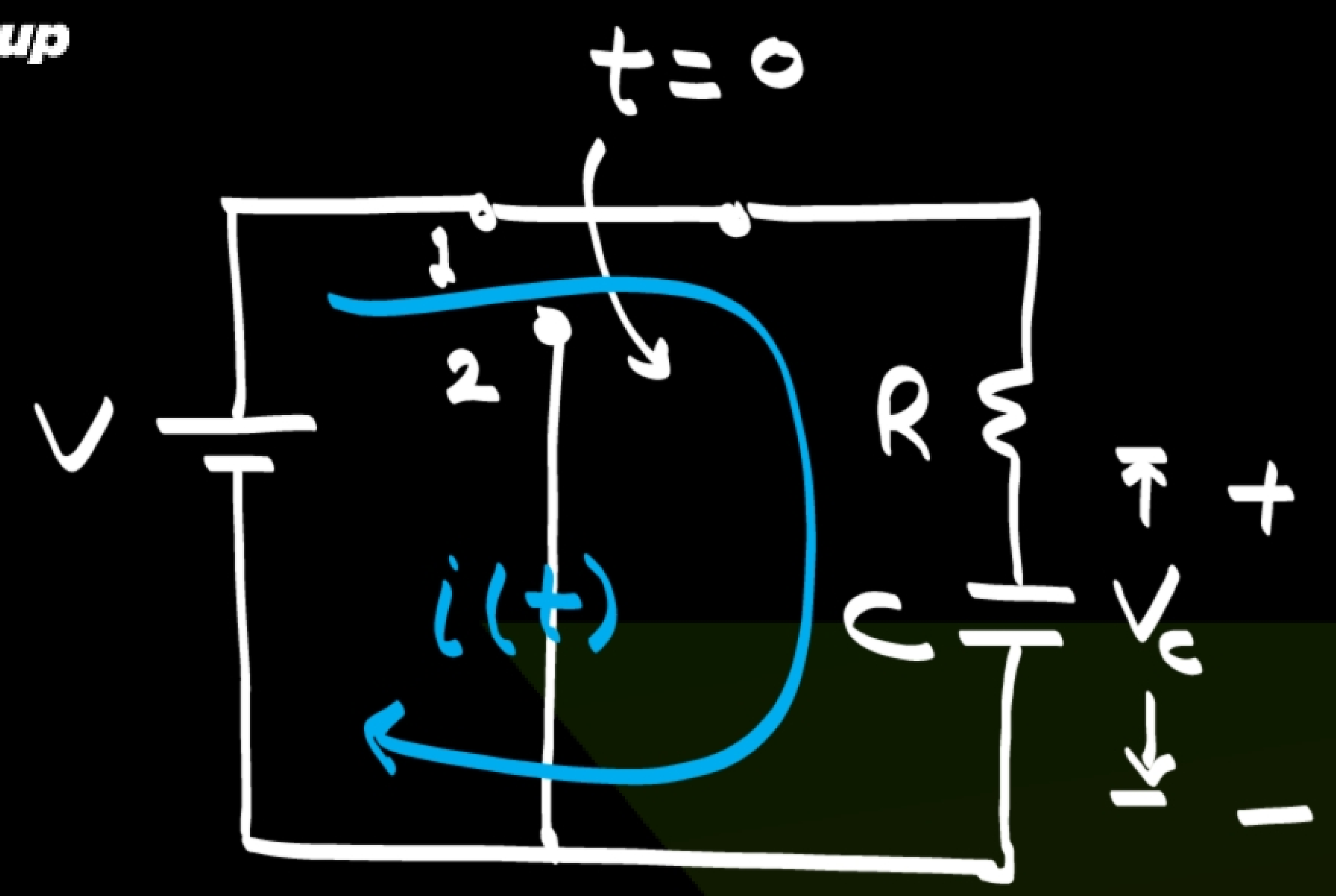
At $t = \infty$, $V_c(t) = V$ volt

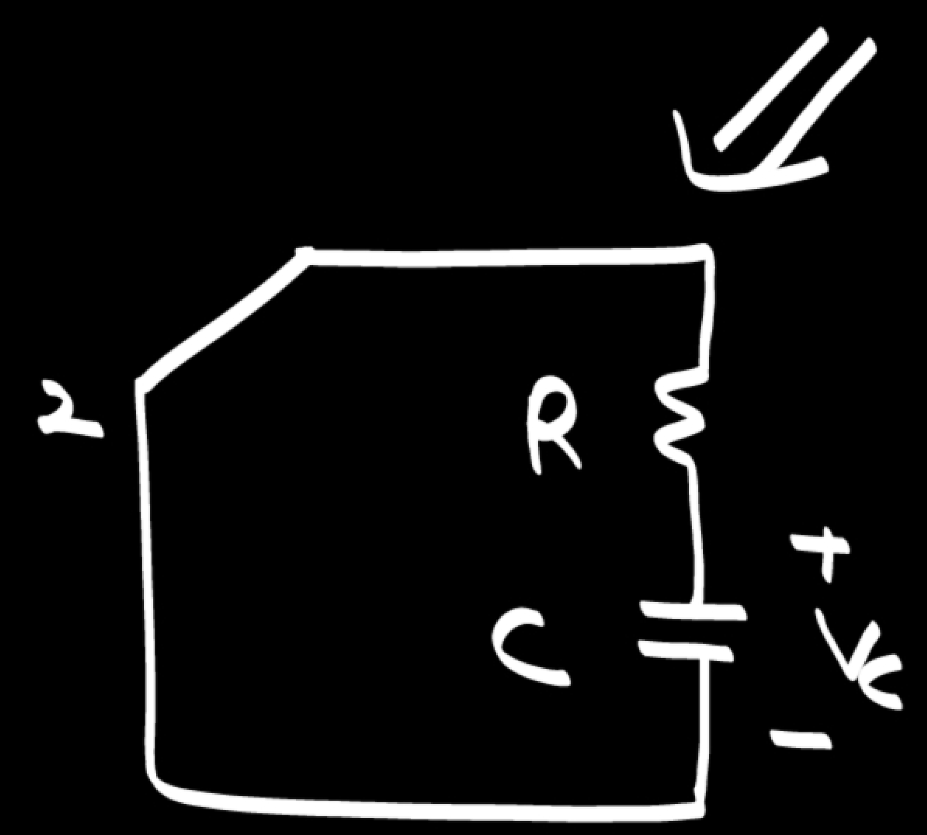
$i(t) = \frac{V - V}{R}$

$i(t) = 0$ Amp

Capacitor → open circuit

Case-IV





Inductor - do not accept change in current.

Capacitor - do not accept change in voltage.

At $t = 0^-$, $\boxed{V_c(t) = V_0}$

$t = 0^+$, $\boxed{V_c(t) = V_0}$

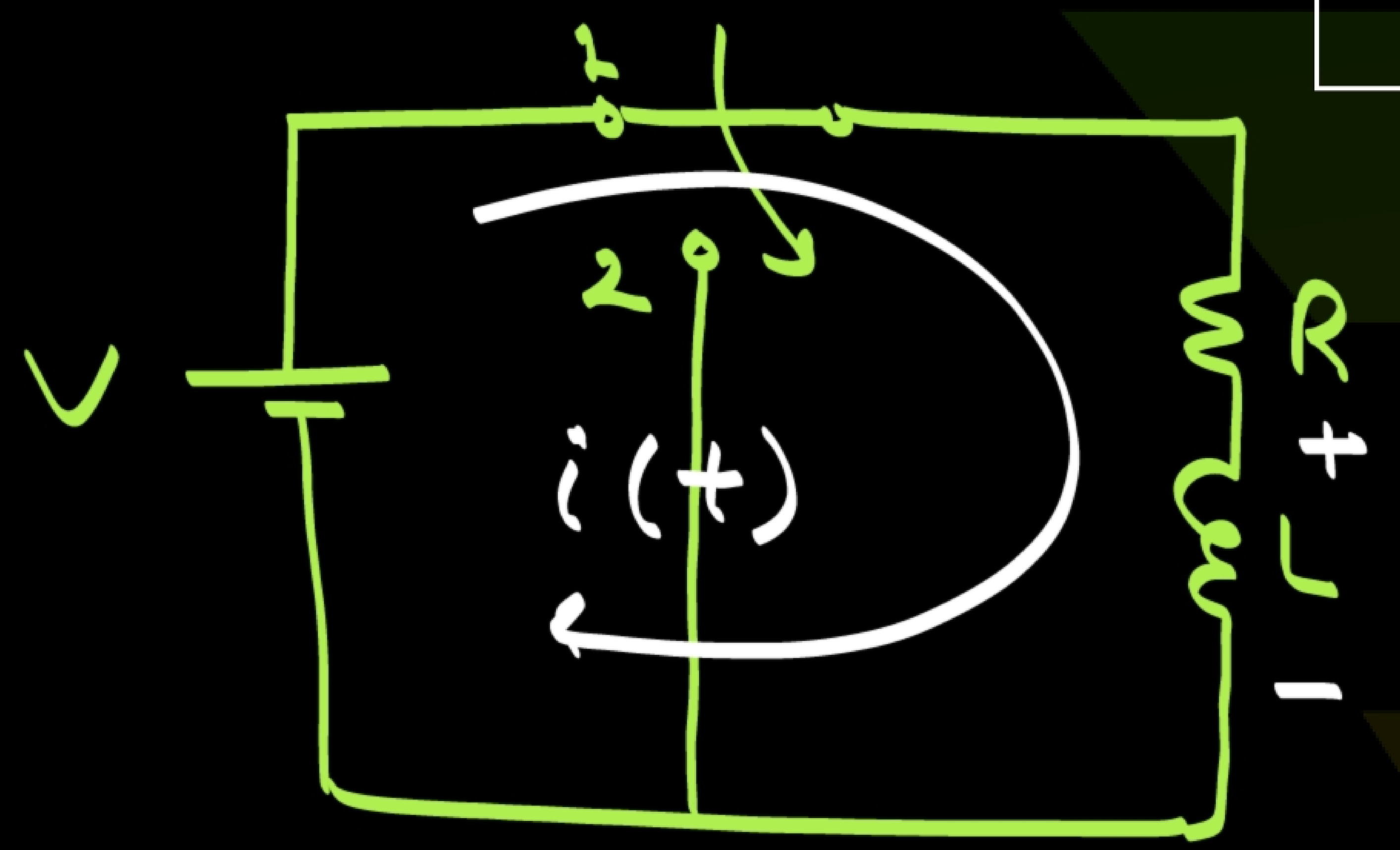
Capacitor → Voltage Source

$t = \infty$, $V_c(t) = 0$

Case - I

Source free RL Circuit

Discharging of Inductor



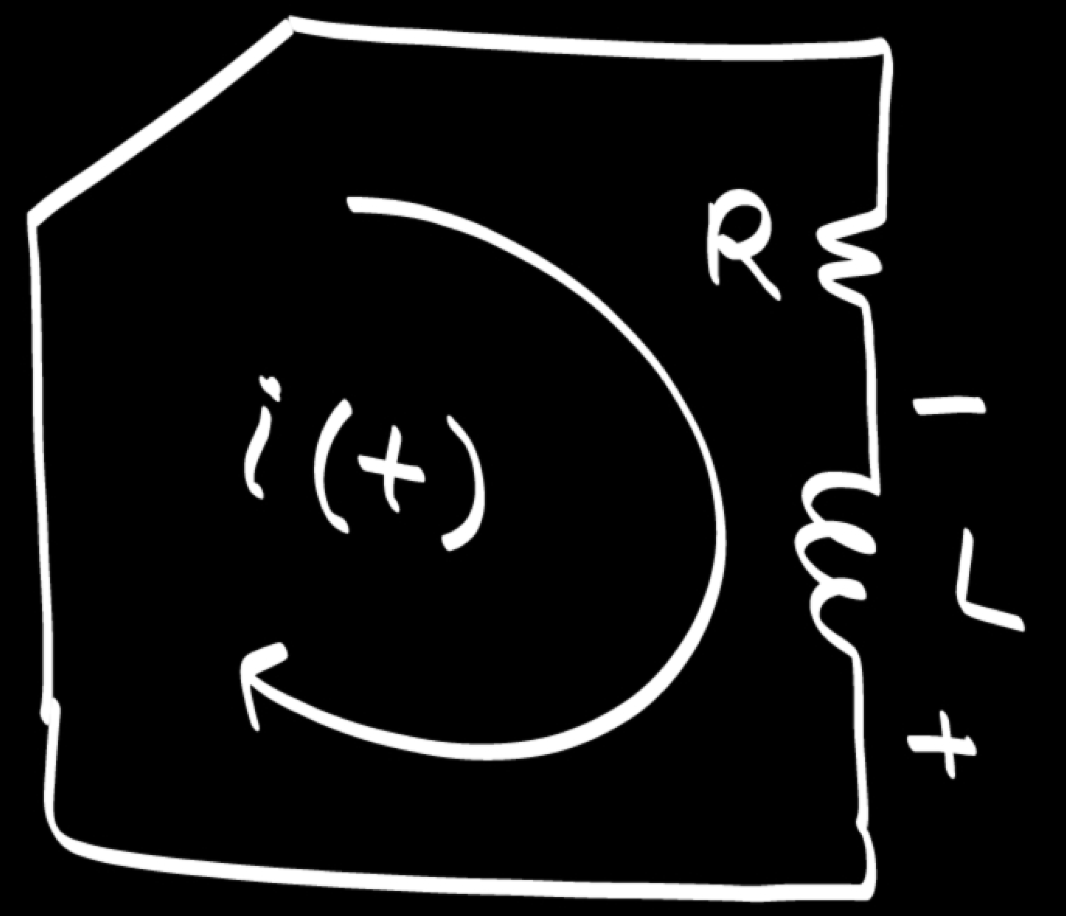
At $t = 0^-$

$i(t) = I_0$

$t = 0^+$,

$i(t) = I_0$

$t = 0 \rightarrow i(t) = I_0$



At $t = 0$, switch is connected to Position 2 →

Apply KVL →

$$R \cdot i(t) + L \cdot \frac{di(t)}{dt} = 0$$

$$R \cdot i(t) = -L \cdot \frac{di(t)}{dt}$$

$$-\frac{R}{L} \cdot dt = \frac{di(t)}{i(t)}$$

$$\int_0^t -\frac{R}{L} \cdot dt = \int_{I_0}^{i(t)} \frac{di(t)}{i(t)}$$

$$-\frac{R}{L} \cdot t = \left[\log i(t)\right]_{I_0}^{i(t)}$$

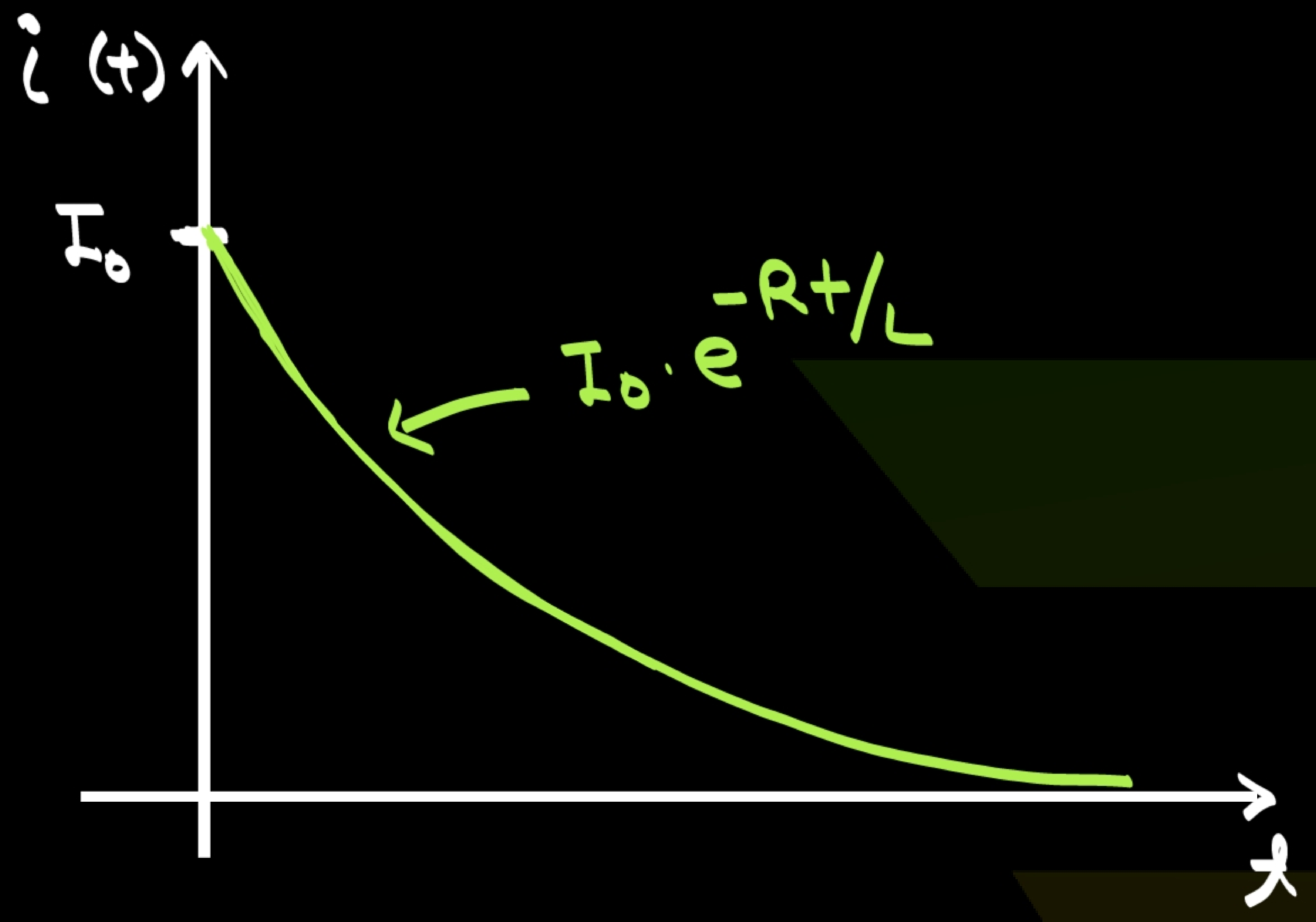
$$-\frac{R}{L} \cdot t = \log(i(t)) - \log(I_0)$$

$$-\frac{R}{L} \cdot t = \log\left(\frac{i(t)}{I_0}\right)$$

take Antilog

$$\frac{i(t)}{I_0} = e^{-Rt/L}$$

$$\boxed{i(t) = I_0 \cdot e^{-Rt/L}}$$



$$\boxed{i(t) = I_o \cdot e^{-Rt/L}}$$

At $t=0$, $\rightarrow \boxed{i(t) = I_o}$

At $t=\infty$, $\rightarrow \boxed{i(t) = 0}$

voltage across inductor -

$$V_L = L \cdot \frac{di(t)}{dt}$$

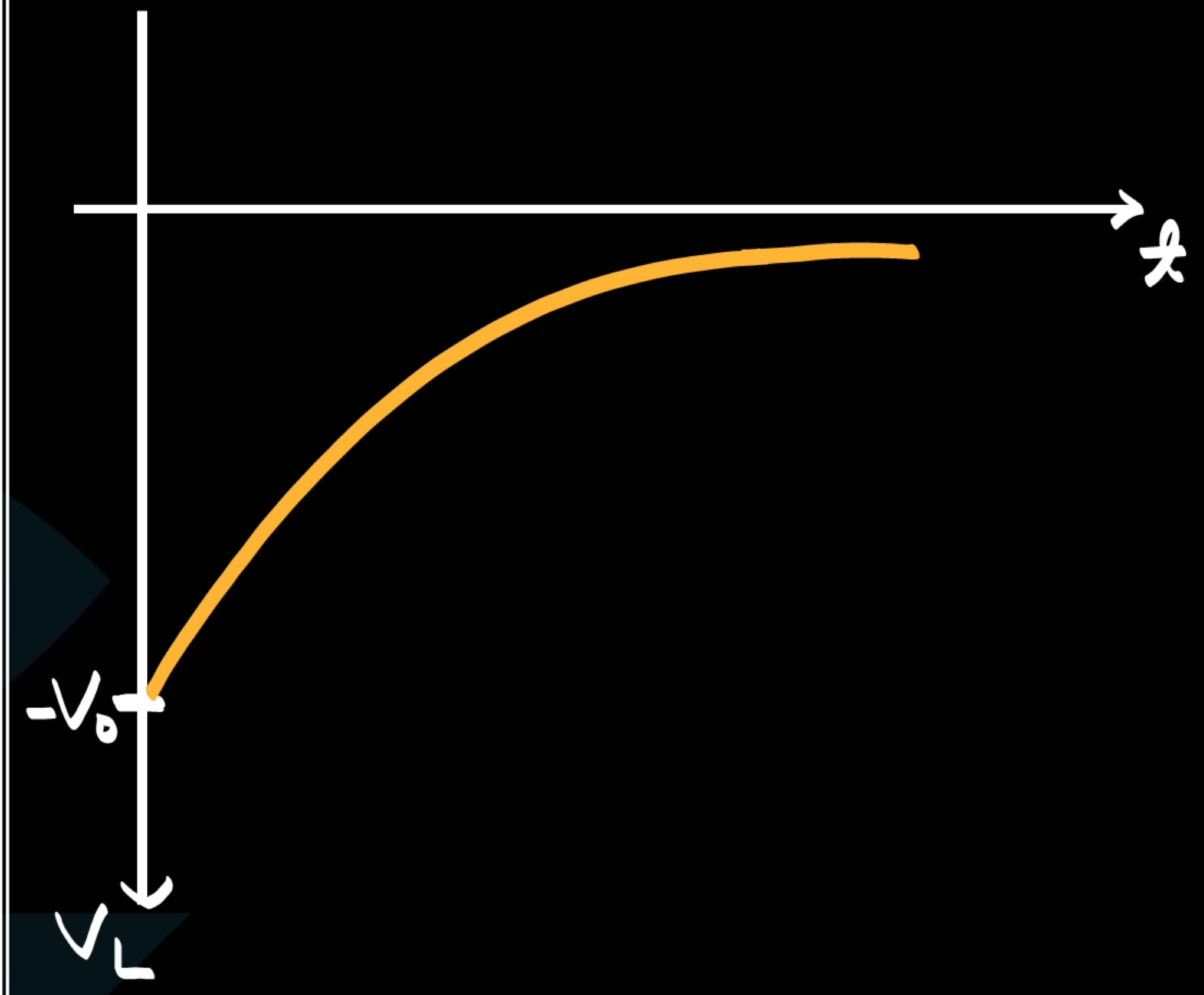
$$V_L = L \cdot \frac{d}{dt} I_o \cdot e^{-Rt/L}$$

$$= I_o \cdot \cancel{L} \cdot \left(-\frac{R}{\cancel{L}}\right) \cdot e^{-Rt/L}$$

$$V_L = -I_o R\, e^{-Rt/L}$$

let $(V_o = I_o R)$

$$\boxed{V_L = -V_o \cdot e^{-Rt/L}}$$



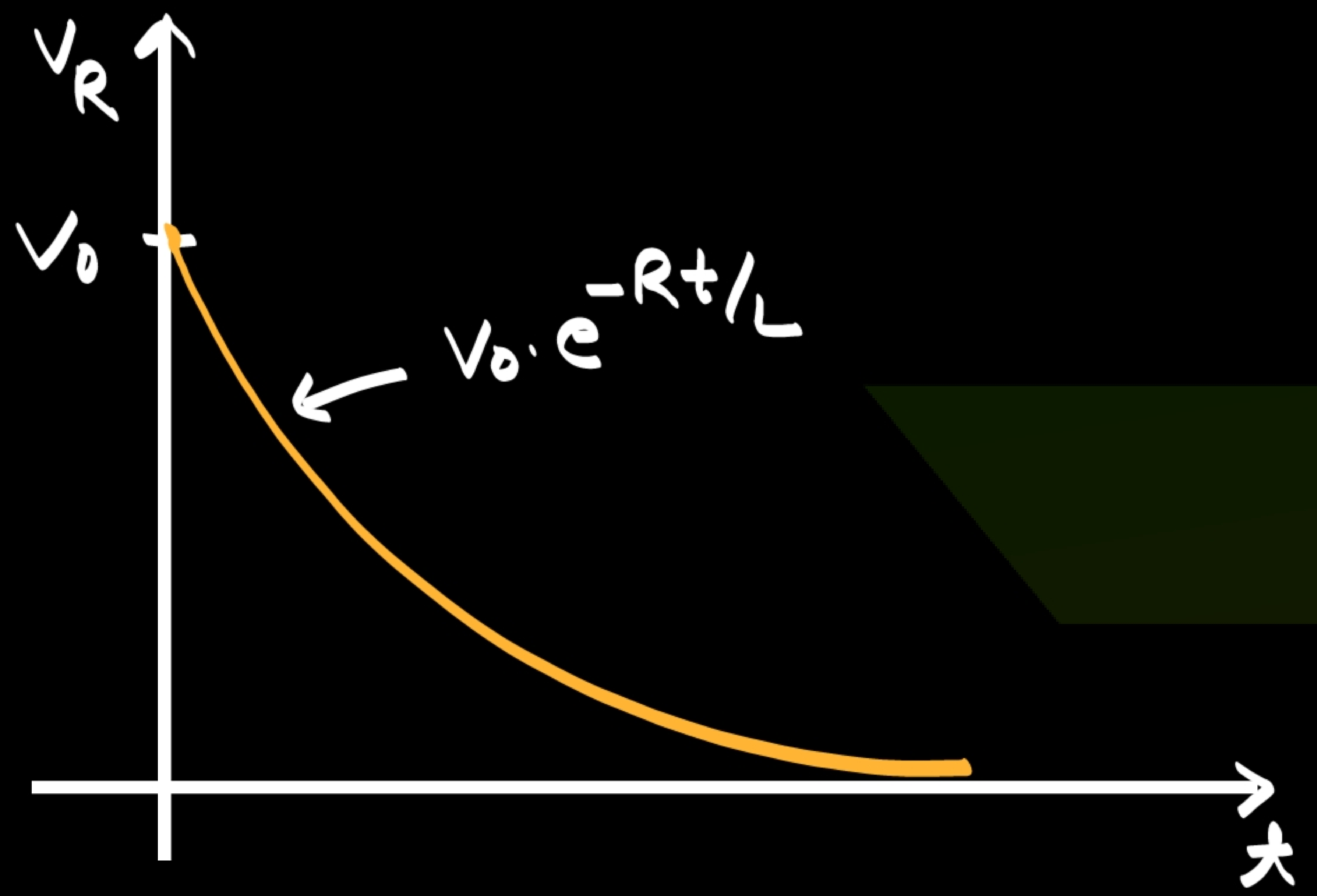
At $t=0$. $V_L = -V_o$

voltage across Resistor

$$V_R = R \cdot I(t)$$

$$= R \cdot I_o \cdot e^{-Rt/L}$$

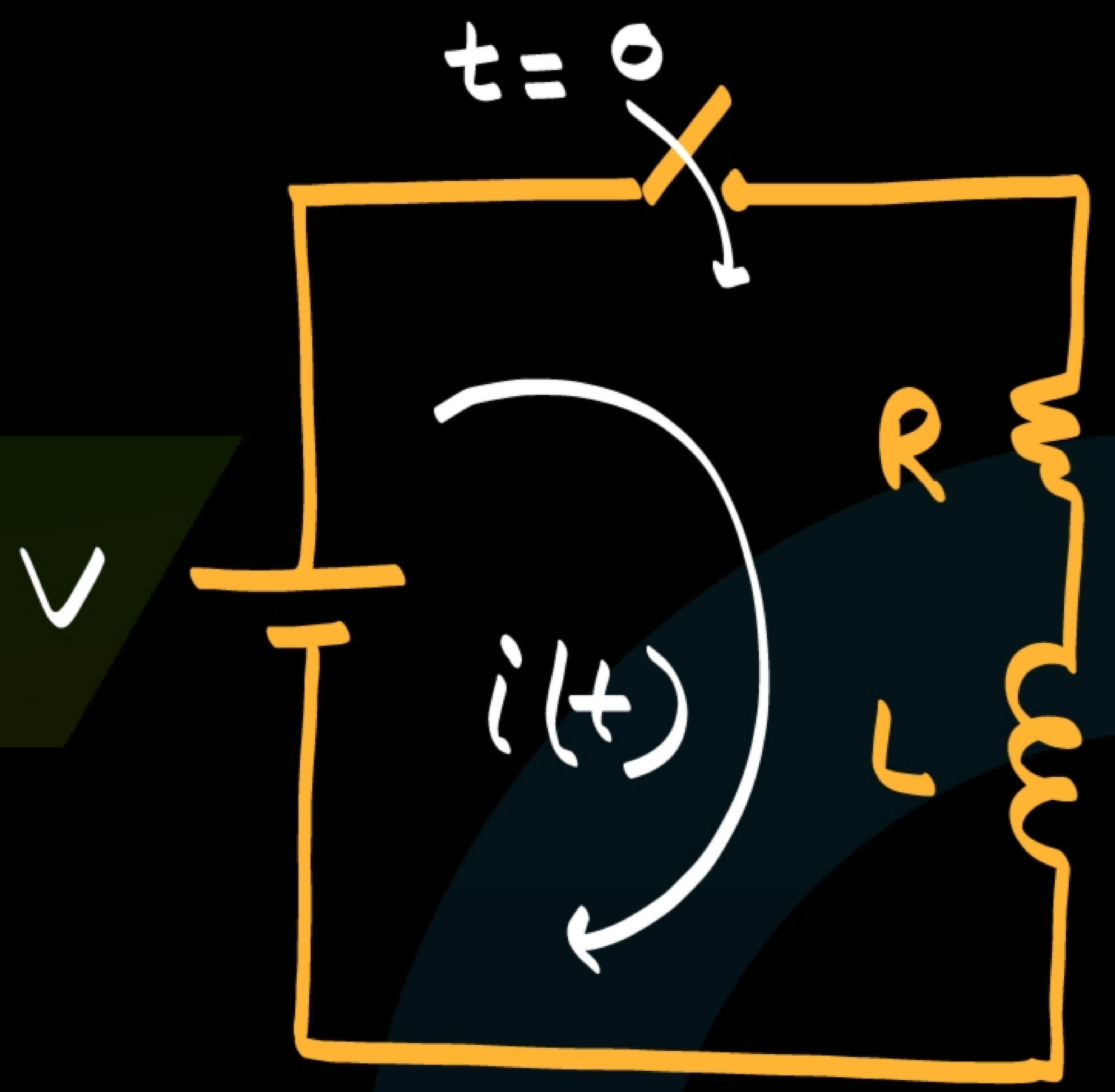
$$\boxed{V_R = V_o \cdot e^{-Rt/L}}$$



Case-II

RL Circuit with Source

Charging



At $t = 0$ → swith will Connect.

Apply KVL →

$$V = R \cdot i + L \cdot \frac{di}{dt}$$

$$V = R \cdot i + L \cdot \frac{di}{dt}$$

Devide above eqn with L →

$$\frac{V}{L} = \frac{R}{L} \cdot i + \frac{L}{L} \cdot \frac{di}{dt}$$

$$\frac{di}{dt} + \frac{R}{L} \cdot i = \frac{V}{L}.$$

$$i(t) = C.F. + P.I.$$

C.F. → transient

P.I. → steady state

C.F. → transient response →

$$\frac{di}{dt} + \frac{R}{L} \cdot i = 0.$$

After solving this.

$$i(t) = A \cdot e^{-Rt/L} \longrightarrow (1)$$

$$\boxed{A \rightarrow \text{Constant}}$$

P.I. → steady state

$$i(t) = \frac{V}{R} \longrightarrow (11)$$

$$i(t) = C.F. + P.I.$$

$$\boxed{i(t) = A \cdot e^{Rt/L} + \frac{V}{R}}$$

At $t = 0^-$, $i(t) = 0$

$t = 0^+$, $i(t) = 0$

$t = 0$, $i(t) = 0$

So, $0 = A \cdot e^{0} + \frac{V}{R}$

$$\boxed{e^0 = 1}$$

$$0 - \frac{V}{R} = A$$

$$\boxed{i(\infty) = \frac{V}{R}}$$

$$\boxed{A = i(0^+) - i(\infty)}$$

So,

$$i(t) = \{i(0^+) - i(\infty)\} e^{\frac{-Rt}{L}} + \frac{V}{R}$$

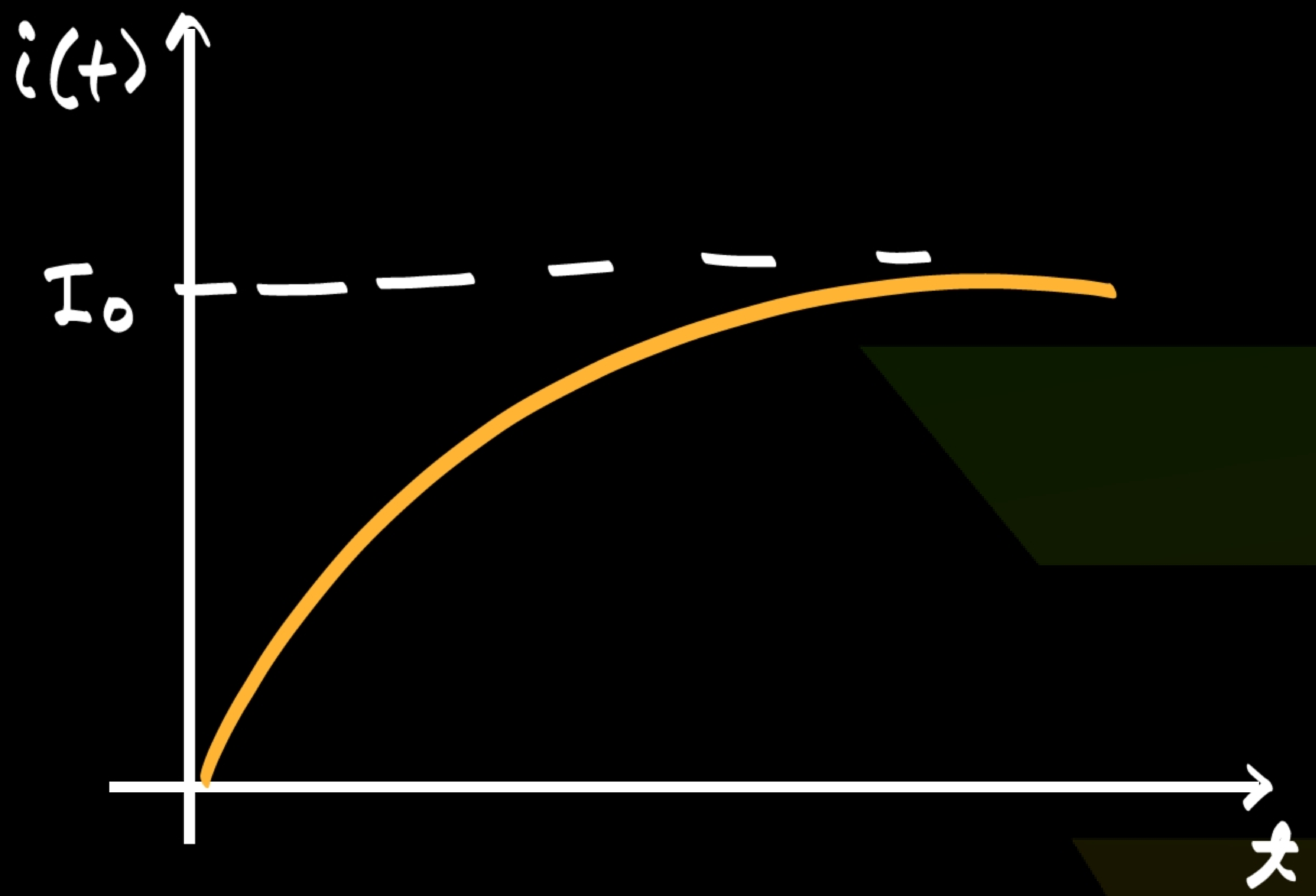
$$\boxed{i(t) = \left(i(0^+) - i(\infty)\right) e^{\frac{-Rt}{L}} + i(\infty)}$$

$$i(t) = \left(0 - \frac{V}{R}\right) e^{-Rt/L} + \frac{V}{R}$$

$$i(t) = \frac{V}{R}\left(1 - e^{-Rt/L}\right)$$

Let $I_0 = \frac{V}{R}$

$$\boxed{i(t) = I_0\left(1 - e^{-Rt/L}\right)}$$



$$i(t) = I_0\left(1-e^{-Rt/L}\right)$$

$t=\infty$

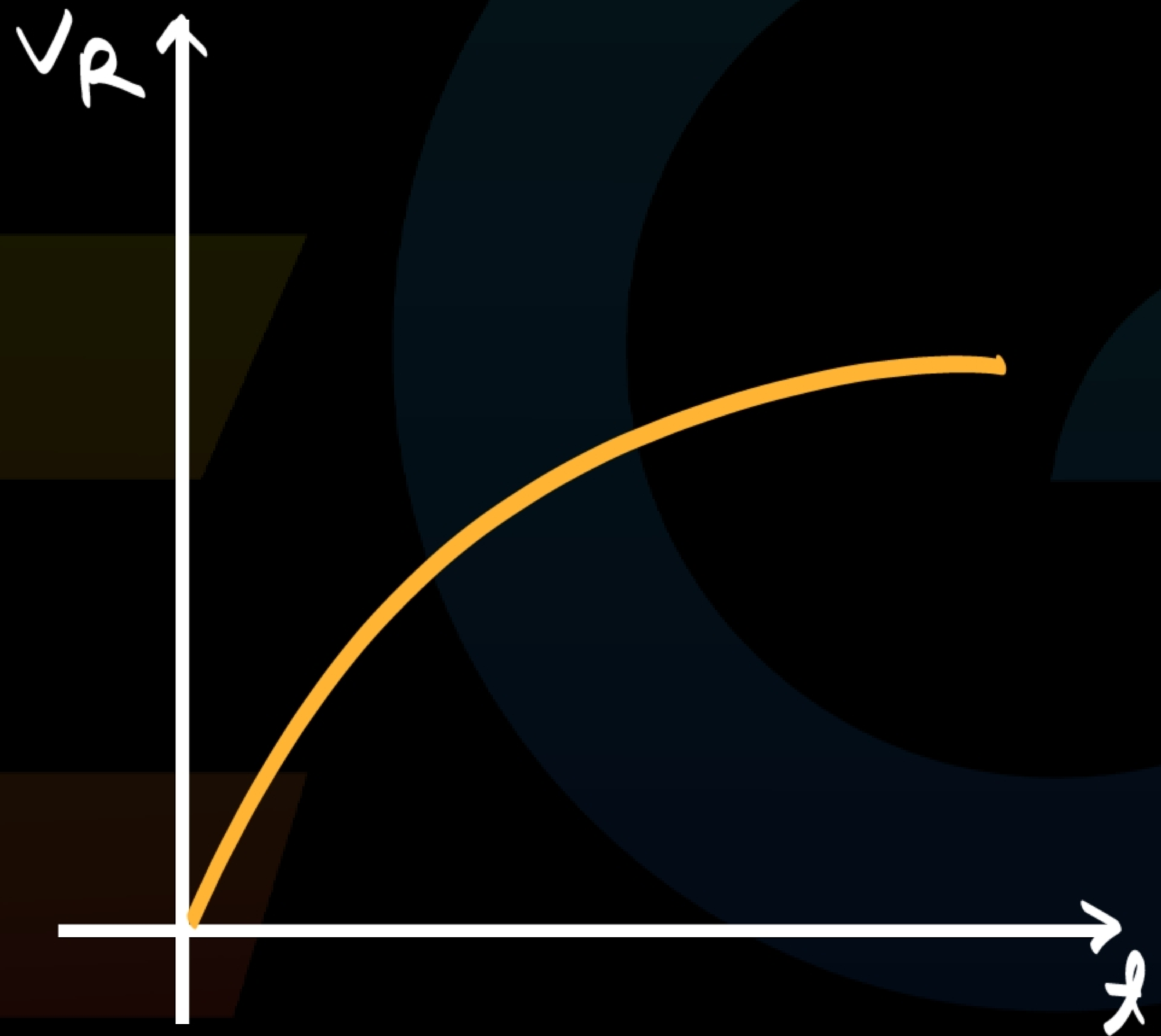
$$\boxed{i(t) = I_0}$$

Voltage across Resistor

$$V_R = R\cdot i(t)$$

$$V_R = R\cdot I_0\cdot\left(1-e^{-Rt/L}\right)$$

$$\boxed{V_R = V_0\cdot\left(1-e^{-Rt/L}\right)}$$



Voltage across inductor.

$$v_L = L\cdot\frac{di(t)}{dt}$$

$$V_L = L\cdot\frac{d}{dt} I_0\left(1-e^{-Rt/L}\right)$$

$$V_L = \cancel{L}\cdot I_0\left(0+\frac{R}{\cancel{L}}\cdot e^{-Rt/L}\right)$$

$$V_L = I_0\cdot R\, e^{-Rt/L}$$

$$\boxed{V_L = V_0\cdot e^{-Rt/L}} \qquad \boxed{V_0 = I_0\cdot R}$$