Investigating the Significance of Special Measurement Matrices for DMD based Frequency Estimation

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Abstract. Power systems are prone to failures such as power quality tripping, resonance, and CMV failures. These failures are infinite and dynamic because of the fluctuations in the power system impedance together with its load demand and harmonic distortions. To have stability and control over the power system together with a quality power supply, it is imperative to monitor the fundamental frequency. Motivated by this, a data-driven approach based on dynamic mode decomposition (DMD) is proposed which is essential to observe parameters such as frequency and amplitude. This methodology works well for extracting fundamental frequency under harmonics, sub-harmonics, and inter-harmonics scenarios in the signal which causes considerable damage to end-user equipment if not monitored properly. In this methodology, appended time-shifted power signals stacked into a measurement matrix are used for extracting the parameters from the signal. DMD relies on dynamics to capture information and predict frequencies. Measurement matrices play a crucial role in identifying the underlying dynamics of the system. Thus it creates a suitable measurement matrix and gives us input to the data-driven system which is vital for the system to perform well. This study investigates the importance of different special matrices such as Hankel, Toeplitz, and Circulant to create the measurement matrices in DMD to extract the signal parameters. DMD can replace conventional Fourier-based techniques by reducing the estimation error percentage. Various experiments are conducted to confirm the potentiality of the proposed methodology.

INTRODUCTION

Smart grids are one of the most needed requirements in today's era to meet the clean, green energy requirements [1]. It allows the transition from a conventional-centralized power grid to a modern-decentralized smart grid. A smart grid can be considered a digital component that responds to information based on the behaviour of the components incorporated in it (supplier and consumer) [2]. It consists of HVDC transmission lines, a distributed generation system, a microgrid, and FACTS devices. Flexible loads are inherently a part of it to meet the increasing power demand [3]. Due to constant, tiny demand variations, electric power systems are never really stable [4]. On the other hand, control devices can keep a system's operating point within a specific range, even during these tiny fluctuations in load, which is known as a pseudo-steady state. Sometimes large disturbances can be caused by both planned and unforeseen events. This could lead to increased oscillatory behaviour and, eventually, to a new state of pseudo-stability [5]. The problems are mostly related to power quality, including waveform distortions and frequency distortions. This results in a deteriorated power supply, issues occurring to end users, power interruptions, and enormous economic losses [6]. End-user equipment is manufactured to work at a specific frequency within a tight tolerance, so it is vital to keep the frequency of the power supply stable. If there is a way to detect these frequency deviations at an early stage, it can help us to compensate for the effect with the help of compensation devices and post-disturbance analysis. Conventional power analyzers fail to detect these frequency changes, which drastically reduces the power grid's performance [7]. Therefore, it is required to monitor such deviations continuously to provide sufficient information about these disturbances.

Frequency is an essential factor in a power system which determines the system's efficiency and performance. Deviation of fundamental frequency from its preferred range is an indicator to know the power system's performance [8]. Various causes of frequency deviations are disconnected from a large block of load or a large source of generation going off-line due to the imbalance between generation and load. This enforces the need to implement a continuous frequency monitoring system with utmost accuracy to find the balance between power generation and demand. Further real-time analysis and interpretation are required to process the huge volume of data coming from advanced smart metering techniques like micro-PMUs. Various techniques based on the Fourier approach [9], adaptive filtering [10], optimization methods [11] and signal processing algorithms [2, 12] are proposed to deal with frequency estimation task in the past. Prony is another method widely used for power quality monitoring [9]. However, this method is not accurate under multiple disturbance scenarios. Later short-term Fourier transform-based approaches are proposed for harmonics monitoring and power quality analysis. An interesting approach for power quality disturbance monitoring using variational mode decomposition (VMD) algorithm is developed in [2]. VMD is a powerful iterative algorithm that helps to identify the dominant frequency components from the data. Harmonics monitoring using standard DMD algorithm and Total-DMD are proposed in [7] and [12] respectively. However many of them fail under scenarios such as fundamental frequency deviations, the presence of noise, integer and non-integer data components.

Motivated by the above facts, in this paper, a data-driven methodology based on dynamic mode decomposition (DMD) is proposed to monitor frequency deviations in power systems. DMD can be considered a data diagnostic algorithm that helps to extract Spatio-Temporal patterns from measured data [13]. This study explores the significance of shift-stacked matrices in DMD-based frequency estimation methodology. The remaining section of the paper is organised as follows: Section 2 explains the mathematical details of the DMD algorithm, and Section 3 describes the importance of stacked matrices in DMD-based estimation techniques. Section 4 explains the proposed estimation procedure, followed by Section 5, which contains major results and related discussions. Finally, section 6 concludes the work.

DYNAMIC MODE DECOMPOSITION ALGORITHM

Dynamic mode decomposition (DMD) is a data-driven technique that identifies low-rank structures and inherent dynamics from the measured data [14]. DMD provides precise reproductions of coherent patterns that appear in dynamic systems. It helps analyze short time future predictions of dynamic systems [1]. DMD captures the underlying dynamics of physical systems using measured data from the system [15]. The main objective of DMD is to estimate frequency deviation before it occurs.

Consider a 1-dimensional voltage signal as $X(t) = [x_1, x_2, ..., x_n]$, $X(t) \in R^n$, measurements taken for time T seconds. Here each data point has a separation of Δt seconds, $(T >> \Delta t)$, and n represents the count of samples for a time period of T. Let A be the linear operator which maps the two matrices that are the snapshots of the original signal at two consequent intervals of time. The two snapshot matrices (measurement matrices) are defined as X_1 and X_2. Assume that X_1 is captured at time t and X_2 is captured at time $t + \Delta t$.

$$X_1 = [x_1, x_2, ..., x_{n-1}] \tag{1}$$

$$X_2 = [x_2, x_3, ..., x_n] \tag{2}$$

Now, DMD computes the leading eigendecomposition of the linear operator which maps to these two matrices X_1 and X_2 as given below.

$$AX_1 = X_2 \tag{3}$$

Here, $X_1, X_2 \in \mathbb{C}^{k \times n-1}$ and $A \in \mathbb{C}^{k \times k}$. Now, to calculate the best fit value for the linear operator A, the pseudo inverse of X_1 is taken as,

$$A = X_2 X_1^{\dagger} \tag{4}$$

where $\dagger$ is the pseudo-inverse operation. But, in a real-world scenario, n can take a very large dimension and hence A will be much larger to compute. Therefore this does not seem to be an optimal solution to proceed further. Thus we need a smaller matrix that would be easy and comfortable to compute and it should also capture the maximum possible characteristics of A [16].To obtain the approximation of A with the lower dimensions (named as A'), the number of modes for the analyses of data matrices is truncated by DMD [12].

In the first step of the DMD algorithm, the SVD of X_1 is computed as follows.

$$X_1 = U\Sigma V^* \tag{5}$$

where, $U \in \mathbb{C}^{k\times m}$, $\Sigma \in \mathbb{C}^{m\times m}$,$V \in \mathbb{C}^{n-1\times m}$. Here m is the number of modes truncated by DMD to produce A'. Now, taking the inverse on both sides results as follows.

$$X_1^{\dagger} = V\Sigma^{\dagger}U^* \tag{6}$$

From equation 4 and 6,

$$A = X_2 V\Sigma^{\dagger}U^* \tag{7}$$

As discussed earlier, the dimension of A is relatively high. This leads to the involvement of Proper Orthogonal Decomposition (POD). This is a statistical method that helps in obtaining low-dimensional abstraction for high-dimensional data. Hence, we produce A' for ease of computation. Here, A and A' are similar matrices. Both the matrices will have the same eigenvalues and also the same number of eigenvectors. With this information, it is possible to compute the similarity transformation matrix A',

$$A' = U^*AU \tag{8}$$

Here $A' \in \mathbb{C}^{m\times m}$, $U^* \in \mathbb{C}^{m\times k}$, $A \in \mathbb{C}^{k\times k}$, $U \in \mathbb{C}^{k\times m}$. Later the eigendecomposition of A' is calculated using the following equation.

$$A'\Omega = \Omega\Lambda \tag{9}$$

By employing diagonalisation, A' can be written as,

$$A' = \Omega\Lambda\Omega^{-1} \tag{10}$$

Hence, from equation 8 to 10,

$$U^*AU = \Omega\Lambda\Omega^{-1} \tag{11}$$

$$A(U\Omega) = (U\Omega)\Lambda \tag{12}$$

$$AW = W\Lambda \tag{13}$$

Here, W contains $(n-1)$ columns where all of them represent the eigenvectors of A and Λ is the diagonal matrix with $(n-1)$ entries which are the eigenvalues of A.

IMPORTANCE OF STACKED MATRICES

The standard DMD algorithm is inaccurate to estimate the periodic oscillations in the measured data [16]. In order to overcome this drawback of DMD and to capture the periodic oscillations in the measured waveform, the concept of shift-stacking of data is introduced [17]. The snapshot matrices are formulated by appending multiple time-shifted copies of the data. The concept is that DMD takes input from snapshot matrices and is a capture of information at discrete points. In order to identify the frequency deviations of power signals we created the snapshot matrices by stacking multiple time-shifted copies of measured data. DMD algorithm uses the concept of vectorization, diagonalization, projection, pseudo-inverse, and similarity matrix to obtain the solution. This algorithm can detect the changes in snapshot matrices, so if there is a way to induce more dynamics inside the X_1 and X_2 matrices which in turn helps DMD to identify the dynamics of the underlying system more accurately. Here we make use of a special type of matrices called measurement matrices to induce more dynamics in the system [18]. This has a significant influence on the accuracy and processing time of the sparse recovery. In particular, semi-deterministic matrices are a subset of measurement matrices used in this study. Semi-deterministic matrices can be created in two steps: the first step is to generate the first row randomly from the input and the second step is to generate entries of the rest with some transformation or shifting of elements. This study explores the use of different types of semi-deterministic matrices including Hankel, Circulant, and Toeplitz matrices and Fig. 1 represents the corresponding matrix notations in general.

$$H = \begin{bmatrix} h_0 & h_1 & \cdots & h_{n-1} \\ h_1 & h_2 & \cdots & h_n \\ h_2 & h_3 & \cdots & h_{n+1} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n-1} & h_n & \cdots & h_{2n-2} \end{bmatrix} \qquad C = \begin{bmatrix} c_0 & c_1 & \cdots & c_{n-1} \\ c_{n-1} & c_0 & \cdots & c_{n-2} \\ c_{n-2} & c_{n-1} & \cdots & c_{n-3} \\ \vdots & \vdots & \ddots & \vdots \\ c_1 & c_2 & \cdots & c_0 \end{bmatrix} \qquad T = \begin{bmatrix} t_0 & t_1 & \cdots & t_{n-1} \\ t_1 & t_0 & \cdots & t_{n-2} \\ t_2 & t_1 & \cdots & t_{n-3} \\ \vdots & \vdots & \ddots & \vdots \\ t_{n-1} & t_{n-2} & \cdots & t_0 \end{bmatrix}$$

FIGURE 1. H,C,T represent that matrix notation for Hankel, Circulant, and Toeplitz

Hankel Matrix: Hankel matrix also referred as per symmetric matrix $(h_{i,j})$ for which the r^{th} entry in diagonal is same i.e $i + j = r$ [19]. It can also be rephrased as elements parallel to the main anti-diagonal are equal. These types of matrices are used in a number of applications some include decomposing non-stationary signals, time-frequency representation etc. The formation of the Hankel matrix is represented below [19].

Circulant Matrix: A circulant matrix is a special case of the Toeplitz matrix and is also a square matrix which is the sum of shift matrices [20]. It is created from a vector as the first row (or column). The following rows are filled with the same elements as the first row where the elements are shifted by one element. The eigenvalues of the circulant matrix are the same as that of the inverse Fourier transform of the first row of circulant matrix. Similar to Hermitian matrices, they also have orthonormal eigenvectors and also we know exactly what their eigenvectors are. Eigenvectors of circulant matrix are closely related to the famous Fourier transform as eigenvectors of circulant matrix are column vectors of Fourier matrix. Circulant matrices can also be created using a linear combination of shift matrices [20].

Toeplitz Matrix: Toeplitz matrix also called diagonal-constant matrix [21]. Circulant matrix (C) is a special case of the Toeplitz matrix, where in which each row is produced by permuting the preceding row $(12 \ldots n)$. Linear convolution can be represented as multiplication by a Toeplitz matrix. Asymptotically, Toeplitz matrices commute. When the row and column dimensions approach infinity, they diagonalize on the same basis [21].

PROPOSED ESTIMATION PROCEDURE

Snapshot matrices (measurement matrices) are stacked with respect to time as column vectors $[x_1, x_2, \cdots, x_m]$. Thus 2-dimensional snapshot matrices of size $R^{N \times M}$, where N is the number of data points and M is the number of time-shifted copies of data which is created. Snapshot matrix can be viewed as the snap of underlying dynamics, each snapshot is captured at time $X_{t+\Delta t}$, and the time $t + \Delta t$ is assumed to be connected with the preceding frame x_t. In this study, semi-deterministic matrices like Hankel, Circulant and Toeplitz are used to induce randomness with respect to data inside the stacked matrix. DMD then acquires the required amount of dynamics upon which it can capture changes occurring in data (frequency-deviation) which apparently helps to improve power quality with reduced distortions and improved efficiency in the power grid. The block diagram of the proposed estimation procedure is given in Fig.2. Visualization of DMD eigenvalues obtained for Hankel, Circulant, and Toeplitz matrices is given in Fig. 3. Further the frequency estimation is done based on the extracted eignevalues. A small discussion about happenings in the block diagram. At first time series signal is obtained as snapshots with very small intervals, which are then stacked. This stacked matrix is then fed into measurement matrices (which are Hankel or Toeplitz or Circulant or a Combination of Toeplitz and Circulant) to induce dynamics. Further the matrix is fed into DMD upon which we obtain the modes with which we can predict the future state.

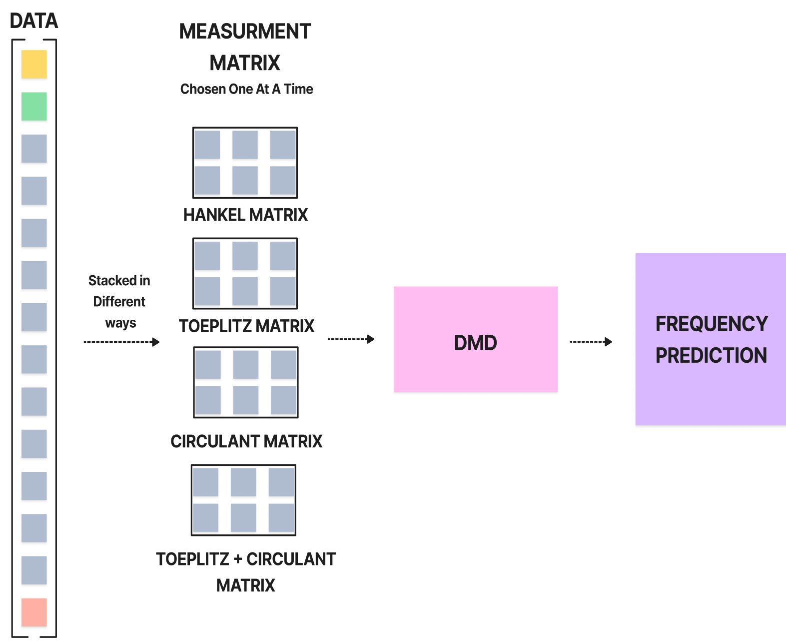

FIGURE 2. Block diagram of the estimation procedure using DMD

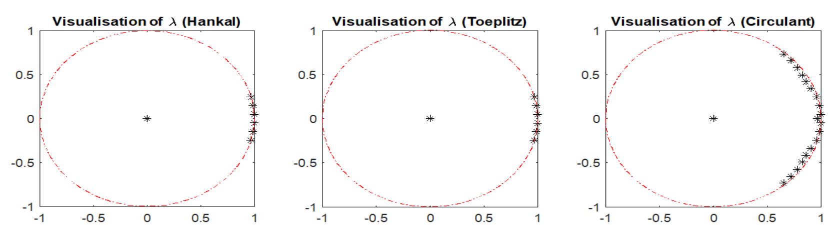

FIGURE 3. Representation of DMD eigenvalues obtained for Hankel, Circulant, and Toeplitz

RESULTS AND DISCUSSIONS

The major results based on this study are presented here. The estimation accuracy of frequency and amplitude using various stacked matrices is calculated in terms of error percentage [22]. Error percentage tells us, how accurate the parameter estimation is. Mathematically, it is expressed as given below.

$$Error\% = \frac{|Predicted - Actual|}{Actual} \times 100$$

The study is conducted by considering various test scenarios. The following subsections briefly explain each of them.

Fundamental Frequency Estimation Under Integer Harmonics Scenario

The test signal is synthetically generated with integer harmonics as mentioned in [12]. The synthetic signal consists of integer harmonics of orders 1,3, and 5. The results of estimation using the proposed strategy are tabulated in Table 1. As given in Table 1, considering different matrices for data stacking like Hankel, Toeplitz, and Circulant and their combination gives comparable results with other state-of-the-art methods. The results prove that by inducing the change in dynamics by shift-stacking the data captures the exact frequency variations from the measured data.

TABLE 1. Estimation of Fundamental Frequency under Integer Harmonics Scenario

$X(t) = cos(wt) + 0.05cos(3wt) + 0.02cos(5wt)$	
Ref.freq(Hz)	50
Adaptive Notch Filtering	50.12
VMD Chebfunction	50.02
DFT+ZC	51.55
DFT+Interpolation	50.00
DFT+Phasor rotation	50.001
Hankel	50.00
Toeplitz	50.00
Circulant	50.00

Further to prove the importance of the shift-stacked matrix concept in DMD-based estimation methodology, a fundamental frequency deviation scenario is considered. The error% for all measurement matrices combinations are below 0.010 and the results are tabulated in Table 2.

TABLE 2. Estimation under Frequency Deviation

Matrices	**Ref.freq(Hz)**	**Pred.freq(Hz)**	**Error%**
DMD+Hankel	50	50	0.00
	49.5	49.5	0.00
	50.5	50.5	0.00
DMD+Toeplitz	50	50	0.00
	49.5	49.5	0.00
	50.5	50.5	0.00
DMD+Circulant	50	50	0.00
	49.5	49.52	0.006
	50.5	50.29	0.002
Toeplitz+Circulant	50	50	0.00
	49.5	50.57	0.010
	50.5	50.36	0.001

Fundamental Frequency Estimation Under Non-integer Harmonics Scenario

The test case considered is a combination of both integer and non-integer harmonics, which adds a little more difficulty for frequency estimation. However, the proposed methodology has no error and is giving the exact estimation of the fundamental frequency. The estimation results are depicted in Fig. 4. As evident from Fig. 4, compared with DFT-based techniques and adaptive filtering methods, the proposed methodology outperforms the estimation even in the presence of non-integer harmonics. The univariate time-series data is converted into a two-dimensional data matrix using different special matrices which helps the DMD algorithm to capture the inherent dynamics of the power data. This leads to the highest accuracy of estimation for the proposed DMD-based strategy. By extending the evaluations under fundamental frequency deviations, the proposed methodology estimated the results with error% less than 0.02.

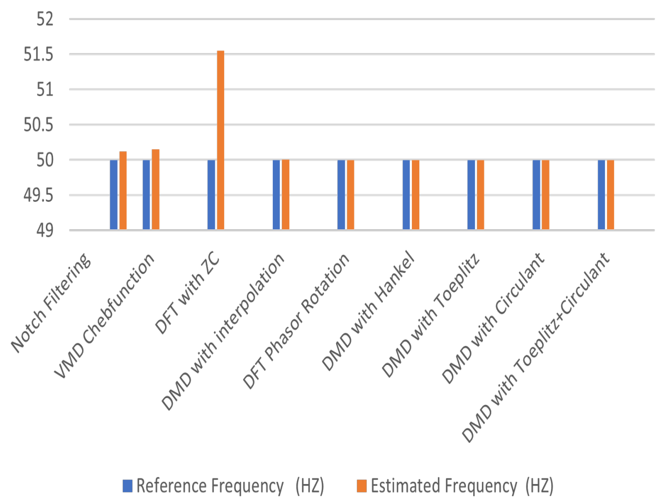

FIGURE 4. Fundamental Frequency Estimation Under Non-integer Harmonics Scenario

Fundamental Frequency Estimation Under Noisy Scenario

In this test case, the presence of integer harmonics, inter harmonics and noise are considered. This test case would bring the complete possible set of harmonics to be considered to generalise the proposed methodology under different circumstances. This is the case in the real-world scenario, where the signal is never normal or ideal. The signal that we obtain always has some noise in it. Thus the effectiveness of the proposed methodology is evaluated under noisy cases and the results of fundamental frequency estimation are tabulated in Table 3. It is evident from the results that the error rate in the noisy cases is almost negligible for each of the different matrices considered.

TABLE 3. Fundamental Frequency Estimation Under Noisy Scenario

Ref.freq(Hz)	Hankel	Toeplitz	Circulant	Toeplitz+Circulant
49.5	49.50	50.35	49.65	49.51
50.0	50	49.98	50.12	50.01
50.5	50.50	49.34	50.58	50.56

Real-time Validation under Disturbance Scenario

In this test case, we tried to evaluate the proposed method with a real-time signal along with noise fed into it. The test signal is a measured voltage waveform from the distribution system with a sampling frequency of 6.4 kHz [23, 24]. The validation results under no noise, 30 dB and 50 dB noisy scenarios are tabulated in Table 4. DMD-based estimation failed to detect the 9th harmonic as it is of very low power [12]. The same scenario is observed in our proposed approach. However, the proposed method performed relatively well in predicting all other frequencies. This experiment highlights the potential of the proposed method in real-time noisy scenarios.

CONCLUSION

This paper investigates the significance of various special measurement matrices called the semi-deterministic matrices in DMD-based estimation methodology. The semi-deterministic matrices are used to induce dynamics in DMD to capture the variations in the univariate time series data. The special type of matrices used in this work includes

TABLE 4. Real-time Validation on no-noise and noisy scenarios

	Hankel			Toeplitz			Circulant			Toeplitz + Circulant		
Ref.freq(Hz)	**No Noise**	**30 dB**	**50 dB**	**No Noise**	**30 dB**	**50 dB**	**No Noise**	**30 dB**	**50 dB**	**No Noise**	**30 dB**	**50 dB**
50	49.99	49.99	49.99	50.16	50.09	50.17	49.99	49.99	49.99	49.95	49.93	49.95
150	149.98	149.99	149.9891	150.12	150.05	150.10	149.99	149.99	149.99	149.74	149.71	149.74
250	249.98	249.97	249.98	250.05	250.03	250.05	249.99	249.99	249.99	249.70	249.70	249.70
350	349.97	349.94	349.97	350.35	350.31	350.34	349.99	349.97	350.00	349.97	350.09	349.97
450	-	-	-	-	-	-	-	-	-	-	-	-
550	549.96	550.01	549.96	550.22	550.37	550.20	549.99	549.99	549.99	549.99	549.87	550.00

Hankel, Toeplitz, Circulant and a combination of the latter two. With this inclusion of stacked matrices DMD based estimation methodology achieves accurate estimation under various power system scenarios. Various test scenarios are considered to evaluate the potentiality of the proposed methodology. Further the results are validated by considering real-time voltage waveform. Thus it is concluded that the usage of special matrices depending on the measured data helps to improve the estimation results. After validating everything with the real waveform, we recommend using the Hankel matrix as the data matrix for DMD, especially for these types of cases since Hankel works the best. However, Toeplitz and circulant also work well enough in some cases which is beyond the scope of this paper.

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