Bayesian Optimization Algorithm*

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Abstract

The Clever Algorithms project aims to describe a large number of Artificial Intelligence algorithms in a complete, consistent, and centralized manner, to improve their general accessibility. The project makes use of a standardized algorithm description template that uses well-defined topics that motivate the collection of specific and useful information about each algorithm described. This report describes the Bayesian Optimization Algorithm using the standardized algorithm description template.

Keywords: `Clever, Algorithms, Description, Bayesian, Optimization, Algorithm`

1 Introduction

The Clever Algorithms project aims to describe a large number of algorithms from the fields of Computational Intelligence, Biologically Inspired Computation, and Metaheuristics in a complete, consistent and centralized manner [1]. The project requires all algorithms to be described using a standardized template that includes a fixed number of sections, each of which is motivated by the presentation of specific information about the technique [2]. This report describes the Bayesian Optimization Algorithm using the standardized algorithm description template.

2 Name

Bayesian Optimization Algorithm, BOA

3 Taxonomy

The Bayesian Optimization Algorithm belongs to the field of Estimation of Distribution Algorithms, also referred to as Population Model-Building Genetic Algorithms (PMBGA) an extension to the field of Evolutionary Computation. More broadly, BOA belongs to the field of Computational Intelligence. The Bayesian Optimization Algorithm is related to other Estimation of Distribution Algorithms such as the Population Incremental Learning Algorithm, and the Univariate Marginal Distribution Algorithm. It is also the basis for extensions such as the Hierarchal Bayesian Optimization Algorithm (hBOA) and the Incremental Bayesian Optimization Algorithm (iBOA).

4 Inspiration

Bayesian Optimization Algorithm is a technique without an inspiration. It is related to the Genetic Algorithm and other Evolutionary algorithms that are inspired by the biological theory of evolution by means of natural selection.

5 Strategy

The information processing objective of the technique is to construct a probabilistic model that describes the relationships between the components of fit solutions in the problem space. This is achieved by repeating the process of creating and sampling from a Bayesian network that contains the conditional dependancies, independencies, and conditional probabilities between the components of a solution. The network is constructed from the relative frequencies of the components within a population of high fitness candidate solutions. Once the network is constructed, the candidate solutions are discarded and a new population of candidate solutions are generated from the model. The process is repeated until the model converges on a fit prototype solution.

6 Procedure

Algorithm 1 provides a pseudo-code listing of the Bayesian Optimization Algorithm for minimizing a cost function. The Bayesian network is constructed each iteration using a greedy algorithm. The network is assessed based on its fit of the information in the population of candidate solutions using either a Bayesian Dirichlet Metric (BD) [8], or a Bayesian Information Criterion (BIC) [3] (Chapter 3).

Algorithm 1: Pseudo Code for the Bayesian Optimization Algorithm.

```
Input: Bits_num, Population_size, Selection_size
Output: S_best
Population ← InitializePopulation(Bits_num, Population_size);
EvaluatePopulation(Population);
S_best ← GetBestSolution(Population);
while ¬StopCondition() do
    Selected ← SelectFitSolutions(Population);
    Model ← ConstructBayesianNetwork(Selected);
    OffSpring ← 0;
    for i to Population_size do
        OffSpring ← ProbabilisticallyConstructSolution(Model);
    end
    EvaluatePopulation(OffSpring);
    S_best ← GetBestSolution(OffSpring);
    Population ← Combine(Population, OffSpring);
end
return S_best;
```

7 Heuristics

- The Bayesian Optimization algorithm was designed and investigated on binary string-base problems, most commonly representing binary function optimization problems.

- Bayesian networks are typically constructed (grown) from scratch each iteration using an iterative process of adding, removing, and reversing links. Additionally, past networks may be used as the basis for the process that are in turn verified.
- A greedy hill-climbing algorithm is used to each algorithm iteration optimize a Bayesian network to represent a population of candidate solutions.
- The fitness of constructed bayesian networks may be assessed using the Bayesian Dirichlet Metric (BD) or a Minimum Description length method called the Bayesian Information Criterion (BIC).

8 Code Listing

A listing of the algorithm is currently not provided.

9 References

9.1 Primary Sources

The Bayesian Optimization Algorithm was proposed by Pelikan, Goldberg, and Cantu-Paz in the technical report [7], that was later published [9]. The technique was proposed as an extension to the state of Estimation of Distribution algorithms (such as the Univariate Marginal Distribution Algorithm and the Bivariate Marginal Distribution Algorithm) that used a Bayesian Network to model the relationships and conditional probabilities for the components expressed in a population of fit candidate solutions. Pelikan, Goldberg, and Cantu-Paz also described the approach applied to decelptive binary optimization problems (trap functions) in a paper that was published before the seminal journal article [8].

9.2 Learn More

Pelikan and Goldberg described an extension to the approach called the Hierarchical Bayesian Optimization Algorithm (hBOA) [6, 4]. The differences in the hBOA algorithm are that it replaces the decision tables (used to store the probabilities) with decision graphs and used a niching method called Restricted Tournament Replacement to maintain diversity in the selected set of candidate solutions used to construct the network models. Pelikan's work on BOA culminated in his PhD thesis that provides a detailed treatment of the approach, its configuration and application [5]. Pelikan, Sastry, and Goldberg proposed the Incremental Bayesian Optimization Algorithm (iBOA) extension of the approach that removes the population and adds incremental updates to the Bayesian network [10].

Pelikan published a book that focused on the technique, walking through the development of probabilistic algorithms inspired by evolutionary computation, a detailed look at the Bayesian Optimization Algorithm (Chapter 3), the hierarchic extension to Hierarchical Bayesian Optimization Algorithm and demonstration studies of the approach on test problems [3].

10 Conclusions

This report described the Bayesian Optimization Algorithm using the standardized algorithm description template. An implementation of the algorithm was adapted although could not be resolved within the time constraints. It is expected that an implementation can be provided with further study of the seminal references, the book, and the sample C++ code provided by Pelikan.

11 Contribute

Found a typo in the content or a bug in the source code? Are you an expert in this technique and know some facts that could improve the algorithm description for all? Do you want to get that warm feeling from contributing to an open source project? Do you want to see your name as an acknowledgment in print?

Two pillars of this effort are i) that the best domain experts are people outside of the project, and ii) that this work is subjected to continuous improvement. Please help to make this work less wrong by emailing the author 'Jason Brownlee' at jasonb@CleverAlgorithms.com or visit the project website at http://www.CleverAlgorithms.com.

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