# Learning Classifier System\*

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### Abstract

The Clever Algorithms project aims to describe a large number of Artificial Intelligence algorithms in a complete, consistent, and centralized manner, to improve their general accessibility. The project makes use of a standardized algorithm description template that uses well-defined topics that motivate the collection of specific and useful information about each algorithm described. This report describes the Learning Classifier System algorithm using the standardized template.

Keywords: Clever, Algorithms, Description, Optimization, Learning, Classifier, System

### 1 Introduction

The Clever Algorithms project aims to describe a large number of algorithms from the fields of Computational Intelligence, Biologically Inspired Computation, and Metaheuristics in a complete, consistent and centralized manner [3]. The project requires all algorithms to be described using a standardized template that includes a fixed number of sections, each of which is motivated by the presentation of specific information about the technique [4]. This report describes the Learning Classifier System algorithm using the standardized template.

### 2 Name

Learning Classifier System, LCS

# 3 Taxonomy

The Learning Classifier System algorithm is both an instance of an Evolutionary Algorithm from the field of Evolutionary Computation and an instance of a Reinforcement Learning algorithm from Machine Learning. The Learning Classifier System is a theoretical system with a number of implementations. Two streams of classifier are the Pittsburgh-style that seeks to optimize whole classifier, and the Michigan-style that optimize responsive rulesets. The Michigan-style Learning Classifier is the most common and is comprised of two versions: the ZCS (zeroth-level classifier system) and the XCS (accuracy-based classifier system).

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# 4 Strategy

The objective of the Learning Classifier System algorithm is to optimize payoff based on exposure to stimuli from a problem-specific environment. This is achieved by managing credit assignment for those rules that prove useful and searching for new rules and new variations on existing rules using an evolutionary process.

### 5 Procedure

The actors of the system include detectors, messages, effectors, feedback, and classifiers. Detectors are used by the system to perceive the state of the environment. Messages are the discrete information packets passed from the detectors into the system. The system performs information processing on messages, and messages may directly result in actions in the environment. Effectors control the actions of the system on and within the environment. In addition to the system actively perceiving via its detections, it may also receive directed feedback from the environment (payoff). Classifiers are condition-action rules that provides a filter for messages. If a message satisfies the conditional part of the classifier, the action of the classier triggers. Rules act as message processors. Messages are defined at a fixed length using a binary alphabet. A classifier is defined as a binary string with a ternary alphabet of 1,0,#, where the # represents do not care (matching both a 1 or 0).

The processing loop for the Learning Classifier system is as follows: i) Messages from the environment are placed on the message list. ii) The conditions of each classifier are checked to see if they are satisfied by at least one message in the message list. iii) All classifiers that are satisfied participate in a competition, those that win post their action to the message list. iv) All messages directed to the effectors are executed (causing actions in the environment). v) All messages on the message list from the previous cycle are deleted (messages persist for a single cycle). The algorithm may be described in terms of the main processing loop and two sub-algorithms: a reinforcement learning algorithm such as the bucket brigade algorithm or Q-learning, and a genetic algorithm for optimization of the system. Algorithm 1 provides a pseudo-code listing of the high-level processing loop of the Learning Classifier System, specifically the XCS as described by Butz and Wilson [6].

### 6 Heuristics

The majority of the heuristics in this section are specific to the XCS Learning Classifier System as described by Butz and Wilson [6].

- Learning Classifier Systems are suited for problems with the following characteristics: perpetually novel events with large amounts of noise, continual, and real-time requirements for action, implicitly or inexactly defined goals, and sparse payoff or reinforcement obtainable only through long sequences of tasks.
- The learning rate  $\beta$  for a classifiers expected payoff, error and fitness are typically in the range  $\in [0.1, 0.2]$ .
- The frequency of running the genetic algorithm  $\theta_{GA}$  should be in the range  $\in [25, 50]$ .
- The discount factor used in multi-step programs  $\gamma$  are typically in the around 0.71.
- The minimum error for whereby classifiers are considered to have equal accuracy  $\epsilon_0$  are typically 10% of the maximum reward.
- The probability of crossover in the genetic algorithm  $\chi$  are typically in the range  $\in [0.5, 1.0]$ .

**Algorithm 1**: Pseudo Code for the Learning Classifier System algorithm.

```
Input: env
   Output: Population
 1 env ← InitializeEnvironment(env);
 2 Population ← InitializePopulation();
 3 ActionSet_{t-1} \leftarrow 0;
 4 Input_{t-1} \leftarrow 0;
 5 Reward_{t-1} \leftarrow 0;
 6 while ¬StopCondition() do
       Input_t \leftarrow env;
 7
       Matchset \leftarrow GenerateMatchSet(Population, Input_t);
 8
       Prediction ← GeneratePrediction(Matchset);
 9
       Action ← SelectionAction(Prediction);
10
11
       ActionSet_t \leftarrow GenerateActionSet(Action, Matchset);
       Reward_t \leftarrow \texttt{ExecuteAction}(\mathsf{Action}, \mathsf{env});
12
       if ActionSet_{t-1} \neq 0 then
13
           Payof f_t \leftarrow \texttt{CalculatePayoff}(Reward_{t-1}, Prediction);
14
           PerformLearning(ActionSet_{t-1}, Payoff_t, Population);
15
           RunGeneticAlgorithm(ActionSet_{t-1}, Input_{t-1}, Population);
16
17
       end
       if LastStepOfTask(env, Action) then
18
           Payof f_t \leftarrow Reward_t;
19
           PerformLearning(ActionSet_t, Payof f_t, Population);
20
           RunGeneticAlgorithm(ActionSet_t, Input_t, Population);
\mathbf{21}
           ActionSet_{t-1} \leftarrow 0;
22
\mathbf{23}
       else
           ActionSet_{t-1} \leftarrow ActionSet_t;
24
           Input_{t-1} \leftarrow Input_t;
25
26
           Reward_{t-1} \leftarrow Reward_t;
       end
27
28 end
```

- The probability of mutating a single position in a classifier in the genetic algorithm  $\mu$  is typically in the range  $\in [0.01, 0.05]$ .
- The experience threshold during classifier deletion  $\theta_{del}$  is typically about 20.
- The experience threshold for a classifier during subsumption  $\theta_{sub}$  is typically around 20.
- The initial values for a classifiers expected payoff  $p_1$ , error  $\epsilon_1$ , and fitness  $f_1$  are typically small and close to zero.
- The probability of selecting a random action for the purposes of exploration  $p_{exp}$  is typically close to 0.5.
- The minimum number of different actions that must be specified in a match set  $\theta_{mna}$  is usually the total number of possible actions in the environment for the input.
- Subsumption should be used on problem domains that are known contain well defined rules for mapping inputs to outputs.

# 7 Code Listing

Listing 1 provides an example of the Learning Classifier System algorithm implemented in the Ruby Programming Language. The problem is an instance of a Boolean multiplexer called the 6-multiplexer. It can be described as a classification problem, where each of the  $2^6$  patterns of bits is associated with a boolean class  $\{1,0\}$ . For this problem instance, the first two bits may be decoded as an address into the remaining four bits that specify the class (for example in 100010, '10' decode to the index of '2' in the remaining 4 bits making the class '1'). In propositional logic this problem instance may be described as  $F = (\neg x_0)(\neg x_1)x_2 + (\neg x_0)x_1x_3 + x_0(\neg x_1)x_4 + x_0x_1x_5$ . The algorithm is an instance of XCS based on the description provided by Butz and Wilson [6] with the parameters based on the application of XCS to Boolean multiplexer problems by Wilson [16, 17]. The population is grown as needed, and subsumption which would be appropriate for the Boolean multiplexer problem was not used for brevity. The multiplexer problem is a single step problem, so the complexities of delayed payoff are not required. A number of parameters were hard coded to recommended values, specifically:  $\alpha = 0.1, v = 5, \delta = 0.1$  and  $P_{\#} = \frac{1}{3}$ .

```
def new_classifier(condition, action, gen)
 1
2
     other = {}
     other[:condition], other[:action], other[:lasttime] = condition, action, gen
3
     other[:prediction], other[:error], other[:fitness] = 0.00001, 0.00001, 0.00001
4
     other[:experience], other[:setsize], other[:num] = 0.0, 1.0, 1.0
5
     return other
 6
 7
    def copy_classifier(parent)
9
     copy = \{\}
10
     parent.keys.each {|k| copy[k] = (parent[k].kind_of? String) ? ""+parent[k] : parent[k]}
11
     copv[:num] = 1
12
     copy[:experience] = 0.0
13
     return copy
14
15
16
17
    def generate_problem_string(length)
     return (0...length).inject(""){|s,i| s+((rand<0.5) ? "1" : "0")}
18
19
20
    def neg(bit)
21
     return (bit==1) ? 0 : 1
22
    end
23
24
   def target_function(s)
25
     ints = Array.new(s.length){|i| s[i].chr.to_i}
26
     x0,x1,x2,x3,x4,x5 = ints
27
     return neg(x0)*neg(x1)*x2 + neg(x0)*x1*x3 + x0*neg(x1)*x4 + x0*x1*x5
28
    end
29
30
    def calculate_deletion_vote(classifier, pop, del_thresh)
31
     vote = classifier[:setsize] * classifier[:num]
32
     avg_fit = pop.inject(0.0)\{|s,c| s+c[:fitness]\}/pop.inject(0.0)\{|s,c| s+c[:num]\}
33
     derated = classifier[:fitness] / classifier[:num]
34
      if classifier[:experience] > del_thresh and derated < 0.1 * avg_fit</pre>
35
       vote *= avg_fit / derated
36
37
     return vote
38
    end
39
40
    def delete_from_pop(pop, pop_size, del_thresh)
41
     total = pop.inject(0) {|s,c| s+c[:num]}
42
     return if total < pop_size</pre>
43
     pop.each {|c| c[:dvote] = calculate_deletion_vote(c, pop, del_thresh)}
44
     vote_sum = pop.inject(0.0) {|s,c| s+c[:dvote]}
45
```

```
point = rand() * vote_sum
46
      vote_sum, index = 0.0, 0
47
      pop.each_with_index do |c,i|
48
        vote_sum += c[:dvote]
49
        if vote_sum > point
50
          index = i
51
52
          break
53
        end
54
      end
      if pop[index][:num] > 1
55
        pop[index][:num] -= 1
56
57
        pop.delete_at(index)
58
      end
59
    end
60
61
    def generate_random_classifier(input, actions, gen)
62
      condition = ""
63
      input.each_char {|s| condition << ((rand<1.0/3.0) ? '#' : s)}
64
65
      action = actions[rand(actions.length)]
      return new_classifier(condition, action, gen)
66
67
68
    def does_match(input, condition)
69
      i = 0
70
71
      condition.each_char do |c|
        return false if c!='#' and c!=input[i].chr
72
73
        i += 1
74
      end
75
      return true
    end
76
77
78
    def get_actions(pop)
      return [] if pop.empty?
79
80
      set = {}
81
      pop.each do |classifier|
82
        key = classifier[:action]
83
        set[key] = 0 if set[key].nil?
84
        set[key] += 1
85
      end
      return set.keys
86
87
88
    def generate_match_set(input, pop, all_actions, gen, pop_size, del_thresh)
89
      match_set = pop.select{|c| does_match(input, c[:condition])}
90
91
      actions = get_actions(match_set)
      while actions.length < all_actions.length do
92
93
        remaining = all_actions - actions
        classifier = generate_random_classifier(input, remaining, gen)
94
95
        pop << classifier
        match_set << classifier
96
        delete_from_pop(pop, pop_size, del_thresh)
97
        actions << classifier[:action]</pre>
98
      end
99
      return match_set
100
101
102
103
    def generate_prediction(input, match_set)
104
      prediction = {}
      match_set.each do |classifier|
105
        key = classifier[:action]
106
        prediction[key] = {:sum=>0.0,:count=>0.0,:weight=>0.0} if prediction[key].nil?
107
        prediction[key][:sum] += classifier[:prediction]*classifier[:fitness]
108
```

```
prediction[key][:count] += classifier[:fitness]
109
110
      prediction.keys.each do |key|
111
        prediction[key][:weight]=prediction[key][:sum]/prediction[key][:count]
112
      end
      return prediction
114
115
    end
116
    def select_action(prediction_array, p_explore)
117
      keys = prediction_array.keys
118
      return true, keys[rand(keys.length)] if rand() < p_explore</pre>
119
      keys.sort!{|x,y| prediction_array[y][:weight]<=>prediction_array[x][:weight]}
120
      return false, keys.first
121
    end
122
123
124
    def update_set(action_set, payoff, l_rate)
      action_set.each do |c|
125
        c[:experience] += 1.0
126
127
        pdiff = payoff - c[:prediction]
        c[:prediction] += (c[:experience]<1.0/l_rate) ? pdiff/c[:experience] : l_rate*pdiff</pre>
128
        diff = pdiff.abs - c[:error]
129
        c[:error] += (c[:experience]<1.0/l_rate) ? diff/c[:experience] : l_rate*diff</pre>
130
        sum = action_set.inject(0.0) {|s,other| s+other[:num]-c[:setsize]}
131
        c[:setsize] += (c[:experience]<1.0/l_rate) ? sum/c[:experience] : l_rate*sum</pre>
132
      end
133
134
    end
135
136
    def update_fitness(action_set, min_error, l_rate)
137
      sum = 0.0
      accuracy = Array.new(action_set.length)
138
      action_set.each_with_index do |c,i|
139
        accuracy[i] = (c[:error]<min_error) ? 1.0 : 0.1*(c[:error]/min_error)**-5.0
140
141
        sum += accuracy[i] * c[:num]
142
143
      action_set.each_with_index do |c,i|
144
        c[:fitness] += l_rate * (accuracy[i] * c[:num] / sum - c[:fitness])
145
      end
    end
146
147
    def can_run_genetic_algorithm(action_set, gen, ga_freq)
148
      total = action_set.inject(0.0) {|s,c| s+c[:lasttime]*c[:num]}
149
      sum = action_set.inject(0.0) {|s,c| s+c[:num]}
150
      if gen - (total/sum) > ga_freq
151
152
        return true
153
154
      return false
    end
    def select_parent(pop)
157
      sum = pop.inject(0.0) {|s,c| s+c[:fitness]}
158
      point = rand() * sum
159
      sum = 0
160
      pop.each do |c|
161
        sum += c[:fitness]
162
        return c if sum > point
163
164
165
    end
166
    def mutation(classifier, p_mut, action_set, input)
167
      classifier[:condition].length.times do |i|
168
        if rand() < p_mut</pre>
169
          if classifier[:condition][i].chr == '#'
170
            classifier[:condition][i] = input[i]
171
```

```
172
            classifier[:condition][i] = '#'
173
174
        end
      end
177
      if rand() < p_mut</pre>
178
        new_action = nil
179
        begin
          new_action = action_set[rand(action_set.length)]
180
        end until new_action != classifier[:action]
181
        classifier[:action] = new_action
182
      end
183
    end
184
185
    def uniform_crossover(string1, string2)
186
187
188
      string1.length.times do |i|
        rs << ((rand()<0.5) ? string1[i] : string2[i])
189
190
191
      return rs
192
    end
193
    def insert_in_pop(classifier, pop)
194
195
      pop.each do |c|
        if classifier[:condition] ==c[:condition] and classifier[:action] ==c[:action]
196
197
          c[:num] += 1
198
          return
199
        end
200
      end
      pop << classifier
201
    end
202
203
204
    def crossover(c1, c2, p1, p2)
      c1[:condition] = uniform_crossover(p1[:condition], p2[:condition])
205
206
      c2[:condition] = uniform_crossover(p1[:condition], p2[:condition])
207
      c1[:prediction] = (p1[:prediction]+p2[:prediction])/2.0
208
      c1[:error] = 0.25*(p1[:error]+p2[:error])/2.0
209
      c1[:fitness] = 0.1*(p1[:fitness]+p2[:fitness])/2.0
210
      c2[:prediction] = c1[:prediction]
      c2[:error] = c1[:error]
211
      c2[:fitness] = c1[:fitness]
212
    end
213
214
    def run_genetic_algorithm(all_actions, pop, action_set, input, gen, p_cross, p_mut, pop_size,
215
         del_thresh)
216
      p1, p2 = select_parent(action_set), select_parent(action_set)
      c1, c2 = copy_classifier(p1), copy_classifier(p2)
217
      crossover(c1, c2, p1, p2) if rand() < p_cross</pre>
218
      [c1,c2].each do |c|
219
        mutation(c, p_mut, all_actions, input)
220
        insert_in_pop(c, pop)
221
        delete_from_pop(pop, pop_size, del_thresh)
222
      end
223
    end
224
225
226
    def search(length, pop_size, max_gens, all_actions, p_explore, l_rate, min_error, ga_freq,
        p_cross, p_mut, del_thresh)
227
      pop, abs = [], 0
228
      max_gens.times do |gen|
        input = generate_problem_string(length)
229
        match_set = generate_match_set(input, pop, all_actions, gen, pop_size, del_thresh)
230
        prediction_array = generate_prediction(input, match_set)
231
        explore, action = select_action(prediction_array, p_explore)
232
```

```
action_set = match_set.select{|c| c[:action] == action}
233
        expected = target_function(input)
234
        payoff = ((expected-action.to_i)==0) ? 300.0 : 1.0
235
        abs += (expected - action.to_i).abs.to_f
236
        update_set(action_set, payoff, l_rate)
237
        update_fitness(action_set, min_error, l_rate)
238
239
        if can_run_genetic_algorithm(action_set, gen, ga_freq)
240
          action_set.each {|c| c[:lasttime] = gen}
          run_genetic_algorithm(all_actions, pop, action_set, input, gen, p_cross, p_mut, pop_size,
241
              del_thresh)
        end
242
        if (gen+1).modulo(50)==0
243
          puts " >gen=#{gen+1} classifiers=#{pop.size}, error=#{abs.to_i}/50 (#{(abs/50*100)}%)"
244
245
246
247
      end
248
      return pop
^{249}
    end
250
    max_gens, length, pop_size = 5000, 6, 150
251
    all_actions = ['0', '1']
252
    l_rate, min_error = 0.2, 0.01
253
    p_explore, p_cross, p_mut = 0.10, 0.80, 0.04
254
    ga_freq, del_thresh = 50, 20
255
256
257
    pop = search(length, pop_size, max_gens, all_actions, p_explore, l_rate, min_error, ga_freq,
        p_cross, p_mut, del_thresh)
    puts "done! Solution: classifiers=#{pop.size}"
```

Listing 1: Learning Classifier System algorithm in the Ruby Programming Language

### 8 References

#### 8.1 Primary Sources

Early ideas on the theory of Learning Classifier Systems were proposed by Holland [7, 10], culminating in a standardized presentation a few years later [8]. A number of implementations of the theoretical system were investigated, although a taxonomy of the two main streams was proposed by De Jong [12]: 1) Pittsburgh-style proposed by Smith [14, 13] and 2) Holland-style or Michigan-style Learning classifiers that are further comprised of the Zeroth-level classifier (ZCS) [15] and the accuracy-based classifier (XCS) [16].

#### 8.2 Learn More

Booker, Goldberg, and Holland provide a classical introduction to Learning Classifier Systems including an overview of the state of the field and the algorithm in detail [1]. Wilson and Goldberg also provide a classical introduction and review of the approach, although take a more critical stance [18]. Holmes, et al. provide a contemporary review of the field focusing both on the approach and application areas to which the approach has been demonstrated successfully [11]. Lanzi, Stolzmann, and Wilson provide a seminal book in the field as a collection of papers covering the basics, advanced topics, and demonstration applications. A particular highlight from this book is the first section that provides a concise description of Learning Classifier Systems by many leaders and major contributors to the field [9], providing rare insight. Another paper from this book by Lanzi and Riolo provides a detailed review of the development of the approach as it matured throughout the 1990s. Bull and Kovacs a second book introductory book to the field focusing on the theory of the approach and its practical application [5].

### 9 Conclusions

This report described the Learning Classifier System as a machine learning technique using both reinforcement learning and genetic algorithms. The content for this report based based on a previous work on the Learning Classifier System by this author [2].

## 10 Contribute

Found a typo in the content or a bug in the source code? Are you an expert in this technique and know some facts that could improve the algorithm description for all? Do you want to get that warm feeling from contributing to an open source project? Do you want to see your name as an acknowledgment in print?

Two pillars of this effort are i) that the best domain experts are people outside of the project, and ii) that this work is wrong by default. Please help to make this work less wrong by emailing the author 'Jason Brownlee' at jasonb@CleverAlgorithms.com or visit the project website at http://www.CleverAlgorithms.com.

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