# Cost Functions

# Explicit costs and Implicit costs

- Explicit costs involve a direct monetary outlay, whereas implicit costs do not.
- For example, an airline's expenditures on fuel and salaries are explicit costs, whereas the income it forgoes by not leasing its jets is an implicit cost.
- The sum total of the explicit costs and the implicit costs represents what the airline sacrifices when it makes the decision to fly one of its planes on a particular route.

# Opportunity Cost

- The opportunity cost of a particular alternative is the payoff associated with the *best of the alternatives that are not chosen*.
- The opportunity cost of an alternative includes all of the explicit and implicit costs associated with that alternative.
- The concept of opportunity cost is forward looking in that it measures the value that the decision maker sacrifices at the time the decision is made and beyond.

# An Example

- Suppose that you own and manage your own business and that you are contemplating whether you should continue to operate over the next year or go out of business. If you remain in business, you will need to spend $100,000 to hire the services of workers and $80,000 to purchase supplies; if you go out of business, you will not need to incur these expenses. In addition, the business will require 80 hours of your time every week. Your best alternative to managing your own business is to work the same number of hours in a corporation for an income of $75,000 per year.
- In this example, the opportunity cost of continuing in business over the next year is $255,000. This amount includes an explicit cost of $180,000—the required cash outlays for labor and materials; it also includes an implicit cost of $75,000—the income that you forgo by continuing to manage your own firm as opposed to choosing your best available alternative.

# ECONOMIC VERSUS ACCOUNTING COSTS

- **Economic costs** are synonymous with opportunity costs and, as such, are the sum of all decision-relevant explicit and implicit costs.
- **Accounting costs**—the costs that would appear on accounting statements—are explicit costs that have been incurred in the past. Accounting statements are designed to serve an audience outside the firm, such as lenders and equity investors, so accounting costs must be objectively verifiable. That's why accounting statements typically include historical expenses only—explicit cash outlays already made (e.g., the amounts the firm actually spent on labor and materials in the past year).
- An accounting statement would *not* include implicit costs such as the opportunity costs associated with the use of the firm's factories because such costs are often hard to measure in an objectively verifiable way. For that reason, an accounting statement for an owner-operated small business would not include the opportunity cost of the owner's time.

## SUNK (UNAVOIDABLE) VERSUS NONSUNK (AVOIDABLE) COSTS

- Some costs have already been incurred and therefore cannot be avoided, no matter what decision is made. These are called **sunk costs**.
- By contrast, **nonsunk costs** are costs that will be incurred only if a particular decision is made and are thus avoided if the decision is not made (for this reason, nonsunk costs are also called *avoidable costs*).

➢ When evaluating alternative decisions, the decision maker should ignore sunk costs and consider only nonsunk costs.

➢ *Whether a cost is sunk or nonsunk depends on the decision that is being contemplated.*

# Example

- Consider a sporting goods firm that manufactures bowling balls. Let's assume that a bowling ball factory costs $5 million to build and that, once it is built, the factory is so highly specialized that it has no alternative uses.
- *In deciding whether to build the factory*, the $5 million is a *nonsunk cost*. It is a cost the sporting goods firm incurs only if it builds the factory. At the time the decision is being considered, the decision maker can avoid spending the $5 million.
- *After the factory is built*, the $5 million is a *sunk cost*. It is a cost the sporting goods firm incurs no matter what it later chooses to do with the factory, so this cost is unavoidable. *When deciding whether to operate the factory or shut it down*, the sporting goods firm therefore should ignore this cost.

## LONG RUN VERSUS SHORT RUN

- A firm that makes a **long run** decision faces a blank slate (i.e., no constraints): Over the long run, it will be able to vary the quantities of all its inputs as much as it desires.
- By contrast, a firm facing a **short-run** decision is subject to constraints: Over the short run, it will not be able to adjust the quantities of some of its inputs and/or reverse the consequences of past decisions that it has made regarding those inputs.

# Short-run costs

Short-run cost can be split into two groups: total fixed costs and total variable costs.

The fixed costs include:

(a) salaries of administrative staff

(b) depreciation (wear and tear) of machinery

(c) expenses for building depreciation and repairs

(d) expenses for land maintenance and depreciation (if any).

The variable costs include:

(a) the raw materials

(b) the cost of direct labour

(c) the running expenses of fixed capital, such as fuel, ordinary repairs and routine maintenance.

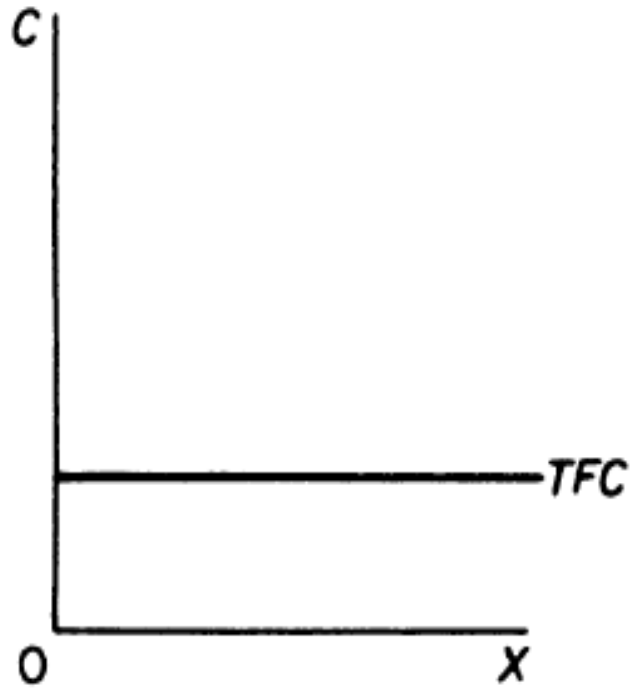



Figure 4.1

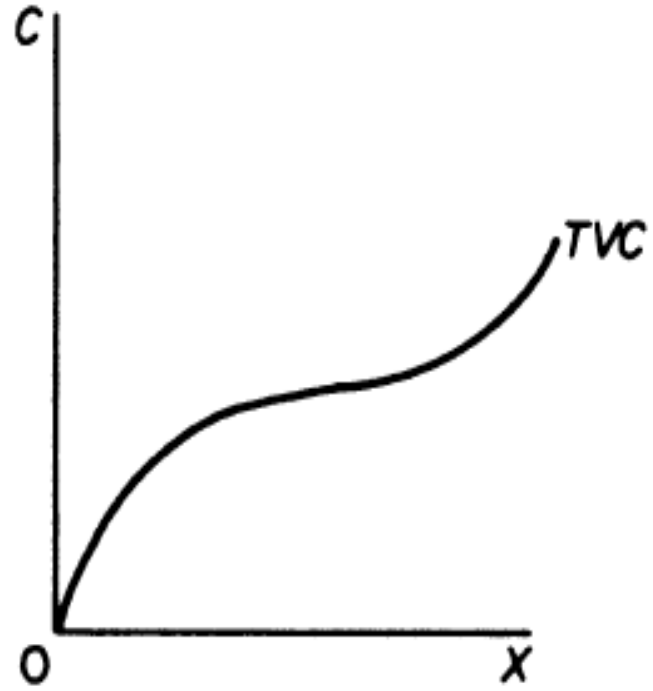



Figure 4.2

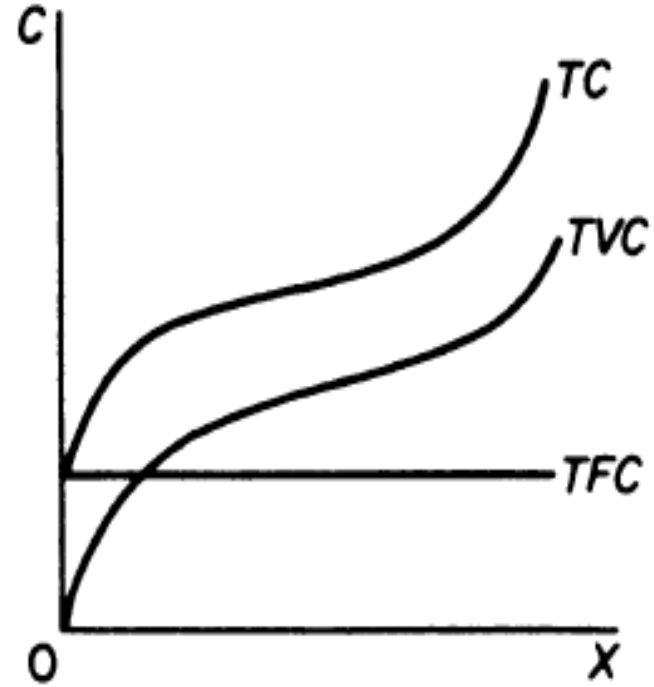


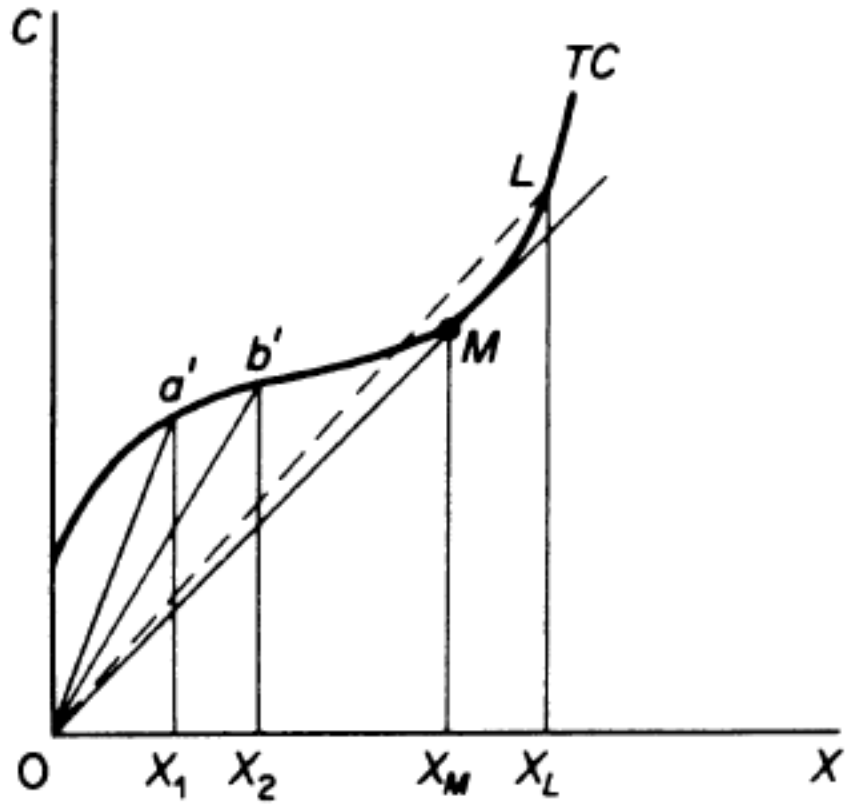

Figure 4.3

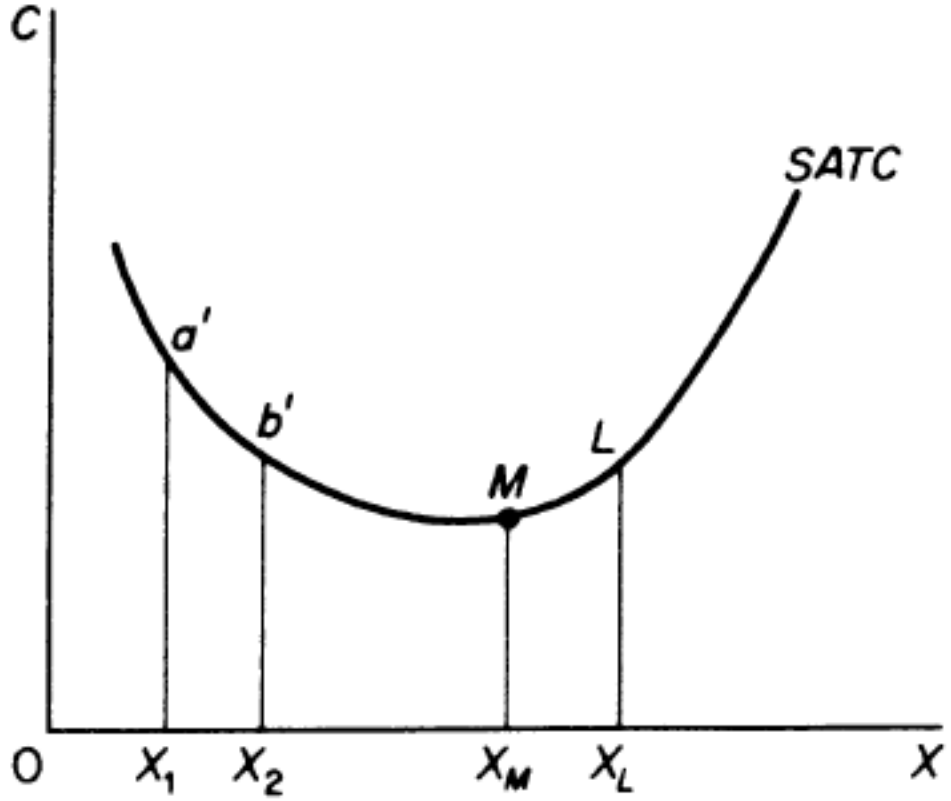
# Average Costs

$$Average\ Total\ Cost\ (ATC) = \frac{TC}{X} = \frac{TFC + TVC}{X} = \frac{TFC}{X} + \frac{TVC}{X} = AFC + AVC$$

AFC = Average Fixed Costs

AVC = Average Variable Costs


C
TC
L
M
a'
b'
O
$X_1$
$X_2$
$X_M$
$X_L$
X


C
SATC
a'
b'
M
L
O
$X_1$
$X_2$
$X_M$
$X_L$
X

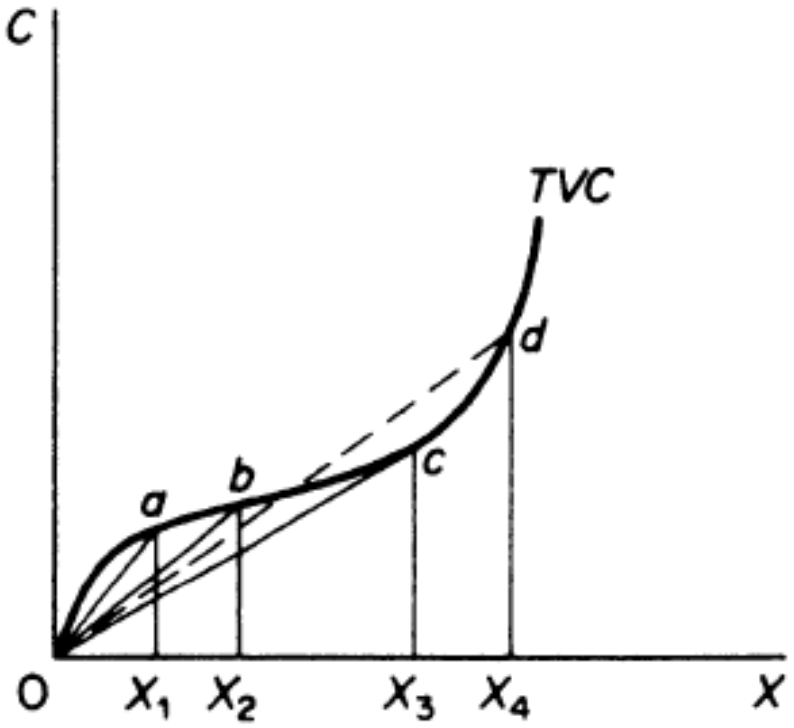



Figure 4.5

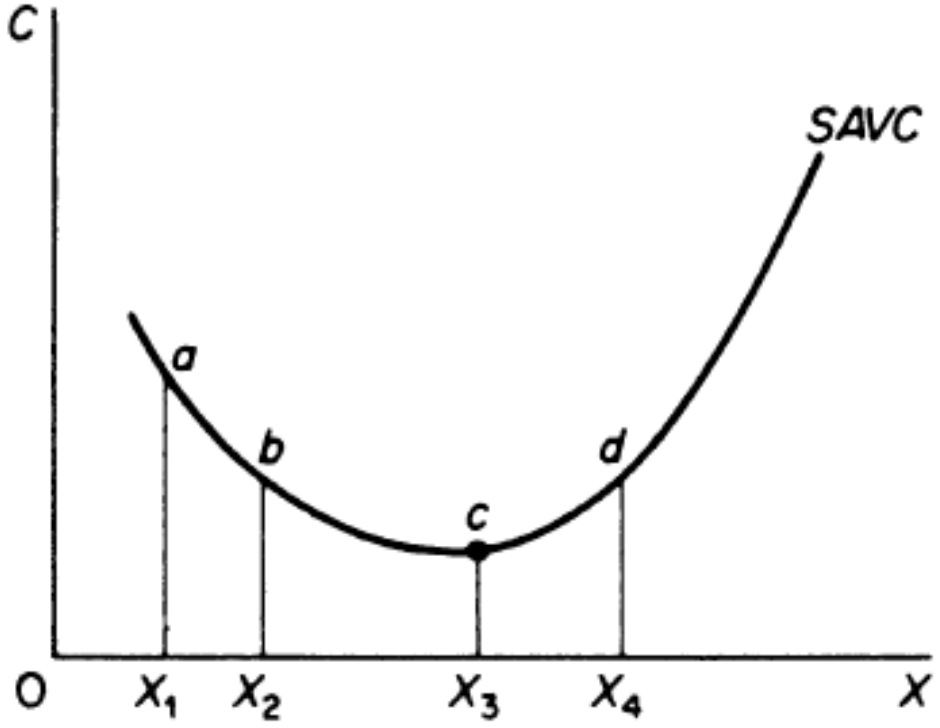
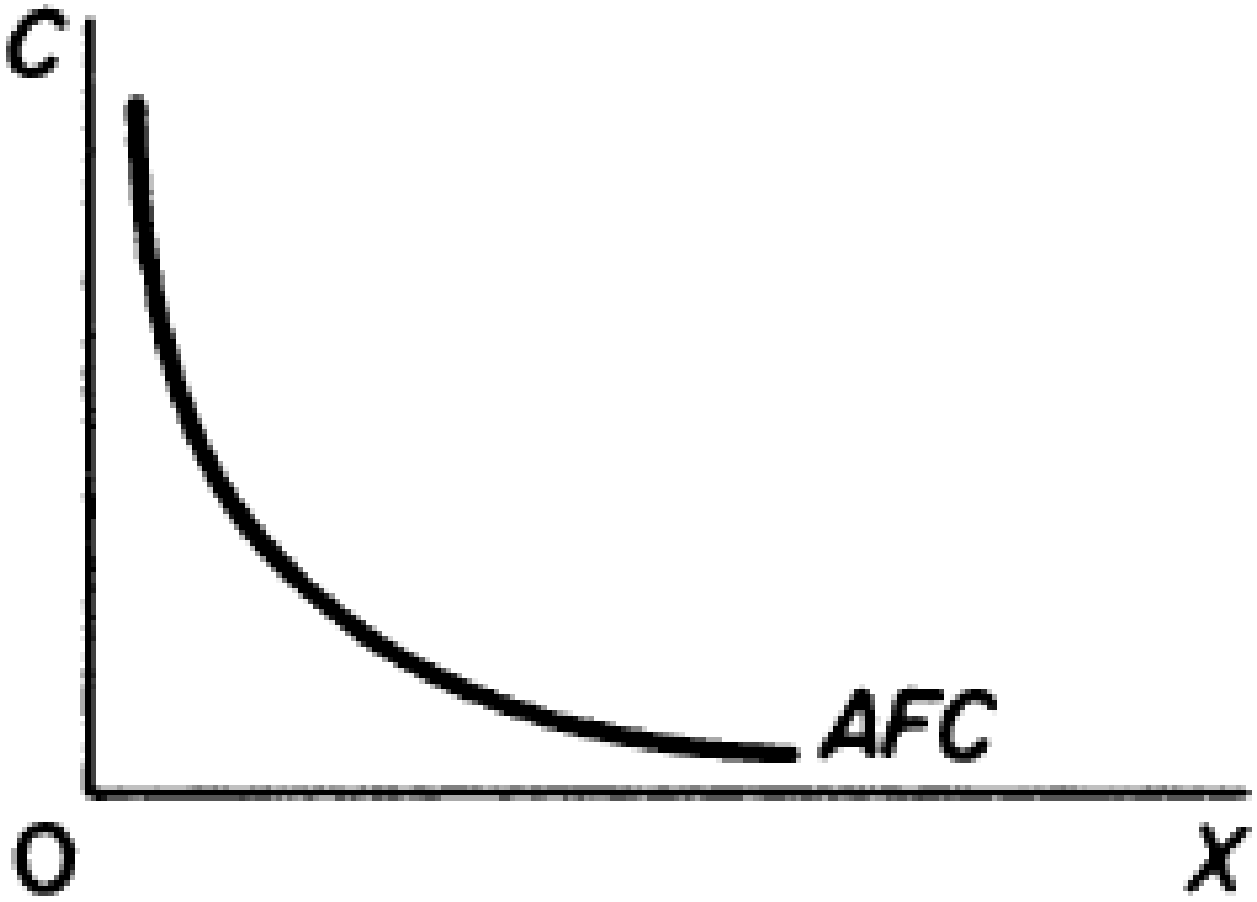
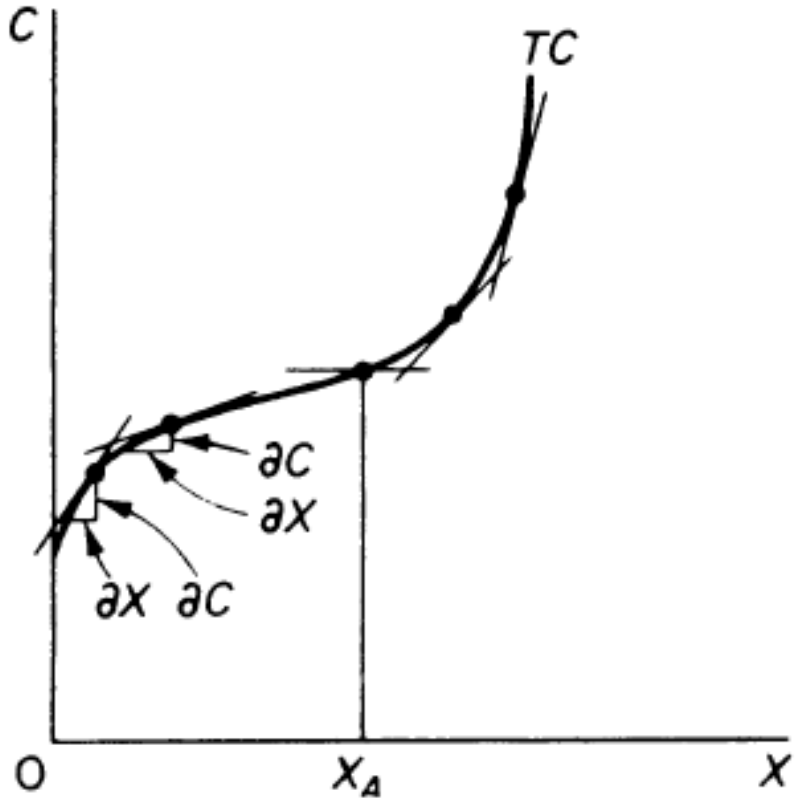
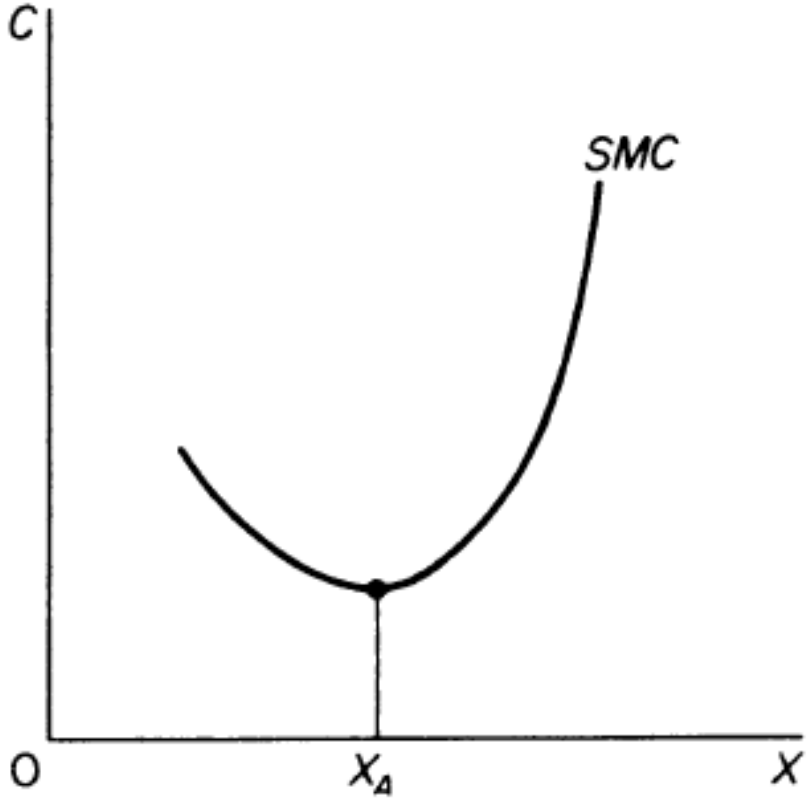


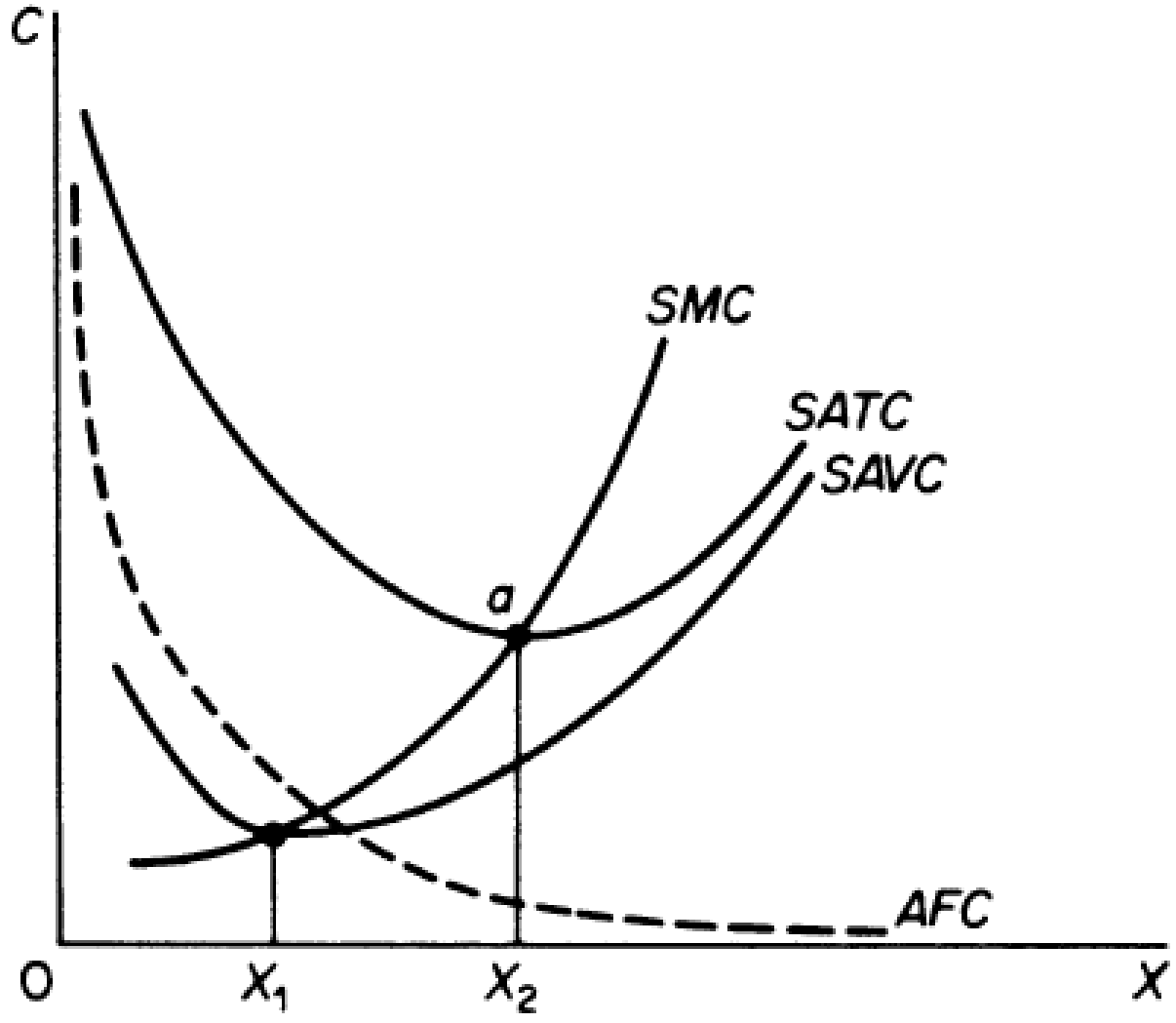

Figure 4.6


C
AFC
O
X

# Marginal costs: $MC = \frac{\partial TC}{\partial X}$




C
SMC
SATC
SAVC
a
AFC
O
$X_1$
$X_2$
X

# Relationship between AC and MC

[1] The relationship between the $MC$ and $AC$ curves becomes clearer with the use of simple calculus. Given $C = zX$, where $z = AC$. Clearly $z = f(X)$. The $MC$ is

$$\frac{\partial C}{\partial X} = \frac{\partial (zX)}{\partial X}$$

Applying the rule of differentiation of 'a function of a function' (which states that if $y = uv$, where $u = f_1(x)$ and $v = f_2(x)$, then $dy/dx = dy/du \cdot du/dx$), we obtain

$$MC = \frac{\partial C}{\partial X} = z\frac{\partial X}{\partial X} + X\frac{\partial z}{\partial X}$$

or

$$MC = AC + (X)(\text{slope of } AC)$$

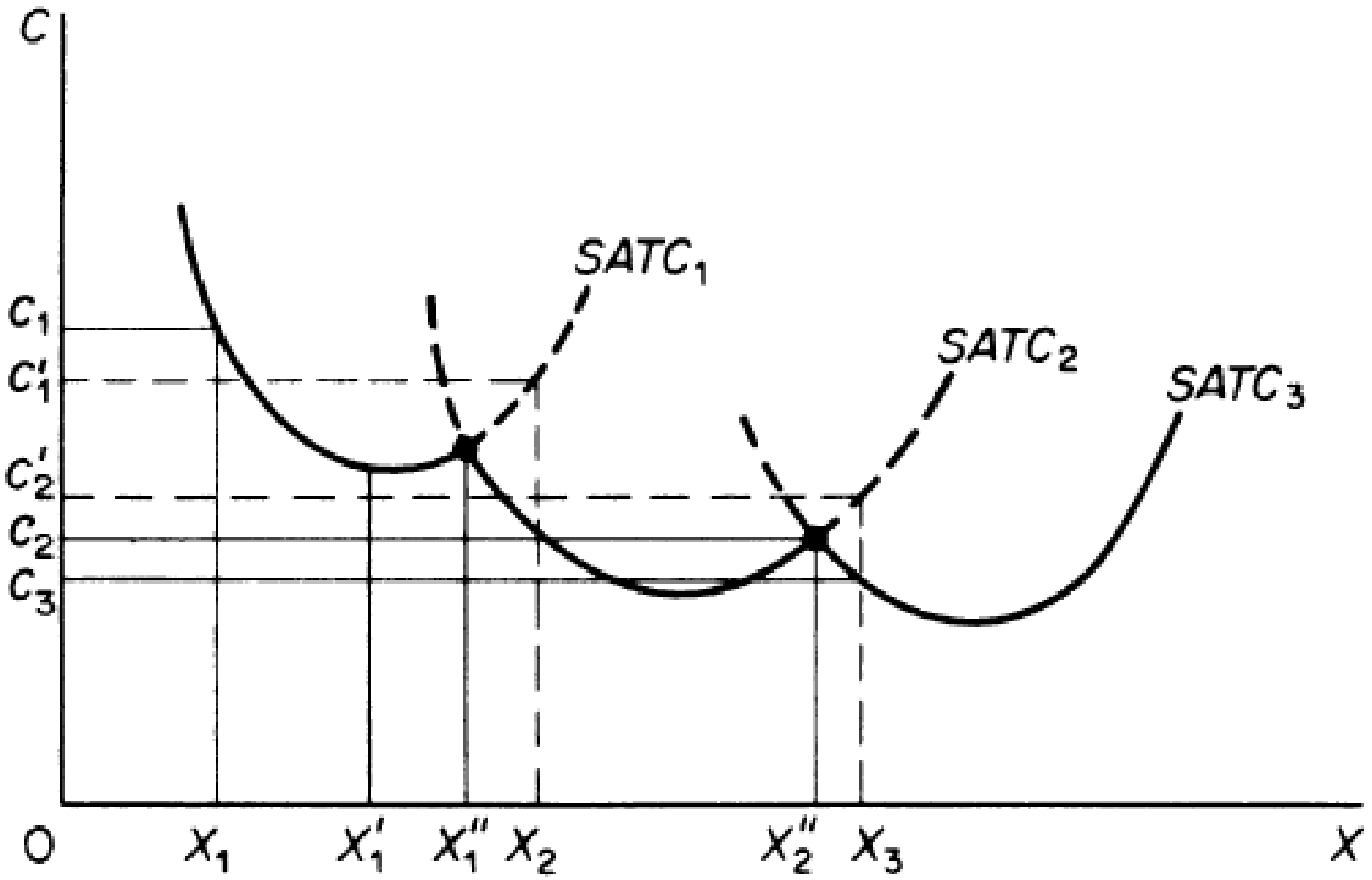
Given that $AC > 0$ and $X > 0$, the following results emerge:

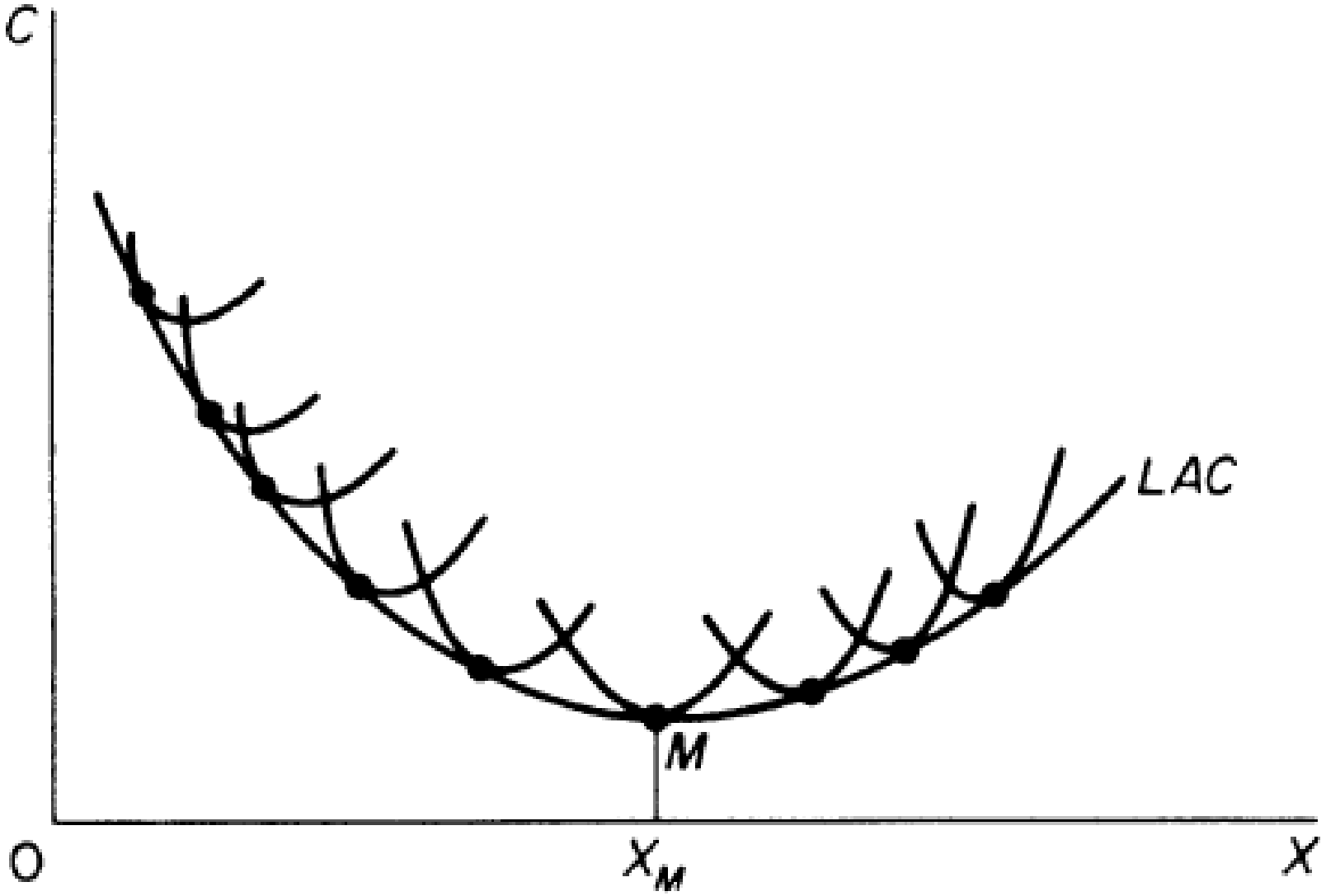
(*a*) if (slope of $AC$) $< 0$, then $MC < AC$
(*b*) if (slope of $AC$) $> 0$, then $MC > AC$
(*c*) if (slope of $AC$) $= 0$, then $MC = AC$

The slope of the $AC$ becomes zero at the minimum point of this curve (given that on theoretical grounds the $AC$ curve is U-shaped). Hence $MC = AC$ at the minimum point of the average-cost curve.

# Long-run Average Cost (LAC)

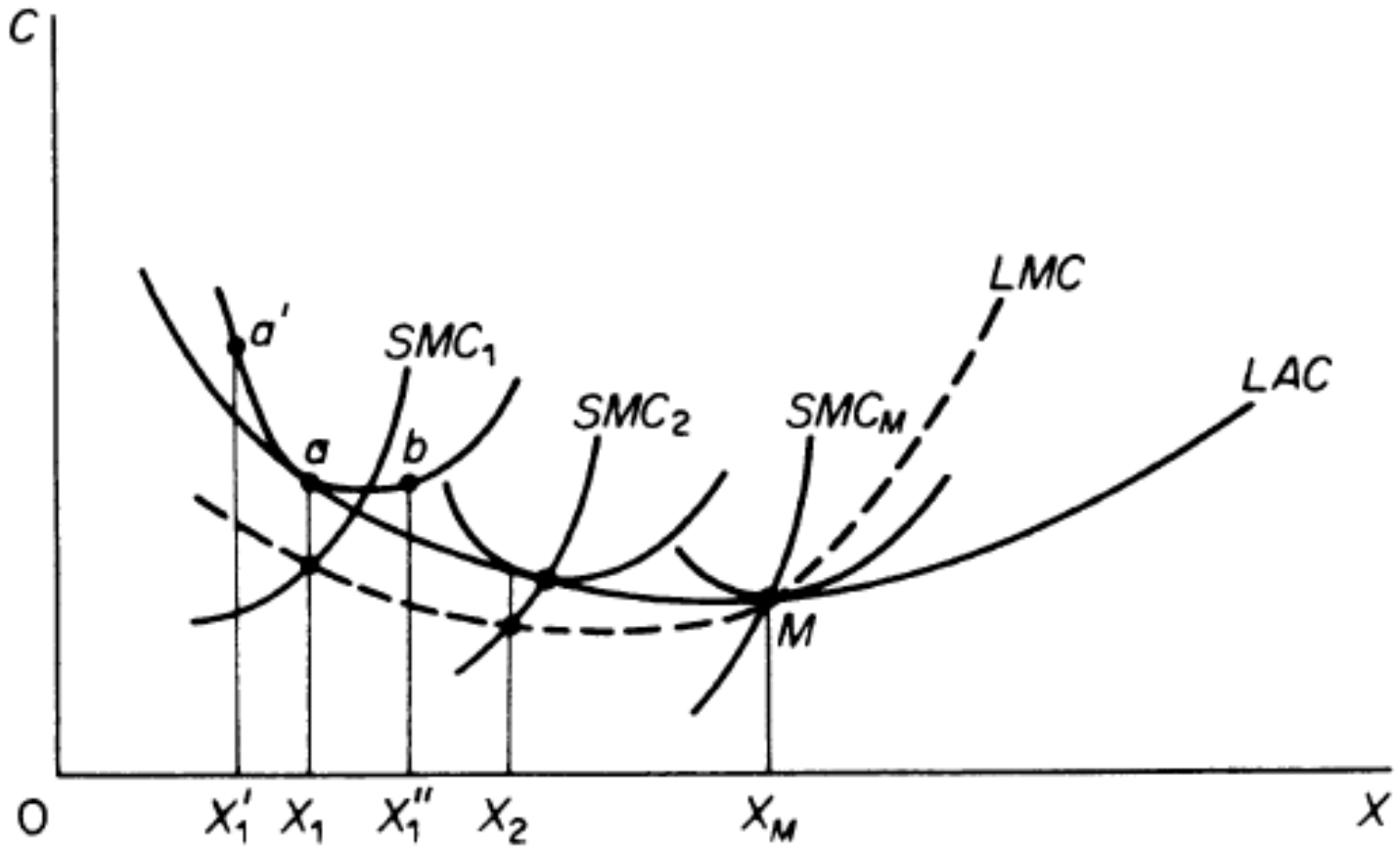
- In the long run all factors are assumed to become variable. We said that the long-run cost curve is a *planning curve*, in the sense that it is a guide to the entrepreneur in his decision to plan the future expansion of his output.
- The long-run average-cost curve is derived from short-run cost curves. Each point on the *LAC* corresponds to a point on a short-run cost curve (SATC), which is tangent to the *LAC* at that point.


C
SATC1
SATC2
SATC3
C1
C'1
C'2
C2
C3
O
X1
X'1
X''1
X2
X''2
X3
X


C
LAC
M
O
$X_M$
X

- The *LAC* curve is the locus of points denoting the least cost of producing the corresponding output. It is a planning curve because on the basis of this curve the firm decides what plant to set up in order to produce optimally (at minimum cost) the expected level of output.
- The firm chooses the short-run plant which allows it to produce the anticipated (in the long run) output at the least possible cost. In the traditional theory of the firm the *LAC* curve is U-shaped and it is often called the 'envelope curve' because it 'envelopes' the *SATC* curves.
- The U shape reflects the *laws of returns to scale*. According to these laws the unit costs of production decrease as plant size increases, due to the economies of scale which the larger plant sizes make possible.

# Long-run Marginal Cost



- The long-run marginal cost is derived from the *SMC* curves, but does not 'envelope' them. The *LMC* is formed from points of intersection of the *SMC* curves with vertical lines (to the *X* -axis) drawn from the points of tangency of the corresponding *SATC* curves and the *LAC* cost curve.
- The *LMC* must be equal to the *SMC* for the output at which the corresponding *SAC* is tangent to the *LAC*.
- At the minimum point *M* the *LMC* intersects the *LAC*. To the right of *M* the *LMC* lies above the *LAC* curve. At point *M* we have

$$C = \underbrace{b_0}_{TFC} + \underbrace{b_1 X - b_2 X^2 + b_3 X^3}_{TVC}$$

The $AVC$ is

$$AVC = \frac{TVC}{X} = b_1 - b_2 X + b_3 X^2$$

The $MC$ is

$$MC = \frac{\partial C}{\partial X} = b_1 - 2b_2 X + 3b_3 X^2$$

The $ATC$ is

$$\frac{C}{X} = \frac{b_0}{X} + b_1 - b_2 X + b_3 X^2$$

# Break-Even Analysis

- Break-even analysis refers to the point at which total costs and total revenue are equal.
- A break-even point analysis is used to determine the number of units or dollars of revenue needed to cover total costs.
- Break-even analysis is important to business owners and managers in determining how many units (or revenues) are needed to cover fixed and variable expenses of the business.

# Break-Even Analysis