Application of Simple Derivatives in Economics

1. Differential Coefficient and Price Elasticity of Demand

In economics, price elasticity of demand is defined as the value of the ratio of the relative (or proportionate) change in demand to the relative (or proportionate) change in price. Precisely, if we suppose that the demand changes from x to $(x + \Delta x)$ when the price changes from (p) to $(p + \Delta p)$, then elasticity of demand as per the definition is given by:

$$\frac{Proportionate\ change\ in\ quantity\ demanded}{Proportionate\ change\ in\ price} = \frac{\frac{\Delta x}{x}}{\frac{\Delta p}{p}} = \frac{p}{x} \times \frac{\Delta x}{\Delta p}$$

Using simple derivatives notation, we can also write it as follows:

$$e_d = \frac{p}{x} \times \frac{dx}{dp}$$

According to the law of demand, if price increases ($i.e.,\ \Delta p > 0$), demand falls ($\Delta x < 0$). This implies that the elasticity of demand is always negative that is:

$$e_d = (-)\frac{p}{x} \times \frac{dx}{dp}$$

The elasticity of demand (e_d) can also be written as:

$$e_d = (-)\frac{\frac{\Delta x}{\Delta p}}{\frac{x}{p}} = (-)\frac{Marginal\ Quantity\ Demanded}{Average\ Quantity\ Demanded}$$

1.1. Degrees of Price elasticity of Demand:

- **Inelastic demand (or Perfectly Inelastic Demand):** If the absolute value of the price elasticity of demand is zero that is $|e_d| = 0$, then the demand for such commodity is said to be price inelastic (or perfectly inelastic demand). Which means that the demand for such commodities is not responsive to changes in price. In other words, the demand curve is vertical.
- **Relatively inelastic demand:** If the absolute value of the price elasticity of demand lies between zero and one that is $0 < |e_d| < 1$, then then the demand for such commodity is said to be relatively price inelastic meaning proportionate change demand less than proportionate change in the price. If the price of a commodity rises by 100% then the demand for the commodity will fall by less than 100%.

- **Unitary elastic demand:**
  If the absolute value of the price elasticity of demand is one that is $|e_d| = 1$, then the demand for such commodity is said to be unitary elastic, meaning proportionate change in demand is equal to the proportionate change in price. In other words, if the price of a product rises by 50% then the quantity demand for the product will fall by the same percentage.
- **Relatively more elastic demand:**
  If the absolute value of the price elasticity of demand is one that is $|e_d| > 1$, then the demand for such commodity is said to be relatively more elastic, meaning proportionate change in demand is more to the proportionate change in price. In the case of a product having more elastic demand, a marginal change in price of a product can result in a large change in the demand. For example, a 5% change in the price of a product can result in a 20% change in demand.
- **Perfectly elastic demand:**
  If the absolute value of the price elasticity of demand is one that is $|e_d| = \infty$, then the demand for such commodity is said to be perfectly elastic.

Example 1:

- Find e_d if the demand function is $x = 25 - 4p + p^2$, where x is the demand for commodity at price p.

Solution:

Marginal function: $\frac{dx}{dp} = -4 + 2p$; and the Average function: $\frac{x}{p} = \frac{25-4p+p^2}{p}$

We know that:

$$|\mathrm{e_d}| = \frac{\text{Marginal function}}{\text{Average Function}} = \frac{(-4+2p)p}{(25-4p+p^2)}$$

The expression gives elasticity as a function of price (*p*). Suppose p is given to be 8; then the elasticity at $p = 8$ is:

$$|\mathrm{e_d}| = \frac{(-4+2(8))8}{(25-4(8)+8^2)} = 1.7\ approx.$$

Since, $|\mathrm{e_d}| > 1$, the demand for x is relatively more elastic.

Example 2:

The demand function is given as $x = 10 - p$ near the point $x = 4$ and $p = 6$. If the price increases by 5%; determine the percentage decrease in demand and hence an approximation to the elasticity of demand.

Solution:

At $p = 6$, price increases by 5%, hence increased price is

$$= \left(6 + \left[6 \times \frac{5}{100}\right]\right) = 6.30$$

$$Rise\ in\ price = 6 - 6.30 = 0.30$$

Corresponding to this increased price, the new demand according to the demand function is: $x = 10 - p = 10 - 6.30 = 3.70$.

The fall in demand is old demand minus new demand.

Old demand (demand at price 6): $x = 10 - p = 10 - 6 = 4$

Fall in demand is: $4 - 3.70 = 0.30$, which is 7.5% fall in demand.

$$e_d = \frac{Percentage\ change\ in\ quantity\ demand}{Percentage\ change\ in\ price} = \frac{-7.5\%}{5\%} = -1.5$$

$|e_d| > 1$, hence demand for commodity x is relatively more elastic.

The same results can be obtained with the help of calculus:

We have to find the elasticity at $x = 4$ and $p = 6$:

$$|e_d| = \left|\frac{p}{x} \times \frac{dx}{dp}\right| = \left|\frac{6}{4} \times \frac{d}{dp}(10 - p)\right| = \left|-\frac{6}{4}\right| = 1.5$$

Similarly, we can calculate the price elasticity demand for various demand functions at various prices using the above formulas of price elasticity of demand. The following are some demand functions for which we can calculate the price elasticity demand at various prices.

Questions for Review:

1. The demand function is given as: $p = \frac{1}{1+x^2}$. Compute the price elasticity of demand at $p = 5\ and\ 10$.
2. The demand function is given as: $x = \frac{8}{p^{3/2}}$. Determine the price elasticity of demand at $p = 2\ and\ 4$.