Viterbi Algorithm

Viterbi Algorithm

Decoding

- Q: what was the second sub-problem in HMM?
- Given $< O_1, O_2, O_3, ...O_T >$ and $\lambda =< A, B, \pi >$ what is the best possible sequence of states $\mathbf{Q}^*$ that generate the given observed sequence?
- $\mathbf{Q}^* =< S_1, S_2, S_3, ..., S_T >$

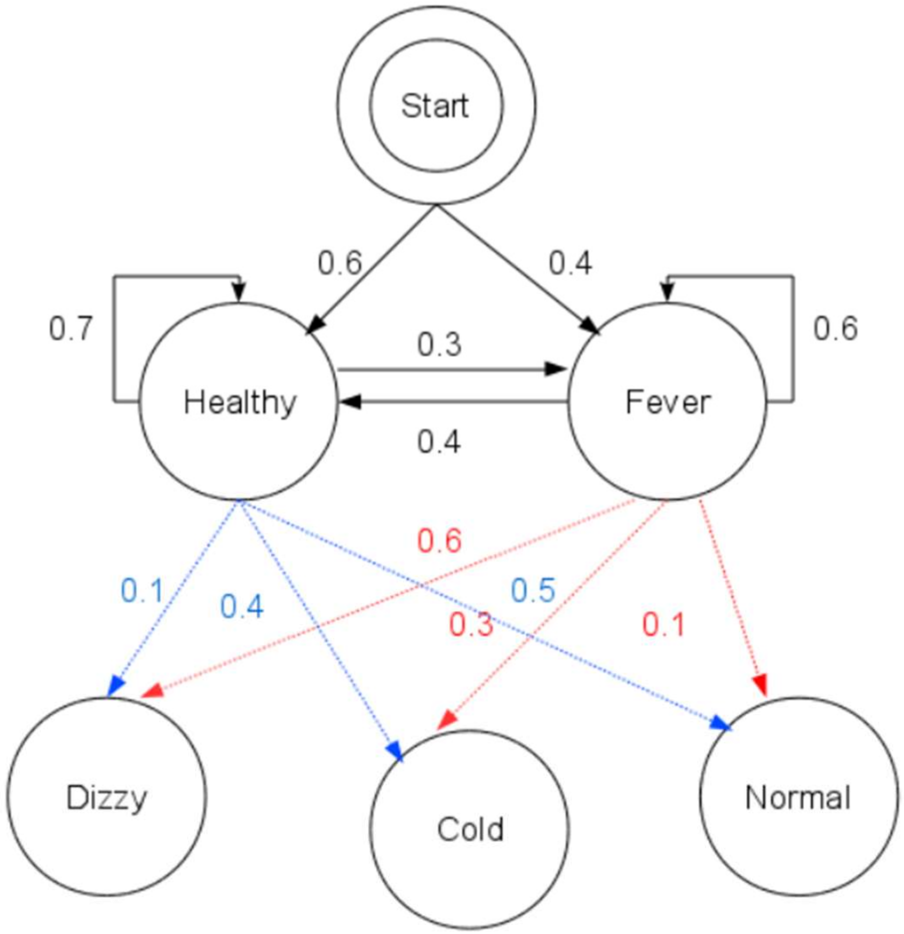
Viterbi Algorithm

Choose a path sequence that maximize $P(O|\lambda)$

$$\begin{aligned}\theta^* &= argmax_{\theta}[p(\theta|O,\lambda] \\ &\approx argmax_{\theta}[p(\theta, O|\lambda] \\ &= argmax_{\mathbf{S}}[p(S_1, S_2, ..., S_t, ..., S_T O_1, O_2, ..., O_t, ..., O_T|\lambda]\end{aligned}$$

Problem

- Consider a village where all villagers are either healthy or have a fever and only the village doctor can determine whether each has a fever. The doctor diagnoses fever by asking patients how they feel. The villagers may only answer that they feel normal, dizzy, or cold.
- There are two states, “Healthy” and “Fever”, but the doctor cannot observe them directly; they are hidden from him. On each day, there is a certain chance that the patient will tell the doctor he is “normal”, “cold”, or “dizzy”, depending on their health condition.
- The observations (normal, cold, dizzy) along with a hidden state (healthy, fever) form a hidden Markov model (HMM).
- The patient visits three days in a row and the doctor discovers that on the first day he feels normal, on the second day he feels cold, on the third day he feels dizzy. The doctor has a question: **what is the most likely sequence of health conditions of the patient that would explain these observations?**


Start
0.6
0.4
0.7
0.6
0.3
Healthy
Fever
0.4
0.6
0.1
0.4
0.5
0.3
0.1
Dizzy
Cold
Normal

Inputs:

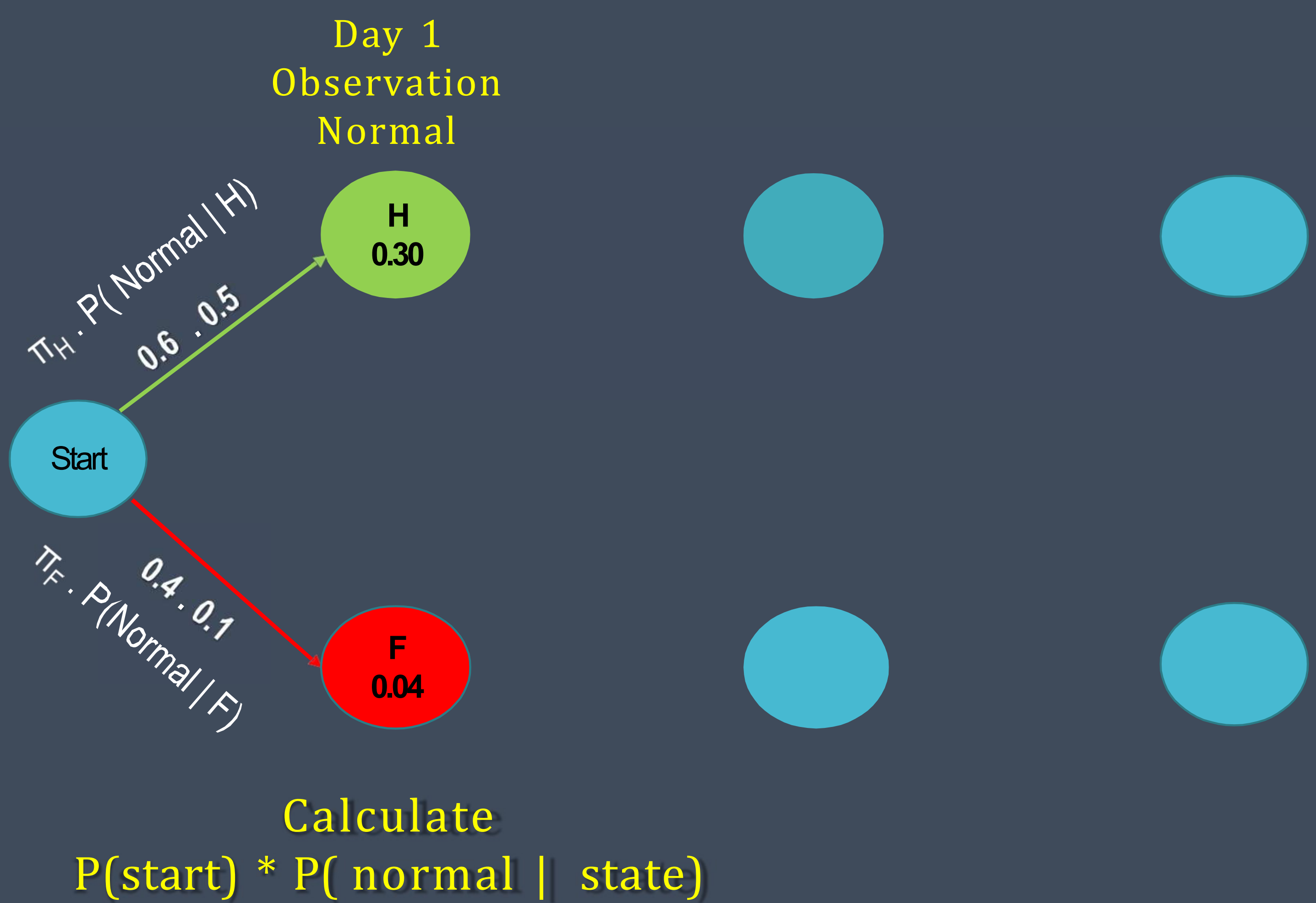
- States (S)='Healthy' , 'Fever'.
- Observation (O)='Normal' , 'Cold' , 'Dizzy'.
- Start_probability (π) = Healthy: 0.6, Fever: 0.4
- Transition Probability(A)=

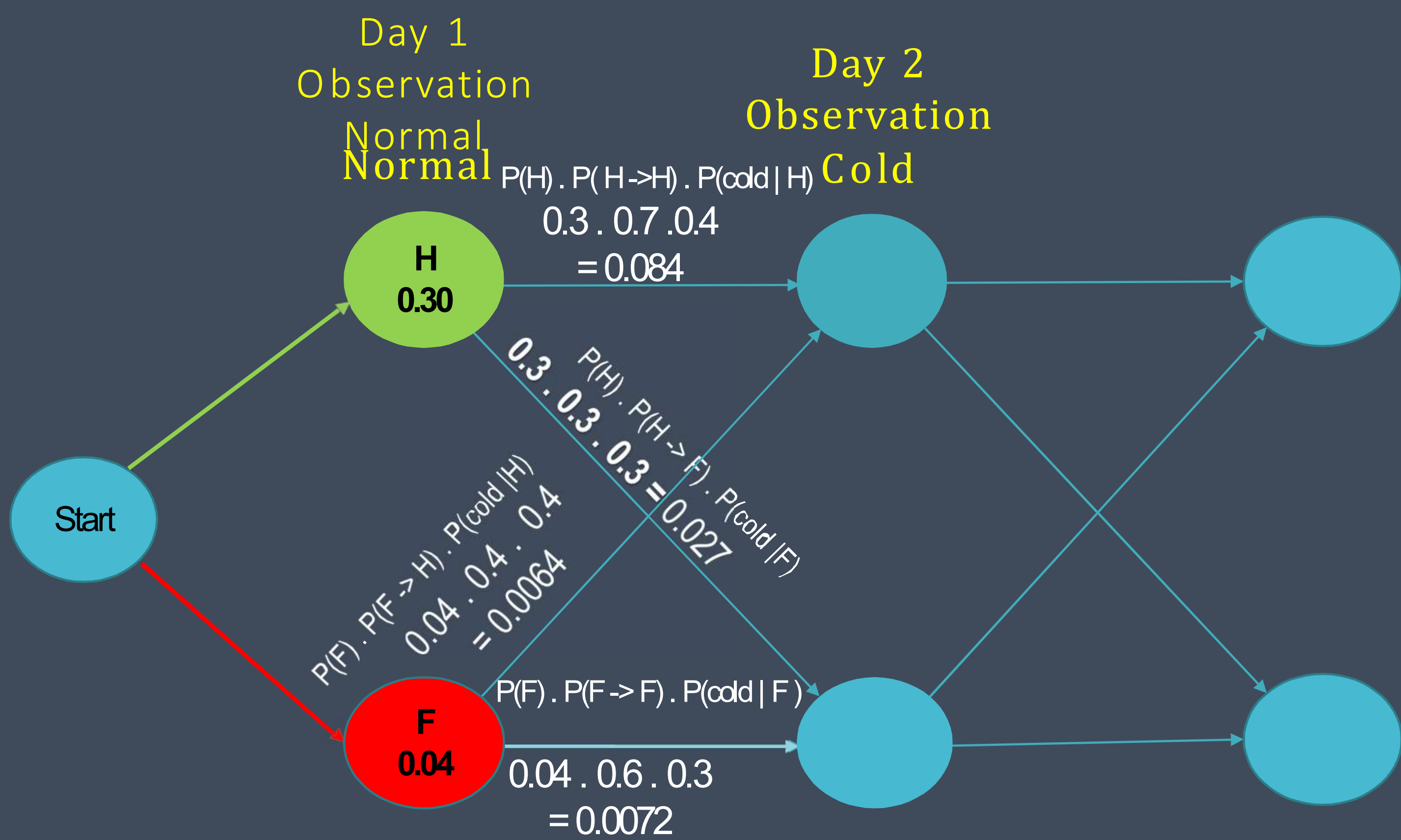
	Healthy	Fever
Healthy	0.7	0.3
Fever	0.4	0.6

- Emission Probability(B)=

	Normal	Cold	Dizzy
Healthy	0.5	0.4	0.1
Fever	0.1	0.3	0.6

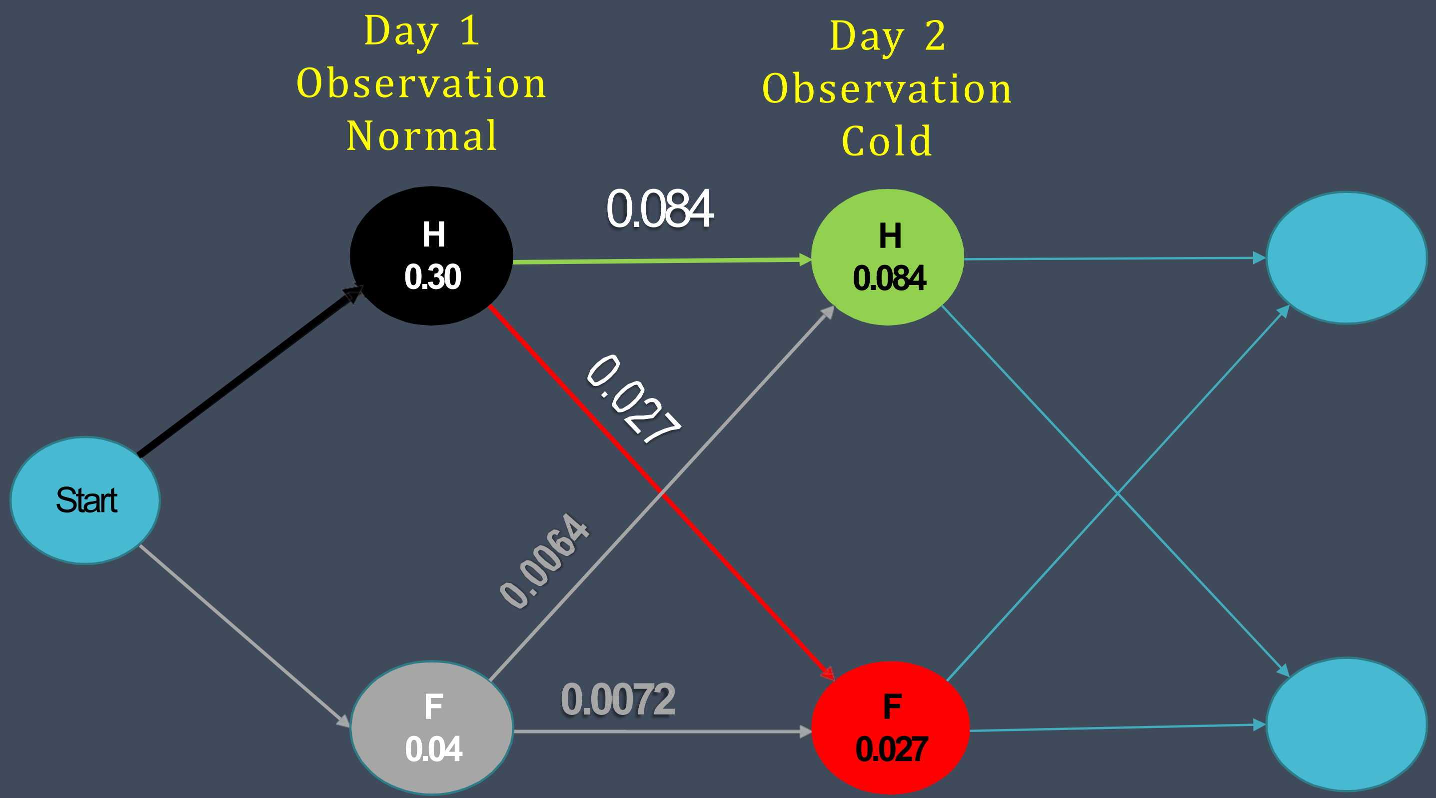
Operations


Day 1
Observation
Normal
π_H . P(Normal | H)
0.6 . 0.5
H
0.30
Start
π_F . P(Normal | F)
0.4 . 0.1
F
0.04
Calculate
P(start) * P(normal | state)

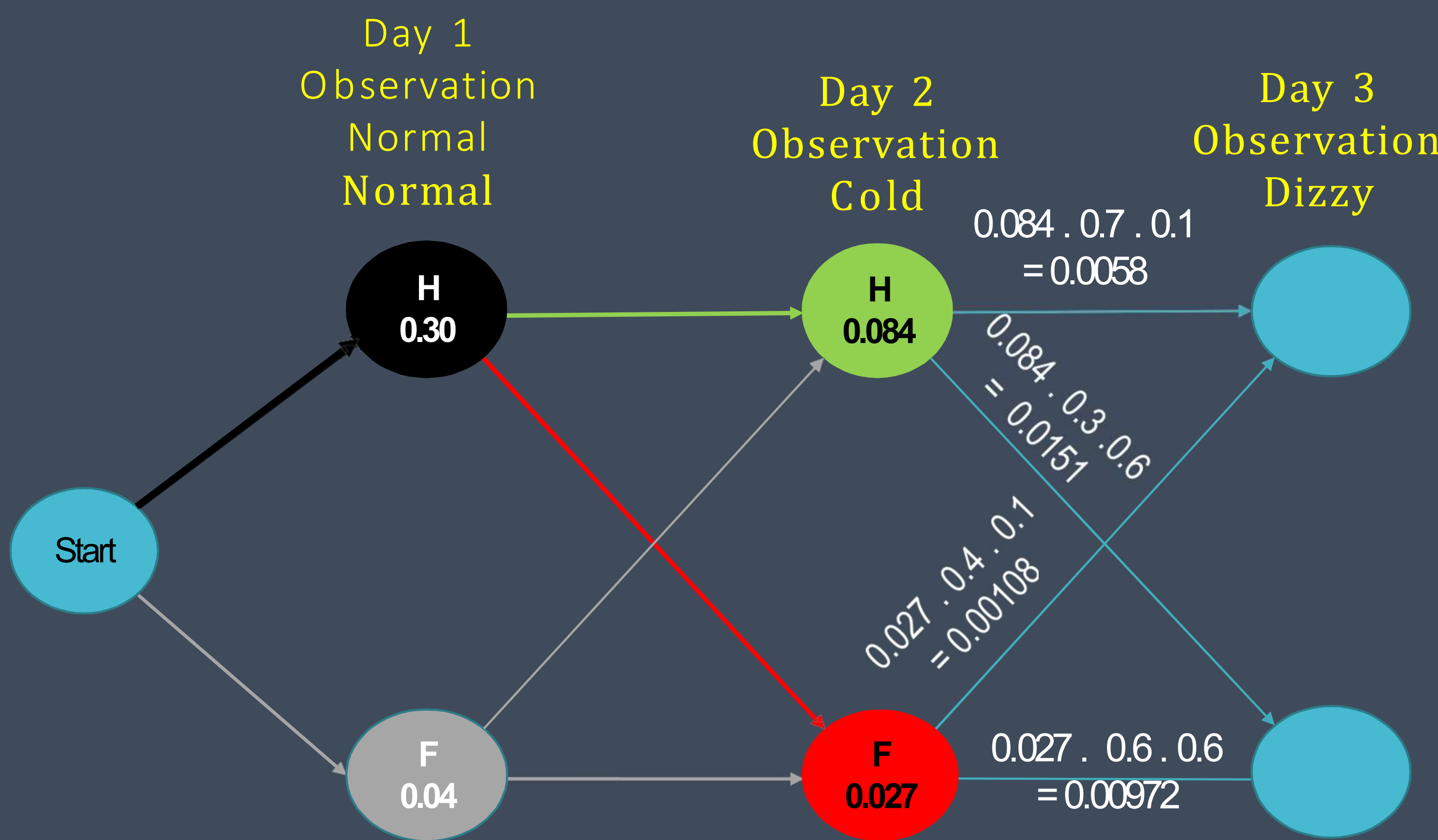


Calculate

P(old_state) * P(old_state -> new_state) * P(cold | new_state)

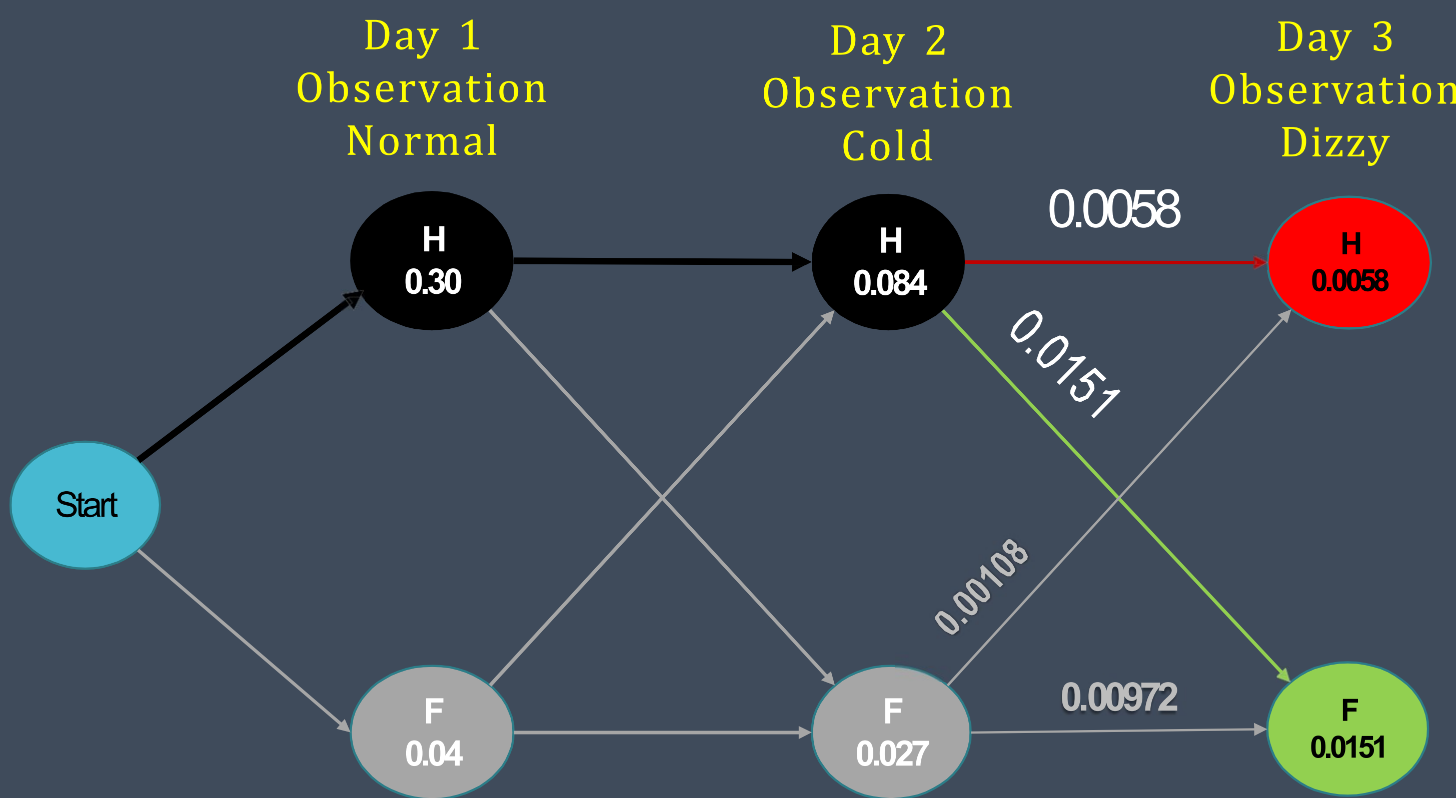


For each State H/F, Select the path with the Highest probability

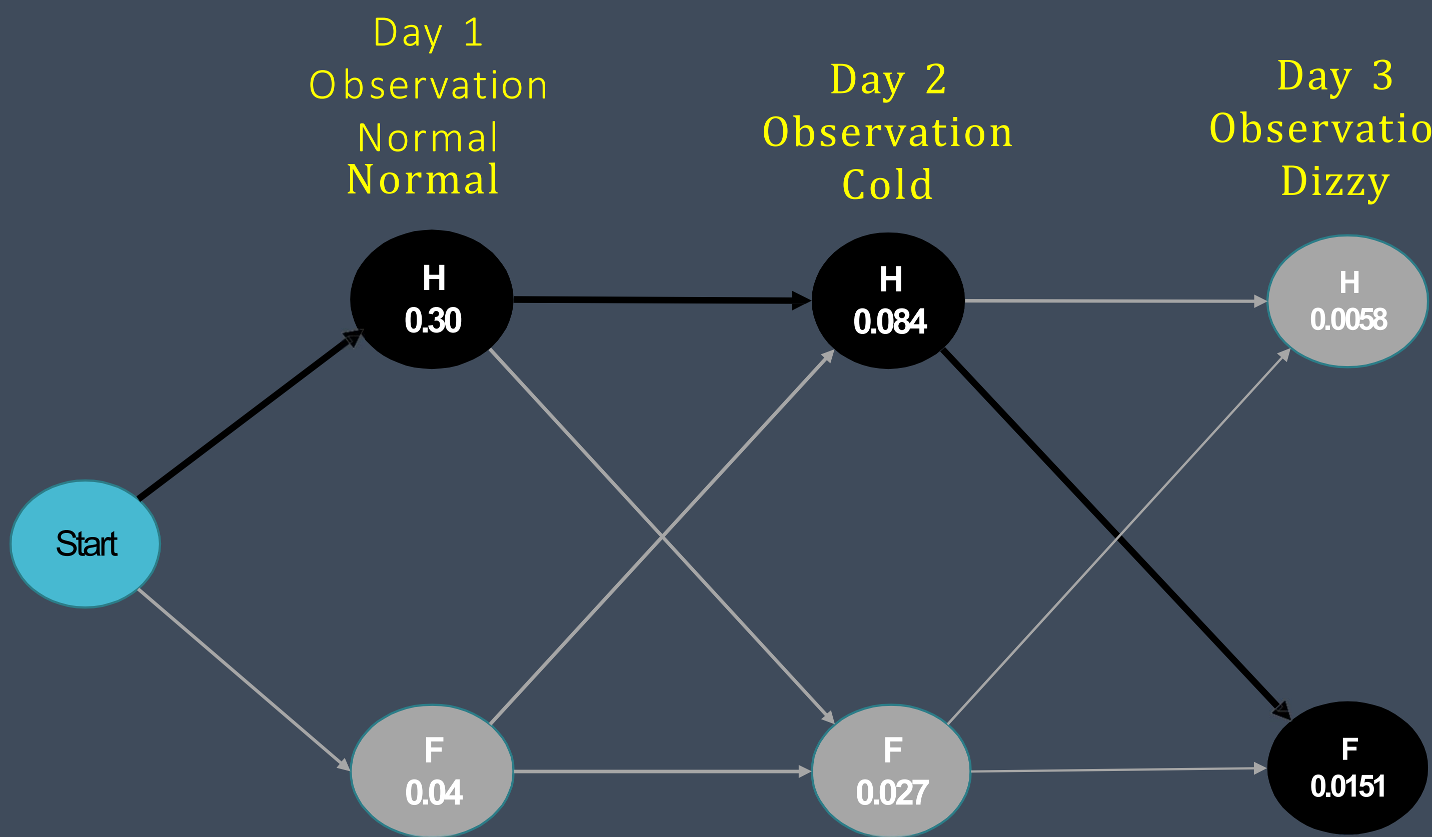


Calculate

P(old_state) * P(old_state -> new_state) *P(Dizzy | new_state)



For each State H/F, Select the path with the Highest probability



For time step T, select the state that has the highest probability and backtrack to the path that produced the highest probability using the backpointer and return the states.

Result

Day 1
Observation
Normal

(0.30)
"HEALTHY"

Day 2
Observation
Cold

(0.084)
"HEALTHY"

Day 3
Observation
Dizzy

(0.0151)
"FEVER"

Compute the hidden state sequence for the given observation.

	B	I	O
B	0	0.5	0.5
I	.1	0	0.9
O	0.2	0	0.8

	United	States	live	in
B	0.8	0.3	0	0
I	0.1	0.6	0.1	0.1
O	0.1	0.1	0.9	0.9

	π
B	0.2
I	0
O	0.8

To decode:
live in United States