Slope of Indifference Curve

Suppose an individual has a utility function $U = f(x, y)$

So, the total differential will be:

$$dU = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy \tag{1}$$

We know that all the points on the indifference curve provides same level of utility, which means that changes utility from one point to another on the indifference curve is zero, meaning $dU = 0$.

Also, we know that marginal utility of x is the changes in total utility due to change in x. Same definition for the marginal utility of y. So, we can define $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ as follows:

$$\frac{\partial f}{\partial x} = Marginal\ utility\ of\ x = MU_x$$

$$\frac{\partial f}{\partial y} = Marginal\ utility\ of\ y = MU_y$$

Substituting the above information in equation (1), we have

$$0 = MU_x dx + MU_y dy$$

Solving this equation, we can obtain the slope of the indifference curve.

$$\frac{dy}{dx} = -\frac{MU_x}{MU_y}$$

Negative sign of the slope indicates that the indifference curve is downward sloping.

Slope of the Budget line

Suppose the individual has a budget constraint as follows:

$$p_x x + p_y y = I$$

where, p_x and p_y are the price of good x and y, respectively and I is the total income.

Solving for, y we have:

$$y = I - \frac{p_x}{p_y}x$$

So, the slope of budget line

$$\frac{dy}{dx} = -\frac{p_x}{p_y}$$

The negative slope indicate that the budget line is downward sloping.

Consumers equilibrium

An induvial maximizes his/her utility when indifference curve is tangent to the budget line. In other words, at the point of equilibrium the slope of indifference curve is equals to the slope of the budget line.

$$-\frac{MU_x}{MU_y} = -\frac{p_x}{p_y}$$