

# 3

# SOLVING PROBLEMS BY SEARCHING

*In which we see how an agent can find a sequence of actions that achieves its goals when no single action will do.*

PROBLEM-SOLVING  
AGENT

The simplest agents discussed in Chapter 2 were the reflex agents, which base their actions on a direct mapping from states to actions. Such agents cannot operate well in environments for which this mapping would be too large to store and would take too long to learn. Goal-based agents, on the other hand, consider future actions and the desirability of their outcomes.

This chapter describes one kind of goal-based agent called a **problem-solving agent**. Problem-solving agents use **atomic** representations, as described in Section 2.4.7—that is, states of the world are considered as wholes, with no internal structure visible to the problem-solving algorithms. Goal-based agents that use more advanced **factored** or **structured** representations are usually called **planning agents** and are discussed in Chapters 7 and 10.

Our discussion of problem solving begins with precise definitions of **problems** and their **solutions** and give several examples to illustrate these definitions. We then describe several general-purpose search algorithms that can be used to solve these problems. We will see several **uninformed** search algorithms—algorithms that are given no information about the problem other than its definition. Although some of these algorithms can solve any solvable problem, none of them can do so efficiently. **Informed** search algorithms, on the other hand, can do quite well given some guidance on where to look for solutions.

In this chapter, we limit ourselves to the simplest kind of task environment, for which the solution to a problem is always a *fixed sequence* of actions. The more general case—where the agent’s future actions may vary depending on future percepts—is handled in Chapter 4.

This chapter uses the concepts of asymptotic complexity (that is,  $O()$  notation) and NP-completeness. Readers unfamiliar with these concepts should consult Appendix A.

## 3.1 PROBLEM-SOLVING AGENTS

Intelligent agents are supposed to maximize their performance measure. As we mentioned in Chapter 2, achieving this is sometimes simplified if the agent can adopt a **goal** and aim at satisfying it. Let us first look at why and how an agent might do this.

GOAL FORMULATION

Imagine an agent in the city of Arad, Romania, enjoying a touring holiday. The agent’s performance measure contains many factors: it wants to improve its suntan, improve its Romanian, take in the sights, enjoy the nightlife (such as it is), avoid hangovers, and so on. The decision problem is a complex one involving many tradeoffs and careful reading of guidebooks. Now, suppose the agent has a nonrefundable ticket to fly out of Bucharest the following day. In that case, it makes sense for the agent to adopt the **goal** of getting to Bucharest. Courses of action that don’t reach Bucharest on time can be rejected without further consideration and the agent’s decision problem is greatly simplified. Goals help organize behavior by limiting the objectives that the agent is trying to achieve and hence the actions it needs to consider. **Goal formulation**, based on the current situation and the agent’s performance measure, is the first step in problem solving.

PROBLEM FORMULATION

We will consider a goal to be a set of world states—exactly those states in which the goal is satisfied. The agent’s task is to find out how to act, now and in the future, so that it reaches a goal state. Before it can do this, it needs to decide (or we need to decide on its behalf) what sorts of actions and states it should consider. If it were to consider actions at the level of “move the left foot forward an inch” or “turn the steering wheel one degree left,” the agent would probably never find its way out of the parking lot, let alone to Bucharest, because at that level of detail there is too much uncertainty in the world and there would be too many steps in a solution. **Problem formulation** is the process of deciding what actions and states to consider, given a goal. We discuss this process in more detail later. For now, let us assume that the agent will consider actions at the level of driving from one major town to another. Each state therefore corresponds to being in a particular town.

Our agent has now adopted the goal of driving to Bucharest and is considering where to go from Arad. Three roads lead out of Arad, one toward Sibiu, one to Timisoara, and one to Zerind. None of these achieves the goal, so unless the agent is familiar with the geography of Romania, it will not know which road to follow.<sup>1</sup> In other words, the agent will not know which of its possible actions is best, because it does not yet know enough about the state that results from taking each action. If the agent has no additional information—i.e., if the environment is **unknown** in the sense defined in Section 2.3—then it is has no choice but to try one of the actions at random. This sad situation is discussed in Chapter 4.



But suppose the agent has a map of Romania. The point of a map is to provide the agent with information about the states it might get itself into and the actions it can take. The agent can use this information to consider *subsequent* stages of a hypothetical journey via each of the three towns, trying to find a journey that eventually gets to Bucharest. Once it has found a path on the map from Arad to Bucharest, it can achieve its goal by carrying out the driving actions that correspond to the legs of the journey. In general, *an agent with several immediate options of unknown value can decide what to do by first examining future actions that eventually lead to states of known value*.

To be more specific about what we mean by “examining future actions,” we have to be more specific about properties of the environment, as defined in Section 2.3. For now,

<sup>1</sup> We are assuming that most readers are in the same position and can easily imagine themselves to be as clueless as our agent. We apologize to Romanian readers who are unable to take advantage of this pedagogical device.

we assume that the environment is **observable**, so the agent always knows the current state. For the agent driving in Romania, it's reasonable to suppose that each city on the map has a sign indicating its presence to arriving drivers. We also assume the environment is **discrete**, so at any given state there are only finitely many actions to choose from. This is true for navigating in Romania because each city is connected to a small number of other cities. We will assume the environment is **known**, so the agent knows which states are reached by each action. (Having an accurate map suffices to meet this condition for navigation problems.) Finally, we assume that the environment is **deterministic**, so each action has exactly one outcome. Under ideal conditions, this is true for the agent in Romania—it means that if it chooses to drive from Arad to Sibiu, it does end up in Sibiu. Of course, conditions are not always ideal, as we show in Chapter 4.



*Under these assumptions, the solution to any problem is a fixed sequence of actions.* “Of course!” one might say, “What else could it be?” Well, in general it could be a branching strategy that recommends different actions in the future depending on what percepts arrive. For example, under less than ideal conditions, the agent might plan to drive from Arad to Sibiu and then to Rimnicu Vilcea but may also need to have a contingency plan in case it arrives by accident in Zerind instead of Sibiu. Fortunately, if the agent knows the initial state and the environment is known and deterministic, it knows exactly where it will be after the first action and what it will perceive. Since only one percept is possible after the first action, the solution can specify only one possible second action, and so on.

SEARCH  
SOLUTION  
EXECUTION

OPEN-LOOP

The process of looking for a sequence of actions that reaches the goal is called **search**. A search algorithm takes a problem as input and returns a **solution** in the form of an action sequence. Once a solution is found, the actions it recommends can be carried out. This is called the **execution** phase. Thus, we have a simple “formulate, search, execute” design for the agent, as shown in Figure 3.1. After formulating a goal and a problem to solve, the agent calls a search procedure to solve it. It then uses the solution to guide its actions, doing whatever the solution recommends as the next thing to do—typically, the first action of the sequence—and then removing that step from the sequence. Once the solution has been executed, the agent will formulate a new goal.

Notice that while the agent is executing the solution sequence it *ignores its percepts* when choosing an action because it knows in advance what they will be. An agent that carries out its plans with its eyes closed, so to speak, must be quite certain of what is going on. Control theorists call this an **open-loop** system, because ignoring the percepts breaks the loop between agent and environment.

We first describe the process of problem formulation, and then devote the bulk of the chapter to various algorithms for the **SEARCH** function. We do not discuss the workings of the **UPDATE-STATE** and **FORMULATE-GOAL** functions further in this chapter.

### 3.1.1 Well-defined problems and solutions

PROBLEM  
INITIAL STATE

A **problem** can be defined formally by five components:

- The **initial state** that the agent starts in. For example, the initial state for our agent in Romania might be described as *In(Arad)*.

```

function SIMPLE-PROBLEM-SOLVING-AGENT(percept) returns an action
  persistent: seq, an action sequence, initially empty
    state, some description of the current world state
    goal, a goal, initially null
    problem, a problem formulation

  state  $\leftarrow$  UPDATE-STATE(state, percept)
  if seq is empty then
    goal  $\leftarrow$  FORMULATE-GOAL(state)
    problem  $\leftarrow$  FORMULATE-PROBLEM(state, goal)
    seq  $\leftarrow$  SEARCH(problem)
    if seq = failure then return a null action
  action  $\leftarrow$  FIRST(seq)
  seq  $\leftarrow$  REST(seq)
  return action

```

**Figure 3.1** A simple problem-solving agent. It first formulates a goal and a problem, searches for a sequence of actions that would solve the problem, and then executes the actions one at a time. When this is complete, it formulates another goal and starts over.

- |                  |  |
|------------------|--|
| ACTIONS          | • A description of the possible <b>actions</b> available to the agent. Given a particular state <i>s</i> , ACTIONS( <i>s</i> ) returns the set of actions that can be executed in <i>s</i> . We say that each of these actions is <b>applicable</b> in <i>s</i> . For example, from the state <i>In(Arad)</i> , the applicable actions are $\{Go(Sibiu), Go(Timisoara), Go(Zerind)\}$ .  |
| APPLICABLE       |  |
| TRANSITION MODEL | • A description of what each action does; the formal name for this is the <b>transition model</b> , specified by a function RESULT( <i>s, a</i> ) that returns the state that results from doing action <i>a</i> in state <i>s</i> . We also use the term <b>successor</b> to refer to any state reachable from a given state by a single action. <sup>2</sup> For example, we have  |
| SUCCESSOR        | $\text{RESULT}(\text{In}(Arad), \text{Go}(Zerind)) = \text{In}(Zerind)$ .  |
| STATE SPACE      | Together, the initial state, actions, and transition model implicitly define the <b>state space</b> of the problem—the set of all states reachable from the initial state by any sequence of actions. The state space forms a directed network or <b>graph</b> in which the nodes are states and the links between nodes are actions. (The map of Romania shown in Figure 3.2 can be interpreted as a state-space graph if we view each road as standing for two driving actions, one in each direction.) A <b>path</b> in the state space is a sequence of states connected by a sequence of actions. |
| GRAPH            |  |
| PATH             |  |
| GOAL TEST        | • The <b>goal test</b> , which determines whether a given state is a goal state. Sometimes there is an explicit set of possible goal states, and the test simply checks whether the given state is one of them. The agent’s goal in Romania is the singleton set $\{\text{In}(Bucharest)\}$ .  |

<sup>2</sup> Many treatments of problem solving, including previous editions of this book, use a **successor function**, which returns the set of all successors, instead of separate ACTIONS and RESULT functions. The successor function makes it difficult to describe an agent that knows what actions it can try but not what they achieve. Also, note some authors use RESULT(*a, s*) instead of RESULT(*s, a*), and some use DO instead of RESULT.

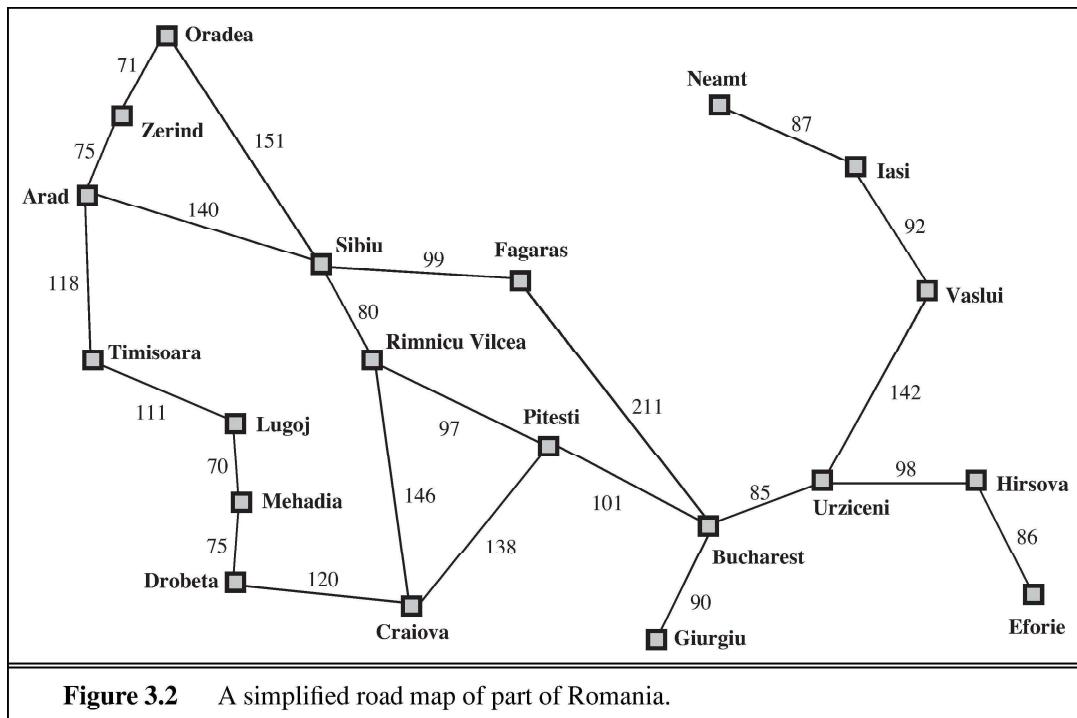


Figure 3.2 A simplified road map of part of Romania.

Sometimes the goal is specified by an abstract property rather than an explicitly enumerated set of states. For example, in chess, the goal is to reach a state called “checkmate,” where the opponent’s king is under attack and can’t escape.

#### PATH COST

- A **path cost** function that assigns a numeric cost to each path. The problem-solving agent chooses a cost function that reflects its own performance measure. For the agent trying to get to Bucharest, time is of the essence, so the cost of a path might be its length in kilometers. In this chapter, we assume that the cost of a path can be described as the *sum* of the costs of the individual actions along the path.<sup>3</sup> The **step cost** of taking action  $a$  in state  $s$  to reach state  $s'$  is denoted by  $c(s, a, s')$ . The step costs for Romania are shown in Figure 3.2 as route distances. We assume that step costs are nonnegative.<sup>4</sup>

#### STEP COST

#### OPTIMAL SOLUTION

The preceding elements define a problem and can be gathered into a single data structure that is given as input to a problem-solving algorithm. A **solution** to a problem is an action sequence that leads from the initial state to a goal state. Solution quality is measured by the path cost function, and an **optimal solution** has the lowest path cost among all solutions.

### 3.1.2 Formulating problems

In the preceding section we proposed a formulation of the problem of getting to Bucharest in terms of the initial state, actions, transition model, goal test, and path cost. This formulation seems reasonable, but it is still a *model*—an abstract mathematical description—and not the

<sup>3</sup> This assumption is algorithmically convenient but also theoretically justifiable—see page 649 in Chapter 17.

<sup>4</sup> The implications of negative costs are explored in Exercise 3.8.