Team notebook

September 27, 2021

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```
#define trace(...) __f(#__VA_ARGS__,
    __VA_ARGS__)
template <typename Arg1>
void __f(const char* name, Arg1&& arg1){
    cerr << name << " : " << arg1 << std::endl;</pre>
template <typename Arg1, typename... Args>
void __f(const char* names, Arg1&& arg1,
    Args&&... args){
    const char* comma = strchr(names + 1,
        ','):cerr.write(names, comma - names)
        << " : " << arg1<<" | ";__f(comma+1,
        args...);
#else
#define trace(...)
#endif
//FILE *fin = freopen("in", "r", stdin);
//FILE *fout = freopen("out","w",stdout);
const int N = int(2e5)+1;
const int M = int(2e5)+1;
const int LOGN = 20;
VI g[N], tree[N], st;//graph in edge-list form.
    N should be 2*N
int
    U[M], V[M], low[N], ord[N], sz[N], depth[N], col[N], ¢, T, compMonfNi pNevel[N], ord[N], DP[LOGN][N];
bool isArtic[N];
int arr[N],dep[N],vis[N];
int adj(int u,int e){
    return u^V[e]^U[e];
//everything from [1,n+C] whose extra[i]=0 is
    part of Block-Tree
//1-Based Graph Input.Everything from [1,C]
    is type B and [C,n+C] is type C.
void dfs(int i){
   low[i]=ord[i]=T++;
    for(int j=0;j<SZ(g[i]);j++){</pre>
       int ei=g[i][j],to = adj(i,ei);
       if(ord[to] ==-1){
           depth[to] = depth[i] + 1;
           st.PB(ei);dfs(to);
           low[i] = min(low[i],low[to]);
           if(ord[i] == 0 | | low[to] >= ord[i]) {
```

```
if(ord[i]!=0||j>=1)
                  isArtic[i] = true;
               ++C:
               while(!st.empty()){
                  int
                       fi=st.back();st.pop_back();
                  col[fi]=C:
                  if(fi==ei)break:
               }
           }
       }else if(depth[to]<depth[i]-1){</pre>
           low[i] = min(low[i],ord[to]);
           st.PB(ei):
       }
   }
}
void run(int n){
   SET(low,-1); SET(depth,-1);
   SET(ord,-1);SET(col,-1);
   SET(isArtic,0);st.clear();C=0;
   for(int i=1;i<=n;++i)</pre>
       if(ord[i]==-1){
           T = 0; dfs(i);
       }
void buildTree(int n){
   VI tmpv;SET(extra,-1);
   tmpv.clear();SET(sz,0);
   for(int i=1;i<=n;i++){</pre>
       tmpv.clear();
       for(auto e:g[i])
           tmpv.PB(col[e]);
       sort(ALL(tmpv));
       tmpv.erase(unique(ALL(tmpv)),
           tmpv.end());
       //handle isolated vertics
       if(tmpv.empty()){
           compNo[i]=C+i;extra[C+i]=0;
           sz[C+i]=1;continue;
       }if(SZ(tmpv)==1){//completely in 1
           compNo[i]=tmpv[0];
           extra[tmpv[0]]=0;
           sz[tmpv[0]]++;
```

```
}else{ //it's an articulation vertex.
           compNo[i]=C+i;
           extra[C+i]=0;sz[C+i]++;
           for(auto j:tmpv){
               extra[i]=0;sz[i]++;
               tree[C+i].push_back(j);
               tree[j].push_back(C+i);
           }
       }
   }
}
int currComp;
void dfs2(int u,int p){
   level[u] = level[p] +1; DP[0][u] = p;
    arr[u]=++T; vis[u]=currComp;
   for(auto w:tree[u])
       if(w!=p)
           dfs2(w,u);
    dep[u]=T++;
int lca(int a,int b){
    if(level[a]>level[b])swap(a,b);
    int d = level[b]-level[a];
   for(int i=0:i<LOGN:i++)</pre>
       if((1<<i)&d)
           b = DP[i][b]:
    if(a==b)return a:
   for(int i=LOGN-1;i>=0;i--)
       if(DP[i][a]!=DP[i][b])
           a=DP[i][a],b=DP[i][b];
    return DP[0][a];
bool anc(int p,int u){
    return (arr[u]>=arr[p] && dep[u]<=dep[p]);</pre>
}
int main()
    int n,m,q;
    si(n);si(m);si(q);
   for(int i=0;i<m;i++){</pre>
       scanf("%d %d",U+i,V+i);
       g[U[i]].PB(i);
       g[V[i]].PB(i);
    buildTree(n);T=0;
```

```
for(int i=1:i<=C+n:i++)</pre>
   if(!vis[i] && !extra[i])
       currComp++,dfs2(i,i);
for(int i=1;i<LOGN;i++)</pre>
   for(int j=1; j<=C+n; j++)</pre>
       if(!extra[j])
           DP[i][j]=DP[i-1][DP[i-1][j]];
while(q--){
   int u,v,w;
   si(u);si(v);si(w);
   if(u==v){
       puts(u==w?"Party":"Break-Up");
       continue;
   u=compNo[u];v=compNo[v];w=compNo[w];
   if(!(vis[u]==vis[w] &&
        vis[w] == vis[v])){
       puts("Break-Up");
       continue:
   int LCA = lca(u,v);
   if(level[u]>level[v])swap(u,v);
   if(sz[w] == 1 && w! = LCA && w! = DP[0][LCA]
        && sz[DP[0][w]]>2) w = DP[0][w]:
   if(sz[u]==1 && u!=LCA &&
        sz[DP[0][w]]>2) u = DP[0][u]:
   if(sz[v]==1 && v!=LCA &&
        sz[DP[0][v]]>2) v = DP[0][v];
   bool ok=false:
   ok = anc(w,u);
   ok = anc(w,v);
   ok&=anc(LCA.w):
   ok = (sz[LCA] > 2 \&\& w == DP[0][LCA]);
   puts(ok?"Party":"Break-Up");
}
return 0;
```

2 Centroid

}

```
#include <bits/stdc++.h>
#define X first
```

```
#define Y second
#define pb push_back
using namespace std;
typedef pair<int, int> pii;
typedef pair<pii, int> ppi;
const int maxn = 2e5 + 17, 1g = 18;
int n = 1, q, par[maxn][lg], cpar[maxn],
    h[maxn], sz[maxn];
set<ppi> s[maxn];
vector<int> g[maxn]. ch[maxn]:
struct Q{
 int t, v, d;
} qu[maxn];
void prep(int v = 0){
 sz[v] = 1:
 for(auto u : g[v]){
   prep(u);
   sz[v] += sz[u]:
}
int get_cent(int root = 0){
 int v = root, size = sz[root];
 bool done = 0:
 while(done ^= 1)
   for(auto &u : g[v])
     if(sz[u] > (size >> 1)){
       v = u, done = 0;
       break;
     }
 int mysz = sz[v];
 for(int u = v; ; u = par[u][0]){
   sz[u] -= mysz;
   if(u == root) break:
 for(auto &u : g[v])
   if(sz[u]){
     int x = get_cent(u);
     //cerr << v << ', ', << x << '\n';
     cpar[x] = v;
     ch[v].pb(x);
   }
 if(v != root){
   int x = get_cent(root);
   //cerr << v << ', ', << x << '\n';
```

```
cpar[x] = v;
    ch[v].pb(x);
  return v;
int dis(int v, int u){
   if(h[u] < h[v]) swap(v, u);
    int ans = h[v] + h[u];
   for(int i = 0; i < lg; i++)</pre>
  if((h[u] - h[v]) >> i & 1)
      u = par[u][i]:
   for(int i = lg - 1; i >= 0; i--)
  if(par[v][i] != par[u][i])
      v = par[v][i], u = par[u][i];
   return v == u ? ans - 2 * h[v] : ans - 2 *
        (h[v] - 1):
}
void add(int v){
  for(int u = v; u != -1; u = cpar[u]){
   if(v == 6)
      ;//cerr << u << '\n';
    int d = dis(u, v);
    auto it = s[u].lower_bound({{d + 1, -1}},
    if(it != s[u].begin() && prev(it) -> X.Y
        >= h[v])
     continue:
    it = s[u].insert({{d, h[v]}, v}).X;
    while(it != s[u].end() && it -> X.Y <=</pre>
        h[v])
      s[u].erase(prev(++it));
int get(int v, int d){
  int ans = -1, cer = -1;
  for(int u = v; u != -1; u = cpar[u]){
    int di = dis(u, v);
   //cerr << u << '\n':
   auto it = s[u].lower_bound({{d - di + 1,}}
        -1}, -1});
   if(it != s[u].begin()){
     it--;
      if(it \rightarrow X.Y > ans)
       ans = it \rightarrow X.Y, cer = it \rightarrow Y;
```

```
}
                                                    int get_cent(int root = 0, int h = 0){
                                                      int v = root, size = sz[root];
 return cer;
                                                      bool done = 0;
                                                      while(done ^= 1)
                                                        for(auto &e : g[v])
                                                         if(e.X != par[v][0] && sz[e.X] > (size
v = e.X. done = 0:
                                                            break:
                                                         }
#include <bits/stdc++.h>
#define X first
                                                      che[v] = h;
#define Y second
                                                      setD(v, h);
#define pb push_back
                                                      mark[v] = 1;
using namespace std;
                                                      int mysz = sz[v];
typedef pair<int, int> pii;
                                                      for(int u = v; ; u = par[u][0]){
typedef pair<pii, int> ppi;
                                                        sz[u] -= mysz;
typedef long long 11;
                                                        if(u == root) break;
const int maxn = 5e5 + 17, 1g = 19;
const ll inf = 1e18;
                                                      for(auto &e : g[v])
int n, q, par[maxn][lg], cpar[maxn], h[maxn],
                                                        if(e.X != par[v][0] && sz[e.X]){
    sz[maxn], che[maxn];
                                                         int x = get_cent(e.X, h + 1);
11 sw[maxn][lg], ns[maxn], sd[maxn][lg];
                                                         //cerr << v << ' ' << x << '\n';
vector<int> ch[maxn]:
                                                         cpar[x] = v;
vector<pii> g[maxn];
                                                          ch[v].pb(x);
bool mark[maxn];
void prep(int v = 0, int p = 0){
                                                      if(v != root){
 sz[v] = 1;
                                                        int x = get_cent(root, h + 1);
 par[v][0] = p;
                                                        //cerr << v << ', ', << x << '\n';
 for(auto e : g[v])
                                                        cpar[x] = v;
   if(e.X != p){
                                                        ch[v].pb(x);
     h[e.X] = h[v] + 1:
     sw[e.X][0] = e.Y;
                                                      return v;
     prep(e.X, v);
     sz[v] += sz[e.X];
                                                    11 dis(int v, int u){
                                                      if(h[u] < h[v]) swap(v, u);
                                                      11 \text{ ans} = 0;
void setD(int v, int lvl, int p = -1, ll cd =
                                                      for(int i = 0; i < lg; i++)</pre>
    0){
                                                        if(h[u] - h[v] >> i & 1){
 if(mark[v])
                                                         ans += sw[u][i];
                                                         //cerr << "$ " << u << ' ' ' << i << ' '
   return :
  sd[v][lvl] = cd:
                                                              << sw[u][i] << '\n':
 for(auto e : g[v])
                                                          u = par[u][i];
   if(e.X != p)
     setD(e.X, lvl, v, cd + e.Y);
                                                      //cerr << ans << '\n';
```

```
if(v == u)
   return ans;
 for(int i = lg - 1; i >= 0; i--)
   if(par[v][i] != par[u][i]){
     ans += sw[v][i], ans += sw[u][i];
     v = par[v][i], u = par[u][i];
  ans += sw[v][0] + sw[u][0]:
 return ans:
void add(int v){
 for(int u = v; u != -1; u = cpar[u])
   ns[u] = min(ns[u], sd[v][che[u]]);
void clear(int v){
 for(int u = v; u != -1; u = cpar[u])
   ns[u] = inf;
11 get(int v){
 11 ans = inf;
 for(int u = v; u != -1; u = cpar[u]){
   ans = min(ans, sd[v][che[u]] + ns[u]);
   //cerr << dis(u, v) << ', ' << ns[u] <<
        '\n':
 return ans:
```

3 ConvexHull

```
#define REMOVE_REDUNDANT

typedef double T;
const T EPS = 1e-7;
struct pt {
   T x, y;
   pt() {}
   pt(T x, T y) : x(x), y(y) {}
   bool operator<(const pt &rhs) const { return
        make_pair(y,x) <
        make_pair(rhs.y,rhs.x); }</pre>
```

```
bool operator==(const pt &rhs) const {
      return make_pair(v,x) ==
      make_pair(rhs.y,rhs.x); }
};
T cross(pt p, pt q) { return p.x*q.y-p.y*q.x;
T area2(pt a, pt b, pt c) { return cross(a,b)
    + cross(b,c) + cross(c,a): }
#ifdef REMOVE REDUNDANT
bool between (const pt &a, const pt &b, const
    pt &c) {
 return (fabs(area2(a,b,c)) < EPS &&
      (a.x-b.x)*(c.x-b.x) <= 0 &&
      (a.y-b.y)*(c.y-b.y) <= 0);
#endif
void ConvexHull(vector<pt> &pts) {
  sort(pts.begin(), pts.end());
 pts.erase(unique(pts.begin(), pts.end()),
      pts.end());
 vector<pt> up, dn;
 for (int i = 0; i < pts.size(); i++) {</pre>
   while (up.size() > 1 &&
        area2(up[up.size()-2], up.back(),
        pts[i]) >= 0) up.pop_back();
   while (dn.size() > 1 &&
        area2(dn[dn.size()-2], dn.back(),
        pts[i]) <= 0) dn.pop_back();</pre>
   up.push_back(pts[i]);
   dn.push_back(pts[i]);
 for (int i = (int) up.size() - 2; i >= 1;
      i--) pts.push_back(up[i]);
#ifdef REMOVE REDUNDANT
  if (pts.size() <= 2) return;</pre>
 dn.clear();
 dn.push_back(pts[0]);
 dn.push_back(pts[1]);
 for (int i = 2; i < pts.size(); i++) {</pre>
```

4 ConvexHullTrick

```
typedef long long int64;
typedef long double float128;
const int64 is_query = -(1LL<<62), inf = 1e18;</pre>
struct Line {
        int64 m, b;
       mutable function<const Line*()> succ;
        bool operator<(const Line& rhs) const {</pre>
               if (rhs.b != is_query) return m
                    < rhs.m:
               const Line* s = succ();
               if (!s) return 0:
               int64 x = rhs.m;
               return b - s \rightarrow b < (s \rightarrow m - m) *
                    x:
       }
};
struct HullDynamic : public multiset<Line> {
    // will maintain upper hull for maximum
       bool bad(iterator y) {
               auto z = next(y);
               if (y == begin()) {
                       if (z == end()) return 0;
```

```
return y->m == z->m &&
                          v->b <= z->b;
              auto x = prev(y);
              if (z == end()) return y->m ==
                  x->m && y->b <= x->b;
              return (float128)(x->b -
                  y->b)*(z->m - y->m) >=
                   (float128)(y->b -
                  z->b)*(y->m - x->m);
       void insert_line(int64 m, int64 b) {
              auto y = insert({ m, b });
              y->succ = [=] { return next(y)
                   == end() ? 0 : &*next(v); };
              if (bad(y)) { erase(y); return;
              while (next(y) != end() &&
                  bad(next(y)))
                   erase(next(y));
              while (y != begin() &&
                  bad(prev(y)))
                   erase(prev(y));
       }
       int64 eval(int64 x) {
              auto 1 = *lower bound((Line) {
                  x, is_query });
              return 1.m * x + 1.b;
       }
};
```

5 Cut

```
stack<int> stak;
inline void add_edge(int v, int u){
    g[v].push_back(u), g[u].push_back(v);
}
int get_cut(int v = 0, int p = -1){
    if(mark[v]) return h[v];
    hi[v] = h[v] = ~p ? h[p] + 1 : 0, mark[v]
    = 1;
```

6 Euclid

```
// returns g = gcd(a, b); finds x, y such
    that d = ax + by
int extended_euclid(int a, int b, int &x, int
    &y) {
        int xx = y = 0;
        int yy = x = 1;
        while (b) {
            int q = a / b;
            int t = b; b = a%b; a = t;
            t = xx; xx = x - q*xx; x = t;
            t = yy; yy = y - q*yy; y = t;
    }
    return a;
}
```

7 FFT

```
#define REP(i, n) for(int i = 0; i < (n); i++)
typedef int llint;
namespace FFT {
    const int MAX = 1 << 17;

    typedef llint value;</pre>
```

```
typedef complex<double> comp;
int N;
comp omega[MAX];
comp a1[MAX], a2[MAX];
comp z1[MAX], z2[MAX];
void fft(comp *a, comp *z, int m = N) {
       if (m == 1) {
               z[0] = a[0];
       } else {
               int s = N/m;
              m \neq 2;
               fft(a, z, m);
               fft(a+s, z+m, m);
               REP(i, m) {
                      comp c =
                          omega[s*i] *
                          z[m+i];
                      z[m+i] = z[i] - c;
                      z[i] += c;
              }
       }
}
void mult(value *a, value *b, value
    *c, int len) {
       N = 2*len;
       while (N & (N-1)) ++N;
       assert(N <= MAX):
       REP(i, N) a1[i] = 0;
       REP(i, N) a2[i] = 0;
       REP(i, len) a1[i] = a[i];
       REP(i, len) a2[i] = b[i];
       REP(i, N) omega[i] = polar(1.0,
            2*M_PI/N*i);
       fft(a1, z1, N);
       fft(a2, z2, N);
       REP(i, N) omega[i] = comp(1, 0)
           / omega[i];
```

```
REP(i, N) a1[i] = z1[i] * z2[i]
                  / comp(N, 0);
              fft(a1, z1, N);
              REP(i, 2*len) c[i] =
                  round(z1[i].real());
       }
       void mult_mod(int *a, int *b, int *c,
           int len, int mod) {
              static llint a0[MAX], a1[MAX];
              static llint b0[MAX], b1[MAX];
              static llint c0[MAX], c1[MAX],
                   c2[MAX];
              REP(i, len) a0[i] = a[i] &
                   0xFFFF;
              REP(i, len) a1[i] = a[i] >> 16;
              REP(i, len) b0[i] = b[i] &
                   0xFFFF;
              REP(i, len) b1[i] = b[i] >> 16;
              FFT::mult(a0, b0, c0, len);
              FFT::mult(a1, b1, c2, len);
              REP(i, len) a0[i] += a1[i];
              REP(i, len) b0[i] += b1[i];
              FFT::mult(a0, b0, c1, len);
              REP(i, 2*len) c1[i] -= c0[i] +
                  c2[i];
              REP(i, 2*len) c1[i] %= mod;
              REP(i, 2*len) c2[i] %= mod;
              REP(i, 2*len) c[i] = (c0[i] +
                   ((long long) c1[i] << 16) +
                   ((long long) c2[i] << 32))
                  % mod;
       }
#undef REP
```

8 Ford Fulkerson

```
// Ford Fulkerson: Runs in O(E * maxflow)
int head[maxn], to[maxm], prv[maxm],
    cap[maxm], cost[maxm], ecnt;
const int maxn = 2e3 + 17, maxm = maxn * maxn
    + 17, inf = 1e9 + 17;
void init() {
       memset(head, -1, sizeof head):
   ecnt = 0:
}
void add(int v, int u, int vu, int uv = 0) {
       to[ecnt] = u, prv[ecnt] = head[v],
            cap[ecnt] = vu, head[v] = ecnt++;
       to[ecnt] = v, prv[ecnt] = head[u],
           cap[ecnt] = uv, head[u] = ecnt++;
}
int dfs(int v, int flow = inf) {
       if (v == sink || flow == 0) return f;
       if (mark[v]) return 0;
       mark[v] = 1;
       for (int e = head[v]; e != -1; e =
           prv[e])
              if (cap[e]) {
                      int x = dfs(to[e]).
                          min(flow, cap[e]));
                      if (x)
                             return cap[e] -=
                                 x, cap[e ^
                                 1] += x, x;
              }
       return 0;
}
int maxflow() {
       int ans = 0;
       for (int tmp; (tmp = dfs(so)); ans +=
            tmp)
              memset(mark, 0, sizeof mark);
       return ans:
```

9 GaussElim

```
// A[0..n-1][0..m-1]*ANS=A[1..n][m]. this
    functions will find ANS and returns
    number of different answer
// which can be 0, 1 or INF.
int gauss (vector < vector <double> > a,
    vector<double> & ans) {
 int n = (int) a.size();
 int m = (int) a[0].size() - 1;
 vector<int> where (m, -1);
 for (int col=0, row=0; col<m && row<n;</pre>
      ++col) {
   int sel = row:
   for (int i=row; i<n; ++i)</pre>
     if (abs (a[i][col]) > abs (a[sel][col]))
       sel = i:
   if (abs (a[sel][col]) < EPS)</pre>
     continue;
   for (int i=col; i<=m; ++i)</pre>
     swap (a[sel][i], a[row][i]);
    where[col] = row;
   for (int i=0; i<n; ++i)</pre>
     if (i != row) {
       double c = a[i][col] / a[row][col];
       for (int j=col; j<=m; ++j)</pre>
         a[i][j] -= a[row][j] * c;
     }
    ++row;
 ans.assign (m, 0);
 for (int i=0; i<m; ++i)</pre>
   if (where[i] != -1)
     ans[i] = a[where[i]][m] / a[where[i]][i];
 for (int i=0; i<n; ++i) {</pre>
   double sum = 0;
   for (int j=0; j<m; ++j)</pre>
     sum += ans[i] * a[i][j];
   if (abs (sum - a[i][m]) > EPS)
     return 0:
```

```
for (int i=0; i<m; ++i)
  if (where[i] == -1)
    return INF;
return 1;
}</pre>
```

10 GaussJordan

```
// Gauss-Jordan elimination with full
    pivoting.
//
// Uses:
// (1) solving systems of linear equations
    (aX=b)
// (2) inverting matrice a (aX=I)
// (3) computing determinants of square
    matrices
11
// Running time: O(n^3)
// INPUT: a[][] = an nxn matrix
//
           b[][] = an nxm matrix
// OUTPUT: X
                  = an nxm matrix (stored in
    b[][])
           a^{-1} = an nxn matrix (stored in
    a[][])
           returns determinant of a[][]
#include <iostream>
#include <vector>
#include <cmath>
using namespace std;
const double EPS = 1e-10;
typedef vector<int> VI;
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;
```

```
T GaussJordan(VVT &a, VVT &b) {
  const int n = a.size():
  const int m = b[0].size();
 VI irow(n), icol(n), ipiv(n);
 T \det = 1:
 for (int i = 0; i < n; i++) {</pre>
   int pj = -1, pk = -1;
   for (int j = 0; j < n; j++) if (!ipiv[i])</pre>
     for (int k = 0; k < n; k++) if (!ipiv[k])</pre>
       if (pj == -1 || fabs(a[j][k]) >
            fabs(a[pj][pk])) { pj = j; pk = k;}
   if (fabs(a[pj][pk]) < EPS) { cerr <<</pre>
        "Matrix is singular." << endl;
        exit(0); }
   ipiv[pk]++;
   swap(a[pj], a[pk]);
   swap(b[pj], b[pk]);
   if (pj != pk) det *= -1;
   irow[i] = pj;
   icol[i] = pk;
   T c = 1.0 / a[pk][pk];
   det *= a[pk][pk];
   a[pk][pk] = 1.0;
   for (int p = 0; p < n; p++) a[pk][p] *= c;
   for (int p = 0; p < m; p++) b[pk][p] *= c;
   for (int p = 0; p < n; p++) if (p != pk) {
     c = a[p][pk];
     a[p][pk] = 0;
     for (int q = 0; q < n; q++) a[p][q] -=</pre>
          a[pk][q] * c;
     for (int q = 0; q < m; q++) b[p][q] -=</pre>
         b[pk][q] * c;
   }
 }
 for (int p = n-1; p >= 0; p--) if (irow[p]
      != icol[p]) {
   for (int k = 0; k < n; k++)
        swap(a[k][irow[p]], a[k][icol[p]]);
 }
```

```
return det;
int main() {
 const int n = 4;
 const int m = 2;
 double A[n][n] = {
      {1,2,3,4},{1,0,1,0},{5,3,2,4},{6,1,4,6}
      }:
 double B[n][m] = \{\{1,2\},\{4,3\},\{5,6\},\{8,7\}\}\};
 VVT a(n) . b(n):
 for (int i = 0; i < n; i++) {</pre>
   a[i] = VT(A[i], A[i] + n);
   b[i] = VT(B[i], B[i] + m);
 double det = GaussJordan(a, b);
 // expected: 60
 cout << "Determinant: " << det << endl;</pre>
 // expected: -0.233333 0.166667 0.133333
      0.0666667
              0.166667 0.166667 0.333333
      -0.333333
              0.233333 0.833333 -0.133333
      -0.0666667
              0.05 -0.75 -0.1 0.2
 cout << "Inverse: " << endl;</pre>
 for (int i = 0; i < n; i++) {</pre>
   for (int j = 0; j < n; j++)
     cout << a[i][j] << ' ';
   cout << endl;</pre>
 // expected: 1.63333 1.3
              -0.166667 0.5
              2.36667 1.7
              -1.85 -1.35
 cout << "Solution: " << endl;</pre>
 for (int i = 0; i < n; i++) {</pre>
   for (int j = 0; j < m; j++)</pre>
     cout << b[i][j] << ' ';
   cout << endl:</pre>
```

}

11 Geometry

```
const double EPS = 1e-12:
struct P {
   double x, y;
   P operator+(const P &p) const { return {x
        + p.x, y + p.y; }
   P operator-(const P &p) const { return {x
        -p.x, y - p.y; }
   P operator*(double c) const { return {x *
        c, y * c; }
   double operator*(P q) const { return x *
        q.y - y * q.x; }
   P operator/(double c) const { return {x /
        c, y / c}; }
   double angle() const {
       return atan2(v, x);
   P RotateCCW90() const { return {-y, x}; }
   P RotateCW90() const { return {y, -x}; }
   P RotateCCW(double t) const {
       return \{x * \cos(t) - y * \sin(t), x * \}
           sin(t) + y * cos(t);
   double size2() const {
       return x * x + y * y;
   double size() const {
       return sqrt(size2());
   }
double dot(P p, P q) { return p.x * q.x + p.y
    * q.v; }
double dist2(P p, P q) { return (p -
    q).size2(); }
double dist(P p, P q) { return (p -
    q).size(); }
ostream & operator << (ostream & os, const P & p) {
   return os << "(" << p.x << "," << p.y <<
        ")";
```

```
// project point c onto line through a and b
// assuming a != b
P project_point_line(P a, P b, P c) {
   return a + (b - a) * dot(c - a, b - a) /
        (b - a).size2():
// project point c onto line segment through
    a and b
P project_point_segment(P a, P b, P c) {
   double r = dot(b - a, b - a):
   if (abs(r) < EPS) return a;</pre>
   r = dot(c - a, b - a) / r;
   if (r < 0) return a:
   if (r > 1) return b;
   return a + (b - a) * r;
// compute distance from c to segment between
double distance_point_segment(P a, P b, P c) {
   return sqrt(dist2(c,
        project_point_segment(a, b, c)));
// compute distance between point (x,y,z) and
    plane ax+by+cz=d
double distance_point_plane(double x, double
    v. double z.
                         double a, double b,
                              double c, double
   return abs(a * x + b * y + c * z - d) /
        sqrt(a * a + b * b + c * c);
// determine if lines from a to b and c to d
    are parallel or collinear
bool lines_parallel(P a, P b, P c, P d) {
   return abs((b - a) * (c - d)) < EPS;
bool lines_collinear(P a, P b, P c, P d) {
   return lines_parallel(a, b, c, d)
          && abs((a - b) * (a - c)) < EPS
          && abs((c - d) * (c - a)) < EPS;
}
// determine if line segment from a to b
    intersects with
```

```
// line segment from c to d
bool segments_intersect(P a, P b, P c, P d) {
   if (lines_collinear(a, b, c, d)) {
       if (dist2(a, c) < EPS || dist2(a, d) <</pre>
           dist2(b, c) < EPS || dist2(b, d) <
               EPS)
           return true:
       if (dot(c - a, c - b) > 0 \&\& dot(d -
           a, d - b) > 0 && dot(c - b, d - b)
           > 0)
           return false;
       return true;
   if (((d - a) * (b - a)) * ((c - a) * (b - a))
        a)) > 0 || ((a - c) * (d - c)) * ((b - c))
        c) * (d - c)) > 0)
       return false;
   return true:
// compute intersection of line passing
    through a and b
// with line passing through c and d,
    assuming that unique
// intersection exists; for segment
    intersection, check if
// segments intersect first
P line_intersection(P a, P b, P c, P d) {
   b = b - a;
   d = c - d;
   c = c - a:
   assert(dot(b, b) > EPS \&\& dot(d, d) > EPS);
   return a + b * (c * d) / (b * d);
}
// compute center of circle given three points
P circle_center(P a, P b, P c) {
   b = (a + b) / 2;
   c = (a + c) / 2;
   return line_intersection(b, b + (a -
        b).RotateCW90(), c, c + (a -
        c).RotateCW90());
}
// determine if point is in a possibly
    non-convex polygon (by William
```

```
// Randolph Franklin); returns 1 for strictly
    interior points, 0 for
// strictly exterior points, and 0 or 1 for
    the remaining points.
// Note that it is possible to convert this
    into an *exact* test using
// integer arithmetic by taking care of the
    division appropriately
// (making sure to deal with signs properly)
    and then by writing exact
// tests for checking point on polygon
    boundary
bool point_in_polygon(const vector<P> &p, P
    a) {
   bool c = false;
   for (int i = 0; i < p.size(); i++) {</pre>
       int j = (i + 1) % p.size();
       if ((p[i].y <= q.y && q.y < p[j].y ||</pre>
           p[j].y \le q.y && q.y \le p[i].y) &&
           q.x < p[i].x + (p[j].x - p[i].x) *
               (q.y - p[i].y) / (p[i].y -
               p[i].y))
           c = !c;
   }
   return c;
// determine if point is on the boundary of a
    polygon
bool point_on_polygon(const vector<P> &p, P
   for (int i = 0; i < p.size(); i++)</pre>
       if (dist2(project_point_segment(p[i],
           p[(i + 1) \% p.size()], q), q) <
           EPS)
           return true;
   return false;
// compute intersection of line through
    points a and b with
// circle centered at c with radius r > 0
// going from a to b, t[1] is the first
    intersection and t[0] is the second
vector<P> circle_line_intersection(P a, P b,
    P c, double r) {
   vector<P> ret;
```

```
b = b - a:
   a = a - c;
   double A = dot(b, b);
   double B = dot(a, b);
   double C = dot(a, a) - r * r;
   double D = B * B - A * C;
   if (D < -EPS) return ret;</pre>
   ret.push_back(c + a + b * (-B + sqrt(D +
        EPS)) / A);
   if (D > EPS)
       ret.push_back(c + a + b * (-B -
           sqrt(D)) / A);
   return ret:
}
// compute intersection of circle centered at
    a with radius r
// with circle centered at b with radius R
// order is counter clock wise
vector<P> circle circle intersection(P a. P
    b, double r, double R) {
   vector<P> ret:
   double d = sqrt(dist2(a, b));
   if (d > r + R \mid \mid d + min(r, R) < max(r, R)
        R)) return ret:
   double x = (d * d - R * R + r * r) / (2 *
   double y = sqrt(r * r - x * x);
   P v = (b - a) / d;
   ret.push_back(a + v * x + v.RotateCCW90()
        * v);
   if (y > 0)
       ret.push_back(a + v * x -
           v.RotateCCW90() * v);
   return ret:
// This code computes the area or centroid of
    a (possibly nonconvex)
// polygon, assuming that the coordinates are
    listed in a clockwise or
// counterclockwise fashion. Note that the
    centroid is often known as
// the "center of gravity" or "center of
    mass".
double signed_area(const vector<P> &p) {
   double area = 0;
```

```
for (int i = 0; i < p.size(); i++) {</pre>
       int j = (i + 1) % p.size();
       area += p[i].x * p[j].y - p[j].x *
           p[i].v; // TODO
   return area / 2.0;
double area(const vector<P> &p) {
   return abs(signed_area(p));
P centroid(const vector<P> &p) {
   P c{0, 0};
   double scale = 6.0 * signed_area(p);
   for (int i = 0; i < p.size(); i++) {</pre>
       int j = (i + 1) % p.size();
       c = c + (p[i] + p[j]) * (p[i].x *
           p[j].y - p[j].x * p[i].y);
   return c / scale:
// tests whether or not a given polygon (in
    CW or CCW order) is simple
bool is_simple(const vector<P> &p) {
   for (int i = 0; i < p.size(); i++) {</pre>
       for (int k = i + 1; k < p.size(); k++)</pre>
           int j = (i + 1) % p.size();
           int 1 = (k + 1) % p.size();
           if (i == 1 || j == k) continue;
           if (segments_intersect(p[i], p[j],
               p[k], p[1]))
              return false:
      }
   }
   return true;
double coef_on_line(P a, P b, P c) {
   if (abs(a.x - c.x) < EPS)
       return (b.y - a.y) / (c.y - a.y);
   return (b.x - a.x) / (c.x - a.x);
void seg_union(vector<pair<double, double >>
    &segs) {
   sort(segs.begin(), segs.end());
   int sz = 0;
```

```
for (auto[1, r] : segs)
       if (1 <= r)
           if (!sz || 1 > segs[sz - 1].second
               + EPS)
               segs[sz++] = \{1, r\};
           else
               segs[sz - 1].second =
                   max(segs[sz - 1].second, r);
   segs.resize(sz);
}
vector<pair<double, double> >
    polygon_segment_intersection(vector<P>
    &pol, P a, P b) {
   vector<pair<double, double> > segs;
   vector<P> impos({a, b});
   for (int k = 0; k < pol.size(); k++)</pre>
       if (segments_intersect(a, b, pol[k],
           pol[(k + 1) % pol.size()]))
           impos.push_back(line_intersection(a,
               b, pol[k], pol[(k + 1) %
               pol.size()]));
   sort(impos.begin(), impos.end(), [&](P x,
        P v) {
       return coef_on_line(a, x, b) <</pre>
           coef_on_line(a, y, b);
   for (int k = 0; k < impos.size() - 1; k++)</pre>
       P \text{ mid} = (impos[k] + impos[k + 1]) / 2;
       if (point_in_polygon(pol, mid))
           segs.emplace_back(coef_on_line(a,
               impos[k], b), coef_on_line(a,
               impos[k + 1], b));
   return segs;
pair < double >
    circle_segment_intersection(P a, P b, P
    c, double r) {
   vector<P> ret =
        circle_line_intersection(a, b, c, r);
   if (ret.size() < 2)
       return {0, 0};
   return {max<double>(0, min(coef_on_line(a,
        ret[0], b), coef_on_line(a, ret[1],
```

12 HLD

```
const int maxn = 1e5 + 17, lg = 17;
int n, q, col[maxn], head[maxn],
    par[lg][maxn], h[maxn], st[maxn],
    ft[maxn], iman[maxn << 2], sina[maxn <<</pre>
vector<int> g[maxn];
pair<int, int> qu[maxn];
int prep(int v = 0, int p = -1){
 if(g[v].empty() || g[v].size() == 1 &&
      f(\alpha == [0][v]g
   col[v] = head[v] = v;
   return 1;
 }
 int sz = 1, big, mx = 0;
 for(int i = 0; i < g[v].size(); i++){</pre>
   int u = g[v][i];
   if(u == p) continue;
   par[0][u] = v;
   h[u] = h[v] + 1;
   int s = prep(u, v);
   sz += s;
   if(s > mx)
     mx = s, big = i;
 col[v] = col[g[v][big]];
 head[col[v]] = v;
 swap(g[v][0], g[v][big]);
 return sz:
}
```

```
void get_st(int v = 0){
 static int time = 0;
 st[v] = time++;
 for(auto u : g[v])
   if(u != par[0][v])
     get_st(u);
 ft[v] = time;
int lca(int v, int u){
 if(h[u] < h[v])
   swap(v. u):
 for(int i = 0; i < lg; i++)</pre>
   if(h[u] - h[v] >> i & 1)
     u = par[i][u];
 for(int i = lg - 1; i >= 0; i--)
   if(par[i][v] != par[i][u])
     v = par[i][v], u = par[i][u];
 return v == u ? v : par[0][v];
}
int dis(int v, int u){
 return h[v] + h[u] - 2 * h[lca(v, u)];
void sadra(int id){
 if(sina[id] == -1)
   return:
 iman[id << 1] = iman[id << 1 | 1] = sina[id
      << 1] = sina[id << 1 | 1] = sina[id]:
 sina[id] = -1;
void majid(int s, int e, int x, int 1 = 0,
    int r = n, int id = 1){
 if(s <= 1 && r <= e){
   iman[id] = sina[id] = x:
   return ;
 if(e <= 1 || r <= s) return ;</pre>
 sadra(id):
 int mid = 1 + r >> 1;
 majid(s, e, x, l, mid, id << 1);
 majid(s, e, x, mid, r, id << 1 | 1);
 iman[id] = max(iman[id << 1], iman[id << 1 |</pre>
     1]):
int hamid(int s, int e, int l = 0, int r = n,
    int id = 1){
```

```
if (s \le 1 \&\& r \le e) return iman[id]:
 if(e <= 1 || r <= s) return 0;
 sadra(id):
 int mid = 1 + r >> 1;
 return max(hamid(s, e, 1, mid, id << 1),</pre>
      hamid(s, e, mid, r, id \ll 1 | 1));
void change(int v, int u, int x){
 //cerr << "changeing " << v << ' ' ' << u << '
      ' << x << '\n':
 if(col[v] == col[u]){
   majid(st[u], st[v] + 1, x);
   return ;
 if(col[v] != col[ par[0][v] ]){
   majid(st[v], st[v] + 1, x);
   change(par[0][v], u, x);
   return :
 majid(st[head[col[v]]], st[v] + 1, x);
  change(par[0][ head[ col[v] ] ], u, x);
void Change(int v, int u, int x){
 int p = lca(v, u);
 change(v, p, x);
  change(u, p, x);
int get_max(int v, int u){
 if(col[v] == col[u])
   return hamid(st[u], st[v] + 1);
 if(col[v] != col[ par[0][v] ])
   return max(hamid(st[v], st[v] + 1),
        get_max(par[0][v], u));
 return max(hamid(st[ head[ col[v] ] ], st[v]
      + 1), get_max(par[0][ head[ col[v] ] ],
      u));
int Get_max(int v, int u){
 int p = lca(v, u);
 return max(get_max(v, p), get_max(u, p));
int main(){
 ios::sync_with_stdio(0), cin.tie(0);
 memset(sina, -1, sizeof sina);
 cin >> n >> q;
```

```
for(int i = 1, v, u; i < n; i++){
   cin >> v >> u;
   v--, u--;
   g[v].push_back(u);
   g[u].push_back(v);
}
prep();
}
```

13 Hungarian

```
typedef long long 11;
const 11 INFL = (1 << 60);</pre>
using Weight = 11;
const Weight InfWeight = INFL;
Weight hungarianMin(const vector
    <vector<Weight>> &A) {
       if (A.empty()) return 0;
       int h = A.size(), n = A[0].size();
       if (h > n) return InfWeight;
       vector <Weight> fx(h), fy(n);
       vector<int> x(h, -1), y(n, -1);
       vector < int > t(n), s(h + 1);
       for (int i = 0; i < h;) {</pre>
              fill(t.begin(), t.end(), -1);
               s[0] = i;
               int q = 0;
               for (int p = 0; p <= q; ++p) {</pre>
                      for (int k = s[p], j =
                           0; j < n; ++j) {
                              if (fx[k] + fy[j]
                                  == A[k][j]
                                  && t[j] < 0
                                  {
                                      s[++q] =
                                          y[j];
                                     t[i] = k;
                                      if (s[q]
                                          < 0) {
                                             for
                                                  (p
```

```
=
                                  j;
                                  р
                                  >=
                                  0;
                                  j
                                  p)
                                                                if (d == InfWeight)
                                      7[j]
                                                                        return InfWeight;
                                                                for (int j = 0; j < n;
                                                                    ++j) {
                                          t[i];
                                                                        if (t[j] >= 0)
                                                                               fy[j] -=
                                                                                    d;
                                          x[k];
                                                                }
                                      k[k]
                                                                for (int k = 0; k \le q;
                                                                     ++k)
                                                                        fx[s[k]] += d;
                                          j;
                              }
                                                         }
                              ++i;
                                                 Weight res = 0;
                              goto
                                  continue_;
                                                 for (int i = 0; i < h; ++i)</pre>
                                                         res += A[i][x[i]];
              }
                                                 return res;
       }
}
if (0) {
continue_:;
} else {
                                               KDTree
       Weight d = InfWeight;
       for (int j = 0; j < n;
            j++)
                                          11
               if (t[j] < 0) {</pre>
                      for (int
                                          // A straightforward, but probably
                          k =
                                              sub-optimal KD-tree implmentation
                           0; k
                                          // that's probably good enough for most
                           <= q;
                                              things (current it's a
                           ++k)
                                          // 2D-tree)
                              if
                                  (A[s[k]],(j]
                                               constructs from n points in O(n lg^2 n)
                                  Inf Weight)
                                             - handles nearest-neighbor query in O(lg
                                              n) if points are well
```

```
distributed
// - worst case for nearest-neighbor may be
    linear in pathological
     case
// Sonny Chan, Stanford University, April 2009
#include <iostream>
#include <vector>
#include <limits>
#include <cstdlib>
using namespace std;
// number type for coordinates, and its
    maximum value
typedef long long ntype;
const ntype sentry =
    numeric_limits<ntype>::max();
// point structure for 2D-tree, can be
    extended to 3D
struct point {
   ntype x, y;
   point(ntype xx = 0, ntype yy = 0) : x(xx),
       v(vv) {}
};
bool operator==(const point &a, const point
    &b)
   return a.x == b.x && a.y == b.y;
// sorts points on x-coordinate
bool on_x(const point &a, const point &b)
   return a.x < b.x;</pre>
// sorts points on y-coordinate
bool on_y(const point &a, const point &b)
```

```
return a.y < b.y;</pre>
}
// squared distance between points
ntype pdist2(const point &a, const point &b)
   ntype dx = a.x-b.x, dy = a.y-b.y;
---return-dx*dx + dy*dy;
// bounding box for a set of points
struct bbox
   ntype x0, x1, y0, y1;
   bbox() : x0(sentry), x1(-sentry),
        y0(sentry), y1(-sentry) {}
   // computes bounding box from a bunch of
        points
   void compute(const vector<point> &v) {
       for (int i = 0; i < v.size(); ++i) {</pre>
           x0 = min(x0, v[i].x); x1 = max(x1,
               v[i].x):
           y0 = min(y0, v[i].y); y1 = max(y1,
               v[i].y);
       }
   }
   // squared distance between a point and
        this bbox, 0 if inside
   ntype distance(const point &p) {
       if (p.x < x0) {
           if (p.y < y0)
                             return
               pdist2(point(x0, y0), p);
           else if (p.y > y1) return
               pdist2(point(x0, y1), p);
           else
                             return
               pdist2(point(x0, p.y), p);
       else if (p.x > x1) {
           if (p.y < y0)
                             return
               pdist2(point(x1, y0), p);
           else if (p.y > y1) return
               pdist2(point(x1, y1), p);
```

```
else
                             return
               pdist2(point(x1, p.y), p);
       }
       else {
           if (p.y < y0)
                             return
               pdist2(point(p.x, y0), p);
           else if (p.y > y1) return
               pdist2(point(p.x, y1), p);
          else
                            return 0;
   }
};
// stores a single node of the kd-tree,
    either internal or leaf
struct kdnode
   bool leaf:
                  // true if this is a leaf
        node (has one point)
                  // the single point of this
   point pt;
        is a leaf
   bbox bound; // bounding box for set of
        points in children
   kdnode *first, *second; // two children of
        this kd-node
   kdnode() : leaf(false), first(0),
        second(0) {}
    ~kdnode() { if (first) delete first; if
        (second) delete second; }
   // intersect a point with this node
        (returns squared distance)
   ntype intersect(const point &p) {
       return bound.distance(p);
   // recursively builds a kd-tree from a
        given cloud of points
   void construct(vector<point> &vp)
       // compute bounding box for points at
           this node
       bound.compute(vp);
```

```
// if we're down to one point, then
           we're a leaf node
       if (vp.size() == 1) {
           leaf = true;
           pt = vp[0];
       }
       else {
           // split on x if the bbox is wider
               than high (not best
               heuristic...)
           if (bound.x1-bound.x0 >=
               bound.y1-bound.y0)
              sort(vp.begin(), vp.end(),
                   on_x);
           // otherwise split on y-coordinate
              sort(vp.begin(), vp.end(),
                   on_y);
           // divide by taking half the array
               for each child
           // (not best performance if many
               duplicates in the middle)
           int half = vp.size()/2;
           vector<point> vl(vp.begin(),
               vp.begin()+half);
           vector<point> vr(vp.begin()+half,
               vp.end());
           first = new kdnode();
               first->construct(v1);
           second = new kdnode();
               second->construct(vr);
};
// simple kd-tree class to hold the tree and
    handle queries
struct kdtree
   kdnode *root;
   // constructs a kd-tree from a points
        (copied here, as it sorts them)
```

```
kdtree(const vector<point> &vp) {
       vector<point> v(vp.begin(), vp.end());
       root = new kdnode();
       root->construct(v);
   "kdtree() { delete root; }
   // recursive search method returns squared
        distance to nearest point
   ntype search(kdnode *node, const point &p)
       if (node->leaf) {
           // commented special case tells a
               point not to find itself
11
             if (p == node->pt) return sentry;
//
              return pdist2(p, node->pt);
       }
       ntype bfirst =
           node->first->intersect(p);
       ntype bsecond =
           node->second->intersect(p);
       // choose the side with the closest
           bounding box to search first
       // (note that the other side is also
           searched if needed)
       if (bfirst < bsecond) {</pre>
           ntype best = search(node->first, p);
           if (bsecond < best)</pre>
               best = min(best,
                   search(node->second, p));
           return best:
       }
       else {
           ntype best = search(node->second,
               p);
           if (bfirst < best)</pre>
               best = min(best,
                   search(node->first, p));
           return best:
       }
   }
```

```
// squared distance to the nearest
   ntype nearest(const point &p) {
       return search(root, p);
};
11
// some basic test code here
int main()
    // generate some random points for a
        kd-tree
   vector<point> vp;
   for (int i = 0; i < 100000; ++i) {</pre>
       vp.push_back(point(rand()%100000,
            rand()%100000));
   kdtree tree(vp);
    // query some points
   for (int i = 0; i < 10; ++i) {</pre>
       point q(rand()%100000, rand()%100000);
       cout << "Closest squared distance to</pre>
            (" << q.x << ", " << q.y << ")"
            << " is " << tree.nearest(g) <<
                endl;
   }
   return 0;
}
//
```

15 MaxFlowMinCost

```
// Running Complexity is about O(SPFA() *
   Max_flow) but better. Can be O(N^3M)
   using dijkstra.
```

```
const int maxn = 1e2 + 17, maxm = 1e4 + 17,
    so = maxn - 1, sink = maxn - 2;
int head[maxn], to[maxm], prv[maxm],
    cap[maxm], cost[maxm], q[maxm * maxn],
void init(){
   memset(head, -1, sizeof head);
   ecnt = 0:
void add(int v, int u, int cst = 0, int vu =
    1. int uv = 0){
   prv[ecnt] = head[v], to[ecnt] = u,
        cap[ecnt] = vu, cost[ecnt] = cst,
        head[v] = ecnt++;
   prv[ecnt] = head[u], to[ecnt] = v,
        cap[ecnt] = uv, cost[ecnt] = -cst,
        head[u] = ecnt++;
}
int d[maxn], par[maxn];
bool mark[maxn];
bool spfa(){
   memset(d, 63, sizeof d);
   d[so] = 0;
   int h = 0, t = 0;
   q[t++] = so, par[so] = -1;
   while(h < t){</pre>
       int v = a[h++]:
       mark[v] = 0;
       for(int e = head[v]; ~e; e = prv[e])
           if(cap[e] && d[to[e]] > d[v] +
               cost[e]){
              d[to[e]] = d[v] + cost[e]:
              if(!mark[to[e]]){
                  mark[to[e]] = 1;
                  q[t++] = to[e];
              }
              par[to[e]] = e;
           }
   }
   return d[sink] < 1e9;</pre>
int mincost(){
   int ans = 0;
   while(spfa())
```

16 MaxMatchingAndIndependentSet

```
int mat[N][2];
bool mark[N];
// I hope this is HopcroftKarp algorithm. O(E
    * sqrt(V)).
// for sparse random graphs, runs in O(E *
    log(v)) with high probability.
bool dfs(int v){
   if(mark[v]) return 0;
   mark[v] = 1;
   for(auto u : adj[v][0])
       if(mat[u][1] == -1 || dfs(mat[u][1]))
           return mat[v][0] = u. mat[u][1] =
               v, 1;
   return 0;
void dfs(int v, int part){
   seen[v][part] = 1;
   for(auto u : adj[v][part])
       if(!seen[u][!part]){
          bad[u] = 1;
           seen[u][!part] = 1;
           dfs(mat[u][!part], part);
}
void maximum_matching() { // can be used to
    find max independent set
   memset(mat, -1, sizeof mat);
   bool br = 0;
   int ans = n:
   while(br ^= 1) {
              memset(mark, 0, sizeof mark);
```

if(bad[i] == 0 && seen[i][0] == 1)

cout << i + 1 << ' ';

$17 \quad NTT$

cout << '\n':

```
namespace NTT {
    const int maxn = 1 << 18;</pre>
    const int p = 998244353;
    const int g = 3;
   int R[maxn], tmp[maxn];
   int pm(int a, int b) {
       int res = 1;
       while (b) {
           if (b & 1)
               res = (11) res * a % p;
           a = (11) a * a % p;
           b >>= 1;
       return res;
   void NTT(int * a, int n, int on) {
       for (int i = 0; i < n; i++)</pre>
           if (i < R[i])</pre>
               swap(a[i], a[R[i]]);
       int wn, u, v;
       for (int i = 1, m = 2; i < n; i = m, m
            <<= 1) {
```

```
wn = pm(g, (p - 1) / m);
       if (on == -1)
           wn = pm(wn, p - 2);
       for (int j = 0; j < n; j += m) {</pre>
           for (int k = 0, w = 1; k < i;
               k++, w = (11) w * wn % p) {
              u = a[j + k], v = (11) w *
                   a[i + j + k] \% p;
              a[j + k] = (u + v) \% p;
              a[i + j + k] = (u - v + p)
                   % p;
           }
       }
   }
   if (on == -1)
       for (int i = 0, k = pm(n, p - 2); i
           < n; i++)
           a[i] = (11) a[i] * k % p;
}
vector < int > operator * (vector < int >
    & A, vector < int > & B) {
   vector < int > C;
   int n = A.size(), m = B.size();
   int 11 = n, 12 = m, L = 0;
   m += n, n = 1;
   while (n \le m)
       n \ll 1, L++;
   for (int i = 0; i < n; i++)</pre>
       R[i] = (R[i >> 1] >> 1) | ((i & 1)
            << (L - 1));
   A.resize(n);
   B.resize(n):
   NTT(A.data(), n, 1);
   NTT(B.data(), n, 1);
   for (int i = 0; i < n; i++)</pre>
       tmp[i] = (11) A[i] * B[i] % p;
   NTT(tmp, n, -1);
   return vector < int > (tmp, tmp + 11 +
        12 - 1);
}
```

18 OrderedSet

```
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
tree<int, null_type, less<int>,
    rb_tree_tag,tree_order_statistics_node_update>
    os;
```

19 SCC

```
bool mark[maxn], in_comp[maxn];
vector<int> g[maxn], rg[maxn];
void dfs(int v, vector<int> *g, vector<int>
    &vec){
   mark[v] = 1:
   for(auto u : g[v])
       if(!mark[u])
           dfs(u, g, vec);
   vec.push_back(v);
}
bool mark[maxn], in_comp[maxn];
int main(){
   vector<int> all:
   for(int i = 0; i < n; i++)</pre>
       if(!mark[i])
           dfs(i, g, all);
   memset(mark, 0, sizeof mark);
   reverse(all.begin(), all.end());
   for(auto v : all){
       if(mark[v]) continue;
       vector<int> comp;
       dfs(v, rg, comp);
       for(auto u : comp) in_comp[u] = 1;
       for(auto u : comp) in_comp[u] = 0;
   }
}
```

20 SuffixArray

```
int sa[maxl], pos[maxl], tmp[maxl], lcp[maxl];
void buildSA(string s) {
    int n = s.size();
   for (int i = 0; i < n; i++)</pre>
       sa[i] = i, pos[i] = s[i];
   for (int gap = 1;; gap *= 2) {
       auto sufCmp = [&n, &gap](int i, int j)
           if (pos[i] != pos[j])
               return pos[i] < pos[j];</pre>
           i += gap;
           j += gap;
           return (i < n && j < n) ? pos[i] <</pre>
               pos[j] : i > j;
       };
       sort(sa, sa + n, sufCmp);
       for (int i = 0; i < n - 1; i++)
           tmp[i + 1] = tmp[i] + sufCmp(sa[i],
               sa[i + 1]);
       for (int i = 0; i < n; i++)</pre>
           pos[sa[i]] = tmp[i];
       if (tmp[n-1] == n-1) break;
   for (int i = 0, k = 0; i < n; ++i)
       if (pos[i] != n - 1) {
           for (int j = sa[pos[i] + 1]; s[i +
               k] == s[j + k];)
               ++k;
           lcp[pos[i] + 1] = k;
           if (k)--k;
       }
```

21 aho

```
int nxt[maxn][z], q[maxn], f[maxn], sz = 1;
int insert(string &s){
  int v = 0;
  for(auto c : s){
    if(!nxt[v][c - 'a'])
      nxt[v][c - 'a'] = sz++;
```

```
v = nxt[v][c - 'a']:
 }
 return v;
}
void aho_corasick(){
  int head = 0, tail = 0;
 for(int i = 0; i < z; i++)</pre>
   if(nxt[0][i])
     q[tail++] = nxt[0][i];
  while(head < tail){</pre>
    int v = a[head++]:
    for(int i = 0; i < z; i++)
     if(nxt[v][i]){
       f[ nxt[v][i] ] = nxt[ f[v] ][i];
       q[tail++] = nxt[v][i];
     else
       nxt[v][i] = nxt[ f[v] ][i];
 }
}
```

22 berlekamp-massey

```
#include<cassert>
#include<vector>
#include<cstdio>
#include<cstring>
#include<iostream>
#include<algorithm>
using namespace std;
const int MOD = 1000000007;
int inverse(int a) {
 return a == 1 ? 1 : (long long)(MOD - MOD /
      a) * inverse(MOD % a) % MOD;
}
// Berlekamp-Massey Algorithm
// Requirement: const MOD, inverse(int)
// Input: vector<int> the first elements of
    the sequence
```

```
// Output: vector<int> the recursive equation
    of the given sequence
// Example: In: {1, 1, 2, 3} Out: {1,
    1000000006, 1000000006 (MOD = 1e9+7)
struct Poly {
 vector<int> a;
 Poly() { a.clear(); }
 Poly(vector<int> &a): a(a) {}
 int length() const { return a.size(); }
 Polv move(int d) {
   vector<int> na(d, 0);
   na.insert(na.end(), a.begin(), a.end());
   return Poly(na);
 int calc(vector<int> &d, int pos) {
   int ret = 0;
   for (int i = 0; i < (int)a.size(); ++i) {</pre>
     if ((ret += (long long)d[pos - i] * a[i]
         % MOD) >= MOD) {
       ret -= MOD;
     }
   }
   return ret;
 Poly operator - (const Poly &b) {
   vector<int> na(max(this->length(),
        b.length()));
   for (int i = 0; i < (int)na.size(); ++i) {</pre>
     int aa = i < this->length() ? this->a[i]
         : 0,
       bb = i < b.length() ? b.a[i] : 0;
     na[i] = (aa + MOD - bb) % MOD;
   return Poly(na);
 friend Poly operator * (const int &c, const
      Poly &p) {
   vector<int> na(p.length());
   for (int i = 0; i < (int)na.size(); ++i) {</pre>
     na[i] = (long long)c * p.a[i] % MOD;
   }
   return na;
};
```

```
vector<int> solve(vector<int> a) {
  int n = a.size():
  Polv s, b;
  s.a.push_back(1), b.a.push_back(1);
  for (int i = 1, j = 0, ld = a[0]; i < n;
      ++i) {
   int d = s.calc(a, i);
   if (d) {
     if ((s.length() - 1) * 2 <= i) {</pre>
       Polv ob = b:
       b = s;
       s = s - (long long)d * inverse(ld) %
            MOD * ob.move(i - j);
       j = i;
       ld = d:
     } else {
       s = s - (long long)d * inverse(ld) %
           MOD * b.move(i - j);
   }
 }
  return s.a;
//end of template
int main() {
  int T = 1000;
  for (int i = 0; i < T; ++i) {</pre>
    cout << "Test " << i + 1 << endl;</pre>
    int n = rand() \% 1000 + 1;
   vector<int> s:
   for (int i = 0; i < n; ++i) {</pre>
     s.push_back(rand() \% (MOD - 1) + 1);
    vector<int> a;
   for (int i = 0; i < n; ++i) {</pre>
     a.push_back(rand() % MOD);
   for (int i = 0; i < n; ++i) {</pre>
     int na = 0:
     for (int j = 0; j < n; ++j) {
       if ((na += (long long)a[n + i - 1 - j])
            * s[j] % MOD) >= MOD) {
```

```
na -= MOD:
    a.push_back(na);
  vector<int> ss = solve(a);
  for (int i = 0: i < n: ++i) {
    printf("\d^{\c}, s[i], i == n - 1 ? '\n' :
        '');
  cout << endl;</pre>
  for (int i = 0; i < n; ++i) {
    printf("%d%c", ss[i + 1], i == n - 1 ?
        '\n' : '');
  assert((int)ss.size() == n + 1);
  assert(ss[0] == 1):
  for (int i = 0; i < n; ++i) {</pre>
    assert((ss[i + 1] + s[i]) \% MOD == 0);
  }
}
cout << "All tests OK!!!" << endl;</pre>
return 0;
```

23 charpoly-matrix

```
// see modnum.cpp file for defining num type.
// Compute the characteristic polynomial of a
    square matrix A over some field.
// Not numerically stable at all.
// Takes argument by value, use std::move if
    you can.
// at the end of method, res[i] will be c_i
    in sigma(i:0->n): c_i*A^i which is
    characteristic polynomial of A and equal
    to det(tI-A).
// so (-1)^n*c_0 is the determinant of A.
// c_n will be 1. for more information, go to
    Parsa Abdollahi :)
```

```
template <typename num> std::vector<num>
    charPoly(std::vector<std::vector<num>> A)
   int N = int(A.size());
   std::vector<num> res; res.reserve(N+1);
   res.push_back(num(1));
   for (int i = 0, deg = 0; i < N; i++) {
       auto& Ai = A[i]:
       int c = i+1:
       while (c < N \&\& Ai[c] == num(0)) c++;
       if (c == N) {
          res.resize(i+2, num(0));
          for (int x = deg; x >= 0; x--) {
              num v = res[x];
              for (int y = x+1, z = i; z >=
                  deg; z--, y++) {
                 res[v] -= v * Ai[z];
              }
          deg = i+1;
          continue;
      }
      num vc = Ai[c]:
      num ivc = inv(vc);
       Ai[c] = Ai[i+1];
       Ai[i+1] = 0;
       std::swap(A[i+1], A[c]);
       auto& Ai1 = A[i+1];
       for (int k = deg; k < N; k++) {
          Ai1[k] *= vc;
      for (int k = i+1; k < N; k++) {
          auto& Ak = A[k];
              auto& x = Ak[i+1];
              auto& y = Ak[c];
              num tmp = y;
              y = x;
              x = tmp * ivc;
```

```
num v = Ak[i+1];
               for (int j = deg; j < N; j++) {
                  Ak[i] -= v * Ai[i];
           }
           if (k > i+1) {
               num v = Ai[k]:
               for (int j = deg; j < N; j++) {
                  Ai1[j] += v * Ak[j];
          }
       }
       for (int k = deg; k <= i; k++) {</pre>
           Ai1[k+1] += Ai[k];
   reverse(res.begin(), res.end());
   return res;
}
// Compute the characteristic polynomial of a
    square matrix A over F2.
// Takes argument by value, use std::move if
    vou can.
template <std::size_t MAXS> std::bitset<MAXS>
    charPoly(std::vector<std::bitset<MAXS>>
    A) {
   using bs = std::bitset<MAXS>;
   int N = int(A.size());
   bs ans; ans[0] = 1;
   int deg = 0;
   for (int i = 0; i < N; i++) {</pre>
           int j = int(A[i]._Find_next(i));
           if (j >= N) {
               bs nans;
               for (; deg <= i; ans <<= 1,</pre>
                   deg++) {
                  if (A[i][deg]) nans ^= ans;
               ans ^= nans;
               continue;
```

```
if (j != i+1) {
          swap(A[i], A[i+1]);
          for (auto& a : A) {
              bool tmp = a[i];
              a[i] = a[i+1];
              a[i+1] = tmp;
          }
       }
   assert(A[i][i+1]);
   bs msk = A[i]; msk.flip(i+1);
   for (int k = 0; k < N; k++) {
       if (msk[k]) A[i+1] ^= A[k];
   for (auto& a : A) {
       if (a[i+1]) a ^= msk;
}
return ans:
```

dinic with low scaling

```
// MaxFlow Dinic algorithm with scaling.
// O(N * M * log(MC)), where MC is maximum
    edge capacity.
// Based on problem
// For not using long long, make all "ll"s
    int and change infs.
struct Edge {
   int a, b;
   11 f, c;
   Edge (int a, int b, ll f, ll c) : a(a),
        b(b), f(f), c(c) {};
};
const int INF_BFS=1e9;
const int MAXN = 550;
const 11 \text{ INF}_{CAP} = (11)1e16;
int d[MAXN], source=MAXN-2, sink=MAXN-1;
int pt[MAXN]; // very important performance
    trick
```

```
vector <Edge> e;
                                                   vector <11> g[MAXN];
                                                   ll lim:
                                                   void add_edge(int a, int b, ll ab_cap, ll
                                                       ba_cap=0) {
                                                       //keep edges in vector: e[ind] - direct
                                                           edge, e[ind ^ 1] - back edge
                                                       g[a].emplace_back(e.size());
                                                       e.emplace_back(Edge(a, b, 0, ab_cap));
                                                       g[b].emplace_back(e.size());
                                                       e.emplace_back(Edge(b, a, 0, ba_cap));
                                                   bool bfs() {
                                                       fill(d, d+MAXN, INF_BFS); // be cautious
                                                           about using this.
                                                       d[source] = 0:
                                                       queue <int> q;
                                                       q.push(source);
                                                       while (!q.empty() && d[sink] == INF_BFS) {
                                                           int cur = q.front(); q.pop();
                                                           for (size_t i = 0; i < g[cur].size();</pre>
                                                               i++) {
                                                              int id = g[cur][i];
                                                              int to = e[id].b:
                                                              //printf("cur = %d id = %d a = %d b
                                                                   = %d f = %d c = %d\n'', cur,
                                                                  id, e[id].a, e[id].b, e[id].f,
                                                                   e[id].c);
http://informatics.mccme.ru/mod/statements/viev3.php?chapterid=2784#1 == INF_BFS && e[id].c -
                                                                   e[id].f >= lim) {
                                                                  d[to] = d[cur] + 1;
                                                                  q.push(to);
                                                          }
                                                       while (!q.empty())
                                                           q.pop();
                                                       return d[sink] != INF_BFS;
                                                   }
                                                   bool dfs(int v, ll flow) {
                                                       if (flow == 0)
```

```
return false:
   if (v == sink) {
       //cerr << v << endl;
       return true;
   for (; pt[v] < g[v].size(); pt[v]++) {</pre>
       int id = g[v][pt[v]];
       int to = e[id].b;
       //printf("v = %d id = %d a = %d b = %d
           f = %d c = %d\n", v, id, e[id].a,
           e[id].b, e[id].f, e[id].c);
       if (d[to] == d[v] + 1 \&\& e[id].c -
           e[id].f >= flow) {
           bool pushed = dfs(to, flow);
           if (pushed) {
               e[id].f += flow;
               e[id ^ 1].f -= flow;
               return true;
          }
       }
   }
   return false;
}
11 dinic() {
   ll flow=0:
   for (lim = (1LL << 62); lim >= 1;) {
       if (!bfs()) {
           lim >>= 1;
           continue;
       fill(pt, pt + MAXN, 0); // be cautious
            about this one.
       while (dfs(source, lim)) {
           flow = flow + lim;
       //cerr << flow << endl;</pre>
   return flow;
```

dinic

```
// Dinic: O(V^2*E).
// Runs in O(E * sqrt(V)) for finding
    matching in bipartite graph (for more
    specification go to parsa's talabarg).
// For not using long long, make all "ll"s
    int and change infs.
const int maxn = 2e3 + 17, maxm = 5e4 + 17;
const 11 INF_CAP = (11)1e10, INF = (11)1e17;
int ptr[maxn], head[maxn], prv[maxm],
    to[maxm], d[maxn], q[maxn], dis[maxn],
    source = maxn - 1, sink = maxn - 2, ecnt;
11 cap[maxm];
void init(){
   memset(head, -1, sizeof head);
   ecnt = 0:
}
void add_edge(int v, int u, ll vu, ll uv = 0){
   to[ecnt] = u, prv[ecnt] = head[v],
        cap[ecnt] = vu, head[v] = ecnt++;
   to[ecnt] = v, prv[ecnt] = head[u],
        cap[ecnt] = uv, head[u] = ecnt++;
}
bool bfs(){
   memset(dis, 63, sizeof dis);
   dis[source] = 0;
   int h = 0, t = 0;
   q[t++] = source;
   while(h < t){</pre>
       int v = q[h++];
       for(int e = head[v]; e \ge 0; e =
           prv[e])
           if(cap[e] && dis[ to[e] ] > dis[v]
               + 1){
              dis[to[e]] = dis[v] + 1,
                   q[t++] = to[e];
              if(to[e] == sink)
                  return 1:
          }
   }
   return 0;
11 dfs(int v, 11 f = INF){
   if(v == sink || f == 0)
       return f;
```

```
ll ret = 0:
   for(int &e = ptr[v]; e >= 0; e = prv[e])
       if(dis[v] == dis[ to[e] ] - 1){
           ll x = dfs(to[e], min(f, cap[e]));
           f -= x, ret += x;
           cap[e] -= x, cap[e ^ 1] += x;
           if(!f)
              break:
       }
   return ret;
ll mf(){
   11 \text{ ans} = 0;
   while(bfs()){
       memcpy(ptr, head, sizeof ptr);
       ans += dfs(source):
   return ans;
// Some of the code of finding cut
bool visited[maxn];
void dfs_cut(int v) {
   visited[v] = true:
   for (int e = head[v]; e >= 0; e = prv[e]) {
       if (cap[e] and !visited[to[e]]) {
           dfs cut(to[e]):
   }
void find cut() {
   cerr << "Left part of cut: ";</pre>
   dfs_cut(source);
   for(int i = 0; i < maxn; i++)</pre>
       if(visited[i])
           cerr << i << ' ';
}
```

26 fwt

```
template <typename T>
struct FWT {
```

```
void fwt(T io[], int n) {
   for (int d = 1; d < n; d <<= 1) {
     for (int i = 0, m = d << 1; i < n; i += m)
       for (int j = 0; j < d; j++) { ///
           Don't forget modulo if required
         T x = io[i+j], y = io[i+j+d];
         io[i+j] = (x+y), io[i+j+d] = (x-y);
             // xor
//
          io[i+j] = x+y; // and
           io[i+j+d] = x+v: // or
//
      }
     }
   }
  void ufwt(T io[], int n) {
   for (int d = 1; d < n; d <<= 1) {
     for (int i = 0, m = d << 1; i < n; i += m)
       for (int j = 0; j < d; j++) { ///
           Don't forget modulo if required
         T x = io[i+j], y = io[i+j+d];
         /// Modular inverse if required here
         io[i+j] = (x+y) >> 1, io[i+j+d] =
             (x-y)>>1; // xor
//
           io[i+j] = x-y; // and
           io[i+j+d] = v-x: // or
//
      }
     }
   }
  // a, b are two polynomials and n is size
      which is power of two
  void convolution(T a[], T b[], int n) {
   fwt(a, n);
   fwt(b, n);
   for (int i = 0; i < n; i++)</pre>
     a[i] = 111 * a[i] * b[i]; //% MOD;
   ufwt(a. n):
 }
  // for a*a
  void self_convolution(T a[], int n) {
   fwt(a, n);
   for (int i = 0; i < n; i++)</pre>
     a[i] = a[i] * a[i]; //% MOD;
```

```
ufwt(a, n);
}
};
```

27 interactive runner

```
from __future__ import print_function
import sys, subprocess, threading
class SubprocessThread(threading.Thread):
 def __init__(self,
             stdin_pipe=subprocess.PIPE,
             stdout_pipe=subprocess.PIPE,
             stderr_pipe=subprocess.PIPE):
   threading.Thread.__init__(self)
   self.p = subprocess.Popen(
       args,
       stdin=stdin_pipe,
       stdout=stdout_pipe,
       stderr=stderr_pipe)
 def run(self):
   try:
     self.return_code = self.p.wait()
     self.stdout = "" if self.p.stdout is
         None else self.p.stdout.read()
     self.stderr = "" if self.p.stderr is
         None else self.p.stderr.read()
   except (SystemError, OSError):
     self.return_code = -1
     self.stdout = ""
     self.stderr = "The process crashed or
         produced too much output."
assert sys.argv.count("--") == 1, (
   "There should be exactly one instance of
       '--' in the command line.")
sep_index = sys.argv.index("--")
judge_args = sys.argv[1:sep_index]
sol_args = sys.argv[sep_index + 1:]
```

```
t_sol = SubprocessThread(sol_args)
t_judge = SubprocessThread(judge_args,
    stdin_pipe=t_sol.p.stdout,
                        stdout_pipe=t_sol.p.stdin)
t_sol.start()
t_judge.start()
t_sol.join()
t_judge.join()
print("Judge return code:",
    t_judge.return_code)
print("Judge standard error:",
   t_judge.stderr.decode())
print("Solution return code:",
    t_sol.return_code)
print("Solution standard error:",
    t sol.stderr.decode())
```

28

```
for ((i = 1; i <= 10000; i++)); do
  echo $i ------
  cmake-build-debug/gen 5 $i >in
  cmake-build-debug/code <in >out
  # cmake-build-debug/naive <in >out2
  # diff out out2 >/dev/null
  if [ $? != 0 ]; then
    echo WA
    exit
  fi
done
```

29 kmp

```
int k = 0;
for(int i = 1; i < p.size(); i++){
      while(k && p[k] != p[i]) k = f[k];
      if(p[k] == p[i]) k++;
      f[i + 1] = k;
}</pre>
```

30 polard-rho

```
#define MAXL (50000>>5)+1
#define GET(x) (mark[x>>5]>>(x&31)&1)
#define SET(x) (mark[x>>5] |= 1<<(x&31))</pre>
int mark[MAXL];
int P[50000], Pt = 0;
void sieve() {
   register int i, j, k; // clang++ >=17
        compile error
   SET(1);
   int n = 46340;
   for (i = 2; i <= n; i++) {</pre>
       if (!GET(i)) {
           for (k = n/i, j = i*k; k >= i; k--,
               j -= i)
              SET(i):
          P[Pt++] = i:
       }
   }
}
long long mul(unsigned long long a, unsigned
    long long b, unsigned long long mod) { //
    can be handled with int128
   long long ret = 0;
   for (a %= mod, b %= mod; b != 0; b >>= 1,
        a \le 1, a = a \ge mod ? a - mod : a) {
       if (b&1) {
           ret += a:
           if (ret >= mod) ret -= mod:
       }
   return ret;
void exgcd(long long x, long long y, long
    long &g, long long &a, long long &b) {
   if (y == 0)
       g = x, a = 1, b = 0;
       exgcd(y, x\%y, g, b, a), b = (x/y) * a;
long long llgcd(long long x, long long y) {
   if (x < 0) x = -x:
   if (y < 0) y = -y;
```

```
if (!x || !y) return x + y;
    long long t;
    while (x%y)
       t = x, x = y, y = t\%y;
    return v;
long long inverse(long long x, long long p) {
    long long g, b, r;
   exgcd(x, p, g, r, b);
   if (g < 0) r = -r;
    return (r\%p + p)\%p;
long long mpow(long long x, long long y, long
    long mod) { // mod < 2^32</pre>
    long long ret = 1;
    while (y) {
       if (v&1)
           ret = (ret * x) \% mod;
       y >>= 1, x = (x * x) \text{/mod};
    return ret % mod;
}
long long mpow2(long long x, long long y,
    long long mod) {
    long long ret = 1;
    while (y) {
       if (v&1)
           ret = mul(ret, x, mod);
       y >>= 1, x = mul(x, x, mod);
    return ret % mod;
int isPrime(long long p) { // implements by
    miller-rabin
    if (p < 2 | | !(p&1))
                              return 0;
    if (p == 2)
                                      return 1;
    long long q = p-1, a, t;
    int k = 0, b = 0;
    while (!(q&1)) q >>= 1, k++;
    for (int it = 0; it < 2; it++) {</pre>
       a = rand()\%(p-4) + 2;
       t = mpow2(a, q, p);
       b = (t == 1) \mid \mid (t == p-1);
       for (int i = 1; i < k && !b; i++) {</pre>
           t = mul(t, t, p);
```

```
if (t == p-1)
              b = 1;
       }
       if (b == 0)
           return 0;
   return 1;
long long pollard_rho(long long n, long long
   long long x = 2, y = 2, i = 1, k = 2, d;
   while (true) {
       x = (mul(x, x, n) + c);
       if (x \ge n) x = n;
       d = llgcd(x - y, n);
       if (d > 1) return d:
       if (++i == k) y = x, k <<= 1;
   return n:
void factorize(int n, vector<long long> &f) {
   for (int i = 0; i < Pt && P[i]*P[i] <= n;</pre>
       i++) {
       if (n%P[i] == 0) {
              while (n\%P[i] == 0)
                      f.push_back(P[i]), n /=
                          P[i]:
       }
   if (n != 1) f.push_back(n);
void llfactorize(long long n, vector<long</pre>
    long> &f) {
   if (n == 1)
       return ;
   if (n < 1e+9) {</pre>
       factorize(n, f);
       return ;
   }
   if (isPrime(n)) {
       f.push_back(n);
       return ;
   long long d = n;
   for (int i = 2; d == n; i++)
```

```
d = pollard_rho(n, i);
llfactorize(d, f);
llfactorize(n/d, f);
}
```

31 poly

```
const int inf=1e9, magic=500;// threshold for
    sizes to run the naive algo
template<typename T>
struct poly {
   vector<T> a;
   // get rid of leading zeroes
   void normalize() { while(!a.empty() &&
        a.back() == T(0)) a.pop_back(); }
   poly(){}
   poly(T a0) : a{a0}{normalize();}
   poly(vector<T> t) : a(t){normalize();}
   poly operator += (const poly &t) {
       a.resize(max(a.size(), t.a.size()));
       for(size_t i = 0; i < t.a.size(); i++)</pre>
           a[i] += t.a[i]:
       normalize();
       return *this;
   poly operator -= (const poly &t) {
       a.resize(max(a.size(), t.a.size()));
       for(size_t i = 0; i < t.a.size(); i++)</pre>
           a[i] -= t.a[i]:
       normalize();
       return *this:
   poly operator + (const poly &t) const
        {return poly(*this) += t;}
   poly operator - (const poly &t) const
        {return poly(*this) -= t;}
   // get same polynomial mod x^k
   polv mod_xk(size_t k) const {k = min(k,
        a.size()); return vector<T>(begin(a),
        begin(a) + k);}
   // multiply by x^k
```

```
poly mul_xk(size_t k) const {poly
    res(*this); res.a.insert(begin(res.a),
    k. 0): return res:}
// divide by x^k, dropping coefficients
poly div_xk(size_t k) const {k = min(k,
    a.size()); return vector<T>(begin(a) +
    k, end(a));}
poly substr(size_t 1, size_t r) const { //
    return mod_xk(r).div_xk(1)
   1 = \min(1, a.size()); r = \min(r,
        a.size()):
   return vector<T>(begin(a) + 1,
        begin(a) + r);
poly inv(size_t n) const { // get inverse
    series mod x^n in O(nlgn)
   assert(!is_zero());
   poly ans = a[0].inv();
   size ta = 1:
   while(a < n) {</pre>
       poly C = (ans * mod_xk(2 *
           a)).substr(a, 2 * a);
       ans -= (ans *
           C).mod_xk(a).mul_xk(a);
       a *= 2;
   return ans.mod_xk(n);
poly operator *= (const poly &t)
    {/*fft::mul(a, t.a);*/ normalize();
    return *this:}
poly operator * (const poly &t) const
    {return poly(*this) *= t;}
poly reverse(size_t n, bool rev = 0) const
    { // reverses and leaves only n terms
   polv res(*this);
   if(rev) // If rev = 1 then tail goes
       res.a.resize(max(n, res.a.size()));
   std::reverse(res.a.begin(),
       res.a.end()):
   return res.mod_xk(n);
}
// when divisor or quotient is small
```

```
pair<poly, poly> divmod_slow(const poly
    &b) const {
    vector<T> A(a):
   vector<T> res;
    while(A.size() >= b.a.size()) {
       res.push_back(A.back() /
           b.a.back());
       if(res.back() != T(0))
           for(size t i = 0: i <</pre>
               b.a.size(); i++)
              A[A.size() - i - 1] -=
                   res.back() *
                   b.a[b.a.size() - i - 1];
       A.pop_back();
   std::reverse(begin(res), end(res));
   return {res, A};
// returns quotiend and remainder of a mod
pair<poly, poly> divmod(const poly &b)
    const {
   if(deg() < b.deg())</pre>
       return {poly{0}, *this};
   int d = deg() - b.deg();
   if(min(d, b.deg()) < magic)</pre>
       return divmod slow(b):
   poly D = (reverse(d + 1) * b.reverse(d
        + 1).inv(d + 1)).mod_xk(d +
        1).reverse(d + 1, 1);
   return {D, *this - D * b};
poly operator / (const poly &t) const
    {return divmod(t).first:}
poly operator % (const poly &t) const
    {return divmod(t).second;}
poly operator /= (const poly &t) {return
    *this = divmod(t).first;}
poly operator %= (const poly &t) {return
    *this = divmod(t).second;}
poly operator *= (const T &x) {
   for(auto &it: a) it *= x:
   normalize();
   return *this:
}
```

```
poly operator /= (const T &x) {
   for(auto &it: a) it /= x;
   normalize():
   return *this;
poly operator * (const T &x) const {return
    poly(*this) *= x;}
poly operator / (const T &x) const {return
    poly(*this) /= x;}
T& lead() { return a.back(); } // leading
    coefficient
int deg() const {return a.empty() ? -inf :
    a.size() - 1;} // degree
bool is_zero() const { // is polynomial
    zero
   return a.empty();
T operator [](int idx) const {return idx
    >= (int)a.size() || idx < 0 ? T(0) :
    a[idx]:}
T& coef(size_t idx) { // mutable reference
    at coefficient
   return a[idx];
bool operator == (const poly &t) const
    {return a == t.a:}
bool operator != (const poly &t) const
    {return a != t.a;}
poly deriv() { // calculate derivative
   vector<T> res:
   for(int i = 1; i <= deg(); i++) {</pre>
       res.push_back(T(i) * a[i]);
   return res:
poly integr() { // calculate integral with
    C = 0
   vector<T> res = {0};
   for(int i = 0; i <= deg(); i++) {</pre>
       res.push_back(a[i] / T(i + 1));
   return res;
size_t leading_xk() const { // Let p(x) =
    x^k * t(x), return k
```

```
if(is zero()) {
       return inf;
   int res = 0;
   while(a[res] == T(0)) {
       res++:
   }
   return res:
}
poly log(size_t n) { // calculate log p(x)
    mod x^n
   assert(a[0] == T(1));
   return (deriv().mod_xk(n) *
       inv(n)).integr().mod_xk(n);
}
poly exp(size_t n) { // calculate exp p(x)
    mod x^n
   if(is_zero()) {
       return T(1):
   assert(a[0] == T(0));
   poly ans = T(1);
   size_t a = 1;
   while(a < n)  {
       poly C = ans.log(2 * a).div_xk(a) -
           substr(a, 2 * a):
       ans -= (ans *
           C).mod_xk(a).mul_xk(a);
       a *= 2;
   }
   return ans.mod_xk(n);
}
poly pow_slow(size_t k, size_t n) { // if
   k is small
   return k ? k % 2 ? (*this * pow_slow(k
        - 1, n)).mod_xk(n) : (*this *
        *this).mod_xk(n).pow_slow(k / 2,
       n) : T(1);
poly pow(size_t k, size_t n) { //
    calculate p^k(n) mod x^n in O(nlgnk)
   if(is_zero()) {
       return *this:
   }
```

```
if(k < magic) {</pre>
       return pow_slow(k, n);
    int i = leading_xk();
   T j = a[i];
    poly t = div_xk(i) / j;
   return bpow(j, k) * (t.log(n) *
        T(k)).exp(n).mul_xk(i *
        k).mod_xk(n);
poly mulx(T x) { // component-wise
    multiplication with x^k
   T cur = 1;poly res(*this);
   for(int i = 0; i <= deg();</pre>
        i++){res.coef(i) *= cur; cur *= x;}
   return res:
polv mulx_sq(T x) { // component-wise
    multiplication with x^{k^2}
   T cur = x, total = 1, xx = x * x;
   poly res(*this);
   for(int i = 0; i <= deg();</pre>
        i++){res.coef(i) *= total;total *=
        cur:cur *= xx:}
   return res;
vector<T> chirpz_even(T z, int n) { //
    P(1), P(z^2), P(z^4), ..., P(z^2(n-1))
    int m = deg();
   if(is_zero()) return vector<T>(n, 0);
    vector<T> vv(m + n);
   T zi = z.inv(), zz = zi * zi, cur =
        zi, total = 1;
   for(int i = 0; i \le max(n - 1, m);
        i++) {
       if(i <= m) {vv[m - i] = total;}</pre>
       if(i < n) \{vv[m + i] = total;\}
       total *= cur; cur *= zz;
    poly w = (mulx_sq(z) * vv).substr(m, m)
        + n).mulx_sq(z);
    vector<T> res(n):
   for(int i = 0; i < n; i++) res[i] =</pre>
        w[i];
    return res;
```

```
// calculate P(1), P(z), P(z^2), ...,
    P(z^{(n-1)}) in O(nlgn)
vector<T> chirpz(T z, int n) {
   auto even = chirpz_even(z, (n + 1) /
   auto odd = mulx(z).chirpz_even(z, n /
   vector<T> ans(n):
   for(int i = 0; i < n / 2; i++){ans[2 *</pre>
        i] = even[i]:ans[2 * i + 1] =
        odd[i];}
   if(n \% 2 == 1) ans[n - 1] =
        even.back():
   return ans;
template<typename iter> // auxiliary
    evaluation function
vector<T> eval(vector<poly> &tree, int v,
    iter 1, iter r) {
   if(r - 1 == 1) {
       return {eval(*1)};
   } else {
       auto m = 1 + (r - 1) / 2:
       auto A = (*this % tree[2 *
           v]).eval(tree, 2 * v, 1, m);
       auto B = (*this \% tree[2 * v +
           1]).eval(tree, 2 * v + 1, m,
       A.insert(end(A), begin(B), end(B));
       return A:
   }
}
// evaluate polynomial in (x1, ..., xn) in
    0(nlg2n)
vector<T> eval(vector<T> x) {
   int n = x.size();
   if(is_zero()) return vector<T>(n,
        T(0)):
   vector<poly> tree(4 * n);build(tree,
        1, begin(x), end(x));
   return eval(tree, 1, begin(x), end(x));
template<typename iter>
```

```
poly inter(vector<poly> &tree, int v, iter
        1, iter r, iter ly, iter ry) { //
        auxiliary interpolation function
       if(r - 1 == 1) return {*ly / a[0]};
       else {
           auto m = 1 + (r - 1) / 2:
          auto my = ly + (ry - ly) / 2;
          auto A = (*this % tree[2 *
               v]).inter(tree, 2 * v, 1, m,
               lv, mv);
           auto B = (*this % tree[2 * v +
               1]).inter(tree, 2 * v + 1, m,
               r, my, ry);
           return A * tree[2 * v + 1] + B *
               tree[2 * v]:
       }
   }
};
template<typename T>
T resultant(poly<T> a, poly<T> b) { //
    computes resultant of a and b
   if(b.is_zero()) return 0;
   else if(b.deg() == 0) return
        bpow(b.lead(), a.deg());
   else {
       int pw = a.deg();a %= b;pw -= a.deg();
       T mul = bpow(b.lead(), pw) *
           T((b.deg() & a.deg() & 1) ? -1:
           1):
       T ans = resultant(b, a);
       return ans * mul:
   }
}
template<typename iter> // computes
    (x-a1)(x-a2)...(x-an) without building
poly<typename iter::value_type> kmul(iter L,
    iter R) {
   if(R - L == 1) {
       return vector<typename
           iter::value_type>{-*L, 1};
   } else {
       iter M = L + (R - L) / 2;
       return kmul(L, M) * kmul(M, R);
   }
```

```
template<typename T, typename iter> // builds
    evaluation tree for (x-a1)(x-a2)...(x-an)
poly<T> build(vector<poly<T>> &res, int v,
    iter L, iter R) {
   if(R - L == 1) {
       return res[v] = vector<T>{-*L, 1};
   } else {
       iter M = L + (R - L) / 2:
       return res[v] = build(res, 2 * v, L,
           M) * build(res. 2 * v + 1. M. R):
   }
}
template<typename T> // interpolates minimum
    polynomial from (xi, yi) pairs in O(nlg2n)
poly<T> inter(vector<T> x, vector<T> y) {
   int n = x.size();
   vector<poly<T>> tree(4 * n);
   return build(tree, 1, begin(x),
        end(x)).deriv().inter(tree, 1,
       begin(x), end(x), begin(y), end(y));
}
```

32 sat.

```
struct Sat {
       int n = maxn, col[maxn] = {}, cnt,
           ver[maxn] = {}, versz, cer[maxn];
       vector<int> g[maxn], rg[maxn];
       bool mrk[maxn] = {};
       void addE(int x, int y) {
              g[x].push_back(y);
              rg[y].push_back(x);
       void addOr(int x, int y) {
              addE(x ^1, y);
              addE(y ^1, x);
      }
       void dfsadd(int v){
              mrk[v]=1: for(auto
                  &u:g[v])if(!mrk[u])dfsadd(u);
              ver[versz++]=v;
```

```
}
       void dfsset(int v){
              col[v]=cnt;
              for(auto &u:rg[v])
                      if(col[u]==-1)
                             dfsset(u):
       bool ok() {
               memset(mrk, 0, n);
              memset(col, -1, n * sizeof)
                   col[0]):
               for(int v = 0; v < n; v++)
                      if(!mrk[v])
                             dfsadd(v):
                      while(versz)if(col[ver[--versz]
                          dfsset(ver[versz]).
                          cnt++;
              for(int v = 0; v < n; v += 2)
                      if(col[v]==col[v^1])
                             return 0;
                      else
                             cer[v] = col[v^1]
                                 < col[v];
              return 1;
       }
} sat:
```

33 treap

```
// In the name of Allah.
// We're nothing and you're everything.
// Ya Ali!

#include <bits/stdc++.h>
using namespace std;

const int maxn = 2e5 + 17;
struct Node{
  int k, p;
  Node *l, *r;
};
typedef Node* Ni;
```

```
void split(Ni t, int k, Ni& 1, Ni& r){
 if(!t)
   1 = r = 0:
  else if(k < t \rightarrow k)
    split(t \rightarrow 1, k, 1, t \rightarrow 1), r = t;
    split(t -> r, k, t -> r, r), l = t;
}
void insert(Ni &t, Ni it){
 if(!t)
    t = it:
  else if(it \rightarrow p < t \rightarrow p)
    insert(it -> k < t -> k ? t -> 1 : t -> r,
  else
    split(t, it \rightarrow k, it \rightarrow l, it \rightarrow r), t =
}
int main(){
 ios::sync_with_stdio(0), cin.tie(0);
}
// Implicit treap // GSS6
// In the name of Allah.
// We're nothing and you're everything.
// Ya Ali!
#include <bits/stdc++.h>
using namespace std;
typedef long long 11;
const int maxn = 1e6 + 17, mod = 998244353;
int nxP(){
 static int cur = 1;
 cur = (11) cur * 3 % mod;
 return cur;
struct Store{
 int pre, suf, sum, ans;
 Store (int val = -mod){
    pre = suf = sum = ans = val;
```

```
Store(int a, int b, int c, int d): pre(a),
       suf(b), sum(c), ans(d) {}
Store operator +(Store 1, Store r){
  if(1.sum == -mod)
    return r;
  if(r.sum == -mod)
    return 1:
  return Store(max(1.pre, 1.sum + r.pre),
      max(r.suf. l.suf + r.sum). l.sum +
      r.sum, max({l.ans, r.ans, l.suf +}
      r.pre}));
struct Node{
  int k, p, val;
  Store ans;
  Node *1, *r;
}:
typedef Node* Ni;
int cnt(Ni i){
  return i ? i -> k : 0;
Store ans(Ni i){
  return i ? i -> ans : Store();
void upd(Ni t){
  if(!t) return;
  t \rightarrow k = cnt(t \rightarrow 1) + cnt(t \rightarrow r) + 1;
  t \rightarrow ans = ans(t \rightarrow 1) + t \rightarrow val + ans(t \rightarrow
      r);
void split(Ni t, int k, Ni& 1, Ni& r){
  if(!t)
   1 = r = 0;
  else{
    if(k <= cnt(t -> 1))
      split(t \rightarrow 1, k, 1, t \rightarrow 1), r = t;
      split(t \rightarrow r, k - 1 - cnt(t \rightarrow 1), t \rightarrow
           r, r), 1 = t;
    upd(t);
void merge(Ni &t, Ni l, Ni r){
```

```
if(!1 || !r)
   t = 1 ? 1 : r;
  else if(l \rightarrow p > r \rightarrow p)
   merge(1 \rightarrow r, 1 \rightarrow r, r), t = 1;
   merge(r \rightarrow 1, 1, r \rightarrow 1), t = r;
 upd(t);
}
Ni root;
void insert(int k, int v){
  split(root, k, root, r);
 Ni nw = new Node({0, nxP(), v, v});
  merge(root, root, nw);
  merge(root, root, r);
void erase(int k){
  // removes kth element
  k++:
 Ni tmp, r;
  split(root, k, root, r);
  split(root, k - 1, root, tmp);
  merge(root, root, r);
int get(int 1, int r){
 Ni gans, ri;
  split(root, r, root, ri);
  split(root, 1, root, qans);
  int ret = ans(qans).ans;
  merge(root, root, qans);
  merge(root, root, ri);
  return ret:
void replace(int k, int v){
  erase(k);
 insert(k, v);
void print(Ni v, int h = 0){
 return:
 if(!v)
 cerr << string(h * 2, ' ') << v -> k << ' '
      << v -> p << \cdot \cdot < v -> val << " (" <<
      v -> ans.pre << '''</pre>
```

```
<< v -> ans.suf << ' ' << v -> ans.sum << '
      ' << v -> ans.ans << ')' << '\n';
 print(v -> 1, h + 1);
 print(v -> r, h + 1);
int main(){
 ios::sync_with_stdio(0), cin.tie(0);
 cin >> n;
 for(int i = 0; i < n; i++){</pre>
   int x;
   cin >> x;
   insert(i, x);
 print(root);
 int q;
 cin >> q;
 while(q--){
   char ty;
   cin >> ty;
   if(ty == 'Q'){
     int 1, r;
```

```
cin >> 1 >> r;
     cout << get(1 - 1, r) << '\n';
   }
   else if(ty == 'I'){
    int p, x;
     cin >> p >> x;
     insert(p - 1, x);
   else if(ty == 'R'){
     int p, x;
     cin >> p >> x;
     replace(p - 1, x);
   }
   else{
     int p;
     cin >> p;
     erase(p-1);
   print(root);
}
```

34 z-function

```
vector<int> z_function(string s) {
   int n = (int) s.length();
   vector<int> z(n);
   for (int i = 1, l = 0, r = 0; i < n; ++i) {
      if (i <= r)
        z[i] = min (r - i + 1, z[i - l]);
      while (i + z[i] < n && s[z[i]] == s[i + z[i]])
        ++z[i];
   if (i + z[i] - 1 > r)
        l = i, r = i + z[i] - 1;
   }
   return z;
}
```