COMPUTATIONAL IMAGING

L1. Imaging: Equipment and Math

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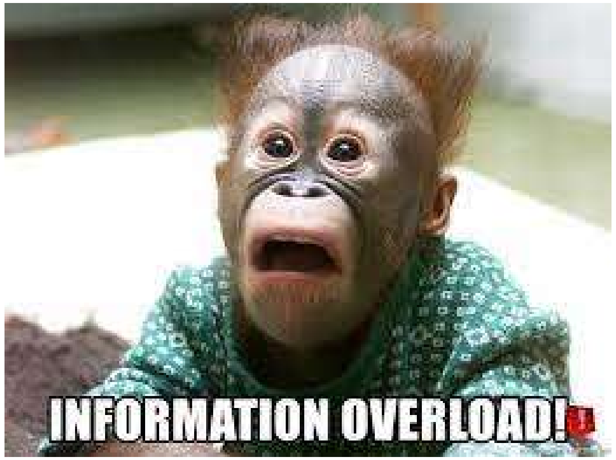
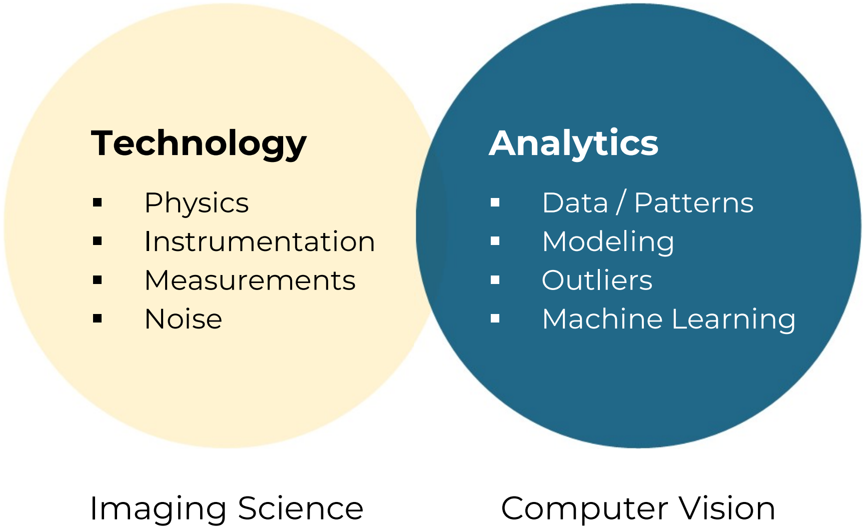
Crash course

- Large volumes of material with practical focus
- Learn while doing something by hands

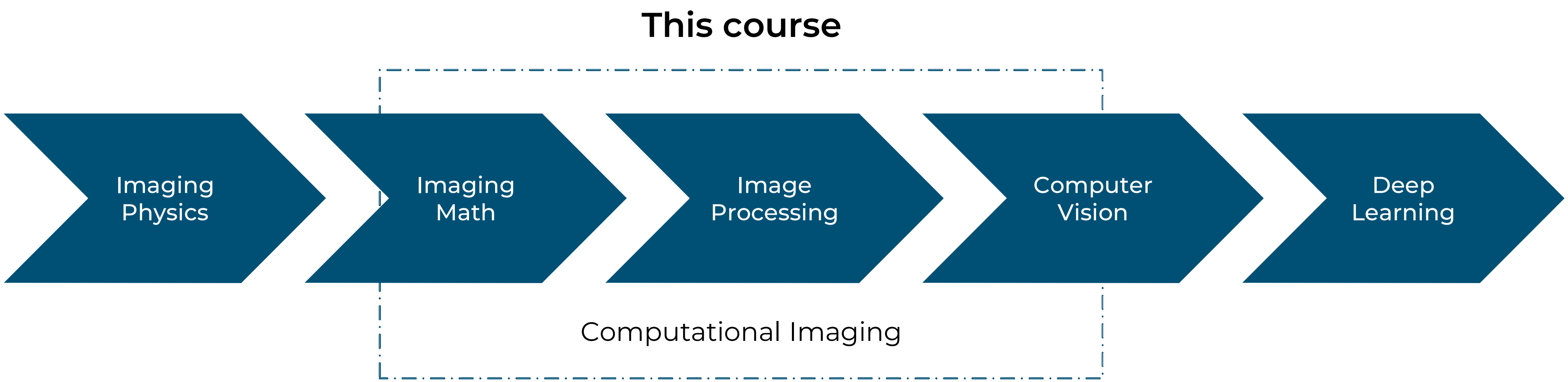


Motivation

Computational Imaging



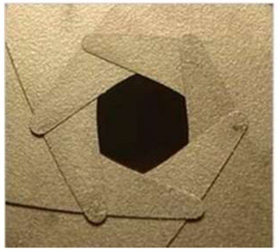
Disciplines touched in this course

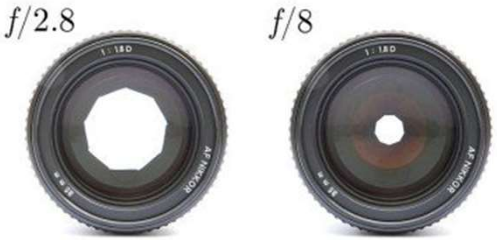


L1. Outline

- **Camera and Image – basics**
  - Imaging math: Formalism of Object and Image. Their relation
  - Point source response function
  - Distortion of signal = convolution
  - Signal restoration = deconvolution
  - Imaging a set of points – Fourier Transform in action
  - Effect of higher frequencies on sharpness. MTF
  - Digital Filtering inspired by physics
- **Course Logistics**
- **Equipment demo**

Camera basics

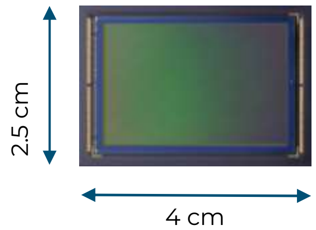




Aperture (size)

Exposure (time)

10 megapixel sensor



ISO (sensitivity)

Camera basics: Exposure

-2EV   -1EV   0EV   +1EV   +2EV

Exposure stops
As measured by expo-meter

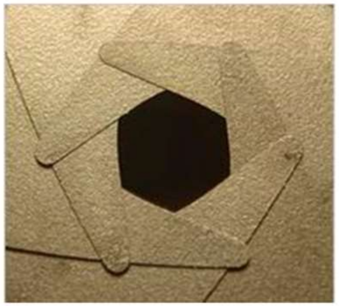
EXPOSURE COMPENSATION

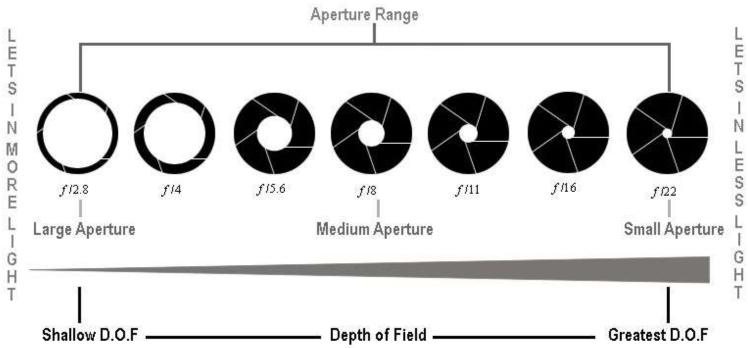
-2 . . 1 . . 0 . . 1 . . +2

Decrease		**Increase**
Exposure Compensation	⟷	Exposure Compensation
IF		***IF***
Photo is dark		Photo is bright
Underexposed		***Overexposed***
(Too Dark)		***(Too Light)***

Camera basics: Aperture

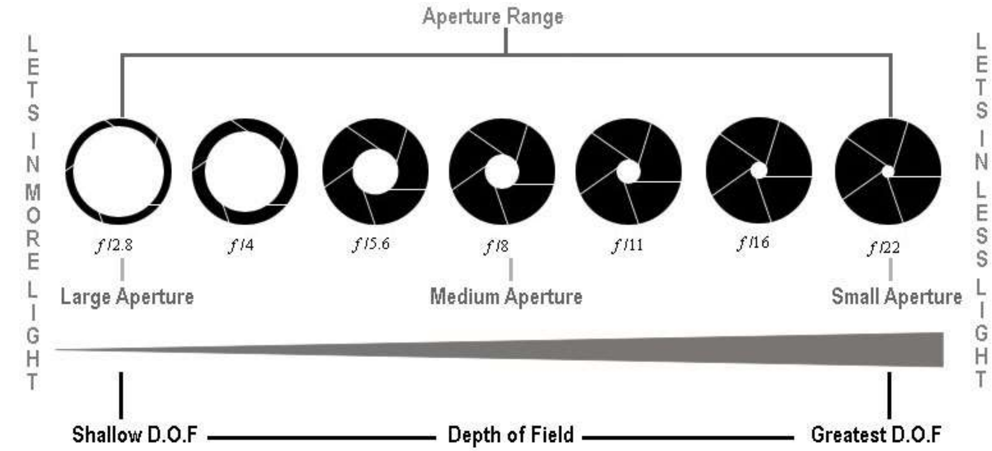
- The camera has a maximum “aperture” which corresponds to letting the full diameter of the outer glass lens gather light for you.
- But inside the lens is a metal aperture of “leaves” which will artificially close down how much light makes it to your sensor





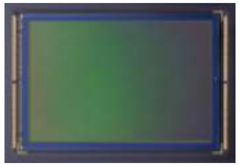
Camera basics: Aperture

Small DOF

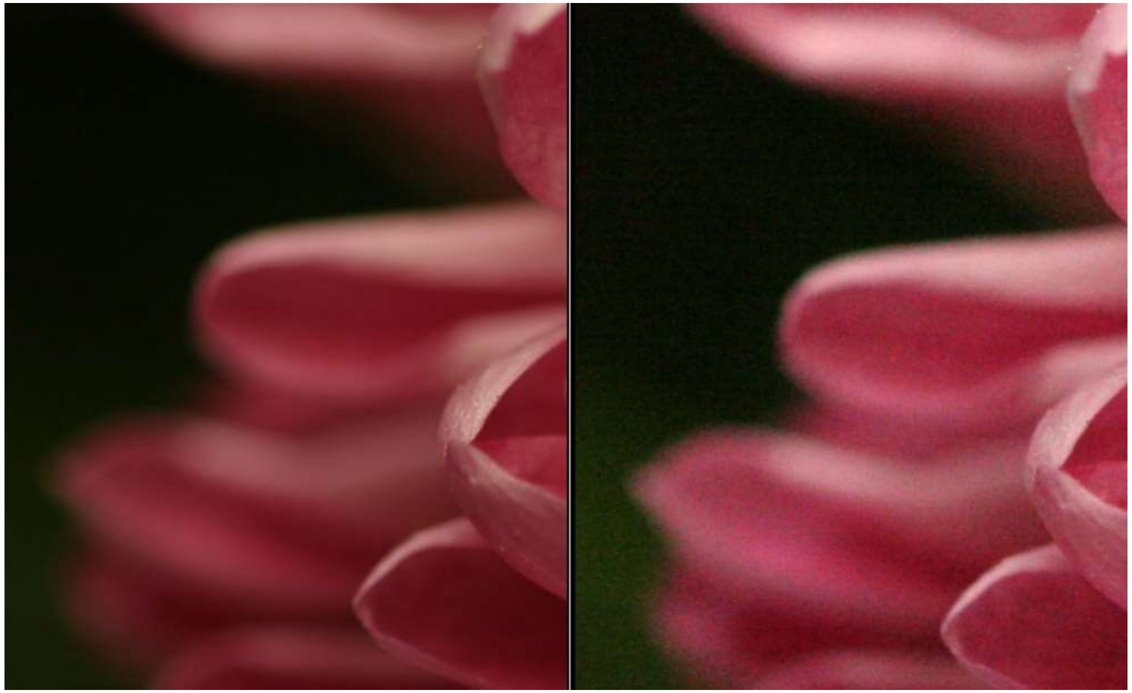


Camera basics: ISO

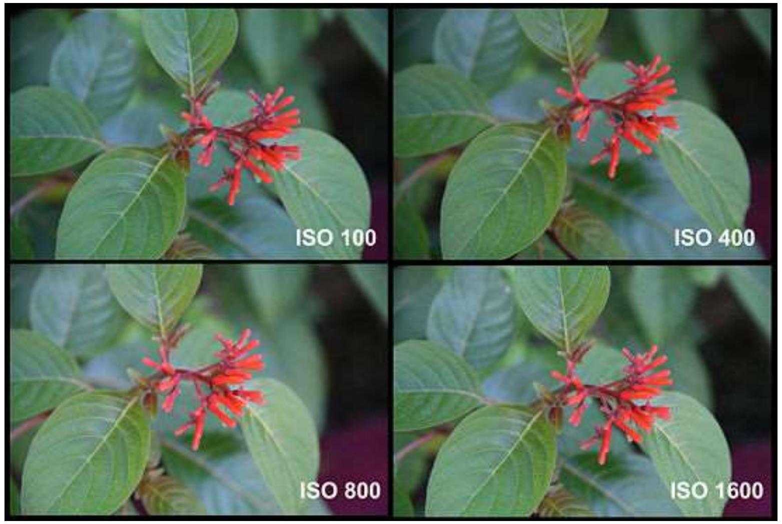
10 megapixel sensor



ISO (sensitivity)



Left: ISO 100 – smooth! Right: ISO 3200 – Noisy!



At high ISO - Colors will also look flat and washed out

Color Intensity

Same hotel, same everything, different camera settings. First crappy shot: New $6,500 camera, as it came from the factory. Second photo. White Balance still on Auto, but set to A6 make image warmer, Nikon [Picture Control](#) set to VIVID to pump colors, saturation set to +3 to pump colors more, and exposure compensation added to lighten to taste.

- Many photos are more interesting if the colors are rendered more intense (or sometimes less intense) than they appear.

Color in Camera: WB Icons May Look Like This…

AWB	Auto White Balance
	Custom
K	Kelvin
	Tungsten
	Fluorescent
	Day light
	Flash
	Cloudy
	Shade

Increasing Color Temperature

Common Situations:

- Tungsten: Will blue your picture
- Fluorescent: Blue it slightly
- Daylight: leave unchanged
- Flash: Redden slightly
- Cloudy: Redden more
- Shade: Redden a lot

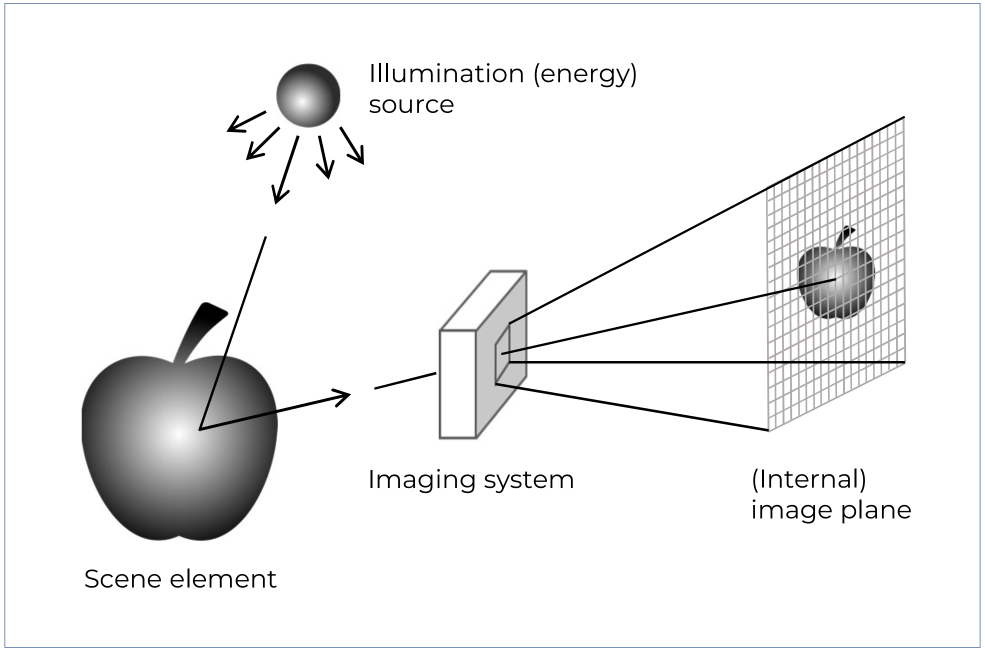
… or – introduce color tint using a filter attached to your illumination source

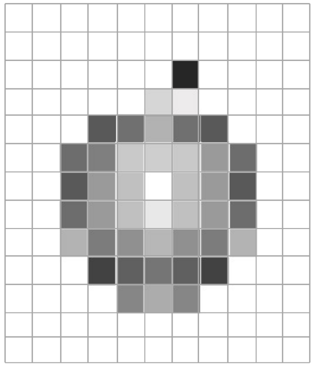
Lab 1: Learn camera parts and settings

What is an image?

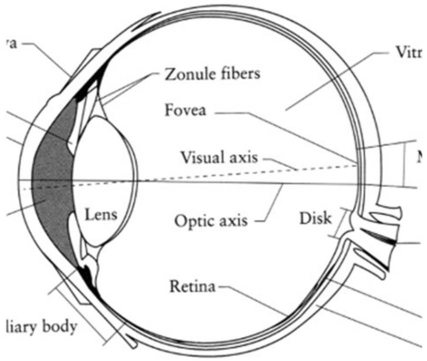
What is the main math operation that forms this image?

What is an image?





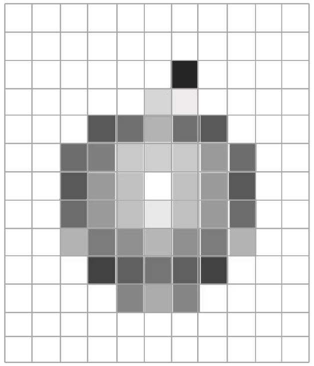
Digital Camera



The Eye

What is an image?

- A grid (matrix) of intensity values



Digital Camera

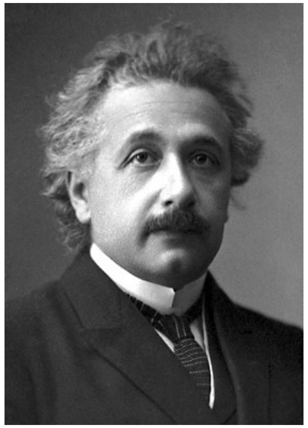
=

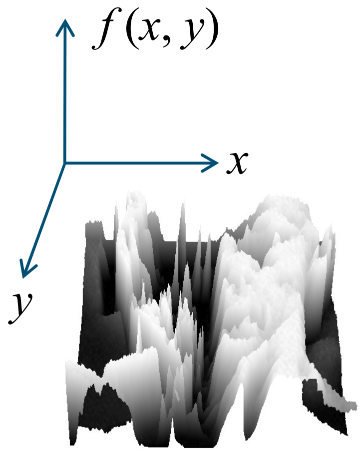
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	20	0	255	255	255	255	255	255	255
255	255	255	75	75	75	255	255	255	255	255	255
255	255	75	95	95	75	255	255	255	255	255	255
255	255	96	127	145	175	255	255	255	255	255	255
255	255	127	145	175	175	175	255	255	255	255	255
255	255	127	145	200	200	175	175	95	255	255	255
255	255	127	145	200	200	175	175	95	47	255	255
255	255	127	145	145	175	127	127	95	47	255	255
255	255	74	127	127	127	95	95	95	47	255	255
255	255	255	74	74	74	74	74	74	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255

(common to use one byte per **value: 0 = black, 255 = white)**

What is an image?

- We can think of a (grayscale) image as a **function**, f, from R^2 to R (2D *signal*):
- $f(x, y)$ gives the **intensity** at position (x, y)

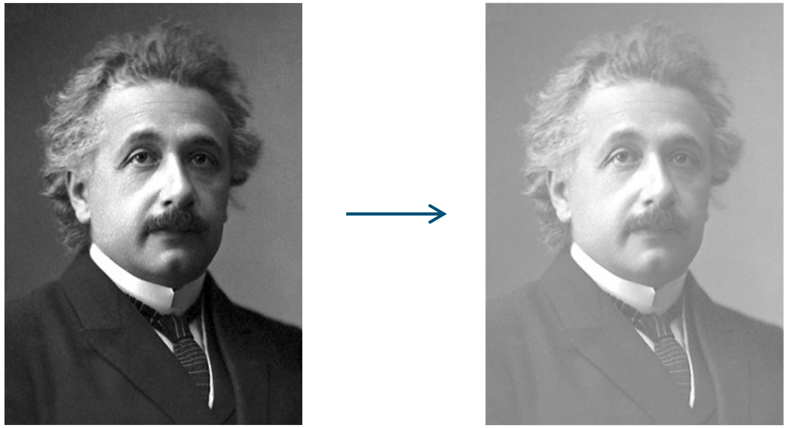




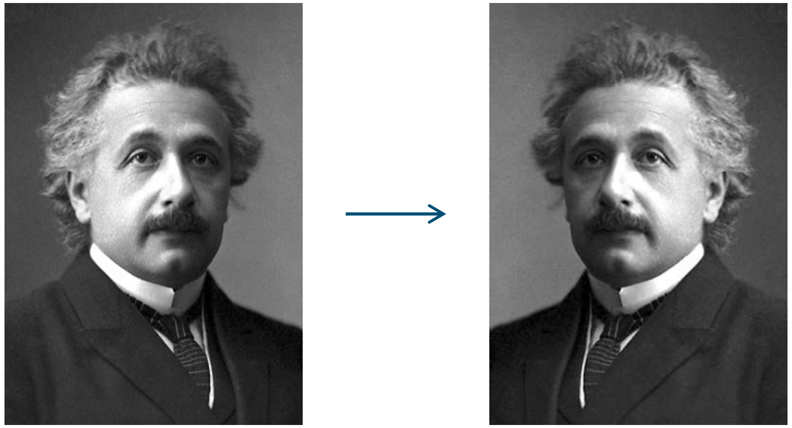
- A **digital** image is a discrete (**sampled**, **quantized**) version of this function

Image transformations

- **As with any function, we can apply operators to an image**



$$g(x,y) = f(x,y) + 20$$



$$g(x,y) = f(-x,y)$$

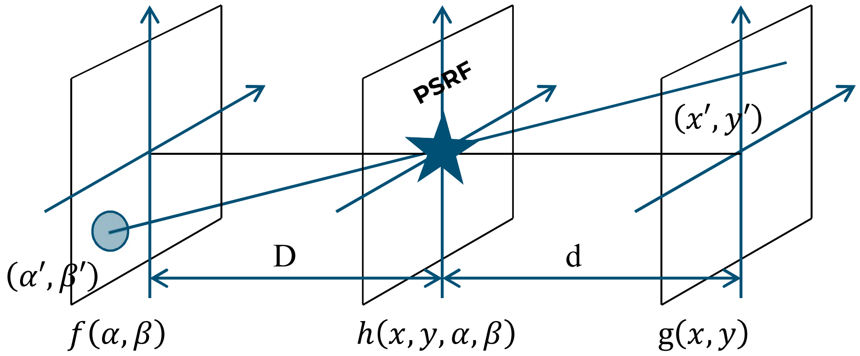
Object and Image

- Can all modalities be approximated by the same set of equations?
- How to describe imperfect images.
- Image processing = restoration + reconstruction + enhancement.
- We denote $f(\boldsymbol{r})$ to be an object; and $g\,(\boldsymbol{r})$ to an image $f(\boldsymbol{r})$ = $g\,(\boldsymbol{r})$ is an ideal
- Photographer's perception: $g(x, y)$ is a representation of $f(\boldsymbol{r})$
- Restrict to 2D imagery (remember of 4D nature): $f(\alpha, \beta) \leftrightarrow g(x, y)$
- Coding 'messengers': optical rays (photo camera) / infrared rays (thermal camera) / gamma-rays (gamma-camera) / x-rays (x-ray camera).
- Physically, they coincide spatially, sometimes at different dimensions, but $f(\boldsymbol{r})$ and $g(\boldsymbol{r})$ relation is the imaging system.

Object and Image Relation

- Restrict to 2D object and image planes: $f(\alpha,\beta) \leftrightarrow g(x,y)$
- $f(\alpha,\beta) \geq 0$ and $g(x,y) \geq 0$
- Both are integrable functions
- Neighborhood principle around each point
- Nonlinear relation: $g'(x,y) = h(x,y,\alpha',\beta',f(\alpha',\beta'))$
- Linear relation: $g'(x,y) = h(x,y,\alpha',\beta').f(\alpha',\beta')$

  (often assumed as the first order)



Point Source Response Function

Object and Image Relation

Linear relation:

$$g'(x,y) + g''(x,y) = h(x,y,\alpha',\beta').(f'(\alpha',\beta') + f''(\alpha',\beta'))$$

$$\boxed{g(x,y) = \iint h(x,y,\alpha,\beta).f(\alpha,\beta)\, d\alpha\, d\beta}$$

Space-invariant PSRF:

$$h(x,y,\alpha,\beta) = h(x-\alpha, y-\beta)$$

$$\boxed{g(x,y) = \iint h(x-\alpha, y-\beta).f(\alpha,\beta) d\alpha\, d\beta}$$

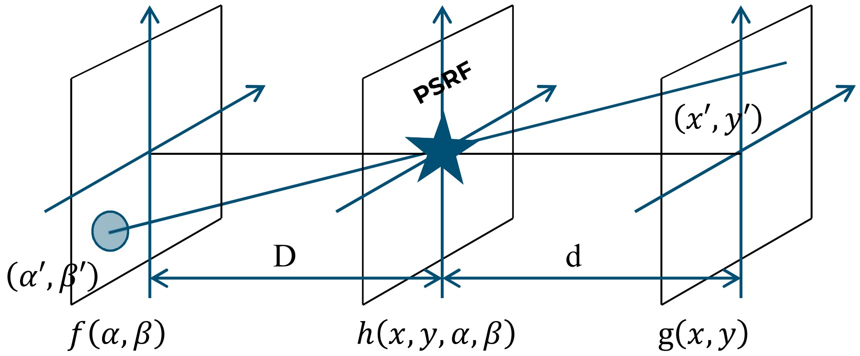
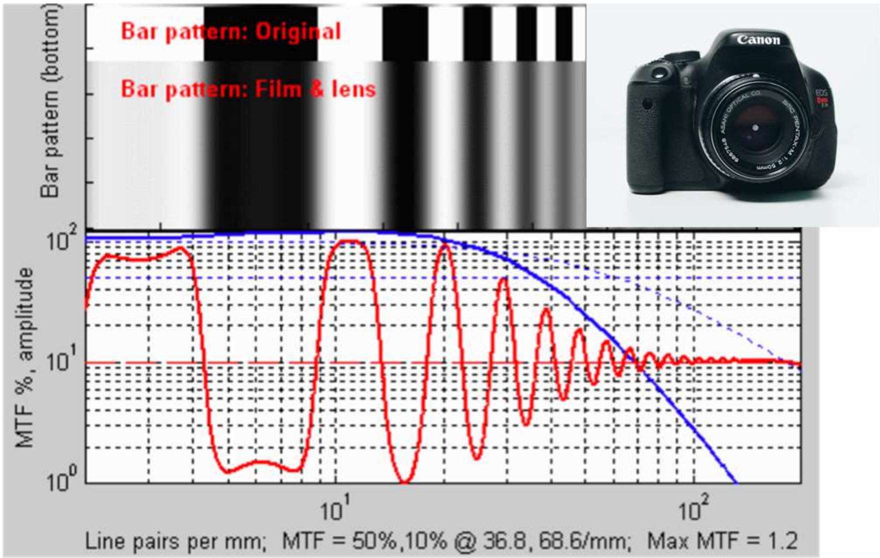


Image is a convolution of object with SIPSRF

$$g(x,y) = h(x,y) * f(x,y)$$

Image Degradation



http://www.normankoren.com/Tutorials/MTF.html

$$g(x,y) = \iint h(x - a, y - \beta) \cdot f(a,\beta) da\, d\beta$$

$$g(x,y) = h(x,y) * f(x,y)$$

$$FT[g(x,y)] = FT[h(x,y) * f(x,y)]$$

$$FT[g(x,y)] = FT[h(x,y)] \cdot FT[(x,y)]$$

$$\mathrm{G}(u,v) = H(u,v)F(u,v)$$

$H(u,v)$ is Modulation Transfer Function (MTF)
Gaussian – Gaussian

Image is a convolution of object with SIPSRF → De-convolution yields actual object

Image Deconvolution

$$\hat{f}(x,y) = \mathrm{FT}[F(x,y)]$$

$$= \left(\frac{1}{N}\right) \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} \frac{G(u,v)}{H(u,v)} \cdot \exp(2\pi j(ux + vy)/N)$$

$$g(x,y) = h(x,y) * f(x,y)$$

$$FT[g(x,y)] = FT[h(x,y) * f(x,y)]$$

$$FT[g(x,y)] = FT[h(x,y)] \cdot FT[(x,y)]$$

$$\mathrm{G}(u,v) = H(u,v)F(u,v)$$

$H(u,v)$ is Modulation Transfer Function (MTF)
Gaussian – Gaussian

Image is a convolution of object with SIPSRF
→ De-convolution yields actual object
→ ! Beware of Noise @ high frequencies

Summarize

PSF contains all the info about your camera

$$g(x,y) = h(x,y) \ * f(x,y)$$

Image is a convolution of object with PSF

$$\mathrm{G}(u,v) = H(u,v)F(u,v)$$

→ **De-convolution recovers actual object**

→ **! Beware of Noise @high frequencies**

$$f(x,y) = \mathrm{F\,T}^{-1}\left[\frac{G(u,v)}{H(u,v)}\right]$$

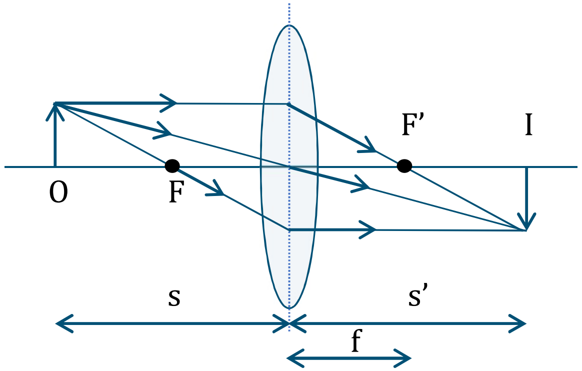
FT(PSF) = “Modulation Transfer Function” shows efficiency of imaging high frequencies

→ **Higher spatial frequencies = better resolution**

Course Logistics

Example: Optical Imaging

CONSTRUCTING RAY DIAGRAMS



s object distance; s' image distance; f focal length

- PRINCIPLE RAYS: (Any 2 are sufficient to construct image)
- Ray passing through the centre of the lens is undeviated.
- Ray parallel to the optical axis passes through a focal point.
- Ray passing towards, or away from, a focal point emerges parallel to the axis.

LENS CALCULATIONS

Thin lens formula

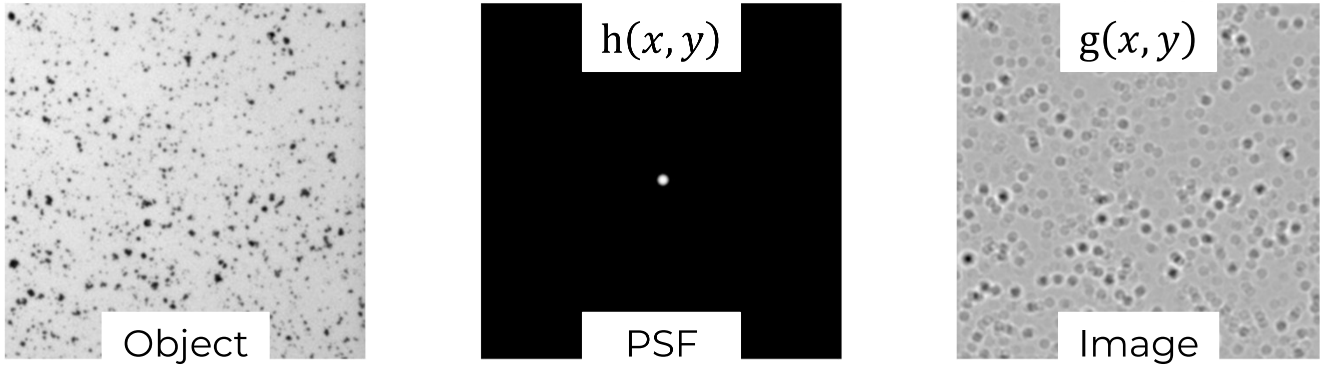
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Magnification formula

$$m = -\frac{s'}{s}$$

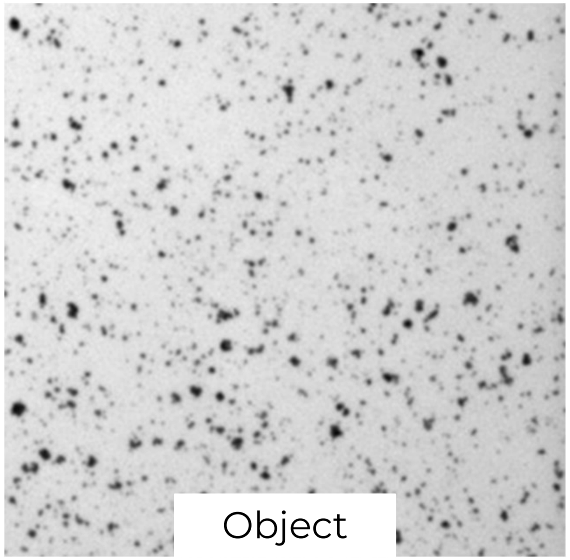
Real lens? (e.g. finite aperture; lens aberration)

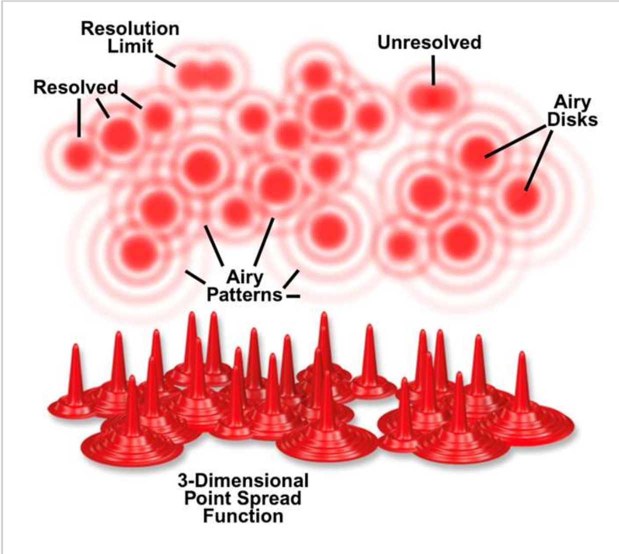
Example: Microscopy & Gaussian PSF



$$f(x,y) = \mathrm{FT}^{-1}\left[\frac{G(u,v)}{H(u,v)}\right]$$

Resolution





Resolution = 0.61 λ/NA
λ = color
NA = Numerical Aperture

Lab 2: “Bokeh” experiments

A little more practical insight: 1D Fourier

$$f(x) \leftrightarrow F(f)$$

$$F(f) = \int_{x=-\infty}^{\infty} f(x).e^{-j2\pi f} \ dx$$

$$\delta(x) \leftrightarrow 1$$

$$F\ (x - x_0) \leftrightarrow e^{-2\pi f x_0} F(f)$$

$$\delta(x - x_0) \leftrightarrow e^{-2\pi f x_0}$$

$$e^{2\pi f x_0} \leftrightarrow \delta(f - f_0)$$

$$f(x) = \int_{f=-\infty}^{\infty} F(f).e^{j2\pi f x} df$$

$$\frac{\partial f(x)}{\partial x} \leftrightarrow j2\pi f F(f)$$

$$\prod(x) \leftrightarrow Sinc(f)$$

$$f(x)^{*}\mathrm{g}(x) \leftrightarrow F(f).G(f)$$

$$\Lambda(x) \leftrightarrow Sinc^2(f)$$

$$f(ax) \leftrightarrow \frac{1}{|a|} F\left(\frac{f}{a}\right)$$

1D Convolution (Shift and Scale)

$$f(x)^{*}g(x) = \int_{x'=-\infty}^{\infty} f(x').g(x-x').dx'$$

$$f(ax+a')^{*}\,g(bx+b') = h(x)^{*}m(x) \;=\; \int_{x'=-\infty}^{\infty} h(x').m(x-x').dx'$$

$$f(ax+a')^{*}\,g(bx+b') = \int_{x'=-\infty}^{\infty} f(ax'+a').g(bx-bx'+b').dx'$$

$$u = ax' + a'; du == adx'$$

$$f(ax+a')^{*}\,g(bx+b') = \int_{u=-\infty}^{\infty} f(u).g\left(bx - \frac{b}{a}u + b' + \frac{ba'}{a}\right).\frac{du}{|a|}$$

$$A{=}b;\ B = -\frac{b}{a};\ C = \frac{ab'+ba'}{a}$$

$$\int_{x'=-\infty}^{\infty} f(x').g(Ax+Bx'+C).dx' = ?$$

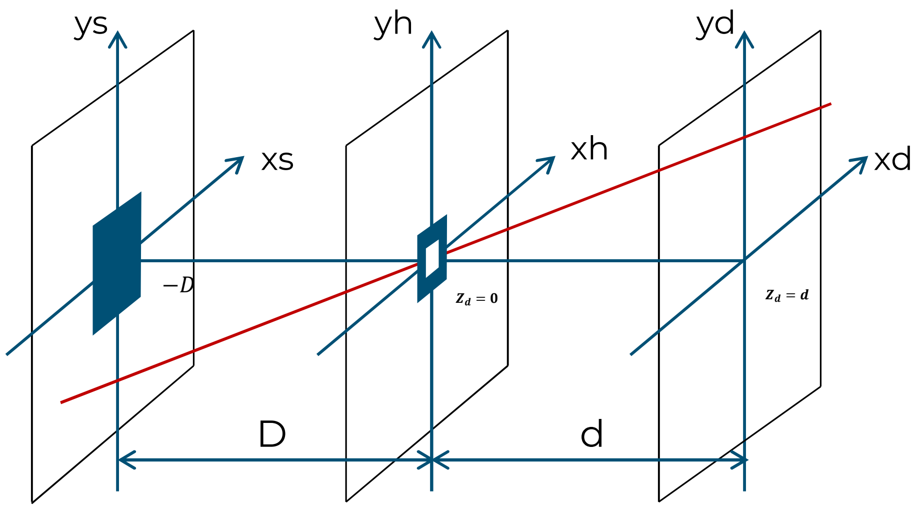
$$b = A; a = -\frac{A}{B}$$

$$a' = 0 \Rightarrow b' = C$$

$$b' = 0 \Rightarrow a' = \frac{aC}{b} = -\frac{C}{B}$$

$$\int_{x'=-\infty}^{\infty} f(x').g(Ax+Bx'+C).dx' = \left|\frac{A}{B}\right| f\left(-\frac{A}{B}x\right)^{*} g(Ax+C)$$

Example 2



Since points s,h,d are on the same line:

$$\frac{xs - x}{xh - x} = \frac{D+d}{d}$$

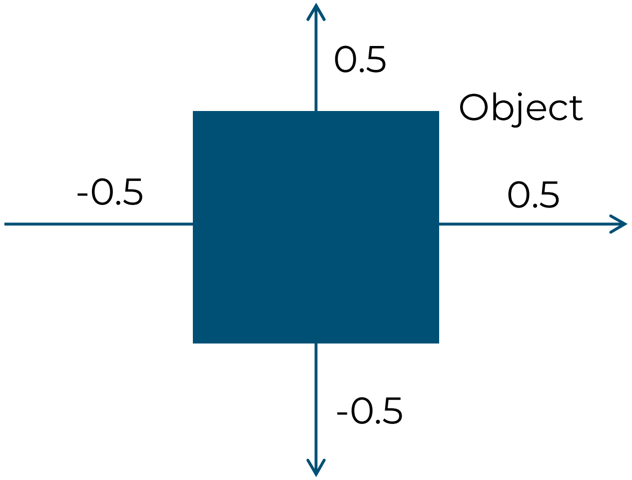
$$xh = \frac{d}{D+a} xs + \frac{D}{D+a} xd$$

$$yh = \frac{d}{D+a} ys + \frac{D}{D+a} yd$$

$$I(xd, yd) = \frac{1}{4\pi (D+d)^2} \int_{xs=-\infty}^{\infty} \int_{ys=-\infty}^{\infty} s(xs, ys).\,h(xh, yh).\,dys.\,dxs$$

Example 2

$$s(xs, ys) = \prod(xs, ys) = \begin{cases} 1 & -0.5<xs< \ .5 \ \& -0.5<ys<0.5 \\ 0 & Others \end{cases}$$



$$h(xh\ yh) = \prod(10xh, 10yh) = \begin{cases} 1 & -0.5<xh-0.5\&<yx<0.05 \\ 0 & Others \end{cases}$$

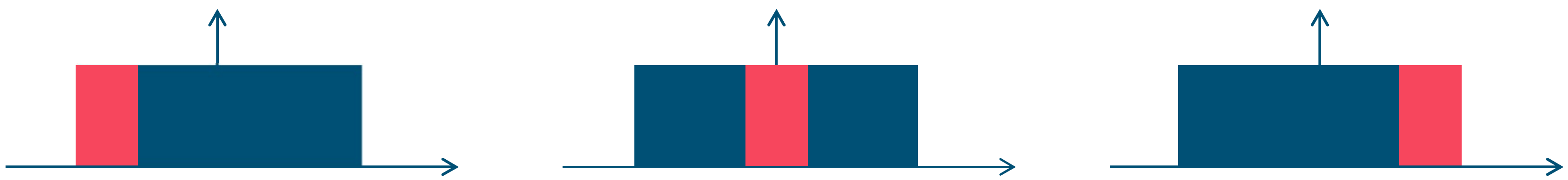
Example 2

$$I(xd,yd) = \frac{D^2}{4\pi(D+d)^2d^2} s\left(-\frac{D}{d}xd, -\frac{D}{d}yd\right) * h\left(\frac{D}{D+d}xd, \frac{D}{D+d}yd\right)$$

$$D = d = 1$$

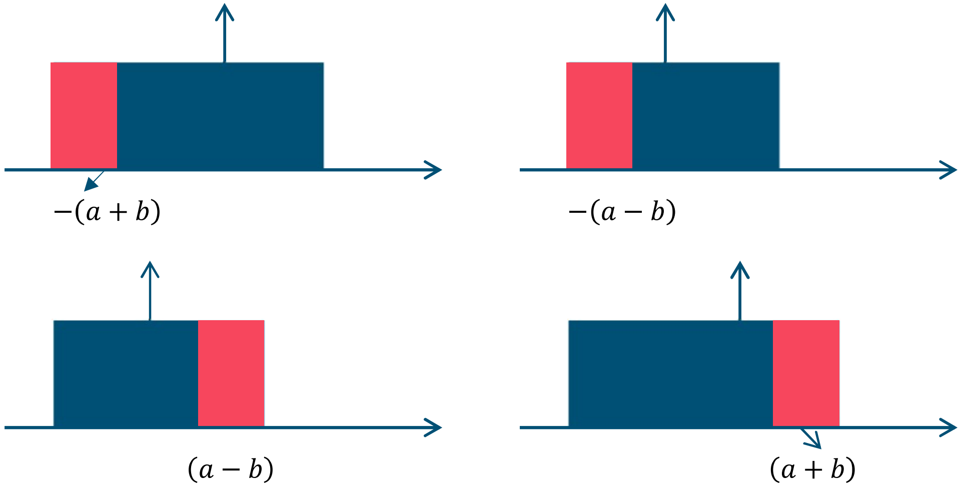
$$I(xd,yd) = \frac{1}{16\pi}\prod(-xd,-yd) * \prod(5xd,5yd)$$

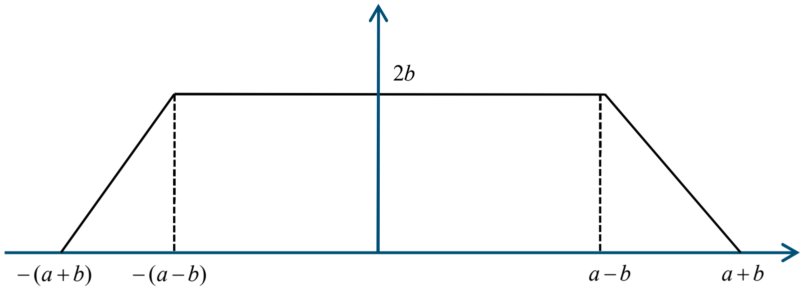
$$I(xd,yd) = \frac{1}{16\pi}\prod(xd)\prod(yd) * \prod(5xd)\prod(5yd)$$



Example 2

$$\prod\frac{x}{2a} * \prod\frac{x}{2b}; a > b > 0$$





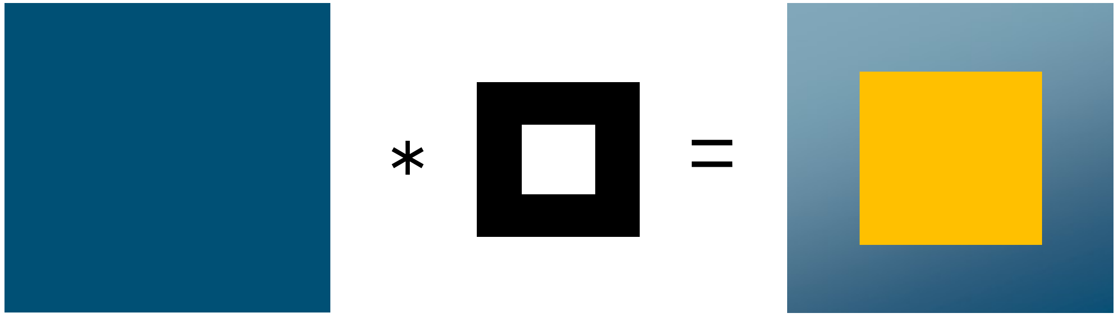
$$(a+b)\Lambda\left(\frac{x}{a+b}\right) - (a-b)\,\Lambda\left(\frac{x}{a-b}\right)$$

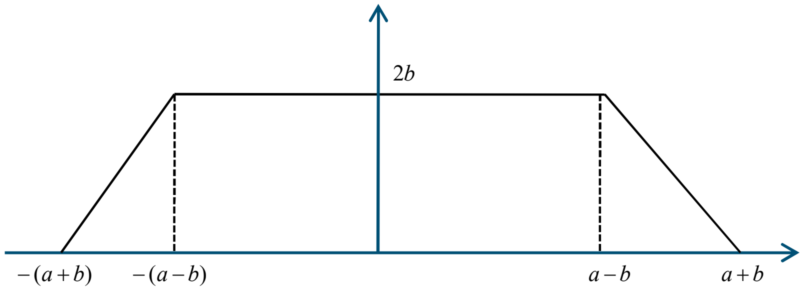
Example 2

$$I(xd, yd) = \frac{1}{16\pi} \prod(xd) \prod(yd) * \prod(5xd) \prod(5yd)$$

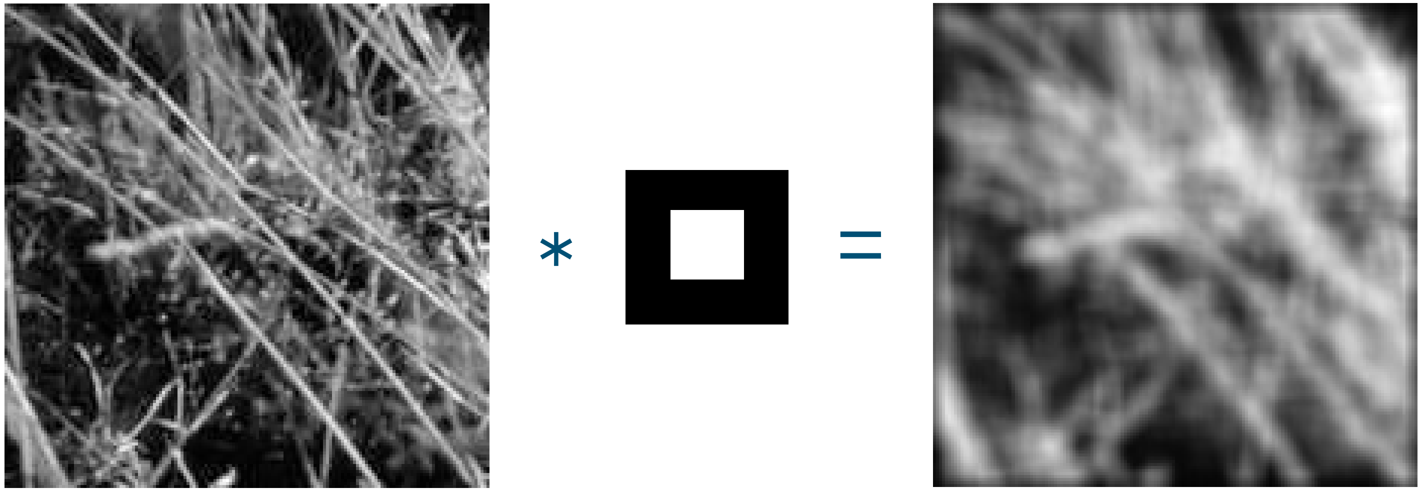
$$a = 0.5; b = 0.1$$

$$I(xd, yd) = \frac{1}{16\pi} \left[0.6\Lambda\left(\frac{xd}{0.6}\right) - 0.4\Lambda\left(\frac{xd}{0.4}\right)\right]\left[0.6\Lambda\left(\frac{yd}{0.6}\right) - 0.4\Lambda\left(\frac{yd}{0.4}\right)\right]$$



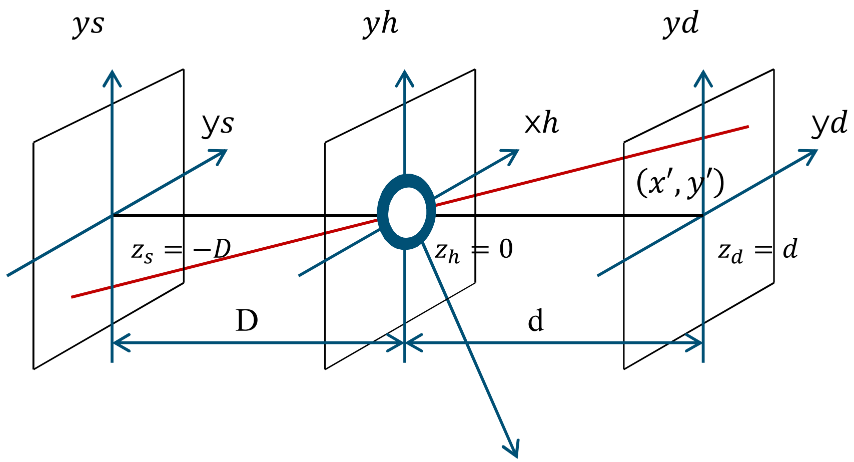


How about real image?



Smoothing with box filter

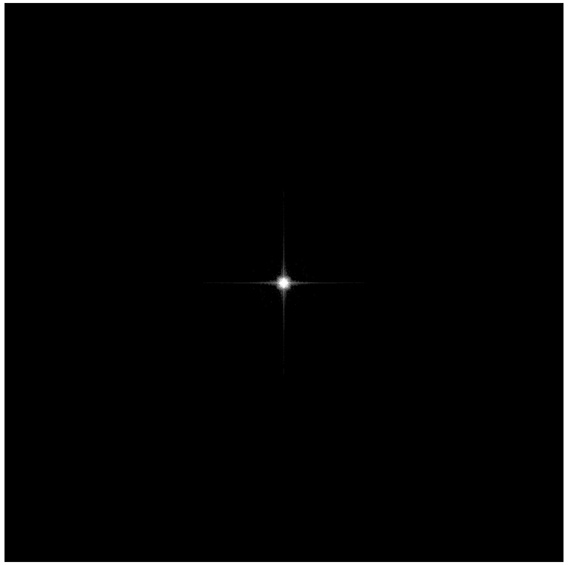
Example 3 (experiment with frequencies)



Iris Aperture

Example 3 (experiment with frequencies)

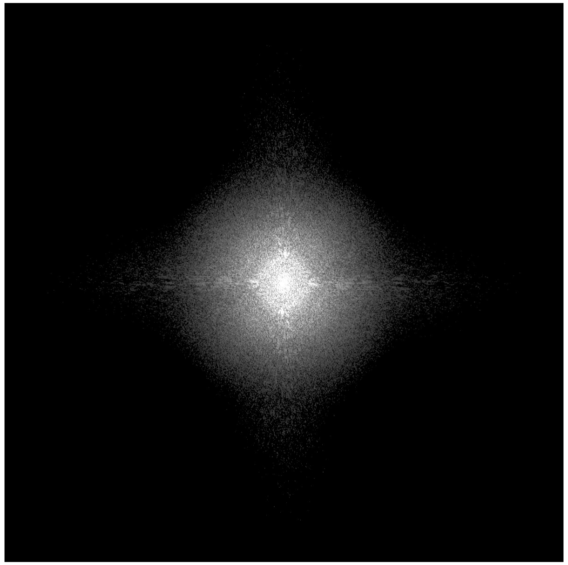
Real Space



Fourier Space

Example 3 (experiment with frequencies)

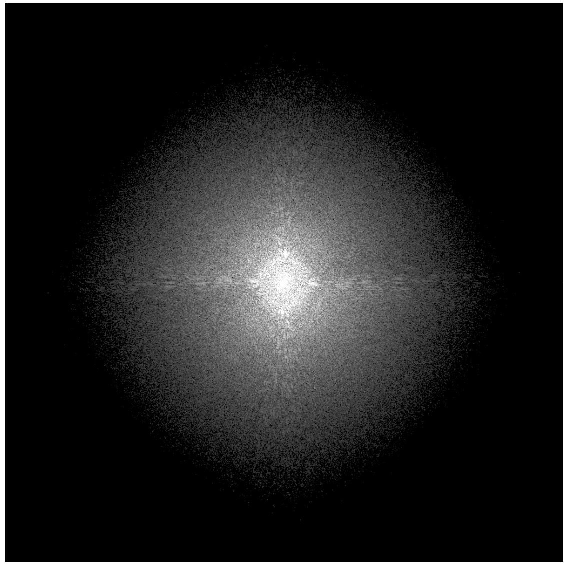
Real Space



Fourier Space

Example 3 (experiment with frequencies)

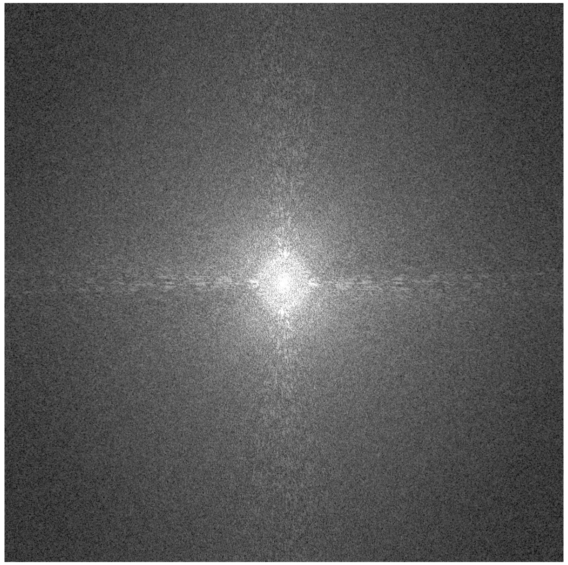
Real Space



Fourier Space

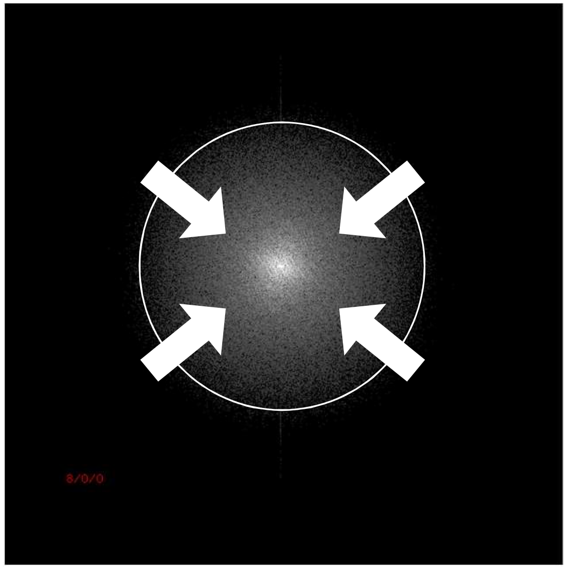
Example 3 (experiment with frequencies)

Real Space



Fourier Space

Game of wide-field Imaging



Want to have as many high frequencies as possible

Summarize again

PSF contains all the info about your camera

$$g(x, y) = h(x, y) * f(x, y)$$

Image is a convolution of object with PSF

→ **De-convolution recovers actual object**

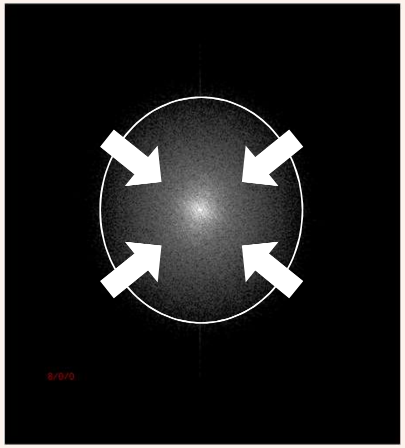
→ **! Beware of Noise @high frequencies**

$$G(u, v) = H(u, v)F(u, v)$$

$$f(x, y) = \mathrm{FT}^{-1}\left[\frac{G(u,v)}{H(u,v)}\right]$$

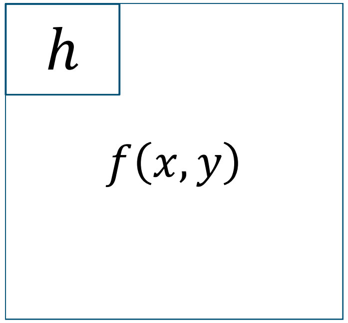
FT(PSF) = "Modulation Transfer Function" shows efficiency of imaging high frequencies

→ **Higher spatial frequencies = better resolution**



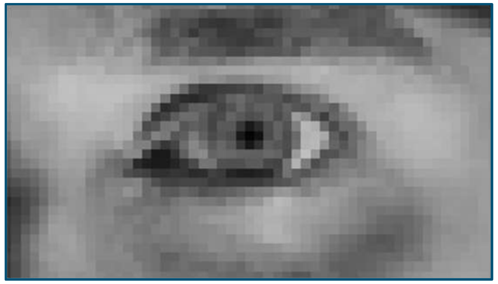
Equipment Demo

Convolution (now, digitally)



$$g(x,y) = h(x,y) * f(x,y)$$

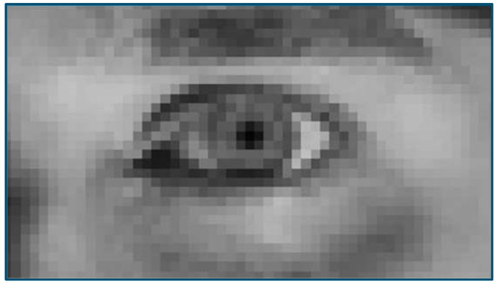
Now that you know what it does, use convolution for various digital processing.



Original

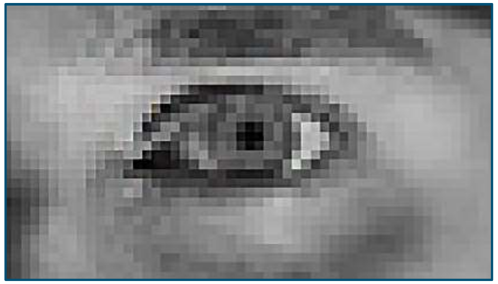
$$* \ \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} =$$

Blur (with a mean filter)



Original

$$* \left(\begin{bmatrix}0&0&0\\0&2&0\\0&0&0\end{bmatrix} - \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\right) =$$



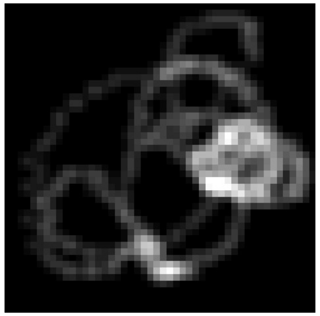
Sharpening filter
(accentuates edges)

Convolution in the real world – Connection to PSF

Camera shake

=

*



Bokeh: Blur in out-of-focus regions of an image.

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- **Slide |3,4,6|** https://en.wikipedia.org/wiki/The_Tower_of_Babel_(Bruegel) Skolkovo Institute of Science and Technology;
- **Slide |9, 10|** Albert Einstein, official 1921 Nobel Prize in Physics photograph
- **Slide |14|** http://www.normankoren.com/Tutorials/MTF.html photo by Andrew Hutchings
- **Slide |20|** https://cheezburger.com/6847687168photo by Denise Johnson
- **Slide |30-34|** https://en.wikipedia.org/wiki/Saint_Basil%27s_Cathedral#/media/File:St._Basil_Cathedral.jpg photo by Anton Zelenov
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- **Slide |66|** Lena Söderberg
- **Slide |67|** source lazebnik
- **Slide |68|** https://en.wikipedia.org/wiki/Pattern#/media/File:Aloe_polyphylla_spiral.jpg
- **Slide |71|** Boston City Hall, designed by Kallmann, McKinnell, & Knowles
- **Slide |77|** Credit W. Freeman et al, “Computer Vision for Interactive Computer Graphics”, IEEE Computer Graphics and Applications, 1998
- **Slide |80|** Fergus, et al. “Removing Camera Shake from a Single Photograph”, SIGGRAPH 2006, MIT Ray and Maria Stata Center

Resources

- Richard Szeliski, “Computer Vision” ISBN-13: 978-1848829343
- Rastislav Lukac, “Computtional Photography” ISBN-13: 978-1439817490
- Rick Nolthenius, Noah Snavely, George Bebis, S. Seitz, L. Zhang, D. Lowe, L. Fei-Fei, Antony Lam
- Pics: Wiki commons & Khan academy
- http://cvcl.mit.edu/hybridimage.htm
- MIT media lab materials, MERL edge detection
- https://vas3k.ru/blog/computational_photography/
- https://www.baslerweb.com/en/vision-campus/vision-systems-and-components/find-the-right-lens/
- http://www.pptsing.com
- https://www.cambridgeincolour.com/tutorials/cameras-vs-human-eye.htm
- https://ai.googleblog.com/2014/10/hdr-low-light-and-high-dynamic-range.html
- http://web.media.mit.edu/~raskar/Talks/ETCVparis08/raskarCompPhotoEpsilonCodedETVC08paper.pdf
- https://graphics.stanford.edu/talks/compphot-publictalk-may08.pdf
- https://www.cambridgeincolour.com/tutorials/cameras-vs-human-eye.htm

Next time: Digital Filtering with Convolution