Algorithms for accelerating vision transformers (ViT) using NLA

Team 7

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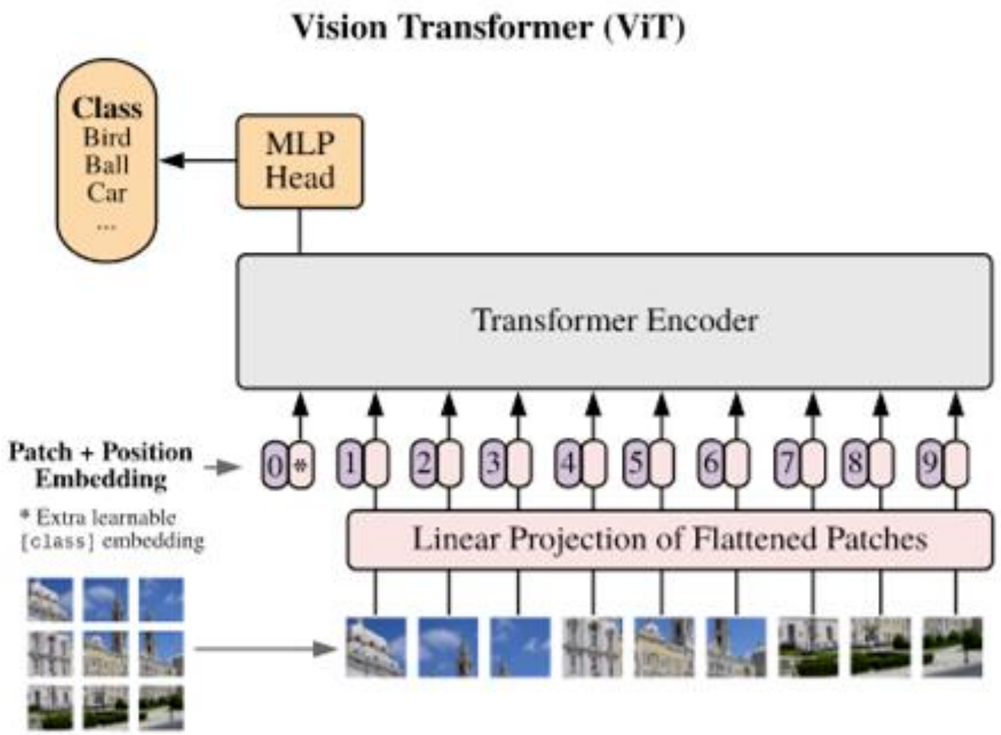
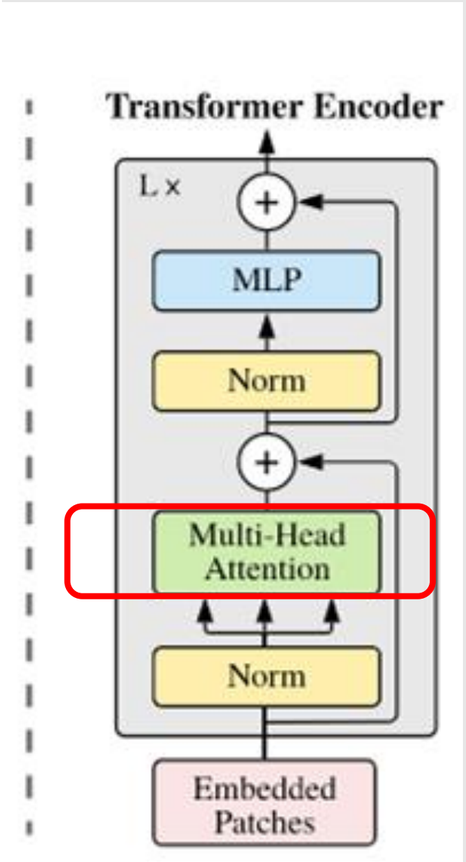
WHY VIT ?!





- **Key Idea — Image Patches Instead of Pixels:**
  By dividing an image into small fixed-size patches (e.g., 8x8 pixels), ViT adapts the transformer model to vision tasks while reducing computational overhead.

Patch Embedding:
The input image is split into fixed-size patches, each of which is flattened and linearly projected into an embedding vector.

Positional Encoding:
Positional embeddings are added to the patch embeddings to retain positional information, as transformers do not inherently account for spatial structure.

Transformer Encoder:
A series of transformer layers (Multi-Head Self-Attention, Feed-Forward Networks, Layer Normalization) processes the patch embeddings, allowing the model to learn global dependencies.

Classification Token:
A special `[CLS]` token is prepended to the sequence of embeddings. The final representation of this token is used for image classification.

MLP Head:
A Multi-Layer Perceptron processes the `[CLS]` token representation to predict the image class.

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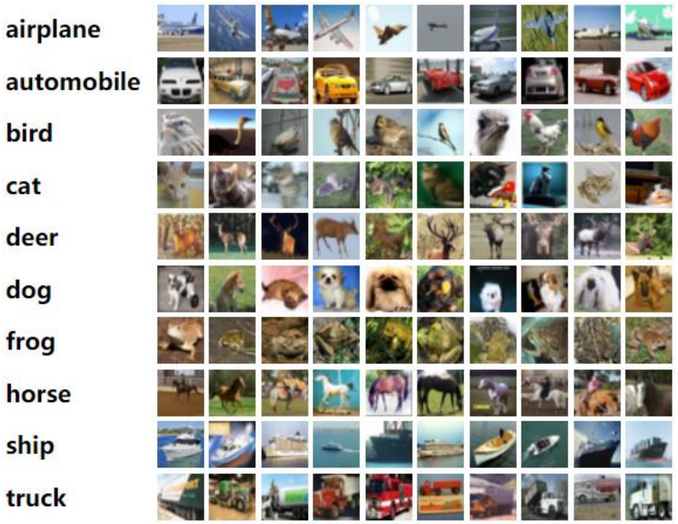
Datasets



Cifar 10

SVHN

FOOD 101

ImageNet

Cifar 100

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WHY MSA ?!

Core Problem: MSA requires computing attention matrices with a complexity of **$O(N^2)$**, where N is the number of patches (i.e., the sequence length).
Solution: The use of decomposition methods, such as **SVD**, sparsity, or random projections, reduces computational complexity and optimizes matrix multiplications.
Application in our case:

Cholesky Decomposition

- **Why Cholesky?**
  Cholesky decomposition is efficient for positive-definite matrices, which often arise in attention computations. It enables faster and more stable matrix factorization compared to general decompositions.
- **Application:** It reduces the cost of matrix operations by decomposing the attention matrix into a lower-triangular form, simplifying further computations.

Singular Value Decomposition (SVD)

- **Why SVD?**
  SVD allows for low-rank approximations of the attention matrix. By retaining only the most significant singular values, we can significantly reduce the matrix rank while preserving essential information.
- **Application:**
  - Compresses attention matrices by removing redundant information.
  - Reduces computational complexity.
  - Improves generalization and reduces overfitting through rank regularization.

Principal Component Analysis (PCA)

- **Why PCA?**
  PCA is a dimensionality reduction technique that identifies principal components capturing the most variance in the data. It simplifies the representation of the attention mechanism.
- **Application:**
  - Reduces dimensionality of key, query, and value matrices.
  - Lowers the cost of attention calculations without significant information loss.

Used methods. Cholesky

We use Cholesky decomposition to the **attention score matrix**. Here's how it's implemented:

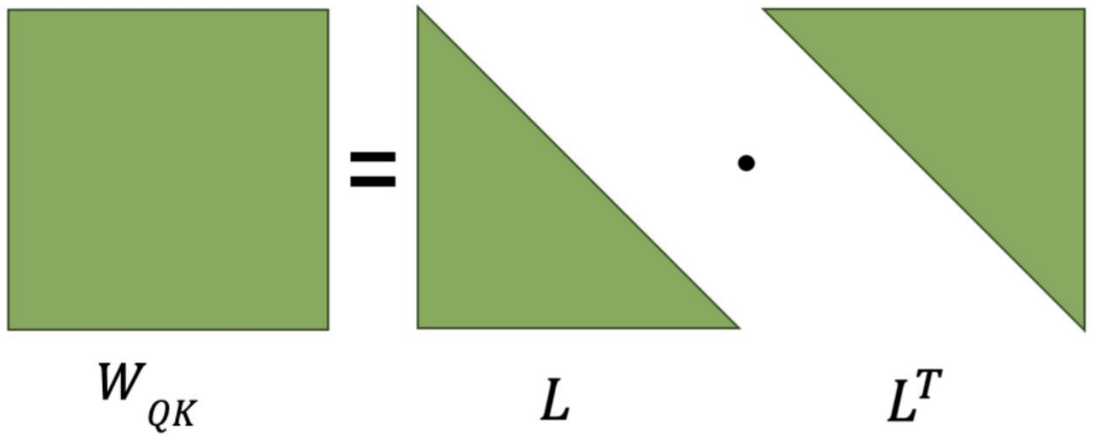
1. **Compute the attention scores matrix:** $S = XW_{QK}X^T$ i.e.query-key tokens similarity**,**

   where W_{QK} — symmetric weight matrix for queries and keys.

1. **Apply Cholesky Decomposition:** $XW_{QK}X^T = LL^T$

2. **Storage:**

Cholesky-Based Symmetric Attention: $\frac{d_k(d_k+1)}{2}$ parameters instead of d_k^2, where d_k is a dimension of a key matrix



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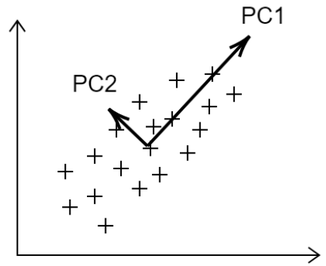
Used methods. PCA

In our code, PCA is used to perform a **low-rank approximation** of the attention scores matrix.

1. **Compute the Attention Matrix:** $S = XW_{QK}X^T$

1. **Apply PCA for Low-Rank Approximation:** $A_{PCA} = XW$

By applying PCA to the attention matrix AA, the model achieves dimensionality and noise reduction:



PC1 = 1st principal component
PC2 = 2nd principal component

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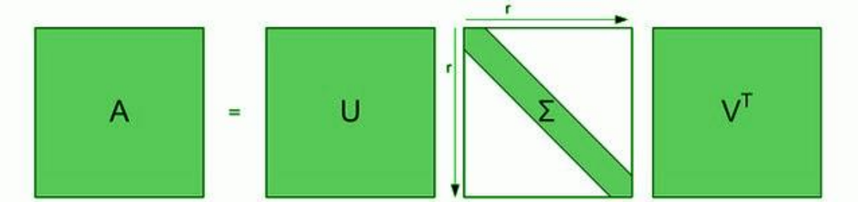
Used methods. SVD

Let consider the matrix multiplication $Y = XW^T + b$ where: $X \in R^{m \times p}$ is the input matrix; $W \in R^{n \times p}$is the weight matrix; $b \in R^n$is the bias vector
Resulting in $Y \in R^{m \times n}$

Full-Rank SVD $A = U\Sigma V^T$

where: $U \in R^{n \times n}$ and $V \in R^{p \times p}$ are orthogonal matrices and Σ is a diagonal matrix of singular values.



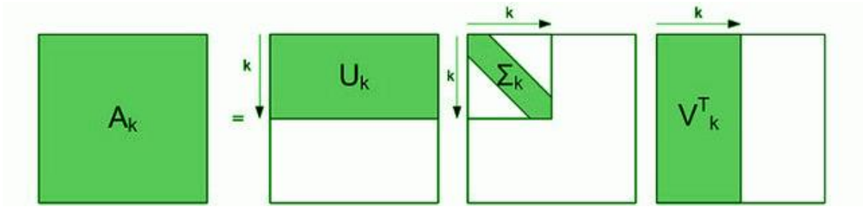
Low-Rank SVD Approximation

To enhance efficiency, retain only the top $r < \min(m, n)$ singular values and their corresponding vectors:

$W \approx U_r \Sigma_r V_r^T$, where $U_r \in R^{n \times r}$, $\Sigma_r \in R^{r \times r}$, $V_r \in R^{p \times r}$

This **low-rank approximation** reduces computational and memory costs while preserving the most significant features

$Y = XV_r \Sigma_r V_r^T + b$



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Goyal, N., & Kumar, S. Lossy color image compression based on singular value decomposition and GNU GZIP // *International Journal of Computer Applications.* 2011. № 12(9). С. 1-5.

Used methods. SVD

Implementation: we enhance Transformer blocks with SVD-based low-rank approximations to improve efficiency and performance

- **TransformerBlockSVD**:
  - Multi-Head Self-Attention: Captures dependencies in the data.
  - **SVD Decomposition**: Applies SVD to the attention output, retaining only the top singular values and vectors for a low-rank approximation
    - Operation: Applies Singular Value Decomposition to the attention output to obtain a low-rank approximation.
    - Benefits**:** we reduces the number of parameters and operations from O(m^2) to O(m*r)
  - Feed-Forward Network: Includes a two-layer network with ReLU activation.
  - Layer Normalization: Stabilizes training by normalizing inputs before attention and feed-forward layers.

- **CustomVisionTransformerSVD**:
  - Embedding Layer: Transforms image patches into embedding vectors.
  - **Stack of TransformerBlockSVD**: Incorporates multiple SVD-enhanced Transformer blocks.
    - Operation: Processes embeddings through multiple Transformer blocks enhanced with SVD-based low-rank approximations.
    - Benefits: When calculate $X' = \text{TransformerBlockSVD}(X)$ L times for L layers with SVD we can maintain efficiency even as the number of layers increases.
  - Normalization & Classification: Applies final layer normalization and a fully connected layer to classify images into categories.

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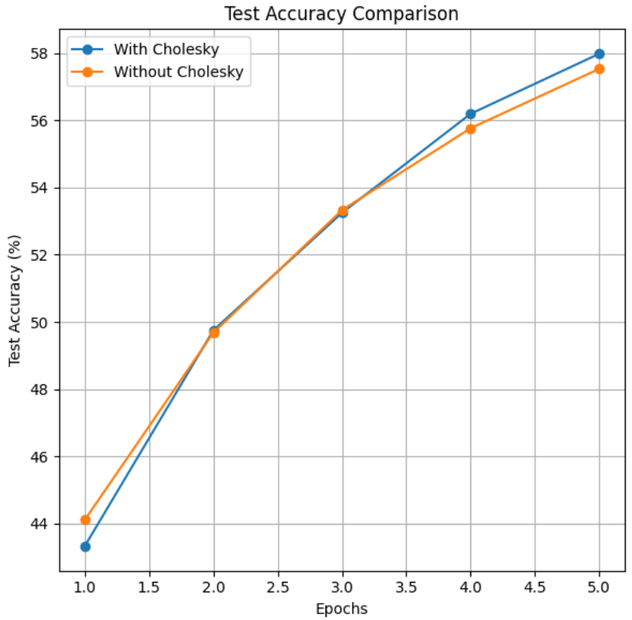
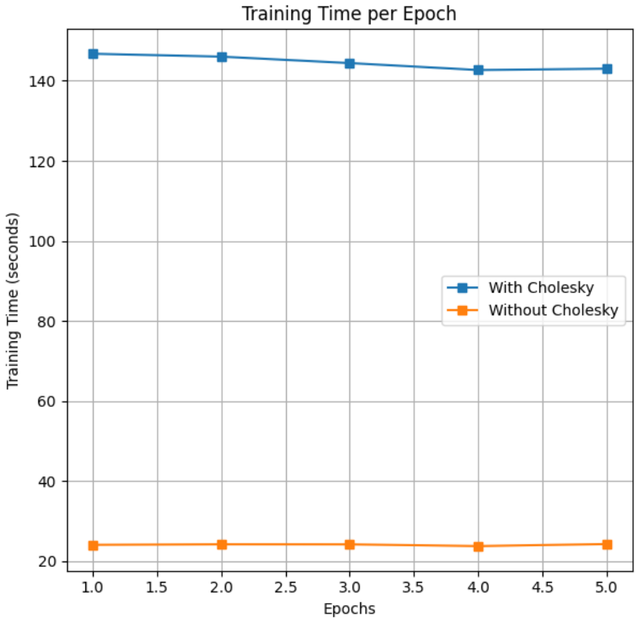
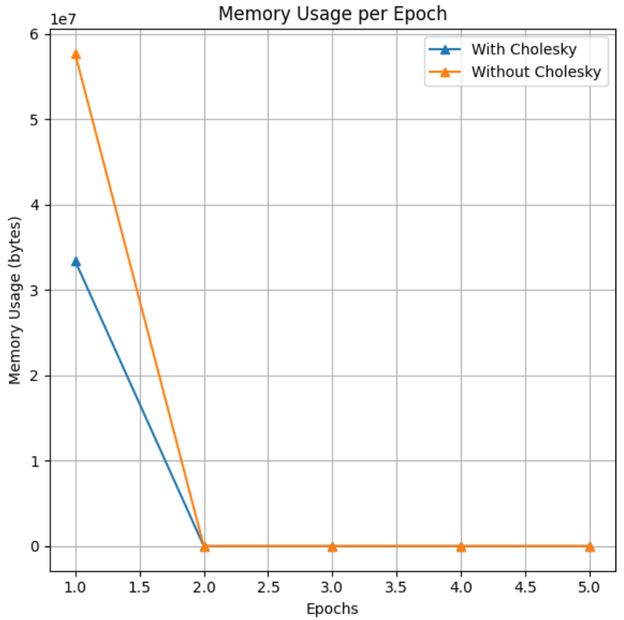
Goyal, N., & Kumar, S. Lossy color image compression based on singular value decomposition and GNU GZIP // *International Journal of Computer Applications.* 2011. № 12(9). С. 1-5.

Methods validation. CIFIR10 - Cholesky







Total Time (without Cholesky): 130.72 seconds
Total Memory (without Cholesky): -12548096 bytes
Average Accuracy (without Cholesky): 21.09%
Total Time (with Cholesky): 132.30 seconds
Total Memory (with Cholesky): 64459776 bytes
Average Accuracy (with Cholesky): 20.42%

Implementing Cholesky decomposition in the transformer model resulted in marginally **increased** training time and significantly **higher** memory usage. Additionally, there was a slight **decrease** in average test accuracy. These outcomes indicate that the Cholesky-based approach **did not enhance** the model's performance and instead introduced additional computational and memory overhead.

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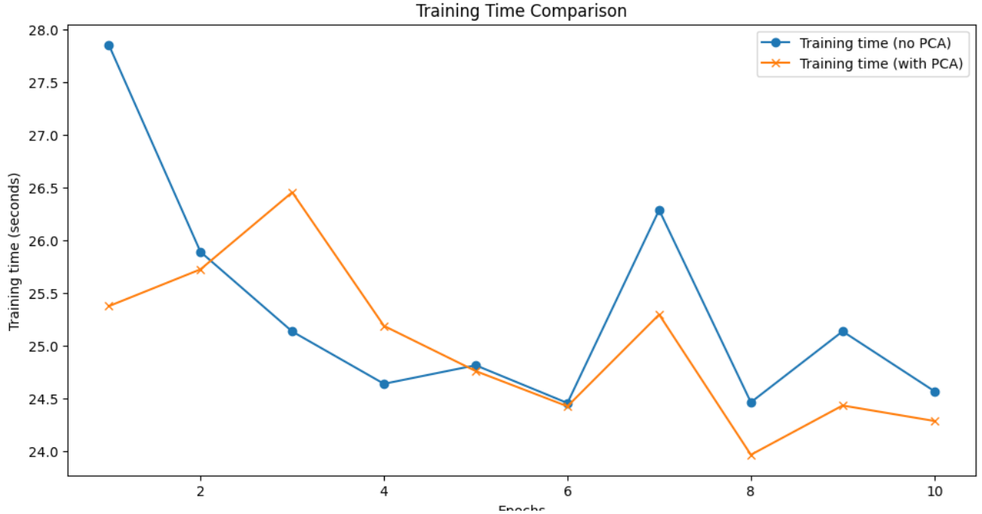
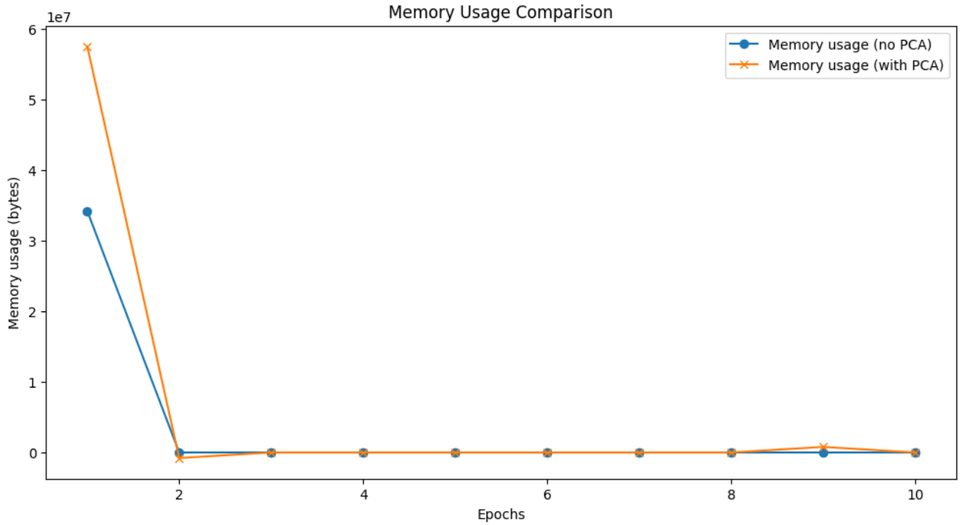
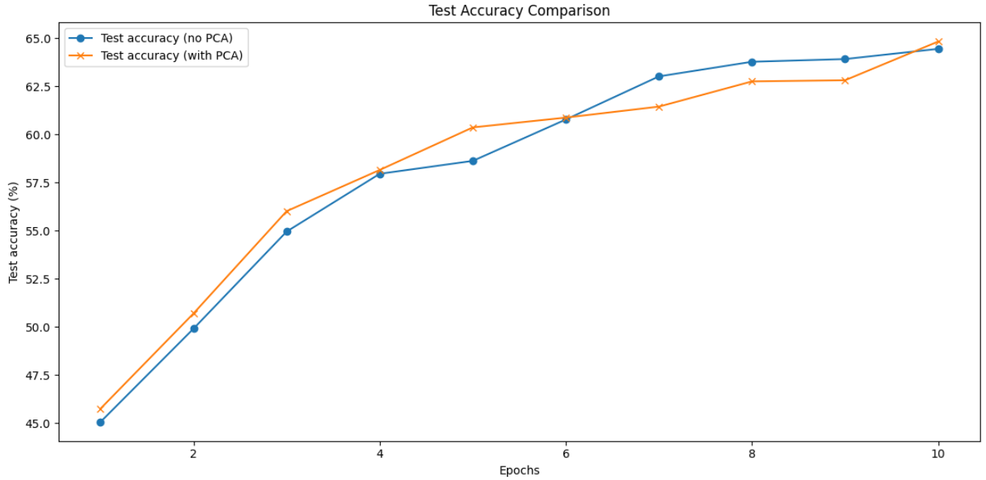
Methods validation. CIFIR10 - PCA







Average Training Time (without PCA): 123.36 seconds/epoch
Average Memory Usage (without PCA): 5682892.80 MB/epoch
Average Test Accuracy (without PCA): 14.81%
Average Training Time (with PCA): 123.36 seconds/epoch
Average Memory Usage (with PCA): 11909017.60 MB/epoch
Average Test Accuracy (with PCA): 14.63%

Using PCA in the transformer model did not yield the desired benefits. The training time remained **unchanged**, indicating that PCA did not enhance computational efficiency. Additionally, memory usage increased.

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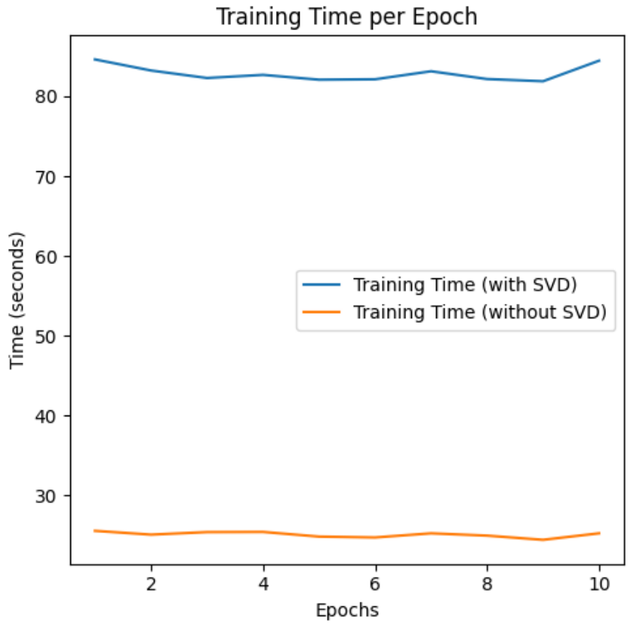
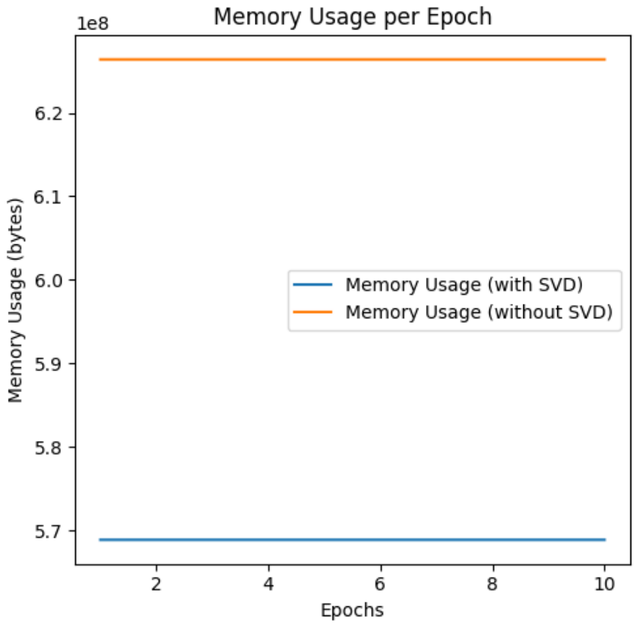
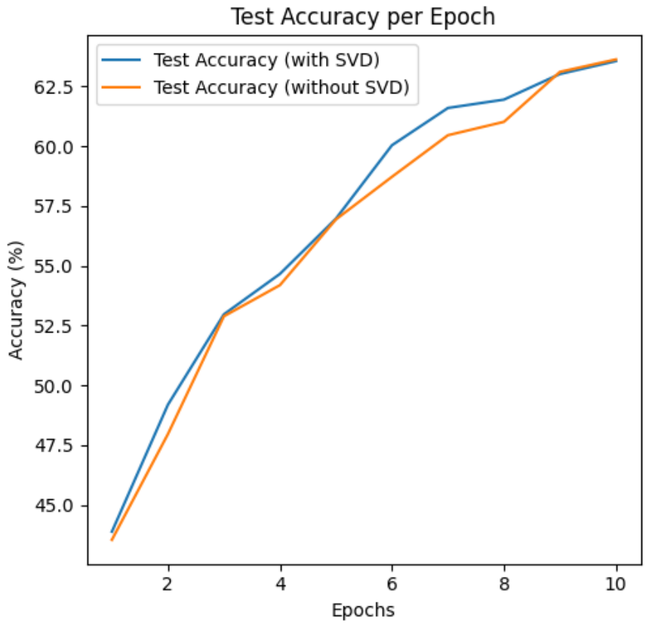
Methods validation. CIFIR10 - SVD







Total Time (without SVD): 192.95 seconds
Total Memory (without SVD): 40953856 bytes
Average Accuracy (without SVD): 20.95%
Total Time (with SVD): 192.95 seconds
Total Memory (with SVD): 40953856 bytes
Average Accuracy (with SVD): 21.21%

The implementation of SVD in the transformer model did **NOT** reduce the learning **time** and the amount of **memory used**, which indicates that this led to additional computing or memory costs.

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RESULTS

1. On the CIFAR-10 and CIFAR-100, the use of SVD shows a gradual improvement in accuracy compared to without SVD, however, the improvement is quite limited, especially on the CIFAR-100, where the difference between models with and without SVD is small. On SVD, SVD significantly improves accuracy throughout all epochs, especially in the early stages of learning. While on FOOD 101 and Imagenet, the improvement using SVD turned out to be less noticeable, perhaps due to the complexity of the data and the scale of the models.

1. Based on the training data across various datasets (FOOD 101, CIFAR 10, SVHN, CIFAR 100, and ImageNet), we observe that the training times for both "with PCA" and "no PCA" approaches are quite comparable, with slight variations depending on the dataset. PCA (Principal Component Analysis) seems to have a marginal impact on the training speed, generally leading to slightly faster epochs in most cases. However, the training speed differences are not drastic, suggesting that PCA does not offer a significant boost in processing speed for these particular datasets, at least in terms of the raw training time for each epoch.

1. For the CIFAR-100 set, the result of testing models without and using Cholesky showed only minor improvements. For example, in the first epoch, the accuracy of testing without Cholesky was 12.04%, and with it — 11.29%, with a gradual increase in the following epochs, where in the fifth epoch the accuracy without Cholesky reached 28.42%, and with Cholesky — 28.05%. The results were better on CIFAR-10, with test accuracy reaching 65.43% without Cholesky and 64.84% with Khaletsky on the tenth epoch. More obvious improvements could be seen on the SVHN set: accuracy without Cholesky in the first epoch was 44.74%, and with Cholesky — 49.42%, with subsequent improvements throughout the training. On ImageNet, the accuracy was significantly lower, with improvements, but still remained at 11%.

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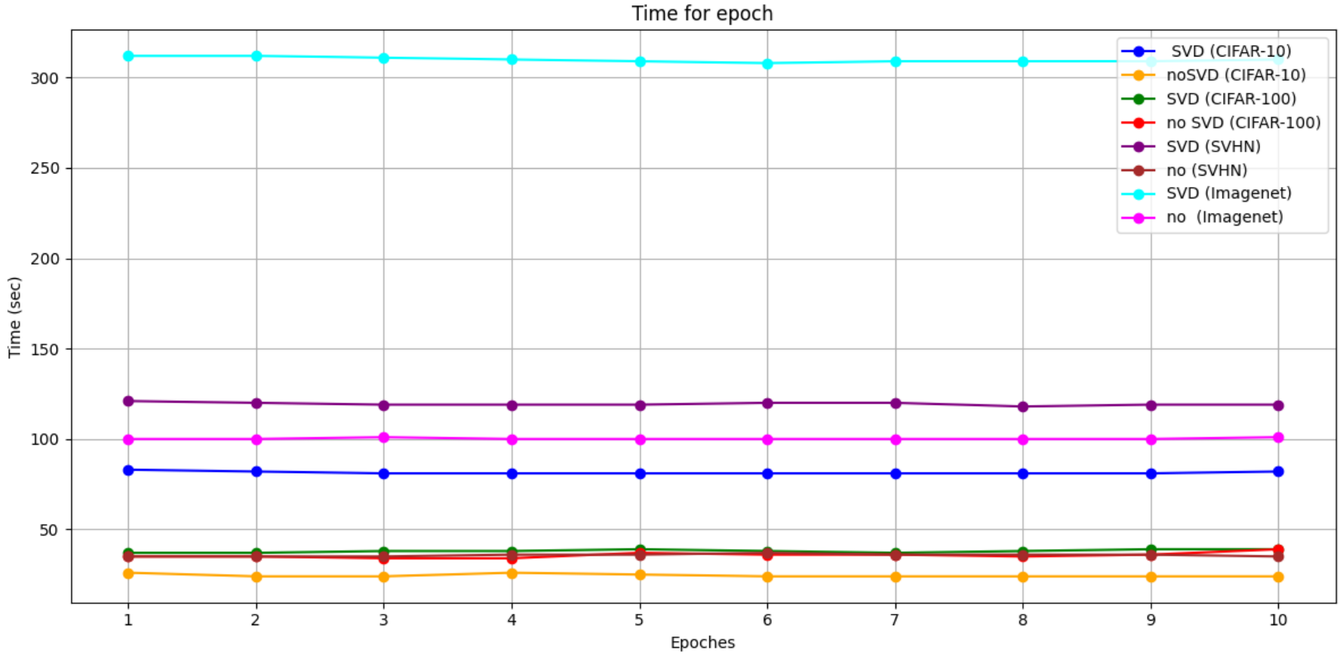
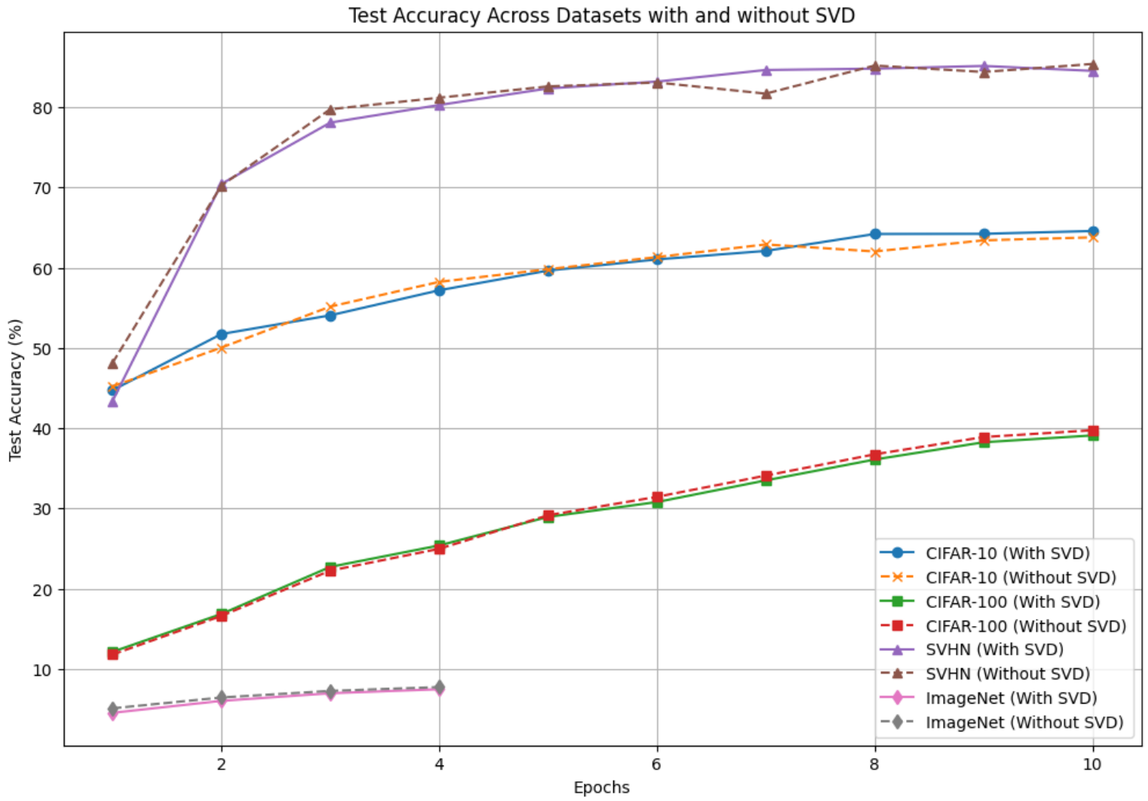

Time for epoch
SVD (CIFAR-10)
noSVD (CIFAR-10)
SVD (CIFAR-100)
no SVD (CIFAR-100)
SVD (SVHN)
no (SVHN)
SVD (Imagenet)
no (Imagenet)
Time (sec)
Epoches


Test Accuracy Across Datasets with and without SVD
CIFAR-10 (With SVD)
CIFAR-10 (Without SVD)
CIFAR-100 (With SVD)
CIFAR-100 (Without SVD)
SVHN (With SVD)
SVHN (Without SVD)
ImageNet (With SVD)
ImageNet (Without SVD)
Test Accuracy (%)
Epochs